COMPUTING WITH CAUSAL THEORIES

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Formalizing commonsense knowledge for reasoning about time has long been a central issue in AI. It has been recognized that the existing formalisms do not provide satisfactory solutions to some fundamental problems, viz. the frame problem. Moreover, it has turned out that the inferences drawn do not always coincide with those one had intended when one wrote the axioms. These issues call for a well-defined formalism and useful computational utilities for reasoning about time and change. Yoav Shoham of Stanford University introduced in his 1986 Yale doctoral thesis an appealing temporal nonmonotonic logic and identified a class of theories, causal theories, which have computationally simple model-theoretic properties. This paper is a study towards building upon Shoham's work on causal theories. We concentrate on improving computational aspects of causal theories while preserving their model-theoretic properties.

Keywords: Causation, causal theories, the frame problem, modal logics, nonmonotonic logics, temporal logics.

1. INTRODUCTION

Reasoning about the commonsense notions of time and change is important in various areas of AI. There have been attempts towards formalizing common sense and various logics have been devised. It has been recognized that reasoning about change requires temporal and nonmonotonic reasoning devices. In this direction, Shoham introduced, in his doctoral dissertation,[1,2] a temporal nonmonotonic logic that he called the logic of Chronological Ignorance (CI). Shoham also identified a class of theories, causal theories, which have computationally simple model-theoretic properties in CI. Other contributions of Shoham to temporal reasoning and nonmonotonic reasoning include Refs. 3–11; as witnessed by Refs. 12–25 these generated considerable interest.

In this paper, after an examination of the preliminary notions of CI and causal theories, it is shown that computing with causal theories is time-dependent. This contradicts with the way human beings reason about consequences of actions and come to conclusions in everyday life. To remove this deficiency, a new class of causal theories containing axiom schemata is introduced and computational aspects of causal theories in this class are investigated. Furthermore, an approach to remove one of the technical limitations imposed by Shoham on causal theories is proposed.

In Sec. 2, our notation and terminology are presented. Section 3 introduces the Yale Shooting Problem (YSP). The weakness of causal theories in representing scenarios similar to YSP and their inefficiency in computing the consequences of these theories are demonstrated. To remove those deficiencies, a new class of causal theories, YSP-like causal theories with axiom schemata, is proposed. Here it should be remarked that the

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essential property of YSP-like causal theories will not be that they allow for axiom schemata (which is only a technical point) but that they have to contain persistence axioms.

Shoham did not permit simultaneous occurrence of cause and effect in his account of causation: he restricted causal theories to have causes strictly precede their effects in time. In Sec. 4, various related ideas from philosophy are mentioned. A modified definition of causal theories that permits simultaneity is given and an algorithm to compute the consequences of such theories is proposed. Section 5 contains the concluding remarks. The Appendix includes the omitted proofs.

2. ESSENTIAL NOTIONS AND TERMINOLOGY

2.1. Notational Conventions

Unless otherwise stated, we follow Shoham's terminology and definitions verbatim. Lower-case letters such as p and p_1 denote propositional symbols; t is used to express a time point variable, and a time point constant when indexed (as in t_1 or t_a).

The symbols $\neg$, $\wedge$, $\supset$, $\equiv$ are used as the standard logical connectives. $\forall$ denotes the universal quantifier. $\Box$ and $\Diamond$ are modal operators described in the following section. ■ is used to denote Q.E.D.

2.2. The Logic of Chronological Ignorance

Nonmonotonic logics can be defined by means of a preference criterion on the interpretations of a standard logic, i.e. (classical or modal) propositional logic or first-order predicate logic. The preference criterion forms a preference relation over the models of the standard logic. CI is a nonmonotonic logic obtained in this way (cf. Def. 2.9). The standard monotonic logic on which CI is based is called the logic of Temporal Knowledge (TK). The syntax and semantics of TK are given below.

We assume the existence of the following:

P: a set of primitive propositions,
TV: a set of temporal variables,
TC: Z (integers) (this characterizes the structure of time),
U: $TC \cup TV$.

Well-formed formulae (wff or formula in short) are defined as follows (t_1 and t_2 are used to denote elements of TC whereas u_1 and u_2 are meta-variables for elements of U):

1. If $u_1, u_2 \in U$, then $u_1 = u_2$ and $u_1 \leq u_2$ are wff.
2. If $u_1, u_2 \in U$ and $p \in P$, then $\mathrm{TRUE}(u_1, u_2, p)$ is a wff.
3. If φ_1 and φ_2 are wff, then so are $\varphi_1 \wedge \varphi_2$, $\neg\varphi_2$, and $\Box\varphi_1$. $\Box\varphi$ reads as "φ is known." $\Diamond\varphi \equiv \neg\Box\neg\varphi$.
4. If φ is a wff and $v \in TV$, then $\forall v\varphi$ is also a wff.

Some abbreviations for wff are used; $\Box\text{TRUE}(t_1, t_2, p)$ is replaced by $\Box(t_1, t_2, p)$, $\Box\neg\text{TRUE}(t_1, t_2, p)$ by $\Box(t_1, t_2, \neg p)$, $\Diamond\text{TRUE}(t_1, t_2, p)$ by $\Diamond(t_1, t_2, p)$, and $\Diamond\neg\text{TRUE}(t_1, t_2, p)$ by $\Diamond(t_1, t_2, \neg p)$. $\text{TRUE}(t_1, p)$ is used as an abbreviation for $\text{TRUE}(t_1, t_1, p)$.

Definition 2.1. A *sentence* is a wff containing no free variables.

Definition 2.2. A *Kripke interpretation* (KI) is a pair $\langle W, M\rangle$ where W is a nonempty universe of possible worlds, and M is a meaning function such that M: $P \rightarrow 2^{W\times Z\times Z}$.

Definition 2.3. A *variable assignment* is a function VA: $TV \rightarrow Z$.

Definition 2.4. A *valuation function* VAL is such that $VAL(u) = VA(u)$ if $u \in TV$ and $VAL(u) = u$ if $u \in TC$.

A KI $= \langle W, M\rangle$ and a world $w \in W$ satisfy a formula φ under VA (written KI, $w \models \varphi[VA]$) if the following hold:

1. KI, $w \models u_1 = u_2[VA]$ iff $VAL(u_1) = VAL(u_2)$.
2. KI, $w \models u_1 \leq u_2[VA]$ iff $VAL(u_1) \leq VAL(u_2)$.
3. KI, $w \models \text{TRUE}(u_1, u_2, p)[VA]$ iff $\langle w, VAL(u_1), VAL(u_2)\rangle \in M(p)$.
4. KI, $w \models \varphi_1 \wedge \varphi_2[VA]$ iff KI, $w \models \varphi_1[VA]$ and KI, $w \models \varphi_2[VA]$.
5. KI, $w \models \neg\varphi[VA]$ iff KI, $w \not\models \varphi[VA]$.
6. KI, $w \models \forall v\varphi[VA]$ iff KI, $w \models \varphi[VA']$, $\forall VA'$ that agree with VA everywhere except possibly on v.
7. KI, $w \models \Box\varphi[VA]$ iff KI, $w' \models \varphi[VA]$, $\forall w' \in W$.

A KI $= \langle W, M\rangle$ and a world $w \in W$ are a *model* for a formula φ (written KI, $w \models \varphi$) if KI, $w \models \varphi[VA]$ for any variable assignment VA. A wff is *satisfiable* if it has a model, and *valid* if its negation has no model. φ_1 *entails* φ_2 (written $\varphi_1 \models \varphi_2$) iff φ_2 is satisfied by all models of φ_1. It should be noted that if φ is true (respectively, is not true) in $w \in W$, this is written KI, $w \models \varphi$ (respectively, KI, $w \not\models \varphi$).

Definition 2.5. *Base formulae* are those wff containing no occurrence of the modal operators.

Definition 2.6. The *latest time point* (ltp) of a base sentence is the latest time point mentioned in it:

1. The ltp of $\text{TRUE}(t_1, t_2, p) = t_2$.
2. The ltp of $\varphi_1 \wedge \varphi_2 = \max(\text{ltp of } \varphi_1, \text{ltp of } \varphi_2)$.
3. The ltp of $\neg\varphi =$ the ltp of φ.
4. The ltp of $\forall v\varphi$ is the minimum among the ltps of all φ' which result from substituting in φ a time point symbol for all free occurrences of v, or $-\infty$ if there is no such earliest ltp.

Definition 2.7. A KI M_2 is *chronologically more ignorant* than a KI M_1 (written $M_1 \subset_{ci} M_2$) if there exists t_0 such that

1. For any base sentence φ with ltp $\leq t_0$, if $M_2 \models \Box\varphi$ then also $M_1 \models \Box\varphi$.
2. There exists a base sentence φ with ltp t_0 such that $M_1 \models \Box\varphi$ but $M_2 \not\models \Box\varphi$.

Definition 2.8. M is said to be a *chronologically maximally ignorant (cmi) model* of φ if $M \models_{\subset_{ci}} \varphi$, i.e. if $M \models \varphi$ and there is no other M' such that $M' \models \varphi$ and $M \subset_{ci} M'$.

Definition 2.9. The *logic of chronological ignorance* CI is the nonmonotonic logic $TK_{\subset_{ci}}$.

2.3. Causal Theories

Definition 2.10. *Formulae* in CI are those base formulae augmented by the modal operators.

Definition 2.11. *A theory* in CI is a collection of sentences in CI.

Definition 2.12. *Base sentences* in CI are those sentences not containing any occurrence of the modal operators.

Definition 2.13. *Atomic base sentences* are either of the form TRUE(t_1, t_2, p) or the form $\neg$ TRUE(t_1, t_2, p).

Definition 2.14. A *causal theory* Ψ is a theory in CI, in which all sentences have the form $\Phi \wedge \Theta \supset \Box\varphi$ where (in the following $[\neg]$ means that the negation sign may or may not appear)

1. $\varphi =$ TRUE(t_a, t_b, $[\neg]p$).
2. $\Phi = \bigwedge_{i=1}^{n} \Box\varphi_i$, where φ_i is an atomic base sentence with ltp t_i such that $t_i < t_a$.
3. $\Theta = \bigwedge_{j=1}^{m} \Diamond\varphi_j$, where φ_j is an atomic base sentence with ltp t_j such that $t_j < t_a$.
4. Φ or Θ (or both) may be empty. A sentence in which Φ is empty is called a *boundary condition*. Other sentences are called *causal rules*.
5. There is a time point t_0 (global for the theory) such that if $\Theta \supset \Box(t_a, t_b, [\neg]p)$ is a boundary condition, then $t_0 < t_a$.
6. There do not exist two sentences in Ψ such that one contains $\Diamond(t_a, t_b, p)$ on its l.h.s. and the other contains $\Diamond(t_a, t_b, \neg p)$ on its l.h.s.
7. If $\Phi_1 \wedge \Theta_1 \supset \Box(t_a, t_b, p)$ and $\Phi_2 \wedge \Theta_2 \supset \Box(t_a, t_b, \neg p)$ are two sentences in Ψ, then $\Phi_1 \wedge \Theta_1 \wedge \Phi_2 \wedge \Theta_2$ is inconsistent. (*Inconsistent* means "implies $p \wedge \neg p$.")

Definition 2.15. The *soundness conditions* of Ψ are the set of sentences $\Diamond(t_a, t_b, p) \supset$ TRUE(t_a, t_b, p) such that $\Diamond(t_a, t_b, p)$ appears on the l.h.s. of some sentence in Ψ. Soundness conditions are implicitly part of the causal theories.

