Fictions at work: the real qualities of fictional quantities in Leibniz's *Specimen Dynamicum*

By

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RESUMÉ

Considérées mathématiquement, les infinitésimales de Leibniz étaient des fictions, des entités qui ont emprunté leur réalité à l'inexhaustabilité nécessaire de la division du continu. Reste que pour Leibniz toutes les entités géométriques ou mathématiques étaient dans tous les cas inactuelles, des produits de l'imagination. Cependant, dans son projet de dynamique, Leibniz a formulé une distinction entre les causes réelles et les effets imaginaires ou géométriques qui constituent les phénomènes de mouvement. En outre, il a donné, de cette cause réelle, une interprétation quantitative, mv^2, produit de la masse et de la vitesse qui sont tous deux des quantités de nature extensive. Mais par crainte de confusion entre le réel et l'imaginaire, un nombre d'interprètes ont jugé problématique que l'écart du modèle mathématique leibnizien du mouvement soit comblé par la force et la nature quantitative qui lui est attribuée. Dans ce qui suit, je soutiens que la compréhension de la force comme cause du mouvement nous oblige non seulement à tenir compte du rôle irréductible joué par les quantités mathématiques dans la nature causale de la force, mais aussi que les mathématiques déployées pour déplier ce rôle causal de la force consistent précisément dans la méthode du calcul infinitésimal. En tant que telles, bien que les quantités infinitésimales soient des fictions, elles jouent un rôle dans le rapport actuel entre la causalité fondamentale et systématique du mouvement et son effet phénoménal et extensionnel.

I. Introduction

Leibniz's *Specimen Dynamicum* presents a basic interpretive challenge. Given that this text was a synthetic sample of his dynamics project, what it prominently features is an alternative to a purely "imaginary" that is, merely geometrical or mathematical approach to corporeal motion, by posing force as the systematic foundation or cause governing the phenomenon of motion. Yet despite the meta- or infra- phenomenal status of force as the substantial and inherent reality of bodies, this very notion of force, understood as cause, plays a direct role in Leibniz's alternative mathematical account of the laws of motion starting with the estimation of *vis viva* as mv^2. The problem is most telling when Leibniz uses infinitesimal quantities in his account and measure of dead forces. Given the fictional status of infinitesimals, we encounter not only a description of force in geometrical terms but also in *fictional* geometrical terms. The *Specimen Dynamicum* not only requires us to understand how forces are to be conceived through the quantities that they engender in physical laws but also how the use of infinitesimals is part and parcel to this conception.

In what follows I argue for the necessity of these fictional infinitesimal quantities in Leibniz's account by demonstrating what it means to understand force as cause. First I briefly take up the seeming illegitimacy of using fictional infinitesimal quantities to describe the actuality of force. With the help of prominent commentators, we see that infinitesimal quantities are not any more problematic than other standard quantities in an account of corporeal motion through force. The reason is that if we isolate force as a foundation of corporeal motion with a metaphysical reality unto itself then any quantitative account would be heuristically adequate but ultimately insufficient. Secondly, given that Leibniz's own conception of force is immediately correlated with the quantity mv^2 as its measure, I argue that we cannot fully evade quantities by reductively separating the metaphysics of force from its purported mathematical measure. By showing the inadequacy of Leibniz's own measurement of force I argue that what remains is nonetheless a positive and irreducible quantitative aspect in his conception of force. Thirdly, I argue that although we might not attach a particular quantity to this measure of force, Leibniz does provide sufficient means, through the architectonic principles of the equipollence of entire effect and full cause and the equivalence of hypotheses, to understand the quantitative nature of force qua cause. This requires us to understand that the causal nature of force is not only efficient but also teleological. It is this teleological dimension that allows us to grasp why forces are irreducibly quantitative in its organization of phenomena. In turn, I argue, through the Leibnizian critique of the "disorder" or incoherency of the quantities involved in the Cartesian account of motion, that to understand the causal nature of force is precisely to understand the ordering of the quantities in phenomena. Finally, I argue that the use of infinitesimals is part and parcel of this very ordering of quantities, a consequence, through the principle of continuity, of force qua cause. Infinitesimals then, despite being fictional and imaginary, are nonetheless an essential feature of what it means for force to be causal. Hence I conclude that although geometrical and mathematical quantities, including fictional infinitesimals, are imaginary, they are nonetheless *at work* in the reality of the causal engendering of phenomena, an actual bridge between causes and effects.

II. The imaginary quality of fictional quantities

Leibniz, in the publication of the first part of the *Specimen Dynamicum* in the April 1695 issue of the *Acta Eruditorum*, adds a clause that was not in the original manuscript. Immediately after having described the *nisus* or solicitation of *vis mortua* (dead force) as *infinite parvum*, Leibniz adds the caveat that: "quanquam non ideo velim haec Entia Mathematica reapse sic reperiri in natura, sed tantum ad accuratas aestimationes abstractione animi faciendas prodesse."[1] This apologetic addition may have been aimed at assuaging his contemporaries but its tardy entrance into the text draws our curiosity. Indeed such a statement, regardless of its explicit and straight-forward manner, does nothing more than to state *that* dead forces are not *actually* infinitely small magnitudes but does not give any clue to *how* or *why* these fictional quantities are nonetheless useful in making "*accuratas aestimationes*". Of course Leibniz's apologetic qualification of the fictional status of infinitesimal magnitudes is not any different from what he says frequently and unequivocally elsewhere. As early as 1676, Leibniz had already used the notion of infinitesimals as "*quantitates fictitiae*" in a clear way.[2] Years later in the early 1700's Leibniz would respond to debates raging in the French Royal Academy concerning the infinitesimal calculus again by

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Outside of abbreviations standard to *Studia Leibnitiana*, abbreviations here follow convention:
AG = *G.W. Leibniz: Philosophical Essays*, ed. Roger Ariew and Daniel Garber (Indianapolis: Hackett Publishing, 1980).
L = *G.W. Leibniz: Philosophical Papers and Letters*, 2nd Edition, ed. Leroy Loemker (Dordrecht: Kluwer Academic Publishers, 1989).
LC = G..W. Leibniz, *The Labyrinth of the Continuum*, ed. Richard T.W. Arthur (New Haven and London: Yale University Press, 2001).

[1] *Specimen Dynamicum* hereafter abbreviated as *Specimen*. GM VI 238; AG 121.
[2] A VII 6, 537; G.W. Leibniz, *Quadrature arithmétique du cercle, de l'ellipse et de l'hyperbole*, ed. Eberhard Knobloch, trans. Marc Parmentier (Paris: Vrin, 2004), 71.

designating infinitesimals as “ideals and well-founded fictions”.[3]

Although infinitesimal quantities can be understood as fictional, this characterization does not do much to clarify their role in the *Specimen*. We see throughout the *Specimen* and in other dynamical writings that Leibniz engages deeply in the structure of his infinitesimal calculus to provide an account of the different aspects of force, their relations to motion and the relations between the quantities involved in motion in the *Specimen*. Leibniz's correlation of dead force with an infinitesimal magnitude is part of a systematic use of the infinitesimal calculus to describe extended motion. With regard to the nature of motion, Leibniz unequivocally states that,

“Porro ut aestimatio motus per temporis tractum fit ex inanitis impetibus, ita vicissim impetus ipse (etsi res momentanea) fit ex infinitis gradibus successive eidem mobili impressis, habetque elementum quoddam, quo non nisi infinities replicato nasci potest.”[4]

Leibniz had previously defined *motus* or extended motion as “change of place” and “instantaneous element of motion” as *motio*.[5] Impetus had also been defined as the momentary quantity of motion or the product of velocity and mass. Impetus is then *mv*, what the Cartesians called “quantity of motion”.[6] Leibniz explains that the quantity of a motion, understood as extended corporeal motion, is made up of the “sum over time” of infinite slices of momentaneous impetus, which are, in turn, the product of momentaneous velocity and mass.[7] Leibniz further says that the impetus themselves are the result of an infinite number of impressions or solicitations (*nisus*). In modern terms, we might say that the impetus is the momentaneous “quantity of motion” and that the *nisus*, impressions or solicitations, are higher-order differentials that integrate into impetuses.[8] This use of higher-ordered

[3] Leibniz letter to Varignon of 20 June 1702, GM IV 110.

[4] GM VI 238; AG 121.

[5] GM VI 237; AG 120.

[6] GM VI 237; AG 120.

[7] GM VI 237; AG 120.

[8] In François Duchesneau’s treatment of these passages in the *Specimen*, echoing Martial Gueroult’s treatment of the same passages in *Dynamique et Metaphysique Leibniziennes*, he ventures to give a mathematical formulation of conatus, nisus and their resulting role in dead and living force. Duchesneau argues,

Une transposition symbolique possible nous donnerait pour le *conatus* (compte tenu qu’il s’agit d’une quantité vectorielle):

Conatus = $dv = gdt$

Pour l’*impetus* réduit à la quantité de mouvement dans l’instant (=quantité de motion), par contraste avec $m|v|$ pour Descartes:

impetus= $\int_0^t gdt = mv$

pour l’*impetus* dans son effet temporal:

summation temporelle d’impetus= $m\int_0^t gtdt = m\int_0^t vdt$

pour la force morte:

vis mortua=$m\int_0^t gdt = mv$

pour la force vive:

vis viva=$m\int_0^t gtdt = m\int_0^t vdt = ms = mv^2$

Although the formalization here, especially the last line on “living force”, makes little mathematical sense, it is one of the possible ways of understanding of Leibniz’s explicit development in these passages of the *Specimen*. It is one possibility of rendering the degrees involved in integrating dead forces into the quantity of living forces. Duchesneau and Gueroult do make explicit both the mathematical intentions of Leibniz and the differentials demonstrates a deep rather than superficial commitment to the methods of the infinitesimal calculus in this account since it is not only the question of arbitrarily defined heuristic metaphors for infinitely small quantities that are at stake here. Rather Leibniz is clear that the very structure put in place to account for these different orders of magnitudes in motion is none other than that of his infinitesimal calculus developed years before.

