# A Dilemma for Lexical and Archimedean Views in Population Axiology

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**Abstract:** According to lexical views in population axiology, there are good lives $x$ and $y$ such that some number of lives equally good as $x$ is not worse than any number of lives equally good as $y$. Such views can avoid the Repugnant Conclusion without violating Transitivity or Separability, but they imply a dilemma: either some good life is better than any number of slightly worse lives, or else the 'at least as good as' relation on populations is *radically* incomplete, in a sense to be explained. One might judge that the Repugnant Conclusion is preferable to each of these horns and hence embrace an Archimedean view. This is, roughly, the claim that quantity can always substitute for quality: each population is worse than a population of enough good lives. However, Archimedean views face an analogous dilemma: either some good life is better than any number of slightly worse lives, or else the 'at least as good as' relation on populations is radically and *symmetrically* incomplete, in a sense to be explained. Therefore, the lexical dilemma gives us little reason to prefer Archimedean views. Even if we give up on lexicality, problems of the same kind remain.

## 1. Introduction

Some populations are better than others. For example, a population in which every person lives a wonderful life is better than a population in which those same people live awful lives. And this betterness relation holds (at least sometimes) between populations that differ in size. A population in which every person lives a wonderful life is better than a slightly bigger population in which every person lives an awful life.

These cases are clear-cut, but others are less certain. Is a population in which one million people live a wonderful life better than a population in which one billion people live a good life? Is a population in which two million people live wonderful lives and one million people live awful lives better than a population

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in which no one lives at all? It would be useful to have a *population axiology* – an 'at least as good as' relation over populations – to adjudicate in cases like these.

Unfortunately, a satisfactory population axiology has proved difficult to find. Many otherwise plausible theories imply what Derek Parfit called the *Repugnant Conclusion*: each population of wonderful lives is worse than some much larger population of lives barely worth living (1984: 388). And many of the remaining theories imply its negative analogue: each population of awful lives is better than some much larger population of lives barely worth *not* living.

The source of the trouble might seem to be *Archimedeanism about Populations*. The positive half of this claim is, roughly, that if adding a life to a population makes that population better, adding enough such lives can make that population better than any other. The negative half is, again roughly, that if adding a life to a population makes that population worse, adding enough such lives can make that population worse than any other. The lesson of the Repugnant Conclusion and its negative analogue seems to be that this kind of outweighing does not always occur. Although each additional life barely worth living might make a population better, no number of lives barely worth living is better than a large number of wonderful lives. And although each additional life barely worth *not* living might make a population worse, no number of lives barely worth not living is worse than a large number of awful lives.

So, many have claimed, we should be non-Archimedean about populations (Parfit 1986; 2016; Griffin 1988: 340, fn.27; Lemos 1993; Rachels 2004; Temkin 2012; Chang 2016; Nebel 2021). Non-Archimedeans claim that some good lives are *weakly noninferior* to other good lives: there is some good life $x$ and some good life $y$ such that a large enough number of lives equally good as $x$ is *not worse* than any number of lives equally good as $y$.[1] We can then avoid the Repugnant Conclusion by claiming that wonderful lives are weakly noninferior to lives barely worth living. A large enough number of wonderful lives is not worse than any number of lives barely worth living. We can avoid the Negative Repugnant Conclusion with a parallel manoeuvre: awful lives are *weakly nonsuperior* to lives barely worth not living. A large enough number of awful lives is *not better* than any number of lives barely worth not living.

However, previous iterations of non-Archimedean views have failed to gain much support, due in large part to their violation of either *Transitivity* or *Separability over Lives*: they imply either that some population $X$ is not at least as good as some population $Z$, even though $X$ is at least as good as some

[1] In my terminology, making this claim is sufficient for qualifying as non-Archimedean. I should note that many of the non-Archimedeans cited above make the stronger claim that some good lives are weakly *superior* to other good lives: a large enough number of lives equally good as $x$ is *better* than any number of lives equally good as $y$. I thank an anonymous reviewer for pressing me to clarify this point.

population $Y$ and $Y$ is at least as good as $Z$, or else they imply that whether some population $X$ is at least as good as some population $Y$ can depend on the existence or welfare of people who are unaffected by the choice of $X$ or $Y$. The latest kind of non-Archimedean view promises to have wider appeal. By representing the value of a life with a vector, these *lexical views* can avoid the Repugnant Conclusion while preserving both Transitivity and Separability (Kitcher 2000; Thomas 2018; Nebel 2021; Carlson forthcoming).

Unfortunately, there's a catch. As we will see, these lexical views, in conjunction with an assumption about the size of the differences between possible lives, imply that some good life is *strongly* noninferior to a life only slightly worse: there is some good life $x$ such that *any* number of lives equally good as $x$ is not worse than any number of lives slightly worse than $x$ (Arrhenius and Rabinowicz 2005; 2015b; Jensen 2008; Nebel 2021). If, in addition, lexicalists claim that the 'at least as good as' ordering on populations is complete – so that for all populations $X$ and $Y$, either $X$ is better than $Y$, $Y$ is better than $X$, or $X$ and $Y$ are equally good – then their view implies that some good life is strongly *superior* to a life only slightly worse: there is some good life $x$ such that any number of lives equally good as $x$ is *better* than any number of lives slightly worse than $x$. If, on the other hand, lexicalists deny that the 'at least as good as' ordering on populations is complete, then it must be incomplete in a worryingly *radical* way (Handfield and Rabinowicz 2018), of which more later.

We might judge that accepting the Repugnant Conclusion is preferable to each horn of this *lexical dilemma*, and so embrace an Archimedean view. However, in this paper I show that Archimedean views face an analogous dilemma. This dilemma arises because Archimedean views also endorse a kind of strong noninferiority: they claim that any number of good lives is not worse than any number of bad lives. This claim, in conjunction with the same assumption about the size of the differences between possible lives, implies that some good life is strongly noninferior to a life only slightly worse: there is some good life $x$ such that any number of lives equally good as $x$ is not worse than any number of lives slightly worse than $x$. If, in addition, Archimedeans claim that the 'at least as good as' ordering on populations is complete, then their view implies that some good life is strongly *superior* to a life only slightly worse: there is some good life $x$ such that any number of lives equally good as $x$ is *better* than any number of lives slightly worse than $x$. If, on the other hand, Archimedeans deny that the 'at least as good as' ordering on populations is complete, then it must be incomplete in a way both radical and *symmetric*. They must claim that, for any arbitrarily good population and any arbitrarily bad population, there is some population that is both not worse than the former and not better than the latter.

The conclusion is that the lexical dilemma gives us little reason to prefer an Archimedean view. Even if we give up on lexicality, problems of the same kind remain.

# 2. The Framework

In this section, I offer definitions and assumptions intended to be uncontroversial in the dispute between Archimedeans and lexicalists. Foundational to this paper is the notion of a life. These lives are individuated, first, by the person whose life it is and, second, by the welfare of that person. Welfare is a measure of how good a person's life is for them. I assume that the 'has at least as high welfare as' relation applied to the set of possible lives is reflexive and transitive. Life $x$ has higher welfare than life $y$ iff $x$ has at least as high welfare as $y$ and $y$ does not have at least as high welfare as $x$. Life $x$ is at the same welfare level as life $y$ iff $x$ has at least as high welfare as $y$ and $y$ has at least as high welfare as $x$.

