

## *Coin flips, credences, and the Reflection Principle*\*

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### *Abstract*

One recent topic of debate in Bayesian epistemology has been the question of whether imprecise credences can be rational. I argue that one account of imprecise credences, the orthodox treatment as defended by James M. Joyce, is untenable. Despite Joyce's claims to the contrary, a puzzle introduced by Roger White shows that the orthodox account, when paired with Bas C. van Fraassen's Reflection Principle, can lead to inconsistent beliefs. Proponents of imprecise credences, then, must either provide a compelling reason to reject Reflection or admit that the rational credences in White's case are precise.

### *Keywords*

precise, imprecise, sharp, mushy, credence, subjective, probability, reflection, Bayesian, epistemology

### *1. Introduction*

Roger White (2010) has introduced an interesting puzzle intended to show that imprecise credences can't be rational, paraphrased as follows:

*Coin Puzzle* Jack has a coin that Mark knows to be fair. In addition, Mark has no idea whether  $p$  but knows that Jack knows whether  $p$ . Jack paints the coin so that Mark can't see which side is heads and which side is tails, then writes ' $p$ ' on one side and ' $\sim p$ ' on the other, explaining to Mark that he has placed whichever is true on the heads side and that he will soon toss the coin so that Mark can see how it lands. Jack tosses the coin, and Mark observes that it has landed with the side marked ' $p$ ' facing up.

Here's the surprising thing: White's analysis seems to show, given some fairly standard assumptions, that Mark can't have an imprecise credence in  $p$  without being inconsistent. James M. Joyce (2010) takes issue with this analysis. He endorses the orthodox view of imprecise credences and the way they're updated, arguing that this treatment handles the Coin Puzzle perfectly well. But I'll argue that Joyce's response is unsatisfactory – it seems likely that the orthodox view *does* lead to an inconsistent set of beliefs, as does *any* account on which Mark has an imprecise credence in  $p$ . We'll see why in §4; first, we'll examine the assumptions leading to the surprising result and take a detailed look at the most interesting – and, I think, most conclusive – argument that White employs in his analysis. Off to the races.

### *2. The assumptions*

Mark, remember, initially has no evidence at all about whether  $p$ . Let's assume that he's maximally rational. Let's further assume (for *reductio*) that Joyce is correct, that the uniquely rational response to Mark's evidential situation is to have a maximally imprecise credence in  $p$ . If

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Mark's initial credence is given by the function  $C$ , then  $C(p) = [0, 1]$ .<sup>1</sup>

Mark also knows the coin is fair. That is, the objective chance that it will land heads on a given toss is 0.5. Lewis's Principal Principle, of course, tells us that, when we don't have direct evidence about a given outcome but do know the objective chance of that outcome, we should make our confidence in the outcome equal to the objective chance (Lewis 1986: 86–87). So  $C(\text{heads}) = 0.5$ .<sup>2</sup> This result is uncontroversial – as White (2010: 175) says, the Principal Principle 'is just intended to accommodate obvious facts like' this one.

Recall that the true statement is written on the heads side of the coin. So, when Mark sees the coin land with the ' $p$ ' side up, he learns that  $p \equiv \text{heads}$ . If Mark's after-the-toss credence is given by  $C_+$ , then  $C_+(p | \text{heads}) = C_+(\text{heads} | p) = 1$ . So  $C_+(p) = C_+(\text{heads})$ . This result is also uncontroversial.<sup>3</sup>

Given these assumptions, we can see that, to remain consistent, Mark needs to change either his credence in  $p$  or his credence in  $\text{heads}$  when he see the coin land with the ' $p$ ' side up.  $C(p)$  and  $C(\text{heads})$  are different, but  $C_+(p)$  and  $C_+(\text{heads})$  are the same. To meet these requirements, Mark can either sharpen his credence in  $p$  to 0.5 or dilate his credence in  $\text{heads}$  to  $[0, 1]$ .<sup>4</sup> But which should he do?

