Modality in Brandom's Incompatibility Semantics

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Abstract

In the fifth of his *John Locke Lectures* Robert Brandom takes up the challenge to define a formal semantics for modelling linguistic contents as established according to his normative analysis of linguistic practices. The project is to exploit the notion of incompatibility in order to directly define a modally robust relation of entailment. Unfortunately, it can be proved that in the original definition the modal system represented by *Incompatibility Semantics* collapses into propositional calculus. In the paper I show how *IS* can be technically amended so to overcome this failure: the required modifications are already known and consist in adapting and including the main notions of Kripke's standard framework of possible worlds. I also show that the modifications do not jeopardize Brandom's original project.

1 Introduction

One of Wilfrid Sellars's characteristic seminal claims was that Truth is not a relation holding between linguistic and non linguistic items.

Many fruits of this thought of can be found in Robert Brandom's normative analysis of linguistic practices. In Brandom (1994) *sapient* beings are described as engaging in practices of *giving and asking for reasons*, whose contents are defined by what speakers are *entitled* and *committed* to endorse: in fact the commitment to one *reason* might rule out the entitlement to others, in the sense that it is *incompatible* with them. Incompatibility Semantics (*IS*) is Brandom's attempt to define a formal semantics as a model for those contents: his basic idea is to define the semantic interpretant of a sentence p as the set of sentences incompatible with it. But *IS* is also part of a wider project. Brandom declares that with his semantics he aims to

Claim 1. "explore the relations between normative and modal vocabulary [...], showing how normative vocabulary can serve both as a *pragmatic metavocabulary* for modal vocabulary and as the basis for a *directly* modal formal semantics for ordinary empirical vocabulary that does not appeal in any way to a notion of truth."[1]

Unfortunately the original definitions of *IS* fail the representation of modality, but *IS* can be modified to overcome this failure, by applying some results from Göcke et al. (2008) and Peregrin (2009). These modifications do not jeopardize Brandom's project expressed in Claim 1.

2 Definitions for *IS*

I recall here the essential definitions of *IS*[2].

Consider a language $\mathcal{L}$ as a set of sentences. Consider an *incoherence relation*, Inc over $\mathcal{L}$, where $X \cup Y \in Inc$ is to be construed as "one can't commit both to X and to Y". Let Inc obey just to the following property:

[1]Brandom (2008), p. 116 (my emphasis).

[2]For any detail however I refer to Lecture V of Brandom (2008).

Persistence $X \subseteq Y \Rightarrow X \in Inc \Rightarrow Y \in Inc$.

This means the only way to solve an incoherence is to discard some commitment.

Then let an *incompatibility* function $I : \wp(\mathcal{L}) \rightarrow \wp(\wp(\mathcal{L}))$ be related to Inc as follows:

Partition $X, Y \in Inc \Leftrightarrow X \in I(Y)$.

Now entailment is defined by exploiting the idea that X incompatibility-entails Y if and only if everything that is incompatible with Y is also incompatible with X:

($\vDash_I$) $X \vDash_I Y$ iff $\bigcap_{p \in Y} I(\{p\}) \subseteq I(X)$.

Eventually connectives are introduced. Since the semantic interpretant of any $p \in \mathcal{L}$ is the set of sentences incompatible with p, the questions to be asked are what it is to be incompatible with *not p* and what it is to be incompatible with *p and q*. Thus:

($\neg$) $X \in I(\neg p)$ iff $X \vDash p$;

($\wedge$) $X \in I(p \wedge q)$ iff $X \in I(\{p, q\})$.

3 Modality

3.1 A first failure

What it is to be incompatible with *necessarily p*? It turns out it's not so obvious to express that in *IS*. Rather than simply stating a definition, what I'm going to tell is a story about how to establish it. Then there are mainly two reasonings one can follow.

The fist one starts from necessary cases and moves forward. Thus, to begin with,

(i) everything that is self-incompatible is incompatible with *necessarily p*,

but also

(ii) everything that is incompatible with p is incompatible with *necessarily p*.

What else? It is tempting to borrow from common knowledge about modality the idea that *not p* rules out *necessarily p*. Thus, given the definition of negation in *IS*, the suggestion is that something is incompatible with *necessarily p* if it *doesn't entail* p. Thus, to put it straightly according to the definition of ($\vDash_I$),

(iii) everything that is compatible with something incompatible with p is incompatible with *necessarily p*.

