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# On Meta-knowledge and Truth

**Abstract.** The paper deals with the problem of logical adequacy of language knowledge with cognition of reality. A logical explication of the concept of language knowledge conceived of as a kind of codified knowledge is taken into account in the paper. Formal considerations regarding the notions of meta-knowledge (logical knowledge about language knowledge) and truth are developed in the spirit of some ideas presented in the author's earlier papers (1991, 1998, 2001a,b, 2007a,b,c) treating about the notions of *meaning*, *denotation* and *truthfulness* of well-formed expressions (*wfes*) of any given categorial language. Three aspects connected with knowledge codified in language are considered, including: 1) syntax and two kinds of semantics: *intensional* and *extensional*, 2) three kinds of non-standard language models and 3) three notions of truthfulness of *wfes*. Adequacy of language knowledge to cognitive objects is understood as an agreement of truthfulness of sentences in these three models.

*Keywords*: Meta-knowledge, categorial syntax, meaning, denotation, categorial semantics, nonstandard models, truthfulness, language knowledge adequacy.

## Introduction

It is commonly realized that the term 'knowledge' is ambiguous. Speaking about knowledge, we disregard psychological knowledge offered through unit cognition, although it is from knowledge of that sort that verbal knowledge codified by means of language arose. Knowledge will be understood as an inter-subjective knowledge preserved in language, where it is formed and transferred to others in cognitive-communicative acts. Representation of this knowledge is regarded as language knowledge.

For our purposes, in this paper we will consider three aspects of language knowledge: one syntactic and two semantic ones: *intensional* and *extensional*. The main aim of the paper is to answer the following well-known, classical philosophical problem:

`When is our language knowledge in agreement with our cognition of reality?`

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In this paper, the problem is considered from a logical and mathematical perspective and is called: *the problem of logical adequacy of language knowledge*. We will consider it as:

1) an adequacy of syntax and two kinds of semantics,
2) concord between syntactic forms of language expressions and their two correlates: meanings and denotations, and
3) an agreement of three notions of truth: one syntactic and two semantic ones.

The main ideas of our approach to meta-knowledge (logical knowledge about language knowledge) and truthfulness of sentences in which knowledge is encoded will be outlined in Section 1. In Section 2 we will give the main assumptions of a formal-logical theory of syntax and semantics which are the basis for theoretical considerations, and in Section 3 we will define three notions of truthfulness of sentences. The paper ends with Section 4 containing a formulation of a general condition for adequacy of language knowledge with regard to these notions.

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The paper is a result of many years of research conducted by the author and a summary of results obtained earlier [47–58]. The synthetic character of the article provides a strong motivation for the conceptual apparatus introduced further. The apparatus employs some formal-logical and mathematical tools. The synthesis being produced does not always allow detailed, verbal descriptions of particular formal fragments of the paper; nor can it allow for development of some formal parts. The author does, however, believe that the principal ideas and considerations in the paper will be clear to the reader.

## 1. Ideas

The notion of meta-knowledge is connected with the relationships defined by the triad: language-cognition-reality (see *Figure 1*).

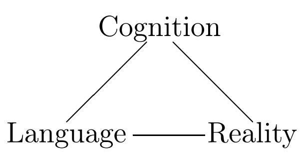



*Figure 1.*

Three different aspects, representing cognitively independent factors, are taken into account at constituting any language $L$ as a tool of communication in which knowledge is formed and transmitted. They are: syntactic, semantic and pragmatic factors.

Reliability of cognition of reality by means of language $L$ and truthfulness of its sentences are given by an agreement of syntactic and two kinds of (*intensional* and *extensional*) semantic knowledge, which correspond to three levels of knowledge about the components of the triad (cf. Wybraniec-Skardowska 2007c).

According to *Figure 1*, following Frege [17], Husserl [25] and other modern followers of *gramatica speculativa*, the meta-knowledge is the *knowledge referring to three realities* (*spaces*):

1. *language reality* $\boldsymbol{S}$ (the set of all *well-formed expressions* of $L$), in which results of cognitive activities such as concepts and propositions are expressed,
2. *conceptual reality* $\boldsymbol{C}$, in which products of cognition of ordinary reality such as logical concepts and logical propositions (*meanings* of language expressions) are considered, and
3. *ontological reality* $\boldsymbol{O}$ which contains objects of cognition, among others, *denotations* of language expressions.

Applying the terms: 'lanuage reality', 'conceptual reality' and 'ontological reality' we aim at distinguishing some models of language $L$ which are necessary to define three different notions of truthfulness of its sentences. Thus, we depart from the classical notion of 'Reality' as an object of cognitive research. In particular, speaking further about indexation reality $\boldsymbol{I}$, we mean certain metalinguistic space of objects (indices) serving the purpose of indication of categories of expressions of $\boldsymbol{S}$, categories of conceptual objects of $\boldsymbol{C}$ and ontological categories of objects of $\boldsymbol{O}$. The reality $\boldsymbol{I}$ forms categorial skeleton of language, conceptual and ontological realities.

Theoretical considerations are based on:

- ***syntax*** – describing ***language reality S*** related to $L$,
  and two kinds of semantics:
- ***intensional*** (**conceptual**) **semantics** – comprising the relationship between $\boldsymbol{S}$ and cognition – describing ***conceptual reality C***, and
- ***extensional*** (***denotational***) ***semantics*** – describing the relationships between $L$ and ordinary reality – ***ontological reality O*** to which the language refers (see Wybraniec-Skardowska 1991, 1998, 2007a, b, c).

The theoretical considerations take into account the **adequacy of the syntax and two kinds of semantics** of language $L$.

The ***language reality S*** is described by a theory of categorial syntax and the ***conceptual*** and ***ontological realities*** by its expansion to a theory of categorial semantics in which we can consider three kinds of ***models*** *of* $L$:

- one syntactic
  and
- two semantic (***intensional*** and ***extensional***).

For these models we can define three notions of ***truthfulness***:

- one syntactic
  and
- two semantic employing the notion of *meaning* (*intension*) and the notion of *denotation* (*extension*), respectively.

## 2. Main Assumptions of the Theory of Syntax and Semantics

### 2.1. Categorial Syntax and Categorial Semantics

Any syntactically characterized language $L$ is fixed if the *set* $\boldsymbol{S}$ *of all well-formed expressions* (briefly *wfes*) is determined. $L$ is given here on the *type*-level, where all *wfes* of $\boldsymbol{S}$ are treated as expression-*types*, i.e. some classes of concrete, material, physical, *identifiable* expression-*tokens* used in definite linguistic-situational contexts. Hence, *wefs* of $\boldsymbol{S}$ are here abstract ideal syntactic units of $L$[1].

