

Extended Modal Dimensionalism

Martin Vacek¹

Received: 14 December 2015 / Accepted: 19 May 2016
© Springer Science+Business Media Dordrecht 2016

Abstract Modal dimensionalism (MD) is realism about spaces, times and worlds—metaphysical indices that make objects spatial, temporal and modal, respectively, and that play the role of alethic relativizers, i.e. items to which matters of truth are relativized. This paper examines several arguments against MD and shows that MD offers a feasible way to understand modal discourse.

1 Introduction

According to modal realism (hereafter MR),¹ possible worlds are concrete, spatio-temporal systems. Extended modal realism (hereafter EMR) goes even further and claims that possible and impossible worlds are concrete spatio-temporal systems. However, it is commonly held that if we are willing to accept impossible worlds, they must not be conceived as real spatio-temporal systems. If we suppose that there are impossible worlds that make certain inconsistencies true and if we suppose that those worlds represent those inconsistencies in a genuine way, then we are committed to the reality of true inconsistencies. In *Worlds and Individuals, Possible and Otherwise*, Takashi Yagisawa presents a different version of modal realism called modal dimensionalism (hereafter MD),² which ‘shares a certain theoretical conviction with David Lewis’s classical modal realist theory and also, superficially, with anti-Lewisian actualist theories’ (Yagisawa 2010, p. 1; see also Yagisawa 2002).

MD is a metaphysical thesis according to which spatial, temporal and modal indices make objects spatial, temporal and modal, respectively. In contrast to modal realism, modal dimensionalism allows for impossible worlds—entities that have proved their utility in various branches of philosophy. In this chapter, I argue that MD, despite having ersatzist features, offers a feasible option when it comes to impossible worlds. In

¹Unless stated otherwise, MR refers to Lewis (1986) as the orthodox version of modal realism.

²Unless stated otherwise, MD refers to Yagisawa (2010) as the orthodox version of modal dimensionalism.

✉ Martin Vacek
martinvacekphilosophy@gmail.com

¹ Institute of Philosophy, Slovak Academy of Sciences, Bratislava, Slovakia

particular, I will try to show how one can be a ‘quasi’ modal realist and still have a consistent ontology of possible and impossible worlds. First, I compare MR and MD. Second, I discuss arguments against MD as a theory of possible worlds. Finally, I respond to objections against MD as a theory of impossible worlds.

2 MD vs. MR

According to MD, worlds are not spatio-temporally closed universes. Nor are they abstract representations of the way the world could have been. Rather, worlds are defined as modal indices that are (but do not exist)³ along the world’s temporal and spatial indices. What MR describes as the actual world, or the universe, MD calls the actual world stage of the universe. The universe is the comprehensive subject of possibility and necessity (Yagisawa 2010, p. 44). Possible worlds are neither concrete nor abstract, and whether they are objects at all is an open question: ‘[I] take moments of time to be real but I am non-committal about whether they are non-concrete objects of some kind. If they are, I will be happy to accept that worlds in my sense are also non-concrete objects of some kind’ (Yagisawa 2010, p. 179, fn. 7). One way or the other, there is a plurality of worlds—a plurality of different world stages of the same universe. Modal space contains many concrete objects, all of which are modal parts of one and the same universe. Some of them may be unified by spatio-temporal relatedness, some may be unified by another relation, and others might not be unified by any relation other than that of being part of the universe and whatever that requires (Yagisawa 2010, p. 45).

Pivotal claims of MD are summarized in the following passage:

Ordinary individuals typically exist at many metaphysical indices of each of the three kinds: time, space, and world. The airplane at Heathrow exists at many temporal points and periods, many spatial points and extended regions, and many possible worlds. Suppose that it exists at different times t_1 and t_2 (for example, yesterday and today), different spatial regions r_1 and r_2 (for example, where its fuselage is and where its wings are), and different possible worlds w_1 and w_2 . The fuselage is not identical with the airplane, but the airplane is where the fuselage is, at r_1 . The airplane is also where the wings are, at r_2 , even though the wings are not identical with the airplane. The airplane is at r_1 but not wholly at r_1 , and at r_2 but not wholly at r_2 . The fuselage is the plane’s spatial part, and so are the wings. The airplane is at every spatial region where some spatial part of the airplane is. Similarly for times and worlds (Yagisawa 2010, p. 53).

All concrete objects are temporal objects because they exist in time. It is times that make them temporal objects. Times make concrete objects temporal by being such that those objects exist at them. To exist at a temporal index means to be a temporal object. Temporal indices exist independent of the events that occur in them. Since temporal indices make objects temporal objects, it is temporal indices that do the representation of temporality. Concrete objects are spatial objects too. They exist in space: a

³ For Yagisawa, reality is fundamental and monadic, whereas existence is domain relative.

metaphysical index responsible for their being spatial objects. Like temporal indices, spatial indices are primitive, although the way they make the object spatial is non-trivial. All concrete objects are also modal objects since they exist at different worlds. It is modal indices that are responsible for concrete objects' being modal. Worlds are makers of modal objects, although they themselves are not modal objects. In sum, MD divides reality into concrete individuals on the one side and metaphysical indices on the other.