Unfortunately this is a wrong suggestion, for the technical reason that this definition would validate both the *S5*-axiom and the converses of the browerian axioms: this situation, as it is well known, produces a collapse of modality in the sense that $p \equiv \Box p$. But let me try to analyze this result. In the standard framework of possible worlds, the only models that satisfy both *S5*-axiom and the converses of the browerian axioms are those containing just one single world. That gives representational sense to the collapse of modality. In the case of *IS*, it is the very semantical definition of necessity that picks up the collapsed case by simply ignoring what differentiates it from the others. To understand why, begin with noticing that, in *IS*, incompatibles behave like *contraries*: one can't commit to both, but can commit to neither. This, conversely, generates inside a language *families of compatible sentences* which do not rule out each other, in the sense that they can in principle be endorsed all together. So, to define what is incompatible with *necessarily p* as what is compatible with *not p* – i.e. what is compatible with something incompatible with *p* – is to narrow the application of modal vocabulary within one single family of compatibles: that makes modal vocabulary superfluous.

But what to do then? The solution at this point is to try to go beyond the boundaries of one single family of compatibles. An obvious way to do that is to require, for something to be incompatible with *necessarily p*, not only that it *doesn't entail p*, but that it *is compatible with something that doesn't entail p*. Thus, formally

($\Box$I) $X \in I(\Box p) \Leftrightarrow X \in Inc$ *or* $\exists Y(Y \cup X \notin Inc \wedge Y \not\models p)$.

This is the definition eventually adopted in *IS* for the modal operator.

Here is where it is important to consider the second reasoning. It starts from sufficient cases and moves backwards. In an intuitive interpretation of necessity, one may say that something is necessary if nothing would prevent it. Thus something is incompatible with *necessarily p* if something is incompatible with *p*. Here however we meet again the suggestion that leads to the collapse of modality. We've just analyzed it and we know how to avoid it: it is not enough to take something that is incompatible with *p*, we have to consider what doesn't imply *p* since the defeasor of *p* might be in another *family* of compatibles. This establishes

($\Box$I′) $X \in I(\Box p) \Leftrightarrow X \in Inc$ *or* $\exists Y(Y \notin Inc \wedge Y \not\models p)$.

Time to take stock.

We followed two reasonings that led us to two different definitions for the introduction of the necessity operator. Now the crucial question is: how are they different? In point of fact, it can be proved that, contrary to the appearances, they are equivalent. And this becomes "the basic observation about modal formulae" in *IS*:

Proposition 2. $X, \Box p \models \varnothing \Leftrightarrow X \models \varnothing$ *or* $\Box p \models \varnothing$.

It basically says that what is incompatible with $\Box p$ has nothing to do with X: either p is necessary or $\Box p$ is self-incompatible. This is what establishes the simplest kind of necessity as represented by **S5** system.

It's worth pausing here to take a deeper look at the proof of this theorem[3]. All the trick is in the ($\Rightarrow$) direction. It says that if something (self-compatible) doesn't imply p then $\Box p$ is self-incompatible. It does it by showing that

$$X \cup \Box p \in Inc \Rightarrow \not\models p \Rightarrow \Box p \in Inc.$$

The X simply disappears. The proof is established by applying one main observation[4]:

$$X \cup Y \notin Inc \Rightarrow X \notin Inc.$$

In fact, this is why $\exists Y(X, Y \not\models \varnothing \wedge Y \not\models p)$ implies $\exists Y(Y \not\models \varnothing \wedge Y \not\models p)$, which is equivalent to $\not\models p$. But the real magic is in the ($\Leftarrow$) direction which 'simply' follows by *Persistence*. Notice that *Persistence* amounts to the contrapositive of the above principle:

$$X \in Inc \Rightarrow X \cup Y \in Inc.$$

The crucial point is that once X is vanished, *Persistence* makes it never come back. In this sense what Proposition 2 shows is that the particular families of compatibles are *irrelevant*, because any proposition may be the defeasor of $\models p$. Thus, *a fortiori*, it doesn't matter if what invalidates $\models p$ is somehow indirectly, i.e. transitively, compatible with X. And this is why *S4*-axiom can't fail.