In view of Leibniz's warning that infinitesimal quantities do not exist in nature, we could safely say that Leibniz limits his use of the infinitesimal calculus in his dynamics to an imaginary setting. We can understand Leibniz's use of imagination here through what Leibniz says in the *Specimen* when he remarks on the insufficiency of approaching corporeal motion by taking bodies as mere extended things, that is, in the abstraction of geometrical or mathematical features, "pure mathematica et imaginationi subjecta".[9] In fact this imaginary status of the infinitesimals is an essential feature of the *Specimen*. The *Specimen* constitutes one of the final and most synthetic texts in a project of articulating a dynamics, a "*Nova Scientia Dynamica*" as Leibniz called it, based on force, which roughly occupied Leibniz from around 1678 to the late 1690's.[10] The project commenced in the mid-1670's as a critique of Cartesian mechanism and evolved through successive stages of Leibniz's development of the notion of force as the cause of motion. The foundational status of force and the naming of dynamics as *special* science was first publically announced in *De primae philosophiae emendatione et de notione substantiae* published in 1694, a year before the *Specimen* but such a formulation was already expressed in a letter to Bodenhausen in 1689.[11] The central idea guiding this development was the conviction, against the Cartesians, that mere attention to the extended or geometrical features of motion is insufficient to provide a coherent theory of corporeal motion including laws of collision and conservation principles. Following an ideal of scientific knowledge of quasi-Aristotelian inspiration, Leibniz held that to know motion is to know its *cause*. The identification of the cause of motion as force then not only constituted a new science but also brought Leibniz to render force a centerpiece of his conception of substance in the 1680's and 1690's. In *De primae philosophiae emendatione* and other texts of the 1690's Leibniz is insistent on rendering *vis viva* (living force) and *vis mortua* (dead force) the two sides (form and matter) of a hylomorphic notion of substance.[12] As such, the dynamics project was one that was staked on the premise that a proper account of motion depends on an extra-geometrical register that is nonetheless capable of providing the grounds for its geometrical or extended features. This extra-geometrical or metaphysical register thus mediates the application of imaginary geometrical terms to the geometrical features of motion since the latter is itself only an imaginary thing. As such we should go a bit further here in noting that what are imaginary are not only the infinitesimal terms but any use of mathematics and geometry. Thus it seems that Leibniz is far from any danger of confounding the use of fictional infinitesimals with their hypostatization as real entities or, as he puts it, entities found in nature.

Although Leibniz's declarations concerning the fictional nature of infinitesimals here and elsewhere are unequivocal, we nonetheless face a major interpretive problem. It seems that what allows Leibniz to safely apply mathematics to phenomena is the imaginary status of both of these domains. Yet, it seems that the purely imaginary interpretation of Leibniz's use of mathematics in the dynamics would render this use inadequate. On the one hand, the use of

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difficulties and multiple confusions therein. François Duchesneau, *La dynamique de Leibniz*, Paris: Vrin, 1994, 223; Cf. Martial Gueroult, *Dynamique et metaphysique Leibniziennes* (Paris, Les Belles Lettres, 1934), 38-39. See also fn. 31 below.

[9] GM VI 241; AG 125

[10] GM VI 234; AG 118.

[11] GP IV 469.

[12] GP IV 470.

mathematics in analyzing the imaginary, that is, extended features of motion does not overstep the bounds of Leibniz's fictional use of infinitesimal quantities. We use something imaginary to describe another imaginary thing. On the other hand, this separation profoundly draws the account of motion away from its cause, force, for which seemingly only a metaphysical account can be given. Hence if the mathematical account of corporeal motion in the *Specimen* and elsewhere in the dynamics project must remain on the extensional level of imagination, separated from the causal level of force, then it seems that Leibniz has not exactly fared better than the Cartesians against whom the project was first conceived. Even if Leibniz could provide a more robust foundation of physics, the account of motion would still remain essentially imaginary, that is, geometrical and extended. At the same time if we attempt to treat forces, and not merely motion, through the infinitesimal mathematical terms provided in the *Specimen*, we seem to face the problem of granting infinitesimal magnitudes, the measure of dead force, for example, a real and actual status. This was what Leibniz was careful to explicitly deny.

A number of commentators have recently responded to this interpretive problem. D. Garber has argued that Leibniz uses mathematics to *represent* forces, but in this we should not mistake this representation as actualizing the quantities, especially infinitesimal ones, involved.[13] Garber argues that mathematical terms "represent" dynamical terms but does not offer much in the way of understanding why and how this representation is adequate to provide the link between force qua cause and the extended features of motion qua effect. F. Duchesneau, a thorough interpreter of Leibniz's dynamics who has a number of articles and a book-length treatment on the subject, offers an epistemological reading by arguing that the mathematical aspects of the *Specimen* constitute the elements of an analysis of motion (as opposed to a synthesis) which does not fully attain the completed science of a dynamics.[14] Duchesneau's position is that a completed (synthetic) science of the dynamics is one that would dispense with these quantities except heuristically and operate synthetically through essential or formal terms.[15] The use of quantities, infinitesimal and otherwise, are thus the elements for the modelization of motion which remain fictional precisely in the sense that they are useful for the analysis of phenomenon but can be dispensed with once the analysis is completed.

My argument here is roughly in agreement with Duchesneau's general interpretation but aims at critically extending this view to provide an account of the crucial role that infinitesimals play in how these quantitative terms would lead to a completed science of the dynamics. That is, we must see the role of infinitesimals not only instrumentally in the analysis of motion but how these very quantities are irreducible in understanding force as the *cause* of motion in Leibniz's dynamics project.

The danger of not clarifying this role of infinitesimals in bridging force and motion is to slip into a reductively imaginary interpretation of the role of infinitesimals. This very danger is reflected in D. Rutherford's recent commentary. Rutherford explicitly shares our interpretive problem stated above. He points out that Leibniz's argument commits us to the position that momentary forces (ie. dead force), like the infinitesimals that describe them, are mere fictions. "[I]f force is real, it cannot be an infinitesimal quantity; if it is an infinitesimal quantity, it cannot be real."[16] As such, Rutherford argues, we must reject the possibility of any

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[13] Daniel Garber, "Dead Force Infinitesimals, and the Mathematicization of Nature", in *Infinitesimal Differences*, edited by Ursula Goldenbaum and Douglas Jesseph (Berlin and New York: Walter de Gruyter, 2008), 281-306.
[14] François Duchesneau, *La dynamique de Leibniz* (Paris: Vrin, 1994); Cf. François Duchesneau, "Rule of Continuity and Infinitesimals in Leibniz's Physics", in *Infinitesimal Differences*, 235-253.
[15] Duchesneau, La dynamique de Leibniz, 260-262.
[16] Donald Rutherford, "Leibniz on Infinitesimals and the Reality of Force" in *Infinitesimal Differences*, 256; A

adequate representation of force through mathematical means and limit the use of mathematics in the dynamics only to the various configurations of extended motion. Force is then an essentially metaphysical notion, something that Leibniz describes as “praeter pure mathematica et imaginationi subjecta, collegi quaedam metaphysica solaque mente perceptibilia esse admittenda” in the *Specimen*.[17] Here, Rutherford adds, the “momentary” feature ascribed to dead forces soliciting movement does not last “for” a moment or any finite or infinitesimal amount of time but rather “at” a moment. As such, the fact that forces constitute the *cause* of motions does not mean that it constitutes the “principle” of such motions. Given the constraints that constitute the dynamics project, that is, the critique of Cartesian mechanism and the separation of a causal level of force from an imaginary level of motions, Rutherford's interpretation attempts to strike a balance between the metaphysical and scientific desiderata of the *Specimen*. In this reading even an epistemologically instrumental interpretation of infinitesimal quantities is misleading. If Leibniz is to be consistent, Rutherford argues, the mathematical terms ascribed to forces must be treated as heuristic in the loosest sense since forces can only be coherently taken as constituting the metaphysical and “internal” changes of a substance and not correlated in any strict way to the nature of motion itself.[18]

Rutherford's sequestering of the status of forces to a metaphysical level is overly reductive. We agree that Leibniz's development of the notion of force in the *Specimen* and elsewhere possess a deeply metaphysical character that must not be ignored. Indeed, in the *Specimen* Leibniz uses the distinction between primitive and derivative forces to underline the *per se* or metaphysical nature of corporeal substance, understood as constant action (primitive), against their situated motions in the phenomena limited by their collision with other bodies (derivative). The measurable sort of force could only be the derivative sort since primitive force understood as the constant and immanent action of corporeal substances is non-phenomenal and escape measurement. Yet the *Specimen* remains a text aimed at employing force to explicate motion in a way that would refute and replace competing Cartesian, Occasionalist and (Cambridge) Platonist theories. As such, Leibniz clearly states the inadequacy of providing merely a theory of primitive force as “ad generales causas pertinet, quae phaenomenis explicandis sufficere non possunt.”[19] The metaphysical import of Leibniz’s dynamics should be asserted but not at the cost of reducing forces to its exclusively metaphysical status. Rather it is precisely through forces understood as both primitive and derivative that Leibniz articulated the importance of providing a coherent foundation for a science of dynamics.

The foundational task of the dynamics was developed throughout the late 1670’s up to the 1690’s, the two decades leading up to the *Specimen*. Part and parcel of this evolution, with metaphysical as well as scientific implications, was Leibniz’s guiding notion, already active in his initial critiques of Descartes in the 1678 *De Corporum Concursu*, of a direct association of force with a quantity, what he would eventually call the conservation of *vis viva* or force as mv^2. This conservation, to be clear, is what Leibniz calls the measurement or estimation of force.[20] This is something that Leibniz adopts from his Dutch-Parisian mentor Christiaan Huygens's theory of elastic collision and a major aspect of his entry into a full-

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related interpretation, largely consonant with Rutherford’s position is that of Robert Sleigh Jr. but applied to §17 of *Discours de Métaphysique*; Cf. Robert Sleigh Jr., *Leibniz and Arnauld: a commentary on their correspondence* (New Haven: Yale University Press, 1990), 117.

[17] GM VI 241; AG 125.

[18] This understanding of causation is also consonant with Rutherford’s more systematic interpretation in *Leibniz and the Rational Order of Nature* (Cambridge: Cambridge University Press, 1995).

[19] GM VI 236; AG 119.

[20] GM VI 243; AG 127.

fledged critique of Cartesian mechanism.[21] The identification of force with a quantity mv^2 is, in short, Leibniz's path from a mere critique of mechanics into the dynamics qua special science.

