Why “17 Gen r” is undecidable: Gödel's proof and the paradox of self-reference

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Abstract

The aim of this text is to offer an explanation of Gödel's Theorem according to the schemes and notations of the original article. There are many good didactic explanations of the theorem that reveal its central points and implications, but these are difficult to recognize when reading the original work, due to the complexity of its formulation and the author's economical style in explaining the steps of his argument. An exposition of the central concepts will be made, as well as a detailed explanation of the main points of the algebraic development of the proof, which will allow the non-specialist[1] reader to find the well-known paradox from them.

Keywords: paradox, incompleteness, self-reference, Gödel.

Introduction

There are two typical situations in which paradoxes arise. The first occurs when the same element shows contradictory consequences for being multiple when it is understood as unique. The twin paradox is a classic example. Under normal conditions, two identical twins after the same time should age in the same way. But both in different circumstances (on a cosmological scale) would each have their own timeline, and this would become clear at their reunion. Time, which *a priori* would be understood as the same everywhere, is in reality different.

The second situation presents itself when an object is defined by a certain property, and from this, another is presented that does not have it. And finally it is revealed then that they are in fact the same object, but that this makes it possess both the property and its negation. We see this in the case of the liar's paradox, about someone who says “I only tell lies.” The statement of someone who utters a possibly true statement, regarding the generalized falsity of the statements of the person being spoken to, has the same subject and object. This creates a contradiction: if he is lying, he sometimes speaks truths; If he is telling the truth, he is lying precisely by denying the statement.

It is possible, however, to find cases in which both situations occur simultaneously. This is what happens in Russell's Paradox (Almeida, 1998), which originates from a problem that arose from the definition of numbers from classes. The question arises when we consider a class of classes that are not members of themselves. Is the class itself a member of the set? If it is a member of itself, it is not a member of itself. The consequence, besides being a paradox of the second kind (the class has and at the same time doesn't have a property), is a case of the first – there is an inconsistency within set theory, one of the foundations of logic, the structure of the definition of functions and concepts in terms of sets is imperfect.

The generating mechanism of this vicious circle, according to Russell, comes from what he called “illegitimate totalities”—those that contain members defined in terms of the totality itself.

1 The author of the text, who is not a mathematician, spent a few months obstinately analyzing the article, looking for the paradoxes so mentioned in the literature on the subject. It was very difficult to find them, since the text is not at all obvious. But as the rule goes, the original is always better than the versions, and this is no exception. The development of the problem in the article is more sophisticated, and raises more questions than the didactic versions, and knowing it in detail is very rewarding. As in the time of research I had to take many notes, and formulate different hypotheses on how to explain it, I ended up gathering a set of ideas that, I believe, can be useful to those who decide to follow in the endeavor.

These are denoted by expressions such as “all propositions”, “all properties”, “all classes”, etc. A case of this fallacy would be the enunciation of the law of the excluded middle in the form “all propositions are true or false” which applied to the law itself results that it itself is true or false. And so, the vicious circle requires a kind of restriction in the formulation of statements: “no function can be contained in the domain of its real or apparent variables” (ibid, p. 142). A function cannot be assumed in its values, and these must be defined before the function itself is. And so, we should state the law of the excluded middle in such a way that it does not apply to itself, even if we have not said so. The big problem is to elaborate the question in such a way as to say that we won't mention something without mentioning it.

But it was through the discovery of a self-referential short circuit that the mathematician Kurt Gödel (1962) showed the impossibility of arriving at a general system of mathematical proof (of which Russell was one of the proponents). He showed that a mathematical system cannot demonstrate its own consistency[2], and so it must be somewhat incomplete. And yet, the attempt to insist on the demonstration of consistency only with the resources it possesses generates types of propositions whose proof or its negative cannot be offered within it. The theorem (Ernst and Nagel, 1973) put an end to the hope that a complete set of axioms could be assembled for an area of mathematics, or that it could be completed by adding a finite number to the original list.

As these authors explain, the theorem exposes an inconsistency in the claim to completeness, but without falling into the fallacy of Richard's Paradox. This originates from a hypothetical system for defining cardinal numbers, in which each number in a list, according to the scenario proposed for the paradox, must be accompanied by a description that is appropriate to the ordering number itself. The number 2 would be "the first prime number", 10, "twice the third prime number". It may happen that a number is accompanied by a description that doesn't apply to it, such as "it's an even number divisible by three" in item 15. In this case, it will be called *richardian*, and if the description matches the number, *non-richardian*.

The expression to be richardian or not is applicable to numbers, and so can have a number applied to it. This begs the question: is the corresponding number of this item richardian or not? The number n, corresponding to the definition, is richardian if and only if it is not related to the definition of being richardian. If it is richardian, it does not have the property designated as defining the number designates the number of the richardian formula. Therefore, it is richardian if it is not richardian. The paradox in this case comes from the mixture of statement and case, but it arises from a fallacy. It comes from the attribution of an extra property (the incidental adequacy or inadequacy of the description) transformed into a legitimate statement within the system.

In Russell's Paradox, this does not occur. The class of trees is not a tree. The class of classes is a class. A class can be a member of itself, and this happens when (for example) the definition of class is part of the characteristics that every class possesses, such as “the class that gathers objects”, “the class of thinkable things”, “the class of abstract entities”. Every class gathers objects, is thinkable, and is an abstract entity, and therefore each class that designates them is a member of the class itself. The definition of classes that are not members of themselves would

2 A consistent system is free of contradictions. One of the goals of Principia Mathematica (Ernst, Nagel, 1973) was to seek techniques to eliminate all self-contradictory forms. This was done by a system of notation in which all mathematical statements and rules of inference are codifiable in a standard way. The entire system is reduced to a calculation of “meaningless marks”, with its formulas obtained by direct transformation of operation rules. A parallel to this was also sought in Hilbert's Program, through “finitary arithmetic” (Shapiro, 2021). Expressions with limited quantifiers can be transformed into computations. This would be a technique for evaluating statements containing infinites, of “ideal” mathematics. And so, “to use an ideal theory of mathematics we have to formalize it and then show, within finitary arithmetic, that the theory is consistent.” (ibid, p.237).

designate those that have elements extrinsic to the definition of class, and that cannot be assigned in any way to it, such as the class of trees, which have no branches or leaves.

And so, in the first type, all are elements of the same nature, such as “being a collection” and “being abstract”; in the second, are they of a different nature, and so the class of these classes is a member of itself? The gathering of these classes (of ships, trees, chess pieces) is a class, but it can only be a member of itself if it adopts its designation, “not to be a member of itself”. The set “definition of class + elements” (such as "statement + cases") is of a different nature, and so the statement of the class makes it distinct from its contents, and therefore it is not a member of itself. But as it is (by definition) the class of classes which are distinct from its contents, it is a member of itself, or, the class of classes that are distinct from their objects is distinct from its object, and therefore is and is not at the same time a member of itself. In Richard's paradox, an incidental property is artificially introduced. In Russell's, the definition of the general class comes from a property of the member classes. The emergence of inconsistency follows naturally as one of the corollaries of the general definition of class, and thus is not built on a fallacy (hence its historical importance).

