Prioritizing Platonism

Kelly Trogdon & Sam Cowling

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Some see *concrete foundationalism* as providing the central task for sparse ontology, that of identifying which concreta ground other concreta but aren't themselves grounded by concreta. There is, however, potentially much more to sparse ontology. The thesis of *abstract foundationalism*, if true, provides an additional task: identifying which abstracta ground other abstracta but aren't themselves grounded by abstracta. We focus on two abstract foundationalist theses—*abstract atomism* and *abstract monism*—that correspond to the concrete foundationalist theses of *priority atomism* and *priority monism*. We show that a consequence of an attractive package of views is that abstract reality has a particular mereological structure, one capable of underwriting both theses. We argue that, of abstract foundationalist theses formulated in mereological terms, abstract atomism is the most plausible.

1. Introduction

We begin with the notion of grounding. In this article, we work with the view that grounding is to be treated in a manner parallel to familiar ideology-driven views concerning instantiation and other central metaphysical notions. So, while we help ourselves to grounding as an element of our best metaphysical theories and an indispensable tool for stating facts about metaphysical structure, for reasons that will become clear later we deny that grounding talk expresses a relation "out there" in the domain of our ontology. In this respect, talk of grounding is markedly different from talk of relations like "is five feet from" or "is better than." Kment captures the idea behind ideology-driven views nicely: "It shouldn't be assumed that all ingredients of reality must be individuals, properties, or relations—or entities of any kind, for that matter. For example, it's possible that in order to describe reality completely, we need to use some primitive piece of ideology that relates to some aspect of reality that doesn't belong to one of these three ontological categories, and which may not be an entity at all" (2014, 150).[1]

[1] For more on such views, see Cowling (2013), Fine (2010), Forrest (2006), Lewis (1983) and (2002), Melia (2008), Nolan (forthcoming), and Sider (2006). Kment (2014, Ch. 6) considers the possibility that grounding in particular is to be given a primitive ideological treatment.

We make four additional assumptions about grounding. First, the notion of grounding is best regimented with a predicate ('grounds').[2] Second, entities are grounded by collections of one or more entities. Third, nothing can ground itself (irreflexivity), and if A grounds B and B grounds C it follows that A grounds C (transitivity). Fourth, there is a distinction between full and partial grounding, and by 'grounds' we always mean 'fully grounds'. Intuitively, a full ground provides a fully satisfactory explanation of what it grounds, while a mere partial ground does not.[3]

As is customary, let's say that an entity is fundamental just in case it isn't grounded. It's an open question whether any ground for a concrete entity is itself concrete. So, from the fact that a concrete entity is grounded, it doesn't follow that it's grounded by concreta in particular, at least in the absence of further argument. It will be helpful, then, to introduce a category-relative conception of fundamentality—let's say that an entity is *c-grounded* just in case it's grounded by concreta, and an entity is *c-fundamental* just in case it isn't c-grounded.

In its most familiar guise, sparse ontology focuses on the organization of concrete reality and the grounding relations among concrete entities. According to *concrete foundationalism*, any c-grounded concrete entity is grounded by c-fundamental concreta. Let a *concrete foundationalist thesis* be any thesis that aims to specify the mechanism for c-fundamentality (what plays the c-fundamentality role) in the concrete realm. Such a thesis has the following form: Δ are the c-fundamental concreta, and entities among Δ ground all the concreta not among Δ. Some—most notably, Schaffer (2010a)—hold that concrete foundationalism is true, and they take the chief task of sparse ontology to be establishing precisely which concrete foundationalist thesis is the most plausible.

It's also an open question whether any ground for an abstract entity is itself abstract. Hence, from the fact that an abstract entity is grounded, it doesn't follow that it's grounded by abstracta in particular, at least in the absence of further argument. Singletons of concreta such as {Socrates} are potential examples of abstracta that

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[2] Fine (2012) claims that the notion of grounding is best regimented with a sentential connective, while Audi (2012) and Rosen (2010) instead go with a predicate understood as expressing a relation between facts. While we work with neither of these approaches in what follows, our discussion could be recast in such a way that it embraces either the sentential connective or the fact relation view. What's important for us (as will become clear later) is that grounding isn't a relation between entities of various ontological categories. Thanks to an anonymous referee for helpful discussion here.