It seems obvious that he should *not* sharpen his credence in  $p$  – after all, the fact that the coin landed with the ' $p$ ' side up doesn't give him any new information at all about whether  $p$ . But let's assume for the moment that he should sharpen his credence in  $p$  to 0.5. Here's the problem: what if the coin had landed with the ' $\sim p$ ' side up instead? Then he should have sharpened his credence in  $\sim p$  to 0.5, and so he should have sharpened his credence in  $p$  to 0.5 as well. But if he knows it's rational to sharpen his credence in  $p$  to 0.5 no matter how the coin lands, shouldn't he just *start* with  $C(p) = 0.5$ ?

Bas C. van Fraassen (1984: 244) codifies this intuition with his Reflection Principle, which states that, for an agent to be rational, his 'subjective probability for proposition  $A$ , on the supposition that his subjective probability for this proposition will equal  $r$  at some later time, must equal this same number  $r$ '.<sup>5</sup> Reflection says that Mark should start with  $C(p) = 0.5$ , but this result is inconsistent with our initial assumption, following Joyce, that  $C(p) = [0, 1]$ . So, to save that initial assumption, we should give up our assumption that Mark should sharpen his credence in  $p$  when he sees that the coin has landed with the ' $p$ ' side up. His credence should instead stay exactly the same. In formal terms,  $C_+(p) = C(p)$ .

So Mark's only remaining option, if he wants to be consistent, is to dilate his credence in

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1 I want to note here that the argument to follow works to show that *any* imprecision in Mark's credence in  $p$  leads to inconsistency. As White (2010: 175) says, 'For simplicity we can focus on the extreme case'.

2 The New Principle, proposed in Lewis 1994 and Hall 1994 as a revised version of the Principal Principle, gives the same answer.

3 From Bayes's Theorem, we have  $C_+(p | \text{heads}) = (C_+(p) / C_+(\text{heads})) * C_+(\text{heads} | p)$ . Then, since each conditional credence is 1,  $C_+(p) / C_+(\text{heads}) = 1$ , and so  $C_+(p) = C_+(\text{heads})$ .

4 He can also change both credences, of course; the important thing is that he has to change at least one of them. The arguments to follow are intended to show that any change at all is inappropriate.

5 As Christensen (1991), Talbott (1991), and others have noted, Reflection is problematic – it can lead to absurd results in cases in which a person is less than certain that her future self will be in no way epistemically compromised. But the Coin Puzzle isn't one of those cases: Mark, an idealized agent, knows what his after-the-toss credences will be not because he knows that he'll lose information or that he'll be unable to properly consider his evidence due to cognitive malfunction, but because he knows he'll respond rationally to *new* information. Briggs (2009), Elga (2007), and others have proposed ways of revising Reflection to deal with its problems, but any such revisions will, in unproblematic cases like the Coin Puzzle, give the same results as van Fraassen's original principle.

*heads* to  $[0, 1]$  when he sees the coin land. And here is where the action is, so to speak. White hopes to convince us that it would be irrational for Mark to dilate, that it must be the case that  $C_+(heads) = C(heads)$ . If he succeeds, then he's shown that Joyce's assumption –  $C(p) = [0, 1]$  – leads inevitably to inconsistency and so must be abandoned. But the orthodox treatment of imprecise updating, which we'll look at in a moment, implies that Mark *should* dilate. White provides several arguments against the orthodox treatment, the most compelling of which we'll examine in the next section.

To recapitulate, the following assumptions are mutually inconsistent:

$$(1) C(p) = [0, 1]$$

$$(2) C(heads) = 0.5$$

$$(3) C_+(p) = C_+(heads)$$

$$(4) C_+(p) = C(p)$$

$$(5) C_+(heads) = C(heads)$$

(1) follows from Joyce's view and is assumed by White for *reductio*. (2) and (3) are uncontroversial. (4), besides being endorsed by both White and (as far as I can tell) Joyce, seems, *prima facie*, quite reasonable, and its denial – together with an uncontroversial application of the Reflection Principle – is inconsistent with (1). (5) is, essentially, the point where the battle over imprecise credences must be waged.

