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Mechanism: Mathematical Laws

Tzuchien Tho
Department of Philosophy,
University of Bristol, Bristol, UK

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Introduction: The Irony of Mathematical Mechanical Laws

The subject of this entry is the emergence of mathematical laws in the development of mechanistic methodology within natural philosophy during the early modern period. The specificity of the emergence of mathematical laws in mechanics should be immediately addressed because of the ambiguity between this and the more general issue of the so-called mathematization of nature that has been used to characterize the essence of the "scientific revolution" during sixteenth and seventeenth century European history of science. To capture the problem of this ambiguity, we note that the very idea of the "laws of nature" is rooted in a very specific idea of *Deus legislator* pertaining to nature (Daston and Stolleis 2008). Those who identified with the label of "mechanical philosophy" and other close allies to the project often employed the idea and terminology of natural "laws" but few of them ventured into giving mathematical descriptions of them. In hindsight, it could be seen that, at least logically or conceptually, the mechanistic ontology of interlocking physical relations (reduced either to a plenum world of impingements and constraints, or to a world system of atoms, etc.) obviated the need for abstract "laws" since the models using mechanical parts made abstract laws redundant in the explanation of nature (Menn 1990; Garber 2016). Excluding creation itself, divine legislation is redundant to the machinic causal form of nature. Hence mechanism does not logically require the *legislation* of God. What room does this leave for mathematization?

The role of mathematical laws in mechanistic philosophy is then a conceptually ironic problem. If one conceptually reduces physics to the nature of machines, then mathematical laws of nature are just the property of machines. It may be striking to the modern reader, accustomed to the "mathematical" meaning of mechanics, that the turn to machines as models for physical motion is not already synonymous with mathematical explanation. After all, the ancient traditions of mechanics (Pseudo-Aristotle, Archimedes, etc.) employed geometry to explain machines.

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However, the classical treatment of machines through geometry made use only of proportions and did not extend beyond the adaptation of the proportions of the lever (the two factors of weight and distance to the fulcrum) to various circumstances (Dijksterhuis 1961, 497–501). Furthermore, the distinction between natural and "violent," or mechanical, motion, upheld into the seventeenth century meant that machines could not, without further mediation or interpretation, stand in for natural phenomena. The general mechanistic methodology of a reduction of natural phenomenon to machines was a *qualitative* reduction. This qualitative reduction obviously included the narrow domain for the geometrical description of machines. However, this methodology of reduction to the machine also implied a restriction on the autonomy and generality of the mathematical content of physical explanation.

Nonetheless what we have sketched here is only a conceptual irony. In practice, this apparent conflict between mechanical reduction and mathematical methods was actually more about mutually reinforcing steps toward an expansion of both mechanics and mathematics. With Stevin, Kepler, Galileo, and others, we find successive strategies explaining the properties of corporeal motion through the use of mechanical models. Mechanical relations were abstracted from the machines that provided the simplified exemplar of these properties (Gabbey 1985; Mahoney 1998; Bertoloni Meli 2006). Hence the incline plane was used to extract mathematical proportions corresponding to the motion of bodies in free fall; pendulums were used to treat the conservation of invariants between magnitude and periodicity; and levers were used to address collision properties. The mathematization of mechanics was part and parcel to the extraction of general principles of motion away from the machines that exemplified them. As principles were generalized, their mathematical descriptions also became generic to physical explanation. This process of abstraction also implied the mechanization of mathematics itself. New mathematical objects like the so-called mechanical curves (the tractrix, etc.) emerged out of this hybridization of disciplines (Bos 1988, 2001). As mechanics became a thoroughly mathematical discipline, the purity of the Euclidian mathematical world was disturbed by objects which were previously inadmissible (Knobloch 2006). This historical transformation makes sense of the irony of mathematical mechanical laws. The reduction of physics to mechanics does not reduce physics to machines per se. The mechanistic reduction provided an avenue for the expansion of the concept of mathematical laws through the abstraction of mathematical principles from the machines that served as the model of explanation.

In what follows we will address this tension between the resistance of mechanistic reduction to abstract mathematical principles and the abstraction of mathematical principles from mechanical exemplars. With the triumph of Newtonian mechanics seen as the beginning of a new chapter in physics where mechanics could be unquestionably be presumed to be unambiguously mathematical, the discipline of mechanics, as the theoretical core of physics writ large, fails to be mechanical in the traditional sense of the word. Namely, action at the distance demonstrated the autonomy of mathematical mechanics from its machinic struts. The irony of the very idea of mechanistic mathematical laws left us with an even more fundamental problem, one that we have yet to fully resolve.