[3] For a general discussion of grounding that touches on these assumptions and related issues, see Trogdon (2013).

have concrete grounds but lack abstract grounds. So let's introduce a further category-relative conception of fundamentality—let's say that an entity is *a-grounded* just in case it's grounded by abstracta, and an entity is *a-fundamental* just in case it isn't a-grounded.[4]

There is an additional, neglected thesis that if true supplies a further task for sparse ontology. According to *abstract foundationalism*, any a-grounded abstract entity is grounded by a-fundamental abstracta. Let an *abstract foundationalist thesis* be any thesis that aims to specify the mechanism for a-fundamentality in the abstract realm. Such a thesis has the following form: Δ are the a-fundamental abstracta, and entities among Δ ground all the abstracta not among Δ. If abstract foundationalism is true, then one crucial project within sparse ontology is to establish precisely which abstract foundationalist thesis is the most plausible.[5]

Are concrete and abstract foundationalism true? We won't attempt to provide a demonstrative argument for either of these theses. In fact, we suspect, following Bliss (2013), that they don't admit of support via valid arguments whose premises have better epistemic credentials than their conclusions. Instead, we view each of them as reasonable starting points for theorizing about the organization of reality.[6] Insofar as they admit of argumentative support, it is likely by way of some broadly abductive considerations. In discussing similar issues, Cameron (2008) observes that if we're going to do metaphysics at all, we seem to have no choice but to begin with some metaphysical principles for which no demonstrative arguments can be supplied. Schaffer (2010a) treats concrete foundationalism in a similar fashion, and we will approach abstract foundationalism in the same spirit.[7]

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[4] See Wang (2016) for discussion of further category-relative conceptions of fundamentality, conceptions that target objects, properties, relations, and states of affairs. Many claim that fundamental entities are modally free in that they are freely recombinable in some sense. Provided that some abstracta both exist necessarily and have all their intrinsic properties essentially, it's unlikely that modal freedom is a useful diagnostic for a-fundamentality among abstracta.

[5] Grounding among abstract entities and between abstract and concrete entities has not gone wholly unexamined. See, for example, Dixon (forthcoming), Donaldson (forthcoming), and Incurvati (2012).

[6] Compare Goodman and Quine on nominalism, who claim that the thesis is "based on a philosophical intuition that cannot be justified by an appeal to anything more ultimate" (1947, 174).

[7] Schaffer (2010a) does sketch an argument for *foundationalism* according to which any grounded entity is grounded by fundamental entities. While Schaffer doesn't defend any particular foundationalist thesis (he instead defends a particular concrete foundationalist thesis), if foundationalism is true, then perhaps the ultimate task of sparse ontology is to

Our focus in this article is abstract foundationalism. In particular, we develop and scrutinize two abstract foundationalist theses—*abstract atomism* and *abstract monism*—that correspond to the concrete foundationalist theses Schaffer (2010a) calls *priority atomism* and *priority monism*. We show that a consequence of an attractive package of views is that abstract reality has a particular mereological structure that is capable of underwriting both abstract atomism and monism. But we argue that, of abstract foundationalist theses formulated in mereological terms, abstract atomism is the most plausible.

Before proceeding we should mark two further assumptions. First, we assume that the notion of being abstract resists analysis in terms of causal, spatiotemporal, or modal notions. Even so, we help ourselves to the notion of *abstract entity* as a theoretical primitive and assume that reality divides, exclusively and exhaustively, into the abstract and the concrete. We take this approach to comport with the most tenable versions of Platonism, theses according to which the very *abstractness* of entities is what accounts for certain of their peculiar features—e.g., their necessary existence and peculiar cognitive accessibility.[8]

The second assumption concerns the type of grounding that abstract foundationalism concerns. Suppose that there are propositions and that they're abstract in nature. Given what we've said so far about grounding, the following question is coherent:

(1) What a-grounds the proposition that Socrates is a philosopher?