The over-emphasis of the metaphysical characterization of force is dangerous because it obliges us to give up too much of what Leibniz attempted to do in the *Specimen*. The role of force in *bridging* a metaphysical and phenomenal level in the account of force qua cause and motion qua effect should be emphasized rather than set aside in favor of a metaphysical reduction. The reason why commentators like Rutherford find this reductive solution attractive is because it is difficult to see how Leibniz could use mathematical terms in the account except in an imaginary manner. In this view since we do not want to treat forces as imaginary, we must reject its correlation with (imaginary) mathematical terms. To overturn this reduction of mathematics to a pure imaginary status, we must see how Leibniz's conservation of force through the quantity mv^2 cannot be reduced to the metaphysical realm. By examining Leibniz's use of quantity in the dynamics, we find that, together with Leibniz's larger metaphysical concerns, the use of mathematics plays an irreducible role in the account of forces without thereby dissolving the actuality of force into mere imagination.

III. Complications in the measure and conservation of *vis viva*

A comprehensive account of Leibniz's use of mv^2 is too complex to enter into here and we shall focus only on its role in the *Specimen*. Yet to slightly contextualize this quantity, we simply note that Leibniz had as early as 1669 noted the controversy over conservation principles of motion occurring through the British Royal Society's *Philosophical Transactions* in 1668 wherein John Wallis, Christopher Wren and Christiaan Huygens published papers concerning the conservation of mv, $m|v|$ and mv^2.[22] Leibniz's serious engagement with the notion of mv^2 as the quantity conserved in motion is an explicit aspect of his 1678 *De Corporum Concursu*. From 1678 onward, Leibniz would return again and again to this quantity and it is no surprise that we find it explicitly in the *Specimen* as a "measure of force".[23]

What is surprising is that the guiding role played by the quantity mv^2 in the evolution of Leibniz's dynamics ends up being presented in the *Specimen* in a different way. His uses of mv^2 in many of the texts leading up to the *Specimen* are associated with the laws of collision, close to the original Huygenian context. Located in the posthumously published second part of the *Specimen*, not only does the explicit discussion of the laws of collision proceed without the explicit use of mv^2 but, in the first part, the quantity becomes attached to another example, that of the motion of a pendulum. In earlier dynamical works such as *De Corporum Concursu*, echoing the works of Edme Mariotte whom he explicitly mentions in the *Specimen*, Leibniz makes use of pendulum examples but as a case of collision between two pendulums at the base of descent.[24]

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[21] Cf. Christiaan Huygens, "The motion of colliding bodies", trans. Richard J. Blackwell, *Isis*, Vol. 68, No. 4 (Dec., 1977), 574-597.

[22] Eric J. Aiton, *Leibniz: A Biography* (Bristol and Boston: Hilger Alexander, 1985), 30; Cf. Philip Beeley, "A Philosophical Apprenticeship: Leibniz's Correspondence with the Secretary of the Royal Society, Henry Oldenburg" in *Leibniz and his Correspondents*, edited by Paul Lodge (Cambridge: Cambridge University Press, 2004) 47-73, 55.

[23] GM VI 243; AG 127.

[24] GM VI 240; AG 123; G.W. Leibniz, *La réforme de la dynamique*, ed. Michel Fichant (Paris: Vrin, 1994), 265; Edme Mariotte, *Traité de la percussion ou du chocq des corps*, Paris 1673, 8-22.

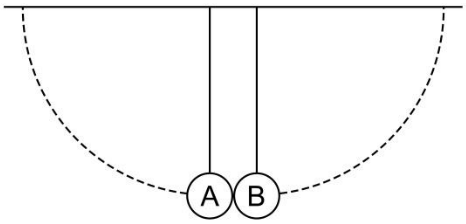

Figure 1[25]

The case in the *Specimen* is different in that it attempts to measure force through the unconstrained motion of the pendulum. This association of mv^2 with the unconstrained motion of the pendulum reveals an important aspect of Leibniz's argument in the *Specimen*. In taking a closer look, we examine the difficulties in establishing the conservation of *vis viva* through the quantity mv^2.

In the passage that immediately follows his announcement for obtaining a “measure of force”, Leibniz appeals to a series of examples involving pendulums. The first and simplest case considers two pendulums, side by side, of equal mass, A and C. Referencing Galileo's law of falling bodies, Leibniz reasons that the pendulum A with a speed of 1 unit ascends to a height of 1 foot. In turn the pendulum C with the speed of 2 units ascends to the height of 4 feet. This means that a pendulum with 2 units of speed has four times as much power, as Leibniz calls it, or *work* in modern mechanical terms, than a pendulum of the same mass moving at 1 unit of speed. Conversely this also means that the speed attained by the second pendulum C at the base of their descent will be twice that of the first pendulum A.

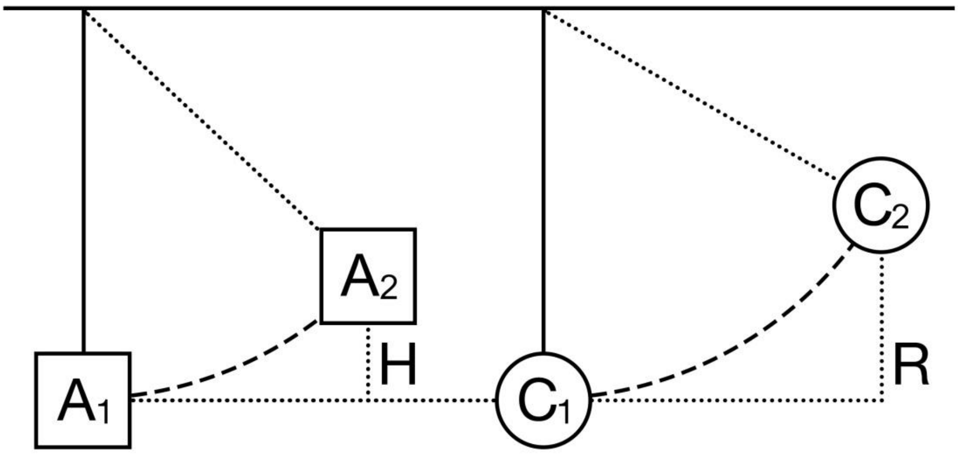

Figure 2[26]

Leibniz draws the conclusion from this demonstration that, “Eodemque modo generaliter colligitur, vires aequalium corporum esse ut quadrata celeritatum, et proinde vires corporum in universum esse in ratione composita ex corporum simplice et celeritatum duplicata.”[27] The veracity of mv^2, as opposed to $1/2mv^2$, for the conservation of energy is not our concern here. Indeed, Leibniz does not need to make explicit the gravitational constant or the factor of ½ since he constructs a suitably simple comparative case between the two bodies A and C (of equal mass). All that needs to be demonstrated is that the quadratic increase of speed (acceleration) drawn from his use of Galileo’s law of falling bodies means that the height (four times) attained by body C directly reflects the principle that the linear difference

[25] The sort of experiment Leibniz describes in *De Corporum Concursu* (section 6-2) models the motion of two pendulums A and B colliding at the base of descent; Cf. Leibniz, *La réforme de la dynamique*, 131.
[26] Figure reconstructed from AG 128.
[27] GM VI 245; AG 128.

of speed between the two masses is proportional to the quadratic difference of height and hence power [*virtus*]. Yet despite providing an adequate measure for the quantities in this sort of example, it has not attained the status of a conservation principle for *vis viva*. I follow Carolyn Iltis here is maintaining that although Leibniz has achieved a measurement for the conservation of potential and kinetic energy, it is far from clear that he has established a conservation for *vis viva* or its universal measure. [28]

Leibniz's argument here, brilliant as it is, does not provide such a conservation. Iltis' critical remarks on the earlier works like *Brevis Demonstratio Erroris Memorabilis Cartesii et Aliorum Circa Legem Naturae* and §17 of *Discours de métaphysique*, are equally applicable to the *Specimen*. The point is that although Leibniz has given a measurement of some quantity conserved in nature, namely energy-work, this measure is not generalizable to other cases and a demonstration of this generalization has not been provided here. In this example above, the conservation of *vis viva* is presumed rather than demonstrated. Iltis further remarks that in the eventual history of the conservation of energy, scientists like Mayer, Joule and Helmholtz were also similarly beholden to a "metaphysical certainty" of the general conservation of energy guiding their empirical work.[29] Duchesneau has argued against Iltis in remarking that there can be no empirical principle adequate to providing such a demonstration of the conservation principle. With respect to the conservation of *vis viva*, there are only, for Duchesneau, "regulative theoretical principles for the interpretation of facts."[30] As such, for Duchesneau, Iltis sets up an impossible criterion in criticizing Leibniz. In reading these interpretations synthetically, we can observe that what Duchsneau and Iltis agree on is that insofar there can be no empirical demonstration for a generalized conservation of *vis viva* without first supposing it, what this pendulum example demonstrates is still at a certain distance from a measure of *vis viva* as such. In this, Leibniz's concept of force should then not be strictly identified with the conservation of mv^2 but rather the very idea of some quantity conserved in the nature of motion. [31] We have further reason then not to separate the "metaphysics" of force from its "empirical" measurement in a radical separation of a non-quantitative primitive force from its quantitative or extended counterpart, derivative force. The notion of force is irreducibly quantitative even if it needs not be correlated to the specific quantity mv^2.

We will examine some aspects of the underlying methodology of measure in this example a little later. What is crucial for now is to show that these complications in establishing a measure of *vis viva* allow us to make clear the real stakes of his arguments here. Before entering into a more direct discussion of this, we should first see the immediate aims of this conservation principle. In the passages that immediately follow the simple pendulum example discussed above, Leibniz provides a more complex and critical analysis.