The second crucial feature of MD rests on taking the analogy between spatial, temporal and modal talk seriously. I existed yesterday, I exist today, and I will (probably) exist tomorrow. Also, I exist where my arms are, where my legs are, where my head is, etc. Analogously, then, I am a PhD student, although I could be a football player. That means that I exist in the actual circumstances as well as in merely possible ones. The truth of the above sentences depends on temporal, spatial and modal indices to which we relativize their truth. More generally, temporal, spatial and modal indices are alethic relativizers—i.e. those items to which matters of truth are relativized. Ontologically, they are on a par, although metaphysically primitive, temporal, spatial and modal indices are further explicable in more graspable terms.

In terms of modality, MD says that, in addition to spatial and temporal dimensions, the universe also spreads out in a modal dimension. The actual world is one of many indices, namely the one at which the universe is the way it actually is. 'Actually' is to be understood in the very same manner as 'now' is—i.e. ostensively. Importantly though, it does not refer to a concrete mereological sum of individuals any more than 'now' refers to something concrete. 'Actual', 'here' and 'now' refer to metaphysical indices.⁴ Generalizing the idea, MD positions the temporal tense in parallel with a spatial and a modal tense, thus introducing a correspondence between tenses on the one hand and metaphysical indices on the other. Times are distinct from events that happen in them. Also, spatial points (or extended regions) are not identical with what occupies them, and worlds are not identical with MR's universes. Rather, they are points in modal space in a way that is analogous to how a temporal instant is a point in temporal space and a spatial point is a point in (at least three-dimensional) spatial space (Yagisawa 2010, p. 27).

A further departure from MR is the analogy between trans-temporal and trans-world identification. Lewis sympathized with the former (which holds that we persist through time by having distinct temporal stages at different times) but formulated several objections to the latter (Lewis 1986, pp. 218–219). MD, on the other hand, accepts such an analogy and posits the so-called 'closest-continuer' relation holding between modal parts of a single individual. The relation is defined along the following lines:

A modal stage x at a possible world w_1 and a modal stage y at a different possible world w_2 are parts of the same modally extended object of a kind K if and only if there is a chain of possible worlds from w_1 to w_2 ordered by the overall similarity relation such that x and some modal stage, $x + 1$, at the next world in the chain are sufficiently similar to each other in relevant respects and are each other's closest continuer at their respective worlds, $x + 1$ and some modal stage, $x + 2$, at the next

⁴ Taking the analogy seriously, Yagisawa introduces a new word, *mau*, combining the temporal 'now' with its (m)odal counterpart (cf. Yagisawa 2002, p. 29).

world in the chain are sufficiently similar to each other in relevant respects and are each other's closest continuer at their respective worlds,..., and $x+n$ and some modal stage, $x+n+1=y$, at the next world, w_2 , in the chain are sufficiently similar to each other in relevant respects and are each other's closest continuer at their respective worlds, where the sufficient similarity, relevant respects, and closeness are relative to the kind K (Yagisawa 2010, p. 109).

This, in a nutshell, is the closest-continuer relation operating on the modal stages.

Furthermore, MD, unlike MR, accepts impossible worlds. Again, such worlds are neither concrete nor abstract but are as real as possible worlds. Some impossible worlds are worlds at which logical impossibilities are obtained. These are logically impossible worlds. Some impossible worlds are worlds at which metaphysical impossibilities are obtained. In addition, there are impossible individuals. They do not exist in the domain of possible objects. They exist in the domain of metaphysically impossible objects; yet given the closest-continuer relation between world stages, they also exist at some possible worlds (by having stages that exist at those worlds). A world is an impossible relative to another world if the two worlds inhabit different logical spaces.⁵ One logical space comprises all and only logically possible worlds, while the other logical space comprises all and only logically impossible worlds: '[t]he logic that governs every world in logical space is the same. So, alternative logics characterize alternative logical spaces' (Yagisawa 2010, p. 184).

However, instead of full modal reductionism, MD prefers soft reductionism, according to which (a) temporal, spatial and modal indices are taken to be metaphysically simple and (b) the at-a-worldness relation is primitive. These features of the theory place it somewhere between modal realism and actualism and, more importantly, between two modes of representation: genuine and ersatz.

Next, MD, as opposed to MR, introduces modal tense. To be a modal tenses might mean several things, but two aspects are especially important for my purposes. First, modal-tensed propositions belong to the everyday terminology used by metaphysicians, and modally tensing verbs are something philosophers already engaged in. Second, though, the modal tense approach is not merely a conceptual approach that would systematize our use of certain words. The modal tense approach is a metaphysical approach. It takes for granted that 'some important modal facts are modal-tensed facts, i.e. they can be designated or quantified over adequately only in modal-tensed terms, and that no important modal facts are modaltenseless facts, i.e. none of them are such that they can be designated or quantified over adequately only in modal-tenseless terms' (Yagisawa 2010, p. 73).

The working hypothesis of MD is the existence of the actuality tense, the mere-metaphysical-possibility tense, the metaphysical-impossibility tense and, rather controversially, a tense specifically for predications concerning modal space at large, subscripts 'a', 'p', 'i' and 'm', respectively. For example, 'Martin is actually a philosopher', where 'actually' is read non-rigidly⁶ and is expressed as 'Martin is_a a philosopher'. By contrast, predications concerning merely possible situations require the mere possibility tense. So, 'Martin could have been a football player' is expressed as 'Martin

⁵ I will postpone the discussion about the metaphysics of logical spaces to section 9.