[3]See Proposition 3.3 in Brandom (2008), p. 144.

[4]Apparently the observations that support Proposition 2 are two:

(i) $X \cup Y \notin Inc \Rightarrow Y \notin Inc$

(ii) $\not\models p \Leftrightarrow \Box p \models \varnothing$

But in fact (ii) holds because of (i):

$$\begin{aligned} \not\models p \quad & \Leftrightarrow \exists Y(Y \notin Inc \wedge Y \not\models p) \\ & \Leftrightarrow \exists Y(Y \cup \varnothing \notin Inc \wedge Y \not\models p) \\ & \Leftrightarrow \Box p \models \varnothing \end{aligned}$$

3.2 'To persist is diabolical'

Hitherto Brandom (2008). Let me now try to tell a story about why the original version of *IS* fails and about how to modify it in order not to fail anymore. So the examination of the proof of Proposition 2 detected the axiom of *Persistence* as the main suspect for the collapse of modality in *IS*. In the previous section I suggested that the problem with *Partition* is a problem of *relevance*. This remark helped me to qualify the problem but now I have to admit I used it also as a bait. Those who might have swollen it, probably resonate to a certain way to construe the logical representation of necessity which has been put forward by relevant logicians. At the opening of **?** the reasons motivating the whole enterprise of *Relevance Logic* are presented as in a par with C.I. Lewis's complaints for the so called "paradoxes of material implication" in Russell's *Principia Mathematica*, in particular,

$$p \to q \to p.$$

Here material implication only represents purely extensional relations between propositional contents and this makes any other relation *irrelevant* for the implication of a true proposition[5]. This is what Lewis avoided with his strict implication:

> "In terms of *material* implication, if $pq. \supset .r$ and p is true then $q \supset r$, since $pq. \supset .r :=: p. \supset .q \supset r$. But in terms of *strict* implication, if two premises, p and q, together imply r, and p is true, it does not follow in general that $q \dashv r$; since $pq. \dashv .r$ is not equivalent to $p. \dashv .q \dashv r$."[6]

Now, it takes but a moment to realize that Lewis and Brandom work with very akin intuitions on entailment, for compare Lewis's definition of strict implication upon the binary operator "$\circ$" for consistency,

[5] See Lewis and Langford (1932), p. 85, and **?**, pp. 3-5.

[6] Lewis and Langford (1932), p. 165 (where Lewis's horseshoe for material implication corresponds to "→").

$$p \prec q =_{Def} \neg(p \circ \neg q),$$

with Brandom's definition of entailment, which can be equivalently expressed as

$$p \vDash_I q =_{Def} \neg\exists X(X \notin I(p) \wedge X \in I(q)).$$

And yet, while Lewis construes strict implication as the proper representation for the necessary character of entailment and then proceeds to define material implication in a different way, Brandom treats his definition as of *the* only notion of implication in his system and then proceeds to define modal operators to express counterfactually robust conditionals.

The first crucial point to notice is that, in spite of the idea of incompatibility as a directly modal notion, in this sense *IS* is a system of material implication[7]: in fact it is trivial to prove $\vDash_I p \rightarrow q \rightarrow p$. But it is important to see why. Now $\vDash_I p \rightarrow q \rightarrow p$ follows from $p, q \vDash_I p$ – which is valid in *IS* – because a standard form of deduction theorem is valid[8]. This is a typical situation you want to avoid if you care about the issue of *relevance*, but the temptation to see it as a stark choice between two obvious principles of implication – deduction theorem and reflexivity – should be resisted because there is more than meets the eye. A quick look to algebras for substructural logics could help[9]. Let me borrow just the essential to make my point. Consider a lattice ordered groupoid $\langle S, \leqslant, \circ \rangle$ and introduce a binary operation "$\rightarrow$" such that it satisfies the following property, usually named *left-residuation*:

$$a \circ b \leqslant c \text{ iff } a \leqslant b \rightarrow c.$$

[7]Here and in what follows I rely on Brandom (2008)'s representation theorem for *IS*.

[8]See Theorem 3.3 in ,Brandom (2008), p. 159.