Theorem 2.1. If Ψ is a causal theory, then

1. Ψ has a cmi model.
2. If M_1 and M_2 are cmi models of Ψ, and φ is any atomic base sentence, then $M_1 \models \Box\varphi$ iff $M_2 \models \Box\varphi$.

Proof. See Ref. 2, pp. 112–113. ■

Definition 2.16. A *time-bounded Kripke interpretation* M/t is a structure which can be viewed as an incomplete Kripke interpretation. Like a Kripke interpretation it assigns a truth value to atomic propositions, but only to those whose ltp $\leq t$. The truth value of an arbitrary sentence whose ltp $\leq t$ is also determined in M/t, according to the usual compositional rules. If a sentence φ with ltp $\leq t$ is *satisfied* by M/t, this is denoted $M/t \models \varphi$.

Definition 2.17. *M/t partially satisfies* a theory Ψ if M/t satisfies all sentences of Ψ whose ltp $\leq t$.

3. COMPUTING THE SENTENCES KNOWN IN THE CMI MODELS

Various nonmonotonic formal systems have been proposed to facilitate commonsense reasoning (e.g. Reiter's default logic[26] and McCarthy's circumscription[27]). *Situation calculus*[28] has initially been used to reason about the effects of actions. In the framework of situation calculus, Hanks and McDermott[29] describe what they call *temporal projection* as follows. Given a description of the current situation, the effects of possible actions, and a sequence of actions to be performed, how do we predict the properties of the world in the resulting situation?

Noticing that this is not a by-product of situation calculus, but is independent of the logic used, they redefine it (in Ref. 16, p. 385)

> "[G]iven an initial description of the world (some facts that are true), the occurrence of some events, and some notion of causality (that an event can cause a fact to become true), what facts are true once all the events have occurred?

Hanks and McDermott[16] applied some of the existing logics (e.g. default logic) to scenarios to see whether the expected results are indeed produced. It turned out that these logics have some flaws (Ref. 16, p. 379):

> "Upon examining the resulting nonmonotonic theories, however, we find that the inferences permitted by the logics are not those we had intended when we wrote the axioms, and in fact are much weaker."

The *Yale Shooting Problem* (YSP) was posed by them[29] as a paradigm to show how the temporal projection problem arises. At some point in time, a person (Fred) is alive. A loaded gun, after waiting for a while, is fired at Fred. What are the results of this action? One expects that Fred would die and the gun would be unloaded. But they[16] demonstrate, in the framework of circumscription,[30] that unintended minimal models are obtained; the gun gets unloaded during the waiting stage and firing it does not kill Fred.

After Hanks and McDermott showed how existing logics fail to produce the expected results for YSP, researchers proposing new formalisms applied their methods to the YSP and other similar scenarios (e.g. McCarthy's *blocks world*[30]) to show how they succeed in avoiding the unintended models.

Hanks and McDermott argue that a solution to the temporal projection problem relies on an answer to two questions (Ref. 16, p. 409):

> "(1) Given a logical theory that admits more than one model, what are the preferred models of that theory (that is, what is the performance criterion) and (2) given a theory and a preference criterion, how do we find the theorems that are true in all 'most preferred' models?"

As they noted, Shoham's[11] preference criterion (see Defs. 2.7–8) provides a satisfactory answer to the first question. Moreover, he gives an algorithm to compute the true sentences in the models preferred under this preference criterion, thus answering the second question.

In this chapter, we argue that Shoham's computational account is not very efficient. Furthermore, since his solution, as Hanks and McDermott also point out (Ref. 16, p. 410) is "very specific to the problem of temporal projection," we demonstrate how its time-dependent nature can be overcome. We also show that causal theories may yield unintended models.

3.1. Time Dependency in Causal Computations

The causal theories of Shoham contain axioms to reason about the effects of actions. Proceeding in time, knowledge about the future is obtained from what is known (and what is not known) about the past. This forms the core of the causal inference mechanism. For example, if you know that a match is struck at time t, and do not know that it is wet at t, then you infer that the match lights at $t + 1$. Causal theories have a nice property: all cmi models agree on what is known (see Theorem 2.1). That is, in all cmi models of a causal theory the same atomic base sentences are known.

Consider the following variant of YSP. A gun, loaded at some point in time, is fired at a later time. We would like to reason about the effect of firing. Shoham (Ref. 2, p. 106) gives a possible axiomatization in which the gun is loaded at time 1 and fired at 5:

1. $\Box(1, \text{loaded})$.
2. $\Box(5, \text{fired})$.
3. $\Box(t, \text{loaded}) \wedge \Diamond(t, \neg\text{fired}) \wedge \Diamond(t, \neg\text{emptied-manually}) \supset \Box(t+1, \text{loaded}), \forall t$.
4. $\Box(t, \text{loaded}) \wedge \Box(t, \text{fired}) \wedge \Diamond(t, \text{air}) \wedge \Diamond(t, \text{firing-pin}) \wedge \Diamond(t, \text{no-marshmallow-bullets})$

$\wedge \ldots \wedge \Diamond$ other mundane conditions
$\supset \Box(t + 1, \text{noise}), \forall t.$

Axioms 1 and 2 are the boundary conditions. The third one is an axiom schema needed for persistence. It says that the gun remains loaded unless certain conditions are obtained. The last one is again an axiom schema. It is a causal rule stating that firing a loaded gun causes a noise unless certain conditions are obtained. In fact, causal theories can only contain axioms, not axiom schemata with time variables (see Def. 2.14). Shoham (personal communication, November 1989) explains:

> "I do assume that all boundary conditions and all causal rules contain only ground atomic sentences. If variables appear it means that this is a schema, standing for all its ground instances. I believe this restriction can be lifted, but I did impose it."

Therefore, the axiom schemata 3 and 4 above must be replicated by replacing the meta-variable t by time points from 1 to 5. This actually corresponds to the finite causal theory below (some $\Diamond$-conditions of schema 4 are omitted to save space):

1. $\Box$(1, loaded).
2. $\Box$(1, loaded) $\wedge$ $\Diamond$(1, $\neg$fired) $\wedge$ $\Diamond$(1, $\neg$emptied-manually) $\supset$ $\Box$(2, loaded).
3. $\Box$(1, loaded) $\wedge$ $\Box$(1, fired) $\wedge$ $\Diamond$(1, air) $\wedge$ $\Diamond$(1, firing-pin) $\supset$ $\Box$(2, noise).
4. $\Box$(2, loaded) $\wedge$ $\Diamond$(2, $\neg$fired) $\wedge$ $\Diamond$(2, $\neg$emptied-manually) $\supset$ $\Box$(3, loaded).
5. $\Box$(2, loaded) $\wedge$ $\Box$(2, fired) $\wedge$ $\Diamond$(2, air) $\wedge$ $\Diamond$(2, firing-pin) $\supset$ $\Box$(3, noise).
6. $\Box$(3, loaded) $\wedge$ $\Diamond$(3, $\neg$fired) $\wedge$ $\Diamond$(3, $\neg$emptied-manually) $\supset$ $\Box$(4, loaded).
7. $\Box$(3, loaded) $\wedge$ $\Box$(3, fired) $\wedge$ $\Diamond$(3, air) $\wedge$ $\Diamond$(3, firing-pin) $\supset$ $\Box$(4, noise).
8. $\Box$(4, loaded) $\wedge$ $\Diamond$(4, $\neg$fired) $\wedge$ $\Diamond$(4, $\neg$emptied-manually) $\supset$ $\Box$(5, loaded).
9. $\Box$(4, loaded) $\wedge$ $\Box$(4, fired) $\wedge$ $\Diamond$(4, air) $\wedge$ $\Diamond$(4, firing-pin) $\supset$ $\Box$(5, noise).
10. $\Box$(5, fired).
11. $\Box$(5, loaded) $\wedge$ $\Diamond$(5, $\neg$fired) $\wedge$ $\Diamond$(5, $\neg$emptied-manually) $\supset$ $\Box$(6, loaded).
12. $\Box$(5, loaded) $\wedge$ $\Box$(5, fired) $\wedge$ $\Diamond$(5, air) $\wedge$ $\Diamond$(5, firing-pin) $\supset$ $\Box$(6, noise).

The first axiom says that "it is known that the gun is loaded at 1." The second one says that "if it is known that the gun is loaded at 1, and it is not known that it is fired at 1 and that it is emptied manually at 1, then it is known that the gun is loaded at 2." The third one says that "if it is known that the gun is loaded at 1 and that it is fired at 1, and it is not known that there is no air and that the gun has no firing pin at 1, then it is known that noise is heard at 2." The remaining axioms are analogous. Shoham's algorithm steps through each axiom and computes the base sentences known in all cmi models of this causal theory. It produces the expected atomic base sentences: TRUE(1, loaded), TRUE(2, loaded), . . . , TRUE(5, loaded), TRUE(5, fired), and TRUE(6, noise).

This cmi model is computed by stepping over each axiom of the causal theory in ordered form, and checking whether the l.h.s. of the axioms are satisfied. Shoham (Ref. 2, pp. 113–114) suggested improving the efficiency of the algorithm by "focus[ing] the attention on the interesting time points, those that are potentially ltps of known atomic base sentences." In other words, "in constructing the cmi model, one can skip the time points which are not the ltp of the r.h.s. in any sentence of the causal theory: at those points no atomic base sentences are known" (Ref. 2, p. 114).

Measuring the size of causal theory in terms of the number of base sentences in the axioms, the size of the causal theory above turns out to be 47. (There exist two boundary conditions. Schema 3 contains four base sentences and schema 4 contains five base sentences. Axiom schemata 3 and 4 are replicated for all time points from 1 to 5, resulting in 45 base sentences.)

Now assume that the gun is loaded at time 1, and instead of at 5 it is fired at 5000. The size of the causal theory describing this scenario is 45 002. Consequently, the later the gun is fired, the larger the size of the corresponding causal theory becomes. Hence, more computation time and space are needed to reason about the effect of firing the gun.

Again to measure the size of a causal theory in terms of the number of base sentences in it, assume that the size of a causal theory with axiom schemata is n. Then the size of the corresponding finite causal theory must be $Tmax\ n$, where $Tmax$ denotes the number of time points (5 in this example) between the time points of the boundary conditions having the earliest ($\Box$(1, loaded)) and the latest time points ($\Box$(5, fired)), respectively. Shoham computes the atomic base sentences known in all cmi models of a finite causal theory. Assuming that this finite causal theory corresponds to the one with axiom schemata shown above, the time complexity of his algorithm becomes $O(Tmax\ n \log(Tmax\ n))$.

3.2. YSP-like Causal Theories

In temporal projection scenarios, there exist two types of axiom schemata. The first takes care of the persistence of facts, permitting inferences about what remains unchanged. This corresponds to axiom schema 3 in our shooting scenario. Such axiom schemata will be called *persistence axiom schemata*.

The second type of axiom schemata represent what changes occur in the environment. They will be called *causal axiom schemata*. More specifically, these schemata allow one to infer what changes actions bring about. In the shooting scenario, number 4 is a causal axiom schema.

It will be assumed in the sequel that scenarios are formalized with a persistence axiom schema and a causal axiom schema, along with two boundary conditions. The boundary condition having the greatest ltp generally represents an action whose consequences are to be determined.