⁶ For the difference between rigid and non-rigid uses of the actuality tense, see Yagisawa (2010, pp. 76–77).

is_p a football player'. Finally, the metaphysical impossibility tense helps us to articulate goings-on at some impossible worlds. Thus, 'it is impossible that Martin is a PhD student and not a PhD student at i_1 ' gets a modal-tensed interpretation by using 'i' in the tense: Martin is a PhD student and is not a PhD student at i_1 . Suppose now that the above sentences are uttered at the actual world. Then, the modal tensing approach interprets 'Martin is_p a football player' as true as it is evaluated at the actual index, the actual world, if and only if Martin is a football player at some non-actual possible index and Martin is_i a PhD student and not a PhD student at some non-actual impossible world.

To sum up, MD presents a two-categorical ontology: it posits concrete world stages and, in addition, metaphysical indices. The universe and its parts have spatial, temporal and modal dimensions as they extend in time, space and worlds. The universe and its parts have temporal, spatial and modal stages, and it is those stages that *represent* temporality, spatiality and, most importantly for our purposes, modality. One thing that MD has going for it is thus that the analogous theoretical roles of temporal, spatial and modal indices pave the way for a systematic and unified metaphysics of modality.

3 Some Problems for MD

In this section, I discuss a dilemma directed against MD's account of possible worlds. The dilemma comes from Jago (2012) and aims to show that MD's metaphysics is incoherent. I outline both horns of the dilemma and argue that MD as a whole, rather than the fragment used in the dilemma, is not subject to it.

4 Possible Worlds

To repeat, MD claims that ordinary things (including the whole universe) are trans-temporal, trans-spatial and trans-modal sums. That means that the *de dicto* possibility schema has the form of P:

P: It is possible that P if and only if there is a universe modal stage, u_1 , such that P holds at u_1 ,

and *de re* modality is expressed as P*:

P*: An object has a modal property, G, if and only if it has a world stage that has G as one of its properties.

Note that P and P* 'localize' possibilities to modal indices. Although worlds are neither concrete (as MR takes them to be) nor abstract (as modal ersatzists would insist), world stages of concrete objects are concrete. They are possible localizers in the same way in which times are temporal localizers and spaces are spatial localizers.

Jago's starting point is that MD's analysis of *de re* modality goes against widely accepted opinions about contingent matters. The objection is as follows: any possible entity whatsoever that might be F has a world stage that is F necessarily or contingently. Although I am actually a PhD student, I could have been a football player ($\neg P_m \wedge \diamond P_m$). According to MD, there is a possible world, w , at which my football player stage exists. Call this stage f_w . Now, the question is: is f_w necessarily or contingently a football player? Suppose it is the former. This entails that I am possibly necessarily a football

player, $\diamond\Box Pb$. Since ' $\diamond\Box A \rightarrow \Box A$ ' is a theorem of modal logic, it follows that I am necessarily a football player. But I am actually not a football player.

Put more formally the following:

1. Martin is not a football player, but he could have been a football player ($\sim F_M$ and $\diamond F_M$).
2. For some possible world w , some Martin's w -stage is a football player [1 by MD].
3. One of Martin's w -stages is necessarily a football player.
4. Martin is possibly necessarily a football player ($\diamond\Box F_M$) [2, Martin has a necessarily football player world stage].
5. Martin is a football player (F_M) [4, given the S5 theorem $\diamond\Box A \rightarrow A$].
6. Martin is not a football player and Martin is a football player ($\sim F_M$ and F_M) [1 and 5] (contradiction).

The second horn of the dilemma takes my football player stage to be a football player only contingently. By definition, my world stages do not have their own stages, and their modal profile is explained by counterpart relations they bear to other world stages. Suppose now that two world stages, m and m^* , are worldmates if and only if they both exist at the same world index. Now consider any two world stages, n and n^* , for which it holds that they are not worldmates ($\sim W_{nn^*}$). Next, what is true is possibly true, and so it is possible that they are not worldmates: $\diamond\sim W_{nn^*}$. Since, according to MD, world stages do not have stages by means of which we analyse their modal profile, the analysis must proceed via a counterpart relation. Thus, there is a world u and there are u stages nu and n^*u such that nu is a counterpart of n and n^*u is a counterpart of n^* and $\sim W_{nu\ n^*u}$. But since nu and n^*u are both u stages, by definition, they are worldmates: $W_{nu\ n^*u}$ (contradiction).

The formal representation is as follows:

1. Martin is not a football player, but he could have been a football player ($\sim F_M$ and $\diamond F_M$).
2. For some possible world w , some of Martin's w -stage is a football player [1 by MD].
- 3*. Martin's w -stage is contingently a football player.
 - 4*. For some world w , some of Martin's w -stage is a football player and is a counterpart of Martin's @-stage.
 - 5*. Martin's w -stage and Martin's @-stage are not worldmates ($\sim W_{\text{orldmate}M_wM_{@}}$).
 - 6*. Possibly, Martin's w -stage and Martin's @-stage are not worldmates ($\diamond\sim W_{\text{orldmate}M_wM_{@}}$) [from $A \rightarrow \diamond A$].
7. For some world v and some v -stages $M1_v$ and $M2_v$, $M1_v$ is a counterpart of Martin's w -stage, $M2_v$ is a counterpart of Martin's @-stage, and $M1_v$ and $M2_v$ are not worldmates ($\sim W_{\text{orldmate}M1_vM2_v}$) [6*].
8. $M1_v$ and $M2_v$ are worldmates and $M1_v$ and $M2_v$ are not worldmates (contradiction).