Leibniz moves from his simple pendulum case to critique what he takes to be the main

[28] Carolyn Iltis, "Leibniz and the Vis Viva Controversy", *Isis*, Vol. 62, No. 1 (1971), 21-35, 26.
[29] Carolyn Iltis, "Leibniz and the Vis Viva Controversy", 27.
[30] [Author's translation] Duchesneau, *La dynamique de Leibniz*, 137.
[31] It is worth noting here that Leibniz's estimation of living force as mv^2 has led to a host of problems in his lifetime and for recent commentators that these two points above cannot encapsulate. Its actual role in the continuity of the history of mechanics is also widely debated. I follow the general consensus argued by R. Westfall and I. Szabó that Leibniz could not fully justify his assertion of this quantity due to his confounding of static and kinetic principles in such crucial texts as *Brevis Demonstratio Erroris Memorabilis Cartesii et Aliorum Circa Legem Naturae* and §17 of *Discours de Métaphysique*. An additional difficulty is the instantiation of living force, understood as a metaphysical entity, in time. My position here is neutral on these debates and my argument needs only to draw from the consensus concerning the errors of the estimation of living force as mv^2 and the idea that, despite these errors, *some* quantity is conserved in motion; Cf. Richard S. Westfall, *Force in Newton's Physics: The Science of Dynamics in the Seventeenth Century* (New York: Elsevier, 1971), 299-301; Cf. Istvan Szabó, *Geschichte der mechanischen Prinzipen* (Basel: Birkhauser, 1979, 3rd Ed.), p. 70-71.

flaw, among others, of Cartesian mechanism, the alleged conservation of the "quantity of motion" *mv* or *impetus*. Here Leibniz modifies his initial pendulum example to one that more resembles, but does not coincide, with his argument, mentioned above, in *De Corporum Concursu*.[32] This time, body A will be 2 units in size and C will be 1 unit in size. Their maximum speeds at the base of descent remains the same as the previous, A will be traveling at 1 unit of speed and B will be traveling at 2 units. Now according to his construal of the Cartesian laws of motion, the "quantity of motion" (*mv*) conserved in both bodies will be the same. The quantity in A will be 2(=2·1) and the quantity in C is also 2(=1·2). In this case, Leibniz can show, drawing from the fact that the Galilean principle of the rate of fall is independent of mass, that since the speeds of the two pendulums are the same as the earlier example, they would behave in the same way. The body A would ascend to the height of 1 foot and C would ascend 4 feet. The quantities of work in this example will be different from the earlier example but assuming that the Cartesian "quantity of motion" *mv* was conserved, we could substitute one body for the other at the base of the descent of A and the start of the ascent of C since the "quantity of motion" between the two bodies would be identical at this state. Here Leibniz points to the absurdity that the "quantity of motion" conserved in body A would be capable of raising body C to a height of 4 feet. Not only is this empirically inadequate but Leibniz argues that this would result in a perpetual motion machine, an *a priori* demonstration of absurdity.[33] This is, in Leibniz's terms, an *ad absurdum* argument against the idea that the quantity conserved in motion is indeed *mv*.

Although saddled with some of the similar problems in the previous example, this case shows more clearly the stakes involved. Its explicit taking aim at the Cartesians is obvious but in this Leibniz also reveals how he sought to put a theory of motion on the right footing. The conservation of *mv* is demonstrated as false since it leads to an absurdity but this demonstration does not sufficiently render the conservation of *vis viva* in alternative quantity mv^2. We see that the argument only requires the hypothesis that some quantity is conserved in motion and with this it follows from the demonstration that the quantity conserved is not *mv*. Reading this back to the earlier example, we see that Leibniz can assert that some quantity is conserved in the motion of the pendulum between its height of ascent and its speed at the base of the descent. Yet without already positing such a quantity at the outset such a quantity conserved cannot be demonstrated except through unwarranted generalization. Yet Leibniz nonetheless provides evidence that *some* quantity is conserved in motion and that this quantity is not *mv*. This is not only a judicious critique of the Cartesians but more importantly shows the hypothetical status of Leibniz's use of mv^2 as the quantity conserved in this case of the motion of the pendulum.

What we can draw from this analysis is that Leibniz's notion of force has a quantitative aspect and cannot be reduced to its metaphysical status. Indeed, Leibniz is committed to the idea that it is through the superiority of this quantitative measure that allows us, at least in part, to deem force the "cause" of motion. Earlier, we reasoned, with Rutherford, that mathematical terms could only be adequate to the extensional features of motion and not to the causal and actual reality underlying such phenomena. We also emphasized that the non-phenomenal nature of force was important to its conception and development precisely because it forms the core of his criticism of the purely mechanical and geometrical focus of Cartesian mechanics. Yet we also see that part and parcel to this

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[32] Leibniz, *La réforme de la dynamique*, 129-137; Cf. François Duchesneau, *La dynamique de Leibniz*, 123-132.
[33] The problem of a machine capable of perpetual motion is an underlying theme in Leibniz's dynamics. The *a priori* absurdity here is due to the fact that such a mechanism would violate the law of equipollence of entire effect and full cause: effectum integrum aequipollere suae causae. Leibniz provides a direct demonstration of the *a priori* absurdity of the perpetual machine in the second demonstration of the "preliminary specimen" of his *Dynamica: De Potentia et Legibus Naturae Corporeae*, dated to 1691. GM VI 289-290. Also see fn. 36 below.

conception of force is quantitative and the reduction of force to metaphysics would result in the independence of this quantitative aspect. We noted that the conservation of the quantity mv^2 was developmentally central to the evolution of the notion of force since the mid 1670's. Despite the incompleteness of Leibniz's argument for this quantity in the *Specimen*, we see that the idea that there is some quantity conserved in motion is a different from the question of *what* this quantity is.

IV. *Stuctura systematis*, architectonics and final cause

Our general interpretive problem can now be posed differently. We have suggested that Leibniz's notion of force has an irreducible quantitative aspect but just as we are wary of reducing force to a metaphysical notion, we are just as wary of reducing it into purely extensional qualities. Moving forward, if we succeed in showing a non-phenomenal (non-empirical) use of quantity in Leibniz's notion of force, we can strike a different kind of balance between the metaphysical and scientific desiderata of this notion in the *Specimen*.

The key to unlocking this use of quantity is to look at the method behind Leibniz's measure of force. Recall that in the second pendulum example, critical of the Cartesians, Leibniz treats two pendulums, A with the mass of one unit and C with mass of two units. Under the Cartesian hypothesis, these two bodies possess the same amount of “power”, conserved as the quantity *mv*. As it occurs, body C will still attain a height of 4 feet and body A will attain 1 foot. In this way, Leibniz argues that the results are paralogistic and would, under appropriate arrangements of the pendulums, lead to a perpetual motion machine. Instead of directly arguing through the phenomenon of the work and the conservation of energy, Leibniz's *ad absurdum* argument here relies on the more general principle of what he calls the equipollence of cause and effect. In the *Specimen*, Leibniz defines this principle by remarking that, “naturam nunqua sibi viribus inaequalia substituere, sed effectum integrum semper causae plenae aequalem esse.”[34] By effect here Leibniz means the entire phenomenon of motion understood as the entire range of the extended or geometrical features (size, shape and speed) in a motion. In turn, cause is to be understood as what engenders this range of quantities in a motion. In the terms of cause and effect, Leibniz's demonstration of a paralogism is in fact the argument that the “cause” proposed by the Cartesians cannot be adequate to the entire series of effects rendered quantitatively.

Our aim here is to stick as close as we can to the *Specimen* but we cannot ignore the immense accumulation of work that led to such a synthetic text. In brief we can underline the importance that the principle of the equipollence of entire effect and full cause played in the conception and development of the entire dynamics project. [35] Already noted by Leibniz in the 1676 *Elementa philosophiae arcanae*, he designates this principle as the primary axiom notion governing physics or physical things.[36] Following commentators like Gueroult and Duchesneau, we see how the deployment of the principle allowed Leibniz to move from an a posteriori method of analyzing motion in his earlier dynamical writings to an a priori method of analysis in the matured view starting around the 1689 *Phoranomus seu de potentia et legibus naturae* and *Dynamica de potentia et legibus naturae corporeae*.[37] The problem that

[34] GM VI 245; AG 129.
[35] A VI 3, 399.
[36] A VI 3 427; “Tria axiomata primaria: *Mathematicae*, totum esse aequale omnibus partibus. *Physicae*, effectum integrum aequipollere suae causae. *Scientiae Civilis*, Mundum esse optimam Rempublicam, sive omnia in mundo fieri optimo modo.”
[37] Cf. Martial Gueroult, *Dynamique et metaphysique Leibniziennes*, 110-111; Cf. François Duchesneau, “The Theoretical Shift in the *Phoranomus* and *Dynamica de Potentia*”, *Perspectives on Science* 6: 1&2 (1998), 77-109.

Leibniz faced in his earlier dynamical work in the late 1670's was that although his arguments were sufficient to show that the Cartesian laws of nature were erroneous and already contained arguments for the superiority of the conservation of mv^2, not only was this insufficient to understand this quantity conserved as the conservation of force but it was also far from establishing force as the immanent cause of motion inherent in corporeal substances. What marks Leibniz's maturation was the use of the equipollence of cause and effect to develop a *formal* understanding of cause and effect. Developed in the *Dynamica*, the a priori determination of formal effect is understood as the *power to act* and the formal cause understood as the cause of this power, that is, force. This methodological shift accounts for why examples concerning unconstrained motions, say a pendulum, play a central role in his estimation of *vis viva*. These are examples where formal effects are shown through the exhaustion of motion attaining rest, what Leibniz calls "violent" effects since in such motions power is measured by the "consumption" of power resulting in rest. As we have seen, this form of measurement is precisely the sort that Leibniz uses in the *Specimen*. This shift also accounts for why these measurements of forces can only be the result of comparison. A collision of bodies preserves the power to act redistributed between the colliding bodies. As such force is more directly measured by comparing the exhaustion of different motions or, in modern terms, the transfer of kinetic into potential energy.

We follow Gueroult and Duchesneau in pointing out that the development of a formal treatment of the equipollence of cause and effect constitute the aim of such an *a priori* turn in the dynamics.[38] This move to a formal understanding of cause and effect is what allows Leibniz to definitively move away from the insufficiencies of his earlier empirically driven reform of what remained essentially a "Cartesian" mechanics. Leibniz could, through this turn, impute formal cause and effect to the inherent nature of bodies themselves and hence remove himself from a range of possible kinematic (Cartesian, occasionalist or otherwise mechanistic) interpretations of his reformed mechanics. In his separate correspondences to Papin in 14 April 1698 and to De Volder in 23 March/3 April 1699 (a few years after the *Specimen*), Leibniz could even make a syllogistic argument to define the capacity to act through the measurement of the exhaustion of a motion according to a ratio of the square of velocity and time.[39] Such an argument leads to issues beyond our scope here but these developments contextualize the *Specimen* insofar as we can better appreciate how the metaphysical understanding of force goes hand in hand with a shift in the manner in which Leibniz conceived of the problem of measuring force. In brief, the developmental conceptualization of force as the cause of motion coincides with a *formalization* of the quantities in motion, the means through which measurements are put into meaningful correspondence. Rather than a separation of the quantitative aspects of motion from their metaphysics, the methodological separation of phenomena and metaphysics was precisely what Leibniz had to reject and overcome in order to lend credence to the metaphysical reality of forces via its determining causal role in ordering the phenomenon of motion.