Definition 3.1. A *YSP-like causal theory* ζ is a theory in CI containing

$$\Box\varphi_s.$$

$$\Box\varphi_f.$$

$$\Box\varphi_p \wedge \Theta_p \supset \Box\varphi_p, \forall t .$$

$$\Phi_c \wedge \Theta_c \supset \Box\varphi_c, \forall t .$$

where

1. $\Box\varphi_s$ is the *initial boundary condition* where φ_s is of the form TRUE(t_1, [$\neg$]p).
2. $\Box\varphi_f$ is the *final boundary condition* where φ_f is of the form TRUE(t_2, [$\neg$]p), $t_1 < t_2$.
3. $\Box\varphi_p \wedge \Theta_p \supset \Box\varphi_p$ is a *persistence axiom schema* where
   (i) φ_p is of the form TRUE(t, [$\neg$]p) (on the l.h.s.) or TRUE($t + 1$, [$\neg$]p) (on the r.h.s.). (There is a slight abuse of notation here.)
   (ii) Θ_p is a (possibly empty) conjunction of sentences $\Diamond\varphi_i$, where φ_i is of the form TRUE(t, [$\neg$]q).
4. $\Phi_c \wedge \Theta_c \supset \Box\varphi_c$ is a *causal axiom schema* where
   (i) Φ_c has two conjuncts one of which must be $\Box\varphi_p$.
   (ii) Θ_c is a (possibly empty) conjunction of sentences $\Diamond\varphi_k$, where φ_k is of the form TRUE(t, [$\neg$]q).
   (iii) φ_c is of the form TRUE($t + 1$, [$\neg$]r).
5. If $\Diamond(t, p)$ (respectively $\Diamond(t, \neg p)$) is a conjunct of Θ_p, then Θ_c does not contain $\Diamond(t, \neg p)$ (respectively $\Diamond(t, p)$).
6. If φ_p and φ_c are of the forms TRUE($t + 1, p$) (respectively TRUE($t + 1, \neg p$)) and TRUE($t + 1, \neg p$) (respectively TRUE($t + 1, p$)) then $\Box\varphi_p \wedge \Theta_p \wedge \Phi_c \wedge \Theta_c$ is inconsistent.
7. If φ_s (φ_f) is of the form TRUE(t_1, p) (respectively TRUE($t_2, \neg p$)) and φ_p is of the form TRUE($t + 1, \neg p$) (respectively TRUE($t + 1, p$)) then $\Box\varphi_p \wedge \Theta_p$ is inconsistent.
8. If φ_s (φ_f) is of the form TRUE(t_1, p) (respectively TRUE($t_2, \neg p$)) and φ_c is of the form TRUE($t + 1, \neg p$) (respectively TRUE($t + 1, p$)) then $\Phi_c \wedge \Theta_c$ is inconsistent.

Theorem 3.1. If ζ is a YSP-like causal theory, then ζ has cmi models and in all of these cmi models the same atomic base sentences are known.

Proof. The proof is similar to the proof of Theorem 2.1 and is omitted here; cf. Ref. 31, pp. 59–61. ■

Proposition 3.1. Any YSP-like causal theory ζ corresponds to a finite causal theory Ψ if each time variable t in axiom schemata in ζ is replaced by the time constants in the range t_1 to t_2, where t_1 and t_2 are the time points mentioned in the initial and final boundary conditions of ζ, respectively.

Proof. The causal theory obtained in this way will contain the following sentences ordered with respect to their ltps. ("Rewriting" a formula at $t = t_i$ means replacing all occurrences of t in that formula with t_i.)

$\Box\varphi_s$.

$\Box\varphi_p \wedge \Theta_p \supset \Box\varphi_p$ (rewritten for $t = t_1$ until $t = t_2 - 1$) .

$\Phi_c \wedge \Theta_c \supset \Box\varphi_c$ (rewrite for $t = t_1$ until $t = t_2 - 1$) .

$\Box\varphi_f$.

$\Box\varphi_p \wedge \Theta_p \supset \Box\varphi_p$ (rewrite at $t = t_2$) .

$\Phi_c \wedge \Theta_c \supset \Box\varphi_c$ (rewrite at $t = t_2$) .

The resulting theory is actually a finite causal theory of type Ψ (see Def. 2.14). ∎

Theorem 3.2. If ζ is a YSP-like causal theory of size n, then the unique set of atomic base sentences known in any cmi model of ζ can be computed in time $O(n)$.

Proof. Appendix. ∎

Consider the causal theory with axiom schemata given in Sec. 3.1. It is a YSP-like causal theory since it contains an initial boundary condition (axiom 1), a final boundary condition (axiom 2), a persistence axiom schema (schema 3) and a causal axiom schema (schema 4). Given this YSP-like causal theory (some mundane conditions are omitted), the algorithm produces the sentences: TRUE(1, loaded), TRUE(2, loaded), . . . , TRUE(5, loaded), TRUE(5, fired), and TRUE(6, noise). These are exactly the sentences Shoham's algorithm yields.

Now the final boundary condition is replaced by $\Box(10^{10}$, fired). Both algorithms produce TRUE(1, loaded), TRUE(2, loaded), . . . , TRUE(10^{10}, loaded), TRUE(10^{10}, fired), and TRUE($10^{10} + 1$, noise). Since Shoham's algorithm must step through each time point between 1 and 10^{10}, it takes too long for it to jump to the conclusion that the gun will be loaded at 10^{10}, and then infer that there will be a loud noise at $10^{10} + 1$. However, if one knows that the gun is loaded and that nothing has happened until the time of reasoning about the effect of firing the gun, one will immediately conclude that the gun is still loaded. Then, one will reason about the effect of firing the gun with this knowledge. In fact, this is what the $O(n)$ algorithm does; knowing that the gun is loaded at 1, and that nothing interferes with the gun's being loaded, it concludes that the gun will remain loaded until it is fired at 10^{10}.

Now let the scenario change. The gun is loaded at 1 but is emptied manually at 9. Shoham's algorithm and the $O(n)$ algorithm both produce TRUE(1, loaded), TRUE(2, loaded), . . . , TRUE(9, loaded), and TRUE(9, emptied-manually).

3.3. Multi-agents and a Broader Class of YSP-like Causal Theories

Restricting theories so that they contain a persistence axiom schema and a causal axiom schema does not provide the full power to represent realistic scenarios.

Consider the YSP. Fred's being alive and the gun's being loaded at time 1 form the initial description. Furthermore, assume that the gun is fired at 10.

1. $\Box(1, \text{alive})$.
2. $\Box(1, \text{loaded})$.
3. $\Box(10, \text{fired})$.
4. $\Box(t, \text{alive}) \wedge \Diamond(t, \neg\text{fired}) \wedge \Diamond(t, \text{air}) \supset \Box(t+1, \text{alive}), \forall t$.
5. $\Box(t, \text{loaded}) \wedge \Box(t, \text{fired}) \wedge \Diamond(t, \text{firing-pin}) \wedge \Diamond(t, \text{no-marshmallow-bullets}) \supset \Box(t+1, \text{dead}), \forall t$.
6. $\Box(t, \text{loaded}) \wedge \Diamond(t, \neg\text{fired}) \wedge \Diamond(t, \neg\text{emptied-manually}) \supset \Box(t+1, \text{loaded}), \forall t$.
7. $\Box(t, \text{loaded}) \wedge \Box(t, \text{fired}) \wedge \Diamond(t, \text{air}) \wedge \Diamond(t, \text{firing-pin}) \wedge \Diamond(t, \text{no-marshmallow-bullets}) \supset \Box(t+1, \text{noise}), \forall t$.

Axioms 1 and 2 describe the initial state. Axiom 3 indicates the occurrence of the firing action. Axiom schema 4 says that Fred remains alive unless the gun is fired or there is no air (and hence he suffocates). Axiom schema 5 says that firing a loaded gun causes Fred's death provided that some conditions are satisfied. Axiom schemata 6 and 7 are used in the usual sense. This theory is not a YSP-like causal theory, because a YSP-like causal theory must contain exactly one persistence schema and one causal axiom schema. Moreover, one initial boundary condition and one final boundary condition are allowed. The theory above however contains two persistence and two causal axiom schemta, two initial boundary conditions, and one final boundary condition. Therefore, scenarios similar to this call for a broader class of YSP-like causal theories which will be introduced in the sequel. Before doing this, Shoham's causal theories will be examined to see whether they succeed in computing the intended models when concurrent actions are introduced.

Causal theories allow concurrent actions. Consider the following blocks world. There is a block initially located at a position (denoted by ''at-center'') on the table. There are two operations ''push-left'' and ''push-right''. Executing ''push-left'' moves the block to a location (denoted by ''at-left''). Executing ''push-right'' causes the block to move to another position (denoted by ''at-right''). It is assumed that the forces applied on the block are of equal magnitude when these operations are performed concurrently. Now, assume that the block is at ''at-center'' at time 1, and ''push-left'' and ''push-right'' are simultaneously executed at 1.

1. $\Box(1, \text{at-center})$.
2. $\Box(1, \text{push-left})$.
3. $\Box(1, \text{push-right})$.
4. $\Box(1, \text{at-center}) \wedge \Diamond(1, \neg\text{push-left}) \wedge \Diamond(1, \neg\text{push-right}) \supset \Box(2, \text{at-center})$.
5. $\Box(1, \text{at-center}) \wedge \Box(1, \text{push-left}) \wedge \Diamond(1, \neg\text{push-right}) \supset \Box(2, \text{at-left})$.
6. $\Box(1, \text{at-center}) \wedge \Box(1, \text{push-right}) \wedge \Diamond(1, \neg\text{push-left}) \supset \Box(2, \text{at-right})$.

Shoham's algorithm computes TRUE(1, at-center), TRUE(1, push-left), TRUE(1, push-right). No other base sentence is known in the cmi models of this causal theory.

This is strange. Since "push-left" and "push-right" are executed concurrently, the block should remain at the center of the table. That is, the sentence TRUE(2, at-center) must be obtained.

This problem can be resolved by introducing additional axioms such as "if it is known that the block is at the center of the table, and that push-right and push-left are simultaneously performed, then it is known that the block remains at the center" and "if it is known that the block is at the center of the table, and that no push-right or push-left operations are performed, then it is known that the block remains at the center". Unfortunately, in more complex domains, the number of such axioms can grow quickly.[36] There must be a way of resolving this problem with a persistence axiom.

Definition 3.2. The *set of counteractions* is the set of actions that prevent each other from being operative when performed concurrently (cf. Sec. 4.3).

Definition 3.3. Let $\Pi = \{\Diamond(t_a, p_i) \mid 1 \leq i \leq n, \text{ for some } t_a\}$ where p_i's are counteractions. Letting M be the unique cmi model of a causal theory Ψ, let us write $M \models \Pi$ iff $M \models \Diamond(t_a, p_i)$, $\forall \Diamond(t_a, p_i) \in \Pi$, or $M \not\models \Diamond(t_a, p_i)$, $\forall \Diamond(t_a, p_i) \in \Pi$. Otherwise, let us write $M \not\models \Pi$.

As an illustration, the fourth axiom in the blocks world example above is replaced with the axiom below, where $\Pi = \{\Diamond(1, \neg\text{push-left}), \Diamond(1, \neg\text{push-right})\}$.

$$\Box(1, \text{at-center}) \wedge \Pi \supset \Box(2, \text{at-center}) .$$

Abusing the notation, Π will be used as if it were a function over its members:

$$\Box(1, \text{at-center}) \wedge \Pi(\Diamond(1, \neg\text{push-left}), \Diamond(1, \neg\text{push-right}) \supset \Box(2, \text{at-center}) .$$

Under the interpretation of Π, in all cmi models of the causal theory for the blocks world example, TRUE(1, at-center), TRUE(1, push-left), TRUE(1, push-right), TRUE(2, at-center) are known.

Now a new class of causal theories with axiom schemata will be defined. It can be looked upon as a broader class of YSP-like causal theories. For this reason, any theory in this class will be called a YSP-like causal theory.