In posing (1), however, there are at least two questions you might have in mind:

(2) What abstracta make it the case that the proposition that Socrates is a philosopher exists?

(3) What abstracta make it the case that Socrates is a philosopher?

When someone asks (2) they want to know what abstracta, if any, make it the case that the proposition exists regardless of whether the proposition is true. So let's say that (2) concerns *existence* a-grounding. In contrast, when someone asks (3) they want to know what abstracta, if any, make it the case that Socrates is a philosopher. Hence, (3) doesn't target the existence of propositions in the same way that (2) does.

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establish which foundationalist thesis is the most plausible. See Trogdon (forthcoming) for critical discussion of Schaffer's argument.

[8] On the concrete/abstract distinction and its significance, see Burgess and Rosen (1997) and more recently Cowling (2017, Ch. 2).

So let's say that grounding claims like (3) target *content* a-grounding rather than existence a-grounding, where questions of this sort concern what abstracta make a proposition true. The abstract foundationalist is primarily concerned with existence a-grounding. When she says that if the proposition that Socrates is a philosopher is a-grounded, then it's grounded by a-fundamental abstracta, what she means is this: if the proposition is *existence* a-grounded, then it's *existence* grounded by abstracta that aren't *existence* a-grounded. Henceforth, by 'grounding' we mean 'existence grounding'.[9]

2. Abstract atomism and monism

How might we characterize the c-fundamental concreta? The orthodox view is that we should do so in mereological terms. The guiding idea is that the cosmos is stratified into "levels" which are ordered by composition, and there is a unique level populated by the c-fundamental concreta.[10] Priority atomism and monism are concrete foundationalist theses of this sort.

Let's say that an entity is *complex* just in case it has proper parts, and an entity is *simple* just in case it isn't complex. And let's say that x is the *cosmos* just in case x is concrete and, for any y, y is concrete and numerically distinct from x just in case y is a proper part of x.[11] We can now formulate priority atomism and monism as follows:

> *Priority atomism*: there are multiple simple concreta, any simple concrete entity is c-fundamental, and any complex concrete entity decomposes into simple concreta such that the latter ground the former.
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> *Priority monism*: there is a cosmos that is c-fundamental, and any concrete entity numerically distinct from the cosmos is grounded by it.[12]

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[9] Just how existence and content grounding are related is an interesting question. Following Sider (2012, Ch. 8), perhaps any proposition that isn't content grounded is such that its non-propositional constituents aren't existence grounded. A natural thought is that thus-and-so is existence grounded just in case the proposition that thus-and-so exists is content grounded.

[10] See Oppenheim & Putnam's (1958) for a seminal articulation and defense of this picture. For more recent discussions, see Kim (1998, Ch. 1) and (2002), Schaffer (2003), and Rueger & McGivern (2010). For an alternative approach, see Giberman (2015a) who characterizes the c-fundamental concreta in topological terms.

[11] We assume that any complex concrete entity only has further concreta as proper parts, and any complex abstract entity only has further abstracta as proper parts.

[12] For a defense of priority monism across a range of papers, see Schaffer (2009), (2010a), (2010b), (2010c), and (2013).

Provided that the c-fundamental concreta are to be characterized in mereological terms, a modest theoretical conservatism suggests that we should pursue a parallel strategy in thinking about the a-fundamental abstracta. It would, after all, be an attractive and economical result if our comprehensive theory of grounding and fundamentality was rightly understood in terms of mereological structure. Additionally, the assumption that abstract reality possesses a mereological structure is bolstered by the familiar assumption that mereology, like identity, is a category-neutral and perfectly general notion rather than a restricted one like mass or membership.[13]

Let a *compositional* abstract foundationalist thesis be any thesis that aims to characterize the mechanism for a-fundamentality in the abstract realm in mereological terms. Let's say that x is the *amalgam* just in case x is abstract and, for any y, y is abstract and numerically distinct from x just in case y is a proper part of x. Among compositional abstract foundationalist theses, we focus on the following theses corresponding to priority atomism and monism:

> *Abstract atomism*: there are multiple simple abstracta, any simple abstract entity is a-fundamental, and any complex abstract entity decomposes into simple abstracta such that the latter ground the former.
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> *Abstract monism*: there is an amalgam that is a-fundamental, and any abstract entity numerically distinct from the amalgam is grounded by it.