We avoid directly imputing such a developmental discussion into the *Specimen* because this text is not one relies on the *formalization* of cause and effect, something left aside for a more condensed explication. However this condensation also reveals how Leibniz understood the architectonic principles such as the formal treatment of the equipollence of cause and effect, in relation to the dynamics in general. That is, the argument that Leibniz pursues in deploying this architectonic principle is a teleological one. Hence in understanding what it means for effect to be equipollent to cause in the *Specimen*, we must emphasize that

[38] Cf. Martial Gueroult, *Dynamique et metaphysique Leibniziennes*, 131-137; Cf. François Duchesneau, "The Theoretical Shift in the *Phoranomus* and *Dynamica de Potentia*", 88.
[39] Leibniz, Letter to Papin 14 April 1689, LBr 714, fol. 136v; Letter to De Volder, 23 March/3 April 1699, GP II 173; Cf. François Duchesneau, "The Theoretical Shift in the *Phoranomus* and *Dynamica de Potentia*", 103.

cause is, at least in his argument here, to be understood teleologically.

In the *Specimen* and other texts of the dynamics project, Leibniz emphasizes a double sense of causation in understanding motion. The world is governed through two interpenetrating but non-conflicting kingdoms, the kingdom of power, that of efficient causality and the kingdom of wisdom, that of final causality. Final causality is not however solely concerned with the determination of normative values, as one might imagine, separating a realm of facts from one of norms. Final causes, as Leibniz argues, have scientific importance and do not reduce metaphysically. After all, the idea that from the wisdom and providence of God the creator follows the best of all possible worlds is a notion that, for Leibniz, plays an ampliative role in many domains. In this vein, Leibniz argues in the 1696 *Tentamen Anagogicum*, around the same period as the *Specimen*, that the inadequacy of geometry for demonstrating the laws of motion is due to the fact that it can only rely on deductive necessity whereas the demonstrations of actuality require appeal to the degree of harmony or perfection in the principles, that is, as Leibniz calls them, « des principes plus sublimes, qui marquent la sagesse de l'auteur dans l'ordre et dans la perfection de l'ouvrage. »[40] In his separation of the governing principles of the two "kingdoms" in this text, this principle of harmony and perfection, or final causality, Leibniz argues, correspond to the determination of *architectonic* principles.[41]

Architectonic principles understood as teleological causes are scientifically important precisely because geometrical reasoning is too deductively strict to provide a sufficient account of phenomena. Hence, on the one hand, laws of motion cannot be deduced geometrically and on the other hand, the geometrical features of motion are not sufficient reason for their actuality but only their possibility. The specific case Leibniz aimed to address in the *Tentamen Anagogicum* was the controversy over reflection (catoptric) and refraction (dioptric) principles in optics. In brief, the Snell-Descartes law, argued by Descartes in his 1637 *Discours de la méthode*, had caused a controversy that pitted Fermat against the Cartesians. Leibniz was well-read on this topic and also noted Huygens' and Molyneux's contributions to the controversy. The controversy was essentially about the principle that governed the motion of light in reflection on a surface and refraction through a medium. On the question of refraction, the Cartesians, sticking with a mechanistic explanation, argued that light would move through the shortest path through the medium and Fermat argued teleologically that light would travel with the fastest speed through the medium. The problem was that it turned out that both Descartes and Fermat provided calculations that were adequate to observation. From the mere description of the geometrical features of the motion of light we could not judge between the appropriate principles that governed this very motion.

Leibniz's contribution to this debate was that both these opposing positions were in some sense right but that neither could grasp the harmony between the efficient and teleological causes. Leibniz reasoned that both approaches could be harmonized not only to give a single explanation that combined efficient and teleological understanding but also give a single principle for reflection and refraction. Leaving out the details here we simply note that Leibniz argues that only an architectonic principle, what he calls the principle of the "effect of the greatest ease" or "most determined path", can allow us a maximally explanatory but unique principle for determining the actual path of light.[42]

In the direct context of the *Specimen*, Leibniz cites, for the sake of authority, that his work on the optics published in the *Acta Eruditorum* of 1682 *Unicum Opticae, Catoptricae, et Dioptricae Principium* had already been commended by Molyneux. The principle of

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[40] GP VII 272.
[41] GP VII 279.
[42] GP VII 274; L 479.

"effect of greatest ease" discussed in the *Tentamen Anagogicum* refers back to this 1682 article for which Molyneux gave a partial translation.[43] Although the problems from the domain of optics play a minor role here in the *Specimen*, the question of this convergence between efficient and final causes is significant. As part of the same discussion in the *Specimen*, Leibniz underlines his previous mistakes in understanding efficient cause. Here, Leibniz recounts the "materialism" of his youth. Leaving out explicit mention of Hobbes, who was no small influence in his youth, he notes the Democritean, Gassendian and Cartesian inspiration of his earlier work like the *Hypothesis Physica Nova*. In recounting his youthful follies, he emphasizes the mistake of thinking that, "omne incurrens suum conatuum dare excipienti seu directe obstanti qua tali."[44] Here Leibniz makes clear that the development of the concept of force allowed him to understand efficient causality without any commitment to this model of the transfer of conatus or *influxus physicus*.[45] That is to say, the efficient causality involved in collision is not the transfer of geometrical quantities from one body to another. Rather, motion is the result of the immanent action of force within a substance. The most obvious implication here is that a body at rest does not simply "receive" the conatus of the body striking it. The quantities of conatus and the effects that result from collision are part of a systematic organization of forces that reflect immanent rather than external causes. This is what Leibniz means when he argues in the *Specimen* that the concept of force is meant to provide a "*structura systematis*" that allows us to avoid "quae per se ex nudis motus legibus a pura Geometria repetitis consequerentur."[46] As such this systematic structure is one where the usual sense of an "efficient cause" is understood as metaphysically and architectonically convergent or harmonious with teleological causation.

What the development of force, through a *stuctura systematis*, adds to the account of motion, regardless of whether we are considering the collision of bodies or the path of light is precisely that efficient cause is to be understood as part and parcel with teleological cause. Efficient cause cannot be understood by itself since it would have to rely on the inadequate resources of geometry. Force qua cause allows us to understand how cause engenders effects qua motions in a manner "*structura systematis*". Efficient cause is not then a question of transferring quantities such as energy from one body to another. Efficient cause allows us to calculate motion and predict its behavior through principles and quantities both of which are governed and ordered teleologically. As such, force is cause insofar as it is the systematic ordering of motion and motion is in turn nothing but the quantitative or geometric values expressed in the change of place. The causal nature of force is thus the systematic ordering of the quantitative values involved in motion.

A further point to draw Leibnizian efficient causality away from intuitive notions of the "transfer" or "influx" of a quantity in one body to another in collision is that the force or "power" qua *potentia* involved in Leibnizian dynamics is not an Aristotelian "potential" that comes to be actualized. For Leibniz, force is in constant action, the immanent reality, qua *actio*, of corporeal substances. In the narration of his youthful errors, Leibniz underlines his ignorance of this notion. Indeed, his maturation through the evolution of the concept of force results in the constant activity of force even in a body considered to be at rest. The idea is that no body is *per se* at rest and that relative rest can only be hypothesized for the purposes of modeling motion. As such the constant action of force in a body, force per se, is present even in the body considered at rest qua resistance. In this we see that the causality of force in motion can neither be intuitively understood mechanically as the efficient "transfer" between potential energy and kinetic energy nor through Aristotle as the actualization of a potential.

[43] Dutens III 145-150.
[44] GM VI 240; AG 123.
[45] A VI 4, 1647; L 269.
[46] GM VI 241; AG 124.

The conservation of *vis viva* is thus nothing but the ordering of the quantities in motion.

We cannot pretend to have given a complete analysis of such a notoriously difficult problem as Leibnizian causality here. Regardless of its complexity, we can nonetheless establish that, for the purposes of dynamics, final causes play a determining (and harmonizing) role in efficient cause.[47] As such, Leibniz remarks in the *Specimen* that once the architectonic or "general and distant" principles have been established, "quoties postea de rerum naturalium causis efficientibus propinquis et specialibus tractatur, animabus aut Entelecheiis locum non damus, non magis quam otiosis facultatibus aut inexplicabilibus sympathiis."[48] That is, once established, the deeper principles for determining efficient causes need not be evoked in using these very causes for purposes of measurement.

As such, how does the conservation of force in motion, as mv^2 or some other quantity, measure force itself? In view of the analysis above, we can only say that this "measure" is a feature of the comparability of the systematic arrangement of phenomena. The epistemological necessity of force as Leibniz emphasizes again and again is due to the inadequacy of geometry to furnish a coherent theory of motion. The architectonic principle behind this is the equipollence of cause and effect. In strictest sense one cannot say that the speed of a pendulum at the base causes it to reach a certain height any more than the speed and mass of a certain ball causes the speed of a ball that it collides with. We understand these quantities as effects, the entire range of motions and the arrangement of the quantities involved therein. As such, the entire range of motions in a pendulum swing is the effect caused by force understood architectonically. What it means to measure force then is to provide a framework for the comparison of effects through the principle of the equipollence of cause and effect.

The quantities involved in this framework aimed at comparing effects in order to compare causes are, as we have argued, relative. Of course, within the account in the *Specimen*, such an organization is given through the conservation of mv^2. We see that this "measure" of force is given architectonically or systematically, that is, by comparison across different cases. There is no *direct* "measure" of force but rather a relative measure through comparison. Once such a "measure" is architectonically given, we can directly treat individual cases through a notion of efficient cause. Yet even in this move from the systematic to the particular cases of efficient cause, the use of quantity remains relative. In the pendulum examples, Leibniz "measures" force by taking the effects exhibited in one case in comparison with the effects exhibited by another. Sticking with the equipollence of effect and cause, the quantitative differences of effects, registered in terms of mass and velocity, imply a difference of force . The role played by the conservation of force mv^2 is thus that of providing a capacity to compare forces through comparing motions or effects. Since force qua cause is non-phenomenal only effects can be compared. In understanding the way in which this comparison occurs, it is important to bear in mind that the quantity conserved in motion is distinct from the idea that some quantity is conserved. The idea that some quantity is conserved provides the condition and framework for the comparability of effects. This means that an essential feature of force is the organization of the effects that it engenders. As such, the comparability of effects, through the quantitative means governed by the equipollence of cause and effect, is part and parcel of what it means for force to cause motion.