Definition 3.4. A *YSP'-like causal theory* ζ' is a theory in CI containing

$$\Box\varphi_{s_i},\ i = 1, \ldots, n.$$

$$\Box\varphi_{f_j},\ j = 1, \ldots, m.$$

and axiom schemata in one of the following forms

$$\Box\varphi_p \wedge \varnothing_p \wedge \Theta_p \supset \Box\varphi_p,\ \forall t .$$

$$\Phi_c \wedge \Theta_c \supset \Box\varphi_c, \forall t .$$

where

1. $\Box\varphi_{s_i}$'s form the (nonempty) set of *initial boundary conditions* where each φ_{s_i} is of the form TRUE(t_a, [$\neg$]p).
2. $\Box\varphi_{f_j}$'s form the (nonempty) set of *final boundary conditions* where each φ_{f_j} is of the form TRUE(t_b, [$\neg$]p), $t_a < t_b$.
3. Any sentence of the form $\Box\varphi_p \wedge \varnothing_p \wedge \Theta_p \supset \Box\varphi_p$ is a *persistence axiom schema* where
   (i) φ_p is of the form TRUE(t, [$\neg$]p) (on the l.h.s.) and TRUE($t+1$, [$\neg$]p) (on the r.h.s.).
   (ii) $\varnothing_p$ is a (possibly empty) conjunction of Π_i, where Π_i is a set of sentences $\Diamond\varphi_j$ such that φ_j is of the form TRUE(t, [$\neg$]q).
   (iii) Θ_p is a (possibly empty) conjunction of $\Diamond\varphi_k$, where φ_k is of the form TRUE(t, [$\neg$]q).
4. Any sentence of the form $\Phi_c \wedge \Theta_c \supset \Box\varphi_c$ is a *causal axiom schema* where
   (i) Φ_c is a nonempty conjunction of sentences $\Box\varphi_i$, where φ_i is of the form TRUE(t, [$\neg$]p). Φ_c must contain at least one sentence of the form TRUE(t, [$\neg$]p) which does not appear on the r.h.s. of any (persistence or causal) axiom schema (as TRUE($t+1$, [$\neg$]p)).
   (ii) Θ_c is a (possibly empty) conjunction of sentences $\Diamond\varphi_j$, where φ_j is of the form TRUE(t, [$\neg$]q).
   (iii) φ_c is of the form TRUE($t+1$, [$\neg$]r).
5. TRUE(t_a, p) and TRUE($t_a, \neg p$) do not appear among the initial boundary conditions together.
6. TRUE(t_b, q) and TRUE($t_b, \neg q$) do not appear among the final boundary conditions together.
7. Let $\Box\varphi_p \wedge \varnothing_p \wedge \Theta_p \supset \Box\varphi_p$ and $\Phi_c \wedge \Theta_c \supset \Box\varphi_c$ be two schemata in ζ'. If $\Diamond(t, p)$ (respectively $\Diamond(t, \neg p)$) is a conjunct of $\varnothing_p \wedge \Theta_p$, then Θ_c does not contain $\Diamond(t, \neg p)$ (respectively $\Diamond(t, p)$) as a conjunct.
8. Let $\Box\varphi_p \wedge \varnothing_p \wedge \Theta_p \supset \Box\varphi_p$ and $\Phi_c \wedge \Theta_c \supset \Box\varphi_c$ be two schemata in ζ'. If φ_p and φ_c are of the forms TRUE($t+1, p$) (respectively TRUE($t+1, \neg p$)) and TRUE($t+1, \neg p$) (respectively TRUE($t+1, p$)) then $\Box\varphi_p \wedge \varnothing_p \wedge \Theta_p \wedge \Phi_c \wedge \Theta_c$ is inconsistent.
9. Let $\Box\varphi_{s_i}$ (respectively $\Box\varphi_{f_j}$) be an initial (respectively final) boundary condition and $\Box\varphi_p \wedge \varnothing_p \wedge \Theta_p \supset \Box\varphi_p$ be a persistence axiom schema. If φ_{s_i} (respectively φ_{f_j}) is of the form TRUE(t_a, p) (respectively TRUE($t_b, \neg p$)) and φ_p is of the form TRUE($t+1, \neg p$) (respectively TRUE($t+1, p$)) then $\Box\varphi_p \wedge \varnothing_p \wedge \Theta_p$ is inconsistent.
10. Let $\Box\varphi_{s_i}$ (respectively $\Box\varphi_{f_j}$) be an initial (respectively final) boundary condition and $\Phi_c \wedge \Theta_c \supset \Box\varphi_c$ be a causal axiom schema. If φ_{s_i} (respectively φ_{f_j}) is of the form TRUE(t_a, p) (respectively TRUE($t_b, \neg p$)) and φ_c is of the form TRUE($t+1, \neg p$) (respectively TRUE($t+1, p$)) then $\Phi_c \wedge \Theta_c$ is inconsistent.

Proposition 3.2. Any YSP′-like causal theory ζ' corresponds to a finite causal theory Ψ if each t in all axiom schemata in ζ' is replaced by constants in the range t_a to t_b, where t_a and t_b are the time points mentioned in the initial and final boundary condition of ζ', respectively.

Proof. Replacing t in axiom schemata with constants gives a finite set of axioms. These axioms, together with the initial and final boundary conditions, form Ψ. ■

Theorem 3.3. If ζ' is a YSP′-like causal theory, then ζ' has cmi models and in all of these cmi models the same atomic base sentences are known.

Proof. Appendix. ■

Theorem 3.4. If ζ is a YSP-like causal theory, then ζ is also a YSP′-like causal theory.

Proof. Consider the initial and final boundary conditions of ζ as the unique members of the sets of initial and final boundary conditions of a YSP′-like causal theory ζ' respectively. The causal axiom schema of ζ, being the only causal axiom schema in ζ', and the persistence axiom schema of ζ (with an empty set of $\Diamond$-conditions for the set of counteractions), being the only persistence axiom schema of ζ', form a YSP′-like causal theory ζ'. ■

Theorem 3.5. If ζ' is a YSP′-like causal theory of size n, then the unique set of atomic base sentences known in any cmi model of ζ' can be computed in time $O(n \log n)$.

Proof. The steps of the construction procedure given in the proof of Theorem 3.3 are followed. The proof is easy but at the same time messy. The reader can refer to Ref. 31, (pp. 66–69) for the algorithm proposed and its time complexity analysis. ■

Let the following YSP′-like causal theory represent the blocks world scenario at the beginning of this section. But now assume that "push-left" and "push-right" are executed concurrently at 10.

1. $\Box(1, \text{at-center})$.
2. $\Box(10, \text{push-left})$.
3. $\Box(10, \text{push-right})$.
4. $\Box(t, \text{at-center}) \wedge \Pi(\Diamond(t, \neg\text{push-left}), \Diamond(t, \neg\text{push-right})) \supset \Box(t+1, \text{at-center}), \forall t$.
5. $\Box(t, \text{at-center}) \wedge \Box(t, \text{push-left}) \wedge \Diamond(t, \neg\text{push-right}) \supset \Box(t+1, \text{at-left}), \forall t$.
6. $\Box(t, \text{at-center}) \wedge \Box(t, \text{push-right}) \wedge \Diamond(t, \neg\text{push-left}) \supset \Box(t+1, \text{at-right}), \forall t$.

The $O(n \log n)$ program first computes the set of base sentences that will be known at 2 from what is known (and what is not known) at 1. It finds out that TRUE(2, at-center) is known by the axiom schema 4. Then, it performs one more iteration to see what is known at 3. Again by axiom schema 4, it is seen that only

TRUE(3, at-center) is known. Since the base sentences that are known at this step of the iteration are only the persistence sentences, it generates the sentences TRUE(4, at-center), TRUE(5, at-center), . . . , TRUE(10, at-center). Finally, it computes the sentences that are known at 11 from the atomic base sentences known at 10. Noticing that "push-left" and "push-right" are counteractions executed simultaneously, it finds out that the l.h.s. of the axiom schema 4 is satisfied. It produces the sentence TRUE(11, at-center). Since the l.h.s. of all other axiom schemata fail due to the occurrence of counteractions at 10, the atomic base sentences that are known in the cmi model of this YSP′-like causal theory are TRUE(1, at-center), TRUE(2, at-center), . . . , TRUE(10, at-center), TRUE(10, push-right), TRUE(10, push-left), and TRUE(11, at-center).

To see the consequences of a more interesting YSP′-like causal theory, consider the shooting scenario. Fred is alive and the gun is loaded at time 1. The gun is fired at Fred at time 10. The theory given for this scenario contains axiom schemata and boundary conditions. It is a typical YSP′-like causal theory. Given this theory, our $O(n \log n)$ algorithm produces the intended model. Shoham's algorithm and this algorithm produce the same sentences: TRUE(1, alive), TRUE(1, loaded), TRUE(2, alive), TRUE(2, loaded), . . . , TRUE(10, alive), TRUE(10, loaded), TRUE(10, fired), TRUE(11, dead), and TRUE(11, noise).

3.4. When is Computation Time-Dependent?

In the previous sections, it has been shown that computing with causal theories is inefficient in the sense that one must step through each axiom in the causal theory to compute the results of some action. To remove this deficiency, new classes of causal theories have been introduced. Restrictions have been imposed on sentences in these classes. One may wonder whether the time-dependent nature of computations can be removed without imposing these restrictions, but still allowing axiom schemata. The answer is not in the affirmative.

For example, consider an electronic circuit which functions as a relay. The output of the relay is directly connected to its input. The output can be either "on" or "off" depending on the input. If the input is "on" (respectively "off") at some time, then the output becomes "off" (respectively "on") at the next instant of time. One can interrupt the system by the operation "interfere". When "interfere" is done, the output of the circuit is delayed. Assume that the output of the circuit is given as "on" at time 1. If "interfere" is executed at time 6, what are the consequences? Below, a causal theory is given as a formalization of this scenario. (This is neither a YSP-like nor a YSP′-like causal theory. For example, $\Box(t, \text{on})$ is the unique $\Box$-condition of the axiom schema 3, but it appears on the r.h.s. of the axiom schema 4.)

1. $\Box(1, \text{on})$.
2. $\Box(6, \text{interfere})$.
3. $\Box(t, \text{on}) \wedge \Diamond(t, \neg\text{interfere}) \supset \Box(t+1, \text{off}),\ \forall t$.
4. $\Box(t, \text{off}) \wedge \Diamond(t, \neg\text{interfere}) \supset \Box(t+1, \text{on}),\ \forall t$.
5. $\Box(t, \text{on}) \wedge \Box(t, \text{interfere}) \supset \Box(t+4, \text{on}),\ \forall t$.

6. $\Box(t, \text{off}) \wedge \Box(t, \text{interfere}) \supset \Box(t + 4, \text{off}), \forall t.$

TRUE(1, on), TRUE(2, off), TRUE(3, on), TRUE(4, off), TRUE(5, on), TRUE(6, off), TRUE(6, interfere), and TRUE(10, off) are obtained as the atomic base sentences known in all cmi models of the corresponding finite causal theory.

Obviously, such a scenario requires examination of each axiom schema in the theory for all time points between 1 and 6. However, by determining regularities one can jump to conclusions. Knowing that the output is initially "on" at time 1 and that the relay produces a regular sequence of "on" and "off" unless "interfere" is executed, one can directly generate the sentences TRUE(2, off), TRUE(3, on), TRUE(4, off), TRUE(5, on), and TRUE(6, off).

4. SIMULTANEITY OF CAUSE AND EFFECT

4.1. Problems with Simultaneous Temporal Propositions

Among the commonly agreed properties of causation three are the touchstones for a formal treatment of causation. These are its properties of being *antisymmetric*, *irreflexive*, and *transitive*. For example, Bunge (Ref. 32, p. 244) proposes a relational approach where a relation R is supposed to hold between the cause and its effect.

