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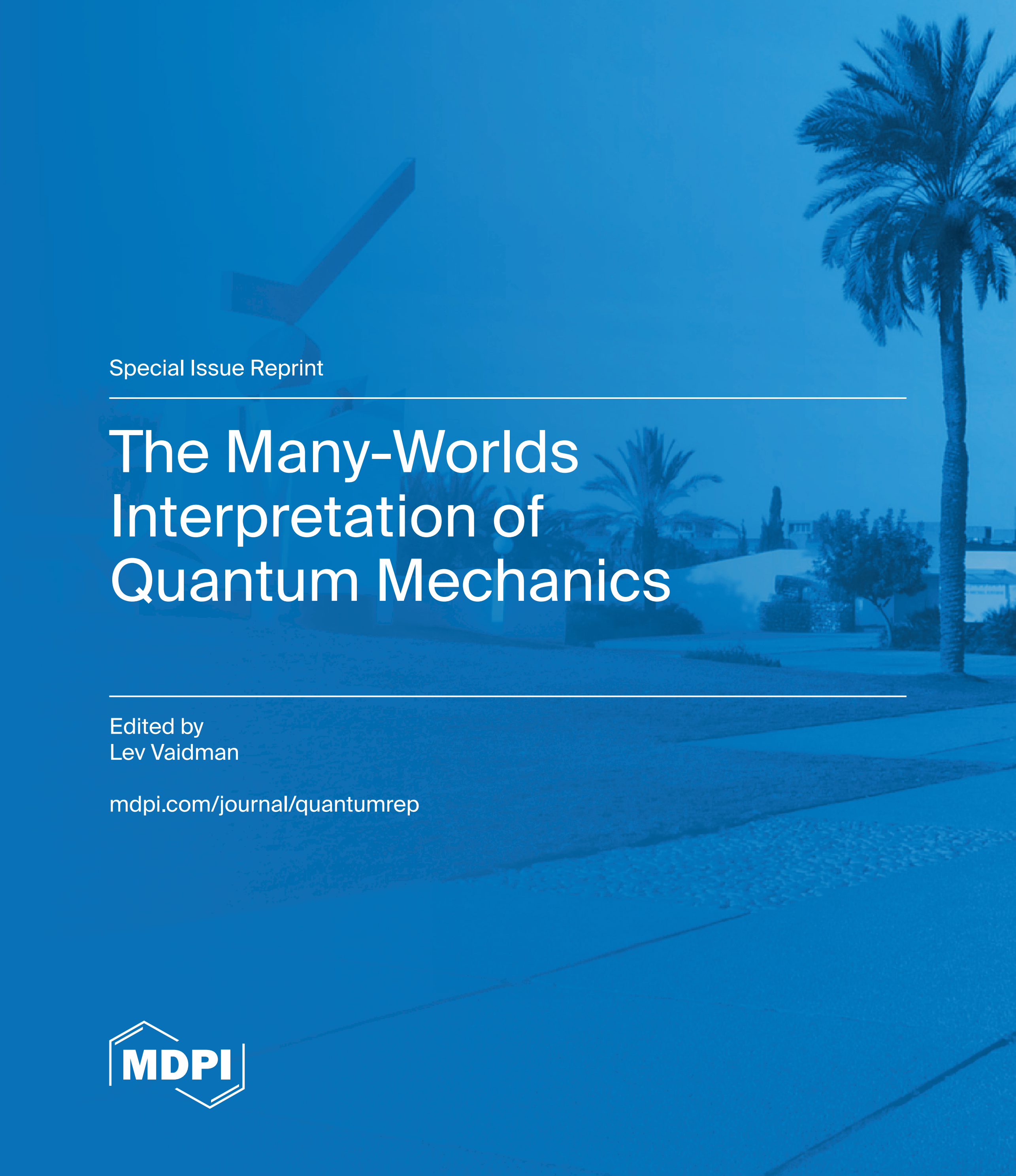

Special Issue Reprint

The Many-Worlds Interpretation of Quantum Mechanics

Edited by
Lev Vaidman

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Editor

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Cover image courtesy of Lev Vaidman

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About the Editor

Lev Vaidman

Lev Vaidman, born in Leningrad, studied physics in Israel. He received a B. Sc. from Hebrew University, an M. Sc. from Weitzmann Institute, and a Ph.D. under the guidance of Yakir Aharonov, with whom he collaborates to this day at Tel Aviv University, where he holds the Alex Maguy-Glass Chair in Physics of Complex Systems. This year, he became an Elected Fellow of the Israel Physics Society. His research is centered on the foundations of quantum mechanics and quantum information. He is a theoretical physicist and many of his proposals have been implemented in laboratories around the world, but recently, he himself has become involved in the experimental realizations of his ideas. Vaidman is mainly known for introducing the teleportation of continuous variables, cryptography with orthogonal states, and novel types of quantum measurement: non-local, weak, protective, and interaction-free, and also for introducing numerous quantum paradoxes. His analyses of the interpretations of quantum mechanics are centered on developing the many-world interpretation, for which he is apparently the strongest proponent.

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Editorial

The Many-Worlds Interpretation of Quantum Mechanics: Current Status and Relation to Other Interpretations

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This is a preface to a Special Issue of *Quantum Reports* devoted to the results of the workshop "The Many-Worlds Interpretation of Quantum Mechanics: Current Status and Relation to Other Interpretations" [1]. As I said in my contribution written before the conference [2], I was optimistic about bringing the MWI closer to consensus. More than two decades ago, I wrote an entry in the Stanford Encyclopedia of Philosophy on the MWI [3]. It is well cited, with very few critical notes, so I gathered the impression that what I presented there is the MWI as viewed by the community. The conference, which brought together a significant part of the community, showed that I was very wrong about that. There were many interesting discussions, but what I learned is that there is a striking diversity of views about the MWI.

I saw many radically different understandings of the concept of a world even among the most enthusiastic proponents of the MWI such as Deutsch, Wallace, Saunders and myself. Contrary to others, I do not consider decoherence with the environment as a definition of world-splitting. In my view, the universe is a superposition of worlds in which all macroscopic objects are well localized; i.e., it is a collection of classical worlds. Deutsch [4], however, writes,

> ... if reality—which in this context is called the multiverse—is indeed literally quantum-mechanical, then it must have a great deal more structure than merely a collection of entities each resembling the universe of classical physics.

Apart from a different semantics—"multiverse" instead of "universe" and "universe" instead of a "world"—this quote presents a very different view.

Saunders [5] views "worlds" within "decoherent histories" formalism rooted in the quantum description itself; this is completely different from my approach, in which a "world" is a concept of an agent that helps to explain her experience. Bigaj [6], who analyzes the consistent histories formalism, writes,

> It seems to me that the only realist, objectual interpretation of a framework is that frameworks refer to some observer-independent and distinct realities. It is hopefully not too far-fetched to call these realities "worlds"

Ridley's [7] counterparts of worlds are

> distinct time-localized 'universes' existing at single times [8] ... built out of parts with opposite time orientations.

Waegell [9] admits that his "local space-time model"

> is fundamentally different from the many-worlds theory of Everett, which is delocalized in the configuration space and describes global worlds in a particular Lorentz frame.

Waegell's contribution complements several works presented in the conference attempting to build local (separable) quantum mechanics. These include Bedard [10], Rubin, Kuypers [11], Tappenden and Faglia.

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My reading of Wallace [12] is that worlds are effectively autonomous branches, the mutual independence of which is ensured by decoherence:

> ... [the universe] must be understood as describing a multiplicity of approximately classical, approximately non-interacting regions which look very much like the 'classical world'.

This is a widespread view; see, for example, the review of Allori [13] from which I took Wallace's last quotation. More than two decades ago, Wallace [14] wrote,

> Everettians [...] may legitimately and meaningfully use the terminology of many worlds without being required to represent these worlds in their formalism.

This resonates with the statement of Lu [15]:

> The existence of a world is approximate and could be vague and indefinite in EQM.

Lazarovici [16] writes,

> there are many different interpretations of the Many-Worlds interpretation ... but even the best-elaborated ones remain vague about how the theory is supposed to make contact with familiar physical reality. I consider this the most serious problem of Everettian quantum mechanics.

The vagueness of the concept of a world in the MWI is a serious problem, but in the workshop, we witnessed even more radical proposals. Cuffaro and Hartmann [17] proposed to change the standard approach in physics which starts with closed systems by taking open systems as the basic concept. In this view, the systems in a world and the worlds themselves are described by density matrices instead of pure states. Chen [18] also prefers the density matrix description, but speculates that the world is "strongly" deterministic, a proposal that resonates with the superdeterminism discussed by Argaman. Zwirn [19] suggested adopting "Convivial Solipsism":

> We need to abandon our usual picture of the world. Reality is entirely relative to each observer, and there exists no absolute reality that could be shared by all observers.

Vedral [20] introduces the q-wave function and states:

> The worlds only emerge fully when we have fully orthogonal states of observers ... each alpha particle tract is orthogonal to every other one, which means that you can think of them as different worlds.

Note that this is very different from my approach, in which worlds should differ by macroscopic changes in macroscopic objects. Papineau and Rowe [21] adopt

> the "fission programme" version of Everettianism. In this version, which was originally adopted by Everett himself and is endorsed by perhaps the majority of his followers, any quantum "collapse" is followed by the macroscopic objects involved, including any observers, "splitting" in a way that results in actual "branches" for all outcomes with a non-zero probability.

Arve [22] writes,

> ... what is real is not directly represented by the wavefunction but by the gauge invariants. ... The success of describing our observations of physical systems and experiments with only the wavefunction gauge invariants demonstrates that a primitive ontology is not necessary.

Tappenden [23] instead suggests

> ... interpreting the universal wavefunction as representing a set of interacting deterministic universes which contain microscopic local beables. Objects in our environment become sets of objects which are macroscopically isomorphic but differ in their microscopic configurations. They are set-theoretically extended in configuration space, so to speak.

Stoica [24] also writes about local beables:

> The local beable ontology of the wavefunctional suggests interpreting these linear combinations as multiple ontic states coexisting in parallel. Since a macrostate is an equivalence class of microstates, probabilities arise by taking into account the possible microstates in each macrostate.

Willhelm [25], in order to solve the probability problem of the MWI, promotes a "centered Everett interpretation", which relies on a particular

> metaphysics of branches and agents: both branches and agents are four-dimensional entities. They extend through time as well as through space. So they are often called 'spacetime worms', and this view of branches and agents is often called the 'worm view'.

The "worm" view corresponds to a divergent world view, which I find meaningless without adding ontology on top of the wavefucntion, which Willhelm denies. He claims to refute my argument by stating that his concept is linguistically coherent, so it cannot be meaningless.

Several interesting talks (which can be viewed on the conference website [26]) were not published in this proceedings because they were published elsewhere. Aharonov [27] preferred an alternative to Everett's MWI. He proposed a solution of the measurement problem that avoids many worlds by postulating the future boundary condition. Gisin [28,29] presented the view that many worlds can be found even in classical physics, which, contrary to consensus, is not a deterministic theory. Maudlin [30] extended his criticism of the ability of the MWI to explain our world due to the onotology of the wavefunction being in the configuration space.

Other speakers questioned the validity of the MWI by analyzing particular experiments which they argued are challenging for the MWI. Elitzur [31] discussed a surprising interferometric experiment with "disappearing" particles. Renner [32] discussed a gedanken Wigner's friend experiment, and Jordan [33] suggested a feasible demonstration of a related interference experiment.

In my opinion, the fact that the MWI avoids action at a distance is its greatest advantage. Ney [34] discussed nonlocality in different metaphysical approaches to worlds in the MWI. Another major issue is probability in the MWI. Page [35] presented an approach inspired by cosmology. Saunders [36] suggested solving the problem by branch counting. I, however, do not see how this can be possible, and moreover, I claim that in the framework of the MWI, one cannot ask the question: What is the probability of a particular outcome of a quantum measurement? The question has no meaning since all outcomes take place. In my understanding of probability, it should be a unique matter of fact to talk about its probability. The comment of Saunders that if this question is meaningless, then the MWI will never be in the consensus does not persuade me, but I admit that it will take a long time. I hope it will be faster than the time it took people to accept that the Sun does not revolve around Earth.

Although I was unable to move the majority towards a consensus on the MWI, the workshop did not make me doubt the superiority of the MWI. I am encouraged by Wallace [37], who compared the MWI favorably with other interpretations, arguing that it has an advantage for generalization to field theory and beyond, and in particular, I am encouraged by the vision of Deutcsh presented at the end of the final session of the workshop. I can conclude with quotes from Huber [38], who conducted an extensive comparative review, writing that

> rival interpretations or theories either face limited applicability/conceptual incoherence, or can be reduced to MWI on closer inspection. ... Finally, I dare to make the following claim: poll results notwithstanding, the majority of the physics community in fact prefer (an unmodified) realist interpretation and are only Copenhagen advocates out of custom and convenience, or because they do not deeply question anti-realist assumptions or hidden-variable theories´

> (limited) applicability. I dare to say that the majority may already subconsciously be ‘many-worlders’, and did not, mostly due to shut-up-and-calculate advice, rigorously reflect on their consciously preferred presuppositions or think them through to their logical endpoint.

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Article

Why the Many-Worlds Interpretation?

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Abstract: A brief (subjective) description of the state of the art of the many-worlds interpretation of quantum mechanics (MWI) is presented. It is argued that the MWI is the only interpretation which removes action at a distance and randomness from quantum theory. Limitations of the MWI regarding questions of probability which can be legitimately asked are specified. The ontological picture of the MWI as a theory of the universal wave function decomposed into a superposition of world wave functions, the important parts of which are defined in three-dimensional space, is presented from the point of view of our particular branch. Some speculations about misconceptions, which apparently prevent the MWI from being in the consensus, are mentioned.

Keywords: many-worlds interpretation, interpretations of quantum mechanics; determinism; action at a distance

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1. Introduction

This is a preface to the Special Issue of *Quantum Reports* devoted to the results of the workshop "The Many-Worlds Interpretation of Quantum Mechanics: Current Status and Relation to Other Interpretations". In my research on this subject [1–15], I find the many-worlds interpretation (MWI) by far the best interpretation of quantum mechanics. For me, the goal of this workshop is to sharpen the MWI by reaching a consensus among supporters of Everett's original idea [16] about what the MWI is. However, as a scientist, I must always be sceptical about my beliefs, so I would also consider the workshop a success if it demonstrates weaknesses of the MWI and shows why the MWI should not be accepted as a leading interpretation. Of course, I hope that the result will be different, that we will reach an understanding that the reason for the MWI not being the consensus is a mistake in the evolution of science due to a long period of observing quantum phenomena without a satisfactory explanation. This apparently led Bohr to persuade the physics community that quantum mechanics can be used, but cannot be understood, a statement which to this day is frequently made in university courses in quantum theory.

My goal in this paper, which will be available before the workshop, is to set the stage for the workshop: to briefly describe what my version of the MWI is, why I view it as the most preferable interpretation, and what might be the reasons for misconceptions about the MWI. I invite participants in the workshop (and not only them) to challenge (or improve) my picture in the workshop and in its proceedings.

2. The MWI Is the Only Solution of the Measurement Problem without Action at a Distance

Let me state here what I view as the measurement problem. Today, we build single-photon sources and single-photon detectors and quantum physics explains well the process of photon emission and detection by describing the wave packet of the photon and the particular wave pattern of the ingredients of the single-photon detector, Figure 1a. If we add a beam splitter and another detector, Figure 1b, the equations of quantum mechanics provide similar wave patterns in the two detectors, A and B. Nevertheless, we never observe two simultaneous detections of a single photon by two detectors. This tension between the empirical evidence (single detector clicks) and the physical picture (both detectors change their quantum states) is the measurement problem.

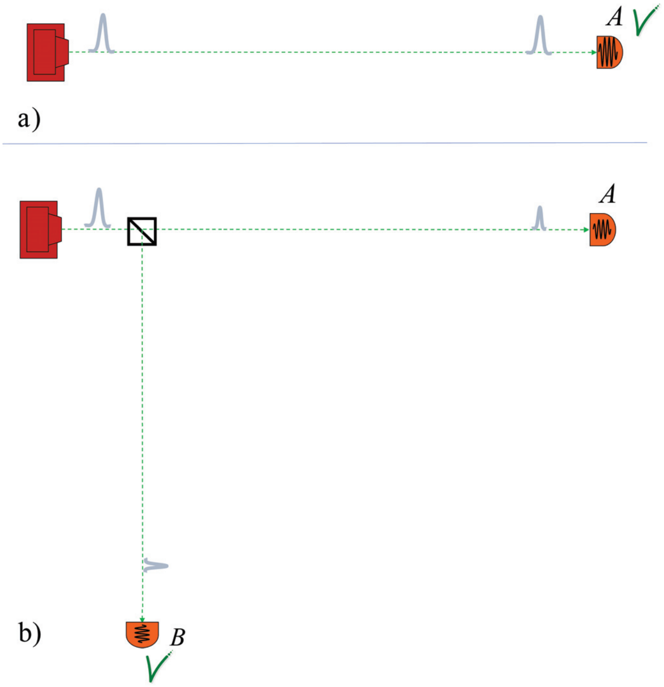

Figure 1. Measurement problem. (**a**) The detection of a single photon is fully understood by the creation of a particular quantum wave of parts of the single-photon detector. (**b**) In the experiment with a single-photon source, beamsplitter, and two detectors, the quantum mechanical equations show a similar (although reduced) change in two detectors. Nevertheless, we never observe simultaneous clicks of the two detectors.

Consider now repeating the experiment with the beam splitter without detectors, see Figure 2. At the time when the wave packet is present in place B, everyone, independently of their preferred interpretation of quantum mechanics, should agree about the following description of place B. Everyone (at least everyone who participates in zero-sum games), should be ready to pay half a dollar for a game in which they get a dollar if a detector placed in B finds the photon. The probability of one half of a detection in B is not an ignorance probability, we know everything relevant, but still we bet based on $p = 0.5$. (In Bohmian mechanics, it is postulated that in the described experiment, the Bohmian position cannot be known.) However, now, by placing a detector shortly before location A, we change the reality in B to probability 0 or 1. An agent near location A will bet with me on the measurement in B if I do not place the detector before A. My action in A can change the behavior of the agent. The betting behavior is changed in A, but since it is the bet about a measurement in B, we witness a superluminal change in B.

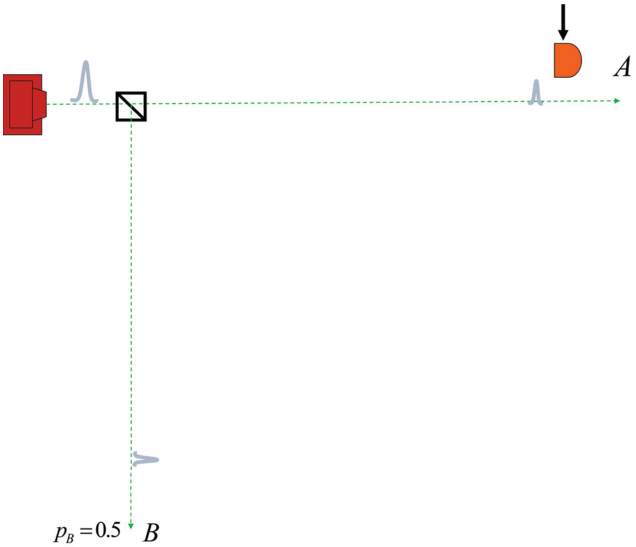

Figure 2. Action at a distance in a single-world universe. If we do nothing at A, then at a particular moment, there will be probability $p = 0.5$ of finding a photon at a spacelike separated region B. Introducing a detector just before A will lead to a superluminal change in B to $p = 0$ or $p = 1$. The change will not be known immediately at B, but it does not change the fact that something in B changed, e.g., the readiness of an agent in A to bet about the result of an experiment in B.

This argument holds only in the framework of a single-world interpretation. A believer in the MWI witnesses the same change, but it represents the superluminal change only in *her* world, not in the physical universe which includes all worlds together, the world with probability 0 and the world with probability 1. Thus, only the MWI avoids action at a distance in the physical universe. The MWI provides a covariant description of the universe made out of quantum particles which allows generalization to field theory, etc.

3. The MWI Is the Most Economical Quantum Theory Regarding the Theory's Laws

The title of Everett's thesis "The Theory of the Universal Wave Function" [17] is a good description of the MWI. In this theory, there are no sophisticated collapse mechanisms, no ontology and equation of Bohmian positions, no "consistent" or "decoherent" histories, no algebras of observables, no "relational" properties with ontological meaning. I consider the wave function and the Hamiltonian, responsible for the evolution of the universal wave function, as the only fundamental entities of the theory, attaching only secondary importance to other operators (observables) by postulating that our experiences supervene directly on the world wave functions.

The only part of our experience which unitary evolution of the universal wave function does not explain is the statistics of the results of quantum experiments we performed. We must add a postulate about the probability of self-location in a world which is a counterpart of the Born rule of the standard interpretation [13]. Although the self-location probability postulate explains the observed statistics, it does so without introducing objective chance in Nature: the postulate quantifies an ignorance probability. Thus, the MWI brings back determinism to scientific description [8]. (Before the quantum revolution, determinism was considered as a virtue of scientific explanation.) We, as agents capable of experiencing only a single world, have an illusion of randomness. This illusion is explained by a deterministic theory of the universe which includes all worlds together.

4. The Paradoxes of the Quantum Theory Are Resolved in the Framework of the MWI Interpretation

The MWI provides simple answers to almost all quantum paradoxes. Schrödinger's Cat is absurd in one world, but unproblematic when it represents one world with a live cat and a multitude of worlds with the cat which died at different times of detection of the radioactive decay.

It is very unfortunate that we do not know what would be the reaction of Einstein to the MWI. It seems that he would adopt it, as it resolves two main difficulties Einstein had with quantum mechanics: randomness and action at a distance.

The paradoxical behavior of Bell-type experiments disappears when quantum measurement does not have a single outcome [9]. Since the spin measurement of one particle of an Einstein–Podolsky–Rosen pair has both results Up and Down, the physical description of the second particle as a mixed state is not changed at the moment of the measurement of the first particle at a spacelike separate location.

The paradoxes in describing collapse in different Lorentz frames [18] do not arise when the theory does not have collapse. The paradox of interaction-free bomb testing [19], in which we get information about a region without a probe being there, is resolved by interaction of the probe in a parallel world. The paradox of the amount of information transferred in teleportation is resolved by the nonlocality of worlds and an observation that the only information remaining to be transferred after the local Bell measurement is the identity of the world we are in [2]. Finally, recent paradoxes appearing in the description of pre- and postselected quantum systems: the three-box paradox [20], Hardy paradox [21], the quantum pigeon holes conundrum [22], and discontinuous traces in nested interferometers [23] are all resolved by the fact that there are parallel worlds with different postselections [14].

5. Conceptual Changes in Our Approach to a Scientific Theory That Should Be Made When We Accept the World Splitting Structure of the Universe

Up until a particular point in time (our present), there is no difference in our experience between the single world of the universe in which quantum mechanics includes collapses at every quantum measurement and the corresponding world of the MWI universe. In Figure 3a, the whole tree of worlds of the MWI is schematically shown, in Figure 3b, our world until the present and all our future worlds, and in Figure 3c, the corresponding world of the universe with collapsing worlds is shown. There is no difference in the description of the past between the MWI and the theory with collapse at every measurement. However, there is a difference for the future. While in the collapsing universe there is a diachronic identity of the world towards the past and future, in the MWI, there is no diachronic identity towards the future.

The difference in the world splitting structure of our universe should be reflected in our attitude towards the past and future. In our memories, there is a single world. In this world, in our past, we can identify deterministic as well as chancy (in the case of observing results of quantum measurements) events. We understand that chancy events are our illusion in a deterministic physical universe due to our construction which does not allow the experience of superpositions. We understand the existence of parallel worlds in the past, but our memories define a unique diachronic identity over the past. By contrast, we do not have diachronic identity in the future. There are multiple worlds (created by future quantum measurements) which are *all* related to our world at present, Figure 3b. Thus, we cannot ask what is the probability for a result of a quantum measurement to be performed. It should not prevent us from behaving in a "normal" way (as believers of a single collapsing world picture). The justification is very different: we care for all future parallel worlds according to their "measure of existence" [3,7], which is proportional to the objective probabilities of the corresponding possible collapsing worlds.

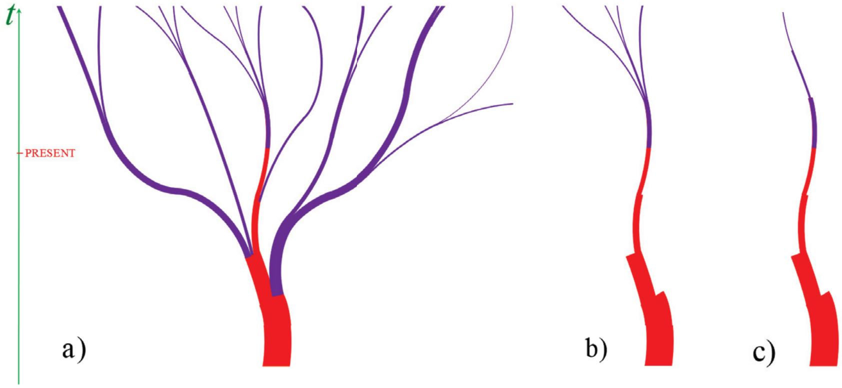

Figure 3. The world structure in the MWI and a single-world universe. (**a**) The whole tree of many worlds in the MWI. (**b**) One world of the MWI until present together with the tree of future worlds splitting out of it in the future. (**c**) One of the corresponding worlds of the theory with collapse.

6. What Is a "World" in the MWI?

The "world" in my MWI is not a physical entity. It is a a term defined by us (sentient beings), which helps to connect our experience with the ontology of the theory, the universal wave function. My definition [4] is:

> *A world is the totality of macroscopic objects: stars, cities, people, grains of sand, etc., in a definite classically described state.*

Just as our experience is vague, so a world is vaguely defined: "macroscopic", "classically definite" are not rigorous terms. For a particular choice of these terms,"world" has a physical counterpart as the world wave function which is a part of the superposition of the universal wave function. Until the next splitting, it autonomously evolves, but contrary to a popular view [24], it has nothing to do with *decoherence due to the environment*. The world wave functions of different worlds do not interfere, unless super-technology à la Wigner with his friend is present, because in real situations, we cannot arrange interference of macroscopic bodies (even if another super-technology switches off the decoherence with the environment).

The MWI is "The Theory of the Universal Wave Function", but the starting point in our description is our world, not the universal wave function. The "emergence" program [25] is not simple, and it is also not needed. In any case, we have very little information about the universal wave function, so the emergence program, even if successful, is of little practical value.

We do know a lot about our world. There is no question of a preferred basis, it is defined by our world, see Figure 3b. Every physicist who does not worry about the interpretation, and accepts the von Neumann process I happening at every quantum measurement, has no difficulty describing the basis in which she observes our single world (so she believes), and the MWI believer uses the same basis. It is an obviously correct statement that the world of a dead cat is stable, while the world "plus", the plus superposition of dead cat and alive cat, in almost no time evolves into equal weight superposition of the worlds "plus" and "minus". However, in my framework, there is no need to analyze this, because the worlds I define have no cats in a superposition.

The MWI believer, being aware of recent quantum measurements, has information about some parallel worlds. It is easier for her to think about coherent splitting in a quantum computing device [26], but the main reason for introducing parallel worlds is to avoid collapse (the von Neumann process I). This makes physics elegant, deterministic, and without action at a distance.

7. Connection between Our Experience and the Universal Wave Function

A popular question is: what is the space in which we consider the world wave function—is it a configuration space or three-dimensional space [27,28]? The answer is a subtle one [15]: the macroscopic objects are well localized and are not entangled within the world wave function, so every macroscopic object is represented by a product of the wave function of some collective variables defined in three dimensions, times the entangled state in the configuration space of degrees of freedom of the microscopic parts of the object.

The theory of our brain is not developed enough, but the hope is that the wave function of some collective variables of its constituents in three-dimensional space directly corresponds to our experience. To avoid dealing with brain science, it is reasonable to assume that our senses faithfully observe the three-dimensional picture of macroscopic objects. Then, the three-dimensional wave function of the collective variables of macroscopic objects is the bridge between the world wave function and our experience.

Instead of collective variables with the wave function in three dimensions, one can consider the spread of the world wave function in three dimensions, which is very similar to the "mass density", the primitive ontology or "local beables" of alternative approaches [29]. However, I do not see the necessity to add an ontic status to the mass density, it is included in the ontology of the world wave function which also allows more efficient ways of describing objects, e.g., the three-dimensional density of organic molecules for obtaining a more precise picture of living organisms. In particular, when such a three-dimensional density looks like me, I postulate that this construction "experiences" my feelings.

8. The (Illusion of) Probability in the MWI

The MWI is a deterministic theory, but the determinism is manifested on the level of all worlds together. This is the level of a mathematically rigorous physical theory. We live (or more precisely, lived) in one world with random probabilistic events (results of quantum measurements). Indeed, the complete knowledge of the wave function of our world, prior to a quantum measurement, does not specify a particular outcome. Usually, the outcome cannot be presented as uncertain due to ignorance of details in the measurement setup. The only way to introduce ignorance is to apply a "sleeping pill" trick [3] which leads to a situation in which an observer splits according to the outcome of the measurement without being aware of the outcome. Then, she (and only she!) is ignorant about what is the world she lives in. The observer does not have a concept of probability of an outcome (she knows that all possible outcomes of the experiment take place), but she has a legitimate concept of probability of self-location in a world with a particular outcome.

A separate issue is the quantitative question: what is the probability of self-location in a particular world? I claim that it has to be postulated in addition to the postulate of unitary evolution of the universal wave function and a postulate of the correspondence between the three-dimensional wave function of an observer within a branch and the experience of the observer. The postulate is that the probability of self-location is proportional to the "measure of existence" [3,7], which is a counterpart of the Born rule of the collapse theories.

Apart from empirical evidence, there are many natural principles which, together with symmetry considerations, suggest the plausibility of the self-location rule. For example, it is enough to postulate that when a quantum measurement performed in one world splits it into several worlds, then the probability of self-location in the first world is equal to the sum of the probabilities of self-location in all the newly created worlds. Never mind how plausible this or other principles taken as a basis of the MWI Born rule proof are (I have a proof based on the impossibility of superluminal signaling), some principle is necessary [13]. The postulate of the unitary evolution of the universal wave function alone is not enough. Note that the necessity of the additional postulate in the framework of the MWI is less obvious than in the framework of collapse interpretations, in which the Born rule is clearly a separate postulate describing a nonunitary process.

9. What Might Be the Reasons for the MWI Not Being in a Consensus?

The reluctance of a human to accept the MWI is natural. We would like to think that we are the center of the Universe: that the Sun, together with other stars, moves around Earth, that our Galaxy is the center of the Universe, and we are unhappy to accept that there are many parallel copies of us which are apparently not less important.

The next issue is the difficulty to apprehend what exactly a parallel quantum world means. It is misleading to view the universe as a multitude of (countable) classical worlds created by a magician. The cosmological multiverse is very different and much easier to understand.

Negative publicity for the MWI comes from the controversial claims about advantages of the MWI relative to other interpretations, e.g., that the Born Rule can be derived instead of postulated [30]. The claim is natural, because it is not simple to postulate the Born Rule in the MWI, but I believe it is false. In any case, the difficulties of this program reflect negatively on the MWI.

Another source of negative publicity is the controversy generated by presenting MWI as a theory of the universal wave function on configuration space [27], obscuring the connection between ontology and our experience. Avoiding non-separability by moving to configuration space [31] is hardly helpful.