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[47] Here I follow Jeffery K. McDonough's work on Leibnizian natural teleology (especially concerning questions in the optics) in asserting the harmony rather than the simple parallelism in Leibniz conception of efficient and final cause. Cf. Jeffery K. McDonough, "Leibniz's Two Realms Revisited", *Nôus* (43:4) 2008: 673-696; Jeffery K. McDonough, "Leibniz and natural teleology and the law of optics", *Philosophy and Phenomenological Research* (78:3) 2009: 505-544.

[48] GM VI 242-243; AG 126.

This analysis leads us to understand the relationship between force and the quantities implied in the phenomenon of motion, qua effects, in a different way. Force is cause in the sense that it arranges the phenomenon of motion in a way that God could bring about the best of all possible worlds. As such, force is a structurally causal principle that governs the organization of phenomena. This organization is none other than the organization of the phenomenon of motion which is itself, in turn, none other than geometrical or extensional displacement (change of place). The explicit way in which this organization occurs is through the conserved quantity mv^2 but this is only hypothetical in Leibniz's understanding and a revisable feature of the relation of force and motion (cause and effect). Hence if we step back from the particular hypothesis of the conservation of mv^2, we find that what is essential in Leibniz's argumentation is the equipollence of cause and effect. This is the architectonic principle responsible for understanding why force is irreducibly quantitative but not essentially tied to particular quantities such as the conservation of mv^2.

What is then the irreducible quantitative nature of force? It is clear that the relation of force to motion is quantitative only in the comparison between effects. This comparability is however not possible without the governing role of force over its motion, as cause and effect. This governing role is however the organization of the relation between quantities, not the quantities themselves. The quantitative feature of force then is not its association with this or that quantity strictly speaking but rather the structural organization of quantities. The structuring of relative quantities is what is irreducible in Leibniz's understanding of force, a quantitative feature that is irreducible to metaphysics.

V. Equipollence, equivalence and continuity

We are now in a position to return to the problem of infinitesimal quantities in the *Specimen*. Infinitesimal quantities like any other quantity in the *Specimen* are relative to each other. We have discussed Leibniz's framework for comparing non-quantitative forces through the quantities available in motion through the equipollence of cause and effect immediately above. To push this framework of comparability a step further, we turn to a more sophisticated framework of the comparison of quantities given through the architectonic principle of the equivalence of hypotheses. Leibniz's use of the equivalence of hypotheses is most known from his correspondence with Clarke, and indirectly Newton, over the relativity of motion in the controversy over Newtonian space-time. The term itself is drawn from larger contemporaneous discussions, from the geocentric-heliocentric debates, about how the same *a posteriori* predictive models for the movement of the celestial bodies can roughly be equivalent despite the contradictory theoretical hypotheses that grounded these models. Although questions of just what kind of space-time position Leibniz held and how Leibniz understood the relation between his affirmation of the equivalence of hypotheses and astronomy are currently debated questions, we shall stick solely to his use of the principle in the *Specimen*.

In the second and posthumously published part of the *Specimen*[49], Leibniz invokes the principle of the equivalence of hypotheses in a rather minimal way. He notes, in criticism of Descartes' laws of motion that,

"aequivalentiam Hypothesium nec per corporum inter se concursus mutari, adeoque tales motuum regulas esse assignandas, ut natura motus respective maneat salva nec ex eventu post concursum divinari possit per phaenomena, ubi ante concursum fuerit quies aut determinatus motus absolutus."[50]

[49] GM VI 246-254; AG 130-138.

[50] GM VI 247; AG 131

Echoing earlier geocentric-heliocentric debates, the basic case of the equivalence of hypotheses is the case of the relation of a moving body with a stationary one. The principle states that the organization of quantities results in the same effect regardless of which body we take to be moving and which one stationary. Since Leibniz holds that no body is actually stationary, the principle can be generalized to allow that all degrees of motion are relative within a motion. As such, the quantities organized through force qua cause not only require comparability but that this very comparability is also understood as relative.

The relativity of the quantities given through the comparison of motion governed by the equivalence of hypotheses entail that the quantities in the phenomena of motion, ranging from rest to accelerated motion, are relative to each other. This relativity is usually taken by commentators to treat problems in Leibniz's relativity of space-time through the lens of Galilean invariance but in the context of the *Specimen* refer directly to questions of continuity. Here however Leibniz uses the equivalence of hypotheses in the *Specimen* to treat the imaginary nature of continuous motion. To clarify this, we start with how the quantities involved in motion can be taken as infinitesimal. Leibniz argues that infinitesimal quantities are taken to be infinitesimal relative to other quantities taken to be finite. In this, Leibniz aimed to make use of Galileo's position by improving on it. Concerning the problem of infinitesimal solicitations or *nisus* described through the use of dead force that we considered at the start of our investigation, Leibniz remarks that, "Et hoc est quod Galilaeus voluit, cum aenigmatica loquendi ratione percussionis vim infinitam dixit, scilicet si cum simplice gravitatis nisu comparetur."[51] Here Leibniz sees himself as improving on this "enigmatic" notion in Galileo by providing a more thorough-going interpretation of the comparison between *nisus*, understood as dead force, and the force of an extended motion, understood as living force. It is neither that dead force is infinitesimal and living force finite nor that dead force is finite and living force infinite. Seen through the lens of the equivalence of hypotheses, insofar as the immobility of dead force is only assumed in the equivalence of hypotheses, it is only a comparison of the effects of these causes qua force in extensional terms that these quantities (finite and infinite) arise. Of course, it is in the comparability of forces, according to the equipollence of cause and effect that requires such a distinction of between orders of infinite, finite and infinitesimal quantities. In this Leibniz obeys the "compendia ratiocinandi" that he had already formulated, through the development of his infinitesimal calculus for the relation between the finite and the infinite, and the finite and the infinitesimal.[52] Here, finite, infinite and infinitesimal are not absolute determinations but rather relative to each other. In Breger's recent explanation of this reasoning per compendia ratiocinandi, he explains that, "second-order differentials are infinitely small compared with first-order differentials […] If first-order differentials have absorbed a logical quantifier, second-order differentials have absorbed two logical quantifiers."[53] For our purposes it is through the same kind of relativity of quantitative order that Leibniz takes up the problem of accurate measurements [*accuratae aestimationes*] in motion.

It seems then that we have a satisfying solution here to how infinitesimals can remain fictional but must nonetheless be strictly and irreducibly involved in Leibniz's dynamics. Any quantity, infinite, finite or infinitesimal, is relative to each other and their measurements are only relative to the comparison of the effects of the causes of motion. This allows us to dissolve any problem regarding his use of infinitesimal quantities in the *Specimen*. From this we are in a position to understand just how such a commitment to the infinitesimal calculus becomes essential to his account of motion in the *Specimen*.

Leibniz's infinitesimal calculus provides the capacity to treat continuity through non-

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[51] GM VI 238; AG 122.
[52] Herbert Breger, "Leibniz's Calculation with Compendia" in *Infinitesimal Differences*, 185-198, 196.
[53] Breger, "Leibniz's Calculation with Compendia", 194.

continuous terms. A major intuitive starting place for Leibniz's work on the infinitesimal calculus through the quadrature of the circle is the idea of treating a circle, fictionally, as an infinitely sided polygon.[54] This comparability, a notion cautiously borrowed from Galileo's discussion of the "Aristotelian wheel" on the first day of the *Discourses and Mathematical Demonstrations Concerning the Two New Sciences*, was already given its demonstrative limits in Leibniz's 1676 work on the quadrature.[55] In the same vein, Leibniz explains to Varignon in the same 1702 letter cited above, that this comparability does not mean that a circle is a polygon. Nonetheless Leibniz argues,

« Et quoyque ces terminaisons soyent exclusives, c'est-à-dire non-comprises à la rigueur dans les varietés qu'elles bornent, neantmoins elles en ont les proprietiés, comme si elles y estoient comprises… qui prend le cercle, par exemple, pour un polygone regulier dont le nombre des costés est infini. Autrement la loy de la continuité seroit violée, c'est à dire puisqu'on passe des polygones au cercle, par un changement continuel et sans faire de saut, il faut aussi qu'il ne se fasse point de saut dans le passage des affections des polygones à celle du cercle. »[56]

The law or principle of continuity is the larger architectonic principle governing the capacity of using discrete terms to treat continuity with falling into the error of taking continuity as discrete. In this, the use of discrete terms to treat the continuous always results in some degree of error. Yet as Leibniz famously relates to De Volder in a 1706 letter, this error nonetheless "semper sit minor quavis assignabili data".[57]

This notion of comparability governed by the principle of continuity is directly salient to the problem of cause and effect in our examination of the dynamics. Recall that force is not a quantity such that we could locate within a motion. Force is something that is constantly active and systematically governs the entire range of quantities involved in a motion. As such, we have motion, something continuous and hence imaginary, and on the other hand, something that is measured (estimated) to the least error, the measure of force as equivalent to the quantity of effects (motion). This equipollence of cause and effect provide a mapping of effects to causes in a continuous way. It is obvious that if we hold the equipollence of cause and effect, the slightest change in effect implies a proportional change in cause. Since changes in motion, measured in terms of extension and time, are measured in continuous mathematical terms, they imply that the forces equipollent to these effects are also to be continuous. Yet the measurement of forces cannot be reduced to their particular effects and the continuity in particular motions does not directly result from the causal nature of forces. Hence, the continuity of *a* motion is not what is invoked here. A differential comparison of forces can only occur across different motions and thus the difference between forces at work in different motions can only be registered through a comparison of effects across different cases. To grasp this systematic organization of the continuous quantities of motion and their relation to non-continuous forces, we should momentarily step out of the *Specimen* to look at another important work within the dynamics project. In looking at this example, we not only reiterate Leibniz's commitment to the method of the infinitesimal calculus in the dynamics but clarify its necessary role in understanding what constitutes force as cause.