5 Diagnoses

Having outlined the structure of both arguments, I now proceed to possible ways of responding to them. In particular, I identify three aspects of MD's theoretical apparatus which, taken together, avoid the undesired consequences. The first has already been

identified in Yagisawa (2015) and uses the strategy of rephrasing the necessity horn of the dilemma by the use of modal tensing. The second reply develops an amodalist response and applies to both parts of the dilemma. Roughly, it takes seriously the idea of forbidding predication of any modal property of any world stage. Finally, I outline a third, more speculative view according to which there is a complex hierarchy of modal spaces. This feature enables MD to meet the contingency horn of the objection.

6 The Necessity Horn

Let me start with the necessity horn of the dilemma. To see how it would work, a brief summary will be helpful. According to MD, Martin could have been a football player if and only if one of his world stages is a football player. If Martin's world stage is a football player necessarily, we get a true proposition: 'Martin is possibly necessarily a football player'. Provided we accept ' $\diamond\Box A \rightarrow \Box A$ ' as a theorem, Martin is necessarily a football player.

I believe that the modal tensing approach can help here. In particular, if we disambiguate the argument in MD's light, the contradiction disappears. The disambiguation would take the following form:

1. Martin is_a not a football player, but he could have been (is_p) a football player ($\sim F_m$ and $\diamond F_m$).
2. For some possible world w , some Martin's w -stage is_p a football player [from 1 by MD].
3. Suppose that Martin's w -stage is *necessarily* a football player.
4. Martin is_a possibly necessarily a football player ($\diamond\Box F_m$) [from 2, Martin has a world stage that necessarily a football player].
5. Martin is_p a football player (F_m) [from 4, given ' $\diamond\Box A \rightarrow A$ '].
6. Martin is_a not a football player and Martin is_p a football player.

Notice, that (6) is a perfectly consistent proposition now. The reason to think so is that occurrences of 'is' in the argument are modally tensed, depending on an index they refer to. The situation is analogous to temporal tensing. 'Martin *was* a child' is understood by MD as 'Martin (simpliciter) has a time-stage, m_t , which is a child'. It is always the case that M_t is a child since M_t is a temporal stage of Martin. Martin now, M_n , is not a child, and it is always the case that M_n is not a child. Does this fact make Martin an inconsistent object? No, since M_t and M_n are Martin's different temporal stages.

Modal tensing in fact does two things in the argument. Positively, it blocks the contradiction in the way temporal tensing does. Negatively, though, the tensed version of the necessity horn of the argument invalidates the $\diamond\Box A \rightarrow A$ theorem, for we do not get an inference from 'Martin is_a possibly necessarily a football player' to 'Martin is_a a football player'. We do get a modified inference from 'Martin is_a possibly necessarily a football player' to 'Martin is_p a football player'. This, however, only underwrites a feature of MD—namely, that world stages lack a modal profile. Recall that world stages make concrete modal objects. World stages per se are modally unextended objects, although they make modally extended objects modal objects. It is therefore not a modal

predication of modally extended objects that is at stake. Rather, MD forbids predication of any modal property of world stages.⁷ In short, concrete objects are modal, whereas world stages are amodal. And this brings us to the second option for MD: the denial of step (3).

7 Amodalism

Amodalism is a negation of modal generalism; it is the view that every proposition has a modal profile. In terms of MD, modal generalism would be the view that every object, whether modally extended or modally unextended, has a modal profile. That is, for any modally extended and modally unextended object, we can predicate a modal property. Amodalism with respect to world stages denies this. Amodalism says that there are some objects—world stages—that lack modal profiles.

Moreover, it turns out that we have independent reasons to prefer amodalism to modal generalism. One of these is the problem of the possibility of the whole logical space's being otherwise.⁸ For example, consider the possibility that logical space might have included more than n -worlds. If the goal of modal reductionism is to explain ordinary and extraordinary⁹ modal facts in terms of possible worlds and logical space, respectively, neither necessity nor contingency can be attributed to logical space. Put differently, the modal reductionist cannot claim that logical space must be such that it contains n -worlds. Nor can she claim that logical space contains n -worlds only contingently. This is because for modal reductionists, 'modal facts—facts about what must and what might be the case—are ontologically and conceptually posterior to facts about the 'shape of logical space'.¹⁰ If this is so, modal reductionists qua modal generalists cannot analyse modal claims about the whole of logical space.

A similar line of argument applies to modal realism; according to which, worlds are concrete universes, for traditional analysis in terms of possible worlds taken for granted that if something exists, it is also possible that it exists ($p \rightarrow \diamond p$). The fact that Martin could have been a football player is represented by his counterpart, Matrin, who exists in another possible world. Therefore, both Martin and Matrin exist, although it is not possible that they both exist, since they do not inhabit the same possible world. But we still want to be able to talk about a possibility: a mereological sum that consists of Martin and Matrin.

Amodalism, on the other hand, appears to handle both of the above limitations. Divers (1999) quite correctly adds that the distinction between ordinary and extraordinary theorizing depends on one's ontological preferences. If the analysis shows that some individuals have modal profiles while others do not, it is only to be expected that the distinction will play a crucial theoretical role in the theory. The case of MD is not an exception. Modal indices make concrete objects modal objects. An object is a modal object in virtue of having world stages. World stages make an object a modal object: a modally extended sum. To require them to be modal goes against explanatory requirements put on metaphysical explanation.