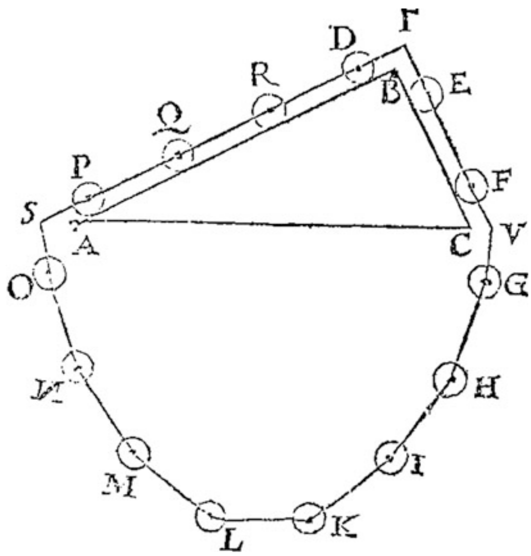
Mechanical Reduction and Geometrical Proportions

To note the reinforcement of the abstraction of mathematical principles from mechanics and the limitation of mechanics for the generalization of mathematical principles, it is valuable to consider an early and crucial waypoint in this move towards mathematical mechanics. We first examine Stevin's "wreath" and consider a few more problems in the conflict between mathematization and the reduction to machines.

Stevin (1548–1620) was Galileo's (1564–1642) contemporary and, like Galileo, is perhaps most remembered for his work on the incline plane and the classical machines of antiquity. Like Galileo, he contributed to the theory of the distribution of the action of gravity on a body in the incline plane. In Stevin's 1586 *De beghinselen der Weeghconst* (*Elements of the Art of Weighing*), he introduces a demonstration for the distribution of weighted masses on an incline plane in terms of its center of gravity. Here he places a "wreath" of equally spaced masses connected by an idealized massless and flexible cord on an incline and notices that the symmetrical hanging of the wreath beneath the triangle implies the symmetry of the distribution of the masses in the incline above (Fig. 1).

The result here is that the action of "gravity," or weight (in the pre-Newtonian context), on these different masses on the sides of unequal length, but with a common height, is the sines of the base angles. This geometrical interpretation identifies the equilibrium according to the center of gravity for the triangle. This allows his machine to be thus reducible to the lever (Mahoney 1998).

What is crucial here is that Stevin's treatment of weight (or pre-Newtonian gravity) does not provide any conceptual movement beyond the reduction to the lever. In fact, it stands as a demonstration of the law of the lever. That is, since the wreath is reducible to the lever, it demonstrates the generality of equilibrium for more complex machines. Yet in this, despite its important ingenuity, the demonstration remains nominally within the earlier tradition of the qualitative translation of a complex machine to a simpler one. Indeed, Stevin's key insight is precisely the mechanical translation of a more complex configuration to a simple and given one. For this to be the case, the hanging chain below the incline figure represents symmetry. The hanging chain pulls equally on both sides of the figure. That is, if one side pulled more "forcefully," a perpetual machine would result since the slipping of the wreath toward either side of the incline would reproduce the same distribution of weights on the triangle and thus that slipping must continue. While there is some controversy about the exact role that perpetual machine arguments have for Stevin in this context, for our purposes it only suffices to say that the rejection of perpetual motion is required for Stevin's argument to hold (Van Dyck 2017).



Mechanism: Mathematical Laws, Fig. 1 Stevin's Wreath (Stevin 1634, 448)

This abstraction of a geometrical distribution of the action of gravity on an incline plane is not unrelated to Galileo's later treatments of freefall through the incline. However, Stevin's demonstration provides a key instance of the extraction of mathematical principles from machines. It identifies the angular proportions for the downward action of bodies on inclines. This principle indeed provided an important avenue for the treatment of the abstracted notion of downward attraction (weight or gravity). However, the demonstration also remains tied to the machine itself. After all, the demonstration and its elaboration required a reduction to the given properties of the lever instead of properly independent physical principles. This is not to say that, in retrospect, such an extrapolation from the machine to abstract principles could not readily be made. However, within the context of demonstration, Stevin bridges the methods of Greek mechanics and the aspirations for an abstracted and general mathematical mechanics.

