Derivative metaphysical indeterminacy and quantum physics*

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Abstract

This chapter argues that quantum indeterminacy can be construed as a merely derivative phenomenon. The possibility of merely derivative quantum indeterminacy undermines both a recent argument against quantum indeterminacy due to David Glick, and an argument against the possibility of merely derivative indeterminacy due to Elizabeth Barnes.

Keywords: Quantum logic; classical logic; logical space; logical realism; bivalence; fundamentality; naturalness.

1 Introduction

It is a near platitude that a sizable part of our utterances are indeterminate in truth value.[1] Because the shirt I am wearing is a shade lying somewhere between green and blue, my utterance of 'this shirt is green' is not quite true, but not false either. And when asked where I live in my hometown, I say 'near the historical center' mainly because everyone knows where it is, and not because I live particularly close to the historical center, although I do not live far from it, either.

In the last century, the philosophical consensus used to be that indeterminacy can only originate in the way we represent the world, never in the nonrepresentational world. Russell [33, p. 85] wrote that "apart from representation, whether cognitive or mechanical, there can be such thing as vagueness or precision: things are what they are." Dummett [19, p. 314] would go as far as to claim that "the notion that things might actually

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[1] That near platitude has been nevertheless denied by those who recommend an epistemicist understanding of vagueness (Williamson [43]).

be vague, as well as being vaguely described, is not properly intelligible." Likewise, according to Lewis [26, p. 212] "the only intelligible account of vagueness locates it in our thought and language. The reason it's vague where the outback begins is not that there's this thing, the outback, with imprecise borders; rather there are many things, with different borders, and nobody has been fool enough to try to enforce a choice of one of them as the official referent of the word 'outback'."

Rhetoric aside, however, little has been provided in the way of arguments against the notion of indeterminacy in the world, aka *metaphysical indeterminacy* (MI). Sure, Evans [21] and Salmon [34, p. 338] have offered a clever and terse disproofs of the possibility of vague objects. But the Evans-Salmon line of argument only shows that an object cannot have indeterminate *de re* identity—it does not rule out objects that are vague in other respects,[2] nor does it rule out ways for reality to display indeterminacy that do not involve any vague objects. And although Williamson [44] has provided a general argument against *de re* indeterminacy, his conclusion rests on the specific limitations of his model theory of choice, rather than on any substantive metaphysical considerations.

My diagnosis is that what kept philosophers from buying into MI was the lack of suitable concepts that would allow them to theorize about it, rather than any specific argument.[3] The general attitude has indeed changed to some degree now that a number of characterizations of MI have been put forward, as in Akiba [2], Barnes & Williams [7], Darby & Pickup [17], Smith & Rosen [37], Torza [40], and Wilson [45].

Potential manifestations of MI include (i) the 'fuzzy' objects of the macroscopic world, such as clouds, mountains and persons; (ii) future contingents and the open future; and (iii) quantum indeterminacy. Putative instances of iii include (iii.a) the failure of value definiteness of quantum observables; (iii.b) the vague identity of quantum objects; and (iii.c) the count indeterminacy arising in quantum field theory. Although a good part of what I will be saying is the result of general features of my favorite way of understanding MI, the focus of this chapter will be on iii.a.

The failure of value definiteness is a typically quantum-mechanical phenomenon whereby a system fails to have any determinate value of an observable at a time. Given the eigenstate-eigenvalue link, the failure of value definiteness follows from the fact that a quantum state which is a superpo-

[2]For example, it has been argued that objects can have indeterminate coincidence (Akiba [1]), and indeterminate distinctness (Akiba [3]).

[3]Barnes [4] has drawn a similar moral.

sition of eigenstates of an observable is in general not an eigenstate of that same observable.[4] For example, a particle in a superposition of position states has no determinate value of position. Although the interpretative details may differ depending on the philosophical methodology being employed, there appears to be a growing consensus that the failure of value definiteness constitutes evidence of MI (see Bokulich [9], Calosi & Mariani [10] [11], Calosi & Wilson [12] [13], Darby [16], Darby & Pickup [17], Fletcher & Taylor [23], Mariani [28], Skow [36], Torza [39] [40]).

