The Limits and Basis of Logical Tolerance: Carnap's Combination of Russell and Wittgenstein
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The aim of this paper is to consider the context and possible influences on Rudolf Carnap's *Principle of Tolerance* (PoT). This principle could be approached from very different angles—either by considering it from the viewpoint of *logic*, and saying something about its technical aspect, or by considering its importance for Carnap's (meta)*philosophy*. I shall take, however, a different approach and say something about its *history*.

One can acknowledge the important influence of Hans Hahn, Karl Menger and Otto Neurath (the left wing of the Vienna Circle), along with Moritz Schlick and Friedrich Waismann (the right wing of the Vienna Circle), and try to place the PoT in the context of Carnap's immediate allies and opponents (Uebel 2009). But, as I claim, one *could* locate it in a broader context, namely in Bertrand Russell and Ludwig Wittgenstein's philosophy of logic.

It is well-known that Russell introduced certain practical elements into the considerations of logic (and the justifications of axioms). But to obtain Carnap's PoT, we have to extend the boundaries of Russell's version of freedom. Carnap did this by relying on Wittgenstein's idea that logic is devoid of empirical content. Since logic is tautological (which Russell denied at various periods), we have the sort of freedom needed for the PoT. Hence Carnap's writings about logic and philosophy in the 1930s could be seen and reconstructed as a *synthesis* (intended or unintended) or special *combination* of Russell's inductive/practical considerations on logic and Wittgenstein's idea of an empty logic.

It is not my aim, of course, to state that this reading fills *all* the gaps in our understanding of the PoT, or manifests itself as an *exclusive* interpretation; there is much more to be said about the principle. One could, for example, review the contribution and influence of Tarski, Gödel, the metamathematical approach of Hilbert, and the conventionalism of Poincaré. My thesis forms only a *necessary* constituent on the road to the PoT and not a *sufficient* one.

The PoT was first presented officially in Carnap's *Logical Syntax of Language* (original German in 1934, English translation in 1937) and it says that "everyone is free to build up his own logic and language as she wishes." There is no fundamentally right and absolute logical system or linguistic framework. And thus the aim of philosophy (or better, the logic of science [Wissenschafstlogik]) is not to construe one final logic and language, but to investigate the linguistic documentations of scientific theories.

The paper is organized as follows. Section 1, prompted by Nikolay Milkov's recent remarks, considers the possible impact and influence of Frege and Russell/Wittgenstein in and on the Vienna Circle. As we will see, both Russell and Wittgenstein played a more important role in the Circle than Frege. For this reason, sections 2 and 3 discuss the main points and concerns of Russell and Wittgenstein regarding the nature of logic. In section 4, I introduce Carnap's PoT and consider its significance and immediate context. Finally, section 5 connects all the dots about Carnap's possible synthesis of Russell and Wittgenstein.

1. In the Vienna Circle: Frege or Russell?

There is (or at least was) a usual narrative about early analytic philosophy which assigns Frege a central position in its history. As the grandfather of analytic philosophy, Frege developed modern formal logic; he demonstrated the significance of logic in the analysis of language and philosophical practice. One cannot imagine a textbook about analytic philosophy, logic, or the philosophy of language which does not start with the works and ideas of Frege.

It is usually thought that this narrative could be upheld also about logical empiricism and the Vienna Circle; i.e., Frege was the intellectual precursor of the Circle, the man who made logic ready for them so that they could go on to eliminate metaphysics, to analyze science, and so on. Recently, however, Nikolay Milkov has suggested that

> [one could say] that the philosophers of the Vienna Circle, excepting Carnap, scarcely ever referred to Frege. This point, however, only suggests that Frege's influence on them came through some indirect channel. Our guess is that this was Wittgenstein's *Tractatus*—so that following the *Tractatus*, Logical Positivists accepted two leading ideas of Frege's, which they occasionally refer to as the "new logic" (Milkov 2003, p. 353).

In the next few pages I will provide examples from the history of the Circle which show that the direct influence of Frege on the Circle was indeed low. The members of the Circle referred to Frege quite rarely, and when they did, Frege's name was just one element in a longer list. As we will see, one should name Russell (and after 1925, Wittgenstein) if one wishes to reconstruct the logical (and philosophical) foreground of logical empiricism and partly even of Carnap.

Though Victor Kraft (1925, p. 65) discussed the nature of logic, mathematics and geometry in his *Die Grundformen der wissenschaftlichen Methode*, he referred to Frege's lesser-known "Über die Grundlagen der Geometrie" only once regarding the self-evidence of axioms. Neither did Kraft pay much attention to Frege in his later writings: in his book from 1947 (*Mathematik, Logik und Erfahrung*), he did not consider Frege at all, while in his historiography of the Vienna Circle (1950, pp. 59, 75) he mentioned Frege's name (only) twice without citing his works, while Wittgenstein and Russell were consistently in the foreground.

Hans Hahn played an interesting role in the story of the Vienna Circle. He taught logic in a department of mathematics; thus, he was always up to date regarding the theoretical and historical importance of "the new logic." Hahn indeed utilized his knowledge and institutional bearings since "[he] directed the interest of the Circle toward logic" (Menger 1980, x). Hahn, however, focused his attention, not on the works of Frege, but on Russell and Whitehead's *Principia Mathematica*. Karl Menger recalled Hahn's course as follows:

> [Hahn] offered a lecture on the algebra of logic in the fall and winter of 1922; and during the academic year 1924-25 he devoted a seminar…to Russell and Whitehead's *Principia Mathematica*. There was a large attendance, and the participants reported on the various chapters of the book…Together with Schlick's courses on the theory of knowledge and on philosophy of science, Hahn's seminar created the background and the basis for the development of the Vienna Circle (Menger 1994, p. 30).

In the 1980 preface to the selected philosophical works of Hahn, Menger (1980, pp. x-xi) recalled the significance of the events in an even more emphatic manner: "This seminar had a very large audience and was of great influence not only on the development of many

Viennese students of mathematics and philosophy but also on the trend of the discussions in the Circle."