Neither is Gödel's theorem, since it is not constructed by illegitimately inserting a property into the system of formulas, but is a natural consequence of a finite proof system, in which the last or most generic element of the list has a dual role. Its detachment from the object of proof, reviewing it through a different set of properties, turns its test into a reconstruction, from which the decision for its provability or not can emerge as a result. And so, the very formula it tests must repeat this distancing in consideration of itself, in a self-referential application, and so its provability (or negative) as a formula will be non-generalizable; or, there is neither a decision by its generalization, nor by the impossibility of generalization for any case.

Self-reference is a type of pattern that generates theoretical perplexities, but in some situations its emergence ends up being an inevitable consequence in areas of scientific development. There are many cases of what is meant by self-reference, such as those highlighted by Hofstadter (2007). These include natural causal or mechanical feedback patterns; repeated cyclical natural events, such as fractals, in which the pattern repeats at different scales; reflective or symmetrical optical phenomena, such as mirrors in opposition, and also in the linguistic field, phrases with references to their own characters, symbols and concepts, or abstract engravings in which the circumstance of the image itself is represented. And so, the precision of the highly formalized mathematics of the early twentieth century could conceal precisely kinds of propositions with this property which, due to their key positions in their respective systems, disrupted their entire rigorous principiological architecture. And so, says the author (ibid, p.168): "(...) what about KG[3] (or any of its cousins) that makes it not probable? In a word, it is its self-referential meaning: if KG were provable, its loopy meaning would flip around and make it unprovable, and PM would be inconsistent, which we know its not".

Gödel's work has important consequences for the theory of knowledge. Popper (1986) explained that a deductive inference is valid if it does not admit counterexamples, and thus is objective. But objectivity does not mean that we can always ascertain whether a statement is true. There are many statements whose truth can be demonstrated, but we don't have a general criterion of truth, and according to Gödel's work, this holds even for arithmetic. We can describe a multitude of rules of inference, but we have no general criterion for deciding whether or not any arithmetic statement legitimately follows from the axioms of arithmetic. And this has an important consequence for epistemology, because, even at a very advanced stage of research in Physics, in

3 Gödel’s formula.

which it is very difficult to emerge new modifications and corrections in the models, the theories will remain incomplete, and we will know this *a priori,* due to Physics being based on Mathematics.

Thomas Nagel (1998), in response to logical and epistemological skepticism, and the subordination of reason to other instances, such as social authority, or linguistic and cultural practices, sought to show how reason is independent of any of these contexts. According to him, rationality is not a core of fundamental truths, but all those propositions for which there is no alternative. Even if some of them can be revised, others still give it meaning. The simple application of mathematical rules leads to consequences that cannot be fully grasped, but only understood (such as the fact that the simple sum rule leads to the concept of infinity) and so a comprehensive grasp of the reality of numbers is impossible. And this incompleteness shows that they are not simply subjective rules. Finally, he presents Gödel's proof as the greatest anti-reductionist argument in history, because it shows that in any case the explanation of reason will always presuppose itself. Even if one tries to reduce mathematics to what is probable, this demand for self-justification will be surrounded by considerations about the nature of mathematical reasoning that do not fit into the proof. In other words, arguments that seek to reduce rationality itself and its criteria will always be circular.

As Kripke (1972) explained, because the theorem proves that no formal system can decide on all mathematical questions, there are always more truths needed than can possibly be included in a unified scientific system. The fundamental standards that define *necessity* and *apriorism* are not always linked and interdependent. Necessity is a logical criterion, and apriorism is epistemological. We cannot prove consistency within a system because, if apriorism is the truth originated from intellectual synthesis, the kinds of truths needed are not, in turn, expressions of a single abstract core — if they were, mathematics would not be productive. And just as the general set of mathematical truths is unapprehensible by a single model, there is no guarantee that there are no hidden mathematizable patterns in the universe that are necessary truths, although they cannot yet be recognized as such *a priori*. The search for consistency is the search for apriorism to encompass the set of the necessary, but this is a never-ending search.

Another consequence of the theorem discussed by Penrose (1993) is the value of abstract reasoning, intuition, and creativity as central to mathematical and scientific understanding and development. Even rigorous mathematical reasoning needs, in its development, abstract leaps and inversions from other perspectives and concepts. A mechanical procedure (symbolic or computational) has clear limitations due to the problem of being a mere expression of sign relationships, and not carrying their true understanding and meaning. That is, the system is unable to abstract itself and step out of its own limits. And the author states that the human capacity for mathematical thinking is due precisely to an essentially non-computable aspect of human intelligence.

The Theorem

To carry out the demonstration, the author proposed what would be a model with all the characteristics of a general system of proof, in a own notation scheme in which mathematical and metamathematical relations are correlated. The metamathematics of the system describes the logical/inferential relationships that occur between the steps of a mathematical development and its formulas (such as the relations of "logical consequence"[4], "proof"[5], "generalization"[6], "negation").

4 The expression Flg(c) (from the expression "Folgerungsmenge") represents the set of consequences of a proof scheme c. A proof in c is one provided only by the resource of its rules.

It also describes the formalization of elementary rules of symbolic and syntactic transformation, such as number types and properties[7], and the very rules of character operation[8].

And he proposes that formulas could be listed in a numbering system in which the basic characters, relationships, and formulas would be related to products of prime numbers. And so, since the product of a prime can always be found exactly its component numbers, each number would represent a formula in an unmistakable way, and this version would be the one used in the calculation. Each character number, relation, or other formula could be a variable of some other formula.

The result is that with the correct formation rules listed systematically, it would be possible to mirror with mechanical procedures a complete mathematical system. And since the proof structures are well defined and explained, any proof can be submitted to this system, which would ideally be a general proof mechanism.

In addition, the author works with the concept of *recursion* in a broad sense, which is one of the main points for his argument. A general sense of recursion is the repeated reapplication of a formal procedure, properties or formula patterns, in new steps of calculation, or the application of series of values in place of their variables, according to defined rules of formation. A series of numbered functions is recursive when it is defined by the previous two, or derived from them by substitution. A formula, which can be selected from a list, has a recursive relation with the preceding and successors. All are expressions developed from the same set of definition rules successively reapplied and made more complex for new cases. Recursion applies both to the relations of equivalence, attribution, disjunction, the application of mathematical concepts in other formulas, and to the metamathematical aspect, such as the rules of formation of a formula by the characters that constitute it. The author explains that function patterns such as x + y, x × y, x/y, and the relations x < y, x = y are recursive, and from them, he will define other functions and relations (numbered from 1 to 46), also containing metamathematical concepts such as “formula”, “axiom”, among others. Here are some examples of these definitions:

11. $n \text{ Var } x \equiv (E\, z)\, [13 < z \leq x \;\&\; \text{Prim}\,(z) \;\&\; x = z^n] \;\&\; n \neq 0$
    x is a *variable of n-th type*.
12. $\text{Var}\,(x) \equiv (E\, n)\, [n \leq x \;\&\; n \text{ Var } x]$
    x is a *variable*.
13. $\text{Neg}\,(x) \equiv R\,(5) * E\,(x)$
    Neg (x) is the *negation* of x.
14. $x \text{ Dis } y \equiv E\,(x) * R\,(7) * E\,(y)$ [183]
    x Dis y is the *disjunction* of x and y.
15. $x \text{ Gen } y \equiv R\,(x) * R\,(9) * E\,(y)$