There are two conditions that any abstract foundationalist thesis should meet in order to be plausible. First, if such a thesis says that Δ are the a-fundamental abstracta, then entities among Δ should be plausible candidate grounds for the abstracta not among Δ. Second, such a thesis shouldn't multiply a-fundamental abstracta beyond necessity. Call these the *grounding* and *parsimony* conditions, respectively. We take it that a compositional abstract foundationalist thesis satisfies the grounding and parsimony conditions only if the entities among Δ (the abstracta that it says are a-fundamental) cover all the abstracta without gaps or overlaps. So, provided that there is an amalgam, such a thesis satisfies the grounding condition only if it satisfies a particular mereological condition, viz. the entities among Δ are such that their fusion is the amalgam. And a compositional abstract foundationalist thesis satisfies the parsimony condition only if it satisfies a related mereological condition, viz. there is no proper sub-collection of Δ such that the fusion of the entities among that sub-collection is the amalgam.

[13] See Lewis (1986a, 212, note 9), (1986b), and (1991, Ch. 1) for discussion of this last point.

Abstract atomism and monism clearly satisfy these particular mereological conditions. As for abstract atomism, provided that every complex abstract entity decomposes into simple abstracta, the fusion of all simple abstracta is the amalgam, and there is no proper sub-collection of the simple abstracta whose fusion is the amalgam. As for abstract monism, the fusion of the amalgam is the amalgam, and there just is no proper sub-collection of the collection consisting of the amalgam, as here we have a collection of one.[14]

There are other compositional abstract foundationalist theses that satisfy these mereological conditions as well. Suppose that there are properties and they're abstract. Consider the compositional abstract foundationalist thesis according to which the property *greenness* and its abstract complement, the fusion of all abstracta mereologically disjoint from *greenness*, are the a-fundamental abstracta. This thesis clearly satisfies the relevant conditions. The fusion of *greenness* and its abstract complement is the amalgam, and there is no proper sub-collection of *greenness* and its abstract compliment such that the fusion of the entities among that sub-collection is the amalgam. Compare: the fusion of an electron and its concrete complement is the cosmos, and there is no proper sub-collection of the electron and its concrete complement such that the fusion of the entities among that sub-collection is the cosmos.

On what basis do we choose between compositional abstract foundationalist theses that satisfy the relevant mereological conditions? There is a simple consideration that in our view takes all such theses out of contention save for abstract atomism and monism. As we've seen, a compositional abstract foundationalist thesis is viable only if it satisfies certain mereological conditions. Another plausibility condition for such a thesis is that it be principled in the sense that the abstracta that it says are a-fundamental are mereologically distinctive in a way that seems relevant to a-fundamentality. The simple abstracta and the amalgam are mereologically distinctive in this way. This is because decomposition in the abstract realm comes to an end with the abstract atoms and decomposition begins with the amalgam. By contrast, there is no distinctive mereological feature of *greenness* and its abstract compliment that seems relevant to a-fundamentality. And we contend that the same is true for any other compositional abstract foundationalist thesis distinct from abstract

[14] See Trogdon (2017) for a corresponding discussion of conditions of adequacy for concrete foundationalist theses and how this relates to priority atomism and monism in particular. For related discussion, see Schaffer (2010a).

atomism and monism that satisfies the relevant mereological conditions. Hence, these views can be safely set aside.[15]

3. Abstract atoms and the amalgam

A distinctive mereological commitment of abstract atomism is that all complex abstracta decompose into simple abstracta. And a distinctive commitment of abstract monism is that there is an amalgam. Why think that these commitments are correct? In the absence of a substantive conception of abstracta, matters would be too theoretically unconstrained to fruitfully address this question. Hence, we embrace as a working hypothesis a set-theoretic conception of abstracta.[16] On such a view, any abstract entity is either a set or something constructed from sets. As for the categories of abstracta, we assume that there are properties (monadic and *n*-adic), propositions, and mathematical entities.[17] While we think that properties, propositions, and mathematical entities just are sets, later we argue that there is an abstract entity (the fusion of all sets) that, while not itself a set, is composed of sets. The set-theoretic conception of abstract reality faces a number of challenges, including the problem of distinguishing between apparently distinct albeit cointensive properties and Benacerraf's problem of multiple reductions.[18] We take as a working assumption that these concerns can be addressed. Platonism, so understood, enjoys most of the explanatory and intuitive benefits claimed by proponents of abstract entities.