In my view, similar damage comes from an attempt to present MWI in the Heisenberg picture with a controversial claim of bringing separability into quantum mechanics [32]. The Heisenberg picture provides not just a description of the present, but also of the past, so it is nonlocal not only in space, but also in time. Assuming the initial state as given, and describing reality by multiplied local Hilbert spaces which include all systems interacting with local systems in the past, achieves formal locality including separability, but for the price of enormous complexity [33].

10. Conclusions

Let me summarize the main points of my approach to the MWI for which I am looking for support/refutation in the upcoming workshop.

(a) The lack of action at a distance is a huge physical advantage which is not present in other interpretations;
(b) Determinism is a huge philosophical advantage which is not considered as such due to an error in the evolution of science (apparently explained by not seeing a deterministic option for physics for too long);
(c) The MWI allows us to view physics in three spatial dimensions within the particular world of the MWI we live in (however, we should not disregard nonlocality of entanglement which requires the configuration space for its description);
(d) Our world defines our world wave function (the alleged preferred basis problem) and the difficult emergence program does not need a solution;
(e) There is only an illusion of probability of outcomes of quantum measurements. It naturally leads to an effective Born Rule via measures of existence of worlds (and can be given an ignorance probability meaning as the probability of self-location in a particular world). Quantum worlds, contrary to classical worlds, might have measures of existence which are not just zero or one.

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Personal Identity and Uncertainty in the Everett Interpretation of Quantum Mechanics

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Abstract: The deterministic nature of EQM (the Everett Interpretation of Quantum Mechanics) seems to be inconsistent with the use of probability in EQM, giving rise to what is known as the "incoherence problem". In this paper, I explore approaches to solve the incoherence problem of EQM via pre-measurement uncertainty. Previous discussions on the validity of pre-measurement uncertainty have leaned heavily on intricate aspects of the theory of semantics and reference, the embrace of either four-dimensionalism or three-dimensionalism of personhood, or the ontology of EQM. In this paper, I argue that, regardless of the adoption of three-dimensionalism or four-dimensionalism of personhood, the overlapping view or the divergence view of the ontology of EQM, the pre-measurement uncertainty approach to the incoherence problem of EQM can only achive success while contradicting fundamental principles of physicalism. I also use the divergence view of EQM as an example to illustrate my analyses.

Keywords: Everett Interpretation of Quantum Mechanics; personal identity; probability in quantum mechanics

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1. The Incoherence Problem

The Everett Interpretation of Quantum Mechanics (EQM) is a deterministic physical theory, but it also involves probability via the Born Rule. (See [1] for an overall introduction. In [2], Everett attempted to reconstruct the Born Rule in Section 5, while assuming full determinism as the underlying principle of Quantum Mechanics.) The deterministic nature of EQM seems to be inconsistent with the use of probability in EQM. This has been called the "incoherence problem" of EQM [3].

Consider the simplest branching process with only two branches. Imagine an observer, Aristotle, measuring the *z*-spin of an electron in a state of superposition of different *z*-spins. The initial state of the entire system is represented by $\frac{1}{\sqrt{2}}(|\uparrow\rangle + |\downarrow\rangle) \otimes |\text{Aristotle } 0\,\rangle$, where $\frac{1}{\sqrt{2}}(|\uparrow\rangle + |\downarrow\rangle)$ represents the initial state of the electron, and $|\text{Aristotle } 0\,\rangle$ is the initial state of Aristotle. After the measurement, the state of the whole system evolves into $\frac{1}{\sqrt{2}}|\uparrow\rangle \otimes |\text{Aristotle } \uparrow\rangle + \frac{1}{\sqrt{2}}|\downarrow\rangle \otimes |\text{Aristotle } \downarrow\rangle$, where $|\text{Aristotle } \uparrow\rangle$ (or $|\text{Aristotle } \downarrow\rangle$) signifies the state of Aristotle seeing the *z*-spin is up (or down). From an "outside" viewpoint, all branches equally exist after the measurement, and both the probabilities of Aristotle seeing the *z*-spin is up and Aristotle seeing the *z*-spin is down are 1. But from an "inside" viewpoint, one can only obtain one single result after the measurement. Consequently, according to the Born Rule, both the probabilities of Aristotle seeing the *z*-spin is up and seeing the *z*-spin is down are $1/2$ [4].

In a deterministic theory, the following principle is commonly held true.

> Ignorance: In order to make propositions such as "the probability that event *E* happens is *p*" meaningful in a deterministic universe, we must be ignorant of some facts about E.

Ignorance is commonly acknowledged in classical physics. In the background of classical mechanics as a deterministic physical theory, whether it will be raining tomorrow is determined by the physical state of a given moment, s. But we cannot discern which physical state it is among a vast array of similar physical states $\{s'\}$. This is the basis for discussions involving probability in classical mechanics. Loosely speaking, if the measure of all states $\{s'\}$ is A, and the measure of those states in $\{s'\}$ that lead to tomorrow's rain is B, then the probability that it will rain tomorrow is B/A given the physical state is s. This probability arises from our ignorance of the precise physical state of this moment.

In this paper, I shall explore one line to solve the incoherence problem via pre-measurement uncertainty. I shall focus on Saunders and Wallace's proposal that some kind of pre-measurement uncertainty, which comes from the lack of specific *indexical knowledge* of observers, can resolve the incoherence problem in EQM [3,5–10]. According to Saunders and Wallace, even though Aristotle knew that the state of the entire system would be $\frac{1}{\sqrt{2}}|\uparrow\rangle \otimes |\text{Aristotle} \uparrow\rangle + \frac{1}{\sqrt{2}}|\downarrow\rangle \otimes |\text{Aristotle} \downarrow\rangle$, he remains uncertain of which person in the future he is identical to. This solution is based on Lewis's account of personal identity (D. Lewis 1976, 1983). This approach is criticized based on the theory of semantics and reference by P. Lewis [11] and Tappenden [12]. In this paper, I will investigate the validity of the pre-measurement uncertainty approach to the incoherence problem and its consequences, while maintaining a more charitable position on the debate in language and semantics.

Pre-measurement uncertainty is not the only attempt to resolve the incoherence problem. Some authors favor post-measurement uncertainty to explain probability in EQM. For instance, Vaidman [13] proposes that, imagining Aristotle is blindfolded during the measurement, he would be uncertain who he is identical to after the measurement until he sees the results of the measurement. This approach is further developed by McQueen and Vaidman [14]. Tappenden [15] argues that this, combined with Sider's account of personal identity [16], explains the use of probability in EQM. Moreover, Papineau [17] and Tappenden [18] reject *Ignorance* as the foundation for understanding probability in EQM. (Instead, in a recent publication, Tappenden [19] embraces pre-measurement uncertainty. However, it bears more similarity to Tappenden's previous approach that rejects *Ignorance*, and remains to be justified whether it truly qualifies as a "pre-measurement uncertainty" approach. For this reason, I do not include Tappenden's recent approach in this paper.) Although I do not find their suggestions unproblematic, this paper will solely focus on pre-measurement uncertainty.

2. Personal Identity and Ontological Structure

Some attempts to understand EQM aim to distinguish different ontological structures of a *world* in order to address the debate of uncertainty. For instance, Wilson [20] argues that the mathematical structure of EQM itself does not decide between the overlapping view or divergence view. If different histories in EQM are not overlapped in the past, namely, they are quantitatively identical but numerically different in the past, EQM should be thought of in terms of divergence; either way, if they are numerically identical in the past, it should be thought of in terms of fission. Wilson [21] further claims that the mathematical structure of EQM remains neutral regarding the view of Individualism, which regards an Everett world (a branch in Saunders and Wallace's terminology) as a metaphysically possible world, or the view of Collectivism, which regards an Everett multiverse (everything described by the quantum state of the universe) as a metaphysically possible world. Following this line of thought, adopting the divergence view can avoid the problem posed by Saunders and Wallace's approach to solving the incoherence problem. This claim relies on a *deep* metaphysical understanding of EQM, namely, that there can be deep and important differences in whether there can be multiple qualitatively identical "worlds" corresponding to one state in EQM and that we should take the identity of *worlds* in EQM very seriously. However, I find this perspective misleading as it may undermine the very spirit of EQM that we do not need any additional structures or postulations of quantum mechanics. The common-sense four-dimensional world we inhabit merely emerges from the quantum state, which is not primary in the ontology of EQM. As Wallace cites Dennett:

> Dennett's criterion: A macro-object is a pattern, and the existence of a pattern as a real thing depends on the usefulness—in particular, the explanatory power and predictive reliability—of theories which admit that pattern in their ontology [22] (p. 93).

The same applies to a *world* in EQM. The existence of a world is approximate and could be vague and indefinite in EQM [9,22–24]. Following this line, there is no deep philosophical inquiry to be made regarding the identity of physical objects in EQM, at least nothing deeper than the identity of physical objects in classic mechanics. The identity of physical objects or *worlds* is not a deep truth underlying the *prima facia* structure of EQM, as Wallace once put it in this way:

> There is a concept of transtemporal identity for patterns, but again it is only approximate. To say that a pattern P_2 at time t_2 is the same pattern as some pattern P_1 at time t_1 is to say something like "P_2 is causally determined largely by P_1 and there is a continuous sequence of gradually changing patterns between them"—but this concept will not be fundamental or exact and may sometimes break down [22] (pp. 95–96).

Consequently, the distinction between overlapped histories and divergent histories is merely a superficial artifact. If adopting the divergence view of EQM can avoid the problem that the overlapping view has in order to solve the incoherence problem, then there must be a substantial difference between understanding one branch in EQM as one world or multiple qualitatively identical but numerically different worlds. We would need to introduce additional structures (possibly only metaphysical rather than physical) to EQM if we want to find any deep differences between them. (Wallace also argues that the difference between overlapping histories and divergent histories is not meaningful for a similar reason [9] (p. 287).) I shall discuss the divergence view in Section 7. Although the divergence view may have its own problems, the aim of this paper is not to reject it. In Section 7, I shall argue that my analyses in this paper apply to the divergence view as well, and supporters of the divergence view will face the same dilemma.

Although there may be no deep ontological questions within EQM, it is still legitimate to inquire whether one *person* is identical to another within the framework of EQM. While it might be commonly agreed that personal identity supervenes the physical reality from a physicalism viewpoint, it is not *part* of our physical theories. As a result, it remains to be investigated *how* personal identity supervenes physical reality, as it allows for the development of different theories of personal identity within the framework of classic mechanics as the background. This inquiry differs from the question "divergence or not" mentioned earlier. Taking personal identity seriously does not necessarily burden the ontology of the underlying physical theory.

Personal identity, as I shall discuss in the following sections, forms the very core of pre-measurement uncertainty in EQM. I will introduce Lewis's account of personal identity in Section 3 and Saunders and Wallace's solution to the incoherence problem involving pre-measurement uncertainty in Section 4. I then will delve into P. Lewis and Tappenden's objection to Saunders and Wallace based on concerns related to reference and semantics. While maintaining a charitable perspective on the debates, I will propose another objection that there are no facts to be uncertain of in a common reading of Saunders and Wallace's proposal. In Section 5, I will present a modified view that suggests the existence of multiple qualitatively identical but numerically different mental states that supervene one physical state before the branching. The modified view can withstand the objections just mentioned. In Section 6, I further argue that this revised view cannot be consistent with physicalism and be successful in addressing the incoherence problem at the same time, unless we introduce some hidden variables into EQM. Finally, in Section 7, I discuss the "divergence view" of EQM, which provides a concrete example that illustrates the analyses presented in Section 6.

3. The Lewisian Account of Personal Identity

The Lewisian account of personal identity, developed by D. Lewis [25,26], is an attempt to preserve personal identity as a definite and transitive relation despite Parfit's destructive arguments through Parfit's personal fission thought experiment [27] (pp. 245–280).

By virtue of the obvious analogy between the brain splitting case and branching in EQM, I will use the branching case in EQM to illustrate both the Parfitian account and the Lewisian account of personal identity here. In our scenario, the quantum state after branching is $\frac{1}{\sqrt{2}}|\uparrow\rangle \otimes |\text{Aristotle }\uparrow\rangle + \frac{1}{\sqrt{2}}|\downarrow\rangle \otimes |\text{Aristotle }\downarrow\rangle$. Let us denote the person represented by $|\text{Aristotle }\uparrow\rangle$ (or $|\text{Aristotle }\downarrow\rangle$) as Aristotle↑ (or Aristotle↓), and the person represented by $|\text{Aristotle } 0\rangle$ before the branching as Aristotle0.

According to Parfit, if we maintain that personal identity is a transitive and definite relation, Aristotle0 can only be identical to at most one of Aristotle↑ and Aristotle↓ since Aristotle↑ and Aristotle↓ cannot interact with each other after branching, and they are distinct agents making their separate decisions. (Here, the term "transitive" means that if person A is identical to person B, and person A is identical to person C, then person A is identical to person C. The term "definite" means it does not admit of degree, for example, we cannot say that person A is 50% identical to person B.) Consequently, Aristotle0 cannot be identical to both Aristotle↑ and Aristotle↓, as it would contradict the transitivity of personal identity. Hence, Aristotle0 is identical to only one of Aristotle↑ and Aristotle↓. If we uphold personal identity as a definite relation, given that the branching is highly symmetric, whether Aristotle0 is identical to Aristotle↑ or Aristotle↓ can only depend on some rather trivial differences between them. Parfit claims that such trivial relations cannot be of significant philosophical importance. Therefore, either there does not exist such a relation as personal identity which is definite and transitive, or such a relation is trivial and lacks significance.

The Lewisian account of personal identity seeks to preserve the definiteness and transitivity of personal identity by positing the existence of (at least) two persons both before and after branching: They coincide before branching but diverge afterward. In the case of EQM, there are *already* two persons present before branching: Aristotle0↑ and Aristotle0↓. Aristotle0↑ (or Aristotle0↓) is identical to Aristotle↑ (or Aristotle↓), but Aristotle0↑ is not identical to Aristotle0↓; hence, the definiteness and transitivity of personal identity can be preserved.

It is important to notice the original Lewisian account has a four-dimensional nature. According to Lewis's account, a person is a four-dimensional entity rather than a three-dimensional entity. The claim that Aristotle0↑ is identical to Aristotle↑ is not of *temporal identity*, but merely a trivial claim that Aristotle0↑ is identical to *itself*. As the same four-dimensional entity, Aristotle↑ is simply an alternative name for Aristotle0↑. Lewis calls the three-dimensional slice of a four-dimensional continuant as a four-dimensional person a *person-stage*, which is usually understood as a fully-present *person* in three-dimensionalism. A person, as a four-dimensional entity according to Lewis, is an aggregate of person-stages that belong to different times. ("A continuant person is an aggregate of person-stages, each one I-related to all the rest (and to itself). (It does not matter what sort of 'aggregate.' I prefer a mereological sum so that the stages are literally parts of the continuant. But a class of stages would do as well, or a sequence or ordering of stages, or a suitable function from moments or stretches of time to stages.)" [25] (p. 22)). In the scenario of this paper, there is only one person-stage before the branching and two person-stages after the branching. Since the quantum states $|\text{Aristotle}0\rangle$, $|\text{Aristotle }\uparrow\rangle$, and $|\text{Aristotle }\downarrow\rangle$ are all (approximately, of course) three-dimensional, we can use them to represent the corresponding person-stages for convenience. These three person-stages can constitute at least two (four-dimensional) persons: $\{|\text{Aristotle}0\rangle, |\text{Aristotle }\uparrow\rangle\}$ and $\{|\text{Aristotle}0\rangle, |\text{Aristotle }\downarrow\rangle\}$. (For simplicity, I have only included two typical person-stages for each person.) The claim that there are already two persons present before branching means that, prior to the branching, the present three-dimensional person-stage $|\text{Aris}tot\text{le}\rangle$ belongs to two four-dimensional persons. One ($\{|\text{Aristotle0}\rangle, |\text{Aristotle}\uparrow\rangle\}$) is identical to the only person who contains $|\text{Aristotle}\uparrow\rangle$, while the other is identical to the only person who contains $|\text{Aristotle}\downarrow\rangle$. (I assume that there is only one person who contains $|\text{Aristotle}\uparrow\rangle$ (or $|\text{Aristotle}\downarrow\rangle$) as its three-dimensional part for simplicity. Strictly speaking, there can be an infinite number of persons containing $|\text{Aristotle}\uparrow\rangle$ (or $|\text{Aristotle}\downarrow\rangle$) considering the possible infinite occurrences of branching in the future. However, this assumption will not affect the results in this paper.) These identity relations between the four-dimensional persons are transitive, but the identity relations between the three-dimensional persons (Lewis calls it *I-relation*, namely, two person-stages are in I-relation if, and only if, there is at least one person containing them) can be intransitive. Both the person-stages represented by $|\text{Aristotle}\uparrow\rangle$ and $|\text{Aristotle}\downarrow\rangle$ share the I-relation with $|\text{Aristotle0}\rangle$, but $|\text{Aristotle}\uparrow\rangle$ does not share the I-relation with $|\text{Aristotle}\downarrow\rangle$.

4. Saunders and Wallace's Lewisian Solution to the Incoherence Problem and Its Objections

Saunders and Wallace [3] utilize the Lewisian account as the foundation of pre-measurement uncertainty in EQM. Before the branching, Aristotle may have been fully aware that the quantum state after branching will be $\frac{1}{\sqrt{2}}|\uparrow\rangle \otimes |\text{Aristotle}\uparrow\rangle + \frac{1}{\sqrt{2}}|\downarrow\rangle \otimes |\text{Aristotle}\downarrow\rangle$, but he lacks knowledge of whether he is Aristotle0↑ or Aristotle0↓. As a result, he is uncertain whether he will observe the electron in state $|\uparrow\rangle$ or $|\downarrow\rangle$. There are no *internal* ways of distinguishing between Aristotle0↑ and Aristotle0↓ before the branching, for they are *physically identical up to the moment of branching*. If this is true, then there can be some *subjective uncertainty* in EQM, although the evolution of the quantum state is deterministic. Aristotle is ignorant of *who he is* before branching.

This solution is objected to by P. Lewis [11] and Tappenden [12]. (P. Lewis did not cite [3] in [11] since it was not published yet by that time. But P. Lewis did argue against a similar line of solution presented by Saunders and Wallace in [5,8])They argue that even if the Lewisian account is correct, neither Aristotle0↑ nor Aristotle0↓ could successfully refer to himself before the branching. Aristotle0↑ and Aristotle0↓ can only successfully refer to the single person-stage represented by $|\text{Aristotle}\,0\rangle$ before branching, which is commonly shared by all persons in this scene. Consequently, they conclude that it makes no sense to claim that Aristotle0↑ is ignorant of some indexical information about himself, as the utterance "I do not know whether I am Aristotle0↑ or Aristotle0↓" fails to express that "Aristotle0↑ does not know whether Aristotle0↑ is Aristotle0↑ or Aristotle0↓". In other words, their argument goes as follows: Before the branching, any singular terms in Aristotle0↑'s expressions cannot singularly refer to Aristotle0↑ but instead refer to all persons who supervene on $|\text{Aristotle}\,0\rangle$ at the same time; thus, the incoherence problem cannot be solved along this line. As P. Lewis argues:

> In particular, I cannot wonder further whether my use of the pronoun 'she' when pointing at the observer picks out she↑ or she↓; since she↑ and she↓ coincide at the moment, I am pointing at both of them [11] (p. 6). (P. Lewis's use of "she↑" and "she↓" is the same as the use of "Aristotle0↑" or "Aristotle0↓" in this paper.)
>
> Tappenden also objects:
>
> But HydraUP and HydraDOWN cannot each indexically refer to her own body via an utterance of 'This is my body' which has a single token sited in a single body-stage at time *T* prior to branching, because that single body-stage is common to the world-tube bodies of both HydraUP and HydraDOWN [12] (p. 311). (Tappenden's use of "HydraUP" and "HydraDOWN" is the same as the use of "Aristotle0↑" or "Aristotle0↓" in this paper.)

Saunders and Wallace attempt to develop a set of semantic rules where one single utterance can be paraphrased as two different propositions to address the objections [3] (pp. 295–296). I do not want to meddle with the somewhat murky issues of language and semantics here. Whether an utterance can successfully refer is, unsurprisingly, sensitive to the context in which it is uttered and the semantic rules we apply. I shall remain neutral in the debate about semantics. Instead, I shall argue that, under some general restrictions, which I shall explicate in the following, there are no *facts* in EQM to be uncertain of. Whether our language can express our uncertainty is one thing, but whether there is *anything* to be uncertain of is another thing.

5. Two Versions of the Solution

In D. Lewis's original writing, the claim that there are two persons before branching is a trivial one. There are no mysterious or philosophical deep facts behind this claim that require investigation. In D. Lewis's original scene and also in Saunders and Wallace's discussions, there exists only one three-dimensional person-stage before branching. To assert that there are two persons *present* before the branching simply means that there are two different ways to combine this particular three-dimensional person-stage with other person-stages to constitute a four-dimensional person. (Tappenden [19] misconstrues Saunders and Wallace's approach as it "reject(s) the concept of splitting, which is arguably Everett's key idea". Everett is not concerned about personhood or personal identity. In [3], Saunders and Wallace do not challenge Everett's idea of split that "the observer state 'branches' into a number of different states. Each branch represents a different outcome of the measurement and the corresponding eigenstate for the object-system state. All branches exist simultaneously in the superposition after any given sequence of observations. [2] (p. 459)". Saunders and Wallace do not alter Everett's conceptual framework as a physical theory; they only introduce four-dimensionalism and an account of personal identity into EQM.) Before the branching, Aristotle's internal mental state and thinking process is single. If Aristotle can be uncertain of something, he must be unaware of some facts. When Aristotle feels uncertain whether he is Aristotle0↑ or Aristotle0↓ in his mind, there should be some facts that determine whether this thinking belongs to Aristotle0↑ or Aristotle0↓. However, it appears that this determination is merely a matter of our choice. It is Aristotle0↑ who is uncertain if we choose to combine the person-stage before the branching with some person-stages that observe the z-spin of the electron as up, and it is Aristotle0↓ who is uncertain if we choose to combine the person-stage before the branching with some person-stages that observe the z-spin of the electron as down. To put it more ironically, it is Aristotle0↑ who is uncertain if we choose that the thought which feels uncertain belongs to Aristotle0↑, and it is Aristotle0↓ who is uncertain if we choose that the thought which feels uncertain belongs to Aristotle0↓. There is something not decided here, and fairly we can say there is some kind of *indeterminacy*; however, such indeterminacy does not come from any further unknown facts, but only from a choice that remains to be made by us. This is not a kind of uncertainty.

(Saunders and Wallace propose that there are two or more thoughts of Aristotle before the branching, as they write: "If persons are continuants, we do better to attribute thoughts and utterances at t to continuants C at t. That is, thoughts or utterances are attributed ordered pairs $\langle C, t\rangle$ or slices of persons $\langle C, S\rangle$, $S \in C$ not to temporal parts S. This is to apply whether or not there is branching. In the absence of branching we obtain the standard worm-theory view; in the presence of branching conclude that there are two or more thoughts or utterances expressed at t, one for each of the continuants that overlap at that time.Is it to be objected that thoughts or utterances have an irreducibly significance? We may grant the point that their tokenings are purely events—And as such, indeed, are identical—But the content of thoughts utterances is another thing altogether. On even the most timid forms of externalism, or functionalism for that matter, meanings are context-dependent. sentences produced pre-branching are likely to play different semantic each person subsequently, and likewise their component terms [3] (p. 295)." They consider *thoughts* as external entities. Their intention is to convey that there exist two or more contents within the agent's single thinking process in mind. Here I use "*thinking*" as the mental process and state of mind in this paper. As the subsequent argument unfolds, however, it is a matter of our choice to decide the semantic content of Aristotle's thinking (according to semantic externalism, as Saunders and Wallace advocate).)

However, with just a few modifications, I will present another version of Saunders and Wallace's solution. If there is more than one three-dimensional entity that supervenes one single physical state $|\text{Aristotle}\,0\rangle$, the previous objections can be addressed. For instance, suppose that there are two three-dimensional person-stages, $(\text{Aristotle}0\uparrow)_3$ and $(\text{Aristotle}0\downarrow)_3$ before the branching, and both of them supervene $|\text{Aristotle}\,0\rangle$. Namely, there is only one singular physical body as Aristotle before branching, but there are multiple mental states, or some other three-dimensional entities, that supervene $|\text{Aristotle}\,0\rangle$. (The requirement that there is only one physical body as Aristotle before the branching can be relinquished if we introduce multiple qualitatively identical "worlds" or multiple physical states before the branching, with each mind of Aristotle situated in a distinct world. I shall discuss this approach in Sections 6 and 7. However, the claim that there are multiple mental states as Aristotle before the branching, which is more essential, shall remain unchanged. The term "three-dimensional" might be a bit perplexing when applied to a mental state. In this context, I am employing the term "three-dimensional" in a broad sense for the sake of convenience, aligning it with the terminology of three-dimensionalism and four-dimensionalism. A three-dimensional person-stage is momentary, while a four-dimensional person is not. From an eternalist perspective, one might uphold that there exists an overarching mental state for a person throughout all time, with their momentary mental states serving as partial "sub-states" of this ultimate mental state. I do not know who exactly upholds this view, but it is important to make a distinction here. In this paper, I call a mental state three-dimensional in the sense that it is momentary.) By having two or more minds that think before branching, which are *qualitatively* identical but *numerically* different, the objection presented in the previous paragraph can be resolved. Before the branching, neither thinking can tell which mental state it belongs to, as both share the same contents. But there are some further facts, though they might be unobservable in principle, that can determine which mental state it belongs to.

P. Lewis and Tappenden's objection concerning reference and semantics can also be resolved. While there is only one singular "physical" utterance, namely, only one string of voices is uttered, this utterance is reflected in two numerically different mental states. When Aristotle utters "I do not know whether I will be Aristotle↑ or Aristotle↓ after the branching", this utterance can be translated into different propositions for different minds. Hence, the pronoun "I" can refer to different entities before the branching. $(\text{Aristotle}0\uparrow)_3$ is uncertain whether $(\text{Aristotle}0\uparrow)_3$ will be Aristotle↑ or Aristotle↓, and similarly, $(\text{Aristotle}0\downarrow)_3$ is uncertain whether $(\text{Aristotle}0\downarrow)_3$ will be Aristotle↑ or Aristotle↓.

This revised solution is similar to some kind of the "Many Minds Interpretation of Quantum Mechanics" (MMI) [28–30]. MMI posits the existence of *indefinite minds* that supervene one singular physical state of ourselves. Some early advocates of MMI do not aim to address the incoherence problem via pre-measurement uncertainty; for instance, Lockwood does not offer any account of personal identity in Lockwood's MMI theory and rejects *Ignorance* as a necessary requirement. Lockwood claims that the idea of multiple minds supervening one physical state itself is consistent with physicalism. As Lockwood writes that "The assumption no more carries any dualistic implications than the conventional assumptions, which even physicalists allow themselves, about what it is like to be in such states [29] (p. 184)". However, in the next section, I shall argue that this option is inconsistent with physicalism if we intend to utilize it as a means to resolve the incoherence problem by pre-measurement uncertainty.

6. The Problem of Supervenience

As we have duplicated the person-stages in the previous section, the so-called "I-relation" between different person-stages is now reestablished as a definite and one-to-one relation. Adopting three-dimensionalism or four-dimensionalism will not influence the conclusions in the following sections. For the simplicity of notations, I will use the notions in three-dimensionalism from now on. If adopting three-dimensionalism, there are already two persons, Aristotle0↑ and Aristotle0↓, before the branching or more. If adopting four-dimensionalism, the argumentation can be restored by replacing "Aristotle0↑" and "Aristotle0↓" with three-dimensional person-stages "(Aristotle0↑)$_3$" and "(Aristotle0↓)$_3$", and replacing "personal identity relation" with "I-relation". This notation shift is purely for convenience and does not imply the adoption of either the three-dimensionalism view or the four-dimensionalism view of personal identity.

I use the term '*physicalism*' to represent the view that human persons are *in essence* physical things. (Peter van Inwagen [31] (p. 225) defines *physicalism* as the thesis that "human persons are physical things". My definition is weaker as it allows some room to interpret what is "in essence" physical. These definitions, though not very precise, suffice for the purpose of my argument here.) Providing a comprehensive and elaborate definition here is both impossible and unnecessary. Instead, I present a relatively weak criterion of physicalism. According to this viewpoint, a human person is essentially a physical entity, and their personal identity can be *determined* if the physical state of the whole universe is determined and can in principle be deduced from the latter. It is fair and reasonable to demand that the following requirement be obtained and fulfilled under physicalism.

> Supervenience: The personal identity relations in a possible universe w' are the same as the personal identity relations in a possible universe w, if w and w' are physically identical. (In simple terms, personal identity in a universe supervenes its physical state. In the terminology of EQM, the term "universe" refers to the entirety of physical existences described by the formulation of Quantum Mechanics. On the other hand, the term "world" is used to denote a specific branch in the universe under decoherence. Therefore, in this paper, I use the term "possible universe" instead of "possible world".)

This requirement is sufficiently lenient as it does not require that we can simply "read off" personal identity relations from the physical state. Such a requirement does not even exclude the possibility that personal identity relations supervene on physical states *nonlocally*. For example, if person A and B supervene on local physical states $|\mathrm{A}\rangle$ and $|\mathrm{B}\rangle$, respectively, whether A is identical to B may not be determined by the properties of $|\mathrm{A}\rangle$ and $|\mathrm{B}\rangle$ themselves. Donald [32] (p. 8) has suggested that a mind in MMI supervenes the entire history, which implies the non-locality of personal identity relations concerning physical states. Nevertheless, physicalism cannot be upheld if *Supervenience* is not satisfied.

The modified view presented in the previous section does not necessarily contradict physicalism. As we discussed earlier, physicalism does not necessarily require that only one mental entity can supervene on one single physical human body, as argued by Lockwood. However, to solve the incoherence problem via pre-measurement uncertainty, a specific kind of identity relation between persons before and after branching is needed. This requires more than multiple mental states to supervene one physical state.

The quantum state before the branching is represented by $\frac{1}{\sqrt{2}}(|\uparrow\rangle + |\downarrow\rangle) \otimes |\text{Aristotle } 0\rangle$. Following the discussions in the previous section, both Aristotle0↑ and Aristotle0↓ supervene on $|\text{Aristotle0}\rangle$ approximately. ($|\text{Aristotle0}\rangle$ is an *instantaneous* physical state. Here, the term "approximately" implies that, strictly speaking, Aristotle0↑ and Aristotle0↓ may supervene the physical states over a small period of time.) $|\text{Aristotle0}\rangle$ represents one single physical state and at least two numerically different mental states, which correspond to different persons (or person-stages). Various accounts can be proposed to explain how these mental states supervene the physical state. The simplest option is that they directly supervene on $|\text{Aristotle0}\rangle$ without any further fine-grained characterizations. We can suppose that Aristotle0↑ before branching is identical to Aristotle↑ after branching (and similarly Aristotle0↓ is identical to Aristotle↓), without loss of generality. This relation as personal identity is either *deterministic* or *indeterministic*. In a deterministic scenario, *which person after branching Aristotle0↑ is identical to* is fully determined by all facts (both physical and non-physical) before branching. In this case, no physical facts can fully explain how this relation is determined. All we know about the relations among Aristotle0↑, Aristotle0↓, and $|\text{Aristotle0}\rangle$ is the *bare* fact that both Aristotle0↑ and Aristotle0↓ supervene on $|\text{Aristotle0}\rangle$, but there are no physical facts to distinguish Aristotle0↑ and Aristotle0↓ from their physical structures or to ground the fact that Aristotle0↑ is identical to one person supervening on one specific physical state while Aristotle0↓ is identical to another. Consequently, non-physical facts must come into play to determine the relations of those states. If, in a different universe, we have these non-physical facts different while keeping the physical state of the universe the same, we would arrive at a different result regarding whether Aristotle0↑ is identical to Aristotle↑. This, however, contradicts *Supervenience*.