Language $L$ can be exactly defined as a *categorial language*, i.e. language in which *wfes* are generated by a *categorial grammar* whose idea goes back to Ajdukiewicz (1935) and Polish tradition, and has a very long history[2]. Language $L$ at the same time may be regarded as a linguistic scheme of

[1] Let us note that the differentiation *token-type* for linguistic objects originates from Charles Sanders Peirce (1931-1935). A formal theory of syntax based on this distinction is given in [49] and [51].

[2] The notion of categorial grammar originated from Ajdukiewicz (1935, 1960) was shaped by Bar-Hillel (1950, 1953, 1964). It was constructed under influence of Leśniewski's theory of semantic (syntactic) categories in his protothetics and ontology systems (1929, 1930), under Husserl's ideas of pure grammar (1900-1901), and under the influence of Russell's theory of logical types. The notion was considered by many authors: Lambek (1958, 1961), Montague (1970, 1974), Cresswell (1973, 1977), Buszkowski (1988, 1989), Marciszewski (1988), Simons (1989) and others. In this paper language $L$ is generated by the so-called classical categorial grammar, the notion introduced and explicated by Buszkowski (1988, 1989) and the author (1985, 1989, 1991).

ontological reality $\boldsymbol{O}$, keeping with Frege's ontological canons (1884), and of conceptual reality $\boldsymbol{C}$.

Considerations are formalized on the ground of author's general formal-logical theory of ***categorial syntax*** and ***categorial semantics*** (1985, 1991, 1998, 1999, 2001a,b, 2006).

Every compound expression of $L$ has a ***functor-argument structure*** and both it and its constituents (the main part – the main ***functor*** and its complementary parts – ***arguments of that functor***) have determined:

- the *syntactic*, the *conceptual* and the *ontological categories* defined by the *functions* $\iota_L$, $\iota_C$, $\iota_O$ *of the indications of categorial indices* assigned to them, respectively,
- *meanings* (*intensions*), defined by the *meaning operation* $\mu$,
- *denotations* (*extensions*), defined by the *denotation operation* $\delta$.

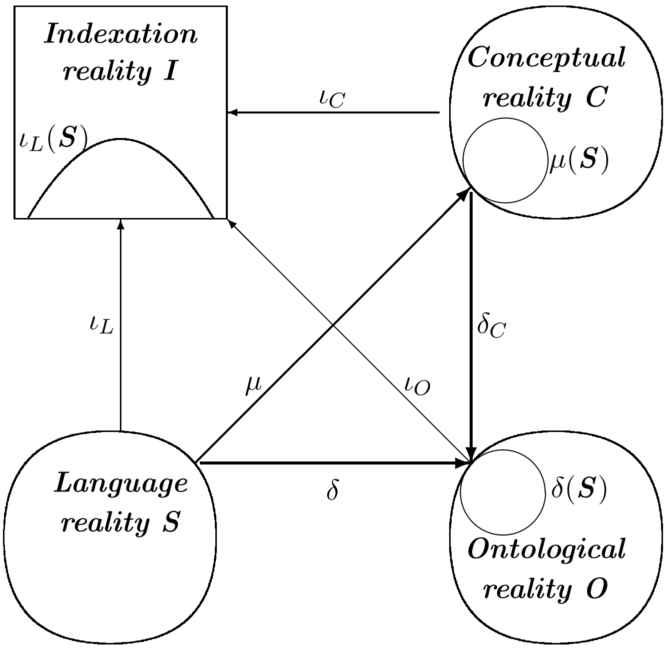



*Figure 2.*

It should be underlined that since *wefs* of $\boldsymbol{S}$ are understood as some abstract syntactic units of $L$, meanings of *wfes* are not their mental signification and denotations of *wfes* are not the same as object references of their concrete, material expression-*tokens* (cf. [57]).

### 2.2. Three referential relationships of *wfes*

We will concentrate on three referential relationships of *wfes of* $\boldsymbol{S}$ to three realities to which *wfes* refer:

- one syntactic: *metalinguistic* relationship connected with the above-mentioned ***indexation reality I***, and
- two semantic: ***conceptual*** (***intensional***) and ***denotational*** (***extensional***) relationships connected with realities $\boldsymbol{C}$ and $\boldsymbol{O}$, respectively. These relationships are illustrated in *Figure 2*.

### 2.3. Categorial indices

The theory of categorial syntax is a theory formalising the basic principles of Leśniewski's theory of semantic (syntactic) categories (1929, 1930) improved by Ajdukiewicz (1935) by introducing *categorial indices* assigned to expressions of language $L$.

*Categorial indices* belong to the indexation reality $\boldsymbol{I}$ and are metalanguage expressions corresponding to expressions of language $L$. They serve to defining the set $\boldsymbol{S}$ of all *wfes* of $L$. The set $\boldsymbol{S}$ is defined according to the *principle* (*SC*) *of syntactic connection* referring to Ajdukiewicz's approach (1935).

(*SC*) is the rule establishing the correspondence between the index of any functor-argument expression of $L$ and indices: the index of its main functor and indices of its successive arguments. It states that:

(*SC*) *The index of the main functor of a functor-argument expression is a complex (functoral) index formed of the index of that expression and the successive indices of the successive arguments of that functor.*

### 2.4. Syntactic Operations

In the theory the *functions:* $\iota_L$, $\iota_C$, $\iota_O$ *of the indications of categorial indices* are certain syntactic operations from reality $\boldsymbol{S}$ or fragments of realties $\boldsymbol{C}$ and $\boldsymbol{O}$ into reality $\boldsymbol{I}$, respectively, i.e.

- the syntactic operation $\iota_L\colon \boldsymbol{S}\to\boldsymbol{I}$,
- the ontological syntactic partial operation $\iota_O\colon \boldsymbol{O}\to\boldsymbol{I}$,
- the conceptual syntactic partial operation $\iota_C\colon \boldsymbol{C}\to\boldsymbol{I}$.

Categorial indices of $\boldsymbol{I}$ also serve to indicate *syntactic, conceptual* (*intensional*) and *ontological* (*denotational*) *categories.* These categories are included in realities $\boldsymbol{S}$, $\boldsymbol{C}$ and $\boldsymbol{O}$, respectively.

If $\xi \in \boldsymbol{I}$ then these categories are defined, respectively, as follows:

(1) $$Cat_\xi = \{e \in \boldsymbol{S} \colon \iota_L(e) = \xi\},$$

(2) $$Con_\xi = \{c \in \boldsymbol{C} \colon \iota_C(c) = \xi\},$$

(3) $$Ont_\xi = \{o \in \boldsymbol{O} \colon \iota_O(o) = \xi\}.$$

In order to define *semantic categories* indicated by categorial indices, and also by conceptual and ontological categories, we have to take into consideration two semantic relationships and use some semantic operations.

### 2.5. Semantic Operations

In the theory of categorial semantics such notions as ***meaning*** and ***denotation*** of a *wfe* of $L$ are considered.