Given our conception of abstracta, the abstract realm has a set-theoretic structure. Hence, one approach to abstract foundationalism is to characterize the a-fundamental abstracta in set-theoretic terms. But remember that our project is to characterize these entities ultimately in mereological terms. How to proceed? Happily, there is a plausible take on the metaphysics of sets that, when combined with a set-theoretic conception of abstracta, has the consequence that the set-theoretic structure of abstract reality just is a certain mereological structure. This package of views also vindicates the distinctive mereological commitments of

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[15] Thanks to an anonymous referee for helpful discussion here.

[16] On competing Platonist options, see Cowling (2017: 92–102, 177–187).

[17] So, we deny that musical works, fictional entities, and directions are among the abstracta, provided that these types of entities are understood non-set-theoretically.

[18] On the former, see Lewis (1986a), Eddon (2010), and Nolan (2014); on the latter, see Benacerraf (1965), Melia (1992), Sider (1996), and Paseau (2009).

priority atomism and monism, provided that composition is unrestricted. The view we have in mind is the one developed by Lewis (1991).[19]

We begin with some further terminology. The *empty set* is the unique set without members. A *non-empty* set is any set distinct from the empty set. So a non-empty set is either a *singleton*—a set with only one member—or a set with more than one member. Let one set be a *subset** of another just in case the former is a subset of the latter and both are non-empty. So, while the empty set is a subset of any set, the empty set isn't a subset* of any set; and, while there is one and only one set that's a subset of the empty set (itself), no set is a subset* of the empty set. Lewis plausibly suggests that subsethood* is a special case of parthood—the parts of a non-empty set are all and only the non-empty subsets of that set.

Lewis' view is the conjunction of the following claims: (i) a non-empty set has its non-empty subsets as parts, and (ii) no non-empty set has any part that isn't a non-empty set. In defending (i), Lewis notes that it conforms to ordinary discourse about sets. It's natural to say, for example, that the set of Canadians is a part of the set of humans, and the set of even numbers is part of the set of natural numbers. And, perhaps more importantly, parthood and subsethood* have analogous formal features like transitivity and non-unique decomposition. As for (ii), Lewis shows that it follows from (i) together with other plausible assumptions, one of which is that reality divides exclusively and exhaustively into things with members (non-empty sets) and things without members that are members (the empty set and individuals).[20]

Given Lewis' metaphysics of sets, a non-empty set is complex just in case it has non-empty *proper* subsets and is simple otherwise. Since singletons have no non-empty proper subsets, they are simple. So on this view {{Socrates}} is no more mereologically complex than {Socrates}—both are simple. And, since the empty set

[19] As we explain below, Lewis (1991) argues that set construction is to be analyzed in terms of composition and singleton formation. (His views on singleton formation are complex and we don't have the space to engage with them here—see Lewis (1993) for further discussion.) There are other approaches to set construction, however. Fine's (2010) approach, for example, is to take set construction as conceptually primitive. If it turns out that there isn't a tight link between set construction and mereology, we should probably abandon the compositional approach to abstract foundationalism given a set-theoretic conception of abstracta.

[20] Here and in what follows we use slightly different terminology than Lewis (e.g. he focuses on classes rather than sets, and he classifies the empty set as an individual while we do not).

has no proper subsets period, it's plausibly taken to be simple as well.[21] Any multi-membered set has the singletons of its members as proper subsets, so any such set is complex. Finally, since a set is just the union of the singletons of its members, any multi-membered set decomposes into singletons.