⁷ Yagisawa (2015, p. 322, fn. 9) thinks that to forbid predication of modal properties of world stages is a radical alternative, although I am not entirely clear on why this ought to be viewed as radical.

⁸ See Yagisawa (1988), Cowling (2011), Divers (1999, 2002), and Jago (2014) for discussion.

⁹ For the distinction between ordinary and extraordinary modalizing, see Divers (1999).

¹⁰ Cf. Cowling (2011, pp. 383–384).

Rather, the amodalism approach stresses the legitimacy of MD when it comes to analysing modality by means of modally unextended objects that lack modal profiles. Such a stance belongs to MD's ideology and is both theoretically justified and methodologically approved.

One worry concerning the modal unextendedness of world stages remains, though. Although amodalism might have some intuitive appeal in certain cases, it is still the case that there is a conflict between modal logic and amodalism. Cowling formulates the argument as follows:

Consider any true proposition, Q . Given the (T)-axiom, we can infer $\diamond Q$ from Q . The (T)-axiom therefore guarantees that any true amodal proposition will have a modal profile by virtue of being possibly true. As a consequence, we seem forced to choose between amodalism and modal logic (or at least any standard modal logic) (Cowling 2011, p. 484).

The leading idea behind the worry is that there is a correspondence between modal logic and possible worlds talk. That means that any limitations of possible worlds talk are reflected in its logical formalizations and vice versa. Put simply, this is the worry that possible worlds theory is subservient to the limited powers of modal logic.

This is not the case, however. The language of boxes and diamonds provides us with formalization of a part of our possible worlds discourse, but that does not mean that the language formalizes every single bit of it. After all, if this language proves a clumsy instrument for talking about modal matters, we do better to follow the resources of MD directly.¹¹ According to this strategy, we restrict the theoretical power of modal logic to modally extended individuals and leave unextended ones outside the expressive resources of standard modal logic.

8 Summary

I thus conclude that the necessity horn of the dilemma can be blocked by modal tensing and by denying that world stages have modal profiles. In the former, we deny the step from (4) to (5). In the latter, we grant to world stages an amodal status and thus deny premise (3).¹² Moreover, such strategies are in accordance with MD's ontological assumptions and present legitimate methodological options.

9 The Contingency Horn

Let us now proceed to the contingency horn. In nutshell, it attacks the position according to which my world stage is a football player only contingently. Put briefly, a modal realistic analysis of contingency introduces a counterpart relation, and the relation as such is incompatible with MD. Yagisawa identifies the structural features of the argument in the following way:

¹¹ Cf. Lewis (1986, pp. 12–13).

¹² The validity of logic is just one part of the problem. The second is about how to build a semantics on amodalism. For a way to meet the worry, see Cowling (2011, pp. 486–491).

...a world-bound object x_1 exists at a possible world w_1 and x_1 is contingently F; so at some possible world w_2 , there is a world-bound object x_2 which bears R to x_1 and which is not F; obviously, $\sim W_{x_1x_2}$ (x_1 and x_2 are not worldmates); hence $\diamond \sim W_{x_1x_2}$; thus, at some possible world, there are y_1 and y_2 —so $W_{y_1y_2}$ —such that y_1 and y_2 bear R to x_1 and x_2 , respectively and $\sim W_{y_1y_2}$; therefore, at some possible world, $W_{y_1y_2}$ and $\sim W_{y_1y_2}$, which is a contradiction (Yagisawa 2015, p. 323).

At least two responses are available to MD. The first response was already mentioned in the previous section: the amodalist's approach to world stages. Put simply, it seems illegitimate to formulate the objection from the modal profile of world stages. World stages make objects modal objects without being modal objects themselves. And a straightforward rejection of (3*) follows from this commitment. The second response is a bit more complex and relies on the hierarchy of modal spaces.

Although Yagisawa does not provide a detailed specification of the notion of 'logical space', he approaches it from several angles. First, a logical space consists of all and only worlds which form an equivalence class under the largest accessibility relation. Second, for any world w , the logical space that includes w includes all and only worlds that are logically accessible from w . Third, within a logical space, any world is logically accessible from (i.e. possible relative to) any world. That means that any world that lies outside a given logical space is not accessible from (possible relative to) any world in that logical space and belongs to a different logical space (Yagisawa 1988, p. 182). Yagisawa (2010) adds a bit more. For example, logical space contains many concrete objects, all of which are modal parts of one and the same universe; the logic that governs every world in logical space is the same, while alternative logics characterize alternative logical spaces. Logical spaces are systematized into a system K, defined in the following way:

For any K (where K stands for a particular kind of possibility),

- (I) K-space is the totality of all K-possible worlds.
- (II) K-space might have been different.
- (III) Possible difference is to be understood in terms of a plurality of alternatives.

The system of K-spaces is hierarchical, complicated and difficult to understand completely. This, however, does not mean that we should give up exploring it. On the contrary, a lack of understanding is an impetus for further investigation.¹³

¹³ Two remarks are in order. MD, even if it postulates possible and impossible individuals, is not a priori committed to primitive modality. Although there is a difference between possibility and impossibility, the difference can be handled non-modally. One way to do this is to analyse any kind of possibility as a restricted modality, while those very restrictions (usually laws) are to be understood non-modally. So, extended modal realism, even if hard to swallow in principle, can be squared with Lewisian reductive ambitions. Second, extended modal realism does not aim to violate our everyday reasoning about actual and possible things. Impossible worlds do not actually exist. They do not exist possibly either, if 'existing possibly' means being restricted to a particular domain. As Yagisawa puts it, '[i]t is certainly impossible for impossibilia to exist under any possible conditions or circumstances. But that does not mean that impossibilia do not exist under any conditions or circumstances whatever. They exist under impossible conditions or circumstances' (Yagisawa 1988, pp. 202–203). It is therefore not the case that MD automatically fails the non-reductive test, and indeed much more should be said about its commonsense test failure. Nonetheless, it still holds that problems regarding representation of logical, metaphysical and mathematical phenomena present strong reason to reject the project. I think, however, that although controversial, extended modal realism might find some resources parallel to or parasitic upon competitive accounts.