The mathematical and technical writings of Hahn (1995; 1996) do not contain any relevant or full-blooded reference to the works of Frege. Neither did his selected philosophical works (1980). There is only one reference to Frege in his "Superfluous Entities, or Occam's Razor" ([1930] 1980a, p. 15) but Hahn put Frege next to Russell just as another scholar who contributed to the logical constitution of numbers.[1]

Hahn's case seemingly supports Milkov's position. Hahn did not cite or discuss Frege, taught the *Principia Mathematica* for years, and in his later works focused only on Wittgenstein's *Tractatus* (see e.g. Hahn [1933] 1987). But even if Frege oozed into the Circle via Wittgenstein (through Hahn), it was only at a relatively late moment, since according to Menger (1980, p. xviii, n. 4), "[c]ontrary to what has been sometimes written about Hahn's seminars, Wittgenstein's name had, certainly up to that time, never been mentioned in them."

It would be important to note that Otto Neurath (1930, pp. 312-313), in his historical remarks for the first issue of *Erkenntnis* (which was an abbreviated version of the Circle's official manifesto), mentioned the name of Frege only twice as part of a broader list, and did not consider it to be of utmost importance. One could object that Neurath is not known for his work in logic or for his competence in that area. But given his historical interest in the topic, and the fact that wrote papers about logic with his second wife (Olga Hahn-Neurath, the sister of Hans Hahn), he certainly could have known about Frege (see Neurath and Hahn 1909a; 1909b; 1910).

Though during the 1910s and 1920s Schlick was working on the philosophy of nature (*Naturphilosophie*) and epistemology, in his *Allgemeine Erkenntnislehre* [*General Theory of Knowledge*] from 1918 (second edition 1925) he dealt with the nature and connections of thinking [Denkprobleme]. He connected the problem of classical logic with the Aristotelian syllogism, writing with great enthusiasm:

> Modern logic (anticipated by Leibniz) is in the process of creating a much more serviceable symbolism than the one fashioned by Aristotle. However, in the discussion that follows, we shall base ourselves on the latter, because it is the one that is most familiar and because in my opinion it *still* provides a means of presenting all logical relationships, and in particular the interconnections of judgements found in syllogistic inference (emphasis in original; Schlick [1925] 1974, pp. 102-103).

Though this remark is admittedly astonishing given the logical empiricists' characteristic dismissal of the "old logic," Schlick ([1925] 1974, p. 107) went even further and claimed that "[t]he Aristotelian theory of inference needs no modification or extension in order to be applicable to modern science."

One could say that perhaps Schlick was not fully aware of the achievements of the new logic in 1918, but given his up-to-date general scientific knowledge (note that he was the first to give an account of the philosophical consequences of Einstein's theory of relativity), this seems somewhat implausible. On the other hand, while the second quotation given above was already in the first edition of Schlick's book, the first quotation, which also shows the preference of the old logic, appeared only in the second edition, from the time when Schlick started to organize the regular meetings of the Vienna Circle. Later, in his review of Carnap's *Aufbau*, Schlick (1929) interestingly suggested that Carnap's most important achievement was that he used the tools of the *new logic* to connect the various special sciences. Schlick's "turn"

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[1] In an earlier review, when Hahn wrote about the development of, and need for, a more precise logic, he did not consider Frege, and while Russell and Whitehead's magnum opus is mentioned as a positive example, he claims that it is too complex for mathematicians to start their work with it. See Hahn ([1919] 1980, pp. 53-54).

might be connected to Hahn's aforementioned seminar (Grattan-Guinnes 200, p. 514) and to Carnap's arrival in Vienna in 1925: in his 1927 review of Russell's *The problems of philosophy*, Schlick already conveyed a more understanding attitude towards the importance of the new logic.

Schlick did not consider Frege's role or possible influence: in the first edition of Schlick's book Frege is not even mentioned, and Schlick connected the achievements of the new logic to Russell (Schlick [1925] 1974, p. 107). He refers to Frege, however, once in the second edition of his book with respect to mathematics ([1925] 1974, p. 356): "That all mathematical propositions can be deduced from a small number of axioms has been conclusively demonstrated by the recent work of Frege, Peano and others."

After the publication of his *Allgemeine Erkenntnislehre*'s second edition, Schlick started to read the *Tractatus*[2] at the Circle's regular meetings and to fall under the influence of Wittgenstein. Later he propagated the Wittgensteinian view of the *practice* of linguistic analysis. Thus, from the late 1920s Schlick was known, alongside Waismann, as the official defender of Wittgenstein.

The only genuine exception to this narrative—one that leaves little room for Frege as compared to Russell and Wittgenstein—might be Carnap,[3] but he is quite hard to categorize. On the one hand, before the First World War, Carnap attended Frege's seminars on logic and mathematics at the University of Jena; thus, he knew of Frege's work and achievements at first hand. He also regularly referred to Frege from the time of his doctoral dissertation (*Der Raum*, published in 1922 in *Kant-Studien*) and he was sort of an authoritative person in the Circle regarding Frege. In a letter from September 11, 1932, for instance, Gödel (2003, pp. 346-348) asked Carnap whether Frege published anything about Russell's paradox after the publication of the second volume of his *Basic Laws of Arithmetic*. Interestingly, though, in the discussions of the Circle, according to Carnap's diaries, he did not lecture about Frege's ideas but instead presented Russell's theory of numbers (November 25, 1926. ASP RC 025-72-05).[4]

Carnap's *Logical Syntax of Language* also has an ambiguous relation with the ideas of Frege (cf. Friedman [1988] 1999). The philosophy of mathematics in *Syntax* embraces the ideas of intuitionism (Carnap's Language I was a constructivist-intuitionist language), the formalism of Hilbert, as well as the logicism of Frege and Russell-Whitehead. Thus, Frege was not at the center of the *Syntax* project. This should come as no surprise. After all, we find the following remark in Carnap's student notes from Frege's 1913 seminar entitled "Begriffsschrift II":

> *Some people think that the symbols are what arithmetic is about*. But that doesn't work in the end. One contradicts oneself continually. Instead, the symbols are just tools for inquiry, not what the inquiry is about; just as the microscope is a tool for

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[2] Schlick wrote to Carnap in 1925 (November 29) that "at our Thursday-Circle we read in this semester Wittgenstein's treatise from sentence-to-sentence" (ASP RC 029-32-34). During the next spring Schlick told Carnap that they did not finish Wittgenstein's book, so they will continue reading it for one more semester. See Schlick's letter to Carnap from March 7, 1926 (ASP RC 029-32-27). In response (March 13, 1926, ASP RC 029-32-24), Carnap said that he would like to participate in the discussion of Wittgenstein's book.