The picture that emerges is this: any set is either complex or simple; a set is simple just in case it's a singleton or the empty set; a set is complex just in case it's a multi-membered set; and, since multi-membered sets decompose into singletons of their members, any complex set decomposes into simple sets. Given Lewis' metaphysics of sets and our assumption that abstract reality is set-theoretic in nature, all complex abstracta decompose into simple abstracta. *Gunky* abstracta—abstract entities such that all their parts have proper parts—are impossible.[22]

Now we turn to the amalgam. You might think that, given Lewis' metaphysics of sets, it's a straightforward matter that there is no amalgam. Suppose that every set is a proper subset of some set, and in particular that every non-empty set is a proper subset of some non-empty set. In this case it follows that every non-empty set is a proper part of some non-empty set. A mereological structure of abstracta is *junky* just in case it's an infinite collection of abstracta, each of which is a proper part of another. If every non-empty set is a proper part of some non-empty set, then the non-empty sets form a junky structure. But, if the non-empty sets form a junky structure, then there is no fusion of all non-empty sets. And there is a fusion of all abstracta only if there is a fusion of all non-empty sets.[23]

There is, however, a clear problem with this argument. It's true, we will suppose, that the non-empty sets form a junky structure. It follows that there is no fusion of all non-empty sets that *is itself a non-empty set*; otherwise, there would be a non-empty set that isn't a proper part of another non-empty set, and in this case the non-empty sets

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[21] It doesn't follow from Lewis' package of views that the empty set is simple—Lewis in fact suggests that the empty set is the cosmos! We agree that the empty set doesn't have abstracta as *proper* parts, but we take the empty set to itself be abstract rather than concrete.

[22] We take the possibility of concrete gunky objects seriously as well as "onion worlds"—roughly, worlds exhibiting infinitely descending qualitative complexity. Note, however, that the former requires no commitment to gunky properties, and, as Williams (2007) argues, neither does the latter.

[23] Compare: a structure of concreta is junky just in case it's an infinite collection of concreta each of which is a proper part of another. If there are junky structures of concreta then there is no fusion of all concreta, provided that any fusion of concreta is itself concrete. See Bohn (2009) for an argument that it's possible that the concreta form a junky structure, and Giberman (2015b) for discussion of the idea that there are proper sub-collections of the concreta that form such structures.

wouldn't form a junky structure. But, for all we've said, there is nevertheless a fusion of all sets that *isn't itself a non-empty set*.

So we don't think that given Lewis' metaphysics of sets it's a straightforward matter that there is no amalgam. Indeed, exactly the opposite is true. We're assuming that composition is unrestricted. So, for any collection of abstracta, there is some whole composed of those entities. Hence, the collection of *all* abstracta composes some entity. Provided that the fusion of all abstracta is itself abstract, the fusion of all abstracta is the amalgam.

A final note about the amalgam is in order before proceeding. Let a *mixed fusion* be any fusion of a non-empty set and something that lacks members but is a member, i.e. either the empty set or some individual. Examples of mixed fusions: the object composed by {Socrates} and the empty set, and the object composed by {Socrates} and Socrates. Are mixed fusions themselves non-empty sets? No—non-empty sets have neither the empty set nor individuals as parts, and mixed fusions have such parts. Is there any mixed fusion that is either the empty set or an individual? No—neither the empty set nor any individual has non-empty sets as parts, and mixed fusions have such parts. It's part of Lewis' package of views that x has a singleton only if x is either a set (empty or not) or an individual. Supposing that Lewis is right about this, it follows that mixed fusions don't have singletons.[24] The amalgam is a mixed fusion, as it's the fusion of all singletons and the empty set. Hence, there is no singleton of the amalgam. As such, the amalgam is ineligible for membership.

Given the set-theoretic conception of properties articulated above, to have a property is to be a member of some set. So in this case something has properties only if it's eligible for membership. Since the amalgam is a mixed fusion and thus ineligible for membership, it follows that it has no properties.[25] What does this mean for abstract monism? Well, remember that the abstract monist claims that the amalgam *grounds* its proper parts. So in making this claim, we shouldn't interpret the abstract monist as claiming that the amalgam stands in a particular relation to its proper parts. This is why it's crucial in this context to treat the predicate 'grounds' as not standing for a relation, recalling one of the assumptions we set out at the

[24] According to Lewis, while mixed fusions are "forced upon us" by universalism (the thesis that composition is unrestricted), there is no principle that is both independently plausible and has the result that mixed fusions have singletons (1991, 8). The corresponding principle of unrestricted set-formation, for example, is false, as there is no set of all non-self-membered sets.