The response to the contingency worry thus proceeds as follows:

...the truth condition for ‘ x_1 and x_2 possibly exist as nonworldmates’ is not that at some possible world x_1 and x_2 have counterparts which are not worldmates, but instead that in some modal space x_1 and x_2 have counterparts which are not worldmates. Since in this modal space x_1 and x_2 themselves exist_m— x_1 exists_p at w_1 and x_2 exists_p at w_2 —and x_1 and x_2 are_m not worldmates, this truth condition is satisfied. And no contradiction comes out of it (Yagisawa, 2015, p. 323).

Notably, the ontology in MD goes hand in hand with modal tensing. Recall that the ideological commitments of modal tense proponents are the actuality tense, the mere possibility tense, the impossibility tense, and the modal tense at large. Every ontological postulate finds its tensed interpretation, whether we talk about actuality, possibility, impossibility or extraordinary modal phenomena. This feature makes MD systematic and theoretically appealing.

To summarize, both the necessity and the contingency horns of Jago’s dilemma ignore crucial features of MD: modal tensing, the amodal status of world stages and the iterative hierarchy of modal spaces. To the degree that we appreciate the complexity of MD, these arguments are revealed as either missing their target or as directed against a different position. In the next section, I turn to objections concerning MD’s account of impossible worlds.

10 Impossible Worlds

In the introduction, I pointed out an important difference between MD and MR with regard to the acceptance of impossible worlds. However, Cameron (2010), Jago (2012, 2014, *Advanced modalizing problems*, *Mind*, forthcoming), Kim (2011) (following Lewis 1986), Yagisawa (1988) and Divers (2002) have formulated arguments according to which MD is an inconsistent hypothesis. The dialectic of the argument proceeds from an assumption that there are real impossible worlds as legitimate objects of quantification and thus as existing in the same manner as the actual world. MD is therefore literally committed to the existence of impossible things. This section presents two such arguments and provides several suggestions as to how MD might respond.

To begin with, provided that we accept the impossible talk and its impossible worlds interpretation, (P) and (P*) transform into their impossibilist counterparts, I and I*, respectively:

I: If it is impossible that P, there is an impossible world, i_1 , such that P holds at i_1 ,
and

I*: If it is impossible for an object to have an impossible property, G, it has an impossible world stage that has G.

For instance, it is not possible for me both to be and not to be a philosopher at the same time, both to be and not to be a football player at the same time or to be and not to be a talking donkey at the same time. If this is so, the modal stage strategy requires that there will be stages such as *Martin-is-and-is-not-a-football-player-at- i_1* , *Martin-is-and-is-not-a-pianist-at- i_2* and *Martin-is-and-is-not-a-talking-donkey-at- i_{99}* . But if impossible

worlds are real, there really are the above-mentioned inconsistent stages. And that is a plain contradiction because inconsistent stages turn out to be actual.

The structure of the argument is as follows:

- 1) It is impossible for Martin to be a football player and not to be a football player at i_1 .
- 2) There is a Martin-is-and-is-not-a-football-player world stage.
- 3) Martin is a football player, and it is not the case that Martin is a football player.¹⁴

Kim (2011) proposes a finer-grained argument against MD. It runs along the following lines:

1. Suppose, for *reductio* that some world w is a logically impossible world of the kind I.
2. There is a logical contradiction that is true at w . Let such a contradiction be schematically represented as ‘P and not P’.
3. That is, P and not P, at w .
4. So, P at w and not P at w .
5. If not P at w , then it is not the case that P at w .
6. P at w , and it is not the case that P at w .
7. At the actual world, the following is the case: P at w , and it is not the case that P at w .
8. So the actual world is a world at which a logical contradiction is true.
9. But the actual world is not a world at which a logical contradiction is true.
10. Therefore, (1) is false. That is, no world is a logically impossible world of the kind I (Kim 2011, pp. 297-298).

What is special about Kim’s argument is that it resists the orthodox response to it. The orthodox response, presented, among others, by Lycan (1979, 1994) and Yagisawa (2010), blocks the inconsistency in the actual world by rejecting the steps from (3) to (4), because ‘we should not expect all logically impossible worlds to behave in accordance with all laws of logic. At a logically impossible world, ‘a conjunction might be true without both conjuncts being true’ (Yagisawa 2010, p. 184). Kim’s argument, although weaker, attacks MD on the basis that it posits an impossible world which does not exist according to MD. Given the principle of plenitude, this counts against MD, for the world Kim has in mind follows logical principles, although it does not follow all of them. The world of kind I is a world at which a contradiction is true because both conjuncts are true. The world violates one law of logic—the law of non-contradiction—but still accepts another principle: a conjunction is true if and only if both conjuncts are true. Given such a world, we do get (4) from (3), and Yagisawa’s response fails.