### 3. White's argument

Before we get to White's attack on the orthodox treatment of imprecise credences, we need to be clear about what that treatment is. The basic idea is as follows: a person's imprecise credal state can be modelled by an infinite set, or committee, of precise credence functions. Mark's initial credal state, for instance, is modelled by a set  $\mathbb{C}$ , which we can think of as a committee of precise credence functions such that, given any value  $x$  in the interval  $[0, 1]$ , there's at least one function  $P \in \mathbb{C}$  for which  $P(p) = x$ .<sup>6</sup> His credal state, then, is imprecise because his committee members disagree – as Joyce (2010: 288) says, 'If all members agree about some matter this reflects a determinate fact about what the person believes'. Mark's initial attitude about *heads* is an example: all the committee members in  $\mathbb{C}$  assign a credence of 0.5 to *heads*, so Mark has a sharp credence in *heads*.

Now,  $\mathbb{C}$  is meant to be, not 'a model of a believer's psychology', but 'a highly formalized representation of her doxastic situation' (Joyce 2010: 288). If  $\mathbb{C}$  were meant to be the former, the view would of course be highly implausible – how could a person be expected to keep track of an infinite number of credence functions? The view is, rather, as follows:

[The believer] will make qualitative or comparative assessments of probability and utility – that  $X$  is more likely than  $Y$ , that  $X$  and  $Y$  are independent, that  $X$  is the evidence for  $Y$ ,

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6 Technical note:  $C$ , on this account, is to be understood as a set-valued function whose value, for any proposition  $p$ , is the set  $\{x: P(p) = x \text{ for some } P \in \mathbb{C}\}$ . In the case of a precise credence, then, the value of  $C$  is really a singleton set, so ' $C(heads) = 0.5$ ' should be read as ' $C(heads) = \{0.5\}$ '.

that  $A$  is a better act than  $A^*$ , *et cetera* – and these concrete judgments are modeled abstractly by requiring that all [the committee members] satisfy certain conditions.... The believer only keeps track of her explicitly held qualitative and comparative beliefs: the formal representation takes care of itself. (Joyce 2010: 288)

What's crucial about this way of representing imprecise credences is the way those credences are updated: standard Bayesian rules are applied *to each committee member in the initial credal state*. And as White shows, this treatment implies that Mark should dilate his credence in *heads*. The reasoning is simple: first, recall that, when Mark sees that the coin has landed with the ' $p$ ' side up, he learns that  $p \equiv \text{heads}$ . Recall also that for any value  $x$  in the interval  $[0, 1]$ , there's some  $P \in \mathbb{C}$  such that  $P(p) = x$ . A simple proof given by White (2010: 177) shows that  $P(\text{heads} \mid p \equiv \text{heads}) = P(p)$ .<sup>7</sup> So we know that, for any value  $x$  in the interval  $[0, 1]$ , there's some  $P \in \mathbb{C}$  such that  $P(\text{heads} \mid p \equiv \text{heads}) = x$ . In other words, if Mark's committee members update according to standard Bayesian procedures, his after-the-toss credence in *heads* will be maximally imprecise.

So why does White think dilation is a problem? He gives several arguments, some of which function as intuition pumps: given a situation  $X$ , the dilation account entails that you should believe  $Y$ , and you don't really believe *that*, do you? As it happens, I share White's intuitions, but Joyce (for example) apparently does not. So the argument we're going to focus on is a formal one based on the Reflection Principle.

