Cognitivist Probabilism

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Abstract: In this article, I introduce the term "cognitivism" as a name for the thesis that degrees of belief are *equivalent* to full beliefs about truth-valued propositions. The thesis (of cognitivism) that degrees of belief are equivalent to full beliefs is equivocal, inasmuch as different sorts of equivalence may be postulated between degrees of belief and full beliefs. The simplest sort of equivalence (and the sort of equivalence that I will discuss here) identifies having a given degree of belief with having a full belief with a specific content. This sort of view was proposed in (Howson & Urbach, 1996). In addition to embracing a form of cognitivism about degrees of belief, Howson and Urbach argued for a brand of probabilism. I call a view, such as Howson and Urbach's, which combines probabilism with cognitivism about degrees of belief "cognitivist probabilism." In order to address some problems with Howson and Urbach's view, I propose a view that incorperates several of modifications of Howson and Urbach's version of cognitivist probabilism. The view that I finally propose upholds cognitivism about degrees of belief, but deviates from the letter of probabilism, in allowing that a rational agent's degrees of belief need not conform to the axioms of probability, in the case where the agent's cognitive resources are limited.

Keywords: bayesianism, probabilism, the Dutch book argument

1 Introduction

In their book *Scientific Reasoning: The Bayesian Approach* (1996), Howson and Urbach propose to treat an agent's beliefs about what betting quotients are fair for wagers as a measure of the agent's degrees of belief. More precisely, Howson and Urbach hold that if r is the unique value such that an agent, A, believes that the betting quotient r is fair for wagers on α, then A's degree of belief in α is r. To the preceding conditional, Howson and Urbach add the idealizing assumption that for each agent, A, and proposition, α, there exists a unique r, such that A believes that the betting quotient r is fair for wagers on α. Given their idealizing assumption, Howson and Urbach's proposed identification of degree of belief with a species of belief entails that an agent's degree of belief in α is r if and only if r is the unique value such that the agent believes that the betting quotient r is fair for wagers on α.[1]

With the proposed equivalence between degrees of belief and beliefs about betting quotients in the background, Howson and Urbach argue that having non-probabilistic degrees of belief (i.e., degrees of belief that do not conform to the axioms of probability) is equivalent to having an inconsistent set of beliefs about what betting quotients are fair. The idea is then that the prescription to have probabilistic degrees of belief (i.e., degrees of belief that conform to the axioms of probability) is a consequence of the prescription that agents should have consistent beliefs.

It is useful to think of Howson and Urbach's brand of probabilism as consisting of two theses. The first thesis is that degrees of belief are equivalent to beliefs about betting quotients (in the manner specified above). The second thesis is that it is inconsistent to regard an assignment of betting quotients as fair if those betting quotients do not conform to the axioms of probability. Howson and Urbach's first thesis reflects their subscription to cognitivism about degrees of belief, while the two theses together yield an argument for probabilism.

Howson and Urbach's argument for their second thesis shares features with traditional arguments for probabilism (cf. Ramsey, 1931, and de Finetti, 1937). Like traditional arguments for probabilism, Howson and Urbach's argument appeals to a consequence of the Dutch Book theorem. The relevant consequence states that if an assignment of betting quotients, p, fails to be a probability function, then there exists a possible set of wagers, W, in accordance with p, such that simultaneously accepting each element of W is *assured* to result in a net loss. The Dutch Book theorem can be stated formally and subjected to rigorous mathematical proof. However, Howson and Urbach's argument for their second thesis relies on two further premises that cannot be subjected to mathematical proof. The two premises are: (1) if acceptance of a given set of wagers is assured to result in a net loss, then acceptance of the set is disadvantageous, and (2) acceptance of a set of wagers whose elements are each individually made at fair betting quotients is not disadvantageous. The second of these two premises has been the subject of criticism (cf. Maher, 1997). In order to avoid such criticisms, the view defended here does not rely on (2).