It is the irreflexivity property of causation that is absent in material implication. Given any proposition p, it *immediately implies* itself (symbolically $p \Rightarrow p$). Hence material implication cannot be regarded as a correct formalization of causal connection. The irreflexive characteristic of causation together with its transitivity property forbids *circular causation*. Causal rules in causal theories are strongly related to material implication. But causal rules are weaker in some respects and stronger in others. Shoham discusses this issue in a related section on the properties of causation (Ref. 2, p. 152 and p. 166). Bunge (Ref. 32, pp. 242–243) also addresses the relation between causation and implication. The discussion is threefold. It centers around causation and the kinds of implication: *material*, *strict*, and *causal*.

Causal theories have antisymmetry and irreflexivity properties by definition since temporal precedence of causes over their effects is taken as the core principle of causal connections expressed by causal sentences. However, the transitivity characteristic is partly missing in causal theories. Temporally ordered sequences of causal relations are permitted. But this does not give a full account of the transitivity relation. A sequence of causes and effects (effects being also the causes of other effects) which are not ordered temporally, but possibly causally, and occurring simultaneously also form a transitive relation. For example, in an isolated environment an event A causes B, which in turn causes C such that there is no time difference between their occurrences and every cause is simultaneous with its effect. Then, it follows that A also (indirectly) causes C since whenever A occurs, B must be thereby causally depending on A, and whenever B occurs, C must be thereby causally depending on B.

It might sound confusing to talk about the conceptual inequality of the *causal order* and *temporal order* of occurrences. There are situations in which two things may happen at the same time. There exists no temporal order between their occurrences. None of them occurs after the other in time. However, the occurrence of one of them can be identified as the cause of the other. In this case, it is said that there is a causal order between them; the cause is *causally* before the caused one, the *effect*.

In causal theories, causal rules can represent causation such that the $\Box$-conditions on the l.h.s. of a causal rule denote causes while the r.h.s. denotes their effects. Under this interpretation, having simultaneous temporal propositions on both sides of causal sentences may result in circular causation (Ref. 2, p. 179): $\Box(t, p_i) \supset \Box(t, p_{i+1})$, $i = 1, \ldots, n-1$, $p_n = p_1$.

Simply, the causal theory may include a sentence of the form $\Box(t, p) \supset \Box(t, p)$. Then, we have *self-causation*. Our object to this is twofold. First, causation is semantic rather than syntactic. But if circularity exists, relating causation to syntactic forms only will not be fair. Instead, causation can take the form of a mere material implication. Furthermore, sentences of the form $\Box(t_1, p) \supset \Box(t_2, p)$, where $t_1 < t_2$, are allowed in causal theories. Does this mean that p causes itself? There can be sentences in the form $\Diamond(t_1, p) \supset \Box(t_2, p)$, where $t_1 < t_2$. Is this rendered as "if $\neg p$ is not known at t_1, then p is known at t_2 for *no reason*"? Through *soundness conditions*, one can write sentences like $\Diamond(t_1, p) \supset$ TRUE(t_1, p). Shoham (Ref. 2, p. 118) says "we now assume that the soundness conditions are implicitly part of the causal theory itself, and are omitted simply for reasons of economy of expression." Moreover, the boundary between $\Box$- and $\Diamond$-conditions in Shoham's account becomes hazy if $\Box$-conditions in a causal rule strictly denote the causes.

The second objection, closely connected to the first objection, is that one is not supposed to look for the causes in the unique cmi model of a causal theory. If this were the case, then there would be difficulties in identifying the causes and computing possible *explanans* of the occurrences. Temporal precedence of causes over effects already provides the necessary criterion to find out the causes of a given set of effects. However, when simultaneous propositions are allowed on both sides of the causal rules, the problem becomes more complex.

As an example for cause-effect distinction, reconsider Taylor's illustration. Assume that the causal theory contains the following:

$$\Box(4, \text{locomotive-moves}) \supset \Box(4, \text{caboose-moves})\ .$$

$$\Box(4, \text{caboose-moves}) \supset \Box(4, \text{locomotive-moves})\ .$$

Looking only at the syntactic forms of these rules, one can say that they permit circular causation. But now add TRUE(4, locomotive-moves) to the causal theory. Then, TRUE(4, locomotive-moves) and TRUE(4, caboose-moves) will be the only sentences known in all cmi models of the causal theory. In this case, if one investigates the cause of the motion of the locomotive and the caboose, one may identify the motion of the locomotive as the cause of the motion of the caboose although $\Box(4, \text{caboose-moves}) \supset \Box(4, \text{caboose-moves})$ implies self-causation.

Permitting causal sentences of the form $\Box(t, p_i) \supset \Box(t, p_{i+1})$, $i = 1, \ldots, n-1$, $p_n = p_1$ introduces no peculiarity at all in constructing cmi models. This will be studied in Sec. 4.4.

4.2. Self-Change

Shoham says (Ref. 2, p. 179):

> "[O]ne might have a set of sentences $\Box(t,p_i) \supset \Box(t,p_{i+1})$, $i=1, \ldots, n-1$, $p_n = \neg p_1$. This would destroy the independence of the past from the future in general, and the 'unique'-model property in particular. Or, as another example, one might have sentences of the form $\Diamond(t,p) \supset \Box(t, \neg p)$, which would have a similarly detrimental effect on the properties of causal and inertial theories."

However, by placing some restrictions on the sentences in the definition of causal theories these problems can be eliminated. In the former case, it is possible to impose some restrictions on the sentences similar to the one in the definition of the original causal theories. Recall that consistency of the causal theories is maintained by the following:

> If $\Phi_1 \wedge \Theta_1 \supset \Box(t_a, t_b, p)$ and $\Phi_2 \wedge \Theta_2 \supset \Box(t_a, t_b, \neg p)$ are two sentences in Ψ, then $\Phi_1 \wedge \Theta_1 \wedge \Phi_2 \wedge \Theta_2$ is inconsistent.

To see what kind of situations yield inconsistency, two possibilities are examined below.

(a) Causal connections can be unidirectional:

$\Box(t, p_1) \supset \Box(t, p_2)$.
$\Box(t, p_2) \supset \Box(t, p_3)$.
$\ldots \quad \ldots \quad \ldots$
$\Box(t, p_n) \supset \Box(t, \neg p_1)$.

If there exists another sentence with $\Box(t, p_1)$ on its r.h.s., then this may result in wrong inferences.

(b) The causal connections can be bidirectional:

$\Box(t, p_1) \supset \Box(t, p_2)$.
$\Box(t, p_2) \supset \Box(t, p_1)$.
$\Box(t, p_2) \supset \Box(t, p_3)$.
$\Box(t, p_3) \supset \Box(t, p_2)$.
$\Box(t, p_3) \supset \Box(t, \neg p_1)$, where there exists at least one causal chain from $\Box(t, p_1)$ to $\Box(t, \neg p_1)$.

In this case, if the r.h.s. of any of these rules is satisfied, due to the existence of causal chain the sentences TRUE(t, p_1) and TRUE$(t, \neg p_1)$ will be obtained.

4.3. Should Simultaneity be Treated by Causal Theories?

Consider the following illustration (Ref. 33, p. 108; Ref. 34). There is a horizontally positioned pipe whose two ends are controlled with two valves. These are connected in such a way that if one is opened at one time, then the other valve is closed at the same time, and vice versa. The opening of one valve and closing of the other occur simultaneously. The pipe also has a top-inlet continuously supplying high pressure water into it. Hence, the pipe, with the valves directing water flow in only one direction at a time, functions as a two-way watering system. Now let the state of the first valve's being open be represented by p while that of the second by q. Then, at any time either $p \wedge \neg q$ or $\neg p \wedge q$. Additionally, let the flow of the water through the first (respectively second) valve be represented by r (respectively s).

Obviously, the cause of r is p and the cause of s is q. Then, a causal theory will contain the sentences:

$$\Box(t, p) \wedge \Theta_1 \supset \Box(t+1, r),\ \forall t, \text{ where } \Theta_1 = \bigwedge_{i=1}^{m} \Diamond(t_{i1}, t_{i2}, a_i)\ .$$

$$\Box(t, q) \wedge \Theta_2 \supset \Box(t+1, s),\ \forall t, \text{ where } \Theta_2 = \bigwedge_{j=1}^{n} \Diamond(t_{j1}, t_{j2}, b_j)\ .$$

Assume that the state of the valves are causally related to a *common cause* (e.g. if there is a possibility for an agent which pushes only the first valve and closes it, this action causes the first valve to close and the second valve to open). In this case, the causal theory above might contain the following sentences where the pushing of the first valve is represented by u:

$$\Box(t, u) \wedge \Theta_3 \supset \Box(t+1, \neg p),\ \forall t, \text{ where } \Theta_3 = \bigwedge_{i=1}^{m} \Diamond(t_{i1}, t_{i2}, a_i)\ .$$

$$\Box(t, u) \wedge \Theta_4 \supset \Box(t+1, q),\ \forall t, \text{ where } \Theta_4 = \bigwedge_{j=1}^{n} \Diamond(t_{j1}, t_{j2}, b_j)\ .$$

It is noted that if $\Theta_3 = \Theta_4$, one cause produces more than one effect. This suggests that causal rules can represent multiple effects.

If the two changes have separate causes, the situation is easy. For example, let the first valve be open and the second closed. If there is an agent pushing the first valve to close it, there may be another agent pulling the second valve to open it. Then, closing of the first valve can be attributed to the pushing of it, and opening of the second valve to the pulling of it. Or, it may well be the case that one agent pushes the first valve while another pushes the second valve. In this case, there are two causes, namely pushing of the first and the second valves, that intervene with each other. Although each cause separately has the efficacy to produce its effect(s), they now prevent the changes that they will bring about. Since each one prevents the other from being operative, these two can be termed *counteracting causes*, following von Wright (Ref. 34, pp. 75–77). These two causes must be involved in the related causal sentences either in the form $\Box(t, u) \wedge \Diamond(t, \neg v)$ or in the form $\Box(t, v) \wedge \Diamond(t, \neg u)$ where u and v denote pushing of the first and the second valves, respectively.

So far, everything is on the side of the temporal precedence of causes over their effects. But, what happens if there is no cause of these two changes? This is quite possible because causal theories allow atomic sentences in the form of boundary conditions to be asserted for no reason. In fact, either p (the first valve's being open) or $\neg q$ (the second valve's being closed) can be asserted at any time into the causal theory. Then, how does one assure that, when only one of them is known, one will know the other occurred at the very same time? A set of sentences in the following form might be helpful:

$$\Box(t, p) \supset \Box(t, \neg q), \ \forall t .$$

$$\Box(t, \neg q) \supset \Box(t, p), \ \forall t .$$

Note that they do not contain a $\Diamond$-condition. This means that occurrence of p (respectively $\neg q$) *unconditionally* necessitates occurrence of $\neg q$ (respectively p). However, there may be cases in which some qualifications must hold for occurrences to be simultaneous.