If this relation is indeterministic (as suggested by Albert and Lower [28], that personal identity in EQM is *irreducibly probabilistic*), it would immediately violate *Supervenience*. The claim that it is indeterministic that Aristotle0↑ is identical to Aristotle↑ entails that in a possible universe, this proposition is false, which contradicts *Supervenience*.

The failure of the previous solution indicates the necessity of providing a more *fine-grained* account of how different persons supervene their physical states. This suggests that we should attempt to divide the state $|\text{Aristotle0}\rangle$ into different parts in its mathematical formulation, each representing (or supervened by) a different person. For instance, we can rewrite the state before branching as follows:

$$\frac{1}{\sqrt{2}}(|\uparrow\rangle+|\downarrow\rangle)\otimes|\text{Aristotle }0\,\rangle=\frac{1}{\sqrt{2}}(|\uparrow\rangle+|\downarrow\rangle)\otimes\frac{1}{2}|\text{Aristotle }0(\uparrow)\rangle+\frac{1}{\sqrt{2}}(|\uparrow\rangle+|\downarrow\rangle)\otimes\frac{1}{2}|\text{Aristotle }0(\downarrow)\rangle$$

where Aristotle0↑ supervenes the state $|\text{Aristotle }0(\uparrow)\rangle$, and Aristotle0↓ supervenes on the state $|\text{Aristotle }0(\downarrow)\rangle$. Treating these as functions over a subset of the overall direct product of configuration spaces in the formulation of Quantum Mechanics, $|\text{Aristotle }0(\uparrow)\rangle$ and $|\text{Aristotle }0(\downarrow)\rangle$ should have the same value due to symmetry. (The quantum state of n particles, known as the "wave function", is a function defined over the direct product of n configuration spaces of the background space manifold). In other words, Aristotle0↑ and Aristotle0↓ are qualitatively identical, so we should expect that $|\text{Aristotle }0(\uparrow)\rangle$ and $|\text{Aristotle }0(\downarrow)\rangle$ have the same value. One might suggest that since $|\text{Aristotle }\uparrow\rangle$ and $|\text{Aristotle}\downarrow\rangle$ are different, $|\text{Aristotle }0(\uparrow)\rangle$ and $|\text{Aristotle }0(\downarrow)\rangle$ should have different values, accordingly. This proposal implies *teleology* or *fatalism*, making it hardly plausible. Suppose Aristotle does not measure the z-spin of an electron, but rather the sum of z-spins of two electrons, and the state $|\text{Aristotle }0\,\rangle$ keeps fixed; it seems that how different mental states supervene on $|\text{Aristotle }0\,\rangle$ should not be influenced by which measurement is going to be performed later. Furthermore, to distinguish $|\text{Aristotle }0(\downarrow)\rangle$ and $|\text{Aristotle }0(\downarrow)\rangle$ as different physical states, we ought to offer a different understanding of *what a physical state is* according to its mathematical formulation. This might require developing a new mathematical formulation of QM to differentiate them mathematically; we could envision reformulating QM as a kind of *fiber bundle* theory, where $|\text{Aristotle }0(\uparrow)\rangle$ and $|\text{Aristotle }0(\downarrow)\rangle$ represent different fibers upon the same element $|\text{Aristotle0}\rangle$ in the base space, or some other alternative approach.

In Section 7, I will discuss a proposal that this can be achieved without introducing any additional mathematical structures, only through a shift of metaphysics. Following this line, it is not necessary to propose that multiple mental states supervene one physical state. Instead, they may supervene on different physical states or different "worlds". However, even if we can distinguish $|\text{Aristotle }0(\uparrow)\rangle$ and $|\text{Aristotle }0(\downarrow)\rangle$ based on their mathematical forms, the challenge of *Supervenience* remains. Physical facts alone cannot ground why the person supervening on $|\text{Aristotle }0(\uparrow)\rangle$ is identical to the person who supervenes on $|\text{Aristotle }\uparrow\rangle$ rather than $|\text{Aristotle }\downarrow\rangle$, given that $|\text{Aristotle }0(\uparrow)\rangle$ and $|\text{Aristotle }0(\downarrow)\rangle$ have the same value. The formulation of a fiber bundle theory still lacks sufficient asymmetry to determine the relation, and the analysis presented in previous paragraphs can be equally applied here.

As Barrett [33] (pp. 185–206) suggests, giving a deterministic law of such identity mentioned above leads to some form of *hidden variable theories*. Such hidden variable theories are ad hoc if their acceptance is only for solving the issues of personal identity, implying that we have special *connecting rules* for mental entities, but not for all physical objects. Moreover, it remains challenging to determine how such connecting rules could be. For example, if we label $|\text{Aristotle} \uparrow\rangle$ with a hidden variable "↑", it could indicate a form of *fatalism* that Aristotle *must* measure the z-spin of the electron before branching. If Aristotle chooses to measure the x-spin of the electron instead, the hidden variable "↑" would hardly be effective in determining personal identity relations. I shall elaborate on this point in the next section with a particular example: the "divergence view" of EQM.

7. The Divergence View

Saunders [6] and Wilson [20,21] have developed the so-called "divergence view" of EQM that there are multiple qualitatively identical but numerically different *worlds* before the branching. The motivation of Saunders's proposal is to avoid the problems of Saunders and Wallace's [3] original solution to the incoherence problem, while the motivation of Wilson's proposal is probably to build a bridge between David Lewis's theory of possible world and EQM. Although Wilson claims that the choice between the divergence view and the overlapping view neutral in terms of the mathematical structure of EQM [20,21], and their proposal requires a *deep* understanding of the ontology of EQM. (In our scenario, for example, the view that there is only *one* world represented by the quantum state $\frac{1}{\sqrt{2}}(|\uparrow\rangle + |\downarrow\rangle) \otimes |\text{Aristotle } 0\,\rangle$, which has two different future branches, is attributed to the overlapping view.) It requires a substantial ontological difference whether $\frac{1}{\sqrt{2}}(|\uparrow\rangle + |\downarrow\rangle) \otimes |\text{Aristotle } 0\,\rangle$ represents one world or two qualitatively different worlds. I do not engage in the debate of whether we should accept the divergence view or the overlapping view in this paper. Instead, I argue that supposing the divergence view is correct, the discussions presented in Section 6 are still applicable to their proposal.

Saunders [6] attempts to make some room for multiple three-dimensional persons before branching by proposing that histories in EQM that share the same past *diverge* rather than *overlap*. (Saunders acknowledges to me that this is the motivation of Saunders's view on 20 October 2022 in [34].) Saunders uses an ordered pair $(\beta,\ |\alpha\rangle)$ to represent a *person*, where β is what Saunders calls a momentary configuration (our $|\text{Aristotle0}\rangle$ is an example), and $|\alpha\rangle$ is an "entire history" consisting of β (*ibid*., pp.191–192). In our case, there are at least two entire histories consisting of $|\text{Aristotle0}\rangle$, whereas they consist of $|\text{Aristotle}\!\uparrow\rangle$ and $|\text{Aristotle}\!\downarrow\rangle$, respectively. I call these histories $|\alpha\!\uparrow\rangle$ and $|\alpha\!\downarrow\rangle$ for convenience. It seems quite natural that $(|\text{Aristotle0}\rangle,\ |\alpha\!\uparrow\rangle)$ is identical to $(|\text{Aristotle}\!\uparrow\rangle,\ |\alpha\!\uparrow\rangle)$ and that $(|\text{Aristotle0}\rangle,\ |\alpha\!\downarrow\rangle)$ is identical to $(|\text{Aristotle}\!\downarrow\rangle,\ |\alpha\!\downarrow\rangle)$.

Following this line, there are multiple (three-dimensional) persons before the branching, and it seems that there can be some facts to ground Aristotle's curiosity about "whether I am Aristotle0↑ or Aristotle0↓" before the branching. But, this proposal still needs to be scrutinized following the analyses in Sections 5 and 6. Again, if Aristotle is uncertain of whether he is $(|\text{Aristotle0}\rangle,\ |\alpha\!\uparrow\rangle)$ or $(|\text{Aristotle0}\rangle,\ |\alpha\!\downarrow\rangle)$, what facts remain unknown for Aristotle? The situation is similar to the discussion in Section 5. Once again, he is $(|\text{Aristotle0}\rangle,\ |\alpha\!\uparrow\rangle)$ if we combine $|\text{Aristotle0}\rangle$ with a z-spin up future, and he is $(|\text{Aristotle0}\rangle,\ |\alpha\!\downarrow\rangle)$ if we combine $|\text{Aristotle0}\rangle$ with a z-spin down future. This is still a matter of choice rather than a kind of uncertainty.

This rejection might be too quick, and probably the core feature of the divergence view is overlooked. The proliferation of persons is grounded in the proliferation of *worlds*. This is more explicit in Wilson's writings that:

> Then the two histories are exactly similar up to and including the penultimate projection operator, but differ on the final projection operator—they agree at all times up to $t_{\mathrm{n}-1}$, but differ at t_{n}. The point at issue between the diverging and branching interpretations is whether the entities represented by the projection operators $\hat{P}_{\alpha_0} \ldots \hat{P}_{\alpha_{\mathrm{n}-1}}$ in $\mathrm{C}_{\underline{\alpha}}$ are numerically identical to the entities represented by the projection operators $\hat{P}_{\alpha'_0} \ldots \hat{P}_{\alpha'_{\mathrm{n}-1}}$ in $\mathrm{C}_{\underline{\alpha}\prime}$, or whether they are (numerically

> distinct) qualitative duplicates. Numerically identical entities give us overlapping worlds; qualitative duplicates give us diverging worlds [20] (p. 73).

Here, Wilson uses symbols of consistent histories. $\hat{P}_{\alpha_0} \ldots \hat{P}_{\alpha_{n-1}}$ and $\hat{P}_{\alpha'_0} \ldots \hat{P}_{\alpha'_{n-1}}$ represent the physical reality before the branching. $C_{\underline{\alpha}}$ and $C_{\underline{\alpha'}}$ represent the complete histories that are the same before the branching. $\hat{P}_{\alpha_0} \ldots \hat{P}_{\alpha_{n-1}}$ and $\hat{P}_{\alpha'_0} \ldots \hat{P}_{\alpha'_{n-1}}$ are exactly the same with respect to mathematical formalism, and Wilson claims that they can be used to represent different ontological realities before the branching: they represent two worlds before the branching. Therefore, there can be two qualitatively identical but numerically different persons Aristotle, Aristotle0↑ or Aristotle0↓, who exist in different worlds, respectively. Aristotle0↑ will see the *z*-spin is up and the future observational result for Aristotle0↓ will be down, making it reasonable for Aristotle to be uncertain whether he is Aristotle0↑ or Aristotle0↓.

This possibility is discussed in Section 6, where it is proposed that distinguishing different physical states before the branching would require some more fine-grained mathematical structures, such as a *fiber bundle*. Wilson's approach does not require a different mathematical structure of EQM, but a different metaphysical structure of it. I do not intend to reject such metaphysical possibility here. However, we still need to address the question raised in Section 5: Is personal identity here, as a relation, deterministic or indeterministic? For simplicity, I suppose without loss of generality that Aristotle that lies in the world $\hat{P}_{\alpha_0} \ldots \hat{P}_{\alpha_{n-1}}$ is Aristotle0↑, and the Aristotle that lies in the world $\hat{P}_{\alpha'_0} \ldots \hat{P}_{\alpha'_{n-1}}$ is Aristotle0↓. Suppose that $C_{\underline{\alpha}}$ is the branch where Aristotle sees the *z*-spin is up, and $C_{\underline{\alpha'}}$ is the branch where Aristotle sees the *z*-spin is down. If the relation (personal identity) is indeterministic, it would violate *Supervenience*. One might argue that the identity of worlds across time is indeterministic, and thus *Supervenience* is preserved: In each case, the identity of Aristotle strictly follows the identity of worlds. According to this view, if the world $\hat{P}_{\alpha_0} \ldots \hat{P}_{\alpha_{n-1}}$ is identical (across time) to the world where Aristotle sees the *z*-spin is up, then Aristotle0↑ is identical to Aristotle↑, not Aristotle↓. However, this introduces indeterminacy of the identity between worlds. Supporters of the divergence view cannot deny that this is an additional character that originally EQM did not have: indeterminacy.

If such a relation (personal identity) is deterministic, it must be grounded in some physical facts that establish a deterministic connection between worlds (or the identity of worlds across time, in other words). In this case, $\hat{P}_{\alpha_0} \ldots \hat{P}_{\alpha_{n-1}}$ is connected to the (future) world where Aristotle sees the *z*-spin is up, and $\hat{P}_{\alpha'_0} \ldots \hat{P}_{\alpha'_{n-1}}$ is connected to the world where Aristotle sees the *z*-spin is down after the branching. This introduces *hidden variables* into EQM: Each qualitatively identical world before the branching is labeled with a hidden variable to determine its future successor. This notion is termed "many-threads theory" by Barrett, as Barrett explains that:

> That is, if one includes the global wave function in the state description of the worlds, then each world might be thought of as being described by a particular hidden-variable theory, where the preferred basis selects the always determinate physical quantity (the hidden variable), the local state of each world at a time gives the value of this quantity in that world, and the connection rule (together with the linear dynamics) determines, in so far as it is determined, how the quantity evolves in each world: A many-threads theory is ultimately just a hidden-variable theory where one simultaneously considers all physically possible worlds [33] (pp. 183–184). (It seems to me that Wilson does not pay much attention to Barrett's alarm in Wilson's writings. Wilson only cites Barrett once in [35] without mentioning this point. I am grateful to Shan Gao who reminds me of Barrett's writing.)

Wilson [20] (p. 69) does acknowledge that "'Many worlds' or 'many minds' theories which posited additional fundamental structure would not be worth the price." It is not necessary to introduce hidden variables into the divergent view in discussing the ontology of EQM, so Wilson does not need to be concerned with that in [20]. However, this is indeed a problem if we want to solve the incoherence problem of EQM via pre-measurement uncertainty. If we want to avoid complicating EQM as a physical theory, we have to introduce a connection rule to determine the successor of different qualitatively identical persons before the branching, which leads to a violation of *Supervenience*. The introduction of the divergence view here serves as an illustration of the various possibilities discussed in Section 6.

8. Conclusions

So far, I have examined approaches to solve the incoherence problem of EQM via pre-measurement uncertainty. Through a comprehensive analysis of Saunders and Wallace's solution based on David Lewis's account of personal identity, I have argued that the pre-measurement solution to the incoherence problem cannot be successful if only one mental state supervenes each observer's physical state in EQM. This need not prove fatal to the pre-measurement approach if there can be multiple qualitatively identical but numerically different mental states supervening each observer's physical state. However, the latter approach can only be successful while violating principles of physicalism. I use the "divergence view" of EQM as an example to illustrate my argumentation. As I have argued in Section 6, this brings us back to the old problems of EQM. Either we need to accept a form of "Many Worlds Theory" by introducing hidden variables into EQM, or we have to develop a kind of "Many Minds Theory" that violates principles of physicalism. My analysis in this paper is impartial regarding the adoption of three-dimensionalism or four-dimensionalism, as well as the overlapping view or the divergence view of EQM. My argument also circumvents the debates on the theory of semantics and reference, upon which previous criticisms of Saunders and Wallace's proposal have rested.

An anonymous reviewer reminds me that "at the Tel Aviv conference [34], several participants argued for the introduction of hidden variables to Many Worlds theory and also for the introduction of objective probability, also distinctly non-Everettian." Indeed, this remains a possibility for EQM. However, after introducing non-Everettian elements into EQM, it still needs to be justified why EQM should be preferred over other interpretations of quantum mechanics. This may be encouraging news for those who favor post-measurement uncertainty or probability without uncertainty in EQM, though I believe that those solutions have their own problems. Discussing these options goes beyond the scope of this paper. For those who are reluctant to complicate our physical theories by adding non-Everettian elements to EQM, embracing non-physicalism remains an option. In this sense, I believe that the Many Minds Interpretation (MMI) deserves more attention than it has received in the literature today.

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Article

Quantum Probability from Temporal Structure

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Abstract: The Born probability measure describes the statistics of measurements in which observers self-locate themselves in some region of reality. In ψ-ontic quantum theories, reality is directly represented by the wavefunction. We show that quantum probabilities may be identified using fractions of a universal multiple-time wavefunction containing both causal and retrocausal temporal parts. This wavefunction is defined in an appropriately generalized history space on the Keldysh time contour. Our deterministic formulation of quantum mechanics replaces the initial condition of standard Schrödinger dynamics, with a network of 'fixed points' defining quantum histories on the contour. The Born measure is derived by summing up the wavefunction along these histories. We then apply the same technique to the derivation of the statistics of measurements with pre- and postselection.

Keywords: Born rule; Keldysh time contour; Everett interpretation

1. Introduction

Textbook formulations of quantum mechanics contain the following:

(a) An *ontological* postulate—The state of a physical system is represented by a wavefunction $|\Psi\rangle$;
(b) A *dynamical* postulate—The state evolves deterministically according to the time-dependent Schrödinger equation (TDSE);
(c) A *composition* postulate—The state space of a composite system is the tensor product of the spaces of its subsystems;
(d) A *statistical* postulate—The probability of each measurement outcome is given by the Born measure.

Perhaps the main obstacles to understanding quantum theory lie in explaining the appearance of the probabilistic element in postulate (d) [1] and deriving the mathematical form of the Born rule. In ψ-ontic quantum theories, the universal wavefunction is in direct correspondence with physical reality [2–4]. There have been several attempts to derive and/or explain (d) from postulates (a–c) within a ψ-ontic framework [5–9], with no reference to the physical 'collapse' of wavepackets, to thereby solve the hard part of the measurement problem. Some derive the Born measure by placing 'rationality' constraints on the beliefs of observers [5,8], but such theories have it backwards—rational beliefs do not determine regularities in nature. Rather, the structure of nature grounds measurement statistics and, therefore, determines what is rational to believe. Other derivations use symmetry arguments [6,7,9], but these rely on auxiliary formal assumptions and a separation of the quantum state into the system plus the environment. Also, these approaches are based upon vigorously-debated concepts of probability, rather than the ontology of the physical theory itself. A new perspective on the problem of probability in quantum mechanics is sorely needed.

The concept of the probability of self-location avoids the need for randomness in quantum mechanics and enables the assignment of probabilities to branches of the wavefunction without recourse to genuine randomness in nature [9–11]. However, in the present work we go further: since the self-location of an observer is carried out with respect to the

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wavefunction itself, the probability of being located in a region of the wavefunction should be literally equated with its relative proportion of the total wavefunction and, therefore, grounded in physical ontology alone, i.e., *the probability measure must therefore emerge from the internal structure of the wavefunction itself*. This invites a ψ-ontic explanation of the appearance of chance in quantum mechanics: an observer is localized to a region of the wavefunction that is consistent with experiments. The Born rule quantifies the relative amount of reality, or the, 'measure of existence', of that region [11,12].

The problem with treating time quantum mechanically is an apparently unrelated foundational question, which, however, has recently attracted a great deal of attention. In particular, time appears as a background parameter in postulate (b), but this way of representing time is at odds with the geometric notion of time in general relativity [13,14]. Proposals for measurements of quantum time generally focus on the theoretical absolute time of a quantum state and the introduction of a background 'quantum clock' degree of freedom to measure 'arrival times' of particles at a detector arising from entanglement between subsystems [15–17]. Traditionally, the problem of defining time in quantum mechanics is presented as the problem of defining an Hermitian operator with monotonically increasing eigenvalues for a system with Hamiltonian bounded from below [18]. It can be shown that such an operator always leads to finite amplitudes for the reverse-time process [19].

Far from being a hindrance to the description of quantum time, however, we may elevate reverse-time causal processes to a central feature of the theory implicit in the unitary evolution of states with complex amplitudes. In 1964 [20], Aharonov et al. published the time-symmetric two-state vector formalism (TSVF) [21,22], describing the probabilities of measurements sandwiched between pre- and postselections with the Aharonov–Bergmann–Lebowitz (ABL) rule. The TSVF was later generalized to a multiple-time formalism, assigning a Hilbert space $\mathcal{H}_t$ and its conjugate space $\mathcal{H}_t^\dagger$ (for backwards-directed states) to each instant of time, i.e., the composition postulate (c) was applied to treat time instants as distinct quantum subsystems [23,24]. The wavefunction is then a global time-extended structure composed of *temporal parts* [25]. It was recently shown that this assignment of two Hilbert spaces to each moment in time is necessary to capture all the correlations in the quantum dynamical evolution of a particle with an equivalent multipartite state [26]. The experimental success of the TSVF [27–29], various explicitly time-symmetric formulations [30–35] and recent demonstrations of indefinite causal ordering [36–39] all provide evidence for a more complex causal structure in nature than a single background time parameter can offer.

By coincidence, the year 1964 saw publication of another time-symmetric formalism by Keldysh [40]. The resulting Nonequilibrium Green's function (NEGF) theory describes the propagation of correlation functions along a time contour C composed of both forwards (f) and backwards (b) time branches [41,42]. Keldysh-based methods have been successfully applied to a vast range of physical problems in fields as diverse as inflationary cosmology, molecular electronics, quantum thermodynamics and photovoltaics [42–49]. Note that this contour time structure itself does not logically presuppose the Born measure, although propagating statistical averages on this contour is equivalent to weighting them with Born probabilities.

In this paper we take advantage of this logical equivalence, showing that the derivation of the Born measure is possible from unitary dynamics and wavefunction structure alone, given a wavefunction-based definition of probability. We incorporate the full Keldysh causal structure of quantum mechanics within the universal wavefunction and model temporally local events in terms of 'fixed point' boundary conditions. We therefore refer to this version of quantum mechanics as the fixed point formulation (FPF). We then introduce a statistical postulate based on our probability definition and derive the correct probability measure from ontological and dynamical postulates, describing unitary evolution in Hilbert space without random collapse. Thus, we reduce the number of independent postulates in the quantum theory—the Born measure follows from ontology, composition and dynamics.

2. The Universal Wavefunction

2.1. General Considerations

We wish to focus on the global temporal structure of wavefunctions composed of both macroscopic and microscopic parts, without approximation or tracing out environmental degrees of freedom. Such a wavefunction represents the observer, the system being observed and the environment in a typical quantum experiment, and it is in this sense that we refer to it as ‘universal’. We start from a strong ψ-ontic standpoint, with the following conceptual desiderata:

Completeness:The wavefunction is all that exists—it contains all physical properties of nature at all moments in time;
Measurement Physicality: Measurements are physical processes occurring within temporal regions of the universal wavefunction;
Event Symmetry: The local description of nature is independent of event location. There are no ontologically privileged spacetime points;
Self-Location: Temporal boundary constraints provide the only information an observer can use to locate themselves within the wavefunction.

The concept of probability developed here utilizes the principle of **Self-Location**.

Definition 1. *(Quantum probability)*
In a temporal region of the wavefunction defined by some set of constraints, process A has probability $p(A) = x$ if and only if A occurs in a fraction x of the total available wavefunction.

Given said constraints, an observer should set their subjective degree of belief that they are located in a region of reality where A occurs corresponding to the fraction of reality, i.e., to the quantum probability. This approach is *logically minimal, physically maximal*—it grounds the mathematical theory of probability in physical ontology. A proponent of **Completeness** must then answer the question:

Which structural feature of the wavefunction implies the Born measure?

To begin to answer this, we observe that recent works in quantum cosmology which describe physical systems with sequences of time-indexed properties (described by projection operators) or ‘histories’ [50,51]. Given a time ordering of N_t times at which physical properties are instantiated, $t_{N_t} > t_{N_t-1} > \ldots > t_1$, reality can be described by a ‘universal’ wavefunction $|\Psi_U\rangle$ specifying the full set of histories defined on these times. In the histories formalism, each value of the time t_i labels a distinct subspace $\mathcal{H}_{t_i}$ of the *history* Hilbert space [52,53]:

$$\mathcal{H}_H \equiv \mathcal{H}_{t_{N_t}} \otimes \ldots \otimes \mathcal{H}_{t_1} \tag{1}$$

In this space, history states can be viewed as ’records’ of all the different stages in a quantum process, indexed by time. Thus, the states at distinct times enter the wavefunction in an atemporal fashion suited to a block universe point of view.

Parallel to the consistent histories approach, products of time-localized Hilbert spaces feature in the time-symmetric approach to quantum mechanics, pioneered by Aharonov et al. [20]. This approach, which became the two state vector formalism (TSVF) [21] and its multiple time generalizations [24,26], treats quantum measurements which include dynamical boundary conditions on past and future times symmetrically.

This is useful in the description of a system defined at time t occurring between preselection and postselection measurements at the times t_1 and t_2, respectively. The preselected state $|\psi(t_1)\rangle$ then travels forwards in time across the interval $[t_1, t]$ in accordance with the TDSE, and the postselected state is represented by a vector in the conjugate space $\langle\phi(t_2)|$ which propagates backwards across the time interval $[t, t_2]$. The two oppositely orientated parts of the system can then be combined into a single ‘two state vector’:

$$\langle\phi(t_2)| \otimes |\psi(t_1)\rangle, \tag{2}$$

which exists in the composite Hilbert space constructed from distinct time-localized 'universes' existing at single times [26]:

$$\mathcal{H}_{t_2}^{\dagger} \otimes \mathcal{H}_{t_1} \tag{3}$$

States in this Hilbert space are fundamentally (i) time non-local objects and (ii) built out of parts with opposite time orientations, which immediately suggests that this is a promising avenue to explore for the development of a quantum theory of events. On this account, the solution to the apparent asymmetry under time reversal in quantum mechanics is to revise the notion of a quantum state itself to include two time degrees of freedom.

According to the TSVF, to obtain the probability of measuring the system in some state $|a_i\rangle$ at the intermediate time $t \in [t_1, t_2]$, the system is propagated in *both* time directions, from $t_1 \to t$ and $t_2 \to t$, such that the amplitude of the i-th outcome is given by sandwiching this state between the forwards and backwards-oriented parts of Equation (2):

$$\langle \phi(t_2)|U(t_2,t)|a_i\rangle\langle a_i|U(t,t_1)|\psi(t_1)\rangle \tag{4}$$

Then, assuming the Born rule, the normalized modulus-square of this yields the probability to obtain outcome a_i:

$$P_{a_i} = \frac{|\langle \phi(t_2)|U(t_2,t)|a_i\rangle\langle a_i|U(t,t_1)|\psi(t_1)\rangle|^2}{\sum_k |\langle \phi(t_2)|U(t_2,t)|a_k\rangle\langle a_k|U(t,t_1)|\psi(t_1)\rangle|^2} \tag{5}$$

This is the ABL probability rule. In the quantum theory, it thus appears that the past and future affect each other symmetrically [28]. However the TSVF relies upon a wavefunction with temporal parts whose behavior depends on time position. Specifically, the preselected state at t_1 is a source of physical processes occurring between t_1 and t, and the postselected state at t_2 is a source for processes connecting t_2 to t. However, the state at time t serves as a unique sink for both types of processes. Clearly, this is a violation of **Event Symmetry**—if, given two connected points in time, one is a source and the other a sink, and the dynamics are allowed to be time symmetric, it must follow that both points are sources and both are sinks for all time regions they are connected to.

2.2. The Universal Wavefunction on the Keldysh Contour

A wavefunction-based theory must contain a representation of the temporal processes occurring in field theories defined on an appropriate time domain. For systems consisting of particles obeying fermionic or bosonic statistics, that is, carried out using the NEGF formalism, which is used to evaluate time-dependent expectation values of quantum observables, $O(t_2)$ propagated from some initial time t_1:

$$O(t_2) = \mathrm{Tr}\big[\rho_1 U(t_1,t_2)\hat{O}(t_2)U(t_2,t_1)\big], \tag{6}$$

where ρ_1 is the density matrix at t_1 and $U(t_2, t_1)$ is the unitary evolution between times t_1 and t_2. The expression in Equation (6) can be evaluated via two separate propagations, the first running forwards in time from t_1 to t_2, at which the operator $\hat{O}$ acts, before the system is propagated backwards from t_2 to t_1. This can be visualized in terms of propagation along the Keldysh time contour shown in Figure 1. The Keldysh contour consists of an 'upper' branch C_f of times t^f on which the wavefunction travels in the forwards direction, and a 'lower' branch C_b of times t^b on which the dynamics is reversed.

We propose a similar physical state space to $\mathcal{H}_H$, with the caveat that temporal degrees of freedom take values on both branches of the Keldysh time contour. Thus, given an ordering of N_t times $t_{N_t} > t_{N_t-1} > \ldots > t_1$, there are two corresponding causal orderings, one on each branch of $C \equiv C_b \oplus C_f$:

$$t^f_{N_t} >_C t^f_{N_t-1} >_C \ldots >_C t^f_1 \tag{7}$$

$$t^b_{N_t} <_C t^b_{N_t-1} <_C \ldots <_C t^b_1 \tag{8}$$

where the contour-ordering notation $>_C$, $<_C$ is introduced as in Ref. [41]. This is the main innovation of the Keldysh contour: ordering in time is distinct from causal ordering, since causal influences propagate in the antichronological direction on the lower branch C_b.

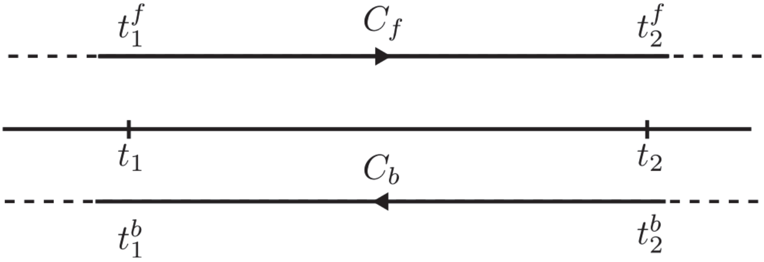


Figure 1. The Keldysh time contour on the time interval $[t_1, t_2]$.

Each of the N_t times in a history possesses two associated Hilbert spaces for the f and b components. Hence, the universal wavefunction has $2N_t$ temporal degrees of freedom and is a member of the *contour* Hilbert space:

$$\mathcal{H}_C = \mathcal{H}^b_{t_{N_t}} \otimes \mathcal{H}^f_{t_{N_t}} \otimes \ldots \otimes \mathcal{H}^b_{t_1} \otimes \mathcal{H}^f_{t_1} \tag{9}$$

A wavefunction in this space is not defined at a single fixed 'present', but at a sequence of moments with oppositely oriented temporal parts acting as 'source' or 'sink' states for processes on the branches C_f and C_b.