A dissenting voice is Glick [25], who has argued that no evidence of MI is to be found in quantum theory. His overarching argument is as follows:

1. Orthodox quantum theory provides no evidence of fundamental MI.
2. The main realist interpretations of quantum theory provide no evidence of fundamental MI.
3. Therefore, the main interpretations of quantum theory provide no evidence of fundamental MI (from 1, 2).
4. MI cannot be derivative.
5. Therefore, the main interpretations of quantum theory provide no evidence of MI (from 3, 4).

A comprehensive assessment of Glick's argument lies outside the scope of this work (see Calosi & Mariani [10], Calosi & Wilson [13] for criticism). I am going to focus my attention on the thesis of line 4.

Glick's belief that there is no derivative MI can be evinced from the following passage, in which he argues that the failure of value definiteness does not bring about MI, if observables are derivative entities: "If, by contrast, one took the properties to be ontologically derivative and quantum states to be fundamental, there would be little room for metaphysical indeterminacy. [. . .] *Any indeterminacy would occur at the non-fundamental level and hence may be viewed as eliminable*" (p. 206, my emphasis).

But as already noted elsewhere, derivative does not amount to eliminable. We can all agree that tables are derivative entities (whatever 'derivative' means), while retaining our belief in the existence of tables. Indeed,

[4] The eigenstate-eigenvalue link, a postulate of so-called 'orthodox' quantum mechanics (Gilton [24]), states that a system has property O with value λ iff the quantum state of the system is in an eigenstate of the associated operator $\hat{O}$ with eigenvalue λ. It is worth mentioning that the orthodoxy of the eigenstate-eigenvalue link has been challenged by Wallace [42]. For discussion of the eigenstate-eigenvalue link *vis-à-vis* quantum indeterminacy, see Calosi & Wilson [12], Fletcher & Taylor [23].

the unpopularity of revisionist ontological doctrines, such as mereological nihilism, is partly explained by the fact that they demand us to give up on the existence of the medium-sized dry goods of naive physics. Perhaps Glick thinks that although not everything which is derivative is eliminable, some things are, and MI is one such thing. If that is the underlying thought, however, we have not been given any arguments.

Here is a different line of reasoning that could be pursued on Glick's behalf. In lieu of 4, one could think that there is derivative MI only if there is fundamental MI—in other words:

4*. MI cannot be merely derivative.

By substituting 4* for 4 in the argument, Glick can still draw his conclusion. For if we have reason to reject fundamental MI, and if lack of fundamental MI entails lack of derivative MI, we have reason to reject MI, period. Interestingly, Barnes [5] has offered a defense of 4*. If her disproof of merely derivative MI turns out to be conclusive, Glick could piggyback on that.

I make two claims: Barnes' argument for 4* is invalid; and, given my preferred characterization of MI, 4* is false. In this paper I will defend both claims, and conclude that for all we know merely derivative MI arises in quantum physics.

2 Metaphysical indeterminacy

As observed in the previous section, run-of-the-mill indeterminacy originates in the way we represent reality. According to the standard account, representational indeterminacy is semantic in character: it is rooted in the meaning of particular subsentential expressions, such as predicates and names. The go-to semantic theory of indeterminacy is the supervaluationism of Fine [22], which characterizes a term as indeterminate just in case its meaning is compatible with different precisifications, that is to say, with different assignments of extensions in actuality.[5] A sentence is said to be *indeterminate in truth value* just in case it is true on some precisifications, and false on others.

The supervaluationist picture can be generalized in a most natural way by taking precisifications to be assignments of intensions, rather than extensions. Accordingly, a term is indeterminate just in case its meaning is

[5] A precisification must be defined for all terms at once, in order to preserve penumbral connections, cf. Fine [22, p. 271].

compatible with different functions from worlds to extensions. A sentence is said to be *indeterminate in content* if its meaning is compatible with multiple functions from worlds to truth values—or, equivalently, with multiple sets of worlds.