4.4. Causal Theories: An Extended Definition

Definition 4.1. An *extended causal theory* Ω is a theory in CI, in which all sentences have the form $\Phi \wedge \Theta \supset \Box\varphi$, where

1. $\varphi =$ TRUE(t_a, t_b, p).
2. $\Phi = \bigwedge_{i=1}^{n} \Box\varphi_i$, where φ_i is an atomic base sentence with ltp t_i, $t_i \leq t_a$.
3. $\Theta = \bigwedge_{j=1}^{m} \Diamond\varphi_j$, where φ_j is an atomic base sentence with ltp t_j, $t_j \leq t_a$.
4. Φ or Θ (or both) may be empty.
5. It is assumed that there exists a global t_0 such that if $\Theta \supset \Box(t_a, t_b, p)$ is in Ω, then $t_0 < t_a$.
6. There do not exist two sentences in Ω such that one contains $\Diamond(t_a, t_b, p)$ on its l.h.s. while the other contains $\Diamond(t_a, t_b, \neg p)$ on its l.h.s.
7. If $\Phi_1 \wedge \Theta_1 \supset \Box(t_a, t_b, p)$ and $\Phi_2 \wedge \Theta_2 \supset \Box(t_a, t_b, \neg p)$ are in Ω, then $\Phi_1 \wedge \Theta_1 \wedge \Phi_2 \wedge \Theta_2$ is inconsistent.
8. There do not exist sentences in Ω of the form $\Phi \wedge \Diamond(t_a, t_b, \neg p) \wedge \Theta \supset \Box(t_b, t_c, p)$ or $\Phi \wedge \Diamond(t_a, t_b, p) \wedge \Theta \supset \Box(t_b, t_c, \neg p)$.

This definition says that causes can occur simultaneously with their effects (they can only coincide at a time point where the cause ceases while its effect starts). If cause and effect overlap for a period of time, the direction of prediction changes: either the past determines the future or the future determines the past.

Theorem 4.1. If Ω is an extended causal theory, then

1. Ω has a cmi model.
2. If M_1 and M_2 are both cmi models of Ω, and φ is any atomic base sentence, then $M_1 \models \Box\varphi$ iff $M_2 \models \Box\varphi$.

Proof. Appendix. ■

Theorem 4.2. If Ω is a finite extended causal theory of size n, then the set of the atomic base sentences known in the cmi models of Ω has size $O(n)$ and can be computed in time $O(n^2)$.

Proof. Appendix. ■

5. CONCLUSION

Shoham's causal theories have computationally simple model-theoretic properties. However, it turns out that computing with causal theories is not very efficient. Axiom schemata are not directly allowed in causal theories. New classes of causal theories have been introduced to capture generality with axiom schemata as well as to efficiently compute the atomic base sentences known in all cmi models of causal theories. It has been shown that computing with these causal theories is not time-dependent. It turned out that causal theories, in general, call for a syntactic sugar to obtain intended models. Such a syntactic sugar has been embedded in our YSP′-like causal theories. A model construction procedure has been proposed to compute the atomic base sentences in all cmi models of YSP′-like causal theories.

There are still some technical problems. One is prohibiting simultaneity of cause and effect. More generally, temporal propositions are not allowed on both sides of sentences in causal theories. In the second part of the paper it has been emphasized that permitting such propositions provides more expressive power and an extended definition has been given. The intervals of the propositions on both sides of sentences in these theories are not allowed to overlap, but meet at certain points in time. Provided that some assumptions hold, it has been shown that extended causal theories have unique cmi models.

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APPENDIX. PROOFS

A.1. Theorem 3.2

The algorithm below follows the construction in the proof of Theorem 3.1.

1. Let KNOWN and CONS be two lists. KNOWN contains $\Box\varphi_s$ and CONS is empty.
2. Find the axiom schema containing $\Box\varphi_s$ as its unique $\Box$-condition. If there exists such an axiom schema, then let CONS contain the r.h.s. of this schema such that the ltp of the r.h.s. becomes $t_1 + 1$ when t is replaced by t_1. Add this atomic base sentence into KNOWN.
3. If CONS is empty, then go to 5.
4. If the atomic base sentence in CONS is a $\Box$-condition in the persistence axiom schema, then add the atomic base sentences into KNOWN such that these atomic base sentences are obtained by replacing t in the $\Box$-condition by constants in the range $t_1 + 2$ to t_2. Then, first empty CONS, and let CONS contain only the atomic base sentence with ltp $= t_2$ obtained from the $\Box$-condition above.
5. Add $\Box\varphi_f$ into CONS and KNOWN.
6. If CONS contains $\Box\varphi_p{}'$ with ltp $= t_2$ and if $\Diamond\neg\varphi_f$ is not a $\Diamond$-condition in Θ_p, then add $\Box\varphi_p$ (by letting its ltp be $t_2 + 1$) into KNOWN.
7. Let $\Phi_c{}' \wedge \Theta_c{}' \supset \Box\varphi_c{}'$ be obtained by replacing t with t_2 in $\Phi_2 \wedge \Theta_c \supset \Box\varphi_c$. Check if each conjunct $\Box(t_i, p)$ of $\Phi_c{}'$ exists in CONS. If so, let the conjuncts of $\Theta_c{}'$ be of the form $\Diamond(t_j, p)$ (respectively $\Diamond(t_j, \neg p)$). If for each $\Diamond(t_j, p)$ (respectively $\Diamond(t_j, \neg p)$), $\Box(t_i, \neg p)$ (respectively $\Box(t_i, p)$) does not exist in CONS, then add $\Box\varphi_c{}'$ into KNOWN.
8. The set of atomic base sentences known in the unique cmi model of the YSP-like causal theory ζ are the ones in KNOWN.

Complexity:

Step 1: O(1) (initialization).
Step 2: O(n) (searching and matching).
Step 3: O(1) (testing).
Step 4: O(1) (matching two atomic base sentences).
Step 5: O(1) (add operation).
Step 6: O(n) (searching).
Step 7: O(n) (searching each condition of the sentence in a list of size at most 2).
Step 8: O(1) (reporting the set of atomic base sentences in KNOWN).

Consequently, the total time complexity of the algorithm is O(n). ■

A.2. Theorem 3.3

Part I. To prove that ζ' has a cmi model, a construction procedure is devised for a model M for ζ'. The construction is built upon the time-bounded Kripke interpretation M/t for a time point t (see Def. 2.16). It starts with a time-bounded Kripke interpretation at ltp t_a of the initial boundary condition and it proceeds by augmenting this interpretation. (Any primed base sentence φ' is obtained by replacing the variable t in φ by a constant. Φ', Θ', $\varnothing'$, and Π_i' denote the conjunction of base sentences obtained by replacing the variable t in each conjunct by a constant.)

1. Let t_a be the ltp of the initial boundary conditions of ζ'. Let $M/t_a \models \Box\varphi_{s_i}$, $i = 1, \ldots, n$, For any other φ appearing on the l.h.s. of the axiom schemata, let $M/t_a \not\models \Box\varphi'$.
2. Augment M/t_a into $M/t_a + 1$:
   $\text{Cons}_{t_a+1} = \{\Box(t_a + 1, p)$: $\Box\varphi_p \wedge \varnothing_p \wedge \Theta_p \supset \Box(t + 1, p) \in \zeta'$ such that $M/t_a \models \Box\varphi_p'$, $M/t_a \models \Theta_p'$ and $M/t_a \models \Pi_i'\forall\Pi_i'$ in $\varnothing_p'$ for $t = t_a$ (Def. 3.3), or $\Phi_c \wedge \Theta_c \supset \Box(t + 1, p) \in \zeta'$ and $M/t_a \models \Phi_c' \wedge \Theta_c'$ for $t = t_a\}$.
   Make the wff in Cons_{t_a+1} true and for any other φ whose ltp $= t_a + 1$, make $\Box\varphi$ false.
3. Cons_{t_a+1} contains the base wff appearing either on the r.h.s. of the persistence axiom schemata or on the r.h.s. of the causal axiom schemata. The sentences in the latter can falsify the l.h.s. of the persistence axiom schemata in which the former appear. Therefore, to find out what base sentences preserve their truth value for the next time point, one more iteration is needed. Then, augmentation of $M/t_a + 1$ into $M/t_a + 2$, $t_a + 2 < t_b$, is done by letting
   $\text{Cons}_{t_a+2} = \{\Box(t_a + 2, p)$: $\Box\varphi_p \wedge \varnothing_p \wedge \Theta_p \supset \Box(t + 1, p) \in \zeta'$ such that $M/t_a + 1 \models \Box\varphi_p'$, $M/t_a + 1 \models \Theta_p'$, and $M/t_a + 1 \models \Pi_i'\forall\Pi_i'$ in $\varnothing_p'$ for $t = t_a + 1\}$,
   making the wff in Cons_{t_a+2} true, and for any other φ whose ltp is $t_a + 2$, making $\Box\varphi$ false.
4. Augmentation of $M/t_a + 2$ into M/t_b is specified first by letting $M/t_b \models \Box\varphi_{f_j}$, $j = 1, \ldots, m$ (note that all final boundary conditions have ltp t_b), and then

letting $M/t_b \models \Box\varphi''\forall\Box\varphi' \in \text{Cons}_{t_a+2}$ such that $\Box\varphi''$ is obtained by replacing the time constants in each $\Box\varphi'$ with the time constants in the range $t_a + 3$ to t_b, For any other φ whose ltp is in the range $t_a + 3$ to t_b, make $\Box\varphi$ false.

5. Finally, M/t_b is augmented into $M/t_b + 1$ by letting
   $\text{Cons}_{t_b+1} = \{\Box(t_b + 1, p)$: $\Box\varphi_p \wedge \varnothing_p \wedge \Theta_p \supset \Box(t + 1, p) \in \zeta'$ such that $M/t_b \models \Box\varphi_p'$, $M/t_b \models \Theta_p'$ and $M/t_b \models \Pi_i'\forall\Pi_i'$ in $\varnothing_p'$ for $t = t_b$, or $\Phi_c \wedge \Theta_c \supset \Box(t + 1, p) \in \zeta'$ and $M/t_a = \Phi_c' \wedge \Theta_c'$ for $t = t_b\}$,
   making the wff in Cons_{t_b+1} true, and for any other φ whose ltp is $t_b + 1$, making $\Box\varphi$ false.

Part II. It must be shown that if there exists another model M' of ζ' which differs from M on the truth value of $\Box\varphi$ for some φ, then M' is not a cmi model for ζ'. Assume that there exists such a model M'. There are two possibilities:

1. $M \not\models \Box\varphi$ while $M' \models \Box\varphi$ for a φ with ltp $\leq t_a$. By Def. 2.7, this means that $M' \supset_{\text{ci}} M$.
2. M and M' differ on the truth value of $\Box\varphi$ with ltp $= t_c + 1$, $t_a \leq t_b$. There are two possibilities:
   (i) $M \models \Box\varphi$ while $M' \not\models \Box\varphi$. Let $\Box\varphi$ be of the form $\Box(t_c + 1, p)$:
   First, if $t_c = t_a$, since $M \models \Box\varphi$, there exists either an axiom schema $\Box\varphi_p \wedge \varnothing_p \wedge \Theta_p \supset \Box(t + 1, p) \in \zeta'$ such that for $t = t_c M \models \Box\varphi_p' \wedge \varnothing_p' \wedge \Theta_p$ or an axiom schema $\Phi_c \wedge \Theta_c \supset \Box(t + 1, p) \in \zeta'$ such that $M \models \Phi_c' \wedge \Theta_c$ for $t = t_c$. Since M and M' agree on the knowledge of all base sentences with ltps $\leq t_a$, by the second construction step $M' \models \Box\varphi$. This is a contradiction.

   If $t_a + 1 < t_c < t_b$, then by the third step of the construction procedure there exists a persistence axiom schema $\Box\varphi_p \wedge \varnothing_p \wedge \Theta_p \supset \Box(t + 1, p) \in \zeta'$ such that $M/t_a + 1 \models \Box(t + 1, p)$ for $t = t_a$. Since there exists no known atomic base sentence with ltp $< t_c$ other than φ, it will always be the case that $M/t \models \Box\varphi_p' \wedge \varnothing_p' \wedge \Theta_p'$, $t_a + 1 < t < t_c$. Then, $M/t_c \models \Box(t_c, p)$ and hence $M/t_c \models \Box\varphi$. Since M and M' agree on the knowledge of all base sentences with ltps $\leq t_a$, $M'/t_a + 1 \models \Box\varphi$. But by the discussion above, this implies that $M'/t_c \models \Box\varphi$. This contradicts the assumption that $M'/t_c \not\models \Box\varphi$.