We make a corresponding first postulate:

Ontological postulate

The universal wavefunction $|\Psi_U\rangle \in \mathcal{H}_C$ *is a 'stack' of* $2N_t$ *temporal parts with fixed ordering on C, dividing time into* $2(N_t - 1)$ *separate regions:*

$$|\Psi_U\rangle = \bigotimes_{i=1}^{N_t} \left|\Psi^b\left(t_i^b\right)\right\rangle \otimes \left|\Psi^f\left(t_i^f\right)\right\rangle \tag{10}$$

Here, $\left|\Psi^\alpha\left(t_i^\alpha\right)\right\rangle$ is restricted to the C_α time branch, and, in general, $\left|\Psi^f\left(t_i^f\right)\right\rangle \neq \left|\Psi^b\left(t_i^b\right)\right\rangle$. The inner product is defined on the Hilbert space $\mathcal{H}^\alpha_{t_i}$ in the usual way, such that $\langle\Psi_U|\Psi_U\rangle = 1$, which implies $\left\langle\Psi^\alpha\left(t_i^\alpha\right)\middle|\Psi^\alpha\left(t_i^\alpha\right)\right\rangle = 1$ for any α. Oppositely-oriented parts of the wavefunction are connected independently on C_f and C_b. We note that Equation (10) can be generalized to contain a summation over all possible tensor products of time-localized states and thereby represent all multiple-time processes on the Keldysh contour, but, for the purposes of the present work, we focus on the case of fixed sequence size N_t, following the histories formulation [50]. We now introduce the second core postulate:

Dynamical postulate

The time derivative of the wavefunction at each point on C is given by the TDSE:

$$i\hbar\partial_{t^\alpha}|\Psi^\alpha(t^\alpha)\rangle = H^\alpha(t^\alpha)|\Psi^\alpha(t^\alpha)\rangle \tag{11}$$

Note that in every case of physical interest the Hamiltonian operator is branch-independent, i.e., it takes on values on the upper/lower branches which are equal for the same physical time, $H^b\left(t^b\right) = H^f\left(t^f\right)$. For simplicity, indices on time arguments are dropped, $|\Psi^\alpha(t^\alpha)\rangle \equiv |\Psi^\alpha(t)\rangle$.

The TDSE in Equation (11) defines a unitary mapping $U^\alpha(t_2, t_1) : \mathcal{H}^\alpha_{t_1} \mapsto \mathcal{H}^\alpha_{t_2}$ between the Hilbert spaces of different times on a single branch $|\Psi^\alpha(t_2)\rangle = U^\alpha(t_2, t_1)|\Psi^\alpha(t_1)\rangle$, where $U^\alpha(t_2, t_1) \equiv U^\alpha\left(t^\alpha_2, t^\alpha_1\right)$ has the form [41]

$$U^\alpha(t_2, t_1) = \hat{T}_C \exp\left[-\frac{i}{\hbar}\int_{t^\alpha_1}^{t^\alpha_2} d\tau H^\alpha(\tau)\right] \tag{12}$$

and $\hat{T}_C$ orders operators chronologically (latest to the left) on C_f and anti-chronologically on C_b.

3. One Fixed Point

A sequence of events in time corresponds to a sequence of time-indexed projectors in the consistent histories language, and we now construct a model of an event on the Keldysh contour suitable for combination into similar history sequences.

We may isolate temporal parts of $|\Psi_U\rangle$ from the main tensor product of Equation (10). A *fixed point* has identical parts on the two contour branches, corresponding to a 'turning point' on the Keldysh contour at time t_1, i.e., to a point at which the time propagation along C switches from the upper to the lower branch [41]:

Definition 2. *(Fixed Point)*
A fixed point at time t is a temporal part of the wavefunction in the $\mathcal{H}^b_t \otimes \mathcal{H}^f_t$ subspace, with equal f and b parts.

Given a specification (via a preparation measurement or theoretical description) of the state $|\psi\rangle$ of a system at some time t_1, all quantum histories in $|\Psi_U\rangle$ consistent with this specification are constrained, regardless of the contour branch. As such, there is a fixed point state at t_1, which is denoted:

$$[\![\psi]\!]_{t_1} \equiv \left|\psi^b(t_1)\right\rangle \otimes \left|\psi^f(t_1)\right\rangle \tag{13}$$

This corresponds to an event in which the state is specified with definite properties at t_1 (or a time-indexed projection, in the consistent histories language). We note that in the context of the TSVF, the existence of a new boundary condition at each measurement event was explicitly denied in Ref. [22], but in fact such boundary conditions are necessary for a full specification of the quantum state at all times.

We may think of the 'present' time t as 'pinched' in between the upper-branch and lower-branch times t^f, t^b. The fixed point state connects to other points on C in both time directions, in accordance with Equation (11). It is represented on C in Figure 2: the forward-directed part of the fixed point defined at t travels to times occurring 'later' than t^f on C_f, and the backward-directed part travels to times occurring 'later' than t^b on C_b. Each fixed point is connected to four temporal regions: it acts as a 'source' of wavefunction in both time directions (the thick black arrows on Figure 2), and a 'sink' for parts of the wavefunction propagating from times lying 'earlier' on C (dashed lines on Figure 2). Thus, for a full description of a measurement connecting times across the region $[t_1, t_2]$, at least two fixed points are required, i.e., $N_t \geq 2$ in Equation (10). A quantum history sequence is defined in these terms:

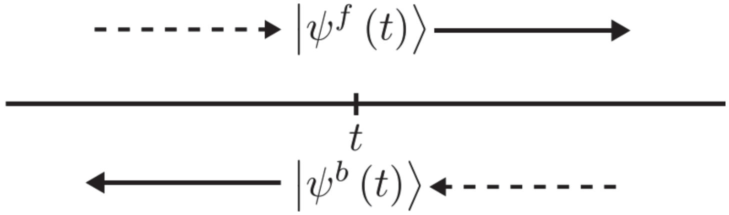


Figure 2. A single fixed point on the Keldysh contour.

Definition 3. *(Quantum history)*

A quantum history $|h_{\mathbf{k}}\rangle$ *extending across the time range* $[t_1, t_2]$ *is a product state constructed from a sequence* $\mathbf{k} = \langle k_1, \ldots, k_{N_t}\rangle$ *of* $N_t \geq 2$ *fixed points:*

$$|h_{\mathbf{k}}\rangle = \bigotimes_{i=1}^{N_t} [\![\psi_{k_i}]\!]_{t_i} \tag{14}$$

connected by unitary mappings and bounded by fixed points at t_1 *and* t_2.

In Equation (14), each k_i in a history $|h_{\mathbf{k}}\rangle$ ranges over a complete basis set spanning $\mathcal{H}^{\alpha}_{t_i}$. To allow us to apply the usual rules of probabilistic reasoning to quantum histories, we define a *family* of quantum histories $\mathcal{F}_H$ by imposing the consistency condition that any pair of histories in a family $\{|h_{\mathbf{k}}\rangle\}$ must be non-overlapping:

$$\langle h_{\mathbf{l}} | h_{\mathbf{k}}\rangle = \delta_{\mathbf{kl}}, \tag{15}$$

where $\mathbf{k} \neq \mathbf{l}$ if $[\![\psi_{k_i}]\!]_{t_i} \neq [\![\psi_{l_i}]\!]_{t_i}$ for at least one value of $i \in [1, \ldots, N_t]$. Each set of quantum histories provides distinct but complementary descriptions of the system over time, which may or may not correspond to measurement events. Note that the consistency condition Equation (15) prevents the overlap of histories composed of different numbers of times N_t.

Now, following the terminology of Vaidman [12], the *measure of existence* of a history may be defined as the relative size of the wavefunction region occupied by that history.

Definition 4. *(Measure of existence)*

The measure of existence $m(h_{\mathbf{k}})$ *of a quantum history* $|h_{\mathbf{k}}\rangle$ *containing* N_t *fixed points in the time range* $[t_1, t_2]$ *is the ratio of the integral of the wavefunction* $\triangle\Psi_{\boldsymbol{k}}$ *along this history, to that of all histories:*

$$m(h_{\boldsymbol{k}}) = \frac{\triangle\Psi_{\boldsymbol{k}}}{\sum\limits_{\mathbf{k}'} \triangle\Psi_{\boldsymbol{k}'}} \tag{16}$$

in a family $\mathcal{F}_H$ *consistent with the fixed point boundary conditions at* t_1 *and* t_2.

Fixed point boundary conditions are imposed by taking the inner product of the integrated wavefunction with the 'sink' state defined at the upper limits of the $2(N_t - 1)$ segment integrals. Definition 4 gives precise meaning to the fraction of wavefunction connecting distinct events and, therefore (by Definition 1), a precise foundation for quantum probability:

Statistical postulate (Vaidman rule):

The quantum probability of a quantum history is equal to its measure of existence in the universal wavefunction.

Note that no explicit formula has been assumed for the measure of existence. The Vaidman rule is a *conceptual* postulate about the physical foundation of measurement statistics. It remains to be proven that this postulate implies the correct *mathematical* formalism in the case of a quantum measurement.

4. The Born Measure

By **Measurement Physicality** and **Self-Location**, the physical process of an experiment occurs within a region of $|\Psi_U\rangle$ subject to the boundary constraints determined by the preparation. The measure of existence is now evaluated for the simplest type of quantum history—a two-time measurement—with $N_t = 2$.

Consider a measurement of $|\phi(t_2)\rangle$ following a preparation of the state $|\psi(t_1)\rangle$. Without loss of generality, the prepared and measured states are taken to be members of complete bases $|\psi\rangle = |\psi_1\rangle \in \{|\psi_i\rangle\}$, $|\phi\rangle = |\phi_1\rangle \in \{|\phi_i\rangle\}$. The measurement then defines a family of histories:

$$\mathcal{F}_H : \left\{ [\![\phi_i]\!]_{t_2} \right\} \otimes [\![\psi]\!]_{t_1} \tag{17}$$

whose measure of existence in $|\Psi_U\rangle$ can be evaluated.

The 'source' term in this measurement process is the following state constructed from two fixed points (using the notation from Equation (13)):

$$\Psi\left(t_2^b, t_2^f, t_1^b, t_1^f\right) = [\![\phi]\!]_{t_2} \otimes [\![\psi]\!]_{t_1} \tag{18}$$

The total change in this wavefunction across the time interval $t \in [t_1, t_2]$ is computed by 'filling in' the Keldysh contour branches connecting the two fixed points. The exact differential of the wavefunction in Equation (18) constrained to this region is as follows:

$$d\Psi = \frac{\partial\Psi\left(t_2^b, t_2^f, t_1^b, t_1^f\right)}{\partial t_1^f} dt_1^f + \frac{\partial\Psi\left(t_2^b, t_2^f, t_1^b, t_1^f\right)}{\partial t_2^b} dt_2^b \tag{19}$$

i.e., one may consider time integrations in both the forwards direction originating at t_1^f and in the backwards direction from t_2^b. The total change in wavefunction is computed from the line integral along C, taking the path $\left(t_1^f, t_2^b\right) \to \left(t_2^f, t_2^b\right) \to \left(t_2^f, t_1^b\right)$:

$$D\Psi = \int_{t_1^f}^{t_2^f} \frac{\partial\Psi\left(t_2^b, t_2^f, t_1^b, x\right)}{\partial x} dx + \int_{t_2^b}^{t_1^b} \frac{\partial\Psi\left(y, t_2^f, t_1^b, t_2^f\right)}{\partial y} dy \tag{20}$$

Applying the branch TDSE in Equation (11) to the independent degrees of freedom in Equation (18) and allowing for cancellations, this becomes the following:

$$D\Psi = \left(U^b\left(t_1^b, t_2^b\right) U^f\left(t_2^f, t_1^f\right) - \hat{\mathbf{I}}\right) \Psi\left(t_2^b, t_2^f, t_1^b, t_1^f\right) \tag{21}$$

where $\hat{\mathbf{I}}$ denotes the identity on the Hilbert space $\mathcal{H}_{t_2}^b \otimes \mathcal{H}_{t_2}^f \otimes \mathcal{H}_{t_1}^b \otimes \mathcal{H}_{t_1}^f$, and the compact notation $U^b\left(t_1^b, t_2^b\right) U^f\left(t_2^f, t_1^f\right) \equiv U^b\left(t_1^b, t_2^b\right) \otimes \hat{\mathbf{I}}_{t_2^f} \otimes \hat{\mathbf{I}}_{t_1^b} \otimes U^f\left(t_2^f, t_1^f\right)$ is used.

Unitary evolution from any fixed point produces quantum superpositions represented by a network structure connecting it to other fixed points in the future and past, as shown in Figure 3. In this figure, the arrows indicate the temporal orientation of quantum processes. Each fixed point in the expansion of the full wavefunction at a given time is a node where processes begin and terminate in the network, similarly to Ref. [54].

The region of wavefunction constrained by two fixed points is represented by the purple region in Figure 3. All the processes consistent with the preparation, defining the family $\mathcal{F}_H$, are represented by black lines. Blue lines represent those processes not connected to the fixed point state $[\![\psi]\!]_{t_1}$. From the birds-eye perspective of the universal wavefunction, there is no difference between the black- and blue-line processes. However, they provide a useful distinction for the observer, who thereby determines the region of wavefunction corresponding to their experiment. Formally, if a single node is connected to N others, then there will be N Keldysh contour regions and $2N$ separate time branches connected to this node. Moreover, we can view each node as both the source and the sink of all processes connected to it, in the following sense: if a fixed point at time t is connected to N_{t_1} nodes at a time $t_1 < t$ and to N_{t_2} nodes at a time $t_2 > t$, then it is the source of exactly $N_{t_1} + N_{t_2}$ branch lines which flow away from it, and is the sink for the same number of lines which flow into it from other times. If N_{t_1} nodes at t_1 are connected to N_{t_2} nodes at t_2, then there are $2N_{t_1}N_{t_2}$ branch lines connecting this pair of times, defining $N_{t_1}N_{t_2}$ regions of the wavefunction, or two-way channels, connecting pairs of fixed points at these times. Every line is in one-to-one mapping with a directed process connecting a pair of fixed points in the wavefunction. The amplitude for a process connecting state $|\beta\rangle$ at time t_a to the state $|\gamma\rangle$ at time t_b following propagation along the Keldysh branch C_α is given as shown:

$$c_{\gamma\beta}^{\alpha}(t_b, t_a) \equiv \langle\gamma^\alpha(t_b)|U^\alpha(t_b, t_a)|\beta^\alpha(t_a)\rangle \tag{22}$$

The integrated wavefunction (the purple region in Figure 3) connects the two fixed points $[\![\psi]\!]_{t_1}$ and $[\![\phi]\!]_{t_2}$, a constraint imposed by taking the inner product of (21) with the 'sink' state

$$\left|\psi^b(t_1)\right\rangle\left|\phi^f(t_2)\right\rangle\left|\psi^b(t_1)\right\rangle\left|\phi^f(t_2)\right\rangle \tag{23}$$

defined at the upper limits of the integration. This gives:

$$\triangle\Psi[\psi(t_1);\phi(t_2)] = c^b_{\psi\phi}(t_1,t_2)c^f_{\phi\psi}(t_2,t_1), \tag{24}$$

where the overlap with the second term in Equation (21) vanishes since $\langle\gamma^\alpha(t_b)|\,\beta^\alpha(t_a)\rangle = 0$, given $|\beta^\alpha(t_a)\rangle \in \mathcal{H}_{t^\alpha_a}$ and $|\gamma^\alpha(t_b)\rangle \in \mathcal{H}_{t^\alpha_b}$ with $t^\alpha_a \neq t^\alpha_b$. Equation (24) is just a scalar-valued function, so contour branch labels can be dropped. We now divide by the normalization factor across all measurement outcomes consistent with the preparation, $\sum_i \triangle\Psi[\psi(t_1);\phi_i(t_2)] = 1$, to give the measure of existence of this history:

$$\begin{aligned} m\left(h_{\langle\psi,\phi\rangle}\right) &= \frac{\triangle\Psi[\psi(t_1);\phi(t_2)]}{\sum_i \triangle\Psi[\psi(t_1);\phi_i(t_2)]} \\ &= |\langle\psi(t_1)|U(t_1,t_2)|\phi(t_2)\rangle|^2 \end{aligned} \tag{25}$$

This gives the relative amount of wavefunction connecting the fixed point $[\![\psi]\!]_{t_1}$ to $[\![\phi]\!]_{t_2}$ as a proportion of the total region of wavefunction at t_2 connected to $[\![\psi]\!]_{t_1}$ on the Keldysh contour.

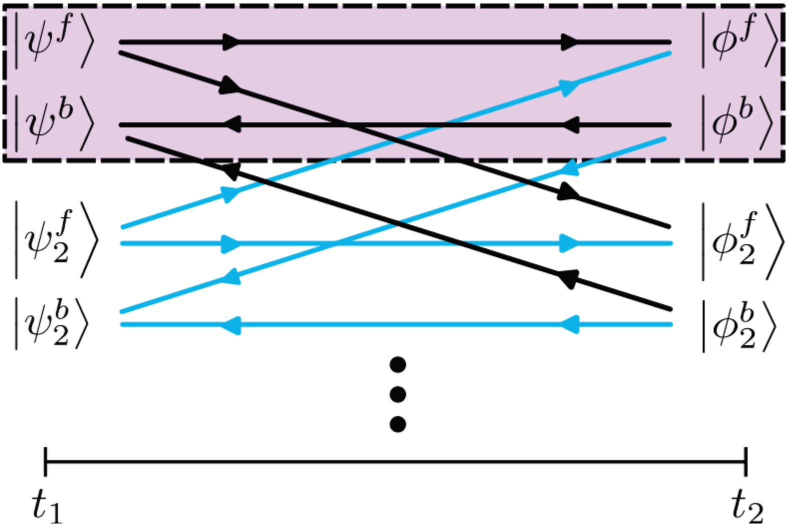


Figure 3. The purple region represents the measure of existence connecting the prepared fixed point state $[\![\psi]\!]_{t_1}$ to a measurement at t_2 described by the fixed point $[\![\phi]\!]_{t_2}$. The black lines represent processes connected to the prepared state. The blue lines represent regions of the universal wavefunction that are incompatible with the preparation.

Equation (25) is the core result of this work: the measure of existence of a quantum history describing a quantum measurement process equals the Born measure. The power of two in this measure is a direct result of the two time branches in C. Instead of postulating the mathematical form of the measure of existence [11], the Born measure has been derived from the temporal structure of the multiple-Keldysh-time wavefunction and the Vaidman rule.

5. Three Fixed Points

The ABL rule may be derived from the Born measure [21], but will now be derived as the measure of existence of the quantum history connecting $N_t = 3$ fixed points. This case is important to consider because it involves all four regions of the Keldysh contour connected to the intermediate fixed point.

Suppose that pre- and postselection measurements at t_1 and t_2 yield the states $|\psi\rangle$ and $|\phi\rangle$, respectively. One is then interested in the probability of measuring a state in some basis, $|a_i\rangle \in \{|a_k\rangle\}$, at the measurement time t, where $t_1 < t < t_2$. This experiment corresponds to the family of histories:

$$\mathcal{F}_H : [\![\phi]\!]_{t_2} \otimes \{[\![a_i]\!]_t\} \otimes [\![\psi]\!]_{t_1} \tag{26}$$

as represented schematically in Figure 4, where the purple region represents the measure of existence corresponding to the measurement of $|a_i\rangle$ at time t, black lines define the history family $\mathcal{F}_H$ (processes consistent with the pre- and postselected boundary values) and blue lines represent wavefunction regions that are incompatible with the preparation.

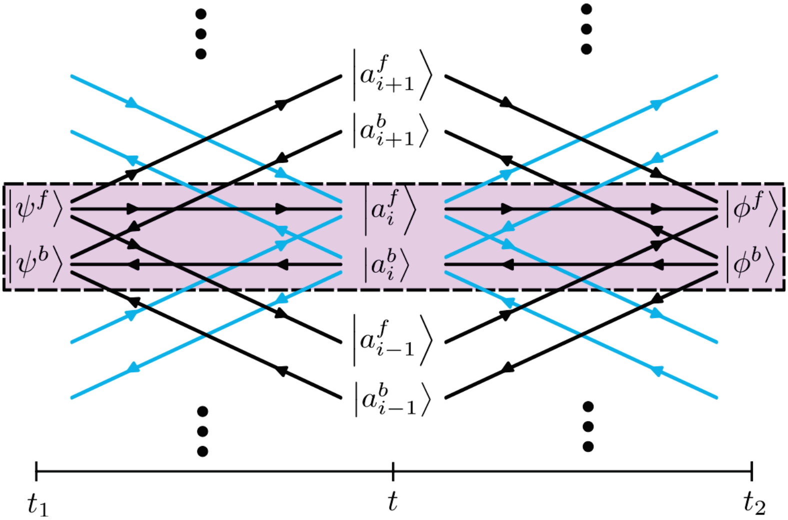
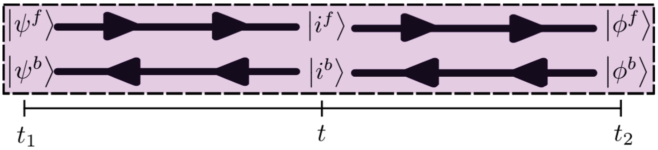


Figure 4. The purple region represents the measure of existence corresponding to the ABL measure in an experiment connecting the pre- and postselection fixed points $[\![\psi]\!]_{t_1}$ and $[\![\phi]\!]_{t_2}$ to a measurement at t corresponding to the fixed point $[\![a_i]\!]_t$.

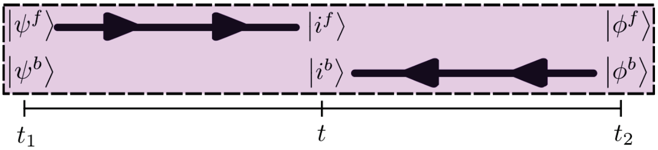
This experimental situation is described by the following 'source' wavefunction constructed from three fixed points:

$$\Psi\left(t_2^b, t_2^f, t^b, t^f, t_1^b, t_1^f\right) = [\![\phi]\!]_{t_2} \otimes [\![a_i]\!]_t \otimes [\![\psi]\!]_{t_1} \tag{27}$$

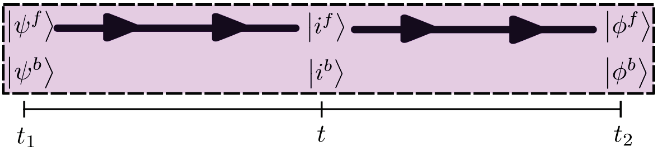
The fixed points in this state are connected via black lines in the shaded region of Figure 4. By contrast, the TSVF divides the universe into 'future' times, described by a state vector traveling backwards from the postselection, and 'past' times, described by future-oriented propagation from the preselection. This restricts the dynamics to the upper black arrow left of the measurement time ($<_C t^f$) and the lower black arrow right of the measurement time ($<_C t^b$) on Figure 4, effectively throwing away half of the wavefunction by treating the fixed point at t as a sink only. The propagation between three boundary constraints in the FPF is illustrated schematically in Figure 5a, where all sections of the contour are covered. This is compared to the situation in the TSVF in Figure 5b, where only half of the available contour is included, therefore violating **Event Symmetry**. For comparison, the standard Schrödinger dynamics used in the consistent histories framework is illustrated in Figure 5c. We also note that the TSVF formalism allows oppositely-oriented states to overlap at the intermediate measurement time (the backwards-travelling vector from the future is represented as a 'bra' state in the conjugate Hilbert space $\mathcal{H}_{t_2}^{\dagger}$) [21], which is prevented by branch-independence in the FPF.



(**a**) FPF



(**b**) TSVF



(**c**) Schrödinger

Figure 5. Schematic representation of the regions and direction of time propagation between three consecutive boundary conditions considered within (**a**) the FPF, (**b**) the TSVF and (**c**) standard Schrödinger dynamics.

The total wavefunction along segments of the Keldysh contour connecting the three fixed points in Equation (27) is a line integral of the exact differential:

$$d\Psi = \frac{\partial\Psi}{\partial t_1^f}dt_1^f + \frac{\partial\Psi}{\partial t^f}dt^f + \frac{\partial\Psi}{\partial t_2^b}dt_2^b + \frac{\partial\Psi}{\partial t^b}dt^b \tag{28}$$

along the path $\left(t_2^b, t^b, t^f, t_1^f\right) \rightarrow \left(t_2^b, t^b, t^f, t^f\right) \rightarrow \left(t_2^b, t^b, t_2^f, t^f\right) \rightarrow \left(t^b, t^b, t_2^f, t^f\right) \rightarrow \left(t^b, t_1^b, t_2^f, t^f\right)$, which is the integral path along the horizontal black lines enclosed by the purple shading in Figure 4. Since the four time degrees of freedom are independent, the total wavefunction in the temporal region $t \in [t_1, t_2]$ is as follows:

$$\begin{aligned} D\Psi = U^b\left(t^b, t_2^b\right)[\![\phi]\!]_{t_2} \\ \otimes U^b\left(t_1^b, t^b\right)U^f\left(t_2^f, t^f\right)[\![a_i]\!]_t \otimes U^f\left(t^f, t_1^f\right)[\![\psi]\!]_{t_1} - \Psi \end{aligned} \tag{29}$$

Taking the inner product of $D\Psi$ with the corresponding 'sink state'

$$\left|a_i^b(t)\right\rangle\left|\phi^f(t_2)\right\rangle\left|\psi^b(t_1)\right\rangle\left|\phi^f(t_2)\right\rangle\left|\psi^b(t_1)\right\rangle\left|a_i^f(t)\right\rangle \tag{30}$$

and then normalizing gives the measure of existence of a history connecting the fixed points $[\![\psi]\!]_{t_1}$, $[\![a_i]\!]_t$ and $[\![\phi]\!]_{t_2}$(the region covered by black lines in Figure 4):

$$m\Big(h_{\langle\psi,a_i,\phi\rangle}\Big) = \frac{\triangle\Psi[\psi(t_1);a_i(t);\phi(t_2)]}{\sum\limits_k \triangle\Psi[\psi(t_1);a_k(t);\phi(t_2)]} \tag{31}$$

$$= \frac{|\langle\phi(t_2)|U(t_2,t)|a_i(t)\rangle\langle a_i(t)|U(t,t_1)|\psi(t_1)\rangle|^2}{\sum\limits_k |\langle\phi(t_2)|U(t_2,t)|a_k(t)\rangle\langle a_k(t)|U(t,t_1)|\psi(t_1)\rangle|^2}$$

Thus, we recover the ABL rule. It has been derived as a ratio of wavefunction regions integrated over C. There is no stochastic 'collapse' process, only unitary evolution and the imposition of constraints..

This analysis is easily extended to a sequence of measurements—each fixed point increases the dimensionality of the line integral in Equation (20) by two, so the corresponding change $\triangle\Psi$ in a N_t-time history is obtained from the $2(N_t-1)$-dimensional line integral along the relevant Keldysh contour segments.

6. Conclusions

In this paper we have derived a direct connection between the temporal structure of the wavefunction and the Born rule of quantum mechanics. Central to our thesis is the concept of a 'fixed point', which replaces the initial condition of standard quantum theory with a state that serves as both 'source' and 'sink' in both directions of time, defined on the Keldysh contour. The FPF has many advantages:

- It is logically parsimonious. The statistical postulate supplies the *meaning* of probability. However, the *mathematical form* of probability is not postulated, but derived from ontic and dynamical structure.
- Unlike derivations which appeal to contingent initial or final conditions of the universe [22,55,56], it explains the ubiquity of the Born measure in nature from temporally local constraints.
- It describes deterministic unitary quantum mechanics with a multiple-event structure which may have implications for quantum gravity [14].
- It makes no theoretical distinction between past, present and future times. A fixed point is simply a crossing point for quantum histories.
- It contains no genuine randomness, only integrals over temporal regions of the wavefunction.
- It is logically simpler than approaches to quantum probability which involve deviations from unitarity [57,58] or the introduction of additional ontological types [56,59].

Hitherto, Zurek's 'envariance'-based approach to quantum probabilities was the leading candidate for a physical derivation [6,7]. This strategy relies upon (i) the Schmidt decomposition into entangled system and environment states via decoherence, (ii) 'envariance' symmetry-based probability assignments and (iii) the modification of the environment with ancilla states satisfying certain 'fine-graining' properties. By contrast, the argument in this paper (i) assumes nothing about the internal composition of states beyond the ontological and dynamical postulates, (ii) assumes nothing about probabilities beyond the statistical postulate and (iii) has no dependence on details of the environment.

Since we are here considering unitary wave mechanics only, the FPF supports an Everettian interpretation of the quantum theory [60] with the caveat that branching of the wavefunction is permitted in both time directions. The other candidate for a time-symmetric quantum theory considered here—the TSVF—omits crucial information contained in the full Keldysh time structure. It is this temporal structure which explains the emergence of quantum probability.

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Consistent Histories and Many Worlds

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Abstract: This paper discusses the fundamental assumptions and background of the consistent histories (CH) approach to quantum mechanics. The focus of the paper is on the concept of frameworks. It is proposed that frameworks should be interpreted objectively as observer-independent realities. Two further options are considered: a hidden-variables variant of the CH approach, and a many-worlds version, which considers each individual history belonging to a given family as describing a separate world. The latter interpretation is subsequently compared and contrasted with the standard many-worlds interpretation. Finally, the solution to the measurement problem offered by the many-worlds variant of CH is analyzed and amended.

Keywords: consistent histories; many worlds; measurement problem; quasi-classicality; interpretations of quantum mechanics

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1. Introduction

The consistent histories (CH) interpretation of quantum mechanics is relatively less well-known than the major contenders in the area: the many-worlds interpretation, Bohmian mechanics, or the spontaneous localization theory. Yet, it deserves to be thoroughly evaluated and contrasted with other interpretations. So far, there have been only a handful of papers that directly and critically analyzed this conception in its entirety, its ontological presuppositions, and consequences. The consistent histories approach was first proposed in [1], and then developed in [2–4]. The fullest exposition of this interpretation can be found in [5], while [6,7] contain a very useful condensed survey of its main assumptions. Early critiques of the CH approach can be found in [8–10], while [11] is one of the most recent polemics focusing on the measurement problem.

While the formalism of consistent histories is occasionally used outside its narrow circle of followers, the specifics of this approach remain obscure for the majority of the philosophers of physics. In this article, I will analyze the basic tenets of the consistent histories approach, as presented in the works of one of its major and most vocal proponents, Robert Griffiths (mostly in [5,6]). The focus of this survey will be on the concept of framework and its possible interpretations. I will defend the claim that the best interpretation of frameworks is in terms of distinct worlds. I will compare the many-worlds variant of consistent histories with the standard many-worlds interpretation, pointing out some of the most crucial differences. Finally, I will address the central issue that prompted the emergence of various interpretations of quantum mechanics in the first place, namely the measurement problem. I will argue that the way the consistent histories interpretation deals with this problem presupposes a weak conception of scientific explanation. For this solution to work, we have to rely on some variant of the anthropic principle.

2. Basic Formalism of Consistent Histories

The fundamental concept of the consistent histories approach is that of a framework. Formally, a framework is constituted by a projective decomposition of the identity, that is a set of projectors $\{P^\alpha\}$ in a Hilbert space $\mathcal{H}$ such that $\sum_\alpha P^\alpha = I$ and $P^\alpha P^\beta = \delta_{\alpha\beta} P^\alpha$. Adding to the set $\{P^\alpha\}$ (a sample space) all linear combinations of the form $\sum_\alpha c_\alpha P^\alpha$, we obtain a Boolean algebra (or, generally, a σ-algebra), on which a classical probability function *Pr* can be defined. Two frameworks $\{P^\alpha\}$ and $\{Q^\beta\}$ are compatible if $P^\alpha Q^\beta = Q^\beta P^\alpha$ for all α, β; otherwise the frameworks are incompatible. As is well known, there is no joint probability function satisfying Kolmogorov's axioms that could be defined on incompatible frameworks (sample spaces). This fact forms the basis for the central postulate of CH, according to which all quantum reasoning involving probabilities has to be done within a single framework (the so-called single framework rule). It is prohibited to combine two incompatible frameworks in one quantum description. For instance, we cannot attribute to a quantum system the conjunction of two properties represented by non-orthogonal projectors (and thus belonging to different, incompatible frameworks), even though each property taken separately may be attributable to the system within a particular framework.