A formula with a generic formation rule, which represents a significant relationship within the system, is called a *class-sign*. It designates a class of elements, which can be types of relations between numbers and characters, formulas, other classes, or, generalizations of the rules of

5 The expression “x Bew c y” (“Beweisbar” - provable) means that the formula x, by and only by means of the rules of c, can prove y.
6 The expression “x Gen y” means that the formula y is generalized by means of the free variable x.
7 The expression “n Gl x” (“Glied”) designates the nth term applied to the variable x.
8 R (11) * x * R(13) - “*” is the concatenation of characters, and the numbers 11 and 13 correspond to the elementary symbols “(” and “)”, resulting in “(x)”.

recursion themselves. In the examples in the figure above, there are class-signs for specifying the degree of the variable, negation, disjunction and generalization operations. The class-sign has one free variable, and expressions with two variables can be simplified to one, and this allows its different cases to be numbered by variables that serve as indexes in recursive structures.

Another important pattern, then, is the *recursive class-sign*. This is a case where a class-sign with a value assigned to the free variable is represented by its recursively enumerable substitute formula, as well as being organized in a relationship with other formulas. On the surface, this notation doesn't seem very practical, but it does have the advantage of allowing the representation of recursive formula chaining.

a) $Sb\left(f \quad \begin{matrix} k \\ b \end{matrix}\right)$ b) $Sb\left(y \quad \begin{matrix} n & f & k \\ v & j & b \end{matrix}\right)$ c) $Sb\left(s \quad \begin{matrix} 17 \\ Z(x) \end{matrix}\right)$ d) $Sb\left(q \quad \begin{matrix} 17 & 19 \\ Z(x) & Z(y) \end{matrix}\right)$

The expression in a) says that from a formula of number b, when the value k is applied in place of its variable, we will have as a result the formula resulting from number f. It is the same as writing b(n) as a general form, and b(k) as a specific case, generating f. Or, rewritten, is a case of k applied to the class-sign b(n). Or it can be rewritten as a class-sign, as for example, in the form k Gen f, and the address of that formula in the list is b.

When a formula is generalized, becoming a class-sign, the variable that generalizes it appears as a kind of index of that formula in this representation of the chained formulas, in their cases of application. And so, in b) the formula y is the one that represents the recursive chaining of the formulas v, j, and b, when the values n, f, and k are applied to them respectively in the places of the variables. In variation c) the formula s is the result of the value of 17 being applied in the formula of *number-sign* Z(x), that is, in the number corresponding to the formula x. In d) shows a relationship q between two formulas, x and y, selected by the numbers of the corresponding formulas, as their free variables. Cases c and d will be used in the proof, as will be seen later.

Furthermore, there are statements presented by the author that are important for the relationship between provability and recursion, and for the notion of consistency that he intends to present in his argument. First, let's consider a class of formulas c, and the smallest set of formulas that are immediate consequences of the relations and axioms established in it, abbreviated by Flg(c). And so, the author presents the notion of ω-consistency[9] of this class (where v is the free variable), which is the condition that there does not exist in c a class-sign such that 1) a recursive case of a formula is a consequence of c, and 2) the general form of this formula has no proof, or, symbolically:

9 As Lourenço (2020, p.734) explains: "Gödel makes use of the concept initially discovered by Tarski, of ω-consistency, which essentially has the following meaning. A theory Z is said to be ω-inconsistent if, and only if, there exists a well-formed formula φ(x) such that for any natural number n one has a demonstration in Z of φ(n) and at the same time a demonstration of the formula ∃x¬φ(x). If, on the contrary, it is not possible in Z to derive for any natural number n the formula φ(x) and at the same time ∃x¬φ(x), then Z is said to be an ω-consistent theory". Shapiro (2021) explains that a formalized arithmetic theory is consistent if we cannot derive a contradictory formula from it by means of its rules and axioms. The notion of ω-consistency in a theory is the condition that it does not occur that a given formula is probable for different numbers, and at the same time, it is demonstrable that it fails for a given value. In the author's words, (p.239) "Gödel showed that there is a sentence G in the language T such that 1) if T is consistent, then G is not a theorem of T, and 2) if T has a slightly stronger property than consistency, the so-called "ω-consistency", then the negation of G is not a theorem of T. That is, if T is ω-consistent, then it does not "decide" G one way or the other".

$$(n)\ [\ Sb\left(a\ \begin{matrix} v \\ Z(n) \end{matrix}\right)\ \varepsilon\ Flg(c)]\ \&\ [Neg(v\ Gen\ a)]\ \varepsilon\ Flg\ (c)$$

We also have the following statements, which mean respectively (accompanied by the article numbering):

$x\ B_c\ y \equiv Bw_c\ (x)\ \&\ [l\ (x)]\ Gl\ x = y$ (6)

"The formula x proves y in c" is equivalent to saying that x is a development of logical/inferential (recursive) steps of c according to which, the end of the proof, the last step of x is the conclusion y.

$Bew_c\ (x) \equiv (Ey)\ y\ B_c\ x$ (6.1)

"The formula x is provable in c" is equivalent to saying that if there is a proof of x, there is a formula number other than x that proves it.

$(x)\ [Bew_c\ (x) \sim x\ \varepsilon\ Flg\ (c)]$ (7)

At formula number x, the proof of x does not belong to the set of consequences of the class of formulas c.

$(x)\ [Bew\ (x) \rightarrow Bew_c\ (x)]$ (8)

If x is provable, it is recursively provable in c.

And with this, the author establishes that, whether it is a function of degree n in a recursive relation, derived from functions of previous degrees, if all the relations up to n can be proved, then at level n it can be decided whether the set of steps up to it can be proved or not.

> Proposition V: To every recursive relation R ($x_1, \ldots x_n$) there corresponds an n-place relation-sign r (with the free variables $u_1, u_2, \ldots u_n$) such that for every n-tuple of numbers ($x_1 \ldots x_n$) the following hold: (p. 55)

$$R\ (x_1 \ldots x_n) \longrightarrow Bew_c\left[Sb\left(r\ \begin{matrix} u_1 & \ldots & u_n \\ Z(x_1) & \ldots & Z(x_n) \end{matrix}\right)\right]$$

$$R\ (x_1 \ldots x_n) \longrightarrow Bew_c\left[Neg\ Sb\left(r\ \begin{matrix} u_1 & \ldots & u_n \\ Z(x_1) & \ldots & Z(x_n) \end{matrix}\right)\right]$$

We then have a case of the above scheme. Formula y can be tested to see whether or not it can be proved within the c[10] formula scheme. The test of the provability of formula y is the recursive relation between it and formula x, and the one that represents this relation is q.

$$x\ B_c\left[Sb\left(y\ \begin{matrix} 19 \\ Z(y) \end{matrix}\right)\right] \longrightarrow Bew_c\left[Sb\left(q\ \begin{matrix} 17 & 19 \\ Z(x) & Z(y) \end{matrix}\right)\right]$$

$$x\ B_c\left[Sb\left(y\ \begin{matrix} 19 \\ Z(y) \end{matrix}\right)\right] \longrightarrow Bew_c\left[Neg\ Sb\left(q\ \begin{matrix} 17 & 19 \\ Z(x) & Z(y) \end{matrix}\right)\right]$$

10 The expression says: if x proves (B) the recursive relation with y, then the formula that recursively represents the proof relation between x and y, is or is not provable (Bew) in c.