White's reasoning here looks just like the argument in the last section showing that Mark shouldn't sharpen his credence in  $p$ . It's simply this: according to the dilation account, Mark should dilate his credence in *heads* to  $[0, 1]$  when he sees that the coin has landed with the ' $p$ ' side up. But the dilation account also says that Mark should have dilated his credence in *heads* to  $[0, 1]$  if he'd seen the coin land with the ' $\sim p$ ' side up. Before the toss, then, Mark knows that, no matter how the coin lands, he'll rationally dilate his credence in *heads* to  $[0, 1]$ . And so, by Reflection, he should dilate his credence *now, before the toss*. Formally,  $C(\text{heads}) = [0, 1]$ . But to assert such a thing is to deny (2), and the denial of such an obvious fact 'is absurd' (White 2010: 178). After all, Mark *knows* that the coin is fair, and it's wildly implausible that the mere fact that Jack has painted the coin could give Mark any reason whatsoever to ignore that knowledge.

The upshot is clear: the orthodox treatment leads to inconsistency. It's uncontroversial that  $C(\text{heads}) = 0.5$ , but according to the orthodox treatment  $C(\text{heads}) = [0, 1]$ . So we must reject the orthodox treatment.

Joyce, of course, doesn't accept this conclusion. He contends that imprecise credences handle the Coin Puzzle perfectly well and that White's argument is founded upon a simple misunderstanding of their nature. In the next section we'll take a look at how Joyce's argument goes.

#### 4. Joyce's response

Recall that, according to Joyce, assumption (5) is false. That is,  $C(\text{heads}) = 0.5$ , and  $C_+(\text{heads}) = [0, 1]$ ; dilation is the rational response to seeing the coin land with the ' $p$ ' side up. But Joyce can't deny that, had the coin landed with the ' $\sim p$ ' side up, it would still have been the case that  $C_+(\text{heads}) = [0, 1]$ . So how can he avoid the above Reflection result?

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7 We should note that White's proof depends on the implicit (but highly plausible) assumption that  $P(\text{heads})$  and  $P(p)$  are independent. Joyce (2010: 299) endorses the dilation result, calling it 'entirely unavoidable on any view that allows imprecise probabilities'.

Joyce's strategy is to argue that, even though Mark's after-the-toss credence in *heads* is maximally imprecise whether the coin lands with '*p*' or ' $\sim p$ ' facing up, the two possible credences are *not* identical:

The beliefs about *heads* you will come to have upon learning  $heads \equiv p$  are *complementary* to the beliefs you will have upon learning  $heads \equiv \sim p$ . There is, in fact, no single belief state for *heads* that you will inhabit whatever you learn about  $heads \equiv p$ . You will inhabit an imprecise state either way, but these state will differ depending on what you learn. (Joyce 2010: 304)<sup>8</sup>

Joyce's claim is based on the following fact: when the coin lands with the '*p*' side up, the committee members with a *high* initial credence in *p* end up with a high after-the-toss credence in *heads* and a low one in *tails*, whereas when the coin lands with the ' $\sim p$ ' side up, the committee members with a *low* initial credence in *p* end up with a high after-the-toss credence in *heads* and a low one in *tails*. So, given any possible credence  $x$ , there will be some committee member  $P_+ \in \mathbb{C}_+$  such that  $P_+(heads) = x$ , but *which* committee member has this value depends on the outcome of the toss.

Joyce is, of course, correct about the mechanics of the orthodox account, but his argument invites questions. Why does it matter *which* function in the committee assigns a certain credence? Given that the 'committee of functions' is a formalized, abstract model of a much messier mental state, how seriously are we supposed to take the model? Are the features that Joyce exploits for his argument grounded in actual facts about Mark's psychological state, or are they just artefacts of the formalization?