11 Diagnoses and Responses

Both arguments follow Lewis’s ‘no difference between a contradiction within the scope of the modifier and a plain contradiction that has the modifier within it’ denial of

¹⁴ Moreover, the argument runs regardless of whether we take ‘is real’ or ‘exists’ to be primitive (see Jago 2012).

impossible worlds, for impossible worlds, or world stages, are real despite the fact that reality is the most fundamental and ultimate subject of reality. Thus, Jago quite correctly points out that, according to MD, the possibility of there being a Martin-is-a-football-player stage implies that there is a Martin-is-a-football-player stage. By the same reasoning, the impossibility of there being a Martin-is-and-is-not-a-football-player-at- i_1 stage implies that there is a Martin-is-and-is-not-a-football-player-at- i_1 stage.

Jago discusses two independent strategies for meeting the charge. One is to draw a distinction between existence and reality,¹⁵ for existence is still relative to an aggregate while reality is not. That, however, seems only to shift the problem somewhere else rather than to solve it. Whichever direction the distinction goes, it will still be the case that Martin-is-and-is-not-a-football player-at- i_2 is actually true, because reality as well as existence are sufficient for the truth of a contradiction. The other option is to bite the bullet and accept that there are true contradictions. The problem with this strategy is that MD accepts the so-called plenitude of *possibilia*. This principle says that for every possibility, there is a world that makes it happen. Qua species of modal realism, MD is forced to accept not only the plenitude of *possibilia* but also a plenitude principle for *impossibilia*. This means that for any arbitrary false proposition, there is a world that instantiates it.

Apparently, the two possible ways of meeting the worries fail. But they do not exhaust our options. In the next section, I propose two strategies available to MD that, if successful, respond to both Jago's and Kim's challenges. The proposed strategies rely on the features of MD that are ignored in the arguments. The ignored features are (again) modal tensing and the iterative conception of logical spaces.

12 Modal Tensing Again

As I have already stressed, a fair criticism should pay attention to every aspect of the criticized theory. For dialectical purposes, it is important for any criticism not to overlook crucial aspects of the criticized theory. Remember that MD draws a parallel between space, time and modality. Spaces, times and worlds are metaphysical relativizers that make concrete objects spatial, temporal and modal, respectively. This is, however, only one part of the story. The second part of the picture reflects another parallel between time and modality. This is the analogous use of temporal and modal tenses. Additionally, MD proposes a hierarchical embedding structure of alternative modal spaces. A world is impossible according to another possible world in case it belongs to a different modal space. Existence in a modal space has a particular predication concerning modal space at large, subscript 'm'. Finally, MD is not MR. Worlds are not concrete mereological sums. Both possible and impossible worlds are indices or modal regions. This feature puts MD somewhere between MR and modal ersatzism. Namely, neither *de re* nor *de dicto* representation is genuine. Taken together, these three features present resources for MD when it comes to avoiding the triviality threat.

¹⁵ See footnote 3.

The first option relies on the modal tensing strategy. With the distinction between modal tenses in mind, the triviality argument is interpreted such that no contradiction arises. This runs as follows:

- 1) It is_a impossible that Martin is_i and is not a football player at i_1 .
- 2) There is_i a Martin-is-and-is-not-a-football-player stage.
- 3) Martin's i_1 -stage is_i a football player and is not a football player.

C: It is_i (not is_a) the case that Martin is a football player and is not a football player.

Premise (1) states that it is (actually) impossible that Martin is a football player and is not a football player. This means that the impossibility is predicated of a stage of Martin. According to MD, the impossibility tense is introduced in (2). The same applies to (3), resulting in the conclusion that it is still the case that Martin is a football player and is not a football player. However, 'is' in C gets an impossibility tensing instead of its actuality counterpart. Consequently, C is not a contradiction. Notice that the impossibility tense belongs to the basic ideological apparatus of MD, and its proponents are thus fully justified in applying it. Thus, arguments like Jago's share the same deficiency. They ignore the distinction between two different tenses: the actuality tense and the impossibility tense. As long as we disambiguate the two, every impossibility merely exists_i but does not exist_a.

What about Kim's challenge? Recall that we are dealing with a special kind of world here: one that obeys every law of logic except the law of non-contradiction. The negative answer Yagisawa proposes is that the kind of world Kim has in mind does not play any theoretical role in any analysis, and MD does better to deny it.¹⁶ Positively speaking, however, such worlds do not render the tensing analysis inapplicable. The special version of the counterargument simply copes with the answer to Jago's original one:

- 1*. Suppose that some world w is_m a logically impossible world of the kind I.
- 2*. There is_i a logical contradiction that is true at w . Let such a contradiction be schematically represented as 'P and not P'.
- 3*. That is, P and not P, at w .
- 4*. So, P at w , and not P at w .
- 5*. If not P at w , then it is_a not the case that P at w .
- 6*. P at w , and it is_i not the case that P at w .
- 7*. At the actual world, the following is_i the case: P at w , and it is not the case that P at w .
- C*. So the actual world is_a not a world at which a logical contradiction is true.

Interestingly, 1* has a special modal tense attached to it—one that goes beyond the local metaphysical space. Although one might think that such a tense is utterly ad hoc, seen from MD's point of view, it simply fits into and reflects MD's ontological picture. MD introduces an iterative hierarchy of modal spaces. Every modal space contains I-possible worlds only where I stands for a certain modality (be it physical, logical or metaphysical possibility). Indeed, the system of logical spaces (I-spaces, for different Is) is hierarchical and difficult to understand completely.

