

[Forthcoming in
Christian Damböck & Georg Schiemer (eds.). *Rudolf Carnap Handbuch*. J.B. Metzler
Verlag, 2024]

Adam Tamas Tuboly

(a) MTA Lendület Values and Science Research Group, Institute of Philosophy, Research Centre for the Humanities

(b) Institute for Transdisciplinary Discoveries, Medical School, University of Pécs
tubolyadamtamas@gmail.com

Carnap's work on modal logic

Though the names of C.I. Lewis and Saul Kripke are often cited in connection with the history of twentieth-century modal logic, it was Rudolf Carnap who played a vital role in keeping the issue alive, rendering the field respectable, and transmitting major insights and elements of his conceptual frameworks for later discussions. In his 1946 paper, "Modalities and Quantification," and his 1947 book, *Meaning and Necessity (MN)*, Carnap formulated the very first, detailed semantic approach to modal logic, solved in detail many of its technical problems, and, most importantly, came up with a philosophical system for all the ontological, epistemological, and linguistic challenges that had been notoriously raised by his friend W.V.O. Quine (see Chapter 30 by Verhaegh). Though much of Carnap's approach (such as the dichotomies of extension/intension and analytic/synthetic, the notion of state-descriptions, and the general linguistic/verbal approach) did not withstand the test of time in their original form, Carnap's legacy continues, and his systems are still frequently subjected to nuanced discussions by logicians (most recently by Max Cresswell).

Remarks on the Prehistory of Modal Logic

In 1918, C. I. Lewis' newly developed logical system enabled him to treat the notions of "necessity," "possibility," and "impossibility" in a comprehensive manner. In those early days of modal logic, the interpretation and even the meaning of modal terms were not at all unambiguous - what Lewis and his critics understood by "possibility" was a general logical conceivability or the absence of self-contradiction. After the appearance of Lewis's 1918 and 1932 works (the latter written together with C.H. Langford), most young logicians and philosophers devoted their energy to tailoring his various systems to modal logic regarding their consistency, the number and reducibility of postulates and theorems, their deducibility, the formal consequence relation, and some of their internal paradoxes.

It was Quine who, in a series of articles published over the years, developed numerous challenges and critiques of modal logic, thereby establishing himself as "a gadfly" among logicians (Orenstein 2002, 160-161) and persistently criticizing all attempts to salvage modal logic through simple technical tailoring. Quine's campaign as *advocatus diaboli* turned out to be highly constructive for the history of modal logic, given that many people, among them Arthur Smullyan, Ruth Barcan Marcus, and Alonzo Church, developed various systems and conceptions of modality to save the field by constructively replying to Quine's criticisms.

Quine's insight was that modal logic violates the most basic idea of classical logic, namely extensionality: In an extensional language (or system), co-extensive terms (having the same extension, such as the same truth value in the case of sentences, or the same referent in case of names) are exchangeable *salva veritate*, that is, without changing the extension of the construction they are part of. In modal contexts, this idea breaks down - for while it is true that 'possibly, the number of planets is smaller than 7,' we get the false statement '9 is possibly small than 7' if we exchange the co-extensive terms 'number of planets' and '9.' Beyond the interchangeability of co-extensive terms, Quine also pointed out problems with other traditional rules of classical logic. He showed that existential generalization also breaks down as a result: If we accept that '9 is greater than 7,' we can arrive at an existential sentence by stating

that 'there is something which is greater than 7' without any issue. But if modalities are involved, problems begin to emerge. By saying that '9 is necessarily greater than 7,' we arrive at the statement 'there is something which is necessarily greater than 7.' As Quine asks, "[f]or, would 9, that is, the number of planets, be one of the numbers necessarily greater than 7?" The sentence would be true, as '9 is necessarily greater than 7,' and false, as 'The number of planets is necessarily greater than 7.' The problem is that "being necessarily or possibly thus and so is in general not a trait of the object concerned, but depends on the manner of referring to the object" (Quine 1953, 148), while at the same time, quantification, taken generally and in the usually accepted sense, abstracts the mode of presenting the designated objects.