Note, however, that the 'has at least as high welfare as' relation need not be complete over the set of possible lives. There may be lives $x$ and $y$ such that $x$ does not have at least as high welfare as $y$ and $y$ does not have at least as high welfare as $x$. In that case, we may say that $x$ and $y$ are incommensurable, on a par, or imprecisely equally good. Although these relations are distinct, their differences are unimportant in this paper.[2] I often let incommensurability stand for all three.[3]

Lives are either personally good, bad, strictly neutral, or weakly neutral. Which category a life falls in depends on how it compares to some standard. Life $x$ is personally good (bad) iff $x$ has higher (lower) welfare than the standard. Life $x$ is personally strictly neutral iff $x$ is at the same welfare level as the standard, and personally weakly neutral iff $x$ is incommensurable with the standard.[4] The standard in question is defined differently by different authors. Some define it as nonexistence (Arrhenius and Rabinowicz 2015a). Others define it as a life

[2] See Chang (2016) for a discussion of the differences, though note that Chang uses 'incomparability' to name the relation I call 'incommensurability.'

[3] There may also be lives $x$ and $y$ such that it is indeterminate whether $x$ has at least as high welfare as $y$ and indeterminate whether $y$ has at least as high welfare as $x$. On some theories of vagueness (like epistemicism and supervaluationism), such instances of indeterminacy do not preclude completeness. On other theories (like many-valued logics), the issue is complex. As Knutsson (2021) notes, departing from classical logic allows for many different versions of completeness and transitivity. Considering all of these versions would take me too far afield, so I assume classical logic in what follows. For more on theories of vagueness, including criticism of non-classical approaches, see Bacon (2018: ch. 1–2). I thank an anonymous reviewer for suggesting that I treat vagueness in this kind of way.

[4] This is Rabinowicz's (2020) terminology. Gustafsson (2020) calls these lives 'neutral' and 'undistinguished' respectively.

constantly at a strictly neutral level of temporal welfare (Broome 2004: 68; Bykvist 2007: 101). Still others define it as a life without any good or bad components: features of a life that are good or bad for the person living it (Arrhenius 2000: 26). My discussion is compatible with all such definitions. Wonderful lives and lives barely worth living are personally good. Awful lives and lives barely worth not living are personally bad.

A population is a set of lives.[5] A population axiology is an 'at least as good as' relation on the set of all possible populations. Population $X$ is better than population $Y$ iff $X$ is at least as good as $Y$ and $Y$ is not at least as good as $X$. Population $X$ is equally good as population $Y$ iff $X$ is at least as good as $Y$ and $Y$ is at least as good as $X$.

The 'at least as good as' relation is reflexive over the set of possible populations, but it need not be complete. Populations $X$ and $Y$ are incommensurable iff $X$ is not at least as good as $Y$ and $Y$ is not at least as good as $X$.[6] For my purposes below, the key feature of incommensurability is its *insensitivity to slight changes*. If $X$ is incommensurable with $Y$, then there is typically some slightly improved version of $X$ – call it $X^+$ – and some slightly worsened version of $X$ – call it $X^-$ – such that $X^+$ and $X^-$ are also incommensurable with $Y$.[7]

I assume *welfarist anonymity*: if two populations feature the same number of lives at each welfare level, then they are equally good. This assumption allows us to represent each population with a distribution – a finite, unordered list of welfare levels, allowing repetitions – so that one population is at least as good as another iff its distribution is at least as good. I denote these distributions with uppercase letters in double-struck square brackets: $[\![X]\!]$ denotes the distribution corresponding to population $X$. I denote welfare levels with lowercase letters in double-struck square brackets: $[\![x]\!]$ denotes the welfare level of life $x$. Distributions and welfare levels can be concatenated, so that $[\![X]\!] \cup [\![Y]\!]$ denotes the distribution comprised of all the welfare levels in $[\![X]\!]$ and $[\![Y]\!]$, $[\![X]\!] \cup [\![x]\!]$ denotes the distribution comprised of all the welfare levels in $[\![X]\!]$ plus the welfare level $[\![x]\!]$, and $m[\![x]\!]$ denotes the distribution comprised of $m$ welfare levels $[\![x]\!]$, where $m$ is some natural number.

This notation is useful in clarifying the notion of a life's *contributive value* relative to a population. Life $x$ is contributively good (bad/strictly neutral/weakly neutral) relative to population $X$ iff $[\![X]\!] \cup [\![x]\!]$ is better than

[5] In this paper, I restrict my attention to finite populations. For discussion of infinite populations, see Bostrom (2011).

[6] There may also be populations $X$ and $Y$ such that it is indeterminate whether $X$ is at least as good as $Y$ and indeterminate whether $Y$ is at least as good as $X$. I assume that this kind of indeterminacy does not preclude completeness. See footnote 3.

[7] Raz (1986: 121) calls this 'the mark of incommensurability.'

(worse than/equally good as/incommensurable with) $[\![X]\!]$. To these absolute classifications of contributive value, we can add comparative ones. Life $x$ is contributively better than (worse than/equally good as/incommensurable with) life $y$ relative to population $X$ iff $[\![X]\!] \cup [\![x]\!]$ is better than (worse than/equally good as/incommensurable with) $[\![X]\!] \cup [\![y]\!]$. The contributive value of lives is my primary concern in this paper, so terms like 'good' and 'weakly neutral' stand for 'contributively good' and 'contributively weakly neutral' unless otherwise stated.

I assume *Separability over Lives.*[8] Roughly, this is the claim that the existence and welfare of unaffected people cannot make a difference to how populations compare. More precisely:

> **Separability over Lives.**
>
> For all populations $X$, $Y$, and $Z$, $X$ is at least as good as $Y$ iff $[\![X]\!] \cup [\![Z]\!]$ is at least as good as $[\![Y]\!] \cup [\![Z]\!]$.

This assumption is contested by some (Carlson 1998: 290–91) and denied by egalitarian, variable value, and average views. But it is *prima facie* plausible and there are strong arguments in its favour (Blackorby, Bossert, and Donaldson 2005: 133; Thomas forthcoming-a). In any case, Separability is agreed upon by many Archimedeans and all lexicalists. Many lexicalists take the satisfaction of Separability to be a major advantage of their view over previous non-Archimedean views (Parfit 2016: 112; Nebel 2021: 16).

Separability entails that each life has the same contributive value relative to all populations. If life $x$ is good (bad/strictly neutral/weakly neutral) relative to some population $X$, it is good (bad/strictly neutral/weakly neutral) relative to all populations. If life $x$ is better than (worse than/equally good as/incommensurable with) life $y$ relative to some population $X$, it is better than (worse than/equally good as/incommensurable with) $y$ relative to all populations. Therefore, I drop the relativisation to particular populations in what follows.

Finally, I assume that the 'at least as good as' relation over populations is *transitive*:

> **Transitivity.**
>
> For all populations $X$, $Y$, and $Z$, if $X$ is at least as good as $Y$ and $Y$ is at least as good as $Z$, then $X$ is at least as good as $Z$.

Although some non-Archimedeans avoid the Repugnant Conclusion by denying Transitivity (Rachels 2004; Temkin 2012), this move strikes most as unduly drastic. In any case, Transitivity is common ground in the debate between Archimedeans and lexicalists.

[8] Blackorby, Bossert, and Donaldson (2005: 132) call this assumption 'existence independence.'

This paper centres around four relations between lives: superiority, inferiority, nonsuperiority, and noninferiority. Each relation has strong and weak versions. The differences are subtle and the names are unwieldy but, unfortunately, the difficulty is unavoidable. The best course of action is to lay them all out here, for initial acquaintance and later reference.