With this brief look at an aspect of Stevin's important contribution to the emergence of mathematical mechanics at the start of the early modern period, we saw a clear example of how mathematical principles emerged out of the conceptual interplay between complex and simple machines. We highlighted the potential abstraction of a general mathematical principle from a case of mechanical reduction. However, we also underlined the limitation of this methodology of a reduction to the law of the lever. What follows in this entry will treat how an expanded mechanics converges with the attempt to abstract and generalize the mathematical content caught within the reduction of complex physical phenomena to simple machines.

The Crystallization of Mathematical Mechanical Laws

We began by briefly examining Stevin's wreath. In some sense, this 1586 work provides a general starting point of early modern mechanics. We have also mentioned Stevin's near contemporary Galileo who also represented a transitional figure who had one foot in the earlier mechanical tradition while reaching toward a more abstract and generalized mathematical physics. We now turn to what we might call a point of crystallization for mathematical laws in mechanics. This is the 1669 volume of the *Philosophical Transactions* where Oldenburg (the general secretary of the Royal Society and the editor of the journal) collected responses to a call for revisiting Descartes' laws of collision presented in the second book of *Principia philosophiae*. The group of three reformers, Wallis, Wren, and Huygens, presented different approaches to either reform or replace the Cartesian laws.

Why should we trace mathematical laws in mechanics to this event? The historiography here is complex and not clearly adjudicated in the scholarly literature. As we have already mentioned, it was surely not Descartes, and hence not Wallis, Wren, or Huygens, who first posited mathematical laws. None of these thinkers was either the first to propose mathematical interpretations of mechanics. As we mentioned, the mathematical (i.e., geometrical) treatment of machines is a rich tradition that goes back at least to antiquity (Pseudo-Aristotle, Archimedes, Pappus of Alexandria, John Philoponus, etc.). Closer to the moderns, Jordanus de Nemore (thirteenth century), Buridan (and other members of the fourteenth century Paris school), the Merton school (fourteenth century), and other commentators of this tradition (Clagett 1959). When Galileo wrote a textbook *Le Meccaniche* around 1598 (first published in French translation by M. Mersenne as *Les mécaniques de Galilée* in 1634) on the subject, he treated the subject in following the basic framework of this tradition. This framework, to be explicit, was the proportional explanation (demonstration) of the behavior of the five simple machines (Galileo and Stevin include a sixth machine: the incline plane). Few of its contents, while innovative in many respects, made it into what he considered "new science," the subject of his famous 1638 dialogue (outside of the incline plane). Galileo took himself in the earlier textbook to be curating pedagogical material and updating the methods of a long tradition in which he was a contributor.

The historiographical issue here is that, at least for the generation of early modern figures like Galileo, Tartaglia, and Stevin, while mechanics had made use of mathematical methods for some time, this has not been undertaken under the aegis of a general mathematization of nature. Confined to the geometrical ratios of the parts of machines (and their movement), the later treatment of machines as isolated models of aspects of nature (the experiment) had not yet been entirely adopted. If one is to follow Koyré (1939), Galileo sought a mathematical project for the explanation of nature under a Platonizing motivation. However, his segregation of the new sciences from the tradition of mechanics is telling. Hence, we can say that in the early seventeenth century, mathematical mechanics partially converged with the mathematization of nature but, due to its own dense and rigorous tradition, cannot be identified as identical with this shared program.

The conceptual irony here has already been mentioned. In a parochial sense, machines qua artifice are precisely the opposite of natural things. This is an idea that Pseudo-Aristotle and Archimedes make clear: machines "cheat" nature for the benefit of the artificer. The "natural" motions of bodies, from Aristotelian physics, are "perverted" into unnatural or "violent" motion through mechanics. Hence if machines determine the target of demonstration and any mathematical method serves to describe and explain their behavior, the issue of natural laws is left aside by definition.

Hence in 1669, something else took place. The subsumption of mechanics, under the rubric of the mathematization of nature, has crystallized enough to be taken as a kind of assumption. Crucially, the three reformers, Wallis, Wren, and Huygens, were explicitly asked to respond to the implications of Descartes' laws of motion and the seven main rules of collision that resulted. Here, ancient mechanics was turned on its head. Instead of a mathematical description of the machine as a whole, the issue was the mathematical description of a particular principle of mechanical constraint: impact. In this context, the parochial understanding of mechanics was loosened. The mathematical rules of impact are understood as the *natural* underpinnings of an aspect of the behavior of machines. This reinterprets the relation between artifice and nature by providing the notion of machines as the fundamental elements of the aggregate and complex machine that is nature itself. The relations between natural laws and mechanical principles were also reshuffled. Descartes's three natural laws are distinct from the rules of impact-collision insofar as the rules are not the laws themselves, but the rules govern the specific cases of their application. Hence, the traditional notion of mechanics is superseded by a new mathematical theory of body-to-body impact, while the notion of the laws of nature is deepened and concretized by the examination of how it regulates body-to-body relations regardless of its classification in the traditional mechanical schema. Those "violent" motions were now casually treated as the constituent causes of nature itself.