[3] One should also mention Karl Menger, who in his influential lecture "The New Logic" discussed alongside Russell such authors as Peano, Peirce, Schröder and Frege. See Menger (1937; originally 1933). It should be mentioned also that Carnap recorded in his diary (January 26, 1925) that when he talked with Menger (presumably in one of their first discussions) about Brouwer and Russell, Menger indicated that he "did not like [Russell's] logic" (ASP RC 025-72-04).

[4] It should be also mentioned that later Carnap defended Russell's approach to identity against Hahn. See his diary entry from May 19, 1927 (ASP RC 025-72-06). The definition of identity was a regular theme of the Circle in the context of Wittgenstein (*Tractatus*, 5.533), and Carnap argued against his conception of the identity-sign in his *Abriss der Logistik* (1929, §7).

botanical inquiry, not what the inquiry is about (emphasis in original; Carnap 2004, p. 133).

In the light of this remark, Frege would surely oppose Carnap's project, for the latter's metalogical ideas in the *Syntax* are simply about the signs and language of logic and mathematics in a way that abstracts from the extra-logical referents.

On the other hand, even though Carnap referred to Frege in the early 1920s and 1930s, he did not embrace they key ideas of Frege, with respect, for example, to logic. Clinton Tolley (2016) argued that from a logical point of view Russell offered much more of an ideal to Carnap in the *Aufbau*. Though Quine's oversimplified narrative (in "Two Dogmas" and "Epistemology Naturalized") about the Russellian lines and aims of the *Aufbau* is considered to be outdated by most scholars (see, however, Pincock 2002), Carnap did indeed choose as his motto for the *Aufbau* ([1928] 2005, p. 5) a quotation from Russell (1914, p. 155): "The supreme maxim in scientific philosophizing is this: Wherever possible, logical constructions are to be substituted for inferred entities."

Carnap also sent his book to Russell with a letter, claiming that:

> I believe [myself] to have made here a step towards the goal that you also bear in mind: clarification of epistemological problems (and the removal of metaphysical problems) with the aid that the new logic, particularly through your own works, provides. I would like already here to indicate two points on which I had to depart from your view. These points of difference do not rest on differences in basic attitude, which appears to me thoroughly in agreement. The differences arise rather just because I have attempted to carry out your basic view in a more consistent way than has happened before. I believe I am here "more Russellian than Russell" (quoted in Pincock 2007, p. 114; for the original letter see ASP RC 102-68-24).[5]

We know from Carnap's diaries that when he met Russell later (October 10, 1934) the latter was quite enthusiastic about the *Aufbau*: "[Russell] said that he followed my earlier works with interest, especially the *Aufbau*, but he did not read the *Syntax* yet properly. I said that the '*Aufbau*' is out-of-date" (ASP RC 025-75-12). Later in his *A History of Western Philosophy*, Russell ([1945] 2004, p. 874) mentioned only the *Syntax* of Carnap when he dealt with the ideas of "The Philosophy of Logical Analysis."

In actuality, Frege became a major source for Carnap only in the 1940s, when he was dealing with the question of modality. This involved the publication in 1947 of his groundbreaking work *Meaning and Necessity*. In that book, alongiside Alonzo Church's intensional approach (based on the notion of sense and denotation) Carnap became the first to introduce Frege's philosophy and logic into mainstream philosophy. But in 1947, already more than ten years passed since the formulation of the PoT and Carnap's emigration to the United States. (The Vienna Circle also disbanded in the late 1930s after the murder of Moritz Schlick.)

All of this seems to strengthen Milkov's claim that Frege did not have much direct and significant influence on the Vienna Circle at its peak. Rather, the texts and contextual informaton of the Circle points toward Russell (and Wittgenstein).[6]

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[5] For the details of the connection between Carnap's *Aufbau* and Russell, see Carus (2007, ch. 6).
[6] Interestingly Neurath recognized the different paths of Frege and Russell already in the early 1930s. See Neurath ([1933] 1987, p. 275, n. 2).

2. Russell on the Nature of Logic:

Russell is notorious for his regular (and sometimes fundamental) changes in his philosophical development. These changes create several intricate difficulties for any coherent and comprehensive reconstruction of his ideas. Nevertheless, three important and recurring ideas of his will be discussed here: (1) logic is the most general and universal science; (2) there is a close connection between logic and metaphysics; (3) logic is not free from intuition and inductive-practical considerations. But first we have to draw out the context for his ideas.

According to Jean van Heijenoort ([1967] 1997), one might draw a distinction with respect to the history of modern logic: there is a *universalist* and a *model-theoretic* tradition.[7] Heijenoort argued that Frege, Russell, and Wittgenstein belong to the universalist tradition, but the early Carnap might also be said to reside in this first camp. Some passages of Carnap, however, indicate otherwise, and his placement problem is still not settled (see e.g. Loeb 2014; Schiemer 2013).

In his characterization of the universalist tradition, van Heijenoort evoked a short passage from Frege:

> I did not want to present an abstract logic with formulas, but to give an expression to a content with written signs more clearly and precisely than is possible with words. In fact, I did not want to create a bare *calculus ratiocinator* but a *lingua character[ist]ica* in Leibniz' sense… (Frege [1883] 1993, pp. 97-98.)