As it was illustrated in *Figure 2* we consider three semantic operations defining ***meanings*** and ***denotations*** *of wfes*:

- the *meaning operation* $\mu \colon \boldsymbol{S} \to \boldsymbol{C}$,
- the *denotation operation* $\delta \colon \boldsymbol{S} \to \boldsymbol{O}$,
- the *conceptual denotation operation* $\delta_C \colon \boldsymbol{C} \to \boldsymbol{O}$.

Let us note that the semantic functions: $\mu$, $\delta$ and $\delta_C$, are defined on abstract objects of $\boldsymbol{S}$ (on *wfes-types*) and of $\boldsymbol{C}$ (on meanings: logical concepts, logical propositions, operations on them, operations on these operations and so on), respectively.

The notion of ***meaning*** as a value of the *meaning operation* $\mu$ on any *wfe* of $L$ is a semantic-pragmatic one and it is defined as a manner of using *wfes* of $L$ by its users in connection to the concept of meaning deriving from L. Wittgenstein (1953) and, independently, from K. Ajdukiewicz (1931, 1934); see Wybraniec-Skardowska (2005, 2007 a,b). So, the notion of meaning of any *wfe* of $L$ is an abstract entity.

We take the standpoint that any *wfe-type* of $\boldsymbol{S}$ has an established meaning which determines its denotation, even if such an expression is understood as an indexical one in natural language (e.g. ‘he’, ‘this’, ‘today’) [3]. In this sense

[3] For example, let us note that the word-*type* ‘today’ understood as a class of all word-*tokens* identifiable with the word-*token*:

today

does not have a fixed meaning, but each of its sub-*types* consisting of *identifiable* tokens (utterances) of the word-*type* ‘today’ formulated on a given day is a meaningful *wfe-type* of English and determines by itself a denotation that is this day.

the approach presented here agrees with the classical Aristotelian position that the context has to be included somehow in the meaning; the manner of using *wfes* of $L$ is in a way built into the meaning (cf. [57]).

The notion of ***meaning*** is differentiated from the notion of ***denotation*** in accordance with the distinction of G. Frege (1892) *Sinn* and *Bedeutung* and R. Carnap's distinction *intension-extension* (1947).

The *denotation operation* $\delta$ is defined as the composition of the operation $\mu$ and the *operation* $\delta_C$ *of conceptual denotation*, i.e.

$$(\delta_C) \qquad \delta(e) = \delta_C(\mu(e)) \quad \textit{for any } e \in \boldsymbol{S}.$$

So, we assume that denotation of the *wfe* $e$ is determined by its meaning $\mu(e)$ and it is the value of the function $\delta_C$ of conceptual denotation for $\mu(e)$. Hence, we can state that:

*If two wfes have the same meaning then they have the same denotation.*

Formally:

FACT 1. $\mu(e) = \mu(e') \Rightarrow \delta(e) = \delta(e'), \qquad \textit{for any } e, e' \in \boldsymbol{S}.$

It is well-known that the converse implication does not hold. So, the operation $\delta_C$ shows that something can differ ***meaning*** from ***denotation***.

### 2.6. Knowledge and Cognitive Objects

The image $\mu(\boldsymbol{S})$ of $\boldsymbol{S}$ determined by the meaning operation $\mu$ is a fragment of conceptual reality $\boldsymbol{C}$ and includes all meanings of *wfes* of language $L$, so all components of ***knowledge*** (logical notions, logical propositions and operations between them, operations on the latter, and so on) and can be regarded as ***knowledge*** of relatively stable users of $L$ about reality $\boldsymbol{O}$, codified by means of *wfes* of $L$.

The image $\delta(\boldsymbol{S})$ of $\boldsymbol{S}$ determined by the denotation operation $\delta$ is a fragment of ontological reality $\boldsymbol{O}$ and includes all denotations of *wfes* of language $L$, so all ***objects of cognition*** of $\boldsymbol{O}$ (things, states of things and operations between them) in cognitive-communicative process of cognition of reality $\boldsymbol{O}$ by relatively stable users of $L$.

We differentiate two kinds of semantic categories: intensional and extensional.

$$(4) \qquad Int_\xi = \{e \in \boldsymbol{S} \colon \mu(e) \in Con_\xi\}.$$

$$(5) \qquad Ext_\xi = \{e \in \boldsymbol{S} \colon \delta(e) \in Ont_\xi\}.$$

So, intensional categories consist of all *wfes* whose meanings belong to suitable conceptual categories, while extensional categories consist of all *wfes* whose denotations belong to suitable conceptual categories.

Adequacy of syntax and semantics required the syntactic and semantic agreement of *wfes* of $L$.

## 2.7. The principles of categorial agreement

In accordance with Frege's-Husserl's-Leśniewski's and Suszko's understanding of the *adequacy of syntax and semantics of language* $L$, syntactic and semantic (*intensional and extensional*) categories with the same index should be the same (see Frege, 1879, 1892; Husserl, 1900-1901; Leśniewski, 1929, 1930; Suszko, 1958, 1960, 1964, 1968).

This correspondence of the ***categorial agreement*** (denoted by $(CA1)$ and $(CA2)$) – is here postulated by means of categorial indices that are the tool of coordination of language expressions and by two kinds of references that are assigned to them:

$(CA1)$ $Cat_\xi = Int_\xi$.

$(CA2)$ $Cat_\xi = Ext_\xi$.

From (1)–(5) and $(CA1)$, $(CA2)$ we get the following variants of the principles:

For any *wfe* $e$

$(C'A1)$ $e \in Cat_\xi$ iff $\mu(e) \in Con_\xi$.

$(C'A2)$ $e \in Cat_\xi$ iff $\delta(e) \in Ont_\xi$.

$(CA3)$ $\iota_L(e) = \iota_C(\mu(e)) = \iota_O(\delta(e))$.

The condition $(C'A2)$ is called the ***principle of categorial agreement*** and it is a formal notation the principle originated by Suszko (1958, 1960, 1964; cf. also Stanosz and Nowaczyk 1976).

So, according to innovative Frege's ideas, the problem of adequacy of syntax and semantics of $L$ is solved if:

*Well formed expressions of L belonging to the same syntactic category correspond with their denotations, and more generally – with their two kinds of references (meanings and denotations) that are assigned to them, which belong to the same ontological, and more generally – to the same conceptual and ontological category.*

## 2.8. Algebraic structures of categorial language and its correlates

The essence of the approach proposed here is considering functors of language expressions of $L$ as mathematical functions mapping some language expressions of $\boldsymbol{S}$ into language expressions of $\boldsymbol{S}$ and as functions which correspond to some set-theoretical functions on extralinguistic objects – indices, meanings and denotations of arguments of these functors.

All functors of $L$ create the set $\boldsymbol{F}$ included in $\boldsymbol{S}$.