And from this he arrives at the central proposition of the article, the assertion that for every ω-consistent recursive class of formulae there is a relation r such that neither it nor its negation belongs to the consequences of c - a formula r of which it cannot be said either that it is or that it is not provable.

> Proposition VI: To every ω-consistent recursive class c of formulae there correspond recursive class-signs r, such that neither v Gen r nor Neg (v Gen r) belongs to Flg (c) (where v is the free variable of r). (p.57)

The author makes an algebraic development of the expression seen above. The formula on the right is separated into its two component parts: the substitute formula q, at number 17, and the relation r, which contains the recursive relation with q, isolated from the formula x.

$$x\ \mathrm{Bc}\left[Sb\left(y\ {19 \atop Z(y)}\right)\right] \longrightarrow \mathrm{Bew}_c\left[Sb\left(q\ {17 \atop Z(x)}\ {19 \atop Z(y)}\right)\right]$$

$$p = 17\ \mathit{Gen}\ q \qquad\qquad r = Sb\left(q\ {19 \atop Z(p)}\right)$$

The term y is replaced by p, to make it easier to identify in the substitutions below, but it is the same formula y, because it is in the same number. In the formula on the left, p is replaced by the corresponding class-sign; and the one on the right is reordered.

$$Sb\left(p\ {19 \atop Z(p)}\right) \qquad\qquad Sb\left(q\ {17 \atop Z(x)}\ {19 \atop Z(p)}\right)$$

$$Sb\left(\left[17\ \mathit{Gen}\ q\right]\ {19 \atop Z(p)}\right) \qquad\qquad Sb\left(q\ {19 \atop Z(p)}\ {17 \atop Z(x)}\right)$$

The same expression is isolated on both sides. It is replaced by the term r, defined above. This term represents the components of the recursion, isolating them from the formula x, in variable 17.

$$17\ \mathit{Gen}\ \boxed{Sb\left(q\ {19 \atop Z(p)}\right)} \qquad\qquad Sb\left(\boxed{Sb\left(q\ {19 \atop Z(p)}\right)}\ {17 \atop Z(x)}\right)$$

$$17\ \mathit{Gen}\ r \qquad\qquad Sb\left(r\ {17 \atop Z(x)}\right)$$

The formula r is the synthesis of the formulas to be tested, y (the object of the test), and q, *which represents the very provability relation between the formulas*. The formula 17 Gen r, which seeks to establish the consistency of c, is the formal type that is undecidable.

$$x\ B_c\left(17\ \mathit{Gen}\ r\right) \longrightarrow \mathrm{Bew}_c\left[\mathit{Sb}\left(r\ \begin{matrix}17\\ Z(x)\end{matrix}\right)\right]$$

$$x\ B_c\left(17\ \mathit{Gen}\ r\right) \longrightarrow \mathrm{Bew}_c\left[\mathrm{Neg}\ \mathit{Sb}\left(r\ \begin{matrix}17\\ Z(x)\end{matrix}\right)\right]$$

The above algebraic developments are reapplied in the expression that says that a formula may or may not be proved. And so we come to the point of why neither proof nor negation of the formula "*17 Gen r*" can be offered within the system.

First, however, what does this formula mean? There is a formula that represents the proof scheme, which tests whether or not others are a consequence of the series of formulas. The relation q summarizes the proof relation between the formulas x and y. The relation r isolates from the formula x a) the provability relation q, and b) the formula to be proved, y. The formula r represents a recursive structure, placed in a formula number.

Each case in which the recursion is isolated in a formula number, one has a case of r. And thus, the general form is the generalization of the recursion itself, and of its validity. And the form after the algebraic transformation above, says "if the general form of recursion is provable, then there is proof in the system of a case of recursion". And so, r is the test of a proof structure, and is an example of a class of formulas r that evaluates the proof properties of the system itself, and precisely whether or not they are probable in the formula scheme.

For every formula r, it must say whether recursive chaining is provable. If it is an operation of calculable general validity, and is the result of the scheme of proofs c, it can be included in the list as the final item of this proof scheme. And so *17 Gen r* is the self-proof test itself, but it can only be transformed into a numerable class-sign *if and only if the formula r can be generalized in an ω-consistent way*.

And we have the following, according to the statements seen above:

a) The formula number of the proof shall be different from that of the formula proved.

b) A formula that proves another has no resources for self-proof in c.

c) Every proof must take place in a recursive step, in which only the previous properties and relations of the system are inherited.

Since recursion is finite, there must be a last formula that evaluates the previous chain without borrowing other relations (c), and to avoid infinite regression, which would make recursion unfeasible, r must finally evaluate itself, contrary to a and b. And so, the proof scheme is included in the recursive step that tests the probability of the previous step (so that the evaluation is complete). The question is precisely: can the formula r itself be included in the evaluation?

Transposing the concept of ω-consistency to set logic, we can consider two examples. The first is the one seen above, in the kind of case Russell called "illegitimate totalities", the example of the formulation of the law of the excluded middle as "every proposition is true or false". If it tells a truth about statements in general, and at the same time does not apply to itself, it has at least one exception, failing precisely in the self-test. And so it is ω-inconsistent because it doesn't follow its own rule.

Let's also consider the maxim “every rule has an exception”, and we will go through the list of rules until we reach it as the last item to be “tested”. If it has no exception, it is the exception of itself, and therefore it is false. If it has an exception, it is an example of the rule itself and is true in one sense, but is false in another because it is not perfectly applicable to the set of other rules. It is thus inconsistent because contradictions always arise from it. But it would be based on the criterion of being faithful to the rule itself, ω-inconsistent in the first case and ω-consistent in the second, and regarding its content, by being correct about what it states about the other rules, ω-consistent in the first, and ω-inconsistent in the second because it requires that, in the case of the rule itself, there be at least some other rule without exception[11].

We then have two ω-inconsistencies: of object (about the correspondence to its content) and of prescription (about being an example of the rule itself). It goes like this:

a) regarding the object: if it has an exception, it follows the rule itself, but it is not true about its content, because it allows there to be some other instance of rule that does not follow. Among their cases, there is at least one contrary to the others. Instances that contradict each other coexist - there is only an exception if there is a case without exception. And so, it has an exception when it shouldn't have.

b) regarding the prescription: it states that every rule has an exception, but because it has no exception, it does not follow what it prescribes, and it is not a case of the rule itself. And so either it cannot be a case of itself, or it is an exception to its applicability. It has no exception when it should have.