Even though propositional modal logic raised important problems concerning its alleged meaningfulness, it was quantified modal logic which led to the major epistemological, linguistic, and even metaphysical problems that prompted Quine to challenge everything modal and intensional. It should be noted, however, that even in the case of Quine, it is not at all obvious how anti-modal he really was. In a letter, Nelson Goodman compared the views of Alonzo Church, Quine, and himself on intensions to differing religious attitudes. "Church believes devoutly and unquestioningly in the gods; Van finds the conception of God unclear but can't cast Him off and hopes to find a meaning for God in human life; I think we'd better recognize that we are going to have to get along without" (June 8, 1947; quoted by White 1999, 342). Carnap knew that Quine could be convinced to a certain degree, and thus forged his system accordingly, always with an eye on what Quine would say.

Carnap on Meaning and Necessity

For a while, Carnap was also a confirmed extensionalist, and his extensionalism culminated during his syntactical period in the 'thesis of extensionality,' where he stated bluntly that "a universal language of science may be extensional" (Carnap 1934/1937, § 67). While Carnap was interested in modal notions at this time, he believed that a simple translation into an extensional language would be possible and that no further major investigations were needed. After finishing his first major semantical work in the early 1940s (see Chapter 50), however, Carnap's attention turned more explicitly and directly towards modal notions, prompted possibly by Quine's intensifying crusade against them. Carnap's book, finished first in 1943 under the title "Extension and Intension," was published in 1947 as *Meaning and Necessity: A Study in Semantics and Modal Logic*.

Compared to the more technical "Modalities and Quantification," which came first, *MN* is a rather informal work, whose main focus was "to state informally some considerations aimed at the discovery of concepts and methods suitable for semantical analysis" (*MN*, § 2). Through semantical analysis, Carnap wanted to give a more precise formulation of cognitive meaning and its related logical concepts, such as analyticity, truth, falseness, contradiction, and equivalence. He calls this process "explication" (see Chapter 52), i.e., replacing a vague notion used in everyday life or at an earlier stage of scientific development with a newly constructed, more exact concept. An explication is not an analysis based on sufficient-and-necessary conditions, hence its aim is not to preserve conservatively the entire original concept; it is rather guided by explicitly stated, liberal and pragmatic conditions, such as simplicity and expediency (though Carnap admits that these are still somewhat flexible terms).

To provide an explication for modal concepts, Carnap introduces the notion of "state-description" (which goes back to Wittgenstein). A *state-description* is a class of sentences in a language *L* that contains, for every atomic sentence, either the given sentence or its negation, but not both (*MN*, § 2). Since a state-description provides a complete story, a full description of our universe, it can be taken as a possible world, as Carnap admits, where state-descriptions consisting only of true atomic sentences (and negations of false ones) equate to the actual world, and those featuring false atomic sentences describe a merely possible world. A certain sentence *S* being true in a state-description means that *S* would be true if the state-description were true. Carnap defines all classical connectives using the concept of state-description ($\neg S$ holds in a state description if and only if *S* does not hold, etc.). In the context of quantification, Carnap follows the substitutional trend of the 1940s (which would be taken up by Ruth

Barcan Marcus, for instance, when she came up with her own semantical ideas later), according to which a universally quantified sentence ' $(x)(Fx)$ ' is true in a state-description if and only if all substitution instances (' Fa ', ' Fb ', ' Fc ' etc.) hold in said state-description.

Using this machinery, Carnap is able to explicate many historically-founded, special philosophical concepts. He puts logical or necessary or analytic truth under one umbrella (a typical move from the first half of the twentieth century) and provides the notion of an "L-truth" as their explication, via the following convention: A given sentence S is L-true in a semantical system D , if and only if S is true in D in such a way that its truth can be established on the basis of the semantical rules of system D alone, without any reference to (extra-linguistic) facts. As a more precise definition, he offers this: A sentence S is L-true (in D), if and only if S holds in every state-description (in D). (*MN*, § 2). If no extra-linguistic facts play a role in determining the truth of a given sentence, then surely it must be true in all possible scenarios - or, as they would be called explicitly a few years later, in all possible worlds. Thus, there is a match between the convention and the definition, hence Carnap's satisfaction with explication, which is strengthened by his further explications of such semantical notions as logical falsity/impossibility via "L-false," entailment via "L-implication," and mutual entailment via "L-equivalence."