Now, let us identify coarse-grained facts (or states of affairs) with sets of worlds. Say that a fact F obtains at world w if $w \in F$; and that F obtains *simpliciter* if it obtains at the actual world @. If sentence p is not indeterminate in content, let $[p]$ be the fact that p; and if it is indeterminate in content, let $[p]_1, [p]_2, \ldots$ be the facts associated with the different precisifications of the language. The following holds:

FACT 1. On the supervaluationist picture, sentence p is indeterminate in truth value just in case (i) it is indeterminate in content, and (ii) $@ \in [p]_i$ and $@ \notin [p]_j$, for some i, j.

The left-to-right direction of FACT 1 highlights that, on the most popular semantic account, all truth-value indeterminacy is indeterminacy in content.

Nevertheless, there appear to be sentences having indeterminate truth value but determinate content, as exemplified by Aristotle's problem of the open future.[6] If there is no fact of the matter now as to whether there will be a sea battle tomorrow, the sentence 'there will be a sea battle' is neither true nor false. Yet each term occurring in it is semantically precise, and so the sentence cannot pick out different intensions on different precisifications. This class of cases suggests that the supervaluationist characterization of truth-value indeterminacy is too restrictive, as it prejudges the possibility of indeterminacy originating in the language-independent world. From now on I will therefore be assuming a definition of truth-value indeterminacy which is neutral as to the source of the indeterminacy, to the effect that p is indeterminate in truth value just in case p is neither true nor false. Truth-value indeterminacy in this sense is entailed by, but does not entail supervaluationist truth-value indeterminacy.

[6] I am saying that there 'appear' to be such cases because, on two prominent characterizations of MI—namely, the metaphysical supervaluationism of Barnes & Williams [7], and the determinable-based account of Wilson [45]—MI does not involve truth-value gaps. However, there are independent reasons for being skeptical of such approaches, since metaphysical supervaluationism is unable to subsume quantum indeterminacy (Darby [16], Skow [36]), whereas the determinable-based theory has a hard time making sense of a number of phenomena such as the open future, indeterminate identity, and indeterminate existence (Barnes & Cameron [6]). Moreover, it has been argued that the determinable-based account is inadequate in the way it deals with quantum indeterminacy as well (Fletcher & Taylor [23], Torza [40]). For a comparison between the present approach and the determinable-based account see Lewis [27] in this volume.

The above considerations suggest a negative characterization of MI as indeterminacy that cannot be eliminated by precisifying the content of our assertions (Torza [39], cf. Barnes [4, p. 604]):

IND$^-$. MI arises if there is a sentence p which is indeterminate in truth value but not in content.

According to IND$^-$, MI occurs just when there is a sentence which is neither true nor false, and yet picks out exactly one fact. However, MI can also be characterized directly, as the phenomenon arising when there is no fact of the matter about something. Fleshing out this alternative characterization will require that we say more about the structure of logical space.

A logical space is a space of possibilities. In order for a class of facts to constitute a logical space, they need to be closed under a number of logical operations such as negation, conjunction etc (cf. Rayo [32]).[7] A caveat: logical operations, as objects in logical space mapping facts to facts, should not be confused with logical operators, which are linguistic items mapping formulas to formulas. For example, the negation operator 'not' is a logical constant having a negation operator as its semantic value. Likewise for conjunction, disjunction etc. Accordingly, if F is the fact that grass is green, the negation of F is the fact that grass is not green.

A logical space can be represented as a structure $\mathcal{S} = \langle S, @, \mathrm{T}_S, -_S, \sqcap_S \ldots \rangle$, such that:

1. S is a set of states (worlds). Among them is a distinguished item @, the actual state.[8]

2. Facts are sets of states. The universal set S is the necessary fact; the empty set is the impossible fact.

3. A fact F is said to obtain at state w (in symbols, $\mathrm{T}_S(F, w)$) if $w \in P$; it is said to obtain *simpliciter* if $\mathrm{T}_S(F, @)$.