[9]I suggest Dunn (1991); Dunn and Hardegree (2001) which are directly connected with the topic.

Now this property is important precisely because it shows the relations holding between operations and in general functions on algebras. For what concerns us here it enables us to see that deduction theorem does nothing but display the relation between "→" and that particular sort of conjunction which is ",":

$$a, b \models c \text{ iff } a \models b \rightarrow c.$$

In other words, material implication residuates extensional conjunction. Notice there's nothing wrong with this. What we want to avoid is that material implication residuates also intensional conjunction, or fusion, "∘". That would force us to accept *Augmentation*, i.e. $p \circ q \models p$, which is unwanted for fusion – compare with "if p is compatible with q then p is true".

To sum up, between the two options of the troubling choice we faced above there's a tenable position: that of requiring both that strict implication doesn't residuate extensional conjunction and that intensional conjunction doesn't validate lower bounds, i.e. $p \circ q \leqslant p$. This is what both the systems of strict implication and system **R** of relevance logic require, by imposing fusion not to be idempotent[10]. Instead Brandom does not prevent that in *IS*. He allows his conditional to be the left-residual of compatibility, but with the axiom of *Partition* he forces compatibility to validate *Augmentation*: as a result his implication behaves materially[11].

[10]Probably neither Lewis nor Belnap and Anderson ever wrote that fusion is not idempotent, and yet to require that is enough both for strict implication and for system **R** to avoid the collapse into material implication *as described above*. Since I refer to his work below, I have to note here that Read (1988), p. 128, explicitly contrasts **R**'s fusion with Lewis's consistency operator and claims that the latter validates *Augmentation*. This however, as far as I can see, needs some clarification. Lewis does define $\Diamond(pq) \dashv \Diamond p$, but this amounts to $p \circ q \dashv p \circ p$, which doesn't imply *Augmentation* for the consistency operator if it is not idempotent: $p \circ p \dashv p$ is not valid. What is valid, as Read remarks, is that "any impossible proposition is inconsistent with any other proposition whatever", that is to say $\neg(p \circ p) \dashv \neg(p \circ q)$: this might be bad for relevance logic but doesn't affect modal logic.

[11]An extended story about this is told in Read (1988), pp. 36-50.

The second crucial point to notice, then, is that Brandom's definition of necessity operator lies directly against this material implication and doesn't impose any other level for the evaluation of another sort of implication. Recall again definition ($\Box$I): it can be also rephrased by saying that something is incompatible with $\Box p$ if it can be conjoined with something that doesn't imply p. Thus what is at play is just conjunction and implication which, as we've just seen, are respectively extensional and material.

3.3 Towards a stable system

If all the problems come from the axiom of *Persistence*, why don't we just drop it?

Indeed, there are encouraging reasons to believe that it would be a good idea. Among these, there's the quite promising fact that all the characterizing formulas of normal modal systems would be easily provable anyway: from rule of *Necessitation* to *K*, and so on. But what's best is that the validity of each single theorem would depend on the expected properties of compatibility relations: *T*-axiom would be valid if and only if compatibility were *reflexive* (which is the case), *B*-axiom would be valid if and only if compatibility were *symmetric* (which is the case), *S4*-axiom would be valid if and only if compatibility were *transitive* (which, presumably, is not the case).

But there are some discouraging facts as well. Suppose in fact we could really drop the axiom of *Persistence* without any unacceptable lost. Well, the bad news are that that wouldn't be enough to avoid its effects in *IS*. Consider the most inescapable and apparently innocuous principle for an entailment relation, *Reflexivity*. The fact is that where, as in *IS*, entailment is defined as a relation between *sets* of sentences, *Reflexivity* becomes: $X \models a$ iff $a \in X$. This immediately gives a form of *Augmentation* since it can't be denied that $X, a \models a$. But things are even worse. Once this is acknowledged, *Weakening* on the left can be re-established in its full generality:

Lemma. $X \models p \Rightarrow X, Y \models p$[12]

Proof. Assume $X \models_I p$. We show $X, Y \models_I p$ for arbitrary Y.

Suppose $Z \in I(p)$. But, as a consequence of *Reflexivity*, $I(p) \subseteq I(Y, p)$.