The Reform of Descartes' Physics and the Mathematization of Mechanics

With this in mind, the 1669 interventions of the three reformers were meant to rectify obvious problems with the Cartesian collision rules. What we have outlined so far is the historiographic and conceptual reason to treat this as the focus of the emergence of mechanical mathematical laws. The reason for this 1669 intervention was due to Descartes's own failure to complete this task. Without giving a comprehensive account of how the Cartesian rules were faulty, we can simply identify two basic issues about the rules themselves and one larger conceptual issue lurking behind these rules but not explicitly addressed by the three reformers. The first basic issue is the distribution of the "quantity of motion" at work in collisions (a problem we understand as the property of elasticity within impact). The second basic issue is the universal conservation of force. The larger conceptual issue that emerges from the two is whether forces really belong to bodies or are merely laws governing the external system of motion. We shall look at these issues briefly in turn.

The Distribution of Motion in Corporeal Collision

In Descartes' own order of reasons in the *Principia*, he first formulates three laws of nature, namely, first, the proto-inertial concept that bodies will remain in their state unless acted on by another cause; second, the essential linearity of corporeal motion; and third, that in the concourse between impacting bodies, the greater force prevails (AT VIII 62–65; see Clarke 1977). The next seven rules follow from these laws by explicitly spelling out the nature of impact stated in the third law. The first rule states that bodies of equal quantities of matter (what Descartes calls size, an imprecise analogue to later concepts of mass) and possessing equal values of speed (impacting velocities) will rebound each other, deflecting each other with resultant equal speeds. This means that Cartesian force, otherwise called the "quantity of motion" is a product of the quantity of matter (i.e., mass) and the speed ($F = m \cdot \cdot v$). The force of each of the colliding bodies in the first rule is equal, and the reciprocal rebounding is an intuitive result. Although Cartesian speed is not precise enough to include direction in his measurement of speed, a problem noticed and rectified by all three reformers in their work, the general point is uncontroversial.

So far so good. However, problems start arising in the next rules. Descartes considers in the second and third rule instances where either the quantity of matter of one body is greater than the other, or the values of speed of one body is greater than the other. In both cases, the Cartesian force of one body (due to the greater value of one of the two factors on the right side of $F = m \cdot \cdot v$) will overcome the other body. Instead of finding a means of redistributing the difference between the two $F = m \cdot \cdot v$ values after collision in different velocities after rebounding, Descartes erroneously argued that the body with the greater force will determine the direction that both bodies will follow after impact. The total force of the system of two bodies will be distributed between the two bodies moving in the same direction post-collision.

The error contained in this thesis of the dominance of the stronger force to determine the direction of both bodies after impact is the main issue for all three reformers (Murray et al. 2011). In the simplest terms, the problem here is that since the quantity of motion is not calculated with direction in mind, the body with the greater quantity of motion will determine the direction of both bodies after impact. This "winner take all" determination of direction was an error immediately motivating all the reformers here. However, beyond the mere issue of whether quantity of motion is meant to be determined by mv, m|v|, or ± mv, all three reformers recognized the deeper point that the quantity of motion (with direction) is redistributed between the bodies in rebounding. The three reformers differed in terms of the principles through which this distribution in rebounding occurs. Wallis and Wren agree that the quantity of motion with direction before collision is conserved between the two bodies after collision (and redistributed between them). Huygens argues for a different measure, the quantity mv^2, conserved before and after collision. In terms of a rectilinear collision, Huygens' theory introduces a new conservation property above and beyond a mere rectification of the Cartesian rules. The two factors of quantity of matter (i.e., mass) and speed (i.e., velocity) do not play the same role in the determination of how force is redistributed after collision. Here, an additional factor of v must be calculated in the redistribution of motion after impact. This, for Huygens, replaces the mv calculation for the quantity of motion in favor for mv^2, which he saw as the better candidate for the meaning of "force" or "power".