Coming from the tradition of Frege, Carnap distinguishes between referents and meanings of expressions, which he calls "extension" and "intension" and applies to all expressions within the language. The extension of a sentence is a truth value (true or false); the extension of an individual expression (including names and descriptions) is the denoted entity (if a description does not satisfy the uniqueness condition - i.e., it has more than one or no denotation - then its extension will be an artificial null entity); and the extension of a predicate is the set of entities to which the predicate applies. Next to extensions, Carnap also defines intensions for all categories (*MN*, §§ 4-9). The intension of a sentence is the *proposition* expressed by it; the intension of an individual expression is the *individual concept* (basically, a function ranging from names to state descriptions, *MN*, §§9, 44) expressed by it, while the intension of a predicate is a *property*.

With this machinery at his disposal, Carnap goes on to introduce the notion of modalities and to develop a modal logic (the majority of the book - after the introductory sections on explication and extension-intension - is essentially devoted to his philosophical journey through what he dubs the "method of name relation," arriving at modal logic only in the last few paragraphs). At the time he was writing, various modal logics abounded in terms of style, format, and detail, with different axioms as foundations for further inquiry. The reason, according to Carnap, was that logicians had different intuitions and definitions of what "necessity" and "possibility" might be, and each of them could find a conception that would support his or her own system. Thus, an explication was needed, and he based the modalities on the semantical L-concepts (*MN*, § 39). This was one of Carnap's most important moves, namely, to offer and go after a more semantical understanding of modalities and work out all the technical and philosophical details of such a program. (At the same - in fact, predating Carnap's investigations - Ruth Barcan Marcus (1946) published important papers about the systematic structure of modal logic, though her logic had only a syntactical dimension; on Barcan Marcus, see Janssen-Lauret (2021))

As a logical or necessary sentence seems to be based on the idea that it follows from purely logical considerations and has nothing to do with the contingencies and peculiarities of the world, Carnap thinks it justified to say that necessity is connected to L-truth, which itself is independent of extra-linguistic facts. Thus, at this point, we arrive at the following convention, which guides the usage of "N," the modal operator of necessity: For any sentence, S , ' $N(S)$ ' is true if and only if S is L-true, which comes down to the idea that (in our current symbolism),

- p is true in a state-description, if and only if p is true in all state-descriptions (in all possible worlds);
- ◇ p is true in a state-description, if and only if p is true in a state-description (in a possible world).

Carnap then defines the sameness of intensions, since two designators have the same intension (in which case they are L-equivalent), if they have the same extension in every state-description – due to the fact that intension determines extension, that is, an intension is a function that connects the extension of a given designator and the state description. The book contains many further details of how to determine extensions, intensions, their sameness, and equality (see especially Parts I and II).

In the case of quantification (which he calls more important because without it, “logicians would probably abandon modal logic entirely” *MN*, § 44), Carnap distinguishes two cases, the so-called *de dicto* and *de re* issues, following the criticism of Quine. The first is more easily solvable for both, as it takes a simple, universally-quantified sentence to say something about the sentence’s modality. If we interpret the modalities linguistically/logically (as Carnap and Quine did), then what we ask is whether *all* the substituted instances of a universally quantified sentence are logically true.

The *de re* cases, where we quantify across a modal operator and thus have a free variable within the operator’s scope, create more difficulties, though Carnap has a suggestion for solving these as well. Taking his example, ‘ $(x)N(Fx)$ ’, ‘for every x , necessarily Fx ’, means, after substitution, loosely speaking, $N(Fa$ and Fb and $Fc \dots)$, that is, ‘necessarily $Fa, Fb, Fc \dots$ ’. By using some of his conventions (*MN*, §40), Carnap argues that the latter equates to ‘ $N(x)(Fx)$ ’, ‘necessarily, for every x , Fx ’. What we see here is that from ‘ $(x)N(Fx)$ ’ (which is considered problematic by many due to Quine’s criticism of ‘9’, ‘7’ and ‘the number of planets’), we move to the less problematic ‘ $N(x)(Fx)$ ’. This is also known as the ‘Barcan formula,’ which permits the move from a modalized, universally quantified sentence to ‘quantifying in’ (for the technical details and the difficulties regarding the formula and its converse, see Williamson 2013).

