Aporia vol. 30 no. 1–2020

An Eternal Society Paradox

Wade A. Tisthammer

An eternal society with the abilities of ordinary humans in each year of its existence would have had the ability to actualize a logical contradiction. This fact casts doubt on the metaphysical possibility of an infinite past. In addition to using this paradox in an argument against an infinite past, one can also use the paradox *mutatis mutandis* as a decisive argument against the sempiternality of God.

1. Background

While José Benardete is often credited as the originator of the Grim Reaper paradox, his original paradoxes are *inter alia* about deafening peals from a gong in which there is no first peal, and later about a series of assassins (whom he does not refer to as Grim Reapers). In his work *Infinity*, Benardete writes:

> Let the peal of a gong be heard in the last half of a minute, a second peal in the preceding 1/4 minute, a third peal in the 1/8 minute before that, etc. *ad infinitum*....Of particular interest is the following puzzling case. Let us assume that each peal is so very

Wade A. Tisthammer graduated from the University of Minnesota in 2007 with a Bachelor of Science in computer science and minor in mathematics. While working full-time as a software developer, he is currently a part-time undergraduate philosophy student at the University of New Orleans with plans to seek a graduate degree in philosophy. His philosophical interests include philosophy of religion, moral philosophy, and philosophy of mind.

> loud that, upon hearing it, anyone is struck deaf—totally and permanently. At the end of the minute we shall be completely deaf (any one peal being sufficient), but we shall not have heard a single peal! For at most we could have heard only one of the peals (any single peal striking one deaf instantly), and which peal could we have heard? There simply was no first peal. We are all familiar with various physical processes that are followed by what are called after-effects. We are now tempted to coin the barbarous neologism of a *before-effect*. The infinite sequence of deafening peals would seem logically to entail the before-effect of total deafness. For we must be in a state of deafness before each peal....The paradox of the before-effect may be generalized over a whole range of cases. A man is shot through the heart during the last half of a minute by A. B shoots him through the heart during the preceding 1/4 minute, C during the 1/8 minute before that, &c. *ad infinitum*. Assuming that each shot kills instantly (if the man were alive), the man must be already dead before each shot. Thus he cannot be said to have died of a bullet wound. Here, again, the infinite sequence logically entails a before-effect.
>
> (Benardete 255, 259)

Even in the case of the assassins, Benardete did not believe a true contradiction emerges here, but rather that the mereological sum of the infinite series of bullets causes the man's death. Something similar applies to the infinite series of peals; at no instant does any individual peal strike one deaf, and yet the peals do make one deaf.

> At twelve o'clock sharp, certainly, we were not yet in a state of deafness, though at any instant after twelve (waiving the infinitesimal tail-end) we are, and have been, deaf. As to the dead man, although he did not die of any *single* bullet wound, his death was certainly *caused* by the infinite fusillade of shots. Here, again, although he is already dead prior to each shot, he remains alive at any assigned instant which is prior to them all. Thus he cannot be said to have died at any moment of time whatever! Nor can we be said to have been struck deaf at any instant of time.
>
> (Benardete 260)

After mentioning some of Benedarte's paradoxes, including the paradox of peals and the unfortunate man getting shot in the heart, John Hawthorne says:

> There is an infinite series of assassins, each tagged with a natural number, no pair tagged with the same number, no number that isn't tagged to some assassin. Assassin 1 is disposed to attack Bob with a machete if Bob is still around at 2 pm. If he attacks, he will take half an hour to kill Bob. It is causally impossible for assassin 1 to attack Bob and fail to kill him within half an hour. Assassin 2 is disposed to attack Bob with a machete if Bob is still around at 1:30 and will take quarter of an hour to do it. It is causally impossible for assassin 2 to attack Bob and fail to kill him within quarter of an hour, and so on. Each assassin is unsurvivable as far as Bob is concerned....For each time which is such that an assassin is disposed to begin attacking Bob at that time, there are infinitely many assassins which are disposed to attack Bob earlier.
>
> (Hawthorne 267)