Howson and Urbach's first thesis depends on the assumption that, for

[1]In order for the entailment to hold it must also be assumed that if an agent's degree of belief in a proposition, α, is r and $r \neq s$, then it is not the case that the agent's degree of belief in α is s.

each agent, A, and proposition, α, there is a unique betting quotient, r, that A regards as fair for wagers on α. Howson and Urbach acknowledge that attributing belief is such "advantage-equilibrating" betting quotients, in all cases, is a "strong" idealizing assumption (1996, p. 75). In order to address possible criticisms that may derive from this assumption, Howson and Urbach assert that the idealizing assumption can be relaxed. In the case where the assumption is relaxed (and interval-valued degrees of belief are defined in a manner analogous to point-valued degrees of belief), Howson and Urbach assert that consistent interval-valued degrees of belief conform to the usual principles that generalize the axioms of probability to interval-values.[2]

Within a cognitivist framework of the sort proposed by Howson and Urbach, I will propose a means to relaxing Howson and Urbach's idealizing assumption. In order to carry out the relaxation, I will appeal to the notions of *favorable* and *unfavorable* betting quotients, as a substitute for Howson and Urbach's notion of a *fair* betting quotient. By appeal to the notions of favorable and unfavorable betting quotients, I will endeavor to defend a view which evades several difficulties with Howson and Urbach view, while at the same time preserving the advantages of cognitivism about degrees of belief.[3]

2 Favorable and Unfavorable Betting Quotients

In this section, I provide a preliminary account of the notions of favorable and unfavorable betting quotients. For the sake of simplicity, the conditions are expressed relative to a propositional language, L, consisting of a countable set of propositional atoms, p_1, p_2, p_3, etc., and truth functional compounds of the atoms via the usual connectives: $\neg, \wedge, \vee, \rightarrow$ (material conditional), and $\leftrightarrow$ (material bi-conditional). Lower case Greek letters, α, β, χ, etc. will be used as meta-logical variables ranging over the elements of L. "$\models$" is used denote the (classical) logical consequence relation.

[2]In fact, Howson and Urbach's original argument for probabilism, which assumes point-valued degrees of belief, does not generalize so neatly to the case of interval-valued degrees of belief. Once one moves to the case of interval-valued degrees of belief, appeal to the Dutch Book theorem is insufficient to derive the conclusion that consistent interval-valued degrees of belief conform to the usual theorems that generalize the axioms of probability to interval-values (cf. Walley, 1991, p. 67).

[3]A significant disadvantage of Howson and Urbach view is its reliance on the package principle (cf. Schick, 1986; Hájek, 2008; Maher, 1997). Further problems with the view were outlined in (Vineberg, 2001).

In expressing the conditions for favorability and unfavorability, I will speak of wager *types* in addition to wager *tokens*. Among other things, this distinction will be used in expressing the fact that, in certain circumstances, agents are rationally required to have identical dispositions toward distinct tokens of the same wager type. Wager types are characterized as follows.

Definition 1 *A wager type is a pair,* $\langle \alpha, s \rangle$*, where* α *is an element of* L*, and* s *is real number.*

Throughout, wealth will be measured in a single currency, denoted "\$", and, for convenience, it is assumed that wagers have a standardized format:

Acceptance of an instance of a *standardized wager* of type $\langle \alpha, s \rangle$ *directly results* in an immediate +\$1-s change in wealth, if α is true, and *directly results* in an immediate -\$$s$ change in wealth, if α is false. Acceptance of an instance of a *standardized wager* has no other *direct results*.[4]

For the purpose of expressing necessary and sufficient conditions for being favorable and unfavorable betting quotients, I will appeal to the notion of a *normalized agent*. Every normalized agent is assumed to possess a body of *primary evidence*, E, which is a set of sentences of L. E is described as a normalized agent's 'primary evidence', in order to distinguish the contents of E from other facts evident to normalized agents, as specified in condition 7 of the definition of *normalized agent* (see below). For a normalized agent, A, it is intended that A's total evidence is comprised of A's primary evidence in combination with the content specified by condition 7 of the following definition.