In standard applications, a particular Hilbert space $\mathcal{H}$ is assumed to contain momentary states of a system taken at a certain instance. If we consider a tensor product of k such spaces, we can represent states taken at k successive instances, and thus constituting a history of a system. A history is formally defined as a tensor product of $N + 1$ projection operators: $Y^\alpha = F_0^\alpha \odot F_1^\alpha \odot \ldots \odot F_N^\alpha$. This operator represents a straightforward situation, in which the system possesses property F_0^α at time t_0, property F_1^α at t_1, and so on. It is also possible to consider histories that are nontrivial superpositions of the projectors of the above kind, even though their physical interpretation is somewhat intricate. A family of histories $\{Y^\alpha\}$ contains orthogonal projectors of the above kind that sum up to unity, exactly as explained in the previous paragraph. Histories Y^α are called "elementary", and they constitute a sample space. In what follows, we will drop the distinction between families of histories and families of elementary histories (sample spaces) if this does not lead to confusion. Thus, such a family constitutes a framework, for which it is possible to define a classical probability function.

In order to introduce probabilities into the formalism, we have to take into account the physical dynamics of the system. The evolution from time t_a to t_b is assumed to be governed by a unitary operator $T(t_b, t_a)$, which depends on the Hamiltonian of the system. Next, we define for any history Y^α its corresponding chain operator $K(Y^\alpha)$, as follows:

$$K(Y^\alpha) = F_N^\alpha T(t_N, t_{N-1}) F_{N-1}^\alpha \ldots T(t_2, t_1) F_1^\alpha T(t_1, t_0) F_0^\alpha. \tag{1}$$

Defining the inner product on the set of chain operators:

$$\langle K(Y^\alpha) | K(Y^\beta) \rangle = \mathrm{Tr}[K^\dagger(Y^\alpha) K(Y^\beta)]. \tag{2}$$

we can introduce the probability function *Pr* to any family of histories, in the standard manner:

$$Pr(Y^\alpha) = \langle K(Y^\alpha) | K(Y^\alpha) \rangle. \tag{3}$$

However, when we want to extend the probability function *Pr* to all linear combinations of histories, a problem occurs. The probability of a linear combination $\sum_\alpha c_\alpha Y^\alpha$ should equal $\sum_\alpha c_\alpha Pr(Y^\alpha)$, but when we apply Formula (3) and use the (anti)linearity of the inner product, we will obtain cross products $c_\alpha c_\beta \langle K(Y^\alpha) | K(Y^\beta) \rangle$. In order to eliminate this possibility, the condition of consistency is imposed on families of histories:

$$\langle K(Y^\alpha) | K(Y^\beta) \rangle = 0 \text{ for } \alpha \neq \beta. \tag{4}$$

In other words, consistent families are built out of histories whose corresponding chain operators are mutually orthogonal. It may be added that the condition of orthogonality depends on the underlying dynamics, and is not a property of a family simpliciter (since the chain operators contain the evolution operator *T*). That is, it is possible to have one and the same family that is consistent under one dynamics and inconsistent under another (for an example see [5], p. 145).

The general Formula (3) for probability reduces to the familiar Born rule in the case when we consider only two moments t_0, t_1 and limit ourselves to histories with a fixed initial state $|\psi_0\rangle$. That is, the selected family of histories consists of the following:

$$\begin{array}{c} Y^\alpha = [\psi_0] \odot [\varphi_1^\alpha] \\ Y^0 = (I - [\psi_0]) \odot I \end{array} \tag{5}$$

where $[\psi_0] = |\psi_0\rangle\langle\psi_0|$, $[\varphi_1^\alpha] = |\varphi_1^\alpha\rangle\langle\varphi_1^\alpha|$, and $|\varphi_1^\alpha\rangle$ are mutually orthogonal and span the entire single-moment Hilbert space $\mathcal{H}_1$. When we apply Formula (2), we can quickly calculate that

$$\begin{array}{c} Pr(Y^\alpha) = |\langle\varphi_1^\alpha|T(t_1,t_0)|\psi_0\rangle|^2 \\ Pr(Y^0) = 0. \end{array} \tag{6}$$

In the subsequent discussions, we will limit ourselves to histories with fixed initial states, however with an arbitrary number of times $N \geq 1$. That is, we will consider any histories of the form

$$[\psi_0] \bigodot F_1^\alpha \bigodot \ldots \bigodot F_N^\alpha \tag{7}$$

A typical misconception associated with the notion of histories is that the above operator (7) should represent a possible evolution of the system that starts with the state $|\psi_0\rangle$ and then develops according to a particular unitary operator. This is incorrect, because, as we can see in the simple two-time example (5), the states $|\varphi_1^\alpha\rangle$ at time t_1 will generally not be the result of a unitary evolution applied to the initial state $T(t_1,t_0)|\psi_0\rangle$. We will return to the question of the proper ontological interpretation of histories later, but for now we can use a conceptual crutch, in the form of a Copenhagen-style explanation, with its irreducible use of measurements. A history of the form (7) may be provisionally interpreted as resulting from a series of measurements, each of which is associated with a particular projector F_i^α. In other words, at every moment t_i where $i > 0$, we ask the experimental question whether the system is in a state corresponding to F_i^α. If the answer each time is "yes", we have physically selected the history (7) out of many alternative possibilities. The probability associated with a particular history Y^α is precisely the probability that appropriate measurements will reveal a string of yes-answers to questions F_i^α. However, we have to stress that the CH approach does not admit the concept of measurement understood as a special physical process different from the standard unitary evolution prescribed by the Schrödinger equation. We will discuss this issue shortly.

Among the families of histories with a fixed initial state, we may of course distinguish a special history that satisfies the above-mentioned intuition. That is, we may consider the following history:

$$[\psi_0] \bigodot [\psi_1] \bigodot \ldots \bigodot [\psi_N] \tag{8}$$

where $|\psi_i\rangle = T(t_i,t_{i-1})|\psi_{i-1}\rangle$ for $i = 1, \ldots, N$. For obvious reasons this history is called unitary, and together with its complementary histories (built out of all remaining combination of projectors $I - [\psi_i]$ and $[\psi_j]$), it constitutes a unitary family. A characteristic feature of this family is that its elements receive only probabilities 0 or 1 under the assumed dynamics. However, as Griffiths stresses, other than that, there is nothing special regarding unitary histories in comparison to other histories. In particular, Griffiths rejects the view that only unitary histories and unitary wave functions ("uniwaves") are ontologically real. A unitary family provides us with one framework within which we can describe a particular system, but alternative frameworks are still available and have no lesser reality. Thus, we should abandon the standard view that a quantum system develops uniquely via the evolution of its wave function, and that at any moment of its evolution the only physical reality associated with this system is its unitarily evolved wave function.

3. Example of a Consistent Family

It should be rather obvious how to use families of histories as frameworks that enable us to consider all sorts of questions regarding the conditional and unconditional probabilities of various quantum events. Given a particular dynamics, each element in a selected family of histories receives its classical probability, according to Formula (3). Then, we can calculate various probabilities of particular occurrences at selected times, on the condition that, at other times, the system has such and such properties. The procedure should be as follows: if we are interested in calculating specific probabilities of some outcomes conditionally on some other outcomes obtained at different times, we should first select a family of histories that contains these outcomes, apply the underlying dynamics, and compute the probabilities. It has to be stressed that this procedure can be executed only within an appropriate framework (family). Selecting an alternative, incompatible framework, we cut ourselves off from the possibility of answering the question of interest.

Let us use a simple example to illustrate this method. Consider a spin-half particle (an electron) entering a Stern–Gerlach magnet that was aligned with the z axis, and select three points in time: t_0 and t_1 before entering the magnet, and t_3 after leaving the magnet (see Figure 1). The initial state at t_0 is assumed to be $|\psi_0\rangle = |x^+\rangle\,|\omega\rangle$, where $|\omega\rangle$ is the spatial wave function associated with the particle at t_0. We assume the standard dynamics in the following form:

$$\begin{array}{c} |z^+\rangle|\omega\rangle \;\to\; |z^+\rangle|\omega'\rangle \;\to\; |z^+\rangle|\omega^+\rangle \\ |z^-\rangle|\omega\rangle \;\to\; |z^-\rangle|\omega'\rangle \;\to\; |z^-\rangle|\omega^-\rangle \end{array} \tag{9}$$

where $|\omega'\rangle, |\omega^+\rangle, |\omega^-\rangle$ indicate the appropriate wave functions, whose spatial supports are depicted on Figure 1, and $|z^+\rangle$, $|z^-\rangle$ are eigenvectors of the z-spin operator σ_z corresponding to eigenvalues +1/2 and $-1/2$. This case, which Griffiths calls "microscopic measurement" is taken from his book [5], pp. 230–233.

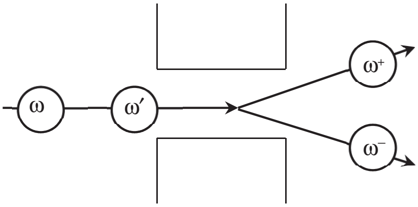

Figure 1. A spin-half particle passing through a Stern–Gerlach apparatus.

A typical way to describe the evolution of the system is by applying the transformations (9) to the initial state and thus obtaining the successive states at t_1 and t_2. That way we will arrive at the following unitary history:

$$[\psi_0] \odot [x^+]\,[\omega'] \odot \left[\frac{1}{\sqrt{2}}\big(|z^+\rangle|\omega^+\rangle + |z^-\rangle|\omega^-\rangle\big)\right] \tag{10}$$

which together with its complement will constitute one possible family of histories (let us symbolize it with $\mathcal{F}_0$). The probability assigned to this history is obviously 1. On the other hand, if we wanted to calculate the probabilities of obtaining definite values of z-spin at time t_2, we would have to use a different family $\mathcal{F}_1$, consisting of the following two histories (plus their complement, which I will ignore):

$$[\psi_0] \odot [x^+]\,[\omega'] \odot \begin{cases} [z^+][\omega^+] \\ [z^-][\omega^-] \end{cases} \tag{11}$$

The algorithm for calculating probabilities produces the straightforward result: each of the above histories has an equal one-half probability. Thus, the conditional probability of finding that the particle has its z-spin "up" at t_2 equals $\frac{1}{2}$, as expected. However, we may be interested in asking a similar question regarding the z-spin at t_1, before the particle enters the magnet. In order to answer this question, we have to select yet another family (let us call it $\mathcal{F}_2$):

$$[\psi_0] \bigodot \begin{cases} [z^+][\omega'] \bigodot [z^+][\omega^+] \\ [z^-][\omega'] \bigodot [z^-][\omega^-] \end{cases} \tag{12}$$

Again, the probabilities associated with these histories equal $\frac{1}{2}$. Thus, the likelihood of finding the z-spin "up" at t_1, conditional on the initial state being $|\psi_0\rangle$, is one-half. However, if we conditionalize on the later values of spins at t_2, the result will be different: the probability of the z-spin being "up" at t_1, given that at t_2 it was "up", equals one.

This conclusion may come as a surprise. Traditionally, we believe that it is the interaction with the magnet that produces the "splitting" of the electrons into two beams with different values of spin. Whether this splitting is treated as a symptom of a mysterious non-unitary "collapse", or as a result of a physical process of decoherence, it remains the case that before the interaction with the magnet, the z-spin is not supposed to be well-defined. However, when we interpret the above probabilistic reasoning literally, it seems that it makes perfect sense to expect that there will be two possible and equally probable ways for the system to evolve, each of which involves a definite value of the z-spin, even before the electron enters the magnet. At least this is true from the perspective of a particular framework. Yet, this is baffling. It seems that we cannot make any progress in our attempt to understand the CH interpretation without delving deeper into the concept of a framework and its role in this interpretation.

4. Frameworks and Worlds

What are frameworks, ontologically speaking, and what exactly is their role? Let us approach this problem by following Griffiths' formulation of the fundamental principles governing the use of frameworks in [6] (p. 98). We have already mentioned the single-framework rule, which states that every instance of quantum reasoning should be performed within one specific framework. To that, Griffiths adds the principle of liberty, which prescribes that the scientist can use any framework he or she deems appropriate. That no framework is better than any other is encompassed in the principle of equality. However, frameworks can be more or less useful for some purposes, and this fact is reflected in the principle of utility: we should use the framework that best suits our goals. Finally, Griffiths mentions the principle of incompatibility, which seems to be a variant of the single-framework rule, since it prohibits the simultaneous use of two incompatible frameworks. As we already know that only one framework can be used in any particular reasoning, this principle appears to be redundant.

It is difficult to shake off the feeling that all these rules and principles have a strongly pragmatic and instrumentalist character. They do not say what frameworks are or what they are supposed to represent, but instead they tell us merely how to use them and what we may or may not do with them. Observe, for example, that the single-framework rule, central to the CH approach, has the form of an unconditional command "you must not use two incompatible frameworks in one reasoning". However, there is no explanation of the source of this postulate (a divine decree?). What would happen if we obstinately ignored this rule? Would we end up with a logical contradiction? Most certainly not, since the rules of the CH interpretation do not have the status of logical laws. Perhaps there would be some other unpleasant consequences, such as consistently losing bets (the Dutch book argument). Alternatively, there may be some ontological reasons for not mixing up different frameworks, if they turn out to describe distinct realities, as will be suggested in what follows.

The emphasis put on the rules of reasoning within particular frameworks seems to show an affinity with quantum Bayesianism (or operationalism) and its antirealist attitude towards quantum theory [12–14]. It is no wonder then that David Wallace has voiced his doubts about whether CH is a realist theory in the conventional sense of the word [15] (p. 39). Griffiths seem to be less pessimistic in regards to this issue, but his clarifications are rather nebulous. First off, he stresses that the choice of a particular framework does not in any way influence reality. This is little consolation to a realist, unless we specify what reality truly consists of and what its relation to the multitude of incompatible frameworks is. Griffiths uses a series of classical analogies that are supposed to throw some light on this problem [6] (p. 99). Unfortunately none of these analogies are complete, since in the classical case there is no analogue of the quantum concept of incompatibility. For instance, he compares the choice of a quantum framework to the selection of an inertial frame of reference in special relativity, only to observe that in the latter case all inferences performed in one frame of reference can be translated into inferences in any other frame, which is not true in the quantum case. Another incomplete analogy drawn by Griffiths likens frameworks to different perspectives adopted when observing an object (for instance viewing a mountain from different sides). However, he quickly admits that these observations can be combined together into a consistent description, in contrast to the case of quantum frameworks. As we can see, it is very difficult, if not outright impossible, to explicate the concept of frameworks under the assumption of the existence of one, unified reality.

It seems to me that the only realist, objectual interpretation of a framework is that frameworks refer to some observer-independent and distinct realities. It is hopefully not too far-fetched to call these realities "worlds". Adopting this interpretation, we can immediately explain the single-framework rule, or its cognate, the principle of incompatibility. Any reasoning has to be done in exactly one framework, because separate worlds do not overlap, and thus their descriptions cannot be combined into one consistent story. This interpretation also accounts for the principle of equality, since among alternative worlds we do not distinguish more or less real ones. The remaining principles (liberty and utility) of course will have to retain their pragmatic character, but this should not be particularly worrying for the realist (pragmatic choices regarding which world we wish to consider do not threaten the objective character of the framework-selected worlds). It may be worth noting that alternative worlds corresponding to distinct and incompatible frameworks do not have to be mutually contradictory, in the sense that there is a quantum-mechanical statement which is true in one of them and false in another. Nevertheless, they are still distinct. Their distinctness may follow from the fact that there are some properties that may be used to characterize objects in one world but not in the other. In other words, different worlds are characterized by different sets of available properties (for instance, in one world, these properties may include spin in the z direction, and in another, spin in the x direction).

In order to explain these things further, we have to delve deeper into the structure of the worlds picked out by appropriate frameworks. As we remember, in the CH approach, frameworks are identified with consistent families of histories. What is the relation between alternative histories from a particular family and the world represented by this family? We can find one possible answer to this question in [6], p. 102, where Griffiths writes:

> *"[. . .] if a single framework, a single consistent family, of histories is in view, the sample space, represented mathematically by an appropriate PD of the history identity, is a collection of mutually-exclusive possibilities, one and only one of which actually occurs."*
>
> [italics mine]

This rather stunning admission seems to indicate that each framework-world contains only one out of the multitude of alternative histories, even if other histories receive non-zero quantum probabilities. To my knowledge, this suggestion has never been worked out in detail, but it definitely looks like a variant of the hidden variable hypothesis. The standard version of this hypothesis asserts that each measurable parameter characterizing a physical system possesses in actuality a well-defined, precise value, which is nevertheless not known to us, hence the use of the probability distribution. This hypothesis is famously vulnerable to a number of no-go theorems, in particular the Bell and Kochen–Specker theorems. However, in combination with the CH approach, the hidden variables escape these problems. The key point is that no framework-world assigns precise values to incompatible observables. Each framework admits only orthogonal projectors that are mutually compatible, and out of these projectors exactly one is selected to represent the actually possessed property. Thus, no violation of Bell's inequalities follows, since all known variants of these inequalities require an assignment of precise values to incompatible properties (such as spins in different directions). For the same reason, the Kochen–Specker "coloring" theorem is not violated [16] (pp. 119–138). Consequently, the hidden-variable version of the CH approach does not need to admit strong non-locality or contextuality.

The Many-Worlds Variant of CH

In spite of the above-mentioned advantage over the standard version, the hidden-variable interpretation of CH may still not be the first choice for many philosophers. It presupposes the existence of a fundamental rift between what is (the actually obtaining history) and what can be known (the probability distribution over alternative histories). Those who prefer not to introduce elements of reality that cannot be known, even in principle, may be compelled to follow a different route. Alternative histories whose probabilities under a given dynamics are non-zero may be assumed to represent distinct and parallel realities, in line with the many-worlds interpretation (MWI) of quantum mechanics. Griffiths himself makes a disparaging remark regarding this approach, but without giving any deeper reason for his preference [6] (p. 102). However, on some occasions he slips into language that may suggest an objectivist interpretation of distinct frameworks. For instance, in [7], he spells out the thesis of unicity, which he subsequently rejects, as follows: "at any point in time there is one and only one state of the universe which is "true", and with which every true statement about the world must be consistent". By negation, if we reject this claim, we have to assume that there are more than one "true" states of the universe, which we may call "worlds". It seems to me that the many-worlds variant of CH (henceforth abbreviated as MWCH) is rather natural, so I will try to analyze it further, in spite of Griffiths' reservations, contrasting it with the well-known Everettian interpretation of quantum mechanics.

According to MWCH, there is not a single world associated with a given family of histories, but a collection of mutually exclusive worlds (except in the case of unitary families). Thus the set of all worlds can be partitioned into families, which then divide up further into individual worlds. We can illustrate this with the help of the example from the previous section. Family $\mathcal{F}_0$ includes just one possible world (let us call it w_0) with the unitary history (10), since this is the only history in this family that receives a non-zero probability. However, another group of worlds contains equally probable histories from family $\mathcal{F}_1$. In one of these worlds (w_1^+), the electron has a well-defined x-spin before entering the magnet and then acquires the value "up" of the z-spin, while simultaneously travelling along the upper trajectory. The alternative world (w_1^-) differs, in that the electron leaves the magnet following the lower trajectory and possesses the "down" value of spin in the z direction. The third considered family $\mathcal{F}_2$, which—it has to be stressed—is equally acceptable, also separates into two worlds. One world w_2^+ contains an electron that already exhibits the "up" value of its z-spin before entering the magnet, and consequently follows the upper trajectory, while in the other world w_2^- the electron consistently possesses z-spin "down" from the moment t_1. All in all, in our simple example we have five distinct worlds w_0, w_1^+, w_1^-, w_2^+, and w_2^- grouped into three families $\mathcal{F}_0$, $\mathcal{F}_1$, and $\mathcal{F}_2$.

Let us observe that worlds belonging to the same family differ with respect to the specific values possessed by the same measurable parameter (z-spin in our example). However, the differences between worlds belonging to distinct families are more subtle. For instance, worlds w_0 and w_1^+ diverge due to the fact that in w_0 at time t_2 the electron is in a superposition with no well-defined z-spin, while in w_1^+ it possesses a definite value z^+. On the other hand, worlds w_1^-, w_2^+ diverge with respect to the definite values of z-spin at t_2, as well as regarding the state of the electron at t_1. In world w_1^-, the electron has a definite value of x-spin at t_1, whereas in w_2^+ the electron is characterized by a definite z-spin at the same moment t_1. In worlds w_1^+ and w_2^+ there are no differences regarding the possessed values of the same parameter, but nevertheless the worlds are different due to their incompatible characterizations of the electron's state at t_1.

We may contrast the MWCH approach with the standard many-worlds interpretation ([17–19]). According to the latter, the evolution of the system is given by the unitary history (10) from family $\mathcal{F}_0$. No other histories are admissible; they do not represent any real physical processes. However, MWI interprets the superposition $\frac{1}{\sqrt{2}}(|z^+\rangle|\omega^+\rangle + |z^-\rangle|\omega^-\rangle)$ characterizing the system at time t_2 as describing two independent realities: one in which the electron has spin "up" in the z direction, and the other in which the z-spin of the electron is "down". Thus, MWI admits the existence of two worlds w_1^+ and w_1^-, even though no history corresponding to these worlds represents a genuine quantum-mechanical process, since these histories clearly violate the universal law of quantum mechanics, i.e., the Schrödinger equation.

This point is rarely made, so it bears repeating. Even though "officially" MWI insists that the Schrödinger equation is universally valid with no exceptions, individual worlds clearly violate it. The law of the unitary evolution applies to the entire multiverse and not the separate worlds constituting it. Observe that, in contrast to MWCH, MWI does not admit the world w_0 as a separate entity. The unitary history refers to the collection of worlds w_1^+ and w_1^- rather than a distinct world. For the proponent of MWI there is no single world in which the electron after leaving the magnet would still be in a superposition of states with distinct locations. Superpositions of states with well-defined locations, by necessity, refer to distinct realities.

Incidentally, we may observe that many authors have combined the assumption of the fundamental reality of the wave function and its unitary evolution with the formalism of CH ([15,20]). That is, they admit a family of histories corresponding to the components of the universal wave function writtenin the preferred basis with respect to which the splitting into separate worlds occurs. However, this is not a full CH (at least not according to Griffiths), since it ignores other available histories. This explains a remark made in [6], p. 95 ft. 2, that "the discussion of consistent histories presented in (Wallace 2008) bears little resemblance to what is found in (Griffiths 2002)".

How about worlds w_2^+ and w_2^-? Here, the thorny issue of the exact moment of the splitting of the worlds comes into view. The "traditional" variant of MWI assumes that the splitting occurs at the precise moment of measurement (I am tempted to call this variant the Copenhagen version of the many-worlds interpretation), when macroscopic outcomes are revealed to us. However, this solution relies on the concept of measurements being fundamentally distinct from other types of physical interactions. An alternative, rather popular view is that the splitting is a result of a physical process of decoherence, which is a physical interaction with the environment possessing a huge number of degrees of freedom (see [21] for a comprehensive physical and philosophical analysis of decoherence). Given some specifics of this interaction, the components of the superposition corresponding to states with distinct spatial locations become "recorded" in approximately orthogonal states of the environment, which leads to the suppression of the interference ("non-diagonal") coefficients in the density operator used to calculate the probabilities of finding the system in particular states. In our simplified example, we assume that the decohering interactions (for instance with air molecules) occur after the electron leaves the magnet and that its unitarily evolved state decomposes into parts with distinct locations. Consequently, the splitting takes place at moment t_2, which eliminates the worlds w_2^+ and w_2^-, since they seem to move the moment of splitting back in time to point t_1.

On the other hand, the many-worlds variant of CH does not require any objective process leading to the splitting of the initial world into several copies, whether in the form of an interaction with a measuring device or as a result of the decoherence with the environment. For the proponent of MWCH, it just does not make sense to ask generally when exactly the electron whose initial state is $|\psi_0\rangle$ splits into a number of copies corresponding to the different outcomes of the measurement down the line. There is one framework in which the splitting seems to occur at the last possible moment, and another in which from the very beginning, the electron evolves in the form of distinct copies associated with the later recorded outcomes. To make matters even more interesting, there is also a framework in which no splitting takes place at all, and the electron is always in the state of superposition. Thus, ultimately, we can get rid of the objective branching of the actual world, replacing it with the multitude of different worlds grouped in various frameworks.

Another related difference between MWI and MWCH that is worth emphasizing is that the latter easily circumvents the problem of the preferred basis that affects earlier versions of MWI. As is well known, MWI requires that there be a unique decomposition of the unitary wave function into mutually orthogonal states that define appropriate worlds. Since formally there is an infinity of ways we can decompose any vector, there has to be an additional rule selecting the preferred orthogonal basis with respect to which the decomposition is made. On the other hand, MWCH admits any decomposition of that sort, according to the principle of liberty. Any decomposition of the unitary wave function corresponds to a set of objectively existing worlds, and no decomposition is considered to be privileged.

It may be asked what is to be gained by introducing the ontologically extravagant hypothesis of the existence of a myriad of distinct worlds, far surpassing the number of the worlds admitted in the standard MW interpretation. My answer to that question would not be in terms of an immediate gain but rather in terms of the lack of a satisfactory alternative. The CH interpretation derives its flexibility from admitting an infinite number of incompatible but equally admissible frameworks in which we can describe a particular quantum process. However, from a realist perspective, these frameworks must correspond to some objective, observer-independent reality. Since incompatible frameworks cannot be combined into a consistent story, the corresponding realities must be in some sense distinct. Calling these realities "worlds" merely reflects the fact that they cannot be summed up to obtain a consistent whole. I do not see any other way to uphold the central postulates of CH while retaining the basic assumptions of scientific realism. The only alternative is to admit that quantum facts are in some sense created by the observer by the very act of selecting a particular framework.

5. The Measurement Problem and Quasi-Classicality

The biggest test of the consistent histories approach is how well it deals with the measurement problem. Here, by the measurement problem I understand the question of how to explain the occurrence of definite macroscopic outcomes in the light of the fact that the unitary evolution of the system consisting of the measured object and the measuring device typically produces a superposition of states involving different outcomes. We may add that, in the literature on the subject, one can find mentions of more than one measurement problem. For instance, [6] distinguishes two measurement problems: one as stated above, and the other expressed by the question of how to relate the alternative positions of the pointer with the earlier microscopic situation that is supposed to be measured. Tim Maudlin, in [22], adds to this a third problem, which he calls the problem of effects. Roughly, it is the question of how to explain that future repeated measurements of the same observable will yield the same results. Maximilian Schlosshauer, on the other hand, mentions two separate but related problems: the nonobservability of interference, and the preferred basis problem ([21], p. 50).

The MWI approach solves the problem of definite outcomes by adopting the many-worlds reading of the final superposition of the combined system. Definite outcomes occur in separate worlds, corresponding to the components of the superposed state with well-defined macroscopic locations of the measuring instrument (the pointer). However, as we have seen, the MWCH approach admits many more possible histories than MWI, and this creates a potential problem. Griffiths optimistically announces that the measurement problem finds its solution in CH, because there is an admissible framework in which measurements indeed produce definite outcomes ([6], p. 105). This specific framework involves the decomposition of the identity in the orthogonal basis corresponding to the states with well-defined positions of the pointer. In this framework it is indeed the case that each history contains a definite outcome, and moreover the probabilities associated with these histories correspond precisely to the observed frequencies of appropriate outcomes.

The problem is, though, that there are many alternative frameworks and alternative worlds in which there are patently no definite outcomes. One such world is the familiar unitary world in which the initial superposition persists even after the interaction with the measuring device. As Elias Okon and Daniel Sudarsky point out in their critique of CH [11], there has to be an additional rule that eliminates all frameworks except the one in which measurements indeed have definite outcomes. However, such a rule is not present in the standard version of CH and, therefore, must be brought in from outside this interpretation. Griffiths seems to disagree with this conclusion. Is there any way to solve this controversy?

One way to reconcile the conflicting positions of Griffiths on the one hand, and Okon and Sudarsky on the other, is to observe that they tacitly base their arguments on different standards of scientific explanations. Okon and Sudarsky seem to presuppose a stronger notion of explanation, according to which, in order to explain a given occurrence, we have to derive it from the applicable laws and assumed initial conditions (this resembles the well-known deductive-nomological conception of explanation of Carl Hempel and Paul Oppenheim [23]). In other words, we explain a particular happening when we show that it must occur in a given situation (we can definitely predict it). On the other hand, for Griffiths it is sufficient that there is an available framework within which measurements can be assigned definite outcomes. That is, he most probably would subscribe to the weaker interpretation of explanation: for an occurrence to be explained we should show that it may happen given the laws and the circumstances (it is consistent with the laws and the initial conditions).

Typically, such weak explanations are employed when we deal with indeterministic events whose occurrences do not follow from the laws and earlier conditions. However, in the case of the many-worlds variant of CH the situation is more subtle. It is not the case that there is some single-world probability that the definite outcomes will occur and some probability that they will not, but rather that they are certain to occur in some but not all of the admissible worlds. Seen from this perspective, a complete explanation of a particular observed phenomenon that occurs only in some worlds requires that we answer the question why we as observers occupy those precise worlds. In our currently considered problem, the question is how to explain that we do not happen to inhabit worlds in which systems, after measurements, continue to occupy superposed states (as is the case in the unitary worlds).

A broad answer to this question may be provided in the form of a familiar anthropic principle of sorts. We may speculate that the effective suppression of macroscopic superpositions is necessary for the emergence of living organisms possessing appropriate cognitive abilities. However, this solution presupposes that we can definitively exclude the possibility of the existence of sentient beings in a universe that admits macroscopic superpositions of positions and other related parameters. While we do not have a satisfactory account of cognitive processes and functions, such as perceptions, in superposed states, the lack of such an account does not, by itself, constitute proof that cognition in the universe without the definite outcomes of experiments is impossible. As Okon and Sudarsky correctly observe, it would be circular to simply start with the thesis that macroscopic objects in our world possess well-defined locations, and then to use this thesis to eliminate all the alternative worlds admissible by our theory. To break this circularity, I suggest that we should propose a general theory of consciousness and cognition which would entail that sentient beings cannot exist in a world with macroscopic superpositions.

A related problem that admits a similar solution to CH is the question of how to generally explain the classical appearance of the macroscopic world. Here, an answer is provided in the form of the so-called quasi-classical frameworks ([5], pp. 356–359; [6], p. 103). These are histories and families of histories, describing possible evolutions of systems with a huge number of particles that, when appropriately coarse-grained, recover classical properties and their approximately classical dynamics. Griffiths suggests that there will be many such families that will give rise to the same classical description. Moreover, he claims that thanks to the process of decoherence some of these quasi-classical frameworks will be consistent. Even though he does not offer a strict proof for this contention, his plausibility argument relies on the formal similarity between the condition of decoherence (the lack of off-diagonal elements in an appropriate density matrix) and the consistency condition (4).

Griffiths is aware of the fact that, typically, there will be many non-classical and yet consistent families of histories in addition to the quasi-classical ones ([5], p. 367). He cites a much-discussed result by Dowker and Kent [8] showing that for any consistent, quasi-classical history of a system up to a certain point, there is an infinity of alternative continuations that do not satisfy the condition of quasi-classicality. Thus, the persistence of the quasi-classical behavior is not guaranteed. Griffiths's answer to this challenge takes us back to his "pragmatic" stance, as constituted by the principle of liberty. The scientist is free to choose whatever framework suits him/her best, and in order to describe the evolution of a macroscopic system, it is most useful to choose a quasi-classical framework. From this perspective, the CH interpretation would be threatened only if Dowker and Kent could prove that there is a quasi-classical history that cannot be extended to preserve its quasi-classicality. However, as long as there is a complete family of histories that retain their quasi-classicality, no problem arises.