It is acknowledged that Frege's *Begriffsschrift* is not just a mere calculus, a mathematical *device*, but a *language*. Since this language codifies the *general* laws of truth, *all rational discourses* have to take place *within* it. This means, van Heijenoort claims, that there is no place for a meta-perspective or a metalanguage in the universalist tradition since every act of rational discourse is internal to it. For that reason, logic is the "skeleton" (Korhonen 2013, p. 8) of all languages which codifies rational and meaningful discourses.

Henry Sheffer pointed out in his review of *Principia Mathematica*'s second edition that a certain *fear of circularity* is detectable in the logicians' practices:

> Just as the proof of certain theories in metaphysics is made difficult, if not hopeless, because of the "egocentric" predicament, so the attempt to formulate the foundations of logic is rendered arduous by a corresponding "logocentric" predicament. *In order to give an account of logic we must presuppose and employ logic*. (emphasis in original; Sheffer 1926, pp. 227-228.)

Whether Scheffer is right or not regarding the motivation lying behind certain practices of logicians, it is true that neither Frege nor Russell usually recognized the usefulness and necessity of a *metalanguage*, or more generally, the possibility of a *metaperspective*. There is no trace of it in Russell's informal *Principles of Mathematics* or in any edition of the *Principia Mathematica*.[8]

It should be noted that Russell claimed in his notorious preface to the English translation of Wittgenstein's *Tractatus* that the hierarchy of languages could solve some problems of the book:

> These difficulties suggest to my mind some such possibility as this: that every language has, as Mr. Wittgenstein says, a structure concerning which, *in the language*,

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[7] Hintikka ([1988] 1997) later generalized this narrative to any language.
[8] Some scholars aim to revisit this view. See Landini (1998); Mares (2013); Blanchette (2013); Korhonen (2012, 2013).

nothing can be said, but that there may be another language dealing with the structure of the first language, and having itself a new structure, and that to this hierarchy of languages there may be no limit (emphasis in original; Russell [1922] 1983, p. 23).

Interestingly though, in his own formulations, Russell did not use this idea in any consequent and direct manner. (At an earlier point, Russell mentioned what seems to be a metaperspective when he claimed that "we need *true* propositions about implication" (emphasis in original; Russell [1913] 1973, p. 290), but he did not act on this claim in any detail.) Peter Hylton formulated this point as follows:

> Logic, for Russell, is a *systematization of reasoning in general*, of reasoning as such. If we have a correct systematization, it will comprehend all correct principles of reasoning. Given such a conception of logic there can be no external perspective. *Any* reasoning will, simply in virtue of being reasoning, fall within logic; any proposition that we might wish to advance is subject to the rules of logic (emphasis added; Hylton [1990] 1992, p. 203).[9]

The other part of my reconstruction of Russell concerns the logic-world relation. I do not want to state that Russell was a logical realist of some kind, but I certainly claim that he was committed to the idea that logic and world are closely connected. When Carnap visited Russell in 1935 (at the end of September) he recorded in his diary talking "with [Russell] about my conception of logic. He has misgivings from the viewpoint of his realistic logic [*realistischen Logik*]" (ASP RC 025-75-13). What was this "realistic logic" of Russell?

Alberto Coffa, in his important *The Semantic Tradition from Kant to Carnap* (1991, pp. 273-280), reconstructed a *monolinguistic project* (pursued by Russell and the early Carnap), according to which there isn't any difference between the object and the metalanguage—but more importantly, there is only *one correct* object language. One language which describes in a right and true manner either the world or the laws of thought (and in the case of Frege, the laws of truth).

Russell periodically emphasized that logic is much more about the *world* than about our cognitive capacities or the norms of our thinking in themselves:

> The name "laws of thought" is also misleading, for what is important is not the fact that we think in accordance with these laws, but the fact that things behave in accordance with them; in other words, the fact that when we think in accordance with them we think *truly* (emphasis in original; Russell 1912, p. 113).

Or, as he put it later (Russell 1912, p. 138), "The belief in the law of contradiction is a belief about things, not only about thoughts…and although belief in the law of contradiction is a thought, the law of contradiction itself is not a thought, but a fact concerning the things in the world." In Russell's highly oversimplified narrative (though it still shows the inner perspective of his), the law of contradiction had its origins in certain worldly experiences; actually it was "discovered by generalizing from instances" which means that "it is both an empirical and a logical premise" in any scientific argumentation (Russell [1907] 1973, p. 274).

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[9] It should be noted, however, that the implicit gist in Hylton's quotation is that *if* one obtains the *correct* systematization of reasoning in general, then one cannot step outside of it and take the metasystematical viewpoint. But if one is just after the correct systematization, then the metasystematical perspective is essential to find out whether the given systematization is the correct one or not. I am indebted to Anssi Korhonen for calling my attention to this.

After all, for Russell, since the rules of logic seems to be also the rules of the world and things within it, there must be a logic and a language based on it, which is the *ultimate* one; here we find some traces of the monolinguistic project. And actually Russell seems to support the first part of this claim[10] when he said in his *Introduction to Mathematical Philosophy* (1919, p. 169) that "logic is concerned with the real world just as truly as zoology, though with its more abstract and general features."

There were, of course, differences over time in Russell's ideas about the objects of logic (and mathematics). Earlier, in *The Principles of Mathematics* (1903, p. vii), he argued that pure mathematics does not deal with actual objects *in* the world but only with "hypothetical objects having those general properties upon which depends whatever deduction is being considered." Though seemingly this is a weaker claim than the 1919 one, but actually in 1911 he already showed the signs, for example, of his later realism regarding universals (documented in *The Problems of Philosophy*). He indicated in a lecture that ([1913] 1973, 293) "[l]ogic and mathematics forces us…to admit a kind of realism in the scholastic sense, that is to say, to admit that there is a world of universals and of truths which do not bear directly on such and such a particular existence." Taking logic and mathematics as such disciplines which commit one to special ontological considerations, the connection between logic and world is just contrary to the later views of Wittgenstein and Carnap.