First of all, we should clarify that the question of *which* committee member assigns a given credence is, on one interpretation, nonsensical. The committee members just *are* credence functions. They have no identity outside of their fully specified credal states. Here's a simplified example: if Mark's committee had only two members – we'll call them  $P_1$  and  $P_2$  – both of which agreed about every atomic proposition besides  $q$ , there would be no real distinction between the case in which  $P_1(q) = 0.7$  and  $P_2(q) = 0.3$  and the case in which  $P_1(q) = 0.3$  and  $P_2(q) = 0.7$ . The two cases would just be different ways of referring to a single committee. And Joyce doesn't claim otherwise. His response relies, rather, on the fact that the two ways Mark could come to have a maximally imprecise credence really do result in entirely different committees. When the coin lands with the '*p*' side up, every function in the committee satisfies  $P_+(p) = P_+(heads)$ . In the other case, every function in the committee instead satisfies  $P_+(p) = 1 - P_+(heads)$ . So the two cases do result in two distinct models. The question, then, is exactly what difference there is between the psychological states – and, in particular, the attitudes toward *p* – represented by those models.

It turns out to be an easy question to answer. In the first case, Mark is certain that *p* if and only if *heads*, and in the second, he's certain that *p* if and only if *tails*. There's a clear psychological difference between the two states. But what does that difference tell us? The mere fact that the states are distinct can't be enough to show that Joyce's analysis is correct. After all, in cases in which credences are precise, there are often psychological distinctions, and *these* distinctions don't prevent us from applying Reflection. Let's look at an example:

*Birthday Predicament* Last night Mark had a birthday party, at which he over-imbibed

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8 Here and below I've replaced Joyce's variables and symbols with the analogous ones from my own presentation.

and made a fool of himself. This morning he'd like to call the friends that were present at the party and apologize for his behaviour, but his memory of the previous evening is hazy – he can't quite remember who was there. If  $a$  is the proposition that Amanda was at the party,  $b$  is the proposition that Becky was at the party, and  $c$  is the proposition that Carl was at the party, Mark has the following credences:  $C(a) = 0.3$ ,  $C(b) = 0.7$ , and  $C(c) = 0.7$ . Jill, who remembers last night perfectly and is amused by Mark's predicament, has decided to teach him a lesson by withholding information from him. She has told him that tomorrow she'll inform him either that Amanda and Becky were together last night or that Amanda and Carl were together last night. That is, he'll learn either that  $a \equiv b$  or that  $a \equiv c$ . Mark has the following conditional credences:  $C(a | a \equiv b) = C(b | a \equiv b) = 0.5$ , and  $C(a | a \equiv c) = C(c | a \equiv c) = 0.5$ . (His  $C(c | a \equiv b)$  and  $C(b | a \equiv c)$  are presumably each  $> 0.7$ .)

Whatever Mark learns, he'll have the updated credence  $C_+(a) = 0.5$ . However, in one case he'll have the conditional credences  $C_+(a | b) = C_+(b | a) = 1$ , and in the other he'll have the credences  $C_+(a | c) = C_+(c | a) = 1$ . Despite this difference, it's clear that, in keeping with Reflection, Mark should update his credence in  $a$  to 0.5 now.

Perhaps we should discuss *why* it seems so clear that Reflection remains applicable despite this kind of psychological distinction. The answer, I think, is simply this: the differences are not differences in  $C_+(a)$  and so are irrelevant to the question at hand. We know, of course, that it isn't the case that *any* psychological difference renders Reflection inapplicable. If it were, Reflection would be applicable only when a person had acquired perfect knowledge of her *entire* future credal state. And no one ever has such knowledge. So, if Reflection is to be at all useful as a principle, some psychological differences must be irrelevant to its applicability.

But why are these particular differences irrelevant? Though it's easy to see why Mark's future credence in a proposition totally independent of  $a$  is irrelevant, a conditional credence like  $C_+(a | b)$  doesn't seem to be such a simple case. But we must remember that the question of the value of  $C_+(a | b)$  is simply separate from the question of the value of  $C_+(a)$ . The point is clear when the questions are asked in informal terms: 'Tomorrow, how confident will Mark be that  $a$ ?' and 'Tomorrow, how confident will Mark be that  $a$  given the assumption that  $b$ ?' are simply two different questions, and our ability to apply Reflection based on the answer to the first question has nothing to do with the answer to the second.