¹⁶ For Yagisawa's response, see Yagisawa (2011, p. 310).

Note, however, that complete understanding is not a sufficient condition for accepting a theory. Nor does it have to be a necessary condition. Among the relevant theoretical virtues that play a role in choosing between modal metaphysics are explanatory power, consistency, simplicity, elegance, strength, and consistency with what we already know. Unless it is shown that MD as a whole violates one of the virtues and does not bring anything in return to the overall picture, commitment to the plurality of modal spaces appears reasonable. Put differently, insofar as we locate some feature of the theory that makes it in some respect superior to its rivals, why not take the theory seriously? One such application is an ability to block the necessity horn of Jago's dilemma. Jago's argument, if valid, shows that any realistic position falls short in the case of analysis of possibilities pertaining to modal space as a whole.¹⁷ This is because such analyses localize possibilities to single worlds, and the possibility of worldmateship and non-worldmateship receive the single-world analysis. MD, on the other hand, does not confine such (extraordinary) possibilities to a single world. Such modalities are understood via relations between modal spaces. A possibility of non-worldmateship switches from world analysis to modal space analysis.

Considering MD as a whole, it is thus not an ad hoc move to have a plurality of worlds, modal spaces and modal tenses that reflect the ontology. When speaking of actuality, we use the actuality tense. Moving to possibility, we switch to the merely possible tense. Extending possibility must be followed by the impossibility tense. Finally, varieties of modal space as such must be mediated by a unique tense: the modal tense at large.¹⁸ Again, these are ontological and ideological elements of MD's toolbox and should not be attacked separately. To be sure, they can be attacked individually, but the extent to which such arguments make their point remains an open question.

Additionally, there is an uncontroversial piece of pre-theoretical modal knowledge, accessible to human beings, that MD accepts and does not aim to revise. There is a general agreement between MD and its rivals about ordinary modal claims. We agree on what actually exists, what is merely possible and what is impossible. The disagreement comes with the interpretation of the modal discourse where incredulous stares not only play a minor role but also have no business being in the game in the first place. And extraordinary modalizing is one such case.

13 Summary

This closes my defence of MD. So far, I do not claim ultimately to have defeated objections against MD. Rather, I have tried to point out that any critique of it must consider the theory as a complex whole. As the arguments were meant to show, MD taken as a complex thesis blocks the arguments outlined above. The theory's essential components are not limited to spatial, temporal and modal indices and consequent spatial-stage,

¹⁷ Cf. Jago, *Advanced modalizing problems*, *Mind*, forthcoming.

¹⁸ There is yet another option available to MD. I have already pointed out that modal indices, unlike Lewis's worlds, are not concrete. This feature puts it somewhere between MR and modal ersatzism. A version of the latter represents modality not genuinely but, to use Lewis's label, by magic. But as I argued in Vacek (2015), there are ways to meet the challenge from magic. If my responses work, I do not detect a serious reason not to apply them to MD as well.

temporal-stage and modal-stage analyses. MD also includes a modal tensing approach to modality, and as the argument above indicates, this feature is ignored in both the possible and the impossible world challenges. To the extent that we are both modal indicers and modal tensors, the objections can be handled on independent grounds.

Acknowledgments I thank the anonymous referees for their helpful comments. I also thank John Divers, Fredrik Haraldsen, Daniel Nolan, Marián Zouhar and especially Takashi Yagisawa for discussions and comments. My work on this paper was supported by research grant Vega No. 2/0049/16, Fictionalism in Philosophy and Science.

References

- Cameron, R. (2010). Worlds and individuals, possible and otherwise: critical notice. *Analysis*, 70(4), 783–792.
- Cowling, S. (2011). The limits of modality. *The Philosophical Quarterly*, 61(244), 473–495.
- Divers, J. (1999). A genuine realist theory of advanced modalizing. *Mind*, 108, 217–239.
- Divers, J. (2002). *Possible worlds*. London: Routledge.
- Jago, M. (2012). Against Yagisawa’s modal realism. *Analysis*, 73, 10–17.
- Jago, M. (2014). *The impossible: an essay on hyperintensionality*. Oxford: Oxford University Press.
- Kim S. (2011). Understanding Yagisawa’s worlds. *Analytic Philosophy*, 52(4), 293–301.
- Lewis, D. (1986). *On the plurality of worlds*. Oxford: Basil Blackwell.
- Lycan, W. G. (1979). The trouble with possible worlds, selected portions repr. in Lycan (1994): 3–24.
- Lycan, W. G. (1994). *Modality and meaning*. Dordrecht: Kluwer.
- Vacek, M. (2015). Modal realism: yet another hybrid view. *Belgrade Philosophical Annual*, 28, 5–20.
- Yagisawa, T. (1988). Beyond possible worlds. *Philosophical Studies*, 53, 175–204.
- Yagisawa, T. (2002). Primitive worlds. *Acta Analytica*, 17(1), 19–37.
- Yagisawa, T. (2010). *Worlds and individuals, possible and otherwise*. Oxford: Oxford University Press.
- Yagisawa, T. (2011). Modal space exploration: replies to Ballarín, Hayaki, and Kim. *Analytic Philosophy*, 52(4), 302–311.
- Yagisawa, T. (2015). Impossibilia and modally tensed predication. *Acta Analytica*, 30(4), 317–323.