4. Logical operations are operations on facts: $-_S F$ is the negation of F; $F \sqcap_S G$ is the conjunction of F and G; etc.

[7]However, Turner [41] has defended the idea that the relations holding between facts are quite different from the familiar logical ones, and are akin to geometrical relations. Although Turner's view is both fascinating and compelling, discussing it would take me far afield. Suffice to say that everything I say here could be restated within Turner's theory.

[8]If the logical space is the space of a dynamical system, @ should be a function of time, rather than a constant. For present purpose, this complication can be set aside.

One caveat: the above characterization is largely independent of questions in modal metaphysics, such as whether worlds are concrete or abstract, or about the nature of facts. All I am assuming is that logical space, whatever it is, instantiates the structure defined above. Likewise, when I speak of states (worlds) as points or vectors in a structure, it is being assumed that states (worlds) play the relevant structural role, and not that they are literally points or vectors.

Armed with those tools we can now state the idea that, relative to a logical space $\mathcal{S}$, MI amounts to there being no fact of the matter about something (Torza [40]):

IND$^+$. MI arises if there is a fact F such that neither F nor $-_{\mathcal{S}}F$ obtains.

Prima facie, IND$^-$ and IND$^+$ provide quite different characterizations of MI, in that the former is semantic in character, whereas the latter defines a property of logical space without making any detour through language. As it turns out, however, given some background assumptions the two characterizations are provably equivalent:

FACT 2. Given a logical space $\mathcal{S}$ and a language $\mathbf{L}$ interpreted on $\mathcal{S}$, there is a sentence p of $\mathbf{L}$ which is indeterminate in truth value but not in content iff there is a fact F in $\mathcal{S}$ such that neither F nor $-_{\mathcal{S}}F$ obtains.[9]

Because of the equivalence between IND$^-$ and IND$^+$, we can speak of MI with no ambiguity.

[9] *Proof.* If p of $\mathbf{L}$ is determinate in content, it picks out a unique fact $[p]$ in $\mathcal{S}$. And if it is indeterminate in truth value, neither $\mathrm{T}_{\mathcal{S}}([p], @)$ nor $\mathrm{T}_{\mathcal{S}}(-_{\mathcal{S}}[p], @)$ is the case. So, $[p]$ is a fact such that neither it nor its negation obtains. Conversely, if F is a fact in $\mathcal{S}$ such that neither it nor its negation obtains, let p be a sentence of $\mathbf{L}$ such that $[p] = F$. Hence, p is not indeterminate in content. Moreover, p is neither true nor false. QED.

This proof hinges on two background assumptions. One is that the object language $\mathbf{L}$ contains no irreferential terms, such as 'Vulcan' or 'God'. For otherwise there could be a sentence p^* such as 'Vulcan is a gas planet' which, by not picking out any fact, is indeterminate in truth value (on some semantic accounts, at least) but not in content (since indeterminacy in content requires that it pick out multiple facts); and yet there would be no fact $[p^*]$ such that neither it nor its negations obtains (Torza [39]). The other background assumption is that every fact F must be expressible in $\mathbf{L}$.

Insofar as one may reject either assumption, and so the equivalence, I take IND$^+$ to be my official characterization of MI. (Also notice that IND$^+$, unlike IND$^-$, does not involve any representational machinery, and so can provide a reductive analysis of MI.) Nevertheless, it is both interesting and illuminating that IND$^+$ can be cast in semantic terms as IND$^-$, given suitable qualifications.

3 Fundamentality and derivativeness

Logical spaces of different kinds correspond to different logics. For example, classical logical spaces differ from intuitionistic logical spaces in that classical negation is involutive, unlike its intuitionistic counterpart. One fact that will play a crucial role in the ensuing discussion is that the class of states of a physical system can live in logical spaces of different kinds that agree about the assignment of values to physical quantities.