However, again the argument by Okon and Sudarsky applies here. The choice of a quasi-classical family seems circular, since we assume from the outset what the result should be—the approximately classical features of the macroscopic world—and we cherry-pick the framework that will achieve this, ignoring the other, equally acceptable frameworks. Again, the main bone of contention here is the standards of scientific explanation. Adopting the many-worlds perspective on CH, we ask whether it is sufficient to explain the observable features of the macroscopic world by pointing out that in some possible worlds these features are guaranteed to occur, or should we find a way to eliminate all non-classical worlds on the basis of some further principles. As in the case of measurements, one way to defend the weaker concept of explanation is to argue that conscious observers cannot exist in worlds where macroscopic objects behave non-classically (persist in macroscopic superpositions).

6. Conclusions

The consistent histories approach to quantum mechanics is based on the concept of a framework, formally identified as a consistent family of histories (sequences of projectors which are mutually orthogonal and sum up to the identity). In this paper, I have suggested interpreting frameworks as referring to distinct worlds rather than to the enigmatic and epistemic "perspectives" or "aspects" of the actual world. Then, there are two further options to consider. One option is to follow the suggestion that in every consistent family of histories exactly one history is selected as actually occurring. This leads to a variant of the hidden variables theory that avoids the known no-go theorems without the necessity of accepting non-locality or contextuality. An alternative proposal, explored in this paper, is to associate each possible history within a given family with a separate world, in a fashion resembling the many-worlds interpretation. According to this approach, there is typically more than one world corresponding to a given family of histories. The many-worlds variant of CH differs in many important respects from the standard many-worlds interpretation: it does not accord a special status to unitary evolution and the universal wave function; it postulates many more alternative worlds than MWI; it does not rely on the special role of measurements or decoherence in bringing about the branching of the actual world; as a matter of fact, it dispenses altogether with the concept of world branching (splitting).

The solution of the measurement problem offered by MWCH is such that the existence of the worlds in which there are definite outcomes of measurements is guaranteed by the fact that decoherence renders appropriate families consistent. This also secures the existence of the worlds in which macroscopic objects display approximately classical behavior (the property of quasi-classicality). The main problem with this solution is that it also admits worlds that do not contain well-defined outcomes of measurements, and do not display classical behavior at the macroscopic level. I have suggested that the proponents of the many-worlds variant of CH may reply that they use a weaker notion of explanation in regard to the observability of the classical behavior of the macroscopic world. However, for this answer to work, we need to offer an argument that sentient, conscious beings can only exist in quasi-classical worlds.

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Article

How Everett Solved the Probability Problem in Everettian Quantum Mechanics

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Abstract: A longstanding issue in the Everettian (Many-Worlds) interpretation is to justify and make sense of the Born rule that underlies the statistical predictions of standard quantum mechanics. The paper offers a reappraisal of Everett's original account in light of the recent literature on the concept of *typicality*. It argues that Everett's derivation of the Born rule is sound and, in a certain sense, even an optimal result, and defends it against the charge of circularity. The conclusion is that Everett's *typicality* argument can successfully ground post-factum explanations of Born statistics, while questions remain about the predictive power of the Many-Worlds interpretation.

Keywords: Foundations of Quantum Mechanics; Many-Worlds interpretation; Everettian quantum mechanics; Born rule; probability; *typicality*

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1. The Probability Problem

Reproducing the probabilistic predictions of standard quantum mechanics, i.e., the *Born rule*, is considered to be one of the major challenges for the Everettian (Many-Worlds) interpretation. In the literature, we find a variety of different proposals for justifying the Born rule (see [1] for an overview), as well as critics expressing serious doubts that any of them are adequate (e.g., [2,3]). This paper will defend the thesis that the original account provided by Hugh Everett III (in his 1956 doctoral thesis [4] and summarized in his 1957 paper [5]) is a satisfying solution to the probability problem—or, at least, as good a solution as one can hope for in a Many-Worlds theory. It has been unjustly dismissed by modern-day Everettians, largely because of a failure to appreciate Everett's derivation of the Born rule as a *typicality argument* in the spirit of Boltzmann's statistical mechanics.

In recent years, a number of publications have elaborated on the concept of *typicality*, its role in Boltzmann's statistical mechanics [6–8], on how it grounds objective probabilities [9–11], and how *typicality* explanations work in general [12–14]. In light of this growing understanding, Everett's *typicality* account of the Born rule deserves a reappraisal.

Probabilities of What?

The problem with grounding quantum probabilities in Everettian quantum mechanics is not that the theory is fundamentally deterministic. Continued philosophical debates notwithstanding, there are very successful strategies for reducing classical statistical mechanics to Newtonian dynamics or even the Born rule to another deterministic quantum theory, Bohmian mechanics [15]. The first and foremost question that arises in the context of the Many-Worlds interpretation is: probabilities of what?

This question can be understood in two ways. With an emphasis on the what part, it points to the *ontology problem* of Everettian quantum mechanics [3], the question of what the theory is fundamentally about and—if it is about the universal wave function or an even more abstract quantum state—how exactly the theory describes localized events in space and time. Bohmian mechanics, for instance, is about the motion of point particles in three-dimensional physical space, and the Born rule, in its most basic form, yields probability distributions for particle configurations, including those composing familiar macroscopic objects. In Everettian quantum mechanics, one would look in vain for a similarly precise answer. Oddly enough, there are many different interpretations of the Many-Worlds interpretation (see, e.g., [16–20]) but even the best-elaborated ones remain vague about how the theory is supposed to make contact with familiar physical reality [3,21,22]. I consider this the most serious problem of Everettian quantum mechanics, but it is not the problem I want to focus on here. For the remainder of this discussion, I will thus grant that it is possible, by whatever means or procedure, to identify "worlds" with more or less familiar macro-objects like cats and physicists and measurement devices indicating measurement results in the wave function of the universe.

The other, more surface-level way of understanding the question "probabilities of what?" is with an emphasis on the probability part. In particular, what could it even mean to ask about the probability of a specific measurement outcome if, according to the Many-Worlds interpretation, *all* possible outcomes actually occur?

An obvious first idea is that the probability of a certain measurement outcome refers to the relative frequency of worlds in which the said outcome occurs. In the "naive" Many-Worlds interpretation, it is assumed that, say, a spin measurement on an electron in the spin state $\psi = \alpha|\uparrow_z\rangle + \beta|\downarrow_z\rangle$ results in exactly two worlds, one in which the outcome is *spin up* and one in which the outcome is *spin down*. The problem is then that this *branch counting* is clearly at odds with the probabilities predicted by standard quantum mechanics. The relative frequency of each outcome would always be $1/2$, which agrees with the Born probabilities $|\alpha|^2$ and $|\beta|^2$ only in the special case $\alpha = \beta = \frac{1}{\sqrt{2}}$. According to more sophisticated, decoherence-based versions of Everettian quantum mechanics (see, in particular, [17]), the number of distinct world branches is not even well-defined and naive branch counting a non-starter. Since decoherence is both ubiquitous and vague, any decomposition of the universal quantum state into (more or less) decoherent branches is, to some extent, arbitrary. (The precise notion of decoherence is largely irrelevant to this point, but it is best to think of decoherent branches as components of the wave function that have essentially no overlap in configuration space.) As Wallace (2012) explains:

> [T]here is no sense in which [decoherence] phenomena lead to a naturally discrete branching process: as we have seen in studying quantum chaos, while a branching structure can be discerned in such systems, it has no natural 'grain'. To be sure, by choosing a certain discretization of (configuration-)space and time, a discrete branching structure will emerge, but a finer or coarser choice would also give branching. And there is no 'finest' choice of branching structure: as we fine-grain our decoherent history space, we will eventually reach a point where interference between branches ceases to be negligible, but there is no precise point where this occurs. As such, the question 'How many branches are there?' does not, ultimately, make sense. ([17], pp. 99–100)

Metaphysically, this view seems even more unsettling than the naive Many-Worlds picture. We must not merely accept the existence of two cats at the end of Schrödinger's experiment but of an indefinite number of cats (and boxes, and experimenters, and, ultimately, worlds). Still, besides reflecting a more honest attempt at identifying worlds in the wave function, the sophisticated picture explains why the theory does not predict the wrong statistics that would result from branch counting (if naive branch counting made sense). Either way, the attempt to identify quantum probabilities with frequencies of worlds fails.

Finding it hard to locate interesting probabilities in the Everettian multiverse, the next obvious idea is to locate them in our minds, i.e., interpret them as subjective probabilities. For instance, after I perform a spin measurement—but before I look at the detector to see the result—I do not know if I find myself on a branch in which the detector registered "spin up" or on a branch in which the detector registered "spin down". What should my credence be for one or the other? If someone offers me a 2:1 bet on "spin up", should I accept? The probabilities, in this case, arise from my *self-locating uncertainty* [23–25]. I do not know what world within the multiverse my present self inhabits, and the goal of a theoretical analysis would be to show that it is rational to assign degrees of belief according to the Born rule. Other authors, most notably Deutsch [26] and Wallace [17], have taken a more decision-theoretic perspective, trying to argue that it is rational to act in accordance with standard quantum probabilities. In this vein, Wallace proposes a set of 10 axioms to justify the use of the branch amplitudes squared for calculating expected utilities in decision problems. Maudlin [27] points out that these axioms do not allow a rational agent to split a payoff among two or more of her future copies, i.e., exploit the option *all of the above* that we would have in a branching multiverse. “If one were mischievous, one might even put it this way: Wallace’s ‘rationality axioms’ entail that one should behave as if one believes that Everettian quantum theory is false” (p. 804).

I will not devote to any of these approaches the detailed discussion they would deserve based on their ingenuity alone. I believe that epistemic probabilities of any kind are ultimately missing the point of vindicating the *empirical adequacy* of Everettian quantum mechanics. First and foremost, the theory must account for the very robust statistical regularities described by the Born rule, not for physicists’ beliefs or betting behaviors. Certain credences and decisions might be rational in virtue of the theory’s physical predictions, but we first need to understand, in objective terms, what the relevant predictions are.

Everett’s account of the Born rule is based on a *typicality* argument—and objective probability assignments—analogous to the derivations of statistical laws in Boltzmannian statistical mechanics. His goal is to show that the Born rule describes *frequencies within* typical *world branches*, i.e., along *nearly all* Many-Worlds histories, in a natural sense of “nearly all”. A similar *typicality* argument, although with respect to *possible* worlds, underlies the quantum equilibrium analysis that grounds the Born rule in Bohmian mechanics (Dürr et al. [15]; see also Bell ([28], Ch. 15) who anticipates the result).

In this paper, I will not engage in a general philosophical discussion of *typicality* (for that, see [10–14]), but continue with a concrete example that provides a good classical analog for the Everettian analysis.

2. The Galton Board

Our model “universe” is the Galton board. (For other discussions of the Galton board in terms of *typicality*, see [29,30].) It consists of a vertical board with a top receptacle holding solid balls, interleaved rows of pins that the balls fall through (bouncing off the pins), and a series of bins at the bottom in which the balls are finally collected (Figure 1). As a large number of balls fall through the board, we find it resulting in an approximately symmetric binomial distribution of balls over the bins (which, in turn, approximates a normal distribution).

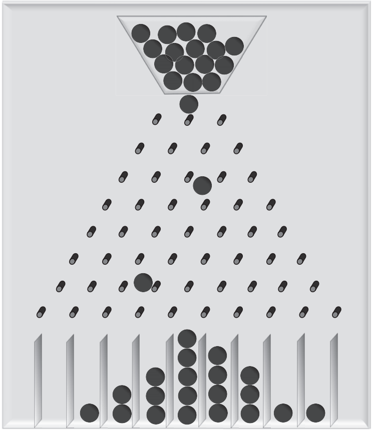

Figure 1. Illustration of a Galton board.

Suppose our Galton board has M rows of pins and $M+1$ bins at the bottom (labeled $0, 1, \dots, M$). And suppose the board starts out at $t=0$ as a perfectly isolated system with $N \gg M$ balls ("particles") placed in the top receptacle. From there on, everything runs its deterministic course. The particles fall through the board and collide (nearly) elastically with the pins before coming to rest in one of the bottom bins, all following laws of classical mechanics. If $r^N(k)$ denotes the fraction of particles ending up in bin $k \in \{0, \dots, M\}$, we find

$$r^N(k) \approx P(k) \ \text{ with } \ P(k) := B(k; M; \tfrac{1}{2}) = \binom{M}{k} 2^{-M}. \tag{1}$$

This binomial distribution is, of course, exactly what we would expect if a particle has a 50:50 chance of bouncing left or right at each pin. But what exactly does this mean? And how is it supposed to explain the observed statistics? There is nothing intrinsically random about the bounces. The trajectory that each particle takes through the Galton board is completely determined by the dynamics and initial conditions of the system. That we do not *know* whether a given collision will deflect the particle to the left or the right (because the dynamics are quite chaotic) and have no reason to favor one possibility over the other (because of the symmetry of the setup) might be a correct observation, but it does not amount to a physical explanation of the statistical phenomenon. What do little metal balls care about our ignorance or indifference?

Let us take the (idealized) Galton board seriously as a physical system, a classical N-particle system with phase space $\Omega^N \cong \mathbb{R}^{6N}$. At time $t=0$, the system starts out with an initial condition $X^N \in \Omega_0^N \subset \Omega^N$ for which all particles are at the top of the board. For any $X^N \in \Omega_0^N$, the relevant (Hamiltonian) equations of motion determine a unique evolution $\Phi_{t,0}^N(X), t \geq 0$, where $\Phi_{t,0}^N$ is the Hamiltonian flow. After a time T, which is sufficiently long for all the particles to have passed through the board, we consider their distribution over the $M+1$ bins. Mathematically, we introduce the macro-variables χ_k^i, $i \in \{1, \dots, N\}, k \in \{0, \dots, M\}$ such that $\chi_k^i(Z) = 1$ if, in the microstate $Z \in \Omega^N$, particle i is at rest in bin number k and $\chi_k^i(Z) = 0$, otherwise. The final (time T) distribution of particles, as a function of the initial microstate X^N, is then given by

$$r^N(k)[X^N] := \frac{1}{N} \sum_{i=1}^{N} \chi_k^i(\Phi_{T,0}^N(X^N)). \tag{2}$$

Now, a *typicality* explanation of the binomial distribution would be a result of the form

$$\lambda \Big\{ X^N \in \Omega_0^N : r^N[X^N] \approx P \Big\} \approx \lambda(\Omega_0^N) \tag{3}$$

for sufficiently large N, where λ denotes the Liouville measure on Ω^N. A little more precisely, for instance,

$$\lambda \Big\{ X^N \in \Omega_0^N : \max_k |r^N(k)[X^N] - P(k)| \geq \epsilon \Big\} = \lambda(\Omega_0^N)(1 - \delta(\epsilon, N)), \tag{4}$$

where $\delta(\epsilon, N)$ goes quickly to zero for large N and any given $\epsilon > 0$. The convergence is, in fact, more mathematical abstraction than needed. What matters physically is that $\delta(\epsilon, N) \approx 0$ for the *actual* particle number N and reasonably small ϵ (consistent with our observation of an *approximately* binomial distribution).

Proving (4) for realistic micro-dynamics can, of course, be very hard and is beyond the scope of this paper. I want to focus on how such a mathematical result (if true) should be interpreted and what it accomplishes. Equation (4) should be read as saying that *nearly all* possible initial conditions result in an *approximately* binomial distribution of particles over the bins. While there exist initial conditions for which some of the bins would end up with significantly more particles—and others with significantly fewer—such initial states are very special ones, forming a set of vanishingly small measure ($\lambda(\Omega_0^N)\delta(\epsilon, N) \approx 0$). A binomial distribution is, in other words, *typical* for the Galton board.

Compare this with Boltzmann's explanation of the Maxwellian velocity distribution in an ideal gas:

> The ensuing, most likely state, which we call that of the Maxwellian velocity distribution, since it was Maxwell who first found the mathematical expression in a special case, is not an outstanding singular state, opposite to which there are infinitely many more non-Maxwellian velocity distributions, but it is, on the contrary, distinguished by the fact that by far the largest number of possible states have the characteristic properties of the Maxwellian distribution, and that compared to this number the amount of possible velocity distributions that deviate significantly from Maxwell's is vanishingly small. The criterion of equal possibility or equal probability of different distributions is thereby always given by Liouville's theorem. ([31], p. 252)

A general definition of the concept of *typicality* is as follows.

Definition 1. *Let Ω be a set (the domain or reference set of the typicality statement) and F a property that the members of Ω can possess or not. We say that F is typical within Ω if nearly all members of Ω instantiate F. The property is atypical within Ω if $\neg F$ is typical, i.e., if nearly none of the members of Ω instantiate F.*

For instance, the property of *being black* is *typical* among ravens. The property of being *irrational* is *typical* within the set of real numbers. In the context of fundamental physics and statistical mechanics, the most interesting *typicality* statements are those with a reference set of (possible) worlds as described by a physical theory (in which case a "property" is technically a proposition that can be true or false at each world).

For the Galton board, we note that every possible initial condition $X^N \in \Omega_0^N$ corresponds to a possible trajectory or micro-history of the N-particle system (with the boundary condition that all particles start at the top of the board). That is, if we regard the Galton board as our model universe, every X corresponds to a physically possible world. The desired *typicality* result thus states that the statistical "law" expressed by P—or, if we prefer, by saying that particles bounce left or right with a probability of 0.5—holds in *nearly all possible worlds* allowed by the microscopic laws of motion.

"Nearly all" is made precise in terms of the Liouville measure λ, which corresponds to the intuitive phase space volume and is distinguished as the simplest measure on Ω that is stationary under the Hamiltonian dynamics. Stationarity is exactly the criterion Boltzmann appeals to in the above quote as he references Liouville's theorem (although it's good to note that *typicality* statements are extremely robust against variations in the measure [11,29]). As a *typicality* measure, the role of λ is not to express frequencies, or propensities, or degrees of belief, but only to characterize very large (resp. very small) sets of possible initial conditions. Stationarity ensures that large sets remain large (and small sets remain small) under time evolution, but also that λ can be understood as a natural measure on micro-histories [11].

It is important to appreciate that, while there are technically three measures involved in the *typicality* result (4), their respective meaning and status is very different. We have:

1. the stationary Liouville measure λ as a *typicality* measure on phase space;
2. the empirical distribution $r^N(k)[X]$ that results from the deterministic dynamics and initial conditions $X \in \Omega_0$;
3. the theoretical probability distribution $P(k) = B(k; M; \frac{1}{2})$ that approximates the empirical distribution for *typical* initial conditions.

What we can call objective probabilities are the *typical relative frequencies* described by P. It is neither necessary nor meaningful to interpret the phase space measure λ as probability distribution over "possible worlds". Note, in particular, that we did not assume at any point that the Galton board experiment is repeated. As our model universe, it has one and only one *actual* history, but this history is *typical* with respect to the statistical distribution of particles.

The central claim is that this *typicality* fact provides a conclusive explanation of the observed statistics, grounded in the underlying micro-dynamics, and justifies the probabilistic hypothesis of the binomial law. We can express this as a general *rationality principle* underlying *typicality* explanations (which is related to *Cournot's principle* in probability [11,32]; for a different but equally workable formulation, see [14]).

> **Typicality Principle (TP).** Suppose we accept a theory $\mathcal{T}$ and observe a phenomenon A. If A is *typical* according to $\mathcal{T}$, we should consider it to be conclusively explained. It is irrational to wonder further why our world is, in this particular respect, like nearly all (possible) worlds that the theory and its laws describe. (Conversely, *atypical* phenomena are, in general, the kind that cry out for further explanation and may ultimately compel us to revise or reject our theory.)

While I would argue that TP is an almost inevitable principle of scientific reasoning, the reader who disagrees may simply take it as a basic postulate of *typicality* accounts.

Finally, let me emphasize that TP is a normative principle, expressing epistemic implications of objective *typicality* facts. *Typicality* facts—e.g., the fact that nearly all possible initial conditions of the Galton board lead to an approximately binomial distribution of balls over the bins—do not depend on anyone's ignorance or beliefs. Hence, when the phenomenon in question is a *statistical regularity* that can be described by a probability "law", an objective *typicality* fact can explain objective probabilities.

3. Everett's Typicality Argument

To derive the Born rule in his Many-Worlds theory, Everett provides a *typicality* argument that is quite analogous to the one just discussed for the Galton board and in line with the general strategy for deriving statistical laws in Boltzmannian statistical mechanics. In Everett's account, the $|\Psi|^2$-measure determined by the universal wave function (that is, the branch amplitudes squared) defines a *typicality* measure on world branches which is used to identify statistical regularities that hold in the vast majority of them. Probabilities are once again *typical* relative frequencies, except that *typicality* is now understood relative to an actual "ensemble" of worlds (corresponding to macro- rather than micro-histories) that coexist within the Everettian multiverse. As Everett explained:

> We wish to make quantitative statements about the relative frequencies of the different possible results of observation—which are recorded in the memory—for a *typical* observer state; but to accomplish this we must have a method for selecting a *typical* element from a superposition of orthogonal states. [...] The situation here is fully analogous to that of classical statistical mechanics, where one puts a measure on trajectories of systems in the phase space by placing a measure on the phase space itself, and then making assertions ... which hold for "almost all" trajectories. [...] However, for us a trajectory is constantly branching (transforming from state to superposition) with each successive measurement. To have a requirement analogous to the "conservation of probability" in the classical case, we demand that the measure assigned to a trajectory at one time shall equal the sum of the measures of its separate branches at a later time. This is precisely the additivity requirement which we imposed and which leads uniquely to the choice of square-amplitude measure. ([5], pp. 460–461)

Just like Boltzmann in classical statistical mechanics and Dürr, Goldstein, and Zanghì in Bohmian mechanics [15], Everett appeals to a form of stationarity to justify the choice of *typicality* measure. More precisely, he stipulates three requirements that distinguish the measure uniquely (see [33] for an excellent discussion):

1. It should be a positive function of the complex-valued coefficients associated with the branches of the universal wave function.

2. It should be a function of the amplitudes of the coefficients alone, i.e., not depend on the phases.
3. It should satisfy the following additivity requirement: if a branch b is decomposed into a collection $\{b_i\}$ of sub-branches, the measure assigned to b should be the sum of the measures assigned to the sub-branches b_i.

This last additivity condition can be understood diachronically as *stationarity*; the weight assigned to a world at any given time equals the sum of the weights assigned to its branching histories at later times. This also assures a form of *locality* in that the weight of a world branch is not affected by the splitting of other branches.

Understood synchronically, the additivity condition does away with the problem that the notion of a world branch is unsharp. Whether one regards some component of the wave function as corresponding to one world (in which, let us say, a particular measurement outcome occurs) or further differentiates it into two or ten or a billion distinct world branches (with the same measurement outcome, but possibly different with respect to a finer-grained description), the total measure remains the same. In other words, the amplitude-squared weight assigned to a class of worlds with a certain characteristic is well-defined, even if the number of worlds in that class is not.

As Everett ([5], p. 460) notes, "In order that this general scheme be unambiguous we must first require that the states themselves always be normalized, so that we can distinguish the coefficients from the states". This presupposes, of course, a norm—and here, indeed, the familiar scalar product—with respect to which the branches can be normalized. It does not presuppose, however, that this scalar product has anything to do with a *typicality* measure. This follows from the three very plausible criteria just stated.

The measure defined by the branch amplitudes squared is not tied to ignorance, nor interpreted as a "measure of existence", as [23] proposes. As a *typicality* measure, its role is to provide a natural characterization of very large (resp. very small) classes of worlds, and whether the reader will find Everett's account of the Born rule satisfactory is bound to depend on whether she agrees that his criteria for such a measure are natural and well-justified.

To see how the *typicality* argument proceeds, we consider the paradigmatic example of a series of spin measurements performed on identically prepared electrons in the spin state

$$\varphi = \alpha|\uparrow_z\rangle + \beta|\downarrow_z\rangle, \ \ |\alpha|^2 + |\beta|^2 = 1. \tag{5}$$

It is important to keep in mind that the analysis starts with such wave functions of *subsystems*, to which the Born rule is actually applied, and then proceeds bottom-up, from subsystems to the universe.

We now denote by $|\Uparrow\rangle$ and $|\Downarrow\rangle$ the state of the measurement device (and, in the last resort, the rest of the universe) that has registered "spin up" and "spin down", respectively. After the first measurement, the joint (and ultimately universal) wave function will be of the form

$$\Psi(t_1) = \alpha|\uparrow_z\rangle_1|\Uparrow\rangle + \beta|\downarrow_z\rangle_1|\Downarrow\rangle\,, \tag{6}$$

where the index 1 indicates the first round of the experiment. The "pointer states" $|\Uparrow\rangle$ and $|\Downarrow\rangle$ are well-localized in disjoint regions of configuration space so that we have a decoherent superposition. Note, however, that this decomposition of the wave function corresponds to a very coarse-grained partition of the multiverse. In particular, no assumption is made about how many numerically distinct copies of the measurement device indicating "spin up" a term like $\alpha|\Uparrow\rangle|\uparrow_z\rangle_1$ represents, or even whether there is a well-defined number.

With the second measurement, the wave function splits anew:

$$\Psi(t_2) = \alpha^2|\uparrow_z\rangle_2|\uparrow_z\rangle_1|\Uparrow\Uparrow\rangle + \beta|\downarrow_z\rangle_2|\uparrow_z\rangle_1\alpha|\Downarrow\Uparrow\rangle + \alpha|\uparrow_z\rangle_2|\downarrow_z\rangle_1\beta|\Uparrow\Downarrow\rangle + \beta^2|\downarrow_z\rangle_2|\downarrow_z\rangle_1|\Downarrow\Downarrow\rangle.$$

The first three steps of the branching process are shown in Figure 2. We see the emergence of a structure reminiscent of the Galton board (although the discrete branching structure must be taken with a grain of salt). The conservation of the measure in each branch can be readily verified.

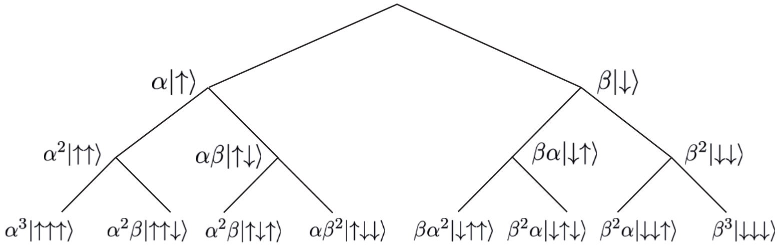

Figure 2. Branching Many-Worlds histories after three spin measurements. Successive arrows indicate successive outcomes. Adapted from Barrett [33].

After n rounds of spin measurements, the total $|\Psi|^2$-weight of branches in which the outcome "spin up" was registered exactly k-times is $\binom{n}{k}|\alpha|^{2k}|\beta|^{2(n-k)}$. Writing $|\alpha|^2 =: p$ and $|\beta|^2 = 1 - p$, we recognize this as a Bernoulli process with n independent trials and "success" probability p. A simple application of the weak law of large numbers thus allows us to conclude that, for large n, the *typical* relative frequency of *spin up* is $\frac{k}{n} \approx p = |\alpha|^2$, matching the Born statistics predicted by quantum mechanics.

The result involves again three different measures that we have to keep apart:

1. the *typicality* measure defined in terms of branch amplitudes of the universal wave function Ψ and uniquely determined by the stationarity condition;
2. empirical distributions (frequencies) that obtain within world branches, here for a sequence of spin measurements on identically prepared particles;
3. the theoretical Born probabilities defined in terms of the quantum state φ (the wave function or perhaps density matrix) of subsystems, e.g., by $P(\text{spin up}) = \langle\varphi|\uparrow\rangle\langle\uparrow|\varphi\rangle = \alpha^2$. They are shown to approximate relative frequencies in *typical* branches.

In conclusion, the Born probabilities describe (to a good approximation) long-term frequencies along *typical* world branches, where "*typical*" is characterized in terms of the stationary *typicality* measure induced by the universal wave function.

4. Living and Dying in the Multiverse

What have we accomplished in regard to the probability problem? Everett's analysis establishes that Born statistics holds across *typical histories* of the constantly branching multiverse. One would now like to conclude with an empirical prediction and say something like, "Hence, *I* should expect to experience a *typical* history consistent with the Born rule". But the indexical *I* does not pick out an individual with a unique future history. My current branch will split repeatedly, and there will be future versions of me who experience very different statistics.

I see no way around the conclusion that the Many-Worlds theory lacks a certain *predictive* quality. When we ask what statistical regularity we will observe, the answer is always that *any* possible sequence of outcomes will be observed by *some* of our "descendants". But I contend that, based on the *Typicality Principle* stated in Section 2, Everett's argument successfully grounds post-factum *explanations*. When I lie on my deathbed and wonder why I have experienced a history consistent with standard quantum mechanics, I will die in peace knowing that this is *typical*, that *nearly all* Many-Worlds histories—in the most natural sense of "nearly all" that the theory allows—manifest phenomena consistent with Born's statistical hypothesis.

Because of the persistent difficulty in making sense of probabilistic predictions, Everett's account does not resolve all doubts about whether the "Many Worlds theory can recover the usual understanding of the implications of Born's Rule" ([3], p. 189). But if one understands the probability problem as one of accounting for objective statistical regularities, a *typicality* result is certainly optimal in the following sense: Born statistics are the empirical regularity we need to explain, and establishing that they obtain in nearly all world branches is the best we can hope for since the Born rule will definitely be false in some.

Those who regard the justification of Born's rule primarily as a problem of decision making should also be happy with Everett's result. It allows us to conclude that we should follow the Born rule to maximize utility for *typical* future selves. This is a reasonable maxim since, for atypical branches, all bets are off anyway. We cannot make rational choices for descendants for whom the fundamental laws of nature will manifest in unpredictable and unrecognizable ways. Even from the perspective of logical parsimony, Everett's account—with three axioms for the *typicality* measure and adding one rationality principle for *typicality*—fares better than most contemporary alternatives.

For critics who insist that "*typicality*" is just another word for "high probability", Wilhelm [13] (in addition to discussing formal, conceptual, and metaphysical differences between *typicality* and probability) makes the interesting observation that Everett's *typicality* explanation is manifestly distinct from a probabilistic explanation—at least if one agrees that probabilistic explanations presuppose only one of multiple alternatives to actually obtain.

> "[I]n Everettian quantum mechanics, the various possible outcomes of any given experiment all obtain. [...] But in probabilistic explanations, that cannot happen. In probabilistic explanations, the event invoked in the explanandum is the only outcome, of the various possible mutually exclusive outcomes, that occurs".

One might try to evade Wilhelm's argument by falling back on self-locating probabilities: only one of the copies of D.L. existing in the multiverse is the branch-indexical *I*. But me being me does not seem like the right explanandum. There is no self-locating uncertainty in the deathbed scenario; I know what life I have lived and hence what branch of the multiverse I have inhabited. For better or worse, the *typicality* explanation ends with the fact that the Born rule holds across the vast majority of world branches. To ask further, for the probability that I find myself on any one of the branches (as if my *ego* had been somehow thrown at random into the multiverse) strikes me as redundant at best and meaningless at worst.