First, strong and weak *superiority*:

> **Strong Superiority.**
>
> Life $x$ is strongly superior to life $y$ iff *any* number of lives at $[\![x]\!]$ is better than any number of lives at $[\![y]\!]$.
>
> **Weak Superiority.**
>
> Life $x$ is weakly superior to life $y$ iff *some* number of lives at $[\![x]\!]$ is better than any number of lives at $[\![y]\!]$.

Strong and weak *noninferiority* are the same, except with 'not worse' in place of 'better.'

> **Strong Noninferiority.**
>
> Life $x$ is strongly noninferior to life $y$ iff any number of lives at $[\![x]\!]$ is not worse than any number of lives at $[\![y]\!]$.
>
> **Weak Noninferiority.**
>
> Life $x$ is weakly noninferior to life $y$ iff some number of lives at $[\![x]\!]$ is not worse than any number of lives at $[\![y]\!]$.

Noninferiority, as distinct from superiority, is important if the 'at least as good as' relation on the set of populations is incomplete. Life $x$ might then be weakly noninferior to life $y$ without being weakly superior to $y$. In that case, some number of lives at $[\![x]\!]$ is *not worse* than any number of lives at $[\![y]\!]$, but there is no number of lives at $[\![x]\!]$ that is *better* than any number of lives at $[\![y]\!]$. For each number of lives at $[\![x]\!]$, there is some number of lives at $[\![y]\!]$ such that the two populations are incommensurable.

Strong and weak *inferiority* are the negative variants of strong and weak superiority:

> **Strong Inferiority.**
>
> Life $x$ is strongly inferior to life $y$ iff any number of lives at $[\![x]\!]$ is *worse* than any number of lives at $[\![y]\!]$.
>
> **Weak Inferiority.**
>
> Life $x$ is weakly inferior to life $y$ iff some number of lives at $[\![x]\!]$ is worse than any number of lives at $[\![y]\!]$.

Strong and weak *nonsuperiority* are the same, except with 'not better' in place of 'worse.'

**Strong Nonsuperiority.**

Life $x$ is strongly nonsuperior to life $y$ iff any number of lives at $[\![x]\!]$ is not better than any number of lives at $[\![y]\!]$.

**Weak Nonsuperiority.**

Life $x$ is weakly nonsuperior to life $y$ iff some number of lives at $[\![x]\!]$ is not better than any number of lives at $[\![y]\!]$.

If the 'at least as good as' relation on the set of populations is incomplete, life $x$ might be weakly nonsuperior to life $y$ without being weakly inferior to $y$. In that case, some number of lives at $[\![x]\!]$ is *not better* than any number of lives at $[\![y]\!]$, but there is no number of lives at $[\![x]\!]$ that is *worse* than any number of lives at $[\![y]\!]$. For each number of lives at $[\![x]\!]$, there is some number of lives at $[\![y]\!]$ such that the two populations are incommensurable.

# 3. The Lexical Dilemma

With all that in mind, we can formulate the Repugnant Conclusion as follows:

**The Repugnant Conclusion.**

Each population consisting only of wonderful lives is worse than some much larger population consisting only of lives barely worth living. (Parfit 1984: 388)

This conclusion strikes many as obviously false. But we cannot avoid it if we accept the following two claims:

**The Equivalence of Personal and Contributive Value.**

A life is personally good (bad/strictly neutral/weakly neutral) iff it is contributively good (bad/strictly neutral/weakly neutral). (Rabinowicz 2009: 391; Gustafsson 2020: 87)

**Archimedeanism about Populations.**

For any population $X$ and any contributively good life $y$, there is some number $m$ such that $m$ lives at $[\![y]\!]$ is better than $X$.[9]

[9] Strictly, this is the positive half of Archimedeanism about Populations. The negative half is as follows: for any population $X$ and any contributively bad life $y$, there is some number $m$ such that $m$ lives at $[\![y]\!]$ is worse than $X$.

The Equivalence of Personal and Contributive Value implies that lives barely worth living are contributively good.[10] Archimedeanism about Populations then implies that enough lives barely worth living can be better than any population of wonderful lives. Non-Archimedeans choose to deny this latter claim (Parfit 1986; 2016; Griffin 1988: 340, fn.27; Lemos 1993; Rachels 2004; Temkin 2012; Chang 2016; Nebel 2021). They claim that some contributively good lives are weakly noninferior to other contributively good lives:[11]

> **Weak Noninferiority Across Good Lives.**
>
> There is some contributively good life $x$, some contributively good life $y$, and some number $m$ such that $m$ lives at $[\![x]\!]$ is not worse than any number of lives at $[\![y]\!]$.

This move allows non-Archimedeans to avoid the Repugnant Conclusion without giving up the Equivalence of Personal and Contributive Value. They simply claim that wonderful lives are weakly noninferior to lives barely worth living.

However, some non-Archimedean views violate Transitivity (Rachels 2004; Temkin 2012). Other non-Archimedean views violate Separability (Hurka 1983; Ng 1989). *Lexical views* incur neither of these costs. By representing welfare levels with vectors, rather than scalars, they can avoid the Repugnant Conclusion while preserving Transitivity and Separability (Kitcher 2000; Thomas 2018; Nebel 2021; Carlson forthcoming).

Here's one example of a lexical view. Welfare levels are given by vectors with two dimensions, each dimension representable by an integer without upper or lower bound. The first dimension quantifies the *higher goods* in that life: perhaps things like autonomy and meaning. The second dimension quantifies the *lower goods*: perhaps things like sensual pleasure. These vectors are ordered lexically, so that $(h_x, l_x)$ is at least as good as $(h_y, l_y)$ iff either $h_x > h_y$ or $h_x = h_y$ and

[10] Advocates of *critical-level* and *critical-range views* deny this claim. Critical-level views raise the level of contributive strict neutrality above the level of personal strict neutrality, so that some personally good lives are contributively bad (Blackorby, Bossert, and Donaldson 2005; Bossert forthcoming). That means that these views avoid the Repugnant Conclusion at the expense of implying the Sadistic Conclusion: each population of awful lives is better than some much larger population of personally good lives. Critical-range views, meanwhile, claim that a range of welfare levels are contributively weakly neutral (Blackorby, Bossert, and Donaldson 1996; Broome 2004; Qizilbash 2007; Rabinowicz 2009). Lives barely worth living fall within this range, so adding them makes a population neither better nor worse. That allows these views to avoid both the Repugnant and the Sadistic Conclusions. As we will see, however, these views imply the second horn of the Archimedean dilemma: radical and symmetric incommensurability. For more discussion of critical-level and critical-range views, see Gustafsson (2020), Rabinowicz (2020), and Thornley (forthcoming).

[11] Indeed, most non-Archimedeans make the stronger claim that some good lives are *weakly superior* to other good lives. See footnote 1.

$l_x \geq l_y$. The value of population $X$ is then given by the vector $(h_X, l_X)$, where $h_X$ is the sum of all the higher goods in the lives in $X$ and $l_X$ is the sum of all the lower goods in the lives in $X$. Populations are ordered lexically in the same way as lives, so that population $X$ is at least as good as population $Y$ iff either $h_X > h_Y$ or $h_X = h_Y$ and $l_X \geq l_Y$.

Kitcher (2000), Thomas (2018), Nebel (2021), and Carlson (forthcoming) offer lexical views along these lines. As they note, these views can be tweaked and generalised in various ways. Lives could be represented by vectors with any number of elements, each element could be represented by any subset of the real numbers, and the ordering could employ thresholds of various kinds. Employing thresholds in the ordering allows lexical views to account for incommensurability between populations and lives. Suppose, for example, that population $X$ is at least as good as population $Y$ just in case $h_X - h_Y > \Delta$ or $h_X \geq h_Y$ and $l_X \geq l_Y$. In that case, it could be that neither of $X$ and $Y$ is at least as good as the other. Lexicalists can also claim that it may be indeterminate whether some life exceeds some threshold, in which case it may be indeterminate whether that life is strongly superior or noninferior to another life.