Thus $Z \in I(Y, p)$ by *Transitivity* of "$\subseteq$".

Then by *Partition*, $Z \cup Y \in I(p)$. Then $Z \cup Y \in I(X)$ by ($\models_I$).

Then $Z \in I(X, Y)$ by *Partition* again. Thus $X, Y \models_I p$. $\square$

The moral to be drawn is that *IS* is too deeply entrenched in set theoretic extensional framework.

4 Possible worlds in *IS*

So far I've talked loosely about 'families of contraries' and correspondent 'families of compatibles'. Let me now formally qualify my loose talk. Fortunately I don't have to look far: if there is an idea deeply entrenched in the whole modern reflection on modality since Leibniz, it is the notion of *compossibility*. As Leibniz himself explains to Bourguet:

> "[N]ot all possibles are compossible. Thus, the universe is only a certain collection of compossibles, and the actual universe is the collection of all existing possibles, that is to say, those which form the richest composite. And since there are different combinations of possibilities, some of them better than others, there are many possible universes, each collection of compossibles making up one of them." (GP III 573/L 662)

This led to the standard definition of possible worlds as *maximally consistent sets of propositions*. This idea can be easily adopted in Brandom's framework:

[12] A correspondent proof was originally provided by Alp Aker.

(Possible World) $PW_{Inc} =_{Def} \{S \mid S \notin Inc$ **and** $\forall X(X \cup S \notin Inc \Rightarrow X \subseteq S)\}$.

Peregrin (2009) notices that a useful fact immediately follows. One of the reasons of discontent with incompatibility-entailment is that it seems to drop, together with the notion of *Truth*, also the idea that one main purpose of a consequence relation is to represent the preservation of a certain semantically relevant status. But now, consider what it means to be true in a possible world in the framework just defined. Given the definition of possible worlds as maximally coherent sets of propositions, for a proposition to be true in a possible world is for it to be part of that world, in the standard sense that it is compatible with it. Notice then that it is equivalent to say that a sentence p is true in a possible world PW_{Inc}, that $p \in PW_{Inc}$, that everything compatible with PW_{Inc} is compatible with p and that $PW_{Inc} \vDash_I p$.

4.1 How to Kripke *IS*

Now that a definition of possible worlds and a notion of truth have been derived, it is obviously tempting to try to do better than Brandom in *IS* by following Kripke's well-trodden path. That this can be done has already been showed by Göcke et al. (2008) and Peregrin (2009).

To begin with, recall that, metaphysical issues apart, the main problem with the reception of this idea of possible worlds as maximally coherent sets of sentences inside the standard truth-functional semantics was that to treat compossibility as consistency in a strictly bipartite evaluation of semantic contents is to crush necessity on logical validity. Kripke's relational semantics, by pivoting on the primitive notion of *accessibility*, disentangled modal possibility from logical possibility and opened the doors to the modern analysis of modality.

Our next goal then is to define the relation of *accessibility* with the resources of *IS* plus the standard definition of possible worlds. Fortunately, the trick to obtain accessibility is common knowledge. Suppose you have a space of possibilities already defined in terms of possible worlds, then, by reversing the basic intuition about necessity as truth in any accessible world, a binary accessibility relation *R* between worlds can be defined by taking world w_1 to be accessible from world w_2 if and only if everything which is necessary in w_1 is true in w_2:

$$w_2 R w_1 \text{ iff } \{p \mid \Box p \in w_1\} \subseteq w_2.$$

In terms of Brandom's definition of the necessity operator, that is to say that PW^1_{Inc} is accessible by PW^2_{Inc} if and only if for any $p \in PW^2_{Inc}$ there is a subset $X \subseteq PW^1_{Inc}$ such that $X \cup p \notin Inc$[13]. Formally,

(Compossibility) $w_2 R w_1$ iff $\forall p(w_2 \vDash p \Rightarrow \exists X(X \subseteq w_1 \wedge X \cup p \notin Inc))$

As Peregrin notices, in *IS* this amounts to treat *accessibility* as a second-level weaker compatibility: while any two possible worlds are incompatible as a whole, it might well be that any piece of the one is compatible with *some* piece of the other. This definition of accessibility very aptly fits with Brandom's own treatment of modal operators. We can simply adapt this idea here by saying that something is incompatible with *necessarily p* if and only if any possible world which contains *p* is *compossible* with a possible world which contains *not p*. Formally,