VI. The principle of continuity and the structure of quantities

[54] A VII 6, 527; Leibniz, *Quadrature arithmétique du cercle, de l'ellipse et de l'hyperbole*, 47.
[55] "Libenter hanc contemplationem persecutus sum, quia specimen exhibet cautionis circa ratiocinia de infinito, et methodum indivisibilium, ostenditque non semper ex partium finitarum perpetuo abscissarum proprietate quadam ad totius infiniti spatii proprietatem posse prosiliri." A VII 6, 583; Leibniz, *Quadrature arithmétique du cercle, de l'ellipse et de l'hyperbole*, 176; Cf. Eberhard Knobloch, "Galileo and Leibniz: different approaches to infinity" in *Archive of the History of the Exact Sciences* 54 (1999): 87-99.
[56] GM IV, 106; L 546.
[57] GP II, 282-283; AG 186.

Up to this point, we have seen why the causal nature of force cannot be divorced from its quantitative aspect, its organization of the quantities registered through the comparison of the phenomena of motion. We have argued that this comparability of effects and the relative nature of quantity within the dynamics can be distinguished from the particular quantities used by Leibniz. In this, the particular way in which Leibniz used the infinitesimal calculus to describe, for example, dead and living force, thus also seems on par with other hypothetical uses of quantities. We have thus argued that the use of infinitesimal quantities in the dynamics are relative notions and employed fictionally in such a way that reflects the capacity of the infinitesimal calculus to compare discrete and continuous things. What we have yet to argue is the necessity of such a quantitative structure, despite the fictionality of such quantities themselves, within the dynamics. This necessity is important to clarify because the assumption of commentators like Rutherford is that since forces get reduced to metaphysics, any mathematics could heuristically play the role of representing motion. As such Rutherford, appealing to Levey, argues for the possibility of "modeling the structure of matter/force through some form of discrete mathematics."[58]

To show the necessary role played by infinitesimals in the dynamics, we turn to an example that Leibniz only eludes to in *Specimen* but which constitutes a milestone within his development of the Dynamics project. This example is from Leibniz's 1692 *Animadversiones in partem generalem Principiorum Cartesianorum*, a lengthy study and close critical reading of Descartes' *Principia philosophiae*.[59] In his commentary on the 53rd article of the 2nd part of the *Principia*, Leibniz provides a refutation of Descartes' laws of motion, outlined in Descartes' 52nd article according to a peculiar criterion.[60] Leibniz considers, as Descartes does, the situation of two bodies of equal mass and equal speeds striking each other as a case of rectilinear elastic collision. To demonstrate his difference with Descartes and show the latter's errors, he showed what happens when we reiterate this scenario with different velocity values assigned to the second body.[61]

We can first imagine what will result from the collision of two bodies, b and c, with equal mass and opposing velocities (±4 m/s). From Descartes' first rule, the bodies, b (-4m/s) and c (+4m/s), will simply exchange velocities after collision, rebounding each other in a perfect elastic collision. But what happens when the speeds are different? Imagine then the body b as having the value of (-4m/s) as before and then a second body c as having the values of (3m/s). According to Descartes' analysis of such a scenario according to his third and seventh rule, the resulting velocities will be the same for both bodies since the more rapid body will carry the other off. The resulting velocities will result from first taking the half of the difference of the speeds and then subtracting the speed of the more rapid and adding to the speed of the less rapid. In such a case if b is traveling at velocity -4 (more rapid) and c traveling at velocity 3 (less rapid), then the half of the difference ((4-3)/2=)0.5 is the quantity that will be subtracted from the speed of b resulting in a final velocity of ((-4+0.5)=)-3.5. In turn, this same quantity will be added to the speed of body c and it will achieve a final velocity of ((-3-0.5)=)-3.5. For clarity's sake, we take another case where b is traveling at velocity -4 and c at -3, moving in the same direction, the more rapid body b will at some

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[58] Rutherford, "Leibniz on Infinitesimals and the Reality of Force", 272; Cf. Samuel Levey, "Leibniz on Mathematics and the Actual Infinite Division of Matter", *The Philosophical Review*, Vol. 107, No. 1 (Jan. 1998), 49-96.
[59] GP IV 350-392.
[60] GP IV 381-384.
[61] My interpretation here is greatly helped and informed by Larry Jorgensen, "The Principle of Continuity and Leibniz's Theory of Consciousness", Journal of the History of Philosophy, Vol. 47, No. 2 (April 2009), pp. 223-248, 230-232.

point also impact c and carry c off. The quantity in exchange between this different configuration would be ((4-3)/2)=)0.5 and this would result in the first body b having the resulting speed at (-4+(0.5)=)-3.5 and the body c moving at (-3-(0.5)=)-3.5. The resulting velocity will, like the previous example, be -3.5 for both bodies.

Leibniz understood as we do, retrospectively, that Descartes' rules are contrary to phenomena. Using the results of Huygen's theory of elastic collision, Leibniz argues that bodies of equal mass will simply exchange their velocities after collision regardless of their differences in speed. But here Leibniz deploys a kind of *reductio ad absurdum* argument.

Leibniz argues by first demonstrating the Cartesian laws of motion. Assuming both elastic collision and that the velocity of body b remains constant at -4, we reiterate the collisions by varying the speed of body c. As such, c will take on the values of -4, -3, -1, 0, 1, etc. We first note that when both bodies are initially traveling at -4, there is no collision. We note that in the case of body b at -4 and body c at 0, Descartes holds that the bodies rebound and this collision is not governed by the same law as that of the collision of a faster and slower body since one body is at rest.

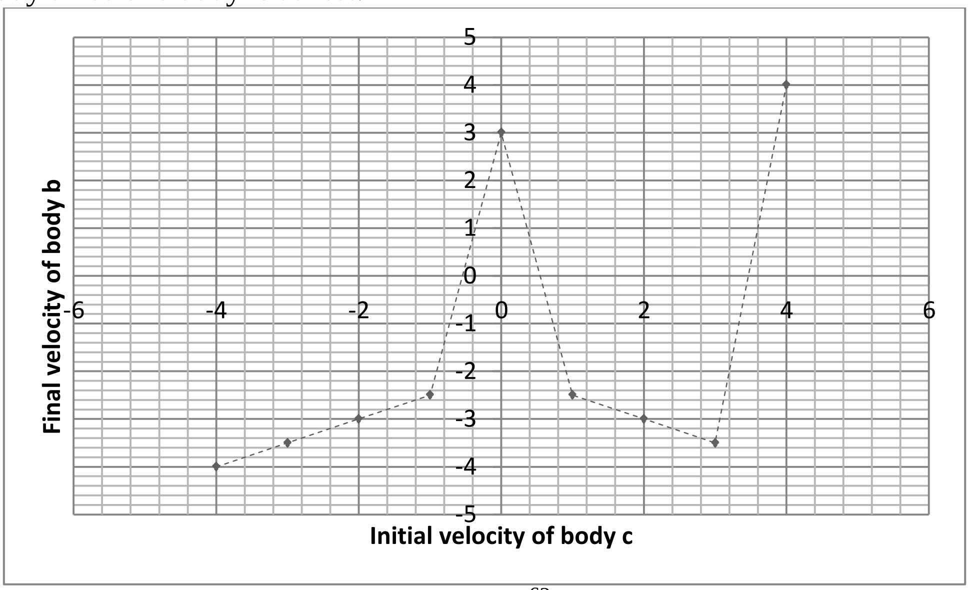

Figure 3[62]

As shown in the graph above, Leibniz argues that Descartes' laws of collision leads to significant gaps especially around the case of the body c at rest and around the case of the equal speed of b and c. Indeed, we know that the major error of Descartes that Leibniz wished to point out was the former's hypothesis that a faster body, regardless of how small this additional speed was, could carry the other body off and that the two bodies would then travel at half the difference of their quantity. This is the reason for the major gap in the correlation of the variation of the initial velocities of c (since the initial velocity of b remains constant throughout the cases) with the final velocity of b after collision.

In Leibniz's alternative to Descartes, employing Huygens' laws of elastic collision, bodies of equal mass simply exchange velocities after collision. As such, if the initial velocity of b is -4 and c is 3, the final velocity after collision would result in b traveling at 3 and c at -4. The resulting graph, using the same reiteration of cases as above, charts out a continuous line. Every variation in the initial velocity of c produces the same value for the final velocity of b.

62 The table and graph above are reconstructions of the Leibniz's argument and figure in this section. GP IV 382.

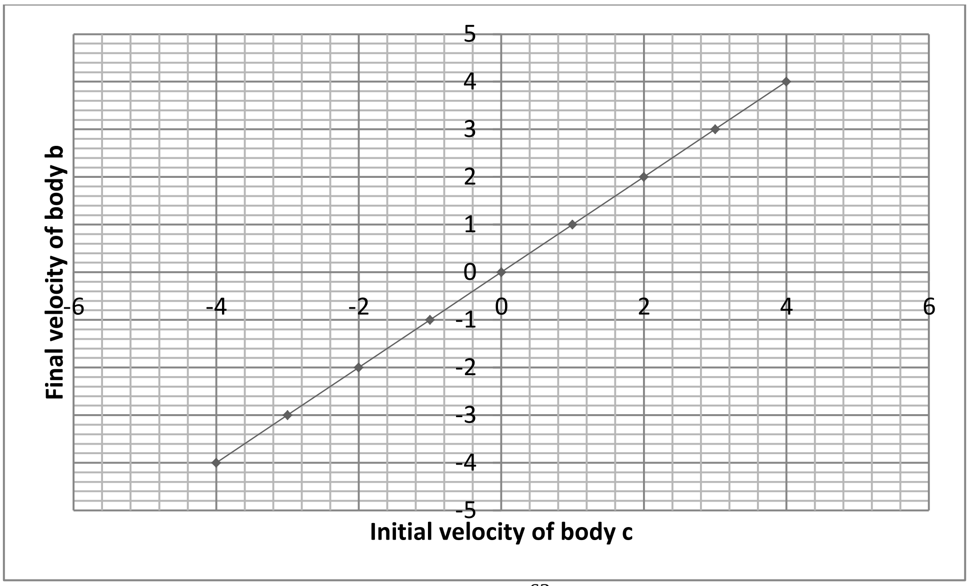

Figure 4[63]

The results here are also more apt than Descartes' in the description of phenomena. Yet this empirical inadequacy is not what Leibniz emphasizes here. Leibniz argues instead that the graph of Descartes' laws of collision presents a *delineatio monstrosa* while the graph of his position presents a *delineatio concinna*.[64] This argument has the sort of teleological import consonant with his argument concerning dioptrics and catoptrics. Leibniz treats the "monstrous" character of the plotting of Descartes' result as a sort of *demonstratio ad absurdum*. In turn Leibniz's continuous graph is more harmonious than Descartes' monstrous one. We might perhaps call this a *demonstratio ad monstrosum*.