This gives us the conservation of the rectified quantity of motion with direction supported by Wallis and Wren (where *A* and *B* are bodies in linear head-on collision, and v and v' are speeds before and after collision):

$$m_A v_A + m_B v_B = m_A v'_A + m_B v'_B$$

Huygens gives us the additional conserved quantity of "force":

$$m_A v_A^2 + m_B v_B^2 = m_A v'^2_A + m_B v'^2_B$$

The Huygensian formulation is consistent with the rectified quantity of motion but is not a trivial result. With a more basic concept of reference frame displacement, we can derive any of these three equations if we have two of them:

$$v_A - v_B = -\left(v'_A - v'_B\right).$$

This system of equations for conservation of both quantity of motion and Huygensian "force" was developed explicitly by Leibniz, a later enthusiastic inheritor of this problem (Darrigol 2014, 17).

The issue of distribution of motion in impact was not only a rectification of the errors in Descartes' rules but also highlighted fundamental problems in mechanics well into the eighteenth and nineteenth century. The problem of elasticity in collision was put on center stage. Between the three reformers only Wallis addressed the cases of inelastic collision (as when a hard ball impacts soft putty). What is the distribution of motion in such cases of hard and soft collisions? The related and more historically famous problem was the ensuing conflict over the two different conservation quantities presented by Wallis and Wren, on one side, and Huygens on the other.

The conservation mv^2 will be later taken up by Leibniz as being not only physically significant but metaphysically as well. It was the debate over this quantity, later coined vis viva by Leibniz, that will dominate a corner of eighteenth century physics with new insights promoted by the Bernoullis, Euler, du Châtelet, D'Alembert, and eventually Lagrange.[1]

What is most important about this first aspect of the three reformers' response to Descartes is the establishment of the distribution and conservation of quantities under impact as an independent *mechanical* problem. Even if Descartes had first given his collision rules under the banner of a universally conserved quantity (the quantity of motion), where since the universe as such (due to God's constant concurrence) conserves the quantity of motion, each local interaction must also conserve this quantity of motion (mv), the three reformers have provided the logical space for impact collisions as a locus of mechanical insights. From this, mechanics drills down to abstract and isolated body-to-body relations. Mathematical mechanics become neither the proportional relation of the parts of machines (even if they involve impact) nor do they reduce to a Platonic mathematical world. Insights about impact and its modalities are given its own genre to provide the basis of other universal hypotheses. Certainly here, the disputes between Wallis, Wren, and Huygens will provide fodder for larger debates about the nature of "force," understood as the conserved quantity in the universe. Yet, these debates will only draw from this domain of the inquiry about impact, rather than being implied by them. In this sense, the mathematization of mechanics is anchored to this point. That is, first, the problems of collisions are not themselves the problems of the mathematization of nature. Secondly, the problems of collisions do not reduce to the traditional mechanics of the proportions of machines. The independence of the collision problem thus highlights the unique case of the mathematical laws in mechanics itself.

[1]The *vis viva* conflict lived on into the twentieth century and receives the currently canonical treatment by Emmy Noether.

The Conservation of the Quantity of Motion

The second aspect in question here is the universal conserved quantity in motion. The origin of the debates about Cartesian force concerns not only its measurement and distribution in collision but more fundamentally its role as the conserved quantity that determines the physical closure of the created world. The term "force" should, in this context, be completely disassociated with the Newtonian-Eulerian idea that $F = ma$ (where F is force, m is mass, and a acceleration), but as a placeholder term for a universal conservation first indicated by Descartes. The very notion of a universally conserved quantity in nature is a curious one. Mechanistic philosophy does not suppose it nor do earlier theories about machines. It is, however, the crucial logical link between natural law and mathematical laws. This link was first developed by Descartes. In Descartes' view, God's constancy is reflected in the natural world through the conservation of a constant quantity of motion in the universe (Gabbey 1971; Hattab 2007). This naturally lends itself to a mechanical image (perhaps *the* mechanical image) of the lever. Motion on one side of the fulcrum is compensated by a corresponding motion on the other side of the fulcrum. The fundamental "balance" of nature is constant while there may be an infinite variation of modal shifts in the pitch of the balance itself. In terms of a model of the universe, this provides the sine qua non of the mechanical world-picture. However, this is an interpretation that would not suit Cartesianism in the strict sense. Here, while Cartesians would have no quarrel with the mechanical metaphor for the natural world and its structure, and even less disagreement about the reduction of natural causes to impact and constraint, the reason for this constancy through conservation is argued from the perspective of a Divine legislator who, through its own constancy, guarantees the constancy of nature. Conservation as a feature of the universe as a total closed and interlocking machine is only a result of the more fundamental concurrence of God on the works of nature.[2] Of course here, the fraught idea that minds can be the cause of physical change deeply upsets the "balance" of a universal conservation based on a closed circuit of changes in motion in the universe (Menn 1990; Gabbey 1985). Do the movements of bodies by minds satisfy universal conservation? We leave this inconvenience aside.