They are two types of ω-consistency, and likewise, the self-evaluation of the r-type formula will always fall into one of them. This is the kind of problem of Gödel's Theorem. Either type r requires a flawed case to be formula, or it does not fail and cannot be a case of itself like the other formulas.

We have two considerations about the function of r in the proof system:

a) A formula generalizes an operation or function, and can be proved or shown to be unprovable.

For the system, proving a formula means showing that it is a consequence of the formula scheme, and that it is valid as a general law without exception, and can be used in the composition of other formulas and relations. Every formula has potentially, however, a case for which it may not be probable, and the proof structure must be open to the possibility of offering proof or not[12].

In the evaluation of every formula, one must be open to the possibility of it failing in some case. Its effectiveness cannot be given as a premise, which means that the evaluation of the function

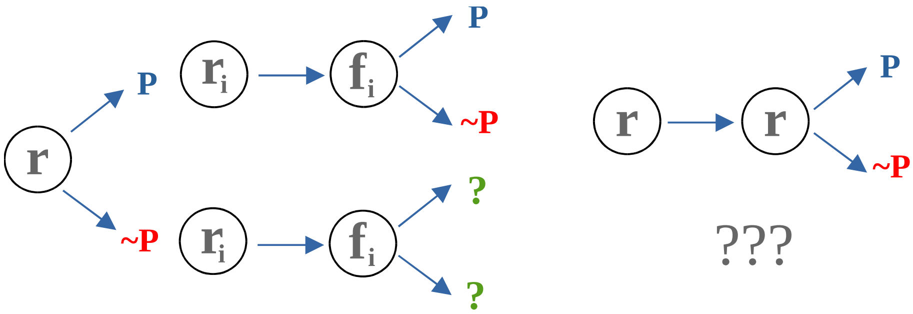
11 The sentence "every sentence articulates concepts" articulates concepts, and so, as well as being right by definition as to its objects, it follows the rule itself. And so it is ω-consistent in both senses. In the case of the statement "I only tell lies", from the liar's paradox, the content and the prescription are always the same case, so both are simultaneously true and false, and the ω-inconsistencies occur together, whether he is lying or telling the truth. In the case of the rule and the exception, there is a difference between the statement and the other rules, and so the two types of ω-inconsistency are quite distinguishable.

12 Presumably there is always some context for formulas in general in which they are no longer perfectly applicable, or this can only be done with the help of other theoretical resources, or an extension. And so there is, at least in theory, some context (or type of value) for which the usual proof does not apply, or for which it can be proved that they are not valid. And in addition, a proof system exists precisely for cases of formulas that are still candidates for consolidation, or to revise the proofs that have been offered so far. And from this comes the need for openness as to whether or not proof is possible.

presupposes a kind of “suspension in the belief of its validity”, so that it can be concluded that it is formally correct, and corresponds with generality and adequacy to the object. The formula and its principles cannot be used in the demonstration itself. And from this judgment, in which these principles are objects of investigation, it is decided whether the formula meets the requirements to be considered proven. In the existing distance between the structure that tests and its object is the opening to the possibilities of its proof or not.

b) The formula r says whether or not any formula can be proved, that is its function, or, the raison d'être of its generalization.

And so, when used in self-evaluation, it must also have a “suspension of belief in its own validity”, and in its own methods - and so, being a case of itself, it can be shown to be probable or not probable. So, if it's just another formula, and is effectively applicable to the others, applied to itself, it must say of itself that it may not decide on provability in some case - it says it may be wrong or incomplete. For this type of formula, to conclude non-probable means that to some extent it will be unable to fulfill its function, that is, to decide on the provable or non-provable character of other formulas. If r is provable, the recursive structure is valid. If it is not, the system does not decide whether the recursion of formulas up to some formula of type q is valid. This is represented in the figure below:



The first version of the expression with the alternatives "Bew (...)" and "Bew (neg ...)" is the test of the probability of the formula q, resulting from the relationship between two formulas with variables 17 and 19. The second, after the algebraic transformation, is the formula representing the recursive structure (where q and the formula to be proved are isolated), at its given recursive address with the respective variable. Whatever developments r represents, it must be a numbered formula, and it must be tested as to the possibility of its generalization, that is, as to its provability. The formula *17 gen r* is the general formula of the r-relations, which summarize the previous recursive structures of the formulas. Since r is a formula, it must be tested to assess its probability - openness to alternatives must be applied to it as well. The solution to evaluate r, since the recursion is finite, is a kind of self-evaluation of r – and so *the generalization of r is a successful outcome of its self-evaluation, and will not be generalizable if it is not able to decide*. The test must, however, consider that it potentially may not be probable, i.e., in its occurrences, *the system must be open to at least one case of the formula r being unprovable*.

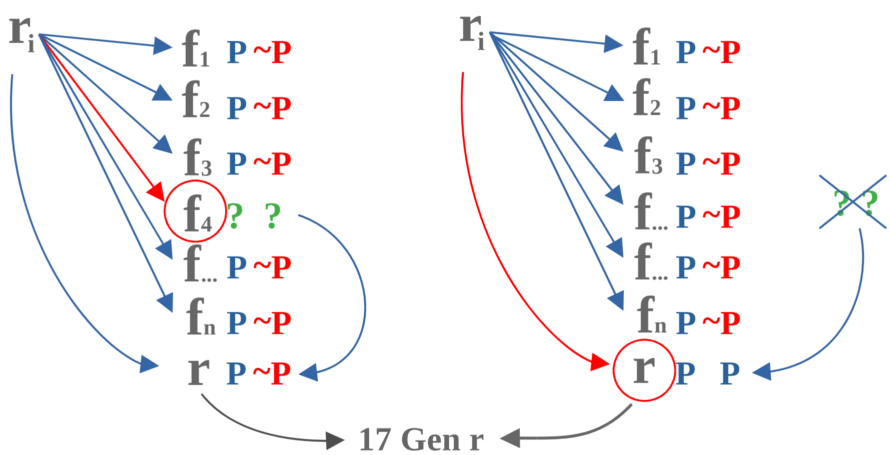
If it claims to be like the other formulas, it must at the same time potentially fail to some extent in evaluating the properties of the other formulas. And so there must always be a possible negative case in one of its recursive applications.

If, however, it is always self-proving, in all instances it will be able to decide (or never, also exclusively), and so the margin of alternation between provability or its negative is not applicable to it, and it is not like the other formulas - it is not one of the cases it prescribes. If it is complete over the other formulas, it is not a typical formula, and cannot be a case in itself.

And so, with respect to the types of ω-inconsistency of the formula r we have:

a) Of object (diagram of the figure below, left): if r is the structure that decides on the provable character of the other formulas (about them being generalizable), being open about the general validity of the function itself, *it says of itself that there is at least one case that will not be able to say whether it is probable or not*. When its rule is applied to itself, it must consider the possibility that it doesn't meet its function. And so, that's the condition for it to be valid as a formula. Whether it should be provable or not probable, there will always be at least one contrary case that coexists with the value of the class-sign. If it is provable it has a non-provable case; if is non-provable it must have a provable case, and both must simultaneously have formula numbers, making the system as a whole at least ω-inconsistent. If it is a case of itself, it must be both unprovable and provable, and vice versa, in some r, and so it says of itself to be inconsistent in some way.

b) Prescription (right): It is the same problem as the law of the excluded middle, which is not “true or false”. If it is demonstrable or indemonstrable for all cases, it does not follow what it prescribes for the rest of the testable formulas. The point is that the formula is not like the other formulas, and does not apply to itself the rule it prescribes. If it proves everything every time, it doesn't follow the rule that any formula must be possibly provable or not. And so, either it is not a formula, or it does not apply to all formulas. And it can't evaluate itself as it does the others.