Definition 2 *A is a normalized agent with evidence, E, just in case:*

(1) A's utility function is a positive linear function of his/her final wealth state,

(2) A's means of changing her wealth state is limited to the direct results of accepting standardized wagers,

(3) A has (finite) wealth sufficient to accept all of the (finite number of) wagers that she is offered,

(4) A has the opportunity to consider all of the wagers that she will be offered before deciding which wagers to accept,

[4]Standardized wagers have a fixed stake of \$1. This limitation in the possible stakes of wagers is made for expository purposes. Wagers for greater stakes can be 'constructed' as multiple tokens of the same wager type.

(5) A's has unlimited and unerring computational resources,

(6) A's primary evidence is E,

(7) A's total evidence consists of E, along with the knowledge that (1) through (6) hold of her, and the knowledge of what wagers she has been offered, and of all other propositions consequent to the full application of her computational resources, and

(8) A believes a proposition, p, iff that belief is supported by A's total evidence.

The present definition is meant to characterize a sort of agent whose interests, abilities, and circumstances are sufficiently fixed, so that agents of the described sort are suitably homogeneous with respect to the prescriptions that apply to them vis-a-vis the acceptance and rejection of wagers.[5]

It is now possible to characterize relevant notions of favorable and unfavorable betting quotients.

Definition 3 *s is a favorable betting quotient for α, given E, just in case for all possible A and w, if A is a normalized agent with evidence, E, and w is a wager of type $\langle \alpha, s \rangle$, and A is offered w, then A is rationally required to accept w.*[6]

Definition 4 *s is an unfavorable betting quotient for α, given E, just in case for all possible A and w, if A is a normalized agent with evidence, E, and w is a wager of type $\langle \alpha, s \rangle$, and A is offered w, then A is rationally required to reject w.*

Although the conditions for being favorable and unfavorable betting quotients are characterized as definitions, it is preferable to think of the 'definitions' as specifying necessary and sufficient conditions that hold in all possible worlds. Indeed, it is clear, for both conditions, that the proposed analysans outstrips the content of the ordinary intuitive conception of what it is for a betting quotient to be favorable or unfavorable.

[5]A similar kind of definition is proposed for a similar purpose in (Christensen, 1996) (cf. Christensen, 2001, 2004).

[6]The sort of rationality appealed to in the definition is inclusive of practical and epistemic rationality. In other words, any agent who fails to abide by the rationality requirement invoked by the condition is guilty of either practical or epistemic irrationality.

3 Postulates concerning Favorability and Unfavorability

The goal of the present section is to propose and defend a series of postulates that partially characterize the nature of favorable and unfavorable betting quotients. In the following section, the principles will be used in showing that betting quotients that are neither favorable nor unfavorable conform to the axioms of probability. I begin by listing seven postulates that are conceptually simple, and (hopefully) uncontroversial, given the conditions for favorability and unfavorability that I stipulated in the previous section.

Postulate 5 (Complete Possibilities) *For every set, E, of sentences of L, and for every ordered set of wager types, $\langle W_1, W_2, \ldots, W_n\rangle$, there exists a possible normalized agent with evidence, E, such that the set of wagers offered to A is $\langle w_1, w_2, \ldots, w_n\rangle$, where for all i, w_i is an instance of W_i.*[7]

Postulate 6 (Consistency of Prescriptions) *s is a favorable betting quotient for α, given E, only if s is not an unfavorable betting quotient for α, given E.*

Postulate 7 (Opposition) *For all r, α, and E, r is a favorable betting quotient for α, given E, just in case $1 - r$ is an unfavorable betting quotient for $\neg\alpha$, given E.*

Note that a wager of type $\langle \neg\alpha, 1 - r\rangle$ simply is the 'other side' of a wager of type $\langle \alpha, r\rangle$. So postulate 7 says that one side of a wager is favorable just in case the other is unfavorable (assuming fixed evidence E).