5. Is Everett's Derivation Circular?

Finally, one must wonder why modern Everettians have almost universally dismissed Everett's account of the Born rule that came with the birth of the Many-Worlds interpretation. The most common objection seems to be that Everett's derivation involves a circularity. Wallace, in his authoritative book *The Emergent Multiverse*, expresses this charge very pointedly:

> In his original paper (1957) [Everett] proved that if a measurement is repeated arbitrarily often, the combined mod-squared amplitude of all branches on which the relative frequencies are not approximately correct will tend to zero. And of course this is circular: it proves not that mod-squared amplitude equals relative frequency, but only that mod-squared amplitude equals relative frequency with high mod-squared amplitude.
>
> Substitute 'probability' for 'mod-squared amplitude', though, and the circularity should sound familiar; indeed, Everett's theorem (as is well known) is just the Law of Large Numbers transcribed into quantum mechanics. So the circularity in Everett's argument is just the circularity in the simplest form of frequentism, disguised by unfamiliar language. That simplest form of frequentism may indeed be hopeless, but so far Everettian quantum mechanics has neither helped nor hindered it. ([17], p. 127)

But Everett does not argue that probability equals relative frequency with high probability. He argues that relative frequencies equal mod-squared amplitudes in nearly all world branches, and there is nothing circular about that. "Probability" is not explained in terms of "probability", nor do we enter an infinite regress of frequencies of frequencies of frequencies. In any case, Everett's argument is not based on the simplest form of frequentism but on *typicality*, a concept he explicitly appeals to. For a detailed discussion of how *typicality* avoids the main objections against (actual and hypothetical) frequentism, I refer the reader to Hubert (2021) [10].

The circularity charge against Everett is easily fueled by the misperception that his derivation is simply "$|\psi|^2$ in, $|\psi|^2$ out", that something tantamount to Born's rule is already assumed by using branch amplitudes squared as the *typicality* measure. To understand why this is wrong, one must appreciate the different meanings and statuses of the measures and quantum states involved. The *typicality* measure is defined in terms of the universal wave function and justified by Everett's three assumptions discussed in Section 3. Its role is to provide a natural and well-defined notion of "nearly all world branches", and its status analogous to that of the stationary Liouville measure as the natural *typicality* measure for classical mechanics. The Born rule is defined in terms of quantum states φ of *subsystems*, e.g., states like (5) that we prepare for particles undergoing a measurement experiment. It turns out to describe relative frequencies (in ensembles of subsystems) along *typical* world branches. The status of the $|\varphi|^2$-distribution is similar to that of the binomial distribution for particles on the Galton board, which we discussed in Section 2, or to the status of the Maxwell distribution $f(\mathbf{v}) \propto \exp\left(-\frac{m\mathbf{v}^2}{2kT}\right)$ that Boltzmann derived as the equilibrium distribution of an ideal gas. Boltzmann showed that for nearly all microstates (with respect to the stationary Liouville measure), the relative frequency of particles with velocity in $A \subset \mathbb{R}^3$ is approximately $\int_A f(\mathbf{v}) d^3v$. Whoever claims that this seminal result is circular, must at least admit that Everett is in good company.

In the Everettian case, we have the mathematically convenient but didactically unfortunate situation that the natural *typicality* measure and the derived probability law describing *typical* frequencies have the same mathematical form. Both are "amplitudes squared"—for world branches of the universal wave function and projections of subsystem wave functions, respectively. But this is a non-trivial feature of the quantum theory and its linear Schrödinger dynamics. It is a result, not a premise, of the statistical analysis. In particular, we could not have run the same argument with branch amplitudes to the power $k \neq 2$ as *typicality* measure and infer that *typical* frequencies are described by a $|\varphi|^k$ probability law. Such a derivation would already fail because the weights of the world branches would not be conserved under the branching process.

In conclusion, Everett's *typicality* account of the Born rule is neither conceptually nor logically circular. And its mathematical simplicity should not blind us to the fact that it is quite profound.

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Article

Centering the Born Rule

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Abstract: The centered Everett interpretation solves a problem that various approaches to quantum theory face. In this paper, I continue developing the theory underlying that solution. In particular, I defend the centered Everett interpretation against a few objections, and I provide additional motivation for some of its key features.

Keywords: Everett interpretation; chance; Born rule; centered propositions; indexicals

1. Introduction

This paper is about a particular solution, to a particular problem, that arises for many different versions of the Everett interpretation of quantum theories. The problem is that these interpretations have trouble reconciling the deterministic evolution of the universe with the indeterministic outcomes of experiments. In this paper, I discuss my preferred solution at length, developing several subtle aspects of it.

Here is the problem in a little more detail. According to many different versions of the Everett interpretation, the complete physical state of the universe evolves deterministically. That state is aptly represented by a wavefunction whose evolution conforms to the deterministic Schrödinger equation. However, according to the Born rule—different versions of which are endorsed by different versions of the Everett interpretation—experimental outcomes are indeterministic. The probability that certain z-spin up electrons will eventually be seen in the x-spin up state, for example, is $\frac{1}{2}$. So the problem is: the deterministic evolution of the universe seems incompatible with the indeterministic outcomes of experiments. It is unclear what should be made of the apparent conflict between (i) the universe being deterministic, and (ii) experimental outcomes being indeterministic. Call this the 'probability problem' for various versions of the Everett interpretation.

To solve this problem, in other work, I formulate and defend the centered Everett interpretation of quantum theories [1]. Roughly put, according to that interpretation, the universe evolves deterministically over one class of propositions, while experimental outcomes are indeterministic over another class of propositions. The indeterminism derives from an unappreciated, and largely overlooked, kind of chance [2,3]: basically, as I argue, objective chances can be assigned to irreducibly subjective propositions such as "I will see the electron in the x-spin up state". The objectively chancy character of those subjective propositions supports a striking, compelling solution to the probability problem; one which, among other things, avoids the shortcomings of standard solutions based on rational credences [1] (pp. 1033–1038). The principle that assigns objective chances to these subjective propositions is what I call the 'centered Born rule'. That rule is the key to the probability problem's solution.

Unfortunately, the centered Everett interpretation is quite complicated. In addition to drawing on theories in the philosophy of science and the philosophy of physics, it also draws on theories in linguistics, philosophy of language, logic, metaphysics, and formal epistemology. All of those moving parts make the details of the centered Everett interpretation hard to absorb.

In addition, the basic posit of the centered Everett interpretation—that apparently subjective propositions such as "I will see the electron in the x-spin up state" have objective

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chances of obtaining—might seem quite odd. It is unclear how the proposition expressed by that sentence, which uses the indexical 'I', could have an objective chance of obtaining. For this reason too, the centered Everett interpretation strikes many as bizarre.

In the present paper, I seek to clarify all this by continuing to develop the theory of the centered Everett interpretation. For starters, I defend the centered Everett interpretation against two objections that are often raised against it. Then I explain why certain posits of the centered Everett interpretation are far more attractive, and far less strange, than they might initially seem to be.

In Section 2, I present the centered Everett interpretation in more detail. In Section 3, I defend the centered Everett interpretation against an objection that, though common, is based on a false assumption about the propositions that indexicals are used to express: the objection mistakenly supposes that sentences such as "I will see the electron in the x-spin up state" sometimes express the same proposition as sentences such as "Susie will see the electron in the x-spin up state". In Section 4, I defend the centered Everett interpretation against an objection that, though also common, is based on a false assumption about how to count branches in the Everettian universe: the objection mistakenly claims that the centered Born rule holds on a negligibly small percentage of branches. In Section 5, I explain why a certain view of the metaphysics of agents—that the centered Everett interpretation endorses—is better, overall, than alternative views of what agents are: I focus, in particular, on a view which utilizes temporal counterpart theory. Finally, in Section 6, I explain why a particular implication of the centered Everett interpretation is not as strange as one might have thought: the centered Everett interpretation implies that some laws only hold relative to particular branches, but as I argue, there is nothing problematic about that.

2. The Centered Everett Interpretation

In this section, I summarize the centered Everett interpretation. To start, I provide a rough, big-picture summary of the key idea with which the centered Everett interpretation solves the probability problem. Then I present the centered Everett interpretation itself.

By way of preparation, it is worth reviewing the distinction between uncentered propositions and centered propositions. Roughly put, uncentered propositions are expressed by sentences that do not contain any indexicals. The sentence "Susie will see the electron in the x-spin up state", for instance, expresses an uncentered proposition since this sentence contains the name 'Susie'. Centered propositions, in contrast, are expressed by sentences that contain indexicals. The sentence "I will see the electron in the x-spin up state", for instance, expresses a centered proposition since this sentence contains the indexical 'I'.

A brief but important aside: my approach to indexicals and centered propositions is based on the theory developed by Kaplan [4,5], as well as on the philosophical and logical foundations of indexicals, demonstratives, centered propositions, and de se content discussed in [6–10]. Now, the physics and philosophy of physics literature—surrounding Everett in particular—discuss a kind of agential subjectivity that is, in some pre-theoretic sense or other, connected to the intuitive ideas that these theories in linguistics, logic, and the philosophy of language make rigorous. However, there is a large gap between the discussions of those intuitive ideas that occur in the physics and philosophy of physics literature, and the corresponding discussions that occur in the linguistics, logic, and philosophy of language literature. This paper, along with [1], helps close that gap.

It is extremely important to note that the proposition expressed by "Susie will see the electron in the x-spin up state" is different from the proposition expressed by "I will see the electron in the x-spin up state". In fact, the propositions are different even when both sentences are uttered by Susie. Of course, when Susie utters the latter sentence, the 'I' ultimately refers to her. However, the proposition she expresses, by uttering the latter sentence, is centered—and so that proposition is different from the uncentered proposition that Susie expresses by uttering the former sentence.

This distinction, between uncentered propositions and centered propositions, can be used to solve the probability problem. In rough outline, the solution is as follows: the universe evolves deterministically with respect to uncentered propositions but indeterministically with respect to centered propositions. More precisely, the universe is deterministic in the following sense: for each time t and each time t' later than t, (i) the uncentered propositions that describe various physical states of the universe at t', are determined by (ii) whatever uncentered proposition completely describes the physical state of the universe at t. The Schrödinger equation captures all of that. However, the universe is indeterministic in the following sense: for each time t and each time t' later than t, (i) the centered propositions that describe various physical states of the universe at t', are generally not determined by (ii) any of the propositions, centered or uncentered, which describe the physical state of the universe at t. That is what the Born rule captures.

Think of it this way. The Schrödinger equation implies that the universe is deterministic over the algebra of uncentered propositions—the algebra, that is, containing one proposition for each possible quantum state of the universe at each time—that describe physical reality. That is perfectly compatible with the universe being indeterministic, in the manner in which the Born rule implies, over the algebra of centered propositions which describe physical reality. So there is no conflict whatsoever between (i) the universe being deterministic and (ii) experimental outcomes being indeterministic. The determinism of the universe is confined to an algebra of uncentered propositions, and the indeterminism of the universe is confined to an algebra of centered propositions. That solves the probability problem.

The centered Everett interpretation provides an account of how, exactly, the Born rule assigns probabilities to centered propositions. In summary, the centered Everett interpretation has three components: a metaphysical account of branches and agents, a version of the Born rule that assigns objective chances to centered propositions, and an analysis of the metaphysics of those chances. Let us consider each component in turn; for more details, see [1].

First, the metaphysics of branches and agents: both branches and agents are four-dimensional entities. They extend through time as well as through space. So they are often called 'spacetime worms', and this view of branches and agents is often called the 'worm view'.

Given the worm view, there is an elegant way to think about how branching works. Each branch is an approximately isolated region of the wavefunction that evolves, more-or-less, like a classical world. Each agent is part of some branch or other. For periods of time, some branches are exact physical duplicates of one another, and some agents are exact physical duplicates of one another too. However, when certain sorts of events occur—quantum experiments, for instance—some branches cease to be exact physical duplicates of some others, and the agents in those branches cease to be exact physical duplicates of each other as well.

The following picture illustrates all this Figure 1.

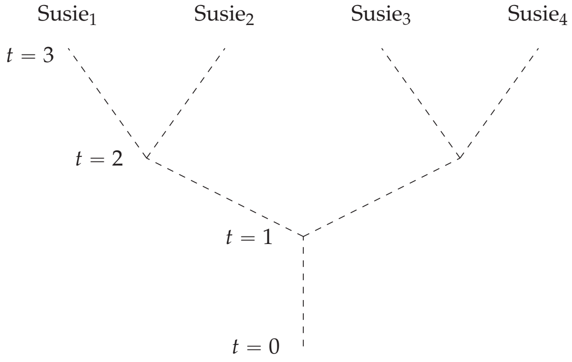

Figure 1. Branching worlds.

In this picture, there are four branches, each represented by a dotted line that begins at $t = 0$ and ends at $t = 3$. From $t = 0$ to $t = 1$, all four branches are exact physical duplicates of one another; this is represented, in the picture, by the four lines overlapping. At $t = 1$, a measurement occurs, and so the branches divide into two groups of two: the left two branches remain exact physical duplicates of each other, the right two branches remain exact physical duplicates of each other, but the left two branches are no longer exact physical duplicates of the right two branches. Then at $t = 2$, measurements occur again: afterwards, none of the four branches are exact physical duplicates of any other branch.

Second, a version of the Born rule: this version assigns ψ-squared chances to centered propositions. In particular, for simplicity, let E be an agent who performs a measurement, or a community of agents who measure something, or a branch that contains some agents who perform some measurements. Let $|\psi\rangle$ be the wavefunction before measurement occurs. Let $|a\rangle$ be a branch, or a collection of branches, into which the wavefunction splits. Let O_a be the centered proposition expressed by the sentence "I am in one of the $|a\rangle$ branches". Let $Ch_{E,\psi}$ be a probability function that assigns chances to centered propositions such as O_a: think of $Ch_{E,\psi}$ as assigning chances to propositions expressed by sentences of the form "I am in one of the thus-and-so branches", where the chances in question are relativized to agents and to wavefunctions before measurement. Suppose that E is unsure of which branch is theirs—as all agents, in the actual world, in fact are. Then

$$Ch_{E,\psi}(O_a) = |\langle a|\psi\rangle|^2$$

Call this the 'centered Born rule'.

Basically, the centered Born rule assigns objective chances to centered propositions. Relative to, for instance, an agent E, and relative to a wavefunction $|\psi\rangle$ before measurement, the chance of the proposition expressed by "I am in one of the thus-and-so branches" obtaining is just the usual ψ-squared probability. These chances, which the centered Born rule associates with centered propositions, are called 'centered chances'.

Third, the metaphysics of centered chance: the centered chances, which the centered Born rule posits, can be analyzed using the ideas that underlie the best system account of lawhood. By way of preparation for that analysis, it is worth reviewing what the best system account of lawhood is. Basically, according to that account, laws are useful summaries. To be a law, in particular, is to be an implication of the best deductive systems, where a deductive system is best just in case it best balances a variety of theoretical virtues that scientists generally value and that good summaries generally have: simplicity, strength, fit, calculational tractability, and more [11–15].

Uncentered chances—that is, objective chances of uncentered propositions—can be analyzed using the best system account of laws. According to that analysis, an uncentered chance is a proposition that (i) assigns a probability to some uncentered proposition, and (ii) follows from the best deductive system. Basically, an uncentered chance contributes to the best overall summary of the world, by summarizing the frequency with which a corresponding uncentered proposition, such as the proposition that a particular electron will be in the x-spin up state, obtains.

Likewise for centered chances. Basically, according to the analysis that I propose, a centered chance is a proposition that (i) assigns a probability to some centered proposition, and (ii) follows from the best deductive system. So think of it like this: a centered chance contributes to the best overall summary of the world, by summarizing the frequency with which a corresponding centered proposition—such as the proposition that I will see a particular electron in the x-spin up state after measurement—obtains.

A striking consequence of this analysis: it implies that the centered Born rule holds on some branches but not others. On some branches, namely branches where propositions expressed by sentences such as "I will see the electron in the x-spin up state" obtain with the standard ψ-squared frequency, the centered Born rule provides a great summary of the frequency facts; so on those branches, the centered Born rule holds. On other branches, however, the summary that the centered Born rule would provide—of the frequency facts on those branches—is terribly inaccurate, so the centered Born rule does not hold. For example, on one such branch, the proposition expressed by the sentence "I will see the electron in the x-spin up state" obtains with a frequency of 1, which of course is not the ψ-squared frequency that the centered Born rule would predict. On another such branch, the proposition expressed by the sentence "I will see the electron in the x-spin up state" obtains with a frequency of 0; that too is incompatible with the centered Born rule. So the centered Born rule is a law on many branches but not on all. The lawhood of the centered Born rule is branch-relative.

To summarize: the centered Everett interpretation says, basically, that the universe is deterministic over uncentered propositions and indeterministic over centered propositions. The centered Born rule captures, in precise detail, the universe's indeterministic evolution. The four-dimensional account of the metaphysics of branches, and of agents, facilitates that. And the best system account of lawhood extends to an analysis of how centered propositions can be objectively chancy in the manner that the centered Born rule implies.

3. Utterances of Indexicals

In this section, I discuss an objection to the chances that the centered Born rule assigns. One might be tempted to make some seemingly natural assumptions about the propositions that utterances of certain sentences—featuring indexicals—express. Those assumptions imply that the chances of the centered propositions at issue must be either 0 or 1. However, as I will explain, those assumptions are false. So contrary to the objection, the relevant centered propositions can have non-null, non-unit chances, as the centered Born rule implies.

Here is the sort of situation that often leads to confusion. In Figure 1, suppose that the splits all correspond to x-spin measurements of a particular electron which, just before time $t = 1$, has z-spin up. In addition, suppose that at $t = 1$, the following situation obtains.

- On Susie$_1$'s branch and Susie$_2$'s branch, the electron is found to have x-spin up.
- On Susie$_3$'s branch and Susie$_4$'s branch, the electron is found to have x-spin down.

Suppose that before $t = 1$, Susie$_1$–Susie$_4$ all utter the sentence "I will see the electron in the x-spin up state": so Susie$_1$'s utterance is true, Susie$_2$'s utterance is true, Susie$_3$'s utterance is false, and Susie$_4$'s utterance is false. In addition, suppose that the centered Born rule holds on all four branches in the figure; and suppose, moreover, that each of Susie$_1$–Susie$_4$ has conducted thousands of experiments on their branch which confirm exactly that. So in the times before $t = 1$, Susie$_1$–Susie$_4$ have found that centered propositions expressed by sentences of the form "I am in one of the branches where systems with thus-and-so quantum state are found, after measurement, to have such-and-such quantum state" obtain with the frequencies that the centered Born rule predicts. Therefore, each of Susie$_1$–Susie$_4$ knows that the centered Born rule holds on their branch.

Note that before $t = 1$, there really are four utterances which Susie$_1$–Susie$_4$ make; so long, that is, as Susie$_1$–Susie$_4$ each have a distinct temporal part at the time of utterance. Susie$_1$–Susie$_4$ are physical duplicates of each other, of course. But their temporal parts, at the time when they each utter the sentence "I will see the electron in the x-spin up state", are distinct. That is why the problem discussed by Tappenden does not arise [16] (p. 311): Susie$_1$–Susie$_4$ successfully refer to themselves, when the utterance of that sentence occurs, because they do not literally share temporal parts at that time. And all this vindicates the view, discussed in detail by Saunders and Wallace, that agents in the Everettian universe can experience *de se* uncertainty before measurement [17] (p. 301).

One might object that the situation here, as I described it, is incoherent. In particular, one might object by claiming that there is a conflict between (i) the fact that Susie$_1$ and Susie$_2$ find the electron to have x-spin up while Susie$_3$ and Susie$_4$ find the electron to have x-spin down, and (ii) the fact that the centered Born rule holds on the branches containing Susie$_1$–Susie$_4$. For the chance assigned to the proposition expressed by the sentence "I will see the electron in the x-spin up state", one might claim, should be 0 or 1, depending on the agent Susie$_1$–Susie$_4$ at issue. After all, on the branches containing Susie$_1$ and Susie$_2$, that proposition obtains: therefore, one might claim that relative to Susie$_1$ and relative to Susie$_2$—and relative to the wavefunction prior to measurement—that proposition's chance is 1. Additionally, on the branches containing Susie$_3$ and Susie$_4$, that proposition does not obtain: therefore, one might claim that its chance—relative to those agents, and to the wavefunction prior to measurement—is 0. So the proposition expressed by the sentence "I will see the electron in the x-spin up state" cannot be assigned chance $\frac{1}{2}$. But that is the chance that the centered Born rule, if true on the branches of Susie$_1$–Susie$_4$, would assign to that proposition. So given (i), it follows that (ii) is false: given what Susie$_1$–Susie$_4$ actually observe, the centered Born rule does not hold on their branches.

This objection contains several mistakes. It is true, of course, that the proposition expressed by the sentence "I will see the electron in the x-spin up state" obtains on the branches containing Susie$_1$ and Susie$_2$. But it does not follow that the chance of this proposition, relative to Susie$_1$ and relative to Susie$_2$—and relative to the wavefunction prior to measurement—must be 1. Similarly, it is true that this proposition does not obtain on the branches containing Susie$_3$ and Susie$_4$. But it does not follow that the chance of this proposition, relative to Susie$_3$ and relative to Susie$_4$—and relative to the wavefunction prior to measurement—must be 0. Each of Susie$_1$–Susie$_4$ utter the sentence "I will see the electron in the x-spin up state". Susie$_1$ and Susie$_2$ speak truly: relative to them, the proposition which that sentence expresses is true. Susie$_3$ and Susie$_4$ speak falsely: relative to them, the proposition which that sentence expresses is false. But it simply does not follow, from any of this, that the chance of this proposition must be 1–relative to Susie$_1$ and Susie$_2$ (and to the wavefunction prior to measurement)–or 0–relative to Susie$_3$ and Susie$_4$ (and to the wavefunction prior to measurement).

I suspect that the confusion underlying this objection derives from a failure to properly distinguish centered propositions from uncentered propositions. By way of illustration, consider Susie$_1$. The objector might think that Susie$_1$'s utterance of the sentence "I will see the electron in the x-spin up state" expresses the proposition that Susie$_1$ will see the electron in the x-spin up state; for after all, Susie$_1$'s utterance of 'I' refers to Susie$_1$. That proposition obtains deterministically: given the initial wavefunction, that proposition must hold. So the objector concludes that the proposition expressed by Susie$_1$'s utterance of the sentence "I will see the electron in the x-spin up state" must have chance 1.

The mistake in the paragraph above is as follows: Susie$_1$'s utterance of the sentence "I will see the electron in the x-spin up state" does not express the proposition that Susie$_1$ will see the electron in the x-spin up state. That proposition is uncentered: it is expressed without using indexicals. But the sentence "I will see the electron in the x-spin up state" contains an indexical and because of that—and for many other reasons too—expresses a centered proposition [18,19]. So the objector is wrong to claim that the proposition expressed by Susie$_1$'s utterance of the sentence "I will see the electron in the x-spin up state" must have chance 1. The proposition that Susie$_1$ will see the electron in the x-spin up state does, of course, have chance 1 of obtaining. But the proposition expressed by Susie$_1$'s utterance of the sentence "I will see the electron in the x-spin up state" is definitely not that proposition; so the proposition expressed by that utterance need not have chance 1, and so that proposition can have the chance assigned by the centered Born rule.

This point is worth belaboring because it is so easy to forget. Suppose that Susie$_1$ utters two sentences:

(1) “I will see the electron in the x-spin up state”

and

(2) “Susie$_1$ will see the electron in the x-spin up state.”

Relative to Susie$_1$, the propositions expressed by those sentences are true. Nevertheless, the propositions expressed by those sentences are, despite the fact that Susie$_1$ is the speaker—and so Susie$_1$’s utterance of ‘I’ refers to herself—different: (1) expresses a centered proposition, and (2) expresses an uncentered proposition. So while the truth value of the proposition expressed by (2) is completely fixed by the deterministic evolution of the wavefunction, the truth value of the proposition expressed by (1) is not.

A crucial, related aside: when each of Susie$_1$–Susie$_4$ utter sentence (1), they each express the very same proposition, despite the fact that their respective uses of the indexical ‘I’ refer to different agents. This is simply how the standard semantics for centered propositions works [4]. If you and I both utter the sentence “I like apple pie”, for instance, then we both express the same proposition, despite the fact that our respective utterances of ‘I’ have different referents. This is related, of course, to the fact that truth values are only ever assigned—to centered propositions—relative to centers.

To summarize: there is no conflict between (i) the fact that Susie$_1$ and Susie$_2$ find the electron to have x-spin up while Susie$_3$ and Susie$_4$ find the electron to have x-spin down, and (ii) the fact that the centered Born rule holds on the branches containing Susie$_1$–Susie$_4$. Relative to Susie$_1$ and Susie$_2$—and relative to the wavefunction prior to measurement—the proposition expressed by the sentence “I will see the electron in the x-spin up state” is indeed true, and relative to Susie$_3$ and Susie$_4$—and relative to the wavefunction prior to measurement—the proposition expressed by this sentence is indeed false. But it is not the case that this sentence, when uttered by Susie$_1$ say, expresses the proposition that Susie$_1$ will see the electron in the x-spin up state. So the proposition expressed by this sentence need not be assigned chance 1. It can be assigned the chance given by the centered Born rule.

One final point: all this connects to a concern that P. Lewis raises for various versions of the Everett interpretation [20]. Basically, P. Lewis claims that no extant version of the Everett interpretation can account for non-trivial probability assignments to measurement outcomes, for the following reason: each such version of the Everett interpretation implies that prior to measurement, every agent knows with certainty (i) where they are, and (ii) what will happen to them in the future [20] (pp. 12–13). Regardless of whether that is correct for other versions of the Everett interpretation, it is not correct for the centered Everett interpretation under discussion here. Because the centered Everett interpretation is consistent with the claim that before measurement, agents cannot tell which branch they are on, the agents’ epistemic states are compatible with being on many, many different branches that are all physical duplicates of one another.

4. Counting Branches

In this section, I discuss an objection regarding the number of branches on which the centered Born rule holds. One might be tempted to assume that the centered Born rule obtains on very few branches; and on that basis, one might object to the centered Born rule. But that assumption is incorrect. Given the proper way of measuring how many branches there are, the centered Born rule obtains on most of the branches.

Here is the objection to which this confusion, about the number of branches in the universe, gives rise. Every outcome of every quantum experiment obtains on some branch or other. So there are branches in which $\frac{1}{2}$ of z-spin up electrons are found to have x-spin up, as the centered Born rule predicts. But there are also branches in which $\frac{1}{3}$ of z-spin up electrons are found to have x-spin up, and there are branches in which $\frac{2}{3}$ of z-spin up electrons are found to have x-spin up, and so on; and on all of these branches, the centered Born rule is false. Therefore, one might claim, the centered Born rule holds on a negligibly small percentage of branches. And therefore, we have no good reason for thinking that the centered Born rule holds on our branch in particular.

The main mistake, in this objection, is the claim that the centered Born rule holds on a negligibly small percentage of branches. It is certainly true that there are branches in which the centered Born rule fails. There are branches where $\frac{1}{3}$ of z-spin up electrons are found to have x-spin up; there are branches where $\frac{2}{3}$ of z-spin up electrons are found to have x-spin up; and so on. But it does not follow, from this alone, that the centered Born rule holds on very few branches. In order to draw any conclusions about the number of branches on which the centered Born rule holds, some sort of measure—over the set of all branches—must be used. For reasons I will explain, the best measure to use is the ψ-squared measure that the centered Born rule invokes. And given that measure, it follows that on the vast majority of branches, the centered Born rule holds. Let us see why.

By way of preparation, note that Figure 1 is a simplified depiction of branches. Strictly speaking, branches do not form discrete, countable units. There are densities of branches. So it is not as if there are exactly four branches, of which Susie$_1$–Susie$_4$ are parts. There are continuum-many branches in the Everettian universe [21] (p. 20).

Because $\mathcal{B}$ is uncountable, it can be equipped with many different measures. That is, the set $\mathcal{B}$, taken on its own—absent any considerations about chance, or the dynamics of the wavefunction, or decoherence, or any such thing—does not select out any one measure as the best way of quantifying the sizes of subsets which $\mathcal{B}$ contains; it is not as if $\mathcal{B}$ has the intrinsic structure of the real numbers, for instance. Of course, if $\mathcal{B}$ were finite, then one measure would clearly be best: the counting measure. However, since $\mathcal{B}$ is uncountable, there are many sigma-algebras and many measures that—again, absent any considerations about chance, or the dynamics of the wavefunction, or decoherence, or any such thing—all do more-or-less equally good jobs of assigning sizes to subsets that $\mathcal{B}$ contains.

All this raises a question: what is the right measure to use, for assigning sizes to subsets of $\mathcal{B}$? Which measure, that assigns sizes to those sets, is best? Which one gets the sizes of those sets right?

Note that for the purposes of evaluating the objection above, these questions really matter. To see why, let $\mathcal{C}$ be the set of branches on which the centered Born rule holds. Some measures will imply that $\mathcal{C}$ is quite large. Other measures will imply that $\mathcal{C}$ is negligibly small. So the right measure to use, for assigning sizes to subsets of $\mathcal{B}$, will determine how large $\mathcal{C}$ is. And that, in turn, will determine whether or not the centered Born rule holds on most branches, or a negligibly small percentage of branches, or something in between.

As shown by Everett, there is a natural measure to use for assigning sizes to subsets of $\mathcal{B}$: the ψ-squared measure [22,23]. That measure satisfies a series of reasonable conditions for quantifying the sizes of sets which $\mathcal{B}$ contains [24,25]. For instance, that measure is a function of the amplitudes of coefficients of branches in certain superpositions. Furthermore, that measure is conserved, in a certain precise sense, under the linear dynamics of the Schrödinger equation. So given reasonable constraints based on the structures of amplitudes and on wavefunction dynamics, that is the best measure for assigning sizes to sets that $\mathcal{B}$ contains.

It follows that the centered Born rule holds on the vast majority of branches. For the ψ-squared measure assigns a size of nearly 1 to $\mathcal{C}$. (A little more precisely: given the ψ-squared measure, as the number of quantum experiments increases without bound, the measure of the set of branches where the outcomes of those experiments conform to the centered Born rule converges to 1. Put in terms of the notion of typicality, which I prefer to use when quantifing the distribution of branches in the Everettian universe: in the set of all branches, the centered Born rule typically holds.) So it is false to claim that the centered Born rule holds on a negligibly small percentage of branches. The centered Born rule obtains on nearly all branches in the Everettian universe.

Let me be clear about what I have not argued. Nowhere did I claim that the centered Born rule holds on our branch because the centered Born rule holds on nearly all branches whatsoever. That sort of justification, for the centered Born rule obtaining on our branch, may be circular. For the notion of 'nearly all', which that sort of justification invokes, is precisified using the ψ-squared measure. And the ψ-squared measure is basically just the measure that the centered Born rule invokes. So the claim in question—that the centered Born rule holds on our branch because the centered Born rule holds on nearly all branches—may amount to a circular explanation: it may be using the centered Born rule to justify the centered Born rule. And that, of course, would be problematic.

What I have argued, rather, is this: the best measure to use, for assigning sizes to subsets of $\mathcal{B}$, is the ψ-squared measure. For that measure concerns the structure of wavefunction amplitudes, the sorts of dynamics which the Schrödinger equation expresses, and so on. Furthermore, according to that measure, the centered Born rule holds on nearly all branches whatsoever. So the original objection, with which this section began, is based on a false assumption.

One might respond to all this by asking: so why does the centered Born rule hold on our branch? The answer: the centered Born rule holds on our branch because it provides the best summary of the frequencies with which, on our branch, propositions expressed by sentences such as "I am in one of the thus-and-so branches" obtain. In other words, the explanation of the centered Born rule's truth, on our branch in particular, appeals to the metaphysical view of centered chance that the centered Everett interpretation provides; the explanation of the centered Born rule's truth, on our branch in particular, does not appeal to any claims about numbers of branches.

5. Metaphysics of Agents

In this section, I provide some motivation for the worm view that the centered Everett interpretation endorses. To do so, I discuss an alternative account of the metaphysics of agents. According to that alternative account, agents are not four-dimensional worms: they only exist at instantaneous moments. In addition, according to that alternative account, the truth conditions for tensed sentences about agents at particular times invoke other agents at other times. If all that were true, of course, then the worm view—and therefore, the centered Everett interpretation—would be false. However, as I argue, that account faces a problem that the worm view avoids, and that is a reason to endorse the worm view.