   For $t_c = t_b$, $M/t_b + 1 \models \Box\varphi$. This is true iff one of the following conditions hold: $M/t_b \models \Box\varphi_p' \wedge \varnothing_p' \wedge \Theta_p'$ for $t = t_b$ or $M/t_b = \Phi_c' \wedge \Theta_c'$ for $t = t_b$. But it is known that M and M' have the same atomic base sentences whose ltps $\leq t_b$. Then, $M'/t_b + 1 \models \Box\varphi$, contradicting the assumption that $M'/t_b + 1 \not\models \Box\varphi$.
   (ii) $M \not\models \Box\varphi$ while $M' \models \Box\varphi$. Again by Def. 2.7, it follows that $M' \supset_{\text{ci}} M$.

■

A.3. Theorem 4.1

We first give some definitions which will be needed in the sequel.

Definition A.3.1. The *earliest time point* (etp) of a base sentence is the earliest time point mentioned in it.

1. The etp of $\text{TRUE}(t_a, t_b, p) = t_a$.
2. The etp of $\varphi_a \wedge \varphi_b = \min\{\text{etp of } \varphi_a, \text{ etp of } \varphi_b\}$.
3. The etp of $\neg\varphi =$ the etp of φ.
4. The etp of $\forall v\varphi$ is the minimum among the etps of all φ' which result from substituting in φ a time point symbol for all free occurrences of v, or $-\infty$ if there is no such minimum etp.

Definition A.3.2. The *temporally meeting set of sentences at time* t, TMS_t, are those sentences in the causal theory Ω such that for any sentence, the etp and ltp of the base sentence on its r.h.s. is the same and equal to t, and the ltp of at least one of the base sentences on its l.h.s. is equal to the etp (ltp) of the base sentence on its r.h.s.

For example, if Ω contains $\Box(t_a, t_b, p) \wedge \Diamond(t_c, t_d, q) \supset \Box(t_d, r)$ and $\Box(t_a, t_d, u) \supset \Box(t_d, w)$, then they are in TMS_{t_d} (assuming that $t_a \le t_b \le t_d$ and $t_c \le t_d$).

Definition A.3.3. The *bounded set of sentences at time* t, BS_t, are those sentences with ltp t.

If $\Box(t_a, t_b, p) \wedge \Diamond(t_c, t_d, q) \supset \Box(t_d, t_f, r)$ and $\Box(t_c, t_f, u) \supset \Box(t_f, w)$ are in Ω, then they are in BS_{t_f}. But note that only $\Box(t_c, t_f, u) \supset \Box(t_f, w)$ is in TMS_{t_f}. Then, a sentence in Ω is always in BS_t for some time t. But it may or may not be in any TMS.

Definition A.3.4. The *temporally dependent set of sentences at time* t, TDS_t, are the sentences in TMS_t of Ω such that if φ is on the r.h.s. of a sentence in TMS_t, then it should be the case that either $\Box\varphi$ or $\Diamond\neg\varphi$ appears on the l.h.s. of other sentences in TMS_t.

For example, let TMS_{t_f} for an extended causal theory Ω be as follows:

$$\text{TMS}_{t_f} = \{\Box(t_a, t_b, p) \wedge \Diamond(t_c, t_f, q) \supset \Box(t_f, r)\ ,$$

$$\Box(t_f, r) \supset \Box(t_f, s)\ ,$$

$$\Box(t_a, t_b, p) \wedge \Diamond(t_e, t_f, u) \supset \Box(t_f, v)\}\ .$$

Then, $\text{TDS}_{t_f} = \{\Box(t_a, t_b, p) \wedge \Diamond(t_c, t_f, q) \supset \Box(t_f, r), \Box(t_f, r) \supset \Box(t_f, s)\}$.

A construction procedure will be needed to build a model M for the extended causal theory Ω. This will be done by augmenting some time-bounded Kripke interpretation. This augmentation, however, cannot be used in the construction of M. There exist some technical defects that can destroy the unique-model property of extended causal theories. Starting with Shoham's augmentation and considering these technical problems, a new augmentation will be specified. Then, it will be used to show that in all cmi models of any extended causal theory the same atomic base sentences are known.

Shoham specifies an augmentation of a time-bounded Kripke interpretation M/t to $M/t+1$ as follows (Ref. 2, p. 112):

$$\mathrm{Cons}_{t+1} = \{\Box(t', t+1, x)\colon \Phi \wedge \Theta \supset \Box(t', t+1, x) \in \Omega \text{ and } M/t \models \Phi \wedge \Theta\} .$$

$M/t+1$ is obtained by making all wff in Cons_{t+1} and all their tautological consequences true, and for any other φ' whose ltp is $t+1$, making $\Box\varphi'$ false.

Now, consider the following sentences in Ω. Note that they are all in TMS_{t_d}, but except the first one they are all in BS_{t_d} as well.

$$\Box(t_a, t_b, s) \supset \Box(t_c, t_d, q) .$$

$$\Box(t_a, t_b, s) \wedge \Diamond(t_d, \neg r) \supset \Box(t_d, v) .$$

$$\Box(t_d, r) \supset \Box(t_d, u) .$$

$$\Box(t_a, t_b, p) \wedge \Diamond(t_c, t_d, q) \supset \Box(t_d, r) .$$

Assume that $M/t_c \models \Box(t_a, t_b, s)$ and $M/t_c \models \Box(t_a, t_b, p)$. We would like to augment M/t_c to M/t_d. Now, if all sentences are examined in the order they are written, Cons_{t_d} will contain $\Box(t_c, t_d, q)$ since $M/t_c \models \Box(t_a, t_b, s)$. However, none of the l.h.s. of the other sentences is satisfied since they contain base sentences with ltp t_d. Then, as a result of the augmentation only $\Box(t_c, t_d, q)$ is obtained. But the l.h.s. of other sentences can also be satisfied. For example, for $\Box(t_a, t_b, s) \wedge \Diamond(t_d, \neg r) \supset \Box(t_d, v)$, $M/t_d \models \Box(t_d, v)$ iff $M/t_c \models \Box(t_a, t_b, s)$ and $M/t_d \not\models \Box(t_d, r)$. Therefore, in order to perform the augmentation successfully, one must also consider the sentences in Cons_{t_d}. That is, a possible augmentation might be:

$\mathrm{Cons}_{t+1} = \{\Box(t', t+1, x)\colon\ \Phi \wedge \Theta \supset \Box(t', t+1, x) \in \Omega$, but $\notin \mathrm{TMS}_{t+1}$ and $M/t \models \Phi \wedge \Theta$, or $\Phi \wedge \Theta \supset \Box(t', t+1, x) \in \Omega$ and $\in \mathrm{TMS}_{t+1}$ such that $\forall \Box\varphi \in \Phi$, $\Box\varphi \in \mathrm{Cons}_{t+1}$ if ltp of φ is $t+1$, $M/t \models \Box\varphi$ otherwise, and $\forall \Diamond\varphi \in \Theta$, $\Box\neg\varphi \notin \mathrm{Cons}_{t+1}$ if ltp of φ is $t+1$, $M/t \not\models \neg\Box\varphi$ otherwise$\}$.

$M/t+1$ is obtained in the same way as in the first specification.

Returning to the example set of sentences above, the augmentation of M/t_c to M/t_d can be obtained:

1. For $\Box(t_a, t_b, s) \supset \Box(t_c, t_d, q)$, $\mathrm{Cons}_{t_d} = \{\Box(t_c, t_d, q)\}$ since $M/t_c \models \Box(t_a, t_b, s)$.
2. For $\Box(t_a, t_b, s) \wedge \Diamond(t_d, \neg r) \supset \Box(t_d, v)$, $\mathrm{Cons}_{t_d} = \{\Box(t_c, t_d, q), \Box(t_d, v)\}$ since $M/t_c \models \Box(t_a, t_b, s)$ and $\Box(t_d, r) \notin \mathrm{Cons}_{t_d}$.

3. For $\Box(t_d, r) \supset \Box(t_d, u)$, $\text{Cons}_{t_d} = \{\Box(t_c, t_d, q), \Box(t_d, v)\}$ since $\Box(t_d, r) \notin \text{Cons}_{t_d}$.
4. For $\Box(t_a, t_b, p) \wedge \Diamond(t_c, t_d, q) \supset \Box(t_d, r)$, $\text{Cons}_{t_d} = \{\Box(t_c, t_d, q), \Box(t_d, v), \Box(t_d, r)\}$ since $M/t_c \models \Box(t_a, t_b, p)$ and $\Box(t_c, t_d, \neg q) \notin \text{Cons}_{t_d}$.

But Cons_{t_d} does not have the right sentences. Since Cons_{t_d} contains $\Box(t_d, r)$, it will cause the l.h.s. of the second sentence to fail and the l.h.s. of the third sentence to be satisfied. Hence, it must be the case that $\text{Cons}_{t_d} = \{\Box(t_c, t_d, q), \Box(t_d, u), \Box(t_d, r)\}$. Therefore, the augmentation specified above is incorrect.

Another technical problem has to do with the *pluri-extensionality* of the nonmonotonic systems. One of the properties of nonmonotonic systems is that they may produce several sets of possible conclusions. For example, consider the following set of premises where Unless(p) is true iff p cannot be inferred:[35]

$$S = \{p, p \wedge \text{Unless}(q) \supset r, p \wedge \text{Unless}(r) \supset q\} .$$

Depending on the order in which inferences have been applied, one can obtain two conclusions: $\{p, r\}$ as a result of the subset $\{p, p \wedge \text{Unless}(q) \supset r\}$ and $\{p, q\}$ as a result of the subset $\{p, p \wedge \text{Unless}(r) \supset q\}$. However, these two conclusions cannot be inferred conjointly; if r is inferred, then q cannot be inferred, and vice versa. If one is mainly interested in constructing only one of these possible sets, then the system is inconsistent in a sense that the intended model may not be obtained.

This is the case with extended causal theories. To illustrate the situation consider the following set of sentences that constitute TMS_{t_d} of an extended causal theory Ω.

$$\text{TMS}_{t_d} = \{\Box(t_d, p) ,$$

$$\Box(t_d, p) \wedge \Diamond(t_d, \neg q) \supset \Box(t_d, r) ,$$

$$\Box(t_d, p) \wedge \Diamond(t_d, \neg r) \supset \Box(t_d, q)\} .$$

Assuming that Cons_{t_d} contains only $\Box(t_d, p)$ and assuming that the sentences in TMS_{t_d} are examined in the order they are written, one finds out that $\text{Cons}_{t_d} = \{\Box(t_d, p), \Box(t_d, r)\}$. If the order of the last two sentences in TMS_{t_d} is changed, then $\text{Cons}_{t_d} = \{\Box(t_d, p), \Box(t_d, q)\}$ is obtained. Thus, the order of these sentences is important.

In the following proof, an augmentation which will not cause such problems will be used.

Let there be two models M and M' such that $M' \supset_{\text{ci}} M$ and they differ on the truth value of some sentence $\Box\varphi$.

1. By definition, there exists a t_0 such that it precedes the ltp of any φ where $\Box\varphi$ appears as in the r.h.s. of a boundary condition in Ω. Then, $M/t_0 \models \Box\varphi$ for any φ with ltp $\leq t_0$, and $M/t_0 \not\models \Box\varphi'$ for any other φ' with ltp $\leq t_0$.