The conservation of Cartesian force, the quantity of motion, is truly a law of nature rather than a mechanical law insofar as this conservation is not an immanent feature of the machine but an imposed legislation ontologically guaranteed by the nature of a concurrent God. However, if this is the case, there would be no strong logical case for inferring the claim that local physical interactions must reflect this global conservation. Nothing prevents that what God adds to one corner of the universe can be subtracted from another. For this, local uniformity must be added to constancy. The reading of force as a conservation quantity (whether we accept the Cartesian quantity of motion or not) in isolated body-to-body impact is presumed but not explicitly argued for. Yet this is what is assumed by all three reformers. Force, even if one disagrees about its measure or fine structure, is the placeholder for the quantity that is conserved in all and any impact, building from body-to-body relations to the universe as such. Hence, while the three reformers do not deal with any of the metaphysics or theology of conservation, the implicit assumption is that global conservation arises from the uniformity of local conservation. The assumed notion is hence, without any explicit argument, that if the laws of mechanics are laws, then they are expressed through this mathematical description of conservation. The conservation of force, either following the Cartesian quantity or otherwise, becomes a mechanical *law* then, instead of a global conservation property of the universe. The Cartesian view of global conservation is left out of the discussion by the mainstream of mechanistic philosophies. It is most conspicuously absent in the Newtonian framework that faced challenge by the Leibnizian view that there is such a global conservation albeit with the measure mv^2 or vis viva.

The work of the three reformers is significant in the respect of representing a shift in how conservation is used in the discussion of impact. The search for a conserved force is transformed from a governing system of the total system of motions in creation to the generic instance of any case of impact. The logical status of conservation thus allows the shift from the mathematization of the total system of nature to the generic description of the systematic unit of mechanical interaction.

The Inherent Properties of Bodies

This shift from nature writ large to local configurations raises a pressing question about mechanical philosophy that would become a raging issue in what follows in the wake of the three reformers. Do forces reside in bodies? Again here, Descartes provides the starting point of the controversy. Since, for Descartes, bodies are nothing but their extension (i.e., size, shape, and motion), the transfer of force between bodies in impact (impetus transfer derided as *influxus physicus*) is written out of the basic theory of motion (O'Neill 1993). In this sense, the notion of "laws" of motion is a theory of the "governance" of the changes of motions between externally related bodies through physical events. Yet, Descartes also freely speaks of the "carrying" and "transfer" of forces in physical interactions (Menn 1990). Worse yet, for the dualist, it seems as if souls must "add" motion to the embodied entities acting in the physical world, breaking the mechanical closure of the world. Are forces external relations or are they properties possessed by individual bodies?

Bracketing the more difficult issue of embodiment, two views about inherence are available to us. First, the force of resistance provides an instance of inherent force.

[2] Cartesian divine concurrence is the idea that God punctually sustains the continued existence of the world. This is contrasted with strict deism where God creates the universe as a machine and allows it to unfold its enfolded properties in time without added intervention. It is also contrasted with strict occasionalism where the world is recreated punctually. Concurrence stands in the middle of a spectrum between no intervention (outside of creation) and complete ontological dependence. See Menn (1990).

In other words, if a massive body at rest resists the force of an impact, this seems to imply a force of resistance (against impact) that is irreducible to mere extensionality. Something "intensive" must reside in the body that causes this resistance. Against this, Descartes originally reasoned that any resistance by a body must be reducible to magnitude (the quantity of extension), therefore this implies that no body at rest can be moved by the impact of a moving body of lesser magnitude (AT IV 183–188). Taken as a whole, Descartes sets up an all or nothing approach to resistance. Either a resting body resists impact and remains immobile (resistance), or bodies in collision enter into an exchange of speeds (not a case of resistance) which fall under one of his seven rules for impact. Resistance is not associated with inertia or elasticity as we tend to think of them in modern terms (AT II 467). Yet, if we consider resistance to be a contributing factor of impact, the interrelation of bodies with varying degrees of mass and speeds renders resistance as a kind of activity of the body. Hence with this activity of resistance, bodies cannot be the Cartesian inert *res extensa*.