[25] See Rosen (2015) for further discussion of the idea that mixed fusions in Lewis' framework lack properties.

beginning of this article. Were we to treat grounding as a relation then abstract monism would be false, as the amalgam doesn't stand in any relations. The same considerations apply to composition.[26]

4. For abstract atomism

We now turn to the task of showing that abstract atomism is more plausible than abstract monism. It seems that there are senses of the terms 'mere aggregate' and 'integrated whole' relevant to grounding, and appealing to this distinction is potentially helpful in this context. Consider the sense of 'mere aggregate' such that a heap of sand counts as a mere aggregate. Mere aggregates in this sense are grounded by their proper parts. And consider the sense of 'integrated whole' such that a molecule counts as an integrated whole. Integrated wholes in this sense are candidate grounds for their proper parts. Call these the *grounding senses* of 'mere aggregate' and 'integrated whole'.[27]

Roughly speaking, if a concrete whole is integrated in the grounding sense, this is because of the *spatiotemporal arrangement* of and *nomic relations* among its parts. The molecule, for example, is an integrated whole partly in virtue of the fact that its atomic constituents causally interact in systematic ways. One potential way of sharpening this conception of integration is this: something is an integrated whole when it and its proper parts are *mutually manipulable*, where manipulation in this context is understood in terms of causal interventionist machinery.[28] The heap of sand, by contrast, is a mere aggregate in the grounding sense because the grains of sand that compose it aren't spatiotemporally arranged or nomically yoked in the right sort of way.

Returning to abstracta, we know that if the amalgam is an integrated whole in the grounding sense, then radically different principles regarding integration would be required given the marked contrast between concrete entities and abstract entities.

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[26] There is another reason to work with a view according to which grounding isn't a relation in this context: in this case we don't face the difficult question of what abstracta if any ground the grounding relation itself. It's true that there is still the question of what makes it the case that particular abstracta ground other abstracta, but notice that this concerns content a-grounding rather than existence a-grounding. As we noted earlier, abstract foundationalist theses target the latter rather than the former.

[27] Indeed, Schaffer (2010b) proposes to *analyze* the concepts of an integrated whole and a mere aggregate in terms of their grounding profiles—here the idea is that something is an integrated whole just in case it has proper parts and grounds them, and something is a mere aggregate just in case it has proper parts and is grounded by them.

[28] See Craver (2007, Ch. 4) for relevant discussion.

After all, the latter seem to be "outside" of spacetime and, on at least some views of the abstract-concrete distinction, stand in no causal relations whatsoever. What we want to suggest is that, regardless of how the parts of the amalgam are arranged, their arrangement provides no good reason to believe that the latter grounds the former. Put differently, our contention is that the amalgam isn't an integrated whole in the grounding sense. It is, instead, a mere aggregate in the grounding sense. This, in turn, suggests that abstract atomism is, all other things being equal, preferable to abstract monism.[29]

Perhaps, however, the proper conclusion to draw instead is that the grounding senses of 'mere aggregate' and 'integrated whole' we're employing simply have no purchase in the abstract realm. And, if the putative distinction between mere aggregates and integrated wholes is essentially spatiotemporal or broadly causal in nature, we might reasonably conclude that it can be of little help in sorting out which abstract entities depend upon which. We concede that it would be better to have considerations that speak more clearly in favor of abstract atomism if we're going to be atomists.

How to proceed? Supposing that the abstract atomist and monist embrace Lewis' metaphysics of sets and the set-theoretic conception of abstracta (as this package of views verifies their distinctive mereological commitments), let's narrow our focus to the abstract atomist's claim that the singletons ground the amalgam, and the monist's claim that the amalgam instead grounds the singletons. Why think that one of these claims is more plausible than the other? Some grounding claims are intuitive in that they strike us as apt or plausible when we consider them. One everyday example that we've already discussed is this: the grains of sand ground the heap they compose. It seems, however, that the grounding claims made by abstract atomists and monists are different—they aren't especially intuitive. If we can show, however, that the atomist's claim is an instance of a substantive and plausible general grounding claim, we will have shown that the atomist's claim is plausible, all things considered. This is what we do below.