The systems:

$$\boldsymbol{L} = \langle \boldsymbol{S}, \boldsymbol{F} \rangle \quad \text{and} \quad \iota_L(\boldsymbol{L}) = \langle \iota_L(\boldsymbol{S}), \iota_L(\boldsymbol{F}) \rangle$$

are treated as some syntactic algebraic structures, while the systems:

$$\mu(\boldsymbol{L}) = \langle \mu(\boldsymbol{S}), \mu(\boldsymbol{F}) \rangle \quad \text{and} \quad \delta(\boldsymbol{L}) = \langle \delta(\boldsymbol{S}), \delta(\boldsymbol{F}) \rangle$$

can be treated as some semantic algebras.

All these algebras are partial algebras[4].

The functors of $\boldsymbol{F}$ differ from other, basic expressions of $\boldsymbol{S}$ in that they have indices formed from simpler ones.

If $e$ is a complex functor-argument *wfe* with the index $a$ and its main functor is $f \in \boldsymbol{F}$ and its successive arguments are $e1, e2, \ldots, en$ with indices $a_1$, $a_2$, $\ldots$, $a_n$, respectively, then the index $b$ of $f$ belonging to the set $\iota_L(\boldsymbol{F})$ is a *functoral* (complex) index formed from the index $a$ and indices: $a_1$, $a_2$, $\ldots$, $a_n$ of its successive arguments.

The index $b$ of the functor $f$ can be noted as the quasi-fraction:

$$\iota_L(f) = b = a/a_1a_2 \ldots a_n = \iota_L(e)/\iota_L(e1)\iota_L(e2) \ldots \iota_L(en).$$

We will show that indices, meanings and denotations of functors of the set $\boldsymbol{F}$ are algebraic, partial functions defined on images $\iota_L(\boldsymbol{S})$, $\mu(\boldsymbol{S})$, $\delta(\boldsymbol{S})$ of the set $\boldsymbol{S}$, respectively.

First we will note that in accordance with the principle $(SC)$ the main functor $f$ of $e$ can be treated as a set-theoretical function satisfying the following formula:

$(Catf)$ $f \in Cat_{a/a_1a_2\ldots a_n}$ iff

$(f)$ $f\colon Cat_{a_1} \times Cat_{a_2} \times \cdots \times Cat_{a_n} \to Cat_a \;\&\; e = f(e1, e2, ..., en) \;\&$

$(\iota)$ $\iota_L(f)\colon \{(\iota_L(e1), \iota_L(e2), \ldots, \iota_L(en))\} \to \{\iota_L(e)\} \;\&$

$(PC1)$ $\iota_L(e) = \iota_L(f(e1, e2, \ldots, en)) = \iota_L(f)(\iota_L(e1), \iota_L(e2), \ldots, \iota_L(en)).$

[4]Ideas about the algebraisation of language can already be found in Leibniz's papers. We can also find the algebraic approach to issues connected with syntax, semantics and compositionality in Montague's 'Universal Grammar' (1970) and in the papers of van Benthem (1980, 1981, 1984, 1986), Janssen (1996), Hendriks (2000). The difference between their approaches and the approach which we shall present here lies in the fact that carriers of the so-called *syntactic* and *semantic algebras* discussed in this paper include functors or, respectively, their suitable correlates, i.e. their $\iota_L-$ or some other semantic-function images. Simple functors and their suitable $\iota_L-$, $\mu-$ or $\delta-$ images are partial operations of these algebras. They are set-theoretical functions determining these operations.

On the basis of the principles of categorial agreement we can state that semantic correlates of the functor $f$ of the expression $e$ are set-theoretical functions too, and deduce that they satisfy the following conditions:

($\mathit{Conf}$) $\mu(f) \in \mathit{Con}_{a/a_1a_2\ldots a_n}$ iff

($\mu$) $\mu(f)\colon \mathit{Con}_{a_1} \times \mathit{Con}_{a_2} \times \cdots \times \mathit{Con}_{a_n} \to \mathit{Con}_a$ &

($\mathit{PC2}$) $\mu(e) = \mu(f(e1, e2, \ldots, en)) = \mu(f)(\mu(e1), \mu(e2), \ldots, \mu(en))$;

($\mathit{Ontf}$) $\delta(f) \in \mathit{Ont}_{a/a_1a_2\ldots a_n}$ iff

($\delta$) $\delta(f)\colon \mathit{Ont}_{a_1} \times \mathit{Ont}_{a_2} \times \cdots \times \mathit{Ont}_{a_n} \to \mathit{Ont}_a$ &

($\mathit{PC3}$) $\delta(e) = \delta(f(e1, e2, \ldots, en)) = \delta(f)(\delta(e1), \delta(e2), \ldots, \delta(en))$.

## 2.9. Compositionality

The conditions ($\mathit{PC1}$), ($\mathit{PC2}$) and ($\mathit{PC3}$) are called the principles of compositionality of syntactic forms, meaning and denotation, respectively (cf. Partee et al. 1990; Janssen 1996, 2001; Hodges 1996, 1998, 2001). They have the following scheme of *compositionality* ($\mathit{Ch}$) for the function $h$ representing:

1) the function $\iota_L$, 2) the operation $\mu$ and 3) the operation $\delta$:

($\mathit{Ch}$) $h(e) = h(f(e1, e2, , en)) = h(f)((h(e1), h(e2), \ldots, h(en))$.

The scheme ($\mathit{Ch}$) says that: *1) the index, 2) the meaning and 3) the denotation of the main functor of the functor-argument expression e is a function defined on 1) indices, 2) meanings and 3) denotations of successive arguments of this functor.*

The suitable variants of compositionality are some requirement of homomorphisms between the mentioned partial algebras:

$$\boldsymbol{L} = \langle \boldsymbol{S}, \boldsymbol{F} \rangle \quad \xrightarrow[\iota_L]{hom} \quad \iota_L(\boldsymbol{L}) = \langle \iota_L(\boldsymbol{S}), \iota_L(\boldsymbol{F}) \rangle,$$

$$\boldsymbol{L} = \langle \boldsymbol{S}, \boldsymbol{F} \rangle \quad \xrightarrow[\mu]{hom} \quad \mu(\boldsymbol{L}) = \langle \mu(\boldsymbol{S}), \mu(\boldsymbol{F}) \rangle,$$

$$\boldsymbol{L} = \langle \boldsymbol{S}, \boldsymbol{F} \rangle \quad \xrightarrow[\delta]{hom} \quad \delta(\boldsymbol{L}) = \langle \delta(\boldsymbol{S}), \delta(\boldsymbol{F}) \rangle.$$

## 2.10. Concord between syntactic forms and their correlates

On the level of metatheory, it is possible to show the agreement between syntactic structures of *wfes* of the language reality $\boldsymbol{S}$ and their correlates in the conceptual reality $\boldsymbol{C}$ and in the ontological reality $\boldsymbol{O}$.

As *wfes* have *function-argument form*: all the functors (all their correlates) precede their arguments (correlates of their arguments as appropriate).