A second view is the inherence of force implied by the Huygensian method. Force can only be the invariance measured by all the shifts of the reference frame according to which we have values for all the moving bodies. This implies that a deeper structural reality governs the motion of any system of bodies through the quantity of "force" (Huygensian force mv^2). In Huygens' demonstration, this is illustrated by the consideration of a set of differing reference frames of colliding bodies understood as one system through the shared center of gravity. No exchange of motion in the system can raise this center or, what is equivalent, increase the total quantity of mv^2 in the system.

These two dimensions of the problem implied the viability of an inherence of force within a system of moving bodies. For the first issue of the "force" of resistance, while a vast number of key figures writing in the wake of this 1669 text like Leibniz saw this as an indication of the foolishness of Cartesian theory of inert bodies, later developments of the theory of inertia, where the active nature of corporeal resistance is absorbed by law-like equal and opposite reaction (as we find in Newton's third law), allowed for an ontologically neutral interpretation of resistance. The second issue, Huygens' notion of force as invariance, also received, in the later work of D'Alembert, an interpretation through a structural distribution of motion, avoiding the implication of the inherence of forces in bodies. Yet what remains pertinent is a logical connection between the system of colliding bodies understood as a machine (with a center of gravity) and the physical world as an interrelated system of aggregations of such machines. Here, the law is not an externally legislated behavior of otherwise free bodies but an explanation of the properties of a machine. More importantly, this is a machine concept abstracted from particular instances of the five simple machines (six, if one includes the incline). From this perspective, it is Huygens that fully delivers a nascent version of mechanical mathematical laws. That is, it is fully mechanical insofar as this vision of physics is predicated not on externally governing laws but on the explanation of machines. However, it is also based on a radically abstracted notion of machines, even to systems of bodies governed only by the center of gravity with no individual body-to-body interactions.

With this recognition of Huygens' important contribution, it should be underlined that this interpretation of collision laws was not fully expounded in his 1669 contribution to the *Philosophical Transactions*. Huygens has submitted only a part of his larger work on collision to Oldenburg (editor of the volume and secretary of the Royal Society). Worse yet, only a summary of this small sample was actually published. Huygens was unsatisfied with the treatment of his work and published a French version of the sample in the *Journal des sçavans* in the same year (1669). However, this text, while standing as a fuller demonstration of his theory, also did not fully display the extent of his insights. His elaborated treatise *De motu corporum ex percussione*, completed some 10 or 15 years before this event, was only published posthumously in the 1703 collection *Opuscula posthuma*. Nonetheless, his long-time mentee

Leibniz was clearly well-acquainted with his views and saw in Huygens' methodological innovations and theoretical results the grounds of a fundamentally new start to physics which he would name the dynamics based on the quantity of force as mv^2 (Duchesneau 1994; Tho 2017).

Leibniz's contribution to the new post-1669 mathematical mechanics was considerable. However, for our purposes here, it is sufficient to underline his development of the Huygensian line of reasoning about the use of the invariant mv^2 and center of gravity arguments in the development of a new kind of physics which he calls dynamics. Leibniz developed a system of metaphysical implications for this dynamics which we will not discuss here. Unfortunately, like his mentor Huygens, the core of these physical theories was not published in their lifetimes. Yet, Leibniz's outlining of the metaphysical stakes in published works like the first part of the *Specimen dynamicum* and his use of Huygensian arguments against the Cartesian faction drew closer attention to a methodology in physics that privileged the mathematical description of machine-like models of fundamental physical behavior. Instead of the behavior of individual free bodies, the developments in this line of thought took as the target of explanation a system of bodies governed by an invariant.

Conclusion: From Machines to Laws

The fundamental historical irony at the end of this trajectory from Stevin to Leibniz-Newton is that while Leibniz pursued a fundamentally mechanical (i.e., machinic) notion of physical systems, the extension of these models to the totality of physical reality required extravagant metaphysical ideas about the interconnectedness of the world as a divine machine. Against this path, the metaphysically parsimonious path taken by Newton and the Newtonians focused on grounding the relation between free bodies. The fundamental laws of Newton and their immediate corollaries explain motion from elemental and isolated free bodies.

Here irony strikes again. Universal gravitational attraction precisely lacks *mechanical* (i.e., efficient) cause. Hence with Leibniz and Newton, both inheritors of the 1669 papers, we find two opposite poles of a spectrum neither of whom satisfies the original Cartesian framing of the debate but which also do not converge to provide a final concluding picture to the meaning of mathematical mechanics at the end of the seventeenth century.