With his “extension-intension” method, Carnap is able to formulate another technically related philosophical point, raised first by Church (1943) in his responses to Quine. As we have seen, in the intensional context, names and variables occur in non-purely designative positions, since they do not refer directly to their customary reference, and certain additional factors thus have to be accounted for in the evaluation of a given construction – it is the ‘form of the name’ (Quine 1943, 114). Because of this, Church introduces, following Frege, a distinction between the *customary* and *indirect* sense and referent of a term. In Carnap’s vocabulary, the customary referent of an expression is its extension (the object denoted) and its customary sense is its intension (the mode of presentation of the denoted object, namely its meaning). Frege has shown that in certain contexts, these traditional semantical relations *shift* – they replace each other, so to speak. For example, when we quote the words of others, these words contribute to the sentence via their *indirect* referent, which is their *customary* sense. In the statement, ‘Mark said that Michael is a secretary,’ the extension ‘Michael is a secretary’ is a truth value (true), while its intension is the proposition ‘*Michael is a secretary.*’ In regular cases, the referent of the whole sentence (its truth value) depends on the referents of its parts – in this case, on the referent of ‘Michael is a secretary’, which is the truth value “true”). But the truth value of our sentence does not depend on the nested expression’s truth value, but on the proposition, or as Frege would have said, that thought which is expressed by ‘Michael is a secretary.’ This expression is used in *an indirect manner*, and its relevant aspect will be its intension – in other words, in certain contexts, the indirect referent of expressions is their customary sense, i.e., their *intension*.

While both Church, and Carnap (who explicitly follows the former up to a point, see *MN*, § 44) find quantification problematic in modal contexts, this does not have to cause any real troubles; it means only that we have to introduce certain modifications. “The conclusion should rather be that in order to [quantify in] a variable must have an intensional range” (Church 1943, 46). Carnap paid heed to Church’s conclusion (with some necessary modifications within his system, of course) and made significant use of it in arguing against Quine. In Carnap’s case, we do not have to worry about the challenges of Quine, since by shifting their semantical relations, names and variables could be in purely designative positions even in modal and intensional contexts.

At the time, there was a lot of work to be done on modal logic – both in technical

and in philosophical terms. As Carnap quoted from a letter of Quine, the latter considered Carnap's approach and system, "an effective way of reconciling quantification and modality" (*MN*, § 44). More was on the table soon, as many logicians in Europe and in the United States turned their attention to modal logic, reacting to Frederic Fitch's 'basic logic,' still to Lewis' systems, and from the 1960s, after the publication of Max Cresswell's textbook, to Saul Kripke's model-theoretic semantics. Carnap was quickly dropped from the mainstream canon, and we cannot find anyone following him in constructing a major research program along the lines of extension-intension. Due to the critiques of Quine and others, linguistic modalities were replaced by metaphysical ones, just like state descriptions gave way to concrete and abstract possible worlds (with Carnap often seen as an early forerunner of what became known as "ersatz modal realism").

The Aftermath of *MN*

After the publication of *MN*, a few reviews appeared in logical and philosophical journals, though most of them focused on Carnap's philosophical ideas, and especially on the "method of the name-relation" (one of the most important critics was Gilbert Ryle (1949)). Carnap's actual ideas on modal logic were taken up by Robert Feys (1963), John Myhill (1963), and Donald Davidson (1963) in the famous Schilpp volume on Carnap's philosophy. All agreed that Carnap's work was a milestone in the modern history of modal logic, but had various reservations about his take on interpretations; the logical-factual distinction; the consequence relation (Feys); changing individuals for individual concepts; the breakdown of existential generalization and universal instantiation (Myhill); propositional attitudes (Myhill and Davidson); the extension and intension method; the status of modal languages in semantical analysis; and finally, the possibility of an extensional metalanguage (Davidson).