It's easy to see that these lexical views satisfy Transitivity. They also satisfy Separability, because the value of a population is the sum of the values of its lives. And they avoid the Repugnant Conclusion if we specify that wonderful lives feature some positive quantity of higher goods and lives barely worth living do not. That's because, in our initial example of a lexical view, lives with welfare $(m, n)$ are strongly superior to lives with welfare $(0, p)$ for all $m > 0$, $n$, and $p$.[12] What's more, representing welfare with a vector seems appealing even independently of securing these formal implications. After all, life is a rich tapestry. Lives vary along many dimensions, and we might doubt that their value can be represented by a single number.[13]

Unfortunately, there's a catch. The weak noninferiority of wonderful lives over lives barely worth living, in conjunction with two assumptions, implies that weak noninferiority holds between lives that differ only slightly in non-evaluative respects. The first assumption is Transitivity, and the second we can call *Small Steps*:

[12] Lexical views also escape Arrhenius's (2011; forthcoming) famed impossibility theorems, as Thomas (2018) and Carlson (forthcoming) prove. For impossibility theorems which lexical views do not escape, see Thornley (2021).

[13] For other ways of representing welfare with more than a single number, see Rabinowicz (2020) and Thornley (forthcoming).

**Small Steps.**

For any two welfare levels, there exists a finite sequence of slight non-evaluative differences between lives at those levels.[14]

What I mean by a 'slight non-evaluative difference' can be made clear enough using examples. Suppose that two lives are identical but for the fact that one of them features one additional second spent in pain. Then the non-evaluative difference between these lives is slight. The same goes for lives identical but for an extra second spent believing some false proposition, or appreciating beautiful music, or conversing with a loved one. Understood in this way, Small Steps seems difficult to deny. By making enough slight non-evaluative changes, we can make lives arbitrarily good or bad.[15]

To see how the weak noninferiority of wonderful lives over lives barely worth living plus Transitivity and Small Steps implies that weak noninferiority holds between lives that differ only slightly, consider a wonderful life $a_1$ and a life barely worth living $a_n$. By Small Steps, a finite sequence of slight differences unites a life at $[\![a_1]\!]$ and a life at $[\![a_n]\!]$. Now suppose, for contradiction, that no life in this sequence is weakly noninferior to its successor. In that case, each number of lives at $[\![a_1]\!]$ is worse than some number of lives at $[\![a_2]\!]$, each number of lives at $[\![a_2]\!]$ is worse than some number of lives at $[\![a_3]\!]$, and so on, all the way down to $[\![a_n]\!]$. Transitivity then implies that each number of lives at $[\![a_1]\!]$ is worse than some number of lives at $[\![a_n]\!]$. But this implication contradicts the lexical claim that wonderful lives are weakly noninferior to lives barely worth living. To avoid this contradiction, lexicalists must claim that some life in the sequence is weakly noninferior to its successor: for some life $a_k$, some number of lives at $[\![a_k]\!]$ is not worse than any number of lives at $[\![a_{k+1}]\!]$, even though $a_{k+1}$ is only slightly worse than $a_k$. Perhaps $a_{k+1}$ features just one extra second of pain. Call this implication *Weak Noninferiority Across Slight Differences.*[16]

Accepting Separability commits the lexicalist to an even stronger conclusion. Given Transitivity and Separability, weak noninferiority collapses into *strong* noninferiority. The lexical view then implies *Strong Noninferiority Across Slight Differences*: *any* number of lives at $[\![a_k]\!]$ is not worse than any number of lives at $[\![a_{k+1}]\!]$.

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[14] This assumption is an amended version of Arrhenius's (2016: 171) *Finite Fine-Grainedness* and Thomas's (2018: 815) *Small Steps*. Their versions refer to slight *welfare* differences rather than slight *non-evaluative* differences. As I note below, Arrhenius's and Thomas's versions are easier for the lexicalist to deny. I thank two anonymous reviewers for pointing this out.

[15] For readability, I drop the 'non-evaluative' in what follows. Unless otherwise specified, 'slight differences' and 'slight changes' refer to non-evaluative differences and changes.

[16] This paragraph draws on Arrhenius and Rabinowicz (2005; 2015b), Jensen (2008), and Nebel (2021).

Here's how. Suppose, for contradiction, that $a_k$ is *not* strongly noninferior to $a_{k+1}$. In that case, some number of lives at $[\![a_k]\!]$ is worse than some number of lives at $[\![a_{k+1}]\!]$. For concreteness, let's say that a single life at $[\![a_k]\!]$ is worse than one million lives at $[\![a_{k+1}]\!]$. Separability implies that adding a life at $[\![a_k]\!]$ to both populations leaves their value-relation unchanged. That means that a population of two lives at $[\![a_k]\!]$ is worse than a population of one million lives at $[\![a_{k+1}]\!]$ and one life at $[\![a_k]\!]$. Separability also implies that adding one million lives at $[\![a_{k+1}]\!]$ to both populations leaves their value-relation unchanged. That means that a population of one life at $[\![a_k]\!]$ and one million lives at $[\![a_{k+1}]\!]$ is worse than a population of two million lives at $[\![a_{k+1}]\!]$. These results, in conjunction with Transitivity, imply that two lives at $[\![a_k]\!]$ are worse than two million lives at $[\![a_{k+1}]\!]$. Repeating the steps above yields the result that three lives at $[\![a_k]\!]$ are worse than three million lives at $[\![a_{k+1}]\!]$ and, indeed, $n$ lives at $[\![a_k]\!]$ are worse than $n$ million lives at $[\![a_{k+1}]\!]$, for all positive integers $n$. But then $a_k$ is not even weakly noninferior to $a_{k+1}$. If $a_k$ is noninferior to $a_{k+1}$ at all, it is *strongly* noninferior: *any* number of lives at $[\![a_k]\!]$ is not worse than any number of lives at $[\![a_{k+1}]\!]$. *A fortiori*, a *single* life at $[\![a_k]\!]$ is not worse than any number of lives at $[\![a_{k+1}]\!]$, even though $a_{k+1}$ is only slightly worse than $a_k$.[17]

Nevertheless, lexical views remain popular. Two responses, not mutually exclusive, are common. The first is to reject an assumption left implicit in my discussion thus far. I write that $a_{k+1}$ is only 'slightly worse' than $a_k$. But lexicalists can claim that, although $a_k$ and $a_{k+1}$ differ only slightly in non-evaluative respects, $a_{k+1}$ is significantly worse than $a_k$ (Thomas 2018; Nebel 2021; Carlson forthcoming).

We can flesh out this response as follows. Recall that, on the lexicalist's representation of welfare levels, wonderful lives feature some positive quantity of higher goods and lives barely worth living do not. That implies that, in any sequence uniting wonderful lives and lives barely worth living, there will be a point at which the quantity of higher goods falls to 0. This fall might correspond to the point at which lives cease to be meaningful or autonomous (Nebel 2021, 11), or the point at which lives no longer instantiate a certain combination of global properties: for example, 'satisfying personal relations, some understanding of what makes life worth while, appreciation of great beauty, the chance to accomplish something with one's life.' (Griffin 1988: 86; see also Carlson forthcoming: 21).[18] Lexicalists can then claim that any life featuring no higher

[17] This paragraph draws on Jensen (2008) and Nebel (2021). Jensen (2020) offers a variant of this argument that does not depend on Small Steps. His argument proves that, on lexical views, a single wonderful life is better than any number of lives barely worth living. He suggests that non-Archimedeans might take this result as a reason to reject Separability.