There are deeper methodological and metaphysical reasons for the divergence of the Leibnizian program, a more conservative project rooted in an expansion of ancient mechanics and Cartesian debates, and the Newtonian one, a progressive reorganization of the basis of mechanics on the notion of the interaction between free bodies. Even if Leibniz did not always appeal to his larger philosophical agenda in his scientific work, he did frequently remind his readers of the importance of its theological and metaphysical stakes. Newton, on the other hand, was always reticent to go beyond the explicit content of experimental results and mathematical demonstrations. The diametric opposition between the two thinkers is deeply striking from the basis of motivation. However, with respect to the commitment to mechanistic uniformity of explanation, Leibniz strangely becomes the champion of mechanical foundation while Newton becomes the adventurer staking a claim in the world of unseen forces.

When Leibniz criticizes Newton for the lack of a mechanical explanation for gravitation, this is made on the basis of a pre-given notion of the universe as a complete, albeit infinite, divine machine. Certainly, both Leibniz and Newton would agree that machines exist and so do aggregates of machines, acting together to produce complex phenomena. However, the obligation to provide a mechanical explanation only holds if the closed mechanical totality of the world is granted. It is on this point that Newton's universal gravitation presents an alternative. While keeping all the mechanical insights gained from predecessors and prudentially seeking mechanical explanation where possible, Newton presented the interactions of the free body as the fundamental generic element of physical interaction.

It should be worth adding here that the lack of mechanical explanation for universal gravity was consistent with the Baconian experimental method. Here we leave ample room for the description of experimental phenomena without speculation on underlying causes. Indeed, with the analogy made between gravity and magnetism, mathematical description of phenomena can be precise without being mechanically explained. Yet this is precisely where "law" stands in to hold up a mathematically precise principle (like universal gravitation) where no mechanical cause can be fully determined.[3] The notion of a mechanical mathematical law, which may be initially seen as an oxymoron, now appears to be the most delicate and efficacious way to sustain and project an unfinished but promising research program.

Newton's program and the subsequent formation of a "classical mechanics" based on the eventual incorporation of Leibniz's vis viva theories by D'Alembert and Lagrange carries out the accomplishment of a notion of mechanics that we are familiar with today. Mechanics is itself divorced from the narrower inquiry into the behavior of machines. As a universal vision, classical mechanics is also dissociated from the metaphysical speculation (made by thinkers like Leibniz) of treating the physical world through the image of an infinitely complex aggregate of machines.[4] Of course, this image was not contradicted and still remained available to thinkers from this period to the work of Mach, late into the nineteenth century. However, the very target of mechanics shifts definitively from an inquiry about the relations between parts of a connected machine to the principles behind those connections. The machine is dissolved thus into principles. Generalized, these principles are laws. Impact, first and foremost, but also resistance, elasticity, inertia, and the like were reduced to elemental laws and definitions from which machines are analyzed. The result here is a classical mechanics where machines have all but disappeared. This represents the apotheosis of the conceptual conflict between mechanical philosophy and the mathematical mechanical laws that we have been tracing throughout.

Cross-References

- ▶ Action at a Distance in Early Modern Natural Philosophy
- ▶ Body and Extension in the Scientific Revolution
- ▶ Corpuscularianism
- ▶ Dead Forces and Live Forces
- ▶ Descartes
- ▶ Descartes, Mathematics and the Science of Motion
- ▶ Descartes' Mechanical Philosophy
- ▶ Gassendi, Pierre
- ▶ Hobbes on First Philosophy and Natural Philosophy
- ▶ Hobbes on Mixed Mathematics
- ▶ Huygens, Christiaan
- ▶ Law of Action and Reaction, the
- ▶ Law of Inertia
- ▶ Locke and Mechanism
- ▶ Mechanical Philosophies
- ▶ Mechanical Philosophy: an Introduction
- ▶ Mechanical Philosophy: Mechanics
- ▶ Mechanical Work
- ▶ Mechanics and Mixed Mathematics in Early Modern Philosophy: an Introduction
- ▶ Mechanism: 18th-Century German Philosophy
- ▶ Wallis, John

[3] Another key instance of this is the behavior of light, where the mathematical principles of reflection and refraction were not in dispute but received widely different mechanical interpretations between Descartes, Fermat, Huygens, and others. See McDonough (2009).

[4] This is also accompanied by the rise of a new genre of "rational mechanics" based on a priori mathematical methods typified by Newton, Varignon, etc. This converges with the use of the term "classical mechanics" where the emphasis is on the physical properties of idealized bodies in mathematical language. See Mahoney (1998).

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