Continuing his long-standing philosophical practice, Carnap (1963, 889-900, 905-914) modified his modal system in accordance with the latest suggestions. He worked on a new system that could easily be seen as parallel to Saul Kripke's model-theoretic semantics, which was published in those years (Carnap worked on his volume mainly in the mid- to late 1950s). When the book came out, he markedly distanced himself from his previous proposals based on verbal entities and now preferred "non-linguistic entities" as the values of variables; analogously, he represented "possible states of the universe of discourse by models instead of state-descriptions, which are sentences or classes of sentences" (Carnap 1963, 891, note 10). It is worth noting that in the literature, only Evert W. Beth (1962) tried to reconcile the method of extension-intension with the model-theoretic interpretation of possible worlds. Be that as it may, however, for Carnap (1963, 895), extra-linguistic entities and models brought about the possibility of translating modal languages into extensional ones, with a view to showing that "these controversial concepts are unobjectionable and acceptable even to those philosophers who profess to understand only an extensional language, provided they are willing to admit class variables of higher types."

Carnap's work on modal logic played an important role in motivating others to delve into the subject. The solutions and tools he developed in the 1940s did not prove to be lasting, mainly due to contextual changes in the logical, mathematical, and philosophical landscape. With Quine, Carnap shared a skeptical attitude towards essentialism and metaphysical approaches, and he would thus be amazed to see the latest developments within "modal logic as metaphysics."

Adam Tamas Tuboly

(a) MTA Lendület Values and Science Research Group

(b) Institute of Transdisciplinary Discoveries, Medical School, University of Pécs
tubolyadamtamas@gmail.com

References

- Barcan Marcus, Ruth (1946), "A Functional Calculus of First Order Based on Strict Implication," *The Journal of Symbolic Logic* 11 (1): 1-16.
- Beth, Evert W. (1962). "Extension and intension." In B.H. Katemier and D. Vuysje (eds.), *Logic and Language: Studies Dedicated to Professor Carnap on the Occasion of his*

- Seventieth Birthday*. Dordrecht: D. Reidel, 64-68.
- Carnap, Rudolf (1934/1937), *Logical Syntax of Language*. London: Routledge and Kegan Paul.
- Carnap, Rudolf (1946), "Modalities and Quantification," *The Journal of Symbolic Logic* 11 (2): 33-64.
- Carnap, Rudolf (1947), *Meaning and Necessity*. Chicago: University of Chicago Press.
- Carnap, Rudolf (1963), "Replies and Systematic Expositions," in P.A. Schilpp (ed.), *The Philosophy of Rudolf Carnap*. La Salle, Ill: Open Court, 859-1013.
- Church, Alonzo (1943), "Review of Quine's 'Notes on existence and necessity'," *The Journal of Symbolic Logic* 8 (1): 46-47.
- Davidson, Donald (1963), "The Method of Extension and Intension," in P.A. Schilpp (ed.), *The Philosophy of Rudolf Carnap*. La Salle, Ill: Open Court, 311-349.
- Feys, Robert (1963), "Carnap on Modalities," in P.A. Schilpp (ed.), *The Philosophy of Rudolf Carnap*. La Salle, Ill: Open Court, 283-297.
- Janssen-Lauret, Frederique (2021), "Ruth Barcan Marcus and quantified modal logic," *British Journal for the History of Philosophy*, forthcoming.
- Lewis, C.I. (1918), *Survey of Symbolic Logic*. Berkeley: University of California Press.
- Lewis, C.I. and Langford, C.H. (1932), *Symbolic Logic*. New York: Denver.
- Myhill, John (1963), "An Alternative to the Method of Extension and Intension," in P.A. Schilpp (ed.), *The Philosophy of Rudolf Carnap*. La Salle, Ill: Open Court, 299-310.
- Orenstein, Alex (2002), *W.V. Quine*. London: Acumen.
- Quine, W.V.O. (1953), "Reference and Modality," in *From a Logical Point of View*. Cambridge, MA.: Harvard University Press, 139-159.
- Ryle, Gilbert (1949), "Discussion of Rudolf Carnap: 'Meaning and Necessity'," *Philosophy* 24 (88): 69-76.
- White, Morton G. (1999), *A Philosopher's Story*. Pennsylvania State University Press.
- Williamson, Timothy (2013), *Modal Logic as Metaphysics*. Oxford: Oxford University Press.