But Russell differed from these authors about the questions of analyticity and tautologies too—and this takes us to my third point. For Russell, logic was not without content. In his book on the philosophy of Leibniz he even argued that

> [a]s regards the meaning of analytic judgments, it will assist us to have in our minds some of the instances which Leibniz suggests. We shall find that these instances suffer from one or other of two defects. Either the instances can be easily seen to be not truly analytic—this is the case, for example, in Arithmetic and Geometry—or they are tautologous, and so not properly propositions at all (Russell 1900, §11).

Being analytical, mathematical and tautologous did not form one group for Russell in 1900. If something is tautological, then it cannot be a proper proposition. (And of course what is not a proposition cannot be meaningful. I do not want to suggest, however, that Russell had such a threefold distinction at that time as did later Wittgenstein—namely, a distinction between meaningful, meaningless, and senseless propositions.) Mathematics, for Russell at that time, was not even analytical, though this changed for Russell later when he started propagating his logicism. Anyways, this idea had some interesting effects on Russell's thinking.[11]

Already in 1900 we found the idea that at the end of our investigations we are faced with the problem of statements that cannot be defined easily or at all. According to Russell,

> [d]efinition, as is evident, is only possible in respect of complex ideas. It consists, broadly speaking, in the analysis of complex ideas into their simple constituents. Since one idea can only be defined by another, we should incur a vicious circle if we did not admit some indefinable ideas (Russell 1900, §11).

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[10] The second part of it is confirmed by the universality and uniqueness of his logic. About the various senses of universality in Russell's early philosophy see Korhonen (2013).

[11] It should be noted, though, that logic was synthetic rather than analytic in *The Principles of Mathematics* (Russell 1903, §434). Later Russell accepted the tautological character of logic (e.g. in Russell 1927, p. 171; 1931, p. 477), but admitted that "[f]or the moment, I do not know how to define 'tautology'" (Russell 1919, p. 205). As a result, it is not clear whether the tautological logic is also without content and factually empty or not in Russell's writings from that time. I am indebted to Anssi Korhonen for these remarks.

One has to account for the *acceptance* of these indefinable elements. Russell distinguished two different ways for doing that. First, when he discussed the methodology of philosophy, he was always against the Bergsonian conception of intuition as a primary source of knowledge (see, e.g., Russell [1917] 1949). But when he considered the rules and axioms of logic he vigorously defended the idea that

> [t]heir truth is evident to us, and we employ them in constructing demonstrations; but they themselves, or at least some of them, are incapable of demonstration. All arithmetic…can be deduced from the general principles of logic, yet the simple propositions of arithmetic, such as "two and two are four," are just as self-evident as the principles of logic (Russell 1912, p. 176).

Most of the rules and propositions of logic and mathematics are thus *self-evident* and *intuitively true*. Given the fact that we have logical/mathematical knowledge, and "since all knowledge must be either self-evident or deduced from self-evident knowledge" (Russell [1913] 1973, p. 293), "self-evidence" oozed into the characterization of logic. For Russell ([1906] 1973, p. 194), "[t]he object is not to banish 'intuition', but to test and systematize its employment." However, Russell did not define or explore what is involved in claiming the self-evidence or intuitiveness of logic and mathematics. Though he did not provide any detailed answer to the question of what is the base of the *evidential character* of logic, he did provide some hints about the nature and role of *obviousness* connected to self-evidence.

First of all, obviousness is not an absolute concept; whether a certain axiom (or rule) is obvious is always a matter of degree (or relative to a context and frame); as Russell ([1907] 1973, p. 273) said: "premises which are ultimate in one investigation may cease to be so in another; that is, we may find logically simpler propositions from which they can be deduced." Secondly, we are not infallible regarding the distribution of obviousness; it may easily happen that what we regarded as obvious ("with the highest degree") turns out to be not obvious after all, or even false. Thirdly, "[a]ssuming the usual laws of deduction, two obvious propositions of which one can be deduced from the other both become more nearly certain than either would be in isolation" (Russell [1907] 1973, p. 279).

Following these guidelines, added Russell, is not enough for the pursuit of science. "[A]lthough intrinsic obviousness is the basis of every science, it is never…the whole of our reason for believing any one proposition of the science" (Russell [1907] 1973, p. 279). Or, as he said earlier ([1906] 1973, p. 194), "[t]he 'primitive propositions' with which the deductions of logistic begin should, if possible, be evident to intuition; but that it is not indispensable, nor is it, in any case, the whole reason for their acceptance."

The second way of justifying axioms and logical propositions comes into play when these elements are either not self-evident or in need of more motivation and justification beyond self-evidence and obviousness. The motivation for this form of justification stems from *inductive-practical reasons*. In *Principia Mathematica*, for example, Russell (and Whitehead) claimed in the context of the axiom of reducibility that

> [t]he reason for accepting an axiom, as for accepting any other proposition, is always largely inductive, namely that many propositions which are nearly indubitable can be deduced from it, and that no equally plausible way is known by which these propositions could be true if the axiom were false, and nothing which is probably false can be deduced from it (Russell and Whitehead 1910, p. 62).

When we choose an axiom we cannot do it simply because we are able to derive it from more fundamental ones. But, as Russell said, we can choose it according to its *usefulness*. If one

knows that there are certain propositions which seem to be or hold to be true in a certain context, then she needs such axioms from which she can derive *those* true statements. If there are seemingly equally good possibilities and choices, then one has to check them for their consequences: we have to know whether something false can be deduced from them or not. In logic and mathematics, "[t]he chief difficulty throughout consists in reconciling the two aims of avoiding the false and keeping what we cannot but think true" (Russell [1907] 1973, p. 280).