Consider a Hilbert space H associated to a given quantum system. The class of all states (unitary vectors) in H can be embedded in a classical logical space $\mathcal{C}$, where facts are arbitrary sets of vectors, negation is set-theoretic complementation, and disjunction is set-theoretic union. The same class of states can also be embedded in a quantum logical space $\mathcal{Q}$, where facts are sets of vectors closed under linear combination, negation is orthocomplementation, and disjunction is span[10] (Birkhoff & von Neumann [8]). Note that the rays in H are maximally specific facts, i.e., facts about the system's having a value of a particular observable. Since $\mathcal{C}$ and $\mathcal{Q}$ on H contain the same rays, the two spaces will agree with respect to the obtaining of facts about the assignment of values to physical quantities (e.g., about wether the system is spin up along a particular direction). In other words, the classical and the quantum logician will only disagree about the truth value of logically complex sentences—in particular, sentences that contain either a negation or a disjunction.

Although a realist attitude towards orthodox quantum theory arguably favors $\mathcal{Q}$ over $\mathcal{C}$ as representing the space of possibilities associated with a quantum system (Torza [40, sec. 3.2], Fletcher & Taylor [23]), there is no conclusive strategy for picking one option over the other on the basis of empirical evidence alone. So, if we think that there is such a thing as the One True logical space of a given quantum system, the choice appears to be underdetermined by the physics.[11] This is relevant to the present discussion because, if it is underdetermined whether the states of a quantum

[10] The orthocomplement of a fact F is the set of vectors that are orthogonal to each vector in F; the span of facts F, G is the closure of the union of F and G under linear combination.

[11] Quantum logic was famously defended as the One True logic in Putnam [31] on the grounds that it provides a solution to the measurement problem. Maudlin [29] has argued against Putnam, and concluded that there is no reason to replace classical with quantum logic. Although I agree with Maudlin that quantum logic is of no help in addressing the measurement problem, I reject his conclusion. Indeed, quantum logic is a consequence of accepting either the eigenstate-eigenvalue link (Fletcher & Taylor [23]) or the EPR criterion of reality (Torza [40, sec. 3.2]).

system live in classical or quantum logical space, the question of quantum MI will also be underdetermined. For if the states of an arbitrary quantum system define a classical logical space, MI cannot arise. Indeed, it is trivially the case that every fact F of a classical logical space $\mathcal{C}$ is such that either it obtains or it does not obtain (i.e., either $\mathrm{T}_C(F, @)$ or not $\mathrm{T}_C(F, @)$). But because the classical negation $-_C$ is complementation, it follows that F is such that either it or its negation obtains (i.e., either $\mathrm{T}_C(F, @)$ or $\mathrm{T}_C(-_C F, @)$). On the other hand, if the states of a quantum system define a quantum logical space, MI can and will arise. For example, when a system composed by a single electron is in a superposition of z-spin states described by the equation $|\psi\rangle = \frac{1}{\sqrt{2}}(|\uparrow_z\rangle_e + |\downarrow_z\rangle_e)$, neither the fact [$e$ is z-spin up] nor the fact [e is z-spin down] obtains. Since the quantum negation $-_Q$ is orthocomplementation, [e is z-spin down]$=-_Q$[e is z-spin up]. Hence, the fact [e is z-spin up] is such that neither it nor its negation obtains (i.e., neither $\mathrm{T}_Q([e \text{ is } z\text{-spin up}], @)$ nor $\mathrm{T}_Q(-_Q[e \text{ is } z\text{-spin up}], @)$).