The diagram shows instances of the r formula evaluating themselves. The general form “*17 Gen r*” is the formula r itself without being in a case, it is its “prototype”. The blue arrow means that it has decided on the structure of the formulas it evaluates, and the red arrow means that it has not been able to decide. On both sides there is at least one version that r cannot alternatively evaluate the proof or the negative, and thus fails to fulfill its function as a formula. If the system were complete and ω-consistent, it could evaluate them all, and thus be generalized.

And so, applying the ω-inconsistencies of object and content to the cases of provability or its negative, we have:

1) Or r is provable,

1.1) but always a non-provable instance should be possible, and so should be provable as a class and not as a case in at least one situation. It must have a provable n, but it must potentially have a non-provable case also with an n, simultaneously.

1.2) and applies to all formulas except itself, because it doesn't follow its own criterion, since it always has provable instances, but *it isn't open to cases in which it isn't provable*.

2) Or r is non-provable,

2.1) and says that every formula can be provable or not, and so in at least one potential case must be provable, and the two situations must have corresponding numbers of formulas, making the scheme of formulas ω-inconsistent.

2.2) and does not see cases in which it is provable, and all its instances are non-provable to it. And so, it fails in the characterization of a formula, because it has no means of encompassing a possible provable instance, and therefore, does not follow the very prescription that is precisely the condition for testing any formula[13].

And so, if r is ω-consistent in terms of the prescription, it will be ω-inconsistent in terms of the object. It says that every formula potentially fails, and it must fail in some case, in order to meet what it says about formulas. For every function of type r numbered as n (added to c), there is at least one recursive version of number m such that it contradicts its character as to demonstrability, and so:

$$\forall r \exists n \exists m\ ((n)\ Bew_c(\ v\ gen\ r) \leftrightarrow (m)\ Bew_c(neg\ sb\ [\ r\ v\ z(m)]))$$
$$\forall r \exists n \exists m\ ((n)\ Bew_c(neg\ (\ v\ gen\ r)) \leftrightarrow (m)\ Bew_c(\ sb\ [\ r\ v\ z(m)]))$$

If it is ω-consistent as to object, it will be ω-inconsistent as to prescription. Every formula can potentially not be generalized, except for itself, and thus does not follow what it determines, and does not count as a case of the rule itself. For every function of type r numbered as n (added to c), any recursive version of number m follows its character as to demonstrability, or:

$$\forall r \forall n \forall m\ ((n)\ Bew_c(\ v\ gen\ r) \leftrightarrow (m)\ Bew_c\ (sb\ [\ r\ v\ z(m)]))$$
$$\forall r \forall n \forall m\ ((n)\ Bew_c(neg\ (\ v\ gen\ r)) \leftrightarrow (m)\ Bew_c(\ neg\ sb\ [\ r\ v\ z(m)\ ]))$$

And from this we can see that, for the first case, if every formula is potentially non-provable, or, not generalizable, being r a formula, it must be potentially non-generalizable as well. It cannot claim that it will always be effective as a function, that is, that it will always decide definitively on the formulas it tests. And on the other hand, it cannot say that it is not probable for any case of r. And so, it says of itself that there is at least some case without decision, that is, that the property r will not be recursively generalizable.

For the second case, if every formula can have proof, or proof of the negative, but r is always probable or not without exception, it is not a typical case of formulas, and does not follow the prescription itself. So it cannot be proved or not proved because it is not a formula, or, it is

13 Gödel's article deals in particular with points 1.1 and 2.1. In them there is a requirement for one of the recursive potential cases of r which contrasts with the general form (with the statement) - it is only generalizable if it is not generalizable. But in unfoldings 1.2 and 2.2 there is also a decision problem - always deciding (or never deciding) is precisely a flawed case, only valid as a general law and as a recursive result intrinsic to the system if it is not a case of the rules of system itself, that is, if it is a foreign object within the system.

precisely the exception to the rule itself, and so, in any case, it cannot be generalized as a formula of the system.

And so, or it will say that there is a case in which it fails, and so, if it is true about the rule itself, its content is false and not generalizable; or, on the other hand, that it never fails when it should – the generality of the rule is overlooked in self-application. And so the formal case r, which decides whether a series of recursive formulas up to it is probable will always be ω-inconsistent, and this means that because it is never generalizable as a formula, neither its proof nor the negation can be generalized[14] (there is no definitive proof that there is a flawed case).

The general version of r (the self-consistency formula) is an illegitimate totality. If it follows what it prescribes, it is inconsistent because it has (at least potentially) a flawed case and may not be likely, if it does not follow, it will be the case that does not apply in the rule itself. And so, *for every formula of type r, there is no number of formula n with the proof or with the negative of its general form*:

$$\forall r\, \neg\exists n\, (((n)\ \mathrm{Bew}_c\ (\ v\ \mathrm{gen}\ r)) \vee ((n)\ \mathrm{Bew}_c\ (\ \mathrm{neg}\ (\ v\ \mathrm{gen}\ r)))$$

Furthermore, if the statement “every rule has an exception”, being a rule, is included in its content in the form “*set = statement + cases”*, we have the following. If it has an exception, it will be one more rule with an exception included in the set, but the set does not change the already existing rule without exception, it does not change the value of the content. And it continues to follow its own prescription. If, however, the rule without exception is included in the set, then, in addition to the content being true, it makes the prescription true because the total set will have one exception, which is the statement itself. All cases have exception except it, and it becomes the exception of the set of content. But this creates an inconsistency. It only has an exception if it has no exception. It is a typical case of self-reference in which opposite values accumulate – it becomes the Russell Paradox.

Transposing this to the formulas, we have that if the formula r can fail to decide a case, it is a typical formula. If it always holds for all cases, then there is one case for which there is no margin of decision, which is itself, for it cannot apply the decision of whether it is probable or not. And so, it would be the flawed case itself, which would itself be as a formula (in the enlarged set cases + statement). And so it is a formula if and only if it is not a formula, which is a full inconsistency, not just a ω-inconsistency. And so, the generalization of r is not only ω-inconsistent, it is genuinely inconsistent. The formula r has the function of evaluating the provability of other formulas, and to decide whether or not their general form is provable, and the inclusion of any of them in an n in the system will always be inconsistent.