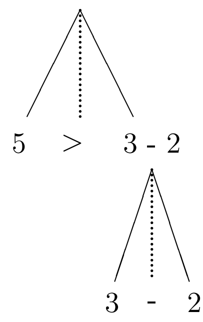
Then the algebraic approach to language expressions corresponds to the tree method.

Example. Let us consider two *wfes* of language of arithmetic:

$$\text{a. } 5 > 3 - 2 \quad \text{and} \quad \text{b. } 3 - 2 > -1.$$

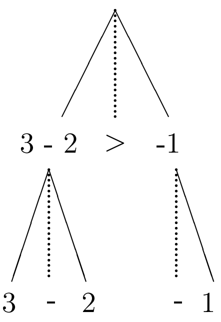
First we present parenthetical recordings a'. and b' for a. and b. and diagrams of trees meant to explicate them. Diagrams $T$a and $T$b show a natural, phrasal, natural *functorial analysis* of these expressions. The dotted lines show functors.

$T$a. $5 > 3$ - $2$




a'. $(5) \;>((3) \;-\; (2))$

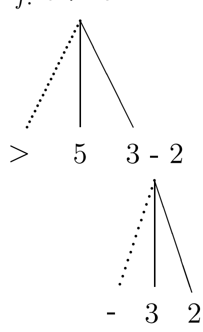
$T$b. $3$ - $2 > $ -$1$




b'. $((3) \;-(2)) > (-\,(1))$

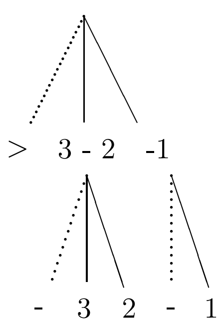
Appropriate function-argument recordings $\mathrm{a}_{f.}$ and $\mathrm{b}_{f.}$ and diagrams of trees: $T\mathrm{a}_{f.}$, $T\mathrm{b}_{f.}$ show "functional analysis" of expressions a., b. in Ajdukiewicz's prefix notation.

$T\mathrm{a}_{f.}$ $5 > 3$ - $2$




$\mathrm{a}_{f.}$ $>\;\; (5, -\,(3,\;\; 2))$

$T\mathrm{b}_{f.}$ $3$ - $2 >$ -$1$




$\mathrm{b}_{f.}$ $> (-\;\; (3,\; 2), -\;\; (1))$

Let us note that the functorial analysis of a. and b. given here provides functional-argument expressions $\mathrm{a}_{f.}$ and $\mathrm{b}_{f.}$. It is unambiguously determined due to the semantic (denotational and intensional) functions of the signs '>' and '−': the first is a sign of two-argument operation on numbers, the second one in a. denotes a two-argument number operation, while in b. it also denotes a one-argument operation.[5] The mentioned signs, as functors, and thus as functions on signs of numbers, have as many arguments as their semantic correlates have.

Comparison of tree method and algebraic method based on compositionality shows one-to-one correspondence of constituents of any *wfe* of $L$ with correlates in order to form and transmit our knowledge on reality $\boldsymbol{O}$ represented by $L$ (see diagrams of trees $T\mathrm{b}_f$. and $T\mathrm{b}$. of the expression b. and corresponding to them diagrams of trees of categorial indices $T_{\iota_L}(\mathrm{b}_f)$ and $T_{\iota_L}(\mathrm{b})$ of b.).

Let us note that from the principle (*PC1*) and in accordance with the principle (*SC*), for $e = f(e_1, e_2, \ldots, e_n) \in S$ and $\iota_L(e) = a, \iota_L(f) = b$, $\iota_L(e_i) = a_i \, (i = 1, 2, \ldots, n)$, we obtain, on the basis of our theory, the following reconstruction of the *rule of cancellation of indices* used by Ajdukiewicz (1935):

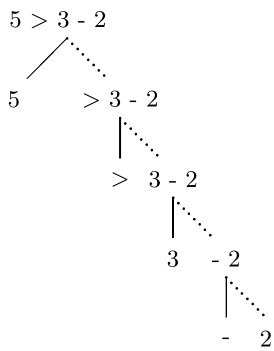
(*rc*) $\quad a/a_1a_2 \ldots a_n(a_1, a_2, \ldots, a_n) = a.$

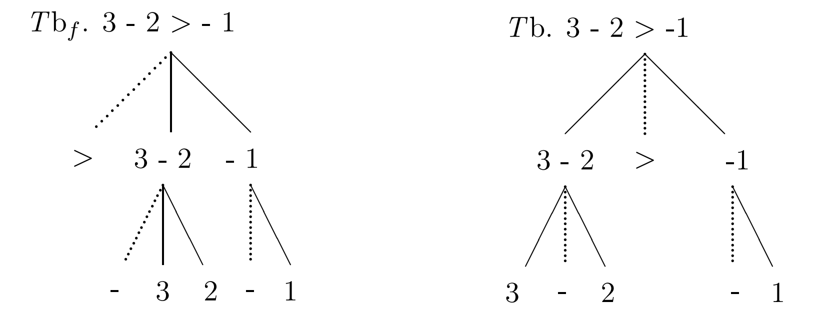
[5]Unambiguous "functorial analysis" is a feature of the languages of formal sciences. In relation to natural languages the analysis depends on linguistic intuition and often allows for a variety of possibilities (see e.g. Marciszewski 1981).

In this conception we do not state that "functoral analysis" of linguistic expressions must be determined unambiguously but we accept the statement that it is connected with expressions of a determined functor-argument structure.

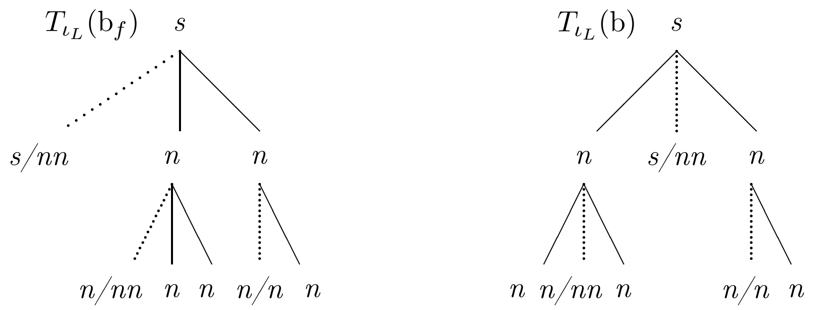
Let us also note that traditional phrasal linguistic analysis, formalized by Chomsky (1957) in his grammars of phrasal structures, takes into consideration grammatical phrasal analysis and only two parts of functoral parsing of expressions.