Since the empirical evidence does not let us select a unique way of carving the logical space associated with a quantum system, we seem to be faced with underdetermination about logic. This raises a challenge to logical realism, the view that there is One True logic, and that this is so in virtue of the way the mind and language-independent world is like. The logical realist could rejoin as follows: we can countenance multiple logical spaces, without thereby surrendering to conventionalism, as long as they are ordered with respect to their relative fundamentality. In that way, a physical system will be associated with One True logical space, namely the fundamental one, as well as a number of nonfundamental logical spaces.[12]

In the quantum case, talk of fundamental vs derivative logical spaces can be cashed out in (at least) two ways. The first strategy involves the realism about structure articulated and defended in Sider [35]—very roughly, the idea that there is a metaphysically primitive and privileged way of carving reality into natural properties, facts, etc. A realist about structure who regards classical logic as fundamental will take there to be a metaphysically privileged way of carving out the class of states of a quantum system, namely the classical one. This brand of realist can countenance the existence of alternative ways of carving the same class of states into a logical space, as long as those ways are nonfundamental. The alternative carvings, despite being metaphysically second-rate, can be helpful relative to specific theoretical or practical goals.

Of course, fundamentality talk is no solution to logical underdetermi-

[12]For a discussion of logical realism see McSweeney [30], Tahko [38].

nation if we have no criteria to identify which logical space is fundamental. Fortunately, there are sensible constraints on fundamentality that the realist can appeal to in order to make progress. One such constraint is that, given a physical system, every nonfundamental logical space must be reducible to the fundamental one in the following sense (Torza [40, sec. 6], cf. Sider [35, ch. 7]):

> COMPLETENESS. Let W be a class of states. If $\mathcal{S}$ is the fundamental logical space on W, and $\mathcal{S}'$ is any logical space on W, every fact in $\mathcal{S}'$ is equivalent to some fact in $\mathcal{S}$.

The rationale for COMPLETENESS is that the facts of a nonfundamental space should be nothing over and above the facts in the fundamental space.[13] In order to avoid irrelevant complications, I am taking fact equivalence to be cointensionality (although more fine-grained notions of fact equivalence could be employed, as in Correia & Skiles [15], Dorr [18]). Thus, facts are equivalent just in case they obtain at the exact same states. Since we are modeling facts as sets of states, and since sets are extensional entities, fact equivalence reduces to strict identity.

Now, given a set W of states of a quantum system, it is compatible with COMPLETENESS that a classical logical space $\mathcal{C}$ on W be fundamental and a quantum logical space $\mathcal{Q}$ on W be nonfundamental, but not the other way around. For recall that in $\mathcal{Q}$ only sets of vectors closed under linear combination count as facts, whereas in $\mathcal{C}$ any arbitrary set of vectors is a fact. So, the facts in $\mathcal{Q}$ are a proper subset of the facts in $\mathcal{C}$, which entails that every fact in $\mathcal{Q}$ is equivalent to a fact in $\mathcal{C}$, but not vice versa. This guarantees that a realist about fundamental structure can regard a system's classical space as fundamental and its quantum space as nonfundamental, but not vice versa.

I now turn to a strategy for justifying fundamentality talk in the quantum case without relying on a metaphysical notion of structure. The reason why in quantum logic facts are sets closed under linear combination is that the quantum logician identifies facts with experimental facts, which is to say, facts whose (non)obtaining can be established by experimental means.[14] But experiments are procedures for determining the value of some quantum

[13] But see Torza [40, sec. 6] for an argument to the effect that COMPLETENESS is too stringent, and should be replaced with a weaker condition of 'collective completeness'. Although that weakening has important consequences in the discussion on quantum MI, I must set it aside for reasons of space.

[14] This is due to the constraints that the Born rule sets on the possible outcomes of quantum experiments (Birkhoff & von Neumann [8], Torza [40, sec. 3.2])

observable (position, momentum, spin, etc). Therefore, the facts that live in a quantum logical space $\mathcal{Q}$ are facts about the possible values of physical observables. For the classical logician, on the other hand, any set of vectors in a Hilbert space is a fact about the relevant system. So, the facts of a classical space $\mathcal{C}$ need not be associated with a possible experiment, nor are they defined by reference to physical observables.