14 A simplified outline of the argument is seen at the beginning of the Gödel’s article, in which he gives a brief exposition of its structure. Consider a list of class-signs numbered and ordered by the R(n) operation. The typical class-sign is designated by a formula, such as [a; n] where the free variable of the class-sign a is replaced by the number n. And so we have the formula n ε K - Bew [R(n),n], which says that if the number n belongs to K, the class-sign applicable to R(n) is provable. Consider K as the class-sign S, whose formula [S; n] says that a number n belongs to K. The formula S has as its own index/variable q. If K lists those class-signs whose index n is provable, and K is also a class-sign designated by S, it must also be testable. But its address in the list is q, and its counterpart among the formulas is R(q). If there is proof of [R(q);q], then n ε K is false, but K is designated by the class-sign S, which is in the relation n ε K, which defines the proof structure. But q does not belong to the set of values K, so [R(q); q] cannot be proved. The proof of all formulas should be proof of itself because it is also a formula, but it is not on the own list. But if the negation of [R(q); q] can be proved, n ε K is valid (that is, the class-sign S applied to the values of n). And yet, if this negative is part of the values of n, S says of itself belonging to the non-probable formulas. If there is no proof of [R(q);q], then n ε K is valid, although S needs to be valid as a formula (without proof in the system) somehow the proposition depends on whether it is held to be valid, and true.

And so the formula *17 Gen r* is neither probable nor unlikely in the system, it is formally undecidable. The goal is that ultimately the system proves its own consistency. And the conclusion is clear, that this proof cannot be offered:

> Proposition XI: If c be a given recursive, consistent class of formulae, then the propositional formula which states that c is consistent is not c-provable; in particular, the consistency of P is unprovable in P, it being assumed that P is consistent (if not, of course, every statement is provable). (p.70)

Recursion is not able to gather a sufficiently comprehensive and generic structure to account for all the rules of formula formation, to the point of unambiguously deciding whether every formula generated by the system is a legitimate mathematical formula, or, if this can only be decided on a case-by-case basis, with different resources[15].

There are two aspects of the theorem, or, two formal problems with respect to a universal proof system. Either it will evaluate the property of an object that mirrors its characteristics, or a separate system turns to it, and reanalyzes its relations and characteristics. And so, either an object of analysis mirrors some of its properties, or, it will evaluate a system that has it as its object. It would thus both be mirrored (and investigated) by systems he would come to evaluate, and would revise theoretical schemes by the same properties as in those they would be found. And so the problem of self-reference will always return.

If we have a system that serves as a general proof, it is not possible for its central aspects to be taken, used, reworked by others, and yet remain as a tool of proof of those. The system does not

15 Gödel states (p.61) that the decision will generate a new class of consistent formulas c', despite being ω-inconsistent. The formula will consistently state something about the set of type r formulas (by failing in one case), but this, at the same time, will be a result of it being unable to evaluate the case itself due to its dual role, although inclusion may be a choice of the mathematician. This choice will be justified by factors and criteria external to the system, and this is the point of the importance of reasoning and intuition in mathematical understanding and development.

There are also two problems in which the two sides of ω-inconsistency generate methodological problems when deciding. The first occurs if we consider that r is a formula, and is a case of the rule itself, and yet we adopt a decision about demonstrability, rejecting the ω-inconsistency of the object. In order to avoid the consequence of r saying of itself "being possibly wrong to follow the rule itself", it must always be open to revision, and must apply a departure, or a "suspension of belief" to the very criteria it applies as a function, which are precisely the criteria of proof, which is an impossibility when we consider a recursive proof system. Recursion only applies to finite operations with well-defined steps, and cannot deal with infinities. Neither decision suspension nor continuous iteration can be generalized to class-signs. Only by defining the terms by which it is not ω-consistent can it be made consistent, and they would have to be incorporated as formulas. But that would be the neutral perspective, which it cannot achieve. The second problem occurs if we proceed to a definitive decision by proof or not, by considering that r can be decided even though it is not a case of the rule itself, rejecting the ω-inconsistency of prescription. The formula of type r contains the principle of proof in its relations, it is the chain that brings together the recursive steps, testing the steps progressively up to a finite limit, which is the formula itself. When proving, r needs to prove the validity of the generality of the proof structure, and then decide on a case of the class in which it is applied. If the logical criterion of a proof is distinct from the logic of its object, the self-proving formula nevertheless has the same logical structure as a criterion and as an object. The formula cannot be used to evaluate a case where it is not demonstrable without assuming the resources of proof themselves to be valid. It must always be right, and it must presuppose that its own criteria (regarding its own structure of proof) will always be valid. The system will not be open to revising its own principles of proof because they are already taken as true, in advance, for any test it makes of itself. And so we have that either it is hyper-provable, and in addition to saying of itself "the consistency formula is always consistent", or it is unprovable and so, in addition to saying this of itself, it tests formulas in c without ever itself being a consequence of c - it is an object foreign to the system, even though it is the result of the previous recursive series. And yet, the fact that it doesn't see a case of non-provability when, being probable, it will always be probable in its cases, and the opposite in the case of the negative, shows the paradoxical situation in which ω-consistency regarding prescription becomes a limitation, leading to the impossibility of demonstrating the principiological integrity of a system (a property that results precisely in the absence of contradictions, or consistency).

serve to evaluate systems that mirror its properties, even if by other paths and with other theoretical purposes.

And yet, is the universal proof system definitely finished, or can it be improved or expanded? So let's imagine that a mathematical development is elaborated on some of its aspects. Will the decision on the suitability of this new development be evaluated by the system itself? And if a proof is offered that it cannot be extended with respect to a domain of mathematics, reaching a definitive state, is that proof an extension or not? To decide whether or not it is an extension, the entire corpus of the system must be submited to the system itself.

Another consequence of the theorem is that there are principles and relations which, because they are of a very general and fundamental order, cannot be proved, for they will always be presumed and used by any candidate to prove them. It will always be the implicit premises, or only expressed, with which one would try to obtain a proof of them. These are relationships that would always reappear as hidden premises, no matter how hard we tried to control them.

What is sought by obtaining the proof of consistency is the attempt to encompass the structure of reality that establishes the rational and mathematical standards that define it, but this already uses some of its manifestations. And in these, the background that establishes them is already given and assumed – it can at most be reiterated in its developments.

Final considerations

Gödel's theorem is a clear exposition of a limit of languages. It shows the paradox of proof, in which a general system of proof must also be applicable to itself. This leads to the problem of the limit of formalization, which is the inability to deal formally with progressive abstraction, or, with the generalization of abstraction itself.

Every formalization of the relationship between things has an intrinsic delimitation, given by the articulation of its concepts and notions. It restricts one relation between elements from all others, and so if a formal system relates certain elements, it will necessarily exclude others. No matter how many relations are included in it, it can always be completed by another aspect. Every relation can always be included, modified or opposed by another, indefinitely. A reasoning can always be more complex and abstract, because another element can always be considered in addition to all those already contained in a given logical universe. Any relationship at its level of generality can be generalized by another.

For a system to be able to establish progressively more relationships between elements, it must be able to evaluate ever more aspects and elements beyond those it relates. And it must be able to be able to judge by what criteria it selects the elements to be considered, and for that it will need other criteria.

A formalization, in order to have the symbolic representation of its relation to another, that is, to extend its own limits and abstract scope (or to have systematic control of them), should be placed in a neutral perspective between them, except that to do this it must consider itself as one of the elements of another relationship, and so it should always be more abstract than it is, but this is a contradiction with the definition of formalization.