This picture of the decision procedure for the selection of axioms and logical rules, however, is highly similar to that employed in other sciences:

> The method of logistic is fundamentally the same as that of every other science. There is the same fallibility, the same uncertainty, the same mixture of induction and deduction, and the same necessity of appealing, in confirmation of principles, to the diffused agreement of calculated results with observation (Russell [1906] 1973, p. 194).[12]

This seems to imply in a way the marriage of epistemology and the idea of truth (which recurred also in the discussions of the Vienna Circle before the mid-1930s) and Russell indeed seems to suggest this:

> Degrees of self-evidence are important in the theory of knowledge, since, if propositions may…have some degree of self-evidence without being true, it will not be necessary to abandon all connexion between self-evidence and truth, but merely to say that, where there is a conflict, the more self-evident proposition is to be retained and the less self-evident rejected (Russell 1912, p. 184).

For Russell, therefore, there was a point regarding questions of logic in which practical freedom came into the picture; this freedom, however, was restrained by close adherence between logic and the world. Though one could build one's logic, at least partly, as one wishes, the majority of the logic is still bound by some previously-determined nodes and considerations which are not apt for revision.

3. Wittgenstein on the Nature of Logic:

Though Wittgenstein is usually grouped together with Frege and Russell, at some points he was against them and called their approach the "old logic" (*Tractatus* 4.126; cf. Ricketts 1996).While Russell said that logic is similar to the other sciences—with its infallibility as well as its inductive and partly intuitive content—Wittgenstein claimed, in a letter to Russell, that "[l]ogic must turn out to be of a totally different kind than any other science."[13]

While Russell and Frege used to claim that logic is the most general and *universal* science in the sense that its statements hold for everything, Wittgenstein argued that "The mark of logical propositions is not their general validity…An ungeneralized proposition can be tautologous [i.e., logical] just as well as a generalized one" (*Tractatus* 6.1231).

On the other hand, while Frege considered logic as the science whose main object is truth, Wittgenstein was against any kind of approach that signified truth values as objectivities: "One could e.g. believe that the words 'true' and 'false' signify two properties

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[12] Cf. Russell ([1907] 1973, p. 272), where he tries to show that "…the close analogy between the methods of pure mathematics and the methods of the sciences of observation." Or a few pages later (*ibid.*, p. 274), where he writes, "the method in investigating the principles of mathematics is really an inductive method, and is substantially the same as the method of discovering general laws in any other science."

[13] Wittgenstein's letter to Russell, June 22, 1912. In McGuinnes ([1995] 2008, p. 30).

among other properties," (*Tractatus* 6.111), but in this case logic would be similar to the natural sciences after all (as Russell thought), and this is "a certain symptom of its being falsely understood" (*Tractatus* 6.111). (It should be mentioned, however, that according to Frege logic was not about truth *per se*, but about the *laws* of truth.)

Since logic is of a different nature than the other sciences, Wittgenstein argued that in the justification of logic one has to abandon the notion of self-evidence and any type of practical/pragmatic elements. "It is remarkable that so exact a thinker as Frege should have appealed to the degree of self-evidence as the criterion of a logical proposition" (*Tractatus* 6.1271).[14] Therefore he characterized logic in a wholly different way; although Russell's logic was contentual in a sense, Wittgenstein argued that "[t]heories which make a proposition of logic appear substantial are always false" (*Tractatus* 6.111). By contrast, in the *Tractatus*, "the propositions of logic are tautologies" (*Tractatus* 6.1). By showing that the propositions of logic are tautologies, hence devoid of empirical content, Wittgenstein held that they say nothing: "all propositions of logic say the same thing. That is, nothing" (*Tractatus* 5.43). For Wittgenstein, it follows that we can also criticize the logicist program; according to him, it would be hard to believe that the infinite number of propositions of logic and of mathematics "should follow from half a dozen 'primitive propositions'" (*Tractatus* 5.43).

Though he did not consider in the *Tractatus* the axiom of choice, Wittgenstein criticized heavily the axioms of reducibility and infinity. He said that "[p]ropositions like Russell's 'axiom of reducibility' are not logical propositions, and this explains our feeling that, if true, they can only be true by a happy chance" (*Tractatus* 6.1232). Note that Russell argued that this proposition or axiom should be regarded as logical due to pragmatic and inductive reasons.

But even if we were to accept that these axioms are logical, logicism would be still a failed approach for Wittgenstein because while logic is tautological, mathematics is not: "The propositions of mathematics are equations, and therefore pseudo-propositions" (*Tractatus* 6.2). It would be an interesting historical project to show how the Vienna Circle and others arrived to the picture that mathematics and logic have the same nature—that neither of them has empirical content, so both are tautologies.

Wittgenstein was against conventions in logic, though the conventional character of logic later played an important role in the Circle (and according to them, they get it from Wittgenstein). He said that "in logic it is not we who express, by means of signs, what we want, but in logic the nature of the essentially necessary signs itself asserts" (*Tractatus* 6.124). Here we find again an important and interesting transition: how the Circle invented the conventionally-based tautologies.

What is more important for now is Wittgenstein's view of the nature of logic. As a consequence of the picture theory, one cannot talk about the language-world relation, i.e., about semantics. But one cannot talk about language either, and here surfaces again the logocentric predicament of Sheffer: in order to talk about language, we have to presuppose language. Hence we cannot talk about logic either; for Wittgenstein, "logic must take care of itself," and logical properties (e.g., being a tautology) can be *shown* but cannot be *said*: "every tautology itself shows that it is a tautology" (*Tractatus* 6.127). Therefore, one cannot develop further, change, or extend logic; it is tied up with an isomorphic one-to-one relation to the world. Seen this way, logic became a prerequisite of meaningfulness; it is transcendental, as Wittgenstein said. It cannot be the subject of rational discourse; logic marks the boundaries of meaningful discourse. This is "Wittgenstein's prison," since "[t]he very nature of language, in

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[14] The same is told about Russell too (*Tractatus* 5.4731): "Self-evidence, which Russell talked about so much, can become dispensable in logic, only because language itself prevents every logical mistake. – What makes logic a priori is the impossibility of illogical thought."