With that being said, here is the sketch of how classical and quantum logic may coexist as two pictures of one and the same reality. According to quantum logic, logical space is the space of experimental facts—the facts involving physical observables. This is the familiar picture suggested by orthodox quantum theory. At the fundamental level, however, reality does not involve either experiments or observables. Fundamentally, the world is isomorphic to a vector in Hilbert space, and facts are regions of that space, i.e., sets of possible positions for the state vector. This picture is exemplified by the *Hilbert Space Fundamentalism* which Sean Carroll defends in this volume:

> Here I want to argue for the plausibility of an extreme position among these alternatives, that the fundamental ontology of the world is completely and exactly represented by a vector in an abstract Hilbert space, evolving in time according to unitary Schrödinger dynamics. Everything else, from particles and fields to space itself, is rightly thought of as emergent from that austere set of ingredients. (Carroll [14])

If something like Hilbert Space Fundamentalism is true, the structure of observables encoded in $\mathcal{Q}$ is nonfundamental, and grounded in $\mathcal{C}$. On this theory, logical space is fundamentally classical and derivatively quantum.

I have outlined two ways of motivating the view that $\mathcal{C}$ is fundamental and $\mathcal{Q}$ is derivative: one from first principles, and one inspired by foundational work in physics. As I am about to argue, such a view bears on our central question. Let us first define what it is for MI to arise fundamentally, derivatively, and merely derivatively:

FMI. Given a class W of states, *fundamental* MI arises if MI arises relative to a fundamental logical space on W; *derivative* MI arises if MI arises relative to a nonfundamental logical space on W; *merely derivative* MI arises if MI arises derivatively but not fundamentally.

Since MI arises in quantum logical spaces, but not in classical logical spaces, we can draw the following corollary:

FACT 3. Let W be the class of possible states of a given quantum system. If $\mathcal{C}$ on W is fundamental, and $\mathcal{Q}$ on W is nonfundamental, MI will arise merely derivatively.

My first main claim is now established: the assumption that

4*. There is no merely derivative MI

which I have put forward on behalf of Glick, is unjustified. Consequently, the revised argument against quantum MI from section 1 is unsound. For all we know, MI arises in quantum mechanics.

4 Against Barnes

I showed in the previous section that MI can be merely derivative, and I did so constructively by providing an example from quantum physics. However, Barnes [5] has offered an argument purporting to show that merely derivative MI cannot possibly arise. The goal of this section is to show that her argument is inconclusive. Although replies to Barnes have already been offered (Eva [20], Mariani [28]), I will put forward a different line of resistance based on the observation that in general there are multiple ways of carving logical space compatibly with the evidence.

Barnes' argument purports to show that derivative MI does not arise unless fundamental MI also arises. Let us start with some definitions, relative to a logical space $\mathcal{S}$:

1. A set $\mathbf{F}$ of facts *entails* a fact F if F obtains whenever each fact in $\mathbf{F}$ obtains.

2. A set $\mathbf{F}$ of facts is said to be *complete* if it entails every fact or its negation.

Here is a sketch of Barnes' proof strategy, reformulated for consistency with the present conventions. First, she supposes that there are a fundamental description of reality, and a derivative description of reality. She also assumes crucially (and implicitly) that both the facts picked out by statements of the fundamental description and the facts picked out by statements of the derivative description—call them *fundamental* and *derivative facts*, respectively—coexist in the same logical space. Finally, she assumes that the set of obtaining fundamental facts is complete.[15] Suppose now that the

[15] This assumption is Barnes' version of the COMPLETENESS condition I discussed in section 3.

fundamental is determinate, i.e., that each fundamental fact is such that either it or its negation obtains. Since the fundamental facts form a complete set, they entail each derivative fact or its negation. The fundamental being determinate, it follows that each derivative fact is such that either it or its negation obtains. Therefore, the derivative is also determinate. It can be concluded that merely derivative indeterminacy cannot arise.

The point where I distance myself from Barnes is the one I have flagged, which is her implicit assumption that both fundamental and derivative facts coexist in one logical space. As the quantum case suggests, such an assumption is unjustified, since different logical spaces can be defined on the same set of states, and different spaces correspond to descriptions of reality differing by their relative fundamentality. For example, we can describe a quantum system as fundamentally isomorphic to a vector in Hilbert space, which defines a classical logical space; and derivatively as made up of observable properties such as position and momentum, which defines a quantum logical space.