The very expansion of the level of abstraction (as a property in itself) cannot be the object of a formalization. The former is the very possibility of linking different relations, while the latter is the restriction to a specific, well-defined relation. It is always possible to contemplate more elements starting from a relation, and so there is no last level of abstraction. But this reasoning, although it is easy for anyone to understand, is not formalizable by definition. In other words, it can be said that the theorem has shown that the self-reference of thought and that of a formal symbolic system have different and incommunicable natures. The proof of 1*7 Gen r* presupposes that the formula must have the generalization of "formula", and its characterization as such, and as the formula of a given system, and of what a system is, and so on. That is, self-abstraction with respect to any logical plateau must be self-included in the formula itself.

There are basically two types of self-reference, which we can call the weak version and the strong version. The first is found in situations and systems such as linguistic patterns with cross-references, recurrent or symmetrical phenomena, or still others that personify by analogy entities capable of strong self-reference. The theorem shows that we can only arrive at a weak version when it was supposed, perhaps, the possibility of arriving at a system powerful enough to equate to strong self-referential structures, consciousnesses.

Consciousness has an apparently antinomic relationship with reference, representation; the relation between them is one of complementarity. The representation is restrictive, it is a particularization of one aspect of reality, to the exclusion of all others. Consciousness is broad and vague, and reconciles any kind of relations by any criterion. It is the extra-logical element that allows all creative synthesis between dispersed relationships. If a representation defines and isolates one aspect of reality from others, the general plane where all aspects are separable and comparable and thus seen as possible perspectives in the face of a non-aspectual reality is the consciousness of an entity. It is the broad sense of apprehension, inexhaustible and indefinable in all its extent—it is the apperception of the whole, from which one arrives at the parts. If representation is directed consciousness, the totality of consciousness is not a simple merely associative or combinatorial sum of representations.

Representations are the focus of attention of consciousness, from objects to thought in abstract schemes. As every focus of attention always highlights aspects, and ignores others, in this lies its representative character. But the existence of a general plane of the possible aspects (which rises above them as a non-aspectual plane), is a condition for every perspective to be seen as such, and it must be included in the constitution of each one. Self-awareness, as self-representation, is not the mere focus reflected by itself, as just another focus of directionality – it is not just another computation at the end of the list. It manifests itself in the very continuous succession of representations, the generalized and self-inclusive reality of the succession of thoughts in aspects.

For Husserl (2015) the representativeness of the universal must be based on the universal itself. Through collections one does not arrive at the idea of the universal as such. It is not reducible to cases, it is the true essence of thinking, before which impressions and images are only particularizations of the situations thought about. If the generalization of one case reveals a property, and the same generalization is taken from several other cases, and is therefore revealed as the same when compared with others arising from their own "histories", it shows itself as universality in itself, and as independent of the circumstances of its formation. And so, only before the consciousness of the representation, the representation seen as such, has its character of universality revealed[16]. The general plane of abstractions is a synthetic continuum of different levels

16 Husserl (2015) differentiates mental reality from the understanding of the concept as such, from its imagistic and psychological manifestations, emphasizing the proper right of universal objects. Representation, its purely abstract component as an idea, is not to be confused either with the collection of experiences (empiricism) or with its imagistic character (psychologism). In these interpretations there is no what he calls "consciousness of universality", but only individual intuitions and a game of conscious and unconscious processes. Understanding and thought have a strictly universal "pure" component, independent of their figurative form or psychological substrate.

of universality. And it is this, when accompanying the specific representation, that precisely gives it the character of being “this representation” and not another.

The transition of representations as a pure synthesis accompanies each particular expression of an abstraction, not only as a symbol, but as a permanent self-inclusive passage between levels of abstraction to accompany every particular representation[17]. The consciousness of the concept, isolated from all others, exists only in the face of the continuous transition between aspects, from which it differs, and before which it is significant. It is intrinsic to the consciousness of the pure concept that it is one among a plane of possible perspectives and abstractions.

The ideas of number, quantity, proportion, equality, and difference are representations, they are pure aspects of reasoning, which can be combined in the identification, formulation, and rational description of reality. All allow a stratum of the universe to be covered, described, and formalized. But new associations are always possible in countless possibilities of new descriptive structures. And the definitions of aspects, occurring in parallel with continuous other changes of perspective, allow the creative combination of new articulations of concepts. Any reasoning can be reversed and considered in relation to any others, productive or not in relation to these.

Reality is made up of aspects of aspects endlessly, and not perfectly representable. At this limit of what is representable, consciousness has a glimpse of the approximation between the broad plane of perspectives and some possible definition, manifested as an intuition. Intuition lies at that subtle boundary between the broad and diffuse perspective of the transition between abstract planes, and the almost clear aspect to be isolated.

Repeated geometric patterns, reflective structures, emergent natural patterns, linguistic cross-references, reiterated computational patterns, are all examples of weak self-reference. None is able to rise as a structure capable of isolating the aspect or pattern repeated, or mirrored in the others, from a broad panorama of possible patterns. This has no relation to the level of knowledge of the genuinely self-referential entity, but to the way in which the possibility of knowledge itself is established.

The universality belonging to the content of meaning is different from psychological universality. There is a distinction between an element A of the collection of A's, and these of the possibility of A in general - A in general is not a collection of A's. Nominalism (which Locke adopted from the medievals) confuses the generality of the universal with representing the specific. The logical possibility of propositions depends on universality, something that must have its existence *a priori*, in the sense of not being confused with occasional psychological acts. Psychologism is an attempt to offer a genetic explanation of intentional content, but it is not a logical explanation. The proposition that representations are only collections of particulars, imaginal impressions, or psychological processes (or the Humean-like notion that the abstract idea is only a faint reminiscence of sense impressions) can only be formulated by means of schemes and general definitions that cannot even be reduced to the thesis, much less proven by it. This explains why different people, each with their own mental history, can agree on the same general principles.

17 The reference, as a pure abstraction, must be included in reality itself as an entity. In other words, the character of being a representation must be part of the representation. The philosopher João Teixeira (2004) discussed this problem when he criticized the classical idea of representation. Every symbol requires an “extra-representative” component that links it to the element of reality it represents. A map is a representation of a place, but its interpretation requires a broader component that links it to its context of application, i.e. it requires the general representation of what a map is to be applied to what a geographical region is, in a progressive synthesis between the two. And this extra-representativeness is not simply the addition of another representation to explain the previous one, which would require yet another in an infinite regress. There is no point in adding more maps and territorial indications, because there is no leap beyond the compositionality of instances of symbols and partial representations. He thus presents the self-location of an agent as a typical situation in which representations are self-generalized. If, for example, we walk on a terrain, even comparing it with its static representation of the map, we will have in each new comparative observation a *representation of the succession of representations*, that is, each representation is seen in this broad perspective.

Understanding is not mediated by categories, but rather, they are the result of the intellective synthesis that passes through them in a self-referential way. And so, there are no limits to the conceptual and categorical articulations by which reality can be thought. Procedural mechanisms are only imitative apparatuses of the true self-referential structure, which is consciousness.

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