[18] To anticipate a little, lexicalists can claim that it is indeterminate whether a life instantiates such properties, and hence indeterminate whether some life is strongly superior or noninferior to

goods is significantly worse than any life featuring some higher goods, so that strong noninferiority across such lives is of little concern.

The second response is to claim that Strong Noninferiority Across Slight Differences is benign. If we find it troubling, that is only because we assume *Trichotomous Completeness*:

> **Trichotomous Completeness.**
>
> For all populations $X$ and $Y$, either $X$ is better than $Y$, $Y$ is better than $X$, or $X$ and $Y$ are equally good.

If we assume Trichotomous Completeness, then Strong Noninferiority Across Slight Differences is tantamount to Strong *Superiority* Across Slight Differences: any number of lives at $[\![a_k]\!]$ is *better* than any number of lives at $[\![a_{k+1}]\!]$. In conjunction with a deontic principle according to which choosing the worse of two available options is impermissible, this consequence implies that creating any number of lives at $[\![a_{k+1}]\!]$ would be impermissible if we could instead create a single life at $[\![a_k]\!]$. That implication seems troubling. However, if we deny Trichotomous Completeness, no such thing follows. Strong noninferiority is no longer tantamount to strong superiority. Lexicalists can claim that, although a single life at $[\![a_k]\!]$ is *not worse* than any number of lives at $[\![a_{k+1}]\!]$, it is nevertheless false that a single life at $[\![a_k]\!]$ is *better* than any number of lives at $[\![a_{k+1}]\!]$. Enough lives at $[\![a_{k+1}]\!]$ may be incommensurable with any number of lives at $[\![a_k]\!]$ (Nebel 2021: 17–19).[19] Typically, lexicalists go on to claim that this move is more than mere evaluative hair-splitting: the distinction has deontic implications. If choosing an option is permissible so long as it is not worse than another available option (Chang 2005: 333; Rabinowicz 2008; 2012; Nebel 2021: 20), then we may permissibly choose $X$ or $Y$ when the two populations are incommensurable. And if $X$ and $Y$ are indeterminately related, then it is indeterminate which of $X$ and $Y$ is permissible to choose.

This strategy seems to offer an attractively conservative way of avoiding the Repugnant Conclusion. It preserves both Separability and Transitivity, and it softens the blow of Strong Noninferiority Across Slight Differences by denying a principle which seems implausible anyway: Trichotomous Completeness. A more general version of this principle – quantifying over all value-bearers, rather than just populations – is impugned by existing Small Improvement Arguments (De Sousa 1974; Chang 2002), and a structurally identical argument tells against the restricted principle. Suppose, for example, that population $X$ features ten people

another (Thomas 2018: 828–29; Nebel 2021: 27–30). As we will see, this indeterminacy must be radical in order to block the Repugnant Conclusion.

[19] Or else enough lives at $[\![a_{k+1}]\!]$ may be on a par with (Chang 2016), imprecisely equally good as (Parfit 1984: 430–32; 2016), or indeterminately related to (Qizilbash 2005; Knapp 2007; Thomas 2018: 828–29) any number of lives at $[\![a_k]\!]$.

each living a 20-year life of ecstasy, and population $Y$ features ten people each living an 80-year life of comfort. Neither $X$ nor $Y$ is better than the other.[20] If we assume Trichotomous Completeness, $X$ and $Y$ must be equally good. But if $X$ and $Y$ are equally good, then any population better than $Y$ is also better than $X$. $Y^+$ – featuring ten people each living an 81-year life of comfort – seems better than $Y$, but not better than $X$. Therefore, it seems, $X$ and $Y$ are not equally good but incommensurable, and Trichotomous Completeness is false. Lexicalists can thus avoid the Repugnant Conclusion and Strong Superiority Across Slight Differences by denying an independently implausible principle.

However, trouble remains. Suppose that we grant the lexicalist's claims about higher goods: in any sequence uniting wonderful lives and lives barely worth living, there will be a point at which the quantity of higher goods falls to 0, and any lives occurring after this point are *significantly* worse than those that come before. We might complain that this move merely masks – and does not solve – the difficulty presented by the $a$-sequence. Once we recall the *non-evaluative* character of the lives in the $a$-sequence, the trouble reasserts itself. The lexical view still implies that there are lives $a_k$ and $a_{k+1}$ such that a single life at $[\![a_k]\!]$ is not worse than any number of lives at $[\![a_{k+1}]\!]$, even though $a_k$ and $a_{k+1}$ differ only slightly in non-evaluative respects. Perhaps this slight difference is as small as an extra second's worth of pain. Strong noninferiority across these near-identical lives might seem tough to accept, even if we go along with the lexicalist's representation of their welfare levels.[21]

Things get worse if we focus on bad lives. The Repugnant Conclusion has a negative analogue:

> **The Negative Repugnant Conclusion.**
>
> Each population consisting only of awful lives is better than some much larger population consisting only of lives barely worth not living.

And if we uphold the Equivalence of Personal and Contributive Value, this conclusion can be avoided only by claiming Weak Nonsuperiority Across Bad Lives:

> **Weak Nonsuperiority Across Bad Lives.**
>
> There is some contributively bad life $x$, some contributively bad life $y$, and some number $m$ such that $m$ lives at $[\![x]\!]$ is not better than any number of lives at $[\![y]\!]$.

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[20] Those who disagree should play around with the numbers and/or nouns.

[21] Henceforth, for brevity's sake, I resume describing the lives in these sequences as 'slightly worse' than their predecessors. Strictly, this phrase should be read as 'slightly different in non-evaluative respects, in a way that makes it worse.' The same goes for my use of 'slightly better.'

But as shown above, this claim – in conjunction with Transitivity and Separability – implies Strong Nonsuperiority Across Bad Lives:

> **Strong Nonsuperiority Across Bad Lives.**
>
> There is some contributively bad life $x$ and some contributively bad life $y$ such that any number of lives at $[\![x]\!]$ is not better than any number of lives $[\![y]\!]$.

And the truth of Small Steps implies Strong Nonsuperiority Across Slight Differences. Suppose $b_1$ is an awful life, $b_2$ is slightly better than $b_1$, $b_3$ is slightly better than $b_2$, and so on, until we reach some life barely worth not living $b_n$. Then there must be some bad life $b_k$ such that *any* number of lives at $[\![b_k]\!]$ is not better than *any* number of lives at $[\![b_{k+1}]\!]$, even though $b_{k+1}$ is only slightly better than $b_k$. Perhaps $b_{k+1}$ features just one extra second of pleasure.

What's more, Handfield and Rabinowicz (2018) prove that the combination of weak noninferiority and the denial of Trichotomous Completeness – along with Transitivity and a weakening of Separability (see 2018: 2385) – has another undesirable implication: to avoid the Repugnant Conclusion, the incommensurability at work has to be *radical*. Here's what that means. Suppose population $A_k$ features only good lives at $[\![a_k]\!]$ and population $A_{k+1}$ features only slightly worse lives at $[\![a_{k+1}]\!]$. If both populations are the same size, then $A_{k+1}$ is worse than $A_k$. According to lexicalists who deny Trichotomous Completeness, increasing the number of lives at $[\![a_{k+1}]\!]$ can take $A_{k+1}$ from worse than $A_k$ to incommensurable with $A_k$. However, *no* number of additional lives at $[\![a_{k+1}]\!]$ on top of that can take $A_{k+1}$ from incommensurable with $A_k$ to better than $A_k$. Indeed, no number of lives at $[\![a_{k+1}]\!]$ can be better than even a *single* life at $[\![a_k]\!]$.