Postulate 8 (Sure Gain) *If acceptance of a standardized wager of type $\langle \alpha, s\rangle$ directly results in an increase in wealth in all possible situations consistent with E, then s in a favorable betting quotient for α, given E.*

Postulate 9 (Sure Loss) *If acceptance of a standardized wager of type $\langle \alpha, s\rangle$ directly results in a decrease in wealth in all possible situations consistent with E, then s is an unfavorable betting quotient for α, given E.*

Postulate 10 (Universality) *For all possible A, E, w, and W, if A is a normalized agent with evidence E, and A is offered w (a wager of type W), then if A is rationally required to accept/reject w, then for all possible A' and w', if A' is a normalized agent with evidence E, and A' is offered w' (a wager of type W), then A' is rationally required to accept/reject w'.*[8]

[7] It is not assumed that $i \neq j$ implies $W_i \neq W_j$, though it is, of course, assumed that $i \neq j$ implies $w_i \neq w_j$.

In essence, postulate 10 says that normalized agents with identical evidence are indistinguishable with respect to the wagers they are required to accept (and reject). The postulate is justified, since the characterization of normalized agents sufficiently fixes the nature and circumstances of such agents, so they cannot differ in a way that is relevant to which types of wagers they are required to accept (and reject). Note especially that condition 8 of the definition of normalized agent assures uniformity in what normalized agents with identical evidence believe. In effect, normalized agents are incredulous, and only believe propositions that are supported by their evidence.

Postulate 11 (Extensionality) *For all E, W_1, W_2, and W_3, if, in all possible situations consistent with E, the payoff for an instance of a wager of type W_1 is identical to the sum of the payoffs for an instance of a wager of type W_2 and an instance of a wager of type W_3, then for all possible A, w_1, w_2, and w_3, if A is a normalized agent with evidence E, and A is offered w_1 (a wager of type W_1), w_2 (a wager of type W_2), and w_3 (a wager of type W_3), then: if A is rationally required to accept/reject w_2 and A rationally required to accept/reject w_3, then A is rationally required to accept/reject w_1.*

Postulate 11 is a watered-down cousin of the package principle (cf. Hájek, 2008). While the package principle is dubitable, Postulate 11 is sufficiently limited as to be unassailable. The operative content of the principle tells us that if a pair of wagers w_1 and w_2 are jointly equivalent to a wager w_3, then if a normalized agent is required to accept (reject) both elements of the pair, then the agent is required to accept (reject) the single wager that is equivalent to the pair.

4 The Axioms of Probability

The point of the present section is to show that, relative to any body of evidence, E, the set of betting quotients that are neither favorable nor unfavorable conform the axioms of probability. The following definitions will

[8] The condition expressed by the postulate is meant to hold in the case where "accept" or "reject" is substituted for "accept/reject".

be used in expressing the theorems which correspond to the axioms of probability.

Definition 12 $P_E(\alpha) = \{s|s$ *is neither a favorable nor unfavorable betting quotient for* α*, given* E $\}$.

Definition 13 $P\lfloor_E(\alpha) =$ *supremum*$\{s|s$ *is a favorable betting quotient for* α*, given* E $\}$.

Definition 14 $P\lceil_E(\alpha) =$ *infimum*$\{s|s$ *is a unfavorable betting quotient for* α*, given* E $\}$.

With no further assumptions (save Postulates 5 through 11), we may prove the following theorems that correspond to the standard Kolmogorov axiomatization of probability (without countable additvity).

Theorem 15 $\forall\alpha, E : P\lfloor_E(\alpha) \geq 0$.

Theorem 16 $\forall\alpha, E : \models \alpha \Rightarrow P_E(\alpha) = \{1\}$.