M/t_0 *partially* satisfies all the boundary conditions of Ω since their ltps are greater than t_0. Obviously, M/t_0 also *partially* satisfies all the causal rules since the truth values of sentences with ltp $> t_0$ depend on the sentences with ltp $\leq t_0$, and the l.h.s. of causal rules with ltp $\leq t_0$ are falsified.

2. The construction progresses iteratively over time.

   $\text{Cons}_{t+1} = \{\Box(t', t+1, x): \Phi \wedge \Theta \supset \Box(t', t+1, x) \in \text{BS}_{t+1}$, but $\notin \text{TMS}_{t+1}$, and $M/t \models \Phi \wedge \Theta\}$,

   $\text{Cons}'_{t+1} = \{\Box(t', t+1, x): \Phi \wedge \Theta \supset \Box(t', t+1, x) \in \text{TDS}_{t+1}$, such that $\forall \Box\varphi \in \Phi$, $\Box\varphi \in (\text{Cons}_{t+1} \cup \text{Cons}'_{t+1})$ if ltp of φ is $t+1$, $M/t \models \Box\varphi$ otherwise, and $\forall \Diamond\varphi \in \Theta$, $\Box\neg\varphi \notin (\text{Cons}_{t+1} \cup \text{Cons}'_{t+1})$ if ltp of φ is $t+1$, $M/t \not\models \neg\Box\varphi$ otherwise$\}$,

   $\text{Cons}''_{t+1} = \{\Box(t', t+1, x): \Phi \wedge \Theta \supset \Box(t', t+1, x) \in \text{TMS}_{t+1}$, but $\notin \text{TDS}_{t+1}$ such that $\forall \Box\varphi \in \Phi$, $\Box\varphi \in (\text{Cons}_{t+1} \cup \text{Cons}'_{t+1} \cup \text{Cons}''_{t+1})$ if ltp of φ is $t+1$, $M/t \models \Box\varphi$ otherwise, and $\forall \Diamond\varphi \in \Theta$, $\Box\neg\varphi \notin (\text{Cons}_{t+1} \cup \text{Cons}'_{t+1} \cup \text{Cons}''_{t+1})$ if ltp of φ is $t+1$, $M/t \not\models \neg\Box\varphi$ otherwise$\}$.

It is assumed that the sentences of Ω are examined in the order they are written. For this reason, although one can obtain more than one possible set Cons'_{t+1}, a unique set is constructed under this assumption. (It must be admitted that this is a very strong assumption.)

$M/t+1$ is obtained by first making all wff in Cons_{t+1} true, then making all wff in Cons'_{t+1} true, and finally making all wff in Cons''_{t+1} true. For any other φ with ltp $t+1$, $\Box\varphi$ is made false.

The last step in the proof of the theorem is to show that this M is chronologically more ignorant than any M' which differs on the truth value of $\Box\varphi$ for some φ. There are two cases:

1. It may be that $M' \models \Box\varphi$ for some φ with ltp $\leq t_0$. But this, by Def. 2.7, implies that $M' \supset_{ci} M$.
2. It may be that there exists a time point t, $t_0 \leq t$, and that either $M' \not\models \Box\varphi$ and $M \models \Box\varphi$, or $M' \models \Box\varphi$ and $M \not\models \Box\varphi$ or for some φ with ltp $= t+1$. Now let $M \models \Box\varphi$. Then two cases must be examined:
   (a) There exists a sentence $\Phi \wedge \Theta \supset \Box\varphi \in \Omega$ such that the ltps of the base sentences in Φ and Θ are $\leq t$, and $M/t \models \Phi \wedge \Theta$. It is known that M and M' agree on the knowledge of all base sentences having ltp $\leq t$. Hence, it follows that $M' \models \Box\varphi$ since $M'/t \models \Phi \wedge \Theta$. But this contradicts $M' \not\models \Box\varphi$.
   (b) There is a sentence of the form $\Phi \wedge \Theta \supset \Box\varphi \in \Omega$, such that the ltps of the base sentences in Φ and Θ are $\leq t+1$. Then, the etp of φ must be equal to its ltp $(t+1)$. This implies that $M/t+1 \models \Phi \wedge \Theta$ and it is known from case (a) that for any φ' in $\Phi \wedge \Theta$ with ltp $t+1$, $M \models \Box\varphi'$ and $M' \models \Box\varphi'$. Since M and M' agree on the knowledge of all base sentences with ltp $\leq t$ and they agree on the knowledge of all base sentences in Φ and Θ with ltp $\leq t+1$, it must be the case that $M' \models \Box\varphi$, contradicting $M' \not\models \Box\varphi$.

Similarly, if $M' \models \Box\varphi$ and $M \not\models \Box\varphi$, in light of the discussion above, for any φ with ltp $= t + 1$, whenever $M' \models \Box\varphi$, it must be the case that $M \models \Box\varphi$.

Consequently, if there is a model M' differing from the model M constructed for Ω, then $M' \supset_{\text{ci}} M$. ∎

A.4. Theorem 4.2

(The reader is referred to Sec. A.3 for some crucial definitions.) The algorithm below organizes the sentences of Ω in ascending order of their ltps. Then, each set of sentences with the same ltp is reordered. This reordering is done by first dividing these sentences into classes and then rearranging these classes among themselves. As a final step, the sentences are examined to see if their l.h.s. are satisfied. If so, their r.h.s. are marked accordingly.

1. Let T be the list of all sentences in Ω. Let S be a list.
2. Gather all atomic base sentences appearing in T into a list S by dropping negation signs.
3. Sort T in ascending order by the ltp of the r.h.s. of the sentences in it. Also sort S in ascending order by the ltp of the base sentences.
4. Remove all duplicates of any atomic base sentence in S. Mark all members UNMARKED.
5. Gather all sentences in T into bounded sets of sentences, BS, such that if say $\Phi \wedge \Theta \supset \Box(t_e, t_f, [\neg]r)$ is a sentence in T, then it must be in the bounded set of sentences at time t_f, BS_{t_f}. At the end, BS contains the ordered sets BS_{t_i} for, say, $i = a_1, \ldots, a_n$.
6. For each BS_{t_i}, divide it into two groups; the temporally meeting set of sentences at t_i, TMS_{t_i}, and the set of other sentences, NMS_{t_i}. Replace BS_{t_i} with these two sets such that NMS_{t_i} appears before TMS_{t_i}. That is, $\text{BS}_{t_i} = \text{NMS}_{t_i} \cup \text{TMS}_{t_i}$. For example, if

$$\begin{aligned}\text{BS}_{t_d} = \{&\Box(t_a, t_b, p) \supset \Box(t_c, t_d, q)\,,\\ &\Box(t_d, u) \wedge \Diamond(t_c, t_d, \neg r) \supset \Box(t_d, v)\,,\\ &\Box(t_d, p) \wedge \Diamond(t_c, t_d, q) \supset \Box(t_d, r)\}\,,\end{aligned}$$

then

$$\text{NMS}_{t_d} = \{\Box(t_a, t_b, p) \supset \Box(t_c, t_d, q)\}\,,$$

and

$$\begin{aligned}\text{TMS}_{t_d} = \{&\Box(t_d, u) \wedge \Diamond(t_c, t_d, \neg r) \supset \Box(t_d, v)\,,\\ &\Box(t_d, p) \wedge \Diamond(t_c, t_d, q) \supset \Box(t_d, r)\}\,.\end{aligned}$$

7. For each TMS_{t_i}, divide it into two groups: the temporally dependent set of sentences at t_i, TDS_{t_i} and the set of other sentences, NDS_{t_i}. Replace TMS_{t_i} with these two sets such that TDS_{t_i} appears before NDS_{t_i}. That is, $\text{TMS}_{t_i} = \text{TDS}_{t_i} \cup \text{NDS}_{t_i}$. For the above example,

$$\text{TDS}_{t_d} = \{\Box(t_d, p) \wedge \Diamond(t_c, t_d, q) \supset \Box(t_d, r)\}\ ,$$

$$\text{NDS}_{t_d} = \{\Box(t_d, u) \wedge \Diamond(t_c, t_d, \neg r) \supset \Box(t_d, v)\}\ .$$

(Now BS contains the sets BS_{t_i}'s such that $\text{BS}_{t_i} = \text{NMS}_{t_i} \cup \text{TDS}_{t_i} \cup \text{NDS}_{t_i}$ for some t_i. All these sentences are still in increasing order of their ltps. From now on, the names BS_{t_i}, NMS_{t_i}, TDS_{t_i}, and NDS_{t_i} will not be used. Since the list BS has all the sentences, each sentence in BS will be examined.)

8. If BS is empty, then halt. The atomic sentences that are known in all cmi models of the extended causal theory are those sentences marked YES in S plus the negation of those marked NO in S.
9. Remove the first sentence of BS, and let this sentence be $\Phi \wedge \Theta \supset \Box(t_a, t_b, [\neg]p)$. For each conjunct $\Box(t_{i1}, t_{i2}, [\neg]p_i)$ of Φ and each conjunct $\Diamond(t_{i1}, t_{i2}, [\neg]p_i)$ of Θ, determine how $\text{TRUE}(t_{i1}, t_{i2}, p_i)$ is marked in S by performing binary search on S. If one of the following conditions is true:
   (a) $\Box(t_{i1}, t_{i2}, p_i)$ is a conjunct of Φ and $\text{TRUE}(t_{i1}, t_{i2}, p_i)$ is not marked YES in S,
   (b) $\Box(t_{i1}, t_{i2}, \neg p_i)$ is a conjunct of Φ and $\text{TRUE}(t_{i1}, t_{i2}, p_i)$ is not marked NO in S,
   (c) $\Diamond(t_{i1}, t_{i2}, p_i)$ is a conjunct of Θ and $\text{TRUE}(t_{i1}, t_{i2}, p_i)$ is marked NO in S,
   (d) $\Diamond(t_{i1}, t_{i2}, \neg p_i)$ is a conjunct of Θ and $\text{TRUE}(t_{i1}, t_{i2}, p_i)$ is marked YES in S,

   then go to 8,
   else mark $\text{TRUE}(t_a, t_b, p)$ in S with YES if the r.h.s. is $\Box(t_a, t_b, p)$, and NO if it is $\Box(t_a, t_b, \neg p)$. Go to 8.

Complexity:

Step 1: O(1) (initialization).
Step 2: $O(n)$ (collection).
Step 3: $O(n \log n)$ (sorting).
Step 4: $O(n)$ (removing duplicates and marking).
Step 5: $O(n)$. Examining the ltp of the r.h.s. of each sentence in T suffices to classify the sentences. Note that the sentences in T are currently sorted with respect to the ltp of their r.h.s.
Step 6: $O(n)$. This step requires determining the sentences that have r.h.s. with the same etp and ltp such that at least one of the conjuncts on the l.h.s. has this etp (ltp) as its ltp. For each class BS_{t_i}, this can be done by examining the ltp of the l.h.s. of

each sentence and comparing it to the etp and ltp of its r.h.s. Since this is done for all sentences in every class BS_{t_i}, this step requires all sentences to be examined at most once.

Step 7: $O(n^2)$. The r.h.s. of each sentence in every class TMS_{t_i} must be on the l.h.s. of all sentences in its corresponding class TMS_{t_i}.

Step 8: O(1) (testing and reporting).

Step 9: $O(n \log n)$. Label checking can be done at most n times. Determination of the label of each conjunct requires binary search. A new labeling can be done in time $O(n \log n)$ since it also requires binary search. There can be at most n new labeling operations during the execution of the algorithm.

Hence, the total time complexity of the algorithm is $O(n^2)$. ∎

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