Like Benedarte, Hawthorne here does not conclude a bona fide self-contradiction emerges and instead believes that the above fusion of the assassins causes Bob's death. The credit for the paradox of the assassins being Grim Reapers—and the adding of crucial details to guarantee a bona fide self-contradiction, such as explicitly stipulating that you can die only via the motion of a Grim Reaper's scythe thereby preventing assassination by mereological summation—goes to David Chalmers after dutifully referencing Benardete and Hawthorne:

> There are countably many grim reapers, one for every positive integer. Grim reaper 1 is disposed to kill you with a scythe at 1pm, if and only if you are still alive then (otherwise his scythe remains immobile throughout), taking 30 minutes about it. Grim reaper 2 is disposed to kill you with a scythe at 12:30 pm, if and only if you are still alive then, taking 15 minutes about it. Grim reaper 3 is disposed to kill you with a scythe at 12:15 pm, and so on. You are still alive just before 12pm, you can only die through the motion of a grim reaper's scythe, and once dead you stay dead. On the face of it, this situation seems conceivable — each reaper seems conceivable individually and intrinsically, and it seems reasonable to combine distinct individuals with distinct intrinsic properties into one situation. But a little reflection

> reveals that the situation as described is contradictory. I cannot survive to any moment past 12pm (a grim reaper would get me first), but I cannot be killed (for grim reaper n to kill me, I must have survived grim reaper n+1, which is impossible). So the description D of the situation is prima facie positively conceivable but not ideally positively conceivable.
>
> (Chalmers 154)

Thus it is Chalmers who gives us the sort of Grim Reaper paradox that many people think of when using the term "Grim Reaper paradox"—a paradox not only of Grim Reapers in a beginningless supertask but also with certain specific conditions that make a self-contradiction unavoidable.

It is worth noting this paradox's ramifications for the metaphysical possibility of supertasks in general, or at least those without a beginning (when it comes to supertasks of countably infinite events, this is known as supertasks of order-type ω^* which are order isomorphic to the set of negative integers, as opposed to order-type ω supertasks that have a beginning but no endpoint and are order isomorphic to the set nonnegative integers) (Fletcher 571, 573). We can envisage each Grim Reaper flipping a lamp switch just before he checks on Fred. It would seem that if a beginningless supertask were possible, then the Grim Reaper paradox would be possible. Or if one prefers supertasks with only lamps, consider this variant. There is a touch lamp that is on at 8:00 and is turned off only if someone touches it. Jane checks on the lamp at 8:00 + $\frac{1}{n}$ hour for all $n \in \mathbb{N}$ (8:00 + $\frac{1}{1}$ hour, 8:00 + ½ hour...) to see whether the lamp is turned off. If it is already turned off, she does nothing (or if one considers a supertask requiring the agent do something in each successive event, imagine her flipping a lamp switch of a different lamp during these instances). If however the lamp is on, she touches the lamp to turn it off. Is the lamp on or off at 9:00? As in the Grim Reaper paradox, we have a contradiction. It cannot still be turned on because e.g. Jane would have turned the lamp off at 8:30 (8:00 + ½ hour) if it were on then. It is equally impossible for the lamp to be off because there is no instance where Jane touches the lamp, and if Jane never touches the lamp it remains turned on.

2. An Argument for a Finite Past

Using the Grim Reaper paradox Alexander Pruss created an ingenious argument for the finitude of the past. This bears relevance to the *kalam* cosmological argument which argues that the universe began to exist

and thus requires a cause. With the help of Hilbert's Hotel, a hotel having infinitely many rooms, Pruss's Grim Reaper argument goes like this:

(1) If there could[1] be a backwards infinite sequence of events, Hilbert's Hotel would be possible.

(2) If Hilbert's Hotel were possible, the Grim Reaper Paradox could happen.

(3) The Grim Reaper Paradox cannot happen.

(4) Therefore, there cannot be a backwards infinite sequence of events.

For (1), Pruss imagines each individual event of an infinite backwards series of events creating a room of the famous hotel (and no such room is destroyed). For (2), Pruss imagines a factory in each room of the hotel creating a Grim Reaper, and the Grim Reapers then go forth on their curious task of paradoxical killing. Robert Koons has taken inspiration from this to develop a more rigorous and mathematical (if not arcane) version of this argument (256–267). In response, one could claim that the Grim Reaper paradox is impossible due to the impossibility of arbitrarily compressible time (or Grim Reapers), rather than the impossibility of an infinite past. In the remainder of the paper, I will show how these concerns can be resolved by considering a thought experiment about an eternal society possessing the abilities of ordinary humans in each year of its existence.