Theorem 17 $\forall\alpha, \beta, E : \{\alpha\} \models \neg\beta \Rightarrow P\lfloor_E(\alpha) + P\lfloor_E(\beta) \leq P\lfloor_E(\alpha \vee \beta) \leq P\lceil_E(\alpha \vee \beta) \leq P\lceil_E(\alpha) + P\lceil_E(\beta)$.

Theorem 18 $\forall\alpha, E : P\lfloor_E(\neg\alpha) = 1 - P\lceil_E(\alpha)$.

One thing that we cannot prove, for arbitrary E and α, is that $P_E(\alpha) \neq \emptyset$. One way to incorporate this assumption is to introduce an additional postulate.

Postulate 19 (Existence) $\forall\alpha, E : P_E(\alpha) \neq \emptyset$.

Postulate 19 corresponds to an assumption that is far weaker than the sort of one that is generally made in discussions of degree of belief (namely, that rational degrees of belief are representable by either point- or interval-values). Yet while arguments can be given on behalf of the Postulate 19, I am willing to concede that it is dubitable. For one, it not obviously incoherent that there be a case where all of the betting quotients for a given proposition are either favorable or unfavorable. One simply imagines that all betting quotients up to some value, s, are favorable, and that all betting quotients greater than s are unfavorable. In any case, I will not fuss over Postulate 19, since the loss in the case where the postulate is rejected is, literally, infinitesimal. Nevertheless, assuming Postulate 19 (in addition to Postulates 5 through 11), one can prove the following:

Theorem 20 $\forall E : \exists p : p$ *is a probability function and* $\forall\alpha : p(\alpha) \in P_E(\alpha)$.

The connection between Theorems 15, 16, 17, and 18, and the axioms of probability become clearest, if we make the additional supposition, for all E and α, that there exists an r, such that $P_E(\alpha) = \{r\}$. In that case, we can recover Kolmogorov's axioms for functions P'_E, where we suppose $P'_E(\alpha) = r$ just in case $P_E(\alpha) = \{r\}$.[9]

5 Imperfect Agents

So far, we have seen, given a particular understanding of what it is for a betting quotient to be favorable or unfavorable, that betting quotients that are neither favorable nor unfavorable conform to the axioms of probability. However, given the proposed understanding of favorability and unfavorability, there appear to be cases where the rational degree of belief in a proposition diverges from corresponding rational beliefs about what betting quotients are neither favorable nor unfavorable for wagers on the proposition. For example, in the case of a mathematical statement, like Goldbach's Conjecture, it appears that a rational agent (in circumstances such as ours) will believe that there is one betting quotient for the Conjecture that is neither favorable nor unfavorable, and that that betting quotient is either *zero* or *one* (cf. Vineberg, 2001). At the same time, the agent may have a high degree of belief, r, in the Conjecture, where $r < 1$.

In order to bridge the connection between rational degrees of belief and rational beliefs about betting quotients for agent's with limited cognitive resources, I will now generalize the notions of favorability and unfavorability that were proposed in section 2. The first step toward expressing these generalizations is to enrich the language that is used to describe a normalized agent's body of primary evidence. To keep things simple, the revised definition of *normalized agent* will be expressed relative to a language L', which is formed by enriching the propositional language, L, by addition of the set of expressions of the form: $S \models \alpha$ (where α is a sentence of L, and where S is a set of sentences of L). The revised definition of *normalized agent*, then assumes that every normalized agent possesses a body of primary evidence,

[9] In the interests of space, I will not present a generalization of the notion of favorable and unfavorable betting quotients for application to conditional wagers. Such a generalization is possible, and given some reasonable postulates about conditional wagers, it is trivial to prove a theorem that corresponds to the standard definition of conditional probability.

E, which is a set of sentences of L'. We must also modify the definition of *normalized agent* (originally provided in section 2), so that it is no longer assumed that normalized agents have unlimited computational and ratiocinative resources.