Wittgenstein's view, prevented us from ever stepping outside it" (Awodey and Carus 2009, pp. 88-89).

4. Carnap's Principle of Tolerance:

Carnap had a long and interesting way of reaching the principle of tolerance. Some aspect of this principle can be found in his *The Logical Structure of the World* ([1928] 2005), but he did not consider there the nature of logic *in detail*. Neither did he do so in his lesser-known *Abriss der Logistik* (1929), which was a short textbook on Russell and Whitehead's *Principia Mathematica* (Carnap 1963, p. 14), with a particular focus on the *application* of the logistic method. He even described his intentions in a letter to Schlick as "to present the Logistic [*Logistik*] as such a *method* which could be used in the various (non-logical) fields."[15]

During the end of the 1920s Carnap was preparing two manuscripts about logic which dealt its nature (*Untersuchungen zur allgeimeinen Axiomatik* and *Neue Grundlegung*), but neither of them contained any hint of the principle of tolerance or of the metalogical approach which characterized the later years of his work.[16]

According to Carnap's quite dramatic narrative, he gave up his earlier projects on the 21st of January, 1931: after a sleepless night he wrote down his ideas on "forty-four pages under the title 'Attempt at a metalogic.' These shorthand notes were the first version of my book *Logical Syntax of Language*" (Carnap 1963, p. 54).[17] Carnap started to lecture about the new "metalogic" at the Circle's discussion nights in the summer of 1931,[18] but the PoT did not surface in them.

One hint of the principle can be found, however, in two articles from 1932. In "Psychology in Physical Language" ([1932] 1959, p. 192), Carnap claimed that we are *free to choose* our methods in a given investigation ("every method of inquiry is justified…we may apply any method we choose"), for what actually matters are the obtained *consequences*, whether the method is fruitful for a given aim or not. In the other article, "On Protocol Sentences," which was Carnap's own contribution to the famous protocol-sentence debate, and which aimed to reconcile (or at least mediate between) Schlick and Neurath's opposed approaches, Carnap said that

> [m]y opinion here is that this is a question, not of two mutually inconsistent views, but rather of *two different methods for structuring the language of science both of which are possible and legitimate*…I now think that the different answers do not contradict each other. They are to be understood as suggestions for postulates; the task consists in investigating the consequences of these various possible postulations and in testing their practical utility (emphasis in original; Carnap [1932] 1987, pp. 457-458).

Schlick and Neurath's approaches are just two different methods—based on different language forms—to consider the protocol-sentences, and we can (re)construct these language forms as we wish. The statements of Schlick and Neurath cannot contradict each other

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[15] Carnap's letter to Schlick, October 7, 1927. ASP RC 029-31-06 (emphasis in the original typed version). It should be noted for reasons of historical curiosity that Springer, the publisher of the *Abriss*, rejected Carnap's *Logical Structure* because of perceived financial risks. Though in the 1930s the *Abriss* was quite a success, nowadays it is less well-known than the *Logical Structure*. See Grattan-Guinnes (2000, p. 502).

[16] The *Untersuchungen* was published posthumously as Carnap (2000), but the *Grundlegung* could be found only as a manuscript in his *Nachlass* as ASP RC 089-64-01 and ASP RC 089-64-02. About these projects see Carus (2007, pp. 191-203).

[17] About the transition from the earlier logical writings to the *Syntax*-project see Awodey – Carus 2007.

[18] The records of Carnap's lectures are preserved at the Pittsburgh Archive as ASP RC 081-07-17, ASP RC 081-07-18, and ASP RC 081-07-19. For the transcription of these lecture-notes see Stadler ([2001] 2015, pp. 107-123).

because they actually do not state anything; they are simply *proposals* which could have such theoretical and practical virtues as fruitfulness, simplicity, inner consistency, etc., but not truth or falseness.

The actual PoT was introduced into the discussion during the summer of 1933, when Carnap admitted the following considerations (or "main points" as he called them):

(1) Our problems concern *language*.
(2) So one always has to specify: *about which language* one talks?
(3) Do not consider what you already know, but make a proposal [*vorgeschlagene*], one has total *freedom* [*völlige Freiheit*] in that! (emphasis in original; ASP RC 110-07-22).

Carnap upheld the idea that "philosophical" discourse is about language throughout his entire career; he claimed in his "Testability and Meaning" (1937, p. 3) that "[i]n the first place we have to notice that this problem [the criterion of meaning] concerns the structure of language. (In my opinion this is true for all philosophical questions…)."[19] But in order to narrow down the range of our inquiries, we have to specify which language concern us, just as the second point above states. But we could extend Carnap's point by making explicit his implicit idea, namely that we must decide *in which metalanguage* we analyze our object language.

Languages are constituted by their logical core (by their logical constants, formation and transformation rules as Carnap called them) and logic is just *tautological*, without any empirical *content*. As Carnap said:

> (Meaningful) statements are divided into the following kinds. First there are statements which are true solely by virtue of their form ('tautologies' according to Wittgenstein…). They say nothing about reality. The formulae of logic and mathematics are of this kind. They are not themselves factual statements, but serve for the transformation of such statements. Secondly there are the negations of such statements ('*contradictions*'). They are self-contradictory, hence false by virtue of their form. With respect to all other statements the decision about truth or falsehood lies in the protocol sentences. They are therefore (true or false) *empirical statements* and belong to the domain of empirical science (emphasis in original; Carnap [1932] 1959, p. 76).