Besides seeming implausible, such radical departures from Trichotomous Completeness lack a key feature shared by other examples of incommensurability in the literature: in those examples, if a change in some good-making feature can take an option $S$ from worse than another option $T$ to incommensurable with $T$, then a further change in that good-making feature can take $S$ from incommensurable with $T$ to better than $T$. This is especially so when, as in the population case, the difference in other respects is slight. Suppose, for example, that your employer offers you a choice between $S$, a contract mandating that you work 39 hours and 59 minutes per week, and $T$, a contract mandating that you work 40 hours per week. If $S$ and $T$ offer the same salary, then $S$ is worse than $T$. Increasing $T$'s salary by some finite amount can render $T$ incommensurable with $S$, and increasing $T$'s salary by some further amount can render $T$ better than $S$. Radical departures from Trichotomous Completeness lack this key feature, so strategies committed to some such departure are not as conservative as they might first seem: lexicalists who avoid the Repugnant Conclusion through the combination of Weak Noninferiority Across Good Lives and the denial of

Trichotomous Completeness are positing a new and controversial phenomenon rather than drawing upon an old and widely accepted one.[22]

I can now summarise the lexical dilemma. If lexicalists uphold Trichotomous Completeness, they are committed to Strong Superiority Across Slight Differences: any number of good lives at $[\![a_k]\!]$ is better than any number of slightly worse lives at $[\![a_{k+1}]\!]$, and any number of bad lives at $[\![b_k]\!]$ is worse than any number of slightly better lives at $[\![b_{k+1}]\!]$. If, on the other hand, lexicalists depart from Trichotomous Completeness, then that departure must be radical. For any number of lives at $[\![a_k]\!]$, there is some number of lives at $[\![a_{k+1}]\!]$ such that the two populations are incommensurable, but there is no number of lives at $[\![a_{k+1}]\!]$ that is better than even a single life at $[\![a_k]\!]$. And the converse is true of bad lives at $[\![b_k]\!]$ and $[\![b_{k+1}]\!]$.

# 4. The Archimedean Dilemma

We might regard the lexical dilemma as strong reason to embrace an Archimedean view. However, this would be a mistake. As we will see, Archimedean views are subject to an analogous dilemma: either a single contributively good life $c_k$ is better than any number of slightly worse lives, or else the departure from Trichotomous Completeness is both radical and *symmetric*: for any arbitrarily good population and any arbitrarily bad population, there is some population that is both not worse than the former and not better than the latter. The conclusion is that the lexical dilemma gives us little reason to prefer Archimedean views. Even if we give up on lexicality, problems of the same kind remain.

To see how the Archimedean dilemma arises, consider the following two claims:

[22] Note that Handfield and Rabinowicz (2018) do not endorse this argument as an objection to radical *indeterminacy*, in the sense compatible with completeness. They point out that 'there is less precedent in the literature for assuming that indeterminacy that arises from a vague threshold in one relevant dimension must eventually be overwhelmed by a large enough difference in a second relevant dimension.' (2018: 2384). Instead, their objection to this kind of radical indeterminacy is that it does not solve the problem: it still implies that there is some life $a_k$ which is strongly superior to a slightly worse life $a_{k+1}$. They claim that this implication remains counterintuitive, even if it is indeterminate where strong superiority sets in (2018: 2385). For claims that indeterminate thresholds are *not* objectionably counterintuitive, see Nebel (2021: 27–30) and Thomas (forthcoming-b).

For the claim that radical incommensurabilities are not objectionably counterintuitive, see Rabinowicz (2019). There Rabinowicz argues that we should interpret the incommensurability along the lines of the fitting-attitudes analysis of value. For the fitting-attitudes interpretation of incommensurability/parity, see Rabinowicz (2008; 2012).

**Contributively Good Life.**

There is some life $a$ and some population $A$ such that $[\![A]\!] \cup [\![a]\!]$ is better than $[\![A]\!]$.

**Contributively Bad Life.**

There is some life $b$ and some population $B$ such that $[\![B]\!] \cup [\![b]\!]$ is worse than $[\![B]\!]$.

Together with Transitivity and Separability, these two claims imply that contributively good lives are strongly noninferior to contributively bad lives.[23] Here's how. Let Ø stand for the empty population, containing no lives whatsoever. Given Separability, if adding $a$ makes some population better, it makes every population better. In that case, any number of lives at $[\![a]\!]$ is better than Ø. Separability also implies that adding $b$ makes every population worse, in which case any number of lives at $[\![b]\!]$ is worse than Ø. By Transitivity, any number of lives at $[\![a]\!]$ is better than any number of lives at $[\![b]\!]$. Life $a$ is thus strongly superior to life $b$. *A fortiori*, life $a$ is strongly noninferior to life $b$: any number of lives at $[\![a]\!]$ is not worse than any number of lives at $[\![b]\!]$.

Adding Small Steps then yields Strong Noninferiority Across Slight Differences. To see how, consider a sequence beginning with a good life $c_1$. We reach $c_2$ by making $c_1$ slightly worse, and so on, until we reach a bad life $c_n$. Now suppose, for contradiction, that no life in this sequence is even weakly noninferior to its successor. In that case, each number of lives at $[\![c_1]\!]$ is worse than some number of lives at $[\![c_2]\!]$, each number of lives at $[\![c_2]\!]$ is worse than some number of lives at $[\![c_3]\!]$, and so on, all the way down to $[\![c_n]\!]$. Transitivity then implies that each number of lives at $[\![c_1]\!]$ is worse than some number of lives at $[\![c_n]\!]$. But this implication contradicts the Archimedean claim that good lives are strongly noninferior to bad lives. To avoid this contradiction, Archimedeans must claim that some life in the sequence is weakly noninferior to its successor: some number of lives at $[\![c_k]\!]$ is not worse than *any* number of lives at $[\![c_{k+1}]\!]$, even though $c_{k+1}$ is only slightly worse than $c_k$. Given Separability and Transitivity, this weak noninferiority collapses into strong noninferiority: *any* number of lives at $[\![c_k]\!]$ is not worse than any number of lives at $[\![c_{k+1}]\!]$.

Now for the first horn of the Archimedean dilemma. If Archimedeans accept Trichotomous Completeness, then Strong Noninferiority Across Slight Differences is tantamount to Strong *Superiority* Across Slight Differences: any number of lives at $[\![c_k]\!]$ is *better* than any number of lives at $[\![c_{k+1}]\!]$.