Definition 21 *A is a normalized* agent with evidence, E, just in case:*

(1) A's utility function is a positive linear function of his/her final wealth state,

(2) A's means of changing his/her wealth state is limited to the direct effects of accepting standardized wagers,

(3) A has wealth sufficient to accept all of the wagers that he/she is offered,

(4) A has the opportunity to consider all of the wagers that he/she is offered before deciding which wagers to accept,

(5) A's primary evidence is E,

(6) A's accessible evidence consists of E, along with the apprehension of what wagers he/she has been offered, and the apprehension that (1) through (5) hold, and

(7) A believes a proposition, α, just in case α is included in A's accessible evidence.

It is not assumed, in general, that E is closed under logical consequences. Rather E will be understood to represent the set of propositions that are 'accessible' to A. The precise content of the present notion of accessibility is left relatively open. The general idea is that some contents implicit in an agent's evidence are accessible, while other implicit contents (conclusions that can only be reached via long derivations from accessible contents, for example) are inaccessible.

Corresponding to the definition of *normalized** agent, the conditions for s being a *favorable** or *unfavorable** betting quotient are identical to the conditions for s being a favorable or unfavorable betting quotient, save that the conditions now apply to *normalized** rather than *normalized* agents.

The postulates that were formerly used in characterizing that nature of favorable and unfavorable betting quotients must be revised before they are

fit to characterize the nature of favorable* and unfavorable* betting quotients. The contents of Postulates 5, 6, 7, and 10 are still reasonable, when expressed in terms of *favorable** and *unfavorable** betting quotients, and remain unchanged, save that they are now expressed in terms of *favorable** and *unfavorable** wagers, and the language L'. The remaining postulates are modified, as follows.

Postulate 22 (Sure Gain*) *If $s < 0$ or ($\emptyset \models \alpha \in E$ and $s < 1$), then s is a favorable* betting quotient for α, given E.*

Postulate 23 (Sure Loss*) *If $s > 1$ or ($\emptyset \models \neg\alpha \in E$ and $s > 0$), then s is an unfavorable* betting quotient for α, given E.*

Postulate 24 (Extensionality*) *For all $E, \alpha, \beta, \chi, s_\alpha, s_\beta$, and s_χ, if $\{\alpha\} \models \neg\beta \in E$ and $\alpha \vee \beta = \chi$ and $s_\alpha + s_\beta = s_\chi$, then for all possible A, w_α, w_β, and w_χ, if A is a normalized agent with evidence E, and A is offered w_α (a wager of type $\langle\alpha, s_\alpha\rangle$), w_β (a wager of type $\langle\beta, s_\beta\rangle$), and w_χ (a wager of type $\langle\chi, s_\chi\rangle$), then: if A is rationally required to accept/reject w_α and A rationally required to accept/reject w_β , then A is rationally required to accept/reject w_χ.*

Next, the functions $P\lfloor_E(\alpha), P\lceil_E(\alpha)$, and $P_E(\alpha)$ must be amended to form the functions $P^*\lfloor_E(\alpha), P^*\lceil_E(\alpha)$, and $P^*_E(\alpha)$, by substituting instances of "*favorable**" and "*unfavorable**" for all instances of "*favorable*" and "*unfavorable*". In that case, the following theorems hold (given Postulates 22, 23, and 24, along with suitably modified versions of Postulates 5, 6, 7, and 10).

Theorem 25 $\forall\alpha, E : P^*\lceil_E(\alpha) \geq 0.$

Theorem 26 $\forall\alpha, E : \emptyset \models \alpha \in E \Rightarrow P^*_E(\alpha) = \{1\}.$

Theorem 27 $\forall\alpha, \beta, E : \{\alpha\} \models \neg\beta \in E \Rightarrow P^*\lfloor_E(\alpha) + P^*\lfloor_E(\beta) \leq P^*\lfloor_E(\alpha \vee \beta) \leq P^*\lceil_E(\alpha \vee \beta) \leq P^*\lceil_E(\alpha) + P^*\lceil_E(\beta).$

Theorem 28 $\forall\alpha, E : P^*\lfloor_E(\neg\alpha) = 1 - P^*\lceil_E(\alpha).$

Theorems 25, 26, 27, and 28 correspond to Kolmogorov's axioms. The four theorems also describe some basic constraints on which betting quotients can be neither favorable* nor unfavorable*, given a body of evidence, E. These constraints have the status of conceptual truths, so that a set of beliefs about betting quotients that are inconsistent with the theorems is thereby inconsistent.