There were a few explicit conclusions that Leibniz wished to draw from this demonstration. Two of these points, the inadequacy of Descartes' collision principles to phenomena and that corporeal collision show bodies to be more solid than liquid are explicitly emphasized by Leibniz after this criticism of Descartes. But both of these points could be argued without this *demonstratio ad monstrosum*. It was simply sufficient to demonstrate the inadequacy of Descartes' law of motion from its inaccurate empirical predictions in order to move toward the different measurements concerning the solidity bodies and elasticity of collision. In adding this layer of explanation involving the comparison between the Descartes' laws and his Huygenian alternative, Leibniz wished to make a different epistemological point. There is a higher principle that governs the quantities manifested at the level of motion. The architectonic principle of continuity is represented in his more "continuous" graph than the leaps exhibited in Descartes' graph. But this is not the usual sense of continuity. When the problem of "*saltus*" in nature is raised in works like *Pacidius Philalethi*, it concerns the question of leaps in a particular extended motion.[65] Yet here we encounter continuity in the sense of a distribution, one that is perhaps more consonant with the generation of animal species evoked in the *New Essays on Human Understanding* than, say, the path of a certain extended motion. [66]

Indeed, n*atura non saltum facit*. A full account of the principle of continuity at work in this example will take us far afield but we should underline how this example reveals Leibniz's mature thinking about the essential role of continuity in physical phenomena. The principle of continuity is argued as an architectonic principle operative in taking a whole

[63] Like the table and graph above, this is my reconstruction of the figure in this section. GP IV 382.
[64] GP IV 382-383.
[65] A VI 3, 560; LC 196-197
[66] A VI 6, 473.

series of quantities that reveal the systematically ordering of phenomena. The variation of the values of body c in the example is the means by which this continuity can be systematically represented. In turn, discontinuity registered on the level of quantities constitutes then, for Leibniz, an error. The nature of this error is first argued on the level of principles and then only subsequently a misrepresentation of phenomena in the usual sense. As such, the principle of continuity is at work in the demonstration of what constitutes a coherent phenomenon by taking continuity as the means by which to determine the cause of an entire range of the quantities distributed in motion.

This use of the principle of continuity is part and parcel of the principle of equipollence of cause and effect. Causality is here understood in the same way as we have analyzed above. Cause is understood through the organization and comparability of a range of quantities in motion and not simply as efficient causality understood as the relation of moments within a motion. However continuity allows us to understand how the equipollence operates quantitatively. Any variation of the quantity of body c is continuously correlated with the variation of the quantity of body b after collision. Continuity hence provides the means to render effects, that is, motions, comparable precisely by creating the framework where the distributed quantities reflect this equipollence of cause and effect. At the same time, this same principle of continuity is also part and parcel with the principle of the equivalence of hypotheses in the narrow sense invoked above. It is continuity that allows us to relativize the set of quantities involved in motion. All the quantities in the example above can be modified without changing the result: the final graph demonstrating the Leibnizian result would still be a continuous line albeit with a different slope.

This example of how Leibniz employs quantities in his dynamics reaffirms their fictional status. Quantitative aspects of motion are only relative to each other and our interest in them, for the purposes of the dynamics, is aimed at demonstrating, through their comparison, the principles of the ordering of these quantities. Continuity then is a principle responsible for the organization of phenomenon. When continuity of the phenomenon is demonstrated to be lacking, as in the case of his refutation of Descartes, something of a *reductio ad monstrosum* is revealed. As such, the principle of continuity applies regardless of the particular quantities measured or estimated, it provides, like the principle of the equipollence of cause and effect and the principle of the equivalence of hypotheses, the general architectonic principles of the dynamics. This example demonstrates not only that the relation between cause and effect in motion relies on the comparability of the quantities registered as effects but that this comparability or this role played by force qua cause in the ordering of motion qua effects, is governed by the principle of continuity.

The concept of continuity at work here casts a different light on why infinitesimals are used in the *Specimen*. We have already established that the infinitesimal, finite and infinite quantities involved in motion are not to be taken *per se* but are relative to each other. By examining the principle of continuity, we demonstrate why the use of these orders of quantities is necessary in his dynamics. The ordering and hence comparability of quantities in motion imply continuity and with this Leibniz needs to draw from the resources of his infinitesimal calculus. This of course does not imply the hypostasization of these fictional quantities any more than it implies the hypostasization of any quantity whatsoever. This use of quantity, finite, infinite or infinitesimal remains within the realm of the comparison of the motive effects of force. Yet what remains is that this feature of continuity is a part of the organization of quantities and hence an irreducible causal feature of the relation between force and its effects.

Although it is easy for Leibniz to step back from his use of infinite and infinitesimal quantities by the addition of a caveat of their fictionality, the reason why these sorts of quantities are part and parcel to the account in the dynamics is not reducible to their fictionality. In order to understand how force causes motion, this compatibility of forces through phenomena, or the quantities of motion, relies on the continuity of these quantities. That is, the ordering of quantities such that they form a coherent motion requires the principle of continuity. Logically then insofar as the causal nature of force is the ordering of phenomenon and this ordering requires the principle of continuity, we must conclude that the principle of continuity is an irreducible feature of what constitutes force as cause. Hence although particular quantities can be abstracted from the account of force, the causal nature of force necessitates their quantitative expression through architectonic principles like the principle of continuity along with the principle of equipollence of cause and effect and the principle of the equivalence of hypotheses. This necessity follows structurally from what it means for force to be causal, the structural determination of the relation of quantities even when particular quantities are abstracted away.

This argument leads us to assert that the necessity of the methods of the infinitesimal calculus in the dynamics as a necessary feature of the architectonic principles that governs it. What is implied here is nonetheless that the particular uses of infinitesimal, finite and infinite terms in measurement refer to fictional entities. Yet, just as particular quantities such as mv^2 can be set aside or rejected without rejecting the idea that some quantity is conserved in motion to preserve the comparability between the relative quantities in motion, we can maintain that specific infinitesimal quantities can be set aside or rejected without rejecting the infinitesimal structure necessary for understanding how forces cause motion. This is a consequence of understanding that the way in which forces cause motions is indirect in the sense that forces are not themselves quantitative.

Forces cause motion through a number of principles which allow them to be comparable through their quantitative effects in motion. There is thus something irreducibly quantitative about forces. Insofar as the causal nature of force is to be found in the ordering of the phenomena of motion, this very ordering is essentially structured through the mathematics of continuity opened up by the infinitesimal calculus that Leibniz had developed years before.

VII. Conclusion

What our argument above does not contest is the rejection that forces can be directly correlated with quantities of either the infinitesimal, finite or infinite sort. In this, not only do we reaffirm this partial agreement with commentators like Rutherford, but give additional reasons for this fundamental agreement. Yet our argument here also shows that this rejection of real infinitesimal quantities must also entail the rejection of a mere reduction of forces to a metaphysical level. Although Leibniz used forces to articulate his ideas about hylomorphic substance in an explicit and central way throughout the 1690’s, the very notion was developed to provide a causal explanation for the realm of physical phenomenon. This very causal nature of force is one that provides consistency and order to physical phenomena. Although counter-intuitive in certain respects, the causal nature of force engenders its effects systematically through the harmony, or convergence, of teleological and efficient causation. As we have shown, the way in which non-geometric or non-extended forces could cause geometric and extended motions can only be understood in this way. That is, dynamical causation is structural. Forces cause motions structurally and indirectly by the *organization* of their extended, geometrical and quantitative features.

As such the crucial disagreement with commentators like Rutherford is in our divergent understanding of the relation between quantity and force. Rutherford argues, as I have noted above, that if forces cannot be represented by a quantity then these forces must be reduced radically to the metaphysical level. This is due to the understanding that since forces are real and motions (qua geometrical) imaginary, a separation must be made in order that, unlike the Cartesian position, forces are not allowed to slip into the geometrical realm. Of course Rutherford's position also entails that since forces are not essentially tied to the quantities of motion, understood as mere imagination, alternative ways of representing these quantities such as with the exclusive use of discrete entities can also be plausible. We have argued that there is another way to understand the role of quantity in Leibniz's treatment of forces. Although Leibniz appears to correlate infinitesimal quantities with moments of (dead) force in the *Specimen*, if we understand that these *measurements*, as Leibniz calls them, are only the results of an interpretation of quantities made structurally or systematically, we find that quantities play an actual and not merely imaginary role in articulating the causal nature of force. As such forces are not themselves quantitative but play the crucial role of providing the structure of these very quantitative features. When all imagination qua quantities are abstracted away, we see that systematic or architectonic features of the relation of force qua cause and motion qua effect entail explicit architectonic principles, like the principle of continuity, through which this causation occurs. In short, Leibniz presents another way of understanding the role of quantity. With the emergence of the concept of force, something that kept Leibniz busy for two long decades, the conceptualization of the geometrical features of motion supersede the merely imaginary mechanistic picture. What arises is the use of quantities in expressing just how causality is to be identified and harmonically synthesized through architectonic principles.

The post-Galilean project of mathematizing nature would have been futile if the aim was merely to make a mere correspondence between mathematics and nature. In this, Leibniz was not the only one in his time to understand that any account of physical motion cannot succeed without being mediated by theory-laden interpretations of how the extensional features are to be made comparable. Leibniz's own solution, one that still conservatively wished to place quasi-Aristotelian causality as the terms through which such an interpretation can be given, nonetheless required a deep commitment to the structure afforded by the mathematical innovations of his infinitesimal calculus. This means, above all, that insofar as there is no direct correspondence between quantities and their underlying real causes, empirical measurements in terms of size, shape and magnitude are relevant in the account of phenomena only through a higher level of theoretization that renders these quantities systematically coherent. As such, when Leibniz measures dead force with an infinitesimal quantity, we should not take this as a correspondence between real forces and imaginary quantities. In this regard even the idea of force as being representable by quantity, infinitesimal or finite, is misleading. Yet forces are however irreducibly quantitative and this quantitative aspect requires a deep commitment to the notion of continuity made available through the infinitesimal calculus. In short, at least for Leibniz, force without its quantitative aspect is empty and quantity without its causal organization is blind. Fictions are thus not blind in the *Specimen* but *work* precisely because they are guided by the concept of force.