Let us consider, for instance, the expression a. and its functorial analysis illustrated by a derivation tree in Chomsky's sense.






b$_f$. > (- (3, 2), - (1)) b'. ((3) - (2)) > (- (1))




$\iota_L(\mathrm{b}_f)$. $s/nn(n/nn(n,n),(n/n,n))$ $\iota_L(\mathrm{b}')$. $((n)n/nn(n))s/nn(n/n(n))$

The agreement between syntactic forms of *wfes* and their correlates is very important whenever we want to know whether our knowledge represented in language $L$ is adequate to our cognition of reality.

Let $e$ is any *wfe* of $L$ and $C_e$ is the *set of all constituents* of $e$. The concord between syntactic structure of $e$ and its correlates is possible because the tree $\boldsymbol{T}(C_e)$ of constituents of $e$ is isomorphic with trees:

$\boldsymbol{T}(\iota_L(C_e))$ of indices of all constituents of $e$,
$\boldsymbol{T}(\mu(C_e))$ of all meanings of all constituents of $e$ and
$\boldsymbol{T}(\delta(C_e))$ of all denotations of those constituents.

These trees are formally defined as graphs by means of the set $C_e$ and corresponding to it sets: $\iota_L(C_e)$, $\mu(C_e)$ and $\delta(C_e)$ of all constituents that are appropriate correlates of constituents of $e$. So,

$$\boldsymbol{T}(C_e) = \langle C_e, \approx\!> \rangle,$$
$$\boldsymbol{T}(h(C_e)) = \langle h(C_e), \approx\!>_h \rangle \quad \text{for } h = \iota_L, \mu, \delta,$$

where $\approx>$ is a linear ordering *relation of an earlier syntactic position in e* defined by means of the *relation* $\rightarrow$ *of syntactical subordination* (see Ajdukiewicz, 1960); $\approx\!>_h$ is $h$-image of the relation $\approx\!>$.

The mentioned isomorphisms of tree graphs are established by the functions $h$ mapping every constituent of $e$ in $C_e$ that occupies in $e$ a fixed syntactic position (place) onto its $h$-correlate that occupies in $h(e)$ the same position (place).

All notions introduced in this part can be defined formally.

Definition 1 (constituent of an expression $e$).

a. $t \in C_e^0 \Leftrightarrow e = t$.
A constituent of the order zero of a given *wfe* $e$ is equal to the expression.

b. $t \in C_e^1 \Leftrightarrow$
$\exists_{n \geq 1} \exists_{f, t_0, t_1, \ldots, t_n \in S} \, (e = f(t_0, t_1, \ldots, t_n) \wedge \exists_{0 \leq j \leq n} (t = f \vee t = t_j))$.
*$t$ is a constituent of the first order of a given expression $e$* iff $e$ is a functor-argument expression and $t$ is equal to the main functor of the expression or to one of its arguments.

c. $k > 0 \Rightarrow \left( t \in C_e^{k+1} \Leftrightarrow \exists_{r \in C_e^k} \, t \in C_r^1 \right)$.
A *constituent of $k$+1-th order* of $e$, where $k > 0$, is a constituent of the first order of a constituent of *$k$-th* order of $e$.

d. $t \in C_e \Leftrightarrow \exists_n \, t \in C_e^n$.
A *constituent of a given expression* is a constituent of a finite order of that expression.

Definition 2 (constituent of $e$ with the fixed syntactic position).

a. $t \in C_e^{(j_1)} \Leftrightarrow e$ is a functor-argument expression $\wedge$ $t$ is the $j_1$-*th* constituent of $C_e^1$.

b. $k > 0 \Rightarrow \Big( t \in C_e^{(j_1, j_2, \ldots, j_{k+1})} \Leftrightarrow$ $t$ is equal to the $j_{k+1}$-*th* constituent of a constituent of the set $C_e^{(j_1, j_2, \ldots, j_k)}) \Big)$.

Definition 3 (relation of an earlier syntactic position in $e$).

a. $s \rightarrow s'$ iff $\underset{k,j}{\exists} \, s \in C_e^k \wedge s' \in C_e^j \wedge k \leq j$.

b. $s \approx\!> s'$ iff $s \rightarrow s' \vee$
$\left( \exists_{j_1, j_2, \ldots, j_k, n, m} \, (s \in C_e^{(j_1, j_2, \ldots, j_k, n)} \wedge s' \in C_e^{(j_1, j_2, \ldots, j_k, m)} \wedge n < m) \right)$.

*$s$ has in $e$ an earlier syntactic position than $s'$* iff $s, s'$ are constituents of $e$ and either $s$ has the order lesser than or equal to the order of $s'$ or $s$ and $s'$ are simultaneously constituents of some part $e'$ of $e$ with the same order $k > 0$ but $s$ has in $e'$ the position $n$ while $s'$ – the position $m > n$.

On the basis of the principles of compositionality it is easy to prove

THEOREM 1. *For* $h = \iota_L, \mu, \delta$

$$\boldsymbol{T}(C_e) = \langle C_e, \approx > \rangle \quad \xrightarrow[isom]{h} \quad \boldsymbol{T}(h(C_e)) = \langle h(C_e), \approx >_h \rangle.$$

Uniformity of algebraic approach and tree approach allows to compare knowledge reference to three kinds of realities and to take into account the problem of its adequacy. It is connected with the problem of truthfulness of sentences of $L$ representing knowledge.

## 3. Three notions of truthfulness

### 3.1. Three kinds of models of language and the notion of truth

We have treated the language reality $\boldsymbol{S}$ and corresponding to it $\iota_L-, \mu-$ and $\delta-$ images of $\boldsymbol{S}$, i.e. $\iota_L(\boldsymbol{S})$ – a fragment of the indexation reality $\boldsymbol{I}$, $\mu(S)$ – a fragment of the conceptual reality $\boldsymbol{C}$ and $\delta(S)$ – a fragment of the ontological reality $\boldsymbol{O}$ as some algebraic structures, as some partial algebras.

Let us distinguish in $\boldsymbol{S}$ the set of all *sentences* of $L$. Models of $L$ are non-standard ***models***. They are the three mentioned algebraic structures (partial algebras) given as homomorphic images of algebraic structure $\boldsymbol{L}=\langle \boldsymbol{S}, \boldsymbol{F} \rangle$ of language $L$:

$$\begin{aligned} \iota_L(\boldsymbol{L}) &= \langle \iota_L(\boldsymbol{S}), \iota_L(\boldsymbol{F}) \rangle, \\ \mu(\boldsymbol{L}) &= \langle \mu(\boldsymbol{S}), \mu(\boldsymbol{F}) \rangle, \\ \delta(\boldsymbol{L}) &= \langle \delta(\boldsymbol{S}), \delta(\boldsymbol{F}) \rangle. \end{aligned}$$

They are determined by the fragments $\iota_L(\boldsymbol{S})$, $\mu(\boldsymbol{S})$ and $\delta(\boldsymbol{S})$ of the realities $\boldsymbol{I}$, $\boldsymbol{C}$ and $\boldsymbol{O}$, respectively. The first of them $\iota_L(\boldsymbol{L})$ is *syntactic* one and the next two are *semantic*: $\mu(\boldsymbol{L})$ – *intensional* and $\delta(\boldsymbol{L})$ – *extensional.*

### 3.2. Three notions of truthfulness

For the three models $\iota_L(\boldsymbol{L})$, $\mu(\boldsymbol{L})$ and $\delta(\boldsymbol{L})$ of the language $L$ we define three notions of truthfulness. For this purpose we distinguish three nonempty subsets $T\iota_L, T\mu, T\delta$ of realities $\boldsymbol{I}$, $\boldsymbol{C}$ and $\boldsymbol{O}$, respectively:

- $T\iota_L$ consists only of the index of any true sentences,
- $T\mu$ consists of all meanings of sentences of $L$ that are true logical propositions and
- $T\delta$ consists of all denotations of sentences of $L$ that are states of affairs that obtain.