6 Conclusion

Theorems 25, 26, 27, and 28 embody a plausible view of the constraints that the axioms of probability place on rational belief. Some consequences of the theorems are as follows:

1. For all propositions, α, it is *inconsistent* to have a degree of belief R in α, if there exists an r in R and $r < 0$.

2. For all propositions, α, if it is *evident* that α is a logical truth, then if $R \neq \{1\}$, then it is *inconsistent* to have a degree of belief R in α.

3. For all propositions, α and β, if it is evident that α and β are logically inconsistent, then, for all sets R_α, R_β, and $R_{\alpha\vee\beta}$, if $inf(R_\alpha) + inf(R_\beta) > inf(R_{\alpha\vee\beta})$ or $sup(R_\alpha) + sup(R_\beta) < sup(R_{\alpha\vee\beta})$, then it is *inconsistent* to simultaneously have a degree of belief R_α in α, degree of belief R_β in β, and degree of belief $R_{\alpha\vee\beta}$ in $\alpha \vee \beta$.

I have defended cognitivism about degrees of belief, and view which deviates a little from probabilism. Call the view defended "cognitivist probabilism*." Cognitivist probabilism* is immune to a number of objections to Howson and Urbach's brand of probabilism. At the same time, the view maintains the principal advantages of cognitivist probabilism, with adjustments made to account for the proposed view's deviation from the letter of probabilism. First, the demand that one's degrees of belief conform to the axioms of probability (modulo one's ability to detect certain inferential relations between propositions) is a consequence of the demand that one have consistent beliefs. Second, we can understand the prescription that degrees of belief conform to the axioms of probability (modulo one's ability to detect certain inferential relations between propositions) independently of specifying the prescriptive connection between degrees of belief and action. Third, given the postulated equivalence between degrees of belief and corresponding full beliefs, degrees of belief are brought within the fold of epistemology and logic.[10]

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References

Christensen, D. (1996). Dutch-book arguments depragmatized: Epistemic consistency for partial believers. *The Journal of Philosophy*, *93*, 450–479.

Christensen, D. (2001). Preference based arguments for probabilism. *Philosophy of Science*, *68*, 356–376.

Christensen, D. (2004). *Putting logic in its place: Formal constraints on rational belief.* Oxford: Oxford University Press.

de Finetti, B. (1937). La prévision: Ses lois logiques, ses sources subjectives. *Annales de l'Institut Henri Poincaré*, *7*, 168. (translated as de Finetti, 1980)

de Finetti, B. (1980). Foresight. Its logical laws, its subjective sources. In J. H. E. Kyburg & H. E. Smokler (Eds.), *Studies in subjective probability.* Robert E. Krieger Publishing Company.

Hájek, A. (2008). Arguments for—or against—probabilism? *Brit. J. Phil. Sci.*, *59*, 793–819.

Howson, C., & Urbach, P. (1996). *Scientific reasoning: The bayesian approach* (2nd ed.). Chicago: Open Court.

Maher, P. (1997). Depragmatized dutch book arguments. *Philosophy of Science*, *64*, 291–305.

Ramsey, F. (1931). *Truth and probability, in foundations of mathematics and other essays.* Routledge & P. Kegan.

Schick, F. (1986). Dutch bookies and money pumps. *Journal of Philosophy*, *83*, 112–119.

Vineberg, S. (2001). The notion of consistency for partial belief. *Philosophical Studies*, *102*, 281–296.

Walley, P. (1991). *Statistical reasoning with imprecise probabilities.* Chapman and Hall.

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