On account of these distinctions, the world, or better, how things are arranged in the world, cannot constrain our logic and language at first place: "*The formal sciences do not have any objects at all*" (emphasis in original; Carnap [1935] 1953, p. 128). Hence regarding possible metalanguages—in which we pursue our theoretical investigations—we have "total freedom," and we construe them as we want: "('*Principle of Tolerance*' (*Toleranzprinzip*): In logical questions there is no moral, no commands [*Gebote*] or prohibitions [*Verbote*]" (ASP RC 110-07-22). Or as it was officially documented in the *Logical Syntax of Language*:

> *It is not our business to set up prohibitions, but to arrive at conventions…In logic there are no morals*. Everyone is at liberty to build up his own logic, i.e. his own form of language, as he wishes. All that is required of him is that, if he wishes to discuss it, he must state his methods clearly, and give syntactical rules instead of philosophical arguments (emphasis in original; Carnap [1934] 1937, §17).

Despite of its name, the principle of tolerance is not a *theoretical principle*, or any kind of thesis, since as such it would require theoretical argumentation, but rather a philosophical *stance*, or *attitude*, which was held and applied by Carnap throughout his whole career. The

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[19] See also Carnap's letter to Neurath (July 24, 1939) where he stated that "I always stressed that the logic of science is the analysis of language [*Wissenschaftslogik Sprachanalyse ist*]." ASP RC 102-53-05.

PoT says that it is superfluous to talk about the correct language, since it lies in our power to formulate different language forms, from which we can choose later with respect to our practical *needs* and *goals*. According to Carnap's own confession ([1934] 1937, p. xv) one of the main motivations behind PoT is the recognition that if "we cast the ship of logic off from the *terra firma* of the classical forms…we reach the boundless ocean of unlimited possibilities." These language forms are on a par regarding their claim for legitimation and validity. None of them is *the* correct one—they just serve different purposes.[20]

So in a more general sense, Carnap claims that regarding the questions of logic, mathematics and even philosophy, we are not faced with such theories and statements that could be true/false or right/wrong (for any of which we have to give a priori or theoretical arguments), but instead we are dealing with *practical decisions*. Carnap's reasoning, again, is that the rules and statements of logic are without empirical content, and hence we have the right to define them as we want. What matters is the *fruitfulness* of our linguistic frameworks, and we have to *test* them against our aims and goals. Firstly, one is required to give precisely one's definitions and rules, then one has to show the consequences that follow from them, and if the results fit into our *space of reasons*, then we can accept them. If the given framework comes with destructive or inconvenient implications, then we give up either the framework or our goals. It's all about costs and benefits, about practical aims.

In the preface of *Syntax* ([1934] 1937, p. xiv) Carnap centered his narrative around Russell: "Up to the present, there has been only a very slight deviation, in a few points here and there, from the form of language developed by Russell which has already become classical." So partly the aim of the PoT is to ensure a path on which one could leave behind the logic of Russell and to support all those who would like to explore "the boundless ocean of unlimited possibilities." Interestingly though a year later Carnap recorded in his diary the followings:

> Discussion with Oppenheim and Hempel in the morning. I explained [to them]: 2 ways to loosen [*Auflockerung*] the Aristotelean class-sentences: 1) Russell's theory of relations; 2) Reichenbach's many valued logic. I think the first is sufficient (ASP RC 025-75-13, April 27, 1935).

Loosening the classical logic of Aristotle is one thing; leaving behind "the new logic" of Russell and entering the land of free possibilities is another. It is still interesting that though it was Russell (and not, for example, Frege) who opened up the gates of the new logic for Carnap, the former never discovered the new land entirely. So one could say that if we enter that gate and depart from the "*terra firma* of classical forms," we need to extend the boundaries of Russellian logic.

5. The Carnapian Synthesis of Russell and Wittgenstein:

Let us recall what we saw in the earlier sections. In the case of Russell, I argued that he accepted that there are certain pragmatic and inductive elements in the justification of logic and mathematics. According to him there isn't any final and ultimately right decision at these points (besides our goals and to-be-preserved true statements). These primitive elements in logic and mathematics are not fixed, and so we have a certain freedom. But this freedom is limited by the need to achieve our fixed goals and aims; it is also restricted by certain

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[20] Due to the PoT, the truth of statements is also context-, or more precisely, linguistic-frame-relative for Carnap (see Carnap 1950) and at this point he seems to be in conflict with Russell, who claimed that "[i]f, finally, we can arrive at a set of principles which recommend themselves to intuition, and which show exactly how we formerly fell into error, we may have a reasonable assurance that our new principles are at any rate *nearer the truth than our old ones*" (emphasis added; Russell [1906] 1973, p. 195).

epistemological ideas. And so in order to get Carnap's principle, we have to extend the boundaries of Russell's freedom.

On the other hand, logic is without empirical content in the Wittgensteinian framework: it is tautological but it is also transcendental, and hence makes it possible to talk about the world while *at the same time* prohibiting talk about logic and language. To capture the one world we have to work with one language and logic. So Wittgenstein's prison does not allow moving freely between alternatives. Wittgenstein also excluded from the domain of logic any pragmatic and conventional moves. This means that we do not have to extend the boundaries of freedom in Wittgenstein's system; we have to introduce freedom first.

I certainly do not want to make the strong thesis that Carnap *intentionally* took Russell and Wittgenstein's ideals of logic (and language), fused the most sympathetic parts of them together, and created the PoT. I do not want to exclude this possibility either; this could be a topic worthy of further research. Nonetheless I want to make that weaker claim that Carnap's PoT *could be read or reconstructed* as a synthesis or combination of these approaches. After all, Carnap was famous for his efforts to make syntheses and to mediate between his allies.

Alternatively, if one would like to make an ever weaker claim, one could state that Russell and Wittgenstein offered different views on the nature of logic (and language). Detaching these ideas from their defenders, it could be seen as a highly plausible idea in *the history of logic and ideas* that these views could be synthetized on a higher level by someone. That "someone" turned out to be Carnap: he was the one who united Wittgenstein's ideal of the emptiness of logic with Russell's ideal of inductive-practical justification in the case of logic. Since logic is empty, we can vary it as we wish and we can justify it with some practical reasons, for "in logic there are no morals."

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