Archimedeans might claim that this implication is of little concern. After all, strong superiority sets in at the point where lives stop being good. Lives at $[\![c_k]\!]$

[23] I once again drop the 'contributively' in what follows; 'good,' 'better,' etc., stand for 'contributively good,' 'contributively better,' etc., unless otherwise stated.

are good and lives at $[\![c_{k+1}]\!]$ are strictly neutral or bad, so it should be no mystery that a single life at $[\![c_k]\!]$ is better than any number of lives at $[\![c_{k+1}]\!]$. However, as with the lexical view, this move merely masks the difficulty. Once we recall the *non-evaluative* character of lives in the $c$-sequence, the trouble is revealed. Suppose, for example, that $c_1$ is a long, turbulent life, featuring soaring highs and crushing lows. Suppose also that $c_1$'s highs just outweigh its lows, so that $c_1$ is good overall. Suppose $c_2$ is identical but for one additional second of pain, and so on for each successive life, until we reach a bad life $c_n$. Archimedeans have to claim that many steps in this sequence are of little consequence – enough lives at $[\![c_2]\!]$ can be better than any number of lives at $[\![c_1]\!]$, enough lives at $[\![c_3]\!]$ can be better than any number of lives at $[\![c_2]\!]$, and so on – but one additional second of pain makes all the difference, so that *any* number of lives at $[\![c_k]\!]$ is better than *any* number of lives at $[\![c_{k+1}]\!]$. Archimedeans and non-Archimedeans alike have found this claim implausible (Broome 2004, 179–80, 251–52; Nebel 2021, 29). It seems absurd to think that one extra second of pain could flip a long, turbulent life from good to either strictly neutral or bad.

Hence the appeal of denying Trichotomous Completeness. That move allows Archimedeans to claim that there is no sharp divide between good and bad lives. Instead, some range of lives in our $c$-sequence is *weakly neutral*. Adding weakly neutral lives to a population renders the new population incommensurable with the original population. Denying Trichotomous Completeness thus allows Archimedeans to avoid the first horn of their dilemma. If lives at $[\![c_{k+1}]\!]$ are weakly neutral, rather than strictly neutral or bad, then Strong Noninferiority Across Slight Differences does not imply Strong *Superiority* Across Slight Differences. Archimedeans can claim that, although any number of good lives at $[\![c_k]\!]$ is *not worse* than any number of weakly neutral lives at $[\![c_{k+1}]\!]$, it is nevertheless false that any number of lives at $[\![c_k]\!]$ is *better* than any number of lives at $[\![c_{k+1}]\!]$. For any number of lives at $[\![c_k]\!]$, there is some number of lives at $[\![c_{k+1}]\!]$ such that the two populations are incommensurable. Archimedeans can also claim that this move is more than mere evaluative hair-splitting because it has deontic implications. If a population of lives at $[\![c_k]\!]$ and a population of lives at $[\![c_{k+1}]\!]$ are incommensurable, then we may permissibly choose either. If the two populations are indeterminately related, then it is indeterminate which is permissible to choose.

As we will see, however, denying Trichotomous Completeness leaves the Archimedean vulnerable to the second horn of their dilemma. To see how, note first that departing from Trichotomous Completeness renders the Archimedean subject to the same objection that Handfield and Rabinowicz (2018) level against the lexicalist: the departure in question has to be *radical*. Here's a reminder of what that means. Suppose population $C_k$ features only lives at $[\![c_k]\!]$ and population $C_{k+1}$ features only lives at $[\![c_{k+1}]\!]$. If both populations are the same size, then $C_k$ is better than $C_{k+1}$. Increasing the number of lives at $[\![c_{k+1}]\!]$ can take $C_{k+1}$ from worse than $C_k$ to incommensurable with $C_k$. However, no further increase in the number of lives at $[\![c_{k+1}]\!]$ can take $C_{k+1}$ from incommensurable with $C_k$ to better than $C_k$. Indeed, no number of lives at $[\![c_{k+1}]\!]$ can be better than even a single life at $[\![c_k]\!]$. Such radical departures from Trichotomous Completeness might seem implausible, and they lack a key feature shared by other examples of incommensurability in the literature: if a change in some good-making feature can take $S$ from worse than $T$ to incommensurable with $T$, then a further change in that good-making feature can take $S$ from incommensurable with $T$ to better than $T$.[24]

Of course, the Archimedean might respond that the objection misses its mark in this case. The objection is effective against the lexicalist because lives at $[\![a_{k+1}]\!]$ are good, so it seems like adding such lives should make a population better. Lives at $[\![c_{k+1}]\!]$, on the other hand, are not good, so there is no reason to think that adding such lives makes a population better. However, this response invites two new objections. The first is that this move casts doubt on the other feature of radical departures from Trichotomous Completeness: if lives at $[\![c_{k+1}]\!]$ are not good, it is puzzling how adding such lives can take a population from worse than a single life at $[\![c_k]\!]$ to not worse.[25] Second, and more seriously, the radical departure from Trichotomous Completeness must then be *symmetric*: for any population of good lives and any population of bad lives, there must be some number of weakly neutral lives that is both not worse than the former and not better than the latter.

To see how, recall that for any weakly neutral life $u$ and any population $X$, $[\![X]\!]$ is incommensurable with $[\![X]\!] \cup [\![u]\!]$. Recall also that incommensurability is typically *insensitive to slight changes*. There will typically be some improved version of $X$ – call it $X^+$ – and some worsened version of $X$ – call it $X^-$ – such that $[\![X^+]\!]$ and $[\![X^-]\!]$ are incommensurable with $[\![X]\!] \cup [\![u]\!]$.

We need not assume that adding a weakly neutral life *always* results in incommensurability that is insensitive to slight changes. The proof can make do with a substantially weaker assumption, which we can call *Insensitivity*:

[24] Gustafsson (2020) and Rabinowicz (2020) argue that this kind of radical incompleteness need not be implausible. If we allow incommensurability between lives, then a single good life can be incommensurable with any number of weakly neutral lives, even if that number is just one. I thank an anonymous reviewer for raising this point.

[25] Many population axiologists do not find this implication puzzling (Rabinowicz 2009; Frick 2017; Gustafsson 2020): they think that lives that are neither good nor bad can nevertheless swallow up goodness and badness, a phenomenon that Broome calls 'greedy neutrality' (2004: 164ff.). My second objection tells against these views, as do many of my objections in Thornley (forthcoming).

**Insensitivity.**

> There exists some sequence of slight differences – running from a good life $d_g$ to a bad life $d_b$ and containing some weakly neutral life $d_0$ – such that for any life in the sequence $d_r$ and any populations $X$ and $Y$, there exists some number $m$ such that, if $[\![X]\!] \cup [\![d_r]\!]$ is incommensurable with $[\![Y]\!]$, then $[\![X]\!] \cup [\![d_{r+1}]\!]$ and $[\![X]\!] \cup [\![d_{r-1}]\!]$ are incommensurable with $[\![Y]\!] \cup m[\![d_0]\!]$.

This assortment of quantifiers is difficult to parse, so here's a rough explanation. We start with two incommensurable populations, represented by the distributions $[\![X]\!] \cup [\![d_r]\!]$ and $[\![Y]\!]$. We then make the life $d_r$ in the first population slightly better. This new life $d_{r+1}$ might feature just one extra second of pleasure. Insensitivity states that adding some number of lives at some weakly neutral welfare level $[\![d_0]\!]$ to the second population can ensure that the resulting populations remain incommensurable. Insensitivity also states that the same is true when we make the life $d_r$ in the first population slightly worse. Perhaps $d_{r-1}$ features just one extra second of pain. Again, adding some number of lives at $[\![d_0]\!]$ to the second population can preserve incommensurability. And Insensitivity states that the above is true for all lives $d_r$ in some $d$-sequence and for all populations $X$ and $Y$ such that $[\![X]\!] \cup [\![d_r]\!]$ and $[\![Y]\!]$ are incommensurable.

Now let $G$ stand for some arbitrarily good population and $B$ stand for some arbitrarily bad population. And recall that Archimedeanism about Populations states that adding enough good lives to a population can make it better than any other, and adding enough bad lives to a population can make it worse than any other. Since the lives $d_g$ and $d_b$ in Insensitivity are good and bad respectively, Archimedeanism implies that there is some $n$ such that $n[\![d_g]\!]$ is better than $[\![G]\!]$ and $n[\![d_b]\!]$ is worse than $[\![B]\!]$.