Back to the Coin Puzzle: Joyce's claim is that, when Mark has imprecise credences, his after-the-toss attitude toward *heads* isn't adequately characterized by  $C_+(\text{heads})$ . (Recall that on the orthodox treatment his credal state is modelled by the *set* of functions  $\mathbb{C}_+$ , not the single set-valued function  $C_+$ .) Let  $C_+$  be  $C_{h \equiv p}$  when Mark sees the coin land with the ' $p$ ' side up and  $C_{h \equiv \sim p}$  when he sees it land with the ' $\sim p$ ' side up. Then  $C_{h \equiv p}(\text{heads}) = C_{h \equiv \sim p}(\text{heads}) = [0, 1]$ , but Joyce claims that Mark's attitude toward *heads* in the two cases is *not* the same. Is this claim accurate? We saw above that there is indeed a psychological difference between Mark's *total* states in the two cases: in the first case he knows that  $\text{heads} \equiv p$  – i.e.,  $C_+(\text{heads} | p) = C_+(p | \text{heads}) = 1$  – and in the second case he knows that  $\text{heads} \equiv \sim p$  – i.e.,  $C_+(\text{heads} | \sim p) = C_+(\sim p | \text{heads}) = 1$ .

But these two cases are clearly quite analogous to the two states in the Birthday Predicament, which are the state in which he knows that  $a \equiv b$  – i.e.,  $C_+(a | b) = C_+(b | a) = 1$  – and the state in which he knows that  $a \equiv c$  – i.e.,  $C_+(a | c) = C_+(c | a) = 1$ . And we saw above that differences in these kinds of conditional credences don't render Reflection inapplicable, at least in cases in

which the relevant credence is precise. Perhaps there's some special feature of imprecise-credence cases that *does* serve to make Reflection inapplicable, but what sort of feature might that be? As we've seen, the feature Joyce exploits is actually present in precise-credence cases in which it's uncontroversial that Reflection is the correct response.

There's even more reason to think that the orthodox account can't be right. We can prove that proponents of the orthodox view are committed to the claim that Mark's initial credence in  $p$  isn't the same as his initial credence in  $\sim p$ . In order to comment on Mark's credences without committing myself to the claim that the functions  $C$  and  $C_+$  adequately characterize his credal states, I introduce the function  $D$  such that  $D(x)$  denotes Mark's initial credence in  $x$ , whatever kind of thing that credence turns out to be. Similarly,  $D_+$  gives his after-the-toss credal state, and  $D_{h \equiv p}$  and  $D_{h \equiv \sim p}$  give his after-the-toss credal states in the case in which the coin lands with the ' $p$ ' side up and the case in which the coin lands with the ' $\sim p$ ' side up, respectively. Recall that, to avoid the applicability of Reflection, proponents of the orthodox account must affirm that Mark's after-the-toss credence in *heads* when the coin lands with the ' $p$ ' side up is different from his credence in *heads* when the coin lands with the ' $\sim p$ ' side up. To begin the proof, then, we assume this claim for *reductio*:

$$(A) D_{h \equiv p}(\text{heads}) \neq D_{h \equiv \sim p}(\text{heads})$$

The following are immediate results of Bayes's Theorem (and are analogous to assumption (3) above):

$$(B) D_{h \equiv p}(\text{heads}) = D_{h \equiv p}(p)$$

$$(C) D_{h \equiv \sim p}(\text{heads}) = D_{h \equiv \sim p}(\sim p)$$

From (A), (B), and (C), we have:

$$(D) D_{h \equiv p}(p) \neq D_{h \equiv \sim p}(\sim p)$$

Recall that assumption (4) above is implied by Reflection in addition to seeming fairly obvious – it's implausible that Mark could learn anything at all about whether  $p$  from the result of the coin flip. Furthermore, Joyce (2010: 297) seems to endorse the assumption: he says that 'for every  $P \in \mathbb{C}$ , the credence that  $P$  assigns to  $p$  upon learning  $\text{heads} \equiv p$  is identical to the *prior* credence that  $P$  assigns to  $p$ '. So we have:

$$(E) D_+(p) = D(p)$$

$$(F) D_+(\sim p) = D(\sim p)$$

But  $D_{h \equiv p}$  and  $D_{h \equiv \sim p}$  are just instances of  $D_+$ , so we have:

$$(G) D_{h \equiv p}(p) = D(p)$$

$$(H) D_{h \equiv \sim p}(\sim p) = D(\sim p)$$

Finally, from, (D), (G), and (H), we get:

$$(I) D(p) \neq D(\sim p)$$

Now, before the toss, Mark has exactly the same evidence for  $p$  and for  $\sim p$  – that is, none. So Joyce has two options: he can reject (A) or claim that there are cases in which identical evidence justifies a certain credence in one case and a different credence in another case. The second option seems obviously absurd – after all, isn't the epistemically rational credence in any given case *determined* by the evidence? – but the first option forces him to admit that  $D_{h \equiv p}(\text{heads}) = D_{h \equiv \sim p}(\text{heads})$ . And if the latter identity holds, Reflection is applicable.

Joyce might protest that, in fact,  $D(p) \neq D(\sim p)$ . After all, even though Mark has no evidence about whether  $p$ , he does have the conditional credences  $D(p | \sim p) = D(\sim p | p) = 0$ . But how could these conditional credences be enough to establish that  $D(p) \neq D(\sim p)$ ? Is it merely that they show that  $p$  and  $\sim p$  aren't true at all the same times? Such a condition would lead to the unacceptably strong result that it's *never* rational to have identical imprecise credences in two propositions  $q$  and  $r$  unless one is certain that  $q \equiv r$ . But it's hard to see what other reasonable condition could be doing the work. Even the fact that  $p$  and  $\sim p$  are *never* true at the same time isn't enough. Consider again an analogous case in which the credences are sharp: Mark also has the conditional credences  $D(\text{heads} | \sim \text{heads}) = D(\sim \text{heads} | \text{heads}) = 0$ , but it's uncontroversial that  $D(\text{heads}) = D(\sim \text{heads})$ . So it looks like  $D(p) = D(\sim p)$ , in which case Reflection is applicable, as we saw above. So the orthodox view is untenable.

### 5. Conclusion

We've seen that, if Reflection, in either its original form or any of its revised forms, is an acceptable principle, White's Coin Puzzle demonstrates that the orthodox treatment of imprecise credences leads to inconsistency. Worse, White's argument shows that *any* view on which Mark's initial credence in  $p$  is at all imprecise leads to inconsistency. Committed believers in the possibility of rational imprecise credences, then, have two options: they can reject Reflection, or they can claim that there's something special about the Coin Puzzle in virtue of which Mark's initial credence in  $p$  should be precise even though imprecise credences can be rational in other cases.<sup>9</sup> If they choose the first option, they must provide a compelling argument showing exactly what's wrong with Reflection as it's applied here. If they choose the second, they must find some special feature of the Coin Puzzle that could plausibly justify a precise initial credence in  $p$ .<sup>10</sup>

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9 One possible such view, which Scott Sturgeon has presented (but not fully endorsed) in conversation, is that the Principle of Indifference can be used to sharpen credences in cases in which the evidence, or lack thereof, makes for an imprecise credence that's determinately symmetrical around a single value.

10 Thanks to David Christensen, Joshua Schechter, Scott Sturgeon, Stephen Emet, Derek Bowman, and Brown University's Philosophy Graduate Forum.



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