But once we account for the existence of multiple logical spaces ordered by relative fundamentality, Barnes' line of reasoning becomes invalid, as can be evinced from the following reformulation of her argument (tailored to the quantum case):

> Consider a quantum system associated with a fundamental classical logical space $\mathcal{C}$ and a nonfundamental quantum logical space $\mathcal{Q}$. Because MI does not arise in $\mathcal{C}$, the set $\mathbf{C}$ of facts that obtain in $\mathcal{C}$ is complete. Now pick a fact F in $\mathcal{Q}$. Because the facts in $\mathcal{Q}$ are a subset of the facts in $\mathcal{C}$, F is also a fact in $\mathcal{C}$. Therefore, $\mathbf{C}$ entails either F or its negation. It follows that either F or its negation obtains. Hence, every fact in $\mathcal{Q}$ is such that either it or its negation obtains. Therefore, merely derivative MI does not arise in $\mathcal{Q}$.

The above argument is invalid because "either F or its negation obtains" is ambiguous between the following two readings:

α. Either F or $-_C F$ obtains.

β. Either F or $-_Q F$ obtains.

Notice that "$\mathbf{C}$ entails either F or its negation" should be read as "$\mathbf{C}$ entails either F or $-_C$ F", since the entailment takes place in $\mathcal{C}$. From that α follows. But it does not follow from α that every fact in $\mathcal{Q}$ is such that either it or its negation obtains, because in $\mathcal{Q}$ the negation of F is $-_Q F$, and the obtaining takes place in $\mathcal{Q}$. On the other hand, whereas it follows from β that every fact in $\mathcal{Q}$ is such that either it or its negation obtains, β does not follow from the fact that $\mathbf{C}$ entails either F or its negation $-_C F$.

The moral should be straightforward. It did not occur to Barnes that logical operations are relative to a logical space, and that one and the same set of states can be embedded in different logical spaces endowed with different sets of operations. In the present case, although classical negation prevents MI from arising in $\mathcal{C}$, quantum negation allows for gaps in $\mathcal{Q}$ and, therefore, for merely derivative MI. Thus, Barnes' argument fails because it rests on the unwarranted assumption that the fundamental and the derivative obey the same logic.

One might object that Barnes and I are talking past each other, since we assume different characterizations of MI. Now, it is true that her own characterization of MI—the *metaphysical supervaluationism* developed for example in Barnes & Williams [7]—neither requires nor postulates gaps in logical space, unlike the view I articulated in section 2. But our disagreement concerning the nature of MI is irrelevant to the question as to whether her argument against merely derivative MI is valid. For although on her theory the failure of bivalence is not a necessary condition for MI, it still is a sufficient condition. Indeed, metaphysical supervaluationism says that MI arises just when there is a fact F such that it neither determinately obtains nor determinately fails to obtain. But if a fact does not obtain, it does not determinately obtain; and if it does not fail to obtain, it does not determinately fail to obtain. So, there is MI in my sense only if there is MI in Barnes' sense (although not vice versa). If MI in my sense arises in $\mathcal{Q}$, it will also arise according to Barnes.[16]

5 Conclusions

I have sketched two ways of motivating the view that every quantum system is associated with a fundamental classical logical space, as well as a nonfundamental quantum logical space—one based on realism about fundamental structure, and one based on foundational work in physics. Since MI arises in quantum logical spaces but not in classical logical spaces, I have concluded that quantum physics can be interpreted as giving rise to merely deriva-

[16]It is also worth mentioning that Darby [16], Skow [36] and Fletcher & Taylor [23] have independently argued that metaphysical supervaluationism is unable to model quantum MI.

tive MI, which is to say, MI arising only at the nonfundamental level. This result has a twofold corollary: it undermines an argument against quantum MI which improves on the one offered in Glick [25]; and it provides a counterexample to an argument against merely derivative MI due to Barnes [5].

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