Consider a population of $n$ lives at $[\![d_0]\!]$. Because lives at $[\![d_0]\!]$ are weakly neutral, the population of $n$ lives at $[\![d_0]\!]$ is incommensurable with the empty population. Insensitivity thus implies that there is some $s_1$ such that $(n-1)[\![d_0]\!] \cup [\![d_1]\!]$ is incommensurable with $s_1[\![d_0]\!]$. That's because we made one of the lives in the first population slightly better – raising it from $[\![d_0]\!]$ to $[\![d_1]\!]$ – so by Insensitivity we can add some number of weakly neutral lives at $[\![d_0]\!]$ to the second population – the empty population – and thereby ensure that the resulting populations remain incommensurable.

We can do the same when we raise a second life up from $[\![d_0]\!]$ to $[\![d_1]\!]$. There is some $s_2$ such that $(n-2)[\![d_0]\!] \cup 2[\![d_1]\!]$ is incommensurable with $s_1[\![d_0]\!] \cup s_2[\![d_0]\!]$. Repeating this process $n-2$ more times, we get the result that $n[\![d_1]\!]$ is incommensurable with $s_1[\![d_0]\!] \cup s_2[\![d_0]\!] \cup ... \cup s_n[\![d_0]\!]$. We can then set about raising each of the lives in the first population up from $[\![d_1]\!]$ to $[\![d_2]\!]$. Again, by Insensitivity, we can preserve incommensurability by adding some number of lives at $[\![d_0]\!]$ to the second population. The same is true of the rise from $[\![d_2]\!]$ to $[\![d_3]\!]$, $[\![d_3]\!]$ to $[\![d_4]\!]$, and so on. Eventually, we'll have raised all $n$ lives up to the good welfare level $[\![d_g]\!]$. Insensitivity thus implies that there is some number $q_1$ such that $n[\![d_g]\!]$ is incommensurable with $q_1[\![d_0]\!]$.

The same is true when we make the lives at $[\![d_0]\!]$ worse rather than better. Since the population of $n$ lives at $[\![d_0]\!]$ is incommensurable with the empty population, Insensitivity implies that there is some $t_1$ such that $(n-1)[\![d_0]\!] \cup [\![d_{-1}]\!]$ is incommensurable with $t_1[\![d_0]\!]$. Because we lowered one life in the first population down from $[\![d_0]\!]$ to $[\![d_{-1}]\!]$, we can preserve incommensurability by adding some number of lives at $[\![d_0]\!]$ to the second population. After enough of these steps, we'll have lowered all $n$ lives down to the bad welfare level $[\![d_b]\!]$. Insensitivity thus implies that there is some number $q_2$ such that $n[\![d_b]\!]$ is incommensurable with $q_2[\![d_0]\!]$.

Letting $q$ represent whichever of $q_1$ and $q_2$ is bigger (or both in the case of a tie), we can conclude that $q[\![d_0]\!]$ is incommensurable with both $n[\![d_g]\!]$ and $n[\![d_b]\!]$. *A fortiori*, $q[\![d_0]\!]$ is not worse than $n[\![d_g]\!]$ and not better than $n[\![d_b]\!]$. Since $n[\![d_g]\!]$ is better than the arbitrarily good population represented by $[\![G]\!]$, Transitivity implies that $q[\![d_0]\!]$ is not worse than $[\![G]\!]$.[26] Since $n[\![d_b]\!]$ is worse than the arbitrarily bad population represented by $[\![B]\!]$, Transitivity implies that $q[\![d_0]\!]$ is not better than $[\![B]\!]$.[27] Coupling up these last two results gives us the second horn of the Archimedean dilemma: for any arbitrarily good population $G$ and any arbitrarily bad population $B$, there is some population of weakly neutral lives that is both not worse than the former and not better than the latter.

I can now summarise the Archimedean dilemma. If Archimedeans uphold Trichotomous Completeness, they are committed to Strong Superiority Across Slight Differences. Many slight changes to lives are of little consequence, but one slight change flips the lives from good to either strictly neutral or bad, and any number of the former lives is better than any number of the latter. This implication is liable to seem especially implausible if both lives are long and turbulent, and the slight change consists in a single extra second of pain. If, on the other hand, Archimedeans depart from Trichotomous Completeness, then that departure must be both radical and symmetric. They are committed to the claim that, no matter how good and numerous the lives in Heaven and no matter how bad and numerous the lives in Hell, there is some number of weakly neutral lives that is both not worse than Heaven and not better than Hell.

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[26] To see how, suppose for contradiction that $q[\![d_0]\!]$ is worse than $[\![G]\!]$. Since $[\![G]\!]$ is worse than $n[\![d_g]\!]$, Transitivity would then imply that $q[\![d_0]\!]$ is worse than $n[\![d_g]\!]$. But that contradicts what was established above.

[27] To see how, suppose for contradiction that $q[\![d_0]\!]$ is better than $[\![B]\!]$. Since $[\![B]\!]$ is better than $n[\![d_b]\!]$, Transitivity would then imply that $q[\![d_0]\!]$ is better than $n[\![d_b]\!]$. But that contradicts what was established above.

That brings us to the conclusion of this paper: the lexical dilemma gives us little reason to prefer an Archimedean view. For recall how the lexical dilemma is derived. We begin with the non-Archimedean claim that some good lives are weakly noninferior to others: there is some good life $x$, some good life $y$, and some number $n$ such that $n$ lives at $[\![x]\!]$ is not worse than any number of lives at $[\![y]\!]$. Adding Transitivity and Separability yields the lexical view. Assuming Small Steps commits the lexical view to Strong Noninferiority Across Slight Differences: a single life at $[\![a_k]\!]$ is not worse than any number of lives at $[\![a_{k+1}]\!]$. If we then assume Trichotomous Completeness, this is tantamount to Strong *Superiority* Across Slight Differences: a single life at $[\![a_k]\!]$ is *better* than any number of lives at $[\![a_{k+1}]\!]$. If, on the other hand, we depart from Trichotomous Completeness, that departure must be *radical*: for any number of lives at $[\![a_k]\!]$, there is some number of lives at $[\![a_{k+1}]\!]$ that is not worse, but no number of lives at $[\![a_{k+1}]\!]$ is better than even a single life at $[\![a_k]\!]$.

The Archimedean dilemma is derived in parallel fashion. We begin with the Archimedean claim that some lives are strongly noninferior to others: there is some life $x$, and some life $y$ such that any number of lives at $[\![x]\!]$ is not worse than any number of lives at $[\![y]\!]$. In particular, good lives are strongly noninferior to bad lives. Adding Transitivity, Separability, and Small Steps commits the Archimedean view to Strong Noninferiority Across Slight Differences: any number of lives at $[\![c_k]\!]$ is not worse than any number of lives at $[\![c_{k+1}]\!]$. If we then assume Trichotomous Completeness, this collapses into Strong *Superiority* Across Slight Differences: any number of lives at $[\![c_k]\!]$ is *better* than any number of lives at $[\![c_{k+1}]\!]$. If, on the other hand, we depart from Trichotomous Completeness, that departure must be both radical and *symmetric*: for any Heaven and any Hell, there is some number of weakly neutral lives that is both not worse than the former and not better than the latter.

The upshot is that the lexical dilemma gives us little reason to embrace an Archimedean view. Even if we give up on lexicality, problems of the same kind remain.[28]

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