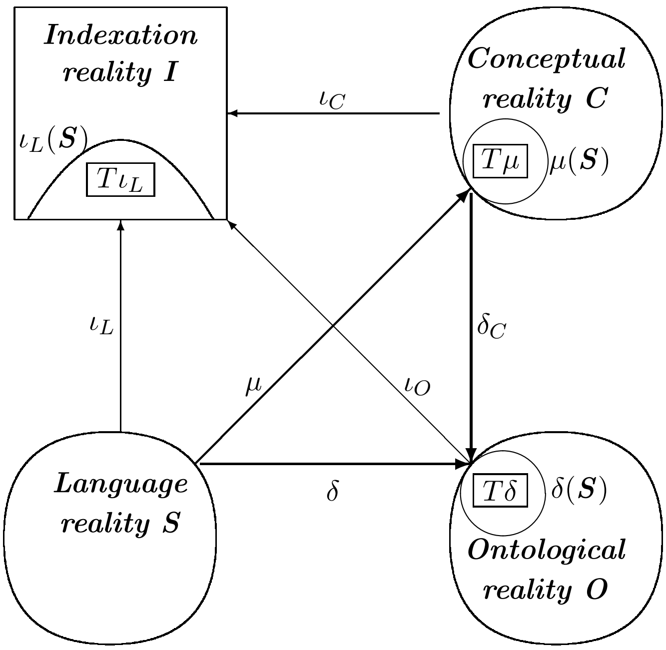



*Figure 3.*

All of the three definitions of a true sentence in one of the models $\iota_L(\boldsymbol{L})$, $\mu(\boldsymbol{L})$ and $\delta(\boldsymbol{L})$ of $L$ are analogous and are substitutions of the following definition scheme:

***Scheme of definitions*** (truthfulness): *For* $h = \iota, \mu$ and $\delta$

*The sentence* $e$ *is true in the model* $h(\boldsymbol{L})$ *iff* $h(e) \in Th$.

The definitions of a true sentence correspond to the ***truth value principle*** (cf. W. Hodges 1996). An expansion of the principle could be formulated as follows:

*The correlate of a sentence* (i.e. its index, meaning or denotation, respectively) *determines whether or not it is true in a suitable model.*

The three definitions of a true sentence can be given as follows:

- $e$ *is syntactically true* iff $\iota_L(e) \in T\iota_L$,
- $e$ *is intensionally true* iff $\mu(e) \in T\mu$,
- $e$ *is extensionally true* iff $\delta(e) \in T\delta$.

From the above scheme of definitions of truthfulness of sentences we can easily get the following scheme of theorems:

Metatheorem 1. *For* $h = \iota_L, \mu, \delta$

*If* $e, e'$ *are sentences and* $h(e) = h(e')$*, then*

$e$ *is true in* $h(\boldsymbol{L})$ *iff* $e'$ *is true in* $h(\boldsymbol{L})$.

Metatheorm 1 is the scheme of the following three theorems our formal theory:
1) *If we have two sentences with the same index then they are syntactically true* iff *they have the same truth value in the syntactic model, i.e. their index is the index of all true sentences*,
2) *If two sentences have the same meanings then they are intensionally true* iff *they have the same truth value in the intensional model, i.e. their meanings are true logical propositions*,
3) *If two sentences have the same denotation then they have the same truth value in the extensional model, i.e. their denotations are the states of affairs that obtain.*

### 3.3. Reliability of cognition of reality

**The main purpose of cognition** is aiming at an agreement of truthfulness of sentences that are results of cognition in all three models: $\iota_L(\boldsymbol{L})$, $\mu(\boldsymbol{L})$ and $\delta(\boldsymbol{L})$ (cf. *Figure 3*).

Let us note that if a sentence is true in the extensional model $\delta(\boldsymbol{L})$ then it does not have to be true in the remaining models. So, in particular, a deductive knowledge that is included in the conceptual reality $\boldsymbol{C}$ cannot be in agreement with knowledge referring to the ontological reality $\boldsymbol{O}$. There can be true sentences in $\delta(\boldsymbol{L})$ that are not deduced from the knowledge accepted earlier and cannot be true in the intensional model $\mu(\boldsymbol{L})$.

Considerations outlined in this paper point to a new aspect of the importance of Gödel's Incompleteness Theorem (1931): it explains why language cognition of reality illustrated by *Figure 3* can be incomplete.

Justification of these statements requires introducing some new notions.

### 3.4. Operations of replacement

The most important theorems which follow from the principles of compositionality (*PC1*), (*PC2*) and (*PC3*) use the syntactic notion of the three-argument *operation* $\pi$ *of replacement of a constituent of a given wfe of* $L$. The operation $\pi$ is defined by means of the *operation* $\pi^n$ *of replacement of the constituents of n-th order.* The expressions $e' = \pi(p', p, e)$ and $e' = \pi^n(p', p, e)$ are read: *the expression* $e'$ *is a result of replacement of the constituent* $p$*,* *respectively, the constituent $p$ of $n$-th order, of $e$ by the expression $p'$.* The definition of the operation $\pi^n$ is inductive (see Wybraniec-Skardowska, 1991).

DEFINITION 4 (operation of replacement). *Let $e, e', p, p' \in \boldsymbol{S}$. Then*

a. $e' = \pi^0(p', p, e)$ iff $p = e$ *and* $p' = e'$,

b. $e' = \pi^1(p', p, e)$ iff $e$ *and* $e'$ *are some functor-argument expressions of the set $\boldsymbol{S}$ with the same number of arguments of their main functors and differ from one another only by the same syntactic position when in $e$ occurs the constituent $p$ and in $e'$ occurs the constituent $p'$,*

c. $e' = \pi^{k+1}(p', p, e)$ iff $\exists_{q,q' \in \boldsymbol{S}} \big(e' = \pi^k(q', q, e) \ \ \& \ \ q' = \pi^1(p', p, q)\big)$,

d. $e' = \pi(p', p, e)$ iff $\exists_{n \geq 0}(e' = \pi^n(p', p, e))$.