3. The Eternal Society Paradox

Among other things, humans possess the remarkable ability to pass complex information along from one generation to the next. This passing of information occurs over a variety of mediums ranging from the written word to electro/digital forms of communication. While it is possible that information obtained by one generation will be preserved by some method for following generations, this transmission is not guaranteed. For each generation that passes, there is at least a slight chance that the transmission of information from one generation to the next will be garbled. Consequently, as time rolls forward, this probability of mis-transmitting information compounds such that the chance that

[1] Throughout this paper let us assume the type of possibility in question is metaphysical possibility, as opposed to e.g. physical possibility.

the information content of any given proposition will be preserved after several quadrillion years is possible, but perhaps unlikely. Still, what matters for our purposes is that it is merely *possible* for a society, possessing the abilities of ordinary humans, to transmit the same propositional content year after year indefinitely. Let us imagine, for the sake of our example, that there exists some society known as the Eternal Society that has existed sempiternally (existing for a beginningless, infinite duration) and has the ability to transmit propositional content indefinitely. The Eternal Society also possesses other abilities common to modern humans along with many of the usual modern societal amenities (coins, books, internet, etc.), in each year of its existence. When I say that the Eternal Society has the abilities of ordinary humans in each year of its existence, by this I essentially mean that in each year of its existence the Eternal Society can do what we humans can do in contemporary society. For example, in each year, members of the Eternal Society can sing songs, flip coins, write books, host marvelous parties, and publish clever paradoxes in philosophy journals. Intuitively, it seems that an Eternal Society would be possible if an infinite past were possible. With the aforementioned abilities of the Eternal Society in mind, consider the following scenario:

> S_1 In the Eternal Society there is a ritual known as the Annual Coin Flipping Tradition during each year of its existence. Each year, as part of the tradition, the members of the society flip a coin such that if the coin comes up heads, they all get together and do a particular chant (the chant being "Grim Reapers do not exist!" repeated seven times) though only if they have not done the chant before; but if the coin does not come up heads, they do not do the chant for that year. Last year, the coin came up heads for the first time and they did the chant.

Ex hypothesi the Eternal Society can transmit propositional content year after year, and so they would be able to keep track of whether they did the chant by doing the following each year:

> If they did the chant this year:
>
> > They transmit the propositional content *We did the chant* to the next year.
>
> If they did not do the chant this year:
>
> > If *We did the chant* was transmitted from the previous year, *We did the chant* is transmitted to

> the next year. Otherwise, *We did not do the chant* is transmitted to the next year.

Thus for any given year, the Eternal Society can keep track of whether they did the chant before. There are many imaginable ways to keep track of this beyond simple oral transmission; e.g. whenever they do the chant, they write in a particular webpage that they have done the chant if no record of doing the chant already exists in the webpage, and the webpage could then be consulted to determine whether they did the chant. The Eternal Society can do anything ordinary humans can do in each year of its existence, and ordinary humans certainly have the ability to flip coins, create webpages, hold annual traditions, etc. So S_1 would be possible if an Eternal Society were possible. Yet S_1 is not possible, for if it were, the following scenario would also be possible:

> S_2 The Eternal Society has the aforementioned Annual Coin Flipping Tradition, but the coin comes up heads each year of the infinite past.

The coin flips are probabilistically independent events. If it is possible for the coin to not come up heads each year until last year, then it would have also been possible for the coin to come up heads each year. But in S_2, did they ever do the chant? They would have had to have done it in some year, for they would have done it last year if they had never done it before, and yet there is no year in which they could have done it, because no matter which year one points to, there is a prior year in which they would have done the chant had they not done the chant before. They had to have done the chant, and yet they could not have done the chant. We thus arrive at a contradiction, and scenario S_2 is impossible, which then makes scenario S_1 impossible. Although scenario S_1 contains no explicit self-contradiction, the scenario is impossible because it necessitates the *possibility* of a self-contradiction if the scenario were actualized via implying that S_2 would have been possible.