Consider the following substantive general grounding claim: mixed fusions (fusions of non-empty sets and either the empty set or individuals) are grounded by the entities that compose them. As the amalgam is a mixed fusion, the atomist's claim is

[29] Schaffer (2010a) and (2013) appeals to the idea that the cosmos is an integrated whole in arguing for priority monism. See O'Conaill & Tahko (2012) and Miller (2013) for critical discussions of these arguments.

an instance of this general grounding claim. And we think that this general grounding claim is highly plausible.

Consider, for a moment, exotic entities such as *trout-turkeys*—individuals composed of trout and turkeys. Notice that the only reason we have to think that there are such things is because we already think that there are trout and turkeys and we accept universalism, the thesis that composition is unrestricted. There are no independent reasons to believe in trout-turkeys. Indeed, these entities can be "mostly ignored" and "left out of the domains of all but our most unrestricted quantifying" (Lewis, 1991, 8). As such, trout-turkeys simply aren't candidate grounds for either trout or turkeys. More generally speaking, they play a substantive role in neither ordinary discourse, our best scientific theories, nor our philosophical theorizing.

This moral extends to mixed fusions. The only reason we have to think that there are such things is because we already think that certain other things exist and we accept universalism. We lack independent reasons to posit them. Consider, for example, *cat-singleton-whiskers*, fusions of cats and singletons of their whiskers. We think that there are such things only because we already think that there are cats and singletons of their whiskers and we accept universalism. Turning to the amalgam, we think that there is such a thing only because we're already on board with the singletons and the empty set and again we accept universalism. As such, mixed fusions are no more candidate grounds for their proper parts than trout-turkeys are.

We assume that either mixed fusions ground the entities that compose them or vice versa. This assumption is plausible from the point of view of sparse ontology and it's something that the abstract atomist and monist would potentially agree on. Provided that mixed fusions don't ground the things that compose them, we get the desired conclusion: mixed fusions are grounded by the entities that compose them.

Summing up, we think that the abstract atomist's claim (that singletons ground the amalgam) is an instance of a substantive and plausible general grounding claim. And it seems that there is no substantive and plausible general grounding claim of which the abstract monist's claim (that the amalgam grounds the singletons) is an instance. While some general whole-to-part grounding claims are plausible, such claims don't suggest that the amalgam grounds the singletons. To take an example, we've seen that there is a sense of 'integrated whole' on which its plausible to regard such wholes as grounding their proper parts. But there's no convincing reason to believe that the amalgam is integrated in the relevant sense. And, while some general whole-to-part grounding claims have the consequence that the amalgam grounds the singletons, none of these claims is plausible. To take an example, the claim that any whole whatsoever grounds its proper parts would, if true, guarantee that the

amalgam grounds the singletons. But this claim is implausible, since some parts ground the wholes that they're parts of (e.g. the grains of sand ground the heap they compose). So we conclude that holistic considerations suggest that abstract atomism is more plausible than abstract monism.[30]

5. Conclusion

At the outset of this article we described an unrecognized task for sparse ontology: deciding which abstract foundationalist thesis is the most plausible. We then set out to make some initial progress in this task. We've shown that there is an attractive (though, of course, non-compulsory) package of views—the set-theoretic conception of abstracta along with Lewis' metaphysics of sets—according to which abstract reality has the mereological structure required by both abstract atomism and monism. We then argued that abstract atomism and monism are the most plausible compositional abstract foundationalist theses, and that the former is ultimately preferable to the latter.[31]

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[30] If composition is "ontologically innocent" in Lewis' (1991, Ch. 3) sense, then mixed fusions don't ground the things that compose them. But with Lewis' principle no fusion whatsoever grounds the things that compose it, and this begs the question against the abstract monist (as well as the priority monist).

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