We can define the operations of replacement $h(\pi)$ for the correlates *wfes* of $\boldsymbol{S}$ $(h = \iota_L, \mu, \delta)$ in an analogous manner.

### 3.5. The most important theorems

In this part we will give some theorems of our deductive, formal-logical theory of syntax and semantics. They are logical consequences of the above-given definitions and principles of compositionality formulated earlier.

It is easy to justify three principles of compositionality with respect to the operation $\pi$. They are a substitution of the following metatheorem:

METATHEOREM 2 (compositionality with respect to $\pi$). *For $h = \iota_L, \mu, \delta$*

$$(PC_\pi) \qquad h(\pi(p', p, e)) = h(\pi)(h(p'), h(p), h(e)).$$

We can also easily state that the theorems that we get from the next scheme are valid:

METATHEOREM 3 (homomorphisms of replacement systems). *For $h = \iota_L, \mu, \delta$*

$$\langle \mathbf{S}, \pi, T \rangle \quad \xrightarrow[hom]{h} \quad \langle h(\mathbf{S}), h(\pi), h(T) \rangle,$$

*where $T$ is the set of all true sentences of $L$.*

We can postulate that $T\iota_L = \iota_L(T), T\mu = \mu(T)$ and $T\delta = \delta(T)$.

Now, we will present theorems called replacement theorems.

FACT 2. *For $h = \iota_L, \mu, \delta$*
*If $e = f(e_1, e_2, \ldots, e_n), e' = f'(e'_1, e'_2, \ldots, e'_n) \in \boldsymbol{S}$*
*then $h(e) = h(e')$ iff $h(f) = h(f')$ and $h(e_i) = h(e'_i)$ for any $i = 1, \ldots, n$.*

By means of Fact 2 we can easily obtain the one *fundamental syntactic replacement theorem* and two *fundamental semantic replacement theorems* which are the suitable substitutions of the following metatheorem of our theory:

METATHEOREM 4 (replacement principles). *For* $h = \iota_L, \mu, \delta$

*If* $e, e' \in \boldsymbol{S}$ *and* $e' = \pi(p', p, e)$ *then* $(h(p) = h(p')$ *iff* $h(e) = h(e'))$.

So: *Two expressions have the same correlate (the same categorial index – the syntactic category, the same meanings, the same denotation, respectively) if and only if by the replacement of one of them by the other in any wfe of L we obtain a wfe of L which has the same correlate (the same categorial index – the same syntactic category, the same meaning, the same denotation, respectively), as the expression from which it was derived.*

COROLLARY 1. *If* $e, e' \in \boldsymbol{S}$ *and* $e' = \pi(p', p, e)$, *then*

$$\begin{aligned} \exists_\zeta(p, p' \in Cat_\zeta) \quad &\textit{iff} \quad \exists_\zeta(e, e' \in Cat_\zeta), \\ \exists_\zeta(p, p' \in Con_\zeta) \quad &\textit{iff} \quad \exists_\zeta(e, e' \in Con_\zeta), \\ \exists_\zeta(p, p' \in Ont_\zeta) \quad &\textit{iff} \quad \exists_\zeta(e, e' \in Ont_\zeta). \end{aligned}$$

The next theorems are connected with the true value principles.

METATHEOREM 5 (referring to the truth value principles). *For* $h = \iota, \mu, \delta$
*If* $e, e'$ *are sentences of* $L$ *and* $e' = \pi(p', p', e)$ *and* $h(p) = h(p')$, *then*
$e$ *is true in* $h(\boldsymbol{L})$ *iff* $e'$ *is true in* $h(\boldsymbol{L})$ .

The three theorems that we get from the above metatheorem together state that:

*Replacing in any sentence its constituent by an expression which has the same correlate (the same index, the same meaning, the same denotation, respectively), never alters the truth value of the replaced sentence in the given syntactic, intensional, extensional, respectively, model.*

If we accept the following axiom:

AXIOM: *If* $e$ *is a sentence and* $\mu(e) \in T\mu$, *then* $\delta(e) \in T\delta$ ,

then from the above metatheorem, for $h = \mu$, we get:

FACT 3. *If* $e, e'$ *are sentences,* $e' = \pi(p', p', e)$ *and* $\mu(p) = \mu(p')$, *then*
*if* $e$ *is true in* $\mu(\boldsymbol{L})$ *then* $e'$ *is true in* $\delta(\boldsymbol{L})$.

So: *Replacing in any true sentence in the intensional model its constituent by an expression that has the same meaning, we get a sentence which is true in the extensional model.*

STRONGER METATHEOREM (referring to truth value principles)
*For* $h = \iota, \mu, \delta$.
*If* $e, e'$ *are sentences and* $e' = \pi(p', p, e)$, *then*
$h(p) = h(p')$ *iff* ($e$ *is true in* $h(\boldsymbol{L})$ *iff* $e'$ *is true in* $h(\boldsymbol{L})$).

The recognition of the above metatheorem requires accepting the three axioms which are connected with Leibniz's principles (cf. Gerhard 1890, p. 280, Janssen 1996, p.463) and have the same scheme:

SCHEME OF LEIBNIZ'S AXIOMS *For* $h = \iota, \mu, \delta$.
*If* $e, e'$ *are sentences and* $e' = \pi(p', p, e)$, *then*
*if* ($e$ *is true in* $h(\boldsymbol{L})$ *iff* $e'$ *is true in* $h(\boldsymbol{L})$) *then* $h(p) = h(p')$.

*Leibniz's Axioms* together state that:

*If replacing in any sentence its constituent* $p$ *by an expression* $p'$ *never alters the truth value of the replaced sentence in the syntactic, in the intensional, in the extensional, respectively, model, then* $p$ *and* $p'$ *have the same categorial index, the same meaning, the same denotation, respectively.*

Three theorems which follow from *Stronger Metatheorem* (referring to truth value principles) together say that (cf. Hodges 1996):

*Two expressions of the language L have the same correlates (the same categorial index – syntactic category or form, the same meaning – intension, the same denotation – extension, respectively), if and only if replacing one of them by another in any sentence never alters the truth value of the replaced sentence in the syntactic, intensional, extensional, respectively, model of the language L.*

## 4. Final remarks

- We have tried to give a description of meta-knowledge in connection with three references of logical knowledge to:
  - language,
  - conceptual reality and
  - ontological reality.
- Thanks to it we could define three kinds of models of language and three kinds of truthfulness in these models.
- These models are not standard models; in particular the notion of truth does not employ the notions of satisfaction and valuation of variables used for formalized languages.

- Adequacy of language knowledge to cognitive objects of reality is understood as an agreement of truthfulness in these three models.
- It is possible to give a generalization of the notion of meta-knowledge in communication systems in order to apply it to knowledge in text systems but the solution of this problem requires more time and is solved by my co-worker Edward Bryniarski.

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