(PW-□I) $X \in I(\Box p)$ iff $\forall w_1(w_1 \vDash X \Rightarrow \exists w_2(w_2 R w_1 \wedge w_2 \nvDash p))$

Notice that *compossibility* inherits all the properties of compatibility, in particular, for what concerns us here, it is *reflexive* and *symmetrical*. Thus, it is easy to show that *IS* with (PW-□I) validates *T*-axiom and *B*-axiom[14]. Instead *S4*-axiom fails because, in general, compossibility is *not transitive*.

[13]Notice that version (iii) of the introduction of necessity is adopted here. This is acceptable now since with the accessibility relation we gain another parameter to play with in order to evaluate compatibility and avoid the collapse of modality.

[14]For the detailed proofs I refer to Göcke et al. (2008).

By mimicking Peregrin (2009)'s labels, I'll call this semantics which implants kripkean framework within Brandom's *IS*, *Extended Incompatibility Semantics* (*EIS*).

4.2 What it means to Kripke *IS*

In this last section I want to claim that the application of Kripke's relational semantics to *IS* doesn't pull any rabbit out of a hat, rather it simply makes explicit some features of modality that remain implicit in *IS* after all. Does that mean that Kripke's framework provide a better *semantic* metavocabulary for incompatibility? Let's see.

Before we even begin with the analysis, it is crucial to ask whether, even with this kripkean implant, modality collapses in *EIS*. The answer is "no". First, *EIS* doesn't verify *S5*-axiom, and that is enough to prevent the collapse. Second, *EIS* doesn't even verify the converses of the browerian axioms. These results were expected: the relation of compossibility produces that second-level compatibility that blocks *Persistence*. This however might raise serious worries about the fulfillment of Brandom's purposes as stated in Claim 1. Thus one may wonder whether the implant of possible worlds, while convenient from a logical point of view, is a step back from the expressive results of *IS* itself. The intended benefit of *IS* would have been the possibility to deploy a directly modal notion of entailment and substitute the metaphysically laden semantic primitive of *truth in a world* with the pragmatically entrenched one of incompatibility. If instead it would be shown that accessibility is nonetheless required to obtain the same expressive results of Kripkean modal logic, then *IS* would need two primitives rather than the one of the standard kripkean framework and its value would quickly get lost. In other words one may wonder whether the indirect path of *compossibility* amounts to declare the failure of the Brandom's project with *IS* after all. But the answer, again, is "no". To see why, it is enough to get clear of what "directly" means in Brandom's Claim 1: while the middle step through possible worlds' vocabulary complicates the elaboration of the practices required to deploy *EIS*, the pragmatic metavocabulary of *incompatibility* is still *sufficient* to express them.

But if this is true, then the expressive advantage of *EIS* over Kripke's relational semantics is patent. Let me try to illustrate. In the previous section it was claimed that *EIS*'s sort of modality is the browerian one of system **B**. Does that mean that according to *EIS*, or in general according to Brandom, system **B** represent the *real* modality? This question might be tricky but the answer negative. *EIS*, as a formal semantics, is a semantic modal metavocabulary to make explicit normative contents implicit in linguistic practices. Kripke's relational semantics for modal logic is a similar metavocabulary. The decisive advantage of *EIS* over Kripke's relational semantics is that it is based on an independent normative analysis of linguistic practices, which provides the pragmatic metavocabulary to express it. In fact, even if *EIS* can't vindicate *incompatibility* as a *technically direct* modal notion, the expressively direct connection with such a normative analysis remains. This advantage pays back not only because it cuts off metaphysical issues about possible worlds – which is not a faint result, by the way –, but also because it puts some normative flesh on the algebraic bones of the accessibility relation. So, is **B** the *real* modality? In Kripke's relational semantics the answer is: "Well, let me check if *accessibility* is reflexive and symmetric but not transitive." How can you tell that? In *EIS* the answer is: "Well, let me check if *compossibility* is reflexive and symmetric but not transitive." How can you tell that? Look at normative linguistic practices.

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