One could claim that the Eternal Society is possible but the annual tradition is not, but this makes little sense given the aforementioned *ex hypothesi* abilities of the Eternal Society in each year of its existence (to write books, to flip coins, etc.). Recall that *ex hypothesi* they can transmit propositional content year after year (by books or otherwise), so that for any given year, the Eternal Society can keep track of whether they did the chant before. They can also surely flip a coin each year to decide whether to do the chant. Given all this, what would have happened if each year the Eternal Society had *tried* to engage in the Annual Coin Flipping Tradition of doing the chant when the coin comes up heads, with the added condition that they do not do the chant if they did it already?

Presumably, there are no gods of metaphysical possibility who would smite the people with lightning should they attempt the annual tradition. It is more plausible that the Eternal Society was never possible in the first place.

Note that if an omnipotent God existed, and He were sempiternal, He would be able to actualize scenario S_1 for himself. Here I am defining "God" in such a way that He is a being who is omnipotent in every year of his existence. Since a sempiternal God is omnipotent in each year of his existence, He must be able to do the Annual Coin-Flipping Tradition each year for the same reasons the Eternal Society can. Call scenario S_1 as applied to God *mutatis mutandis* scenario S_{1G} and scenario S_2 applied to God *mutatis mutandis* S_{2G}. Given these conditions, the following argument presents itself:

(5) If it were possible for God to have existed for an infinite duration, then scenario S_{1G} would be possible.

(6) If scenario S_{1G} would be possible, then S_{G2} would be possible.

(7) S_{G2} is not possible.

(8) Therefore, it is not possible for God to have existed for an infinite duration.

God thus cannot be eternal in the sense of having existed for a beginningless, infinite duration of time, and so the theological doctrine of God's eternity cannot tenably hold to God being sempiternal. If God exists and is eternal, He must have existed timelessly for at least some part of His existence, such as existing for the finite age of physical reality and being timeless *sans* creation.

There is perhaps yet another notable theological implication of the Eternal Society paradox, one that bears relevance to the *kalam* cosmological argument: time itself has a beginning. The Eternal Society argument against an infinite past goes as follows:

(9) If an infinite past were possible, then an Eternal Society would be possible.

(10) If an Eternal Society were possible, then S_1 would be possible.

(11) If S_1 were possible, then S_2 would be possible.

(12) S_2 is not possible.

(13) Therefore, an infinite past is not possible.

While I think there is a sense in which the subjunctive mood is appropriate for (9)–(11), we can also consider the conditionals (9)–(11) as truth-functional material conditionals ("would" counterfactuals imply material conditionals anyway) to lucidly find where the slippery slope stops. Which of the (9)–(11) premises is such that it has a true antecedent with a false consequent? If I were a stubborn adherent of the infinite past, I would deny premise (9) since that seems to be the most vulnerable one. Yet I do not find this denial of (9) entirely satisfactory. While people's intuitions may vary, it does seem like an Eternal Society with its modest human abilities of coin flipping etc. would be possible if an infinite past were possible, and that if such a society is not possible due to its ability to create a logically contradictory situation, so much the worse for an infinite past. Surely there is something metaphysically suspicious about an infinite past if an eternal society with the abilities of ordinary humans can actualize a logical contradiction. By my lights, it is more plausible that an infinite past was never possible in the first place.

Works Cited

Benardete, Jose A. *Infinity: An Essay in Metaphysics*. Clarendon Press, 1964, pp. 255, 259.

Chalmers, David. "Does Conceivability Entail Possibility?" *Conceivability and Possibility*, Clarendon Press, 2002, pp. 154.

Fletcher, Peter. "Infinity." *Philosophy of Logic,* edited by Dale Jacquette, Elsevier, pp. 571, 573.

Hawthorne, John. "Before-Effect and Zeno Causality." *Noûs*, vol. 34, 2000, pp. 267

Koons, Robert. "A New Kalam Argument: Revenge of the Grim Reaper." *Noûs* vol. 48, 2014, pp. 256–267.

Pruss, Alexander. "From the Grim Reaper paradox to the Kalaam argument." alexanderpruss.blogspot.com/2009/10/from-grim-reaper-paradox-to-kalaam.html, October 2, 2009.