The basic idea, underlying the alternative account of the metaphysics of agents, is this: at time $t = 0$, just one agent exists; let us call her 'Susie'. So it is not as if four agents exist at $t = 0$, namely, Susie$_1$–Susie$_4$. Only Susie exists at that time. And so the metaphysics of agents, which I used to formulate the centered Everett interpretation, is false.

There are many different versions of this view. According to one version, each agent is an instantaneous time-slice, and agents that exist at different times are numerically distinct from one another [26,27]. Susie, for instance, is the clump of matter that exists, just for an instant, at time $t = 0$. In addition, at each specific time between $t = 0$ and $t = 1$, exactly one agent exists. But also, between $t = 0$ and $t = 1$, continuum-many agents exist: one for each particular, specific time in that range. Similarly, at each particular, a specific time between $t = 1$ and $t = 2$, two agents exist: one exists on the left branch of Figure 1, and the other exists on the right branch of that figure. But also, between $t = 1$ and $t = 2$, continuum-many agents exist on the left branch and continuum-many agents exist on the right branch: one on the left, and one on the right, for each particular, specific time in that range. And similarly for the range of times between $t = 2$ and $t = 3$.

A brief aside: a similar view is discussed by Vaidman [28]. Vaidman often describes the underlying issue here in terms of the meaninglessness of certain linguistic expressions [28] (p. 254). In my view, there is a better way to describe the underlying issue. The claim that certain linguistic expressions are meaningless is empirically false: it contradicts the empirical science of linguistics. It is better to claim that those linguistic expressions, while meaningful, feature in sentences that—if true—would have certain problematic implications for the metaphysics of agents. The formulation of the issue, in terms of the metaphysics of agents rather than the meaninglessness of utterances, is my focus in this section.

Temporal counterpart theory, which provides an account of the semantics of tensed sentences, can be used to express tensed facts about all these agents [26]. According to this account, temporal sentences about agents are true at the present time because of similar agents—call them 'temporal counterparts'—at other times. These temporal counterparts are defined as follows: given an agent A_t who exists at time t, a temporal counterpart of A_t at time t' is an agent $A_{t'}$ who is at least as similar to A_t as each other agent at t' is. The similarity relation varies from context to context: two agents might count as extremely similar in one context but extremely dissimilar in another.

Now for temporal counterpart theory. Let t be a time, let A_t be an agent who exists at t, let c be a context, and let ϕ be a sentence. Then a sentence of the form "A_t will be such that ϕ" is true at t in c if and only if at some time t' earlier than t, a temporal counterpart of A_t is such that ϕ. Likewise, a sentence of the form "A_t will be such that ϕ" is true at t in c if and only if at some time t' later than t, a temporal counterpart of A_t is such that ϕ [26] (pp. 188–208).

When combined with the Everett interpretation of quantum mechanics, however, temporal counterpart theory—and the corresponding view of the metaphysics of agents—faces a problem. To see why, let $|a_1\rangle$ and $|a_2\rangle$ be any two distinct possible outcomes of a measurement which occurs immediately after time $t = 0$. Let Susie be the sole agent who, according to the theory currently under consideration, exists at that time. Then consider the sentence below.

(3) Susie will be such that she finds herself on branch $|a_1\rangle$, and Susie will be such that she finds herself on branch $|a_2\rangle$.

Intuitively and pre-theoretically, this sentence is false. When physicists say things such as "We only ever see one outcome of a given quantum experiment", they are best understood as claiming that sentences like (3) are falsified, empirically, by experiments in quantum physics. So a good metaphysical theory of agents in the branching Everettian universe, and a good semantic theory for tensed expressions about those agents, would imply the falsity of (3).

The problem, for temporal counterpart theory, is this: it implies that (3) is true. To see why, let c be the present context. Then according to temporal counterpart theory, for each possible outcome $|a\rangle$ of a measurement that occurs immediately after $t = 0$, the sentence "Susie will be such that she finds herself on branch $|a\rangle$" is true at $t = 0$ in c: for given any such $|a\rangle$, a future time t' exists such that a temporal counterpart of Susie exists on branch $|a\rangle$ at t'. Therefore, according to temporal counterpart theory, the sentence "Susie will be such that she finds herself on branch $|a_1\rangle$" is true at $t = 0$ in c, and the sentence "Susie will be such that she finds herself on branch $|a_2\rangle$" is true at $t = 0$ in c. Therefore, (3)—which is the conjunction of those two sentences—is true at $t = 0$ in c as well. And that seems problematic.

Note that this line of argument relies on an assumption about the logical form of (3). To see how, suppose that the language of temporal counterpart theory is a 'Priorian object language', in particular, a first-order language supplemented with the one-place tense operators 'F' and 'P', where 'F' is the analog of the English expression 'It will be the case that' and 'P' is the analog of the English expression 'It was the case that' [29]. Let '$A_1 s$' be the formal analog of the English sentence "Susie will be such that she finds herself on branch $|a_1\rangle$", and let '$A_2 s$' be the formal analog of the English sentence "Susie will be such that she finds herself on branch $|a_2\rangle$". Then there are two reasonable candidate translations of (3) into the Priorian object language: in particular, the sentences below.

(3.1) $F(A_1 s) \wedge F(A_2 s)$
(3.2) $F(A_1 s \wedge A_2 s)$

For the reasons given in the main text, temporal counterpart theory implies that (3.1) is true. In addition, temporal counterpart theory implies that (3.2) is false: for (3.2) is true if and only if there exists a future temporal counterpart of Susie who finds herself on both branches $|a_1\rangle$ and $|a_2\rangle$, and given some plausible assumptions about the initial wavefunction, there is no such counterpart. Now, when I claimed that temporal counterpart theory implies the truth of (3), what I meant was this: temporal counterpart theory implies the truth of the sentence which best translates (3) into the Priorian object language, namely, sentence (3.1). So I was implicitly assuming that (3.1)—not (3.2)—is the best translation of (3) into the language of temporal counterpart theory. And one might claim that (3.2) is a better translation of (3) into the language of temporal counterpart theory than (3.1) is, in which case temporal counterpart theory should be understood as implying the falsity of (3) after all.

This claim is, however, rather implausible. The English analog of (3.2) is "Susie will find herself on both branches $|a_1\rangle$ and $|a_2\rangle$", which is quite different from (3). So the logical form of (3), in the Priorian object language of temporal counterpart theory, seems to be (3.1) rather than (3.2). Therefore, temporal counterpart theory really should be understood as implying, problematically, the truth of (3).

The worm view of agents, which the centered Everett interpretation invokes, does not have this problematic implication. And this is a serious point in favor of the worm view. When supplemented with a standard semantics, the worm view implies that at time $t = 0$ in context c, the sentence "Susie will be such that she finds herself on branch $|a_1\rangle$" is true—given the stipulation that 'Susie' denotes the worm which exists on branch $|a_1\rangle$—and the sentence "Susie will be such that she finds herself on branch $|a_2\rangle$" is false. Therefore, when supplemented with a standard semantics, the worm view implies that at time $t = 0$ in context c, (3) is false as well. So we should endorse the worm view over the alternative view, of the metaphysics of agents, discussed above.

6. Branch Relativity

In this section, I discuss a common concern about the branch-relativity of the centered Born rule. This branch-relativity might strike some as strange. But it is not strange at all. For as I explain below, the branch-relativity of the centered Born rule is an instance of a more general phenomenon: namely, that scientific laws often only hold in some regions, or regimes, of the universe.

By way of illustration, consider Mendel's law of independent assortment. According to this law, alleles for separate traits are passed down, from parents to offspring, independently of each other. The chance of a pea inheriting a particular color from its parents, for instance, is independent of the chance of that pea inheriting a particular shape from its parents.

Quite plausibly, Mendel's law of independent assortment only holds in some regions of the universe. As numerous experiments have confirmed, it holds—to a reasonably high degree of accuracy—for evolutionary processes on Earth. But there may be other planets, out there in the universe, on which Mendel's law of independent assortment does not hold. The evolutionary processes that these worlds support would be quite different, of course, from the evolutionary processes that Earth supports. But those other worlds, with those other evolutionary processes, are still perfectly possible. And in fact, on certain branches in the Everettian universe, there will almost certainly be planets like these: there will almost certainly be planets on which life evolves, but on which Mendel's law of independent assortment is false.

So Mendel's law of independent assortment is region-relative. It holds in some regions of the Everettian universe, but not in others. Its lawhood varies from place to place.

Likewise for the centered Born rule. On some branches in the Everettian universe, the centered Born rule holds. But on other branches, where the frequencies with which certain propositions obtain differ from the ψ-squared probabilities, the centered Born rule is false.

So the lawhood, of the centered Born rule, varies from place to place. That is all it means to claim that the centered Born rule is a law on some branches but not on others. That is all it means to claim that the lawhood, of the centered Born rule, is branch-relative.

So do not be too bothered by the branch-relativity of the centered Born rule. Plenty of special science laws exhibit an entirely analogous kind of relativity. In fact, arguably, pretty much every scientific law ever discovered exhibits a relativity of roughly this sort. For pretty much all such laws only hold in some domain or other. Laws of evolutionary biology only hold for certain sorts of biospheres. Laws of economics only hold for certain sorts of monetary systems. Even laws of quantum field theories, like the equations of motion generated by the Lagrangian density for quantum electrodynamics, only hold in certain energy regimes; for sufficiently short length scales, the physics is unknown.

Note that I have not proposed an account of what it is, exactly, for a law to hold relative to a region of the universe. There are many such accounts with which the centered Everett interpretation is compatible. Let me list three.

First, perhaps lawhood is a two-place relation rather than a one-place property. This two-place relation obtains between (i) certain regularities, and (ii) certain corresponding regions of reality. So for instance, it is incorrect to say that Mendel's law of independent assortment is a law, full-stop. It is correct, instead, to say that Mendel's law of independent assortment stands in the lawhood relation to a particular region, such as Earth. Similarly, it is incorrect to say that the centered Born rule is a law, full-stop. Rather, it is correct to say that the centered Born rule stands in the lawhood relation to our branch. And strictly speaking, it is incorrect to say that the Schrödinger equation is a law, full-stop. Rather, it is correct to say that the Schrödinger equation stands in the lawhood relation to the entire Everettian universe.

Second, perhaps there are many numerically distinct one-place properties of lawhood, one for each region that the universe contains. The regions basically function as indices on different lawhood predicates. So there is the one-place property of being a $\text{law}_{\text{Earth}}$, the one-place property of being a $\text{law}_{\text{our branch}}$, and so on.

Third, perhaps laws always come equipped with certain statements that specify those laws' domains of applicability. So strictly speaking, the sentence "Alleles for separate traits are passed down, from parents to offspring, independently of each other" does not itself express a law. Rather, the relevant law here is expressed by something like this: "On Earth, alleles for separate traits are passed down, from parents to offspring, independently of each other". Similarly, the earlier statement of the centered Born rule does not, itself, express a law. Rather, the relevant law is expressed by something like this: "On our branch, . . . ," where the statement of the centered Born rule goes in for the ellipses. And statements of nomic regularities which hold absolutely everywhere, like the Schrödinger equation, would not themselves express laws. Rather, the relevant laws would be expressed by something like this: "Absolutely everywhere, . . . ," where again, the statement of the law at issue goes in for the ellipses.

The centered Everett interpretation is compatible with each of these accounts of what it is for a law to hold relative to a region of the universe. The reader is welcome to adopt whichever they prefer. Or the reader is welcome to develop their own account of the region-relativity of regularities such as Mendel's law of independent assortment, the centered Born rule, and so on. That would also be compatible with the centered Everett interpretation.

7. Conclusions

According to the centered Everett interpretation, the universe (i) evolves deterministically with respect to uncentered propositions, but (ii) evolves indeterministically with respect to centered propositions. The Schrödinger equation describes the universe's deterministic evolution. The centered Born rule, which assigns objective chances to centered propositions, describes the universe's indeterministic evolution.

There is much to like about the centered Everett interpretation. It avoids a problem connected to the propositions expressed by sentences featuring indexicals. It avoids a problem connected to the number of branches in the Everettian universe. It is based on a view of agents—the worm view—which is better than a standard alternative. And it posits a form of branch relativity that is entirely analogous to the more general phenomenon of some laws only ever holding in some regions of the universe. So the centered Everett interpretation is worth taking seriously.

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quantum reports

Article

Set Theory and Many Worlds

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Abstract: The 2022 Tel Aviv conference on the many-worlds interpretation of quantum mechanics highlighted many differences between theorists. A very significant dichotomy is between Everettian *fission* (splitting) and Saunders–Wallace–Wilson *divergence*. For fission, an observer may have multiple futures, whereas for divergence they always have a single future. Divergence was explicitly introduced to resolve the problem of pre-measurement uncertainty for Everettian theory, which is universally believed to be absent for fission. Here I maintain that there is indeed pre-measurement uncertainty prior to fission, so long as objective probability is a property of Everettian branches. This is made possible if the universe is a set and branches are subsets with a probability measure. A universe that is a set of universes that are macroscopically isomorphic and span all possible configurations of local beäbles fulfills that role. If objective probability is a property of branches, then a successful Deutsch–Wallace decision-theoretic argument would justify the Principal Principle and be part of probability theory rather than specific to many-worlds theory. Any macroscopic object in our environment becomes a set of isomorphs with different microscopic configurations, each in an *elemental* universe (elemental in the set-theoretic sense). This is similar to the many-interacting-worlds theory, but the observer inhabits the set of worlds, not an individual world. An observer has many elemental bodies.

Keywords: Everett; many worlds; multiverse; wavefunction realism; hidden variables

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1. Many Faces of Many Worlds

If a many-worlds interpretation of quantum mechanics is ever to become generally accepted, there first has to be agreement on what *the* many-worlds interpretation is, which is very far from being the case. There is even dispute about what to call it; are we to think in terms of a single branching world or a partitioning multiplicity of worlds? Some theorists work with the Heisenberg picture and a basic ontology of operators, while some work with the Schrödinger picture and a basic ontology of wavefunctions. On both approaches, there is scope for arguing that microscopic local beäbles are needed for a satisfactory physical ontology.

Within the diversity of views, there is a fundamental dilemma that I aim to resolve here. It is between the ideas that an observer may have multiple futures or always has a single future. Everett wrote of *splitting* in quantum measurement situations and it has generally been accepted that a well-informed observer cannot be uncertain about their future prior to Everettian fission. In an attempt to introduce pre-measurement uncertainty, Simon Saunders and David Wallace developed versions of many-worlds theory that reject the concept of splitting, which is arguably Everett's key idea. There shall be more on this in the following section.

To begin with, I will address the thorny matter of understanding the relationship between probability and uncertainty. This will lead to an argument that pre-measurement uncertainty exists for a fission interpretation of branching, where an Everettian observer splits into observers seeing different outcomes. The only reason why that feels counter-intuitive is that we have inherited a folk metaphysics that interprets future probabilities as properties of alternative possibilities. It is this which stands in the way of interpreting probabilities as properties of future coexistent actualities. A thought experiment helps to sugar this pill.

Understanding uncertainty as a cognitive state of assigning partial degrees of belief to coexistent futures requires assigning objective probabilities to those futures equal to the absolute squares of their quantum amplitudes. This calls for an account of how this *branch weight* can be understood to *constitute* objective probability. I shall argue that it can do so if understood to be a subset measure. This leads to interpreting the universal wavefunction as being a set of deterministic universes that contain microscopic local beäbles. Objects in our environment become sets of objects that are macroscopically isomorphic but differ in their microscopic configurations. They are extended in *configuration space*, so to speak. The half-life of an unstable particle thereby becomes a rate of change of a subset measure.

The result is a set-theoretic metaphysics for quantum mechanics that incorporates Everettian fission and microscopic local beäbles. It opens the way to new physics if the interaction between the universes that are the set-theoretic elements of our universe is the source of phase relations. After discussing spin, separability, and locality in the context of this metaphysics, I close with further reflections on Lev Vaidman's *World Splitter* and its implications.

2. Probability and Uncertainty

For classical mechanics, all physical processes are regarded as deterministic. The idea of there being a mind-independent, i.e., *objective*, probability can only be applied to the determination of initial conditions, relegated to an inscrutable past. Probability arises, as in statistical mechanics, from the epistemic condition of ignorance on the part of observers. The lack of Laplacian omniscience as to the exact positions and momenta of particles entails that perfect prediction is impossible, and thus, epistemic probabilities are assigned to fictional *possibilities* on the basis of statistical evidence. The gathering of that evidence involves the measurement of frequencies that can be regarded as surrogate approximations of epistemic probabilities given the assumption of the law of large numbers. Uncertainty about the future is regarded as a mental state that involves the entertaining of partial degrees of belief about future observations equal to the epistemic probabilities assigned to the possibilities of those observations on the basis of measured frequencies.

In the wake of quantum mechanics came the concept of *stochastic* physical processes, which are objectively probabilistic. Continuing to employ the metaphysics of possibility, a stochastic analysis of quantum processes with multiple possible outcomes supposes that one of those outcomes will be actualized by virtue of a random selection constrained by the objective probabilities of the possibilities. Those objective probabilities are determined by the Born rule when interpreted as assigning a quantum amplitude to the fictional possibilities. As in the case of classical mechanics, stochastic theory interprets uncertainty about the future as the entertainment of partial degrees of belief about alternative possible futures but now the partial degrees of belief are equal to the supposed objective probabilities.

The idea of stochastic processes has widely been accepted as plausible by physicists. It can seem plausible that the half-life of an unstable particle is a mind-independent property of that object. However, an air of mystery surrounds the concept, often referred to as propensity. How can propensity be a property of an object? What is the ontic status of propensity?

Hugh Everett III replaced the concept of a stochastic process with that of a *dendritic* process. Consider, for example, Vaidman's *World Splitter* [1]. Connecting with the device via a smartphone, you can choose a setup that will initiate a quantum measurement process with six equal-amplitude outcomes, i.e., a quantum die. The concept of a quantum die simply having six outcomes is an idealization that I shall use for the sake of argument to begin with. Later, I shall consider the implications of abandoning that idealization.

On "rolling" the quantum die, Everett's observer fissions into six observers, each seeing one of six different outcomes, which are all actual [2] (p. 459). Where is uncertainty to be found? Presumably, Everett thought that it was nowhere to be found, which is why he first entitled his thesis *Wave Mechanics Without Probability*. Presumably, the apparent lack of uncertainty did not bother Everett; after all, what has uncertainty to do with *physics*? He was simply suggesting that the histories of quantum processes are typically not linear, they *branch*. They are partially ordered series of events, not well-ordered series. Everett's world was not many, it was one; a single branching world that Saunders once appropriately dubbed the quantum block universe [3]. However, I shall continue to use the term "Many Worlds" since it has become virtually ubiquitous and is harmless enough so long as it is qualified in ways that will become clear as we go on.

As you roll Vaidman's quantum die, believing that you will fission, can you really deny being uncertain about the future? Many theorists have thought so, including Vaidman himself [4] (Section 3). In search of pre-measurement uncertainty, others preferred to replace Everett's concept of fission with those of *overlap* and *divergence*, where the body of an observer at a time is one of a multitude of *doppelgängers* in erstwhile "parallel" worlds [5–7]. However, inasmuch as that is motivated by trying to fill a lacuna left by supposedly absent pre-measurement uncertainty for fission, it is unnecessary, as we shall see.

Content to do without pre-measurement uncertainty, Vaidman kept to the traditional path by following Everett in believing that on rolling the quantum die, you will split into six "successors", each in a different branch and each seeing a different outcome. He wrote:

The quantum world splitter lets you enjoy all the possibilities in life with no need to choose. Why choose one, when you can do it all (AT ONCE!) [1] (original emphasis).

The idea is that you decide in advance to act on each of six different enjoyable options according to which number is observed after the measurement. An obvious first objection is to ask the following: in what sense will it be "you" acting in those different futures? Each of the six successors is a different observer seeing a different number, and thus, it is logically impossible for them all to be the same observer as you. This demonstrates that the metaphysics of persistence needs to be invoked to make sense of Everettian fission even before considering uncertainty.

Vaidman's term *successors* for post-split observers has generally been used by fission theorists and simply fails to meet this objection concerning personal identity. Note that this problem is avoided for the overlap and divergence interpretations of branching because no splitting occurs. Vaidman asserts that *you* can "do it all at once", but you are *not* any of your successors.

What is required is popularly known as *stage* theory, which was introduced by Ted Sider in 1996 [8], first explicitly applied to many-worlds theory in [9], and most recently in [10] (Section 2.1). It is generally accepted that a persisting object is one and the same thing from moment to moment. That is what could be called the folk metaphysics of persistence. However, it is not necessary to think of persistence like that. One can understand the history of an object as consisting of a series of momentary temporal parts or *stages*. What Sider recognized was that an object, at any given moment, could be understood to be a stage of its history and that a persisting object can be understood to be one that has a special relationship with the stages that are called its past and future *temporal counterparts*. A persisting object *was* its past temporal counterparts and *will be* its future temporal counterparts. Contrary to folklore, a persisting object (or observer) does not have to be one and the same thing from moment to moment after all. If he were to adopt stage theory, Vaidman could say, without fear of contradiction, that you will be each of six different observers, each seeing a different number.

What is the ontic status of non-present stages on this account? That depends on one's view of the ontic status of past and future states of affairs. In the *eternalist*, block universe view, which I suggest is most appropriate for many-worlds theory, the past and future counterparts of an object will be objects that exist in the past and future of the present object. In non-eternalist views, the present persisting object will bear the temporal relations *was* and *will be* to objects that *did* and *will* exist.

I mentioned that Vaidman maintained that you cannot be uncertain about the future when you roll the quantum die. Surely you cannot be uncertain about the future when you know that all outcomes will occur! Surprisingly, that is a conviction that also arises from a folk metaphysics from which we can profitably free ourselves.

2.1. The Logic of Uncertainty

For stochastic theory, a quantum die involves an objectively probabilistic process with six possible outcomes. One of those outcomes will be actualized randomly and each possibility can be assigned an objective, mind-independent probability. To put it another way, the quantum die has a propensity. The propensity is such that each of the six possible outcomes has an equal probability of being actualized. There has long been a sense of mystery about propensity, which I hope to dispel.

For stochastic theory, an observer rolling a quantum die is uncertain about the future for the following reason. As with classical mechanics, uncertainty is understood to be a mental state involving the assignment of subjective probabilities and degrees of belief to alternative possible futures. Stochastic theorists derive the values for the degrees of belief by appealing to what has become known as the Principal Principle, which is basically the idea that an observer should assign subjective probabilities to possible outcomes equal to what they believe the objective probabilities of those outcomes to be [11] (Section 2.2). For stochastic theory, the degrees of belief are guided by what are taken to be objective probabilities, whereas, for classical mechanics, the degrees of belief are guided by estimated epistemic probabilities arising from the ignorance of microstates. According to stochastic theory, an observer is uncertain about the future prior to rolling the quantum die because they assign degrees of belief of 1/6 to each of the possible outcomes, whose objective probabilities are 1/6.

Vaidman followed Everett in understanding the process involved in rolling the quantum die to be dendritic rather than stochastic. All six outcomes actually occur, each in a different branch of physical reality. Each branch is assigned the same quantum amplitude as is assigned to the *possible* outcomes of stochastic theory, and since the branches actually exist, quantum amplitude must be a physical property that they possess. The absolute square of quantum amplitude is the quantity that stochastic theorists identify with objective probability and that seems acceptable when amplitudes are assigned to alternative possibilities, but can it be acceptable when amplitudes are assigned to coexistent actualities? Can objective probability be a property of branches?

It is certainly *logically* possible, for if the objective probability of all the outcomes occurring together is 1, then that entails that each of the outcomes will occur but that does not give reason to believe that the objective probabilities of each of those individual outcomes should also be 1. The objective probability of the occurrence of each outcome can be 1/6, contrary to the common belief that if an event will occur, then the objective probability of its future occurrence must be 1. That may seem to involve a contradiction because an observer must be certain that any particular outcome will occur whilst assigning it an objective probability of 1/6. However, the observer is not required to apply the principal principle here, where the future *occurrence* of the outcomes is concerned.

There is as yet no agreed justification for the Principal Principle; it is used by stochastic theorists simply because it seems self-evident. If you believe that a process has six possible outcomes whose objective probabilities are 1/6, then what else can you do but assign a degree of belief of 1/6 to the future occurrence of any particular outcome? However, stochastic theorists are in the habit of applying this idea in the context of multiple futures thought of as *alternatives*, whereas in the context of the dendritic quantum die, the futures are thought of as coexistent. In this context, the application of the Principal Principle is overruled by logical consequence because, again, if the objective probability of all outcomes occurring together is 1, then, necessarily, each outcome will occur, whatever its individual objective probability of occurrence. The observer can assign a subjective probability of 1 to the occurrence of all the outcomes because the objective probability of their combined occurrence is 1. This entails that the observer is certain each outcome will occur, *despite the objective probability of the occurrence of each outcome being 1/6.*

I should mention in passing that this brings an alternative perspective to the Deutsch–Wallace decision theory argument that observers should assign degrees of belief to future measurement outcomes in accordance with the Born rule [12] (pp. 160–189). If, as I am arguing, objective probability can be understood to be a property of future branches, then the decision-theoretic argument, if good, constitutes a justification of the Principal Principle, and thus, belongs to the philosophy of probability rather than specifically to many-worlds theory.

In what sense, then, can an observer be uncertain about the future prior to rolling the dendritic quantum die? They can be uncertain in the sense of assigning a subjective probability of 1/6 to each of the *future observations.* The observer will be each of six observers seeing different outcomes whose objective probabilities are 1/6. Applying the Principal Principle, the observer assigns a degree of belief of 1/6 to the future observation of each outcome. That is exactly what the stochastic theorist does when uncertain about what will be observed. Whether the futures are understood as alternative possibilities or coexistent actualities is beside the point, uncertainty is the very same thing in both cases. The thrall of a folk metaphysics of alternative possibilities can make this hard to grasp.

Should doubt remain, a thought experiment demonstrates that an observer can believe that they are assigning subjective probabilities to alternative possible outcomes whilst they are *in fact* assigning them to coexistent actual outcomes. This involves a set-theoretic metaphysics for physical objects that leads directly to an explanation of how objective probability can be a physical property of Everettian branches.

2.2. Many Worlds without Everett

What cosmologists call the observable universe is a finite region of space that is currently estimated to have a radius of about 46 billion lightyears. Since there is as yet no evidence that space is finite, there may be a countably infinite number of regions that are observationally identical.

Consider an observer who inhabits one of an infinite set of observationally identical universes where quantum dice are, hypothetically, stochastic. On rolling a die, an infinite number of *doppelgängers* in the set of erstwhile "parallel" universes move in concert and an infinite number of quantum dice are rolled. The set of universes subsequently partitions into six subsets whose measures are *necessarily* 1/6. The reason being that what it *means* in stochastic theory for an outcome of a particular type of process to have an objective probability of 1/6 is that the subset measure for that outcome tends to 1/6 as the sample tends to infinity. That is how the *probability measure* on an infinite set gets its name.

Now, drop the ubiquitous assumption of folk metaphysics that there is a one-to-one relation between observers and *doppelgängers*. This requires an exercise in what Donald Davidson has called *radical interpretation* [13]. The idea is that truth values must be preserved for relevant utterances by an observer on the original interpretation and the alternative. On the original interpretation, a single utterance by an observer is tokened by a single noise emitted by a single *doppelgänger*, but on the alternative, a single utterance is tokenized by the infinite number of isomorphic noises emitted by each of the *doppelgängers*. Likewise, for intensional acts; on the original interpretation the act of rolling a die is tokened by the movements of a single *doppelgänger*, whereas on the alternative interpretation, the act of rolling a die is tokened by the parallel movements of all the *doppelgängers.* In the alternative interpretation, a single die is rolled, which is constituted by all the parallel dice. This is the *unitary interpretation of mind* [14] (Section 2).

A novel use of set theory is required [10] (Section 4). Following Willard Van Orman Quine, physical objects in each observable universe are to be construed as self-membered singletons that are each identified with their hierarchy of unit sets [15] (p. 31). Quine spent much energy trying to find a way for mathematics to be understood without an ontic commitment to sets but failed. Having become resigned to the necessity of sets, he noticed that non-sets could be brought into the set-theoretic fold in a way that is harmless in the sense that it does not impair the use of set theory in mathematics. He introduced what are called *Quine atoms* by logicians, though the phrase is sometimes simply used to denote sets that are their own sole element. Quine's definition went further. Take any individual, an apple, say. What it is is the set that contains the apple as its only element, plus the unit set of that set, the unit set of that set, and so on. The singleton set containing the apple is identified with that hierarchy of unit sets as being one thing: the apple. I shall refer to such a thing as a *Quineian individual*.

Thus the body of an observer in the conventional interpretation of the mind–body relation is a single *doppelgänger* that is a Quineian individual. For the alternative unitary interpretation of mind, Quine's idea is extended so that any set of Quineian individuals is also defined as an individual, likewise identified with its unit set, that set's unit set, and so on.

If the observer's body is to be a set of *doppelgängers* in this way, it follows that a set of Quineian individuals must have the properties that its elements share, with some logically necessary exceptions, such as the number of elements and value-definiteness. Therefore, in the conventional interpretation of the setup involving an infinite set of observable universes, each observer has a body of mass M, which is a Quineian individual. In the alternative interpretation, the *single* observer has a body of mass M, which is an infinite set of Quineian individuals. In the alternative interpretation, the single observer inhabits a *single* observable universe, which is an infinite set of *elemental* universes (elemental in the set-theoretic sense). The observer's spatial location is a set of corresponding *elemental* locations. I say more about what correspondence involves below.

Now suppose that in the original interpretation of the setup, each observer believes that they inhabit an observable universe that is a Quineian individual and where quantum dice are stochastic. In this case, they believe that when they roll a quantum die, there will be a single outcome, which is one of six possible outcomes, each of which has an objective probability of 1/6. Switching to the unitary interpretation of mind, the single observer necessarily believes likewise but now they are mistaken because the single observer, unbeknown to them, inhabits an observable universe that is an infinite set of universes, which are Quineian individuals.

When the single observer rolls the quantum die, each of the *doppelgängers* that are elements of the observer's body moves isomorphically so that the parallel quantum dice are caused to roll. In each elemental universe, the outcome gives rise to sensory input to a *doppelgänger* so that as the set of elemental universes partitions into six subsets with different outcomes, the set of *doppelgängers* partitions into subsets with different sensory input. Differences in sensory input give rise to different observations so the single observer fissions into six observers making different observations. The bodies of the six downstream observers are each an infinite set of *doppelgängers* whose subset measures relative to the body of the upstream observer are 1/6, i.e., the probability measure.

For the foregoing non-Everettian cosmological setup, the single quantum die of the unitary interpretation of mind is not stochastic, it is dendritic. The conclusion must be that an observer can be mistaken when believing that their uncertainty prior to rolling a quantum die derives from there being six alternative possible outcomes that all have an objective probability of 1/6. Their uncertainty can derive from there being six coexistent actual outcomes that all have an objective probability of 1/6.

3. A Metaphysics for Everettian Fission

According to Everett, the quantum die splits into six dice, each showing a different number, and the observer splits along with it. As he saw it, *of course*, there can be no probability since there is no uncertainty, thus, his pursuit of a back door to probability via typicality.

Everett's key idea was that the concept of a stochastic process could be replaced by that of a dendritic process. To make it fully intelligible, there has to be an account of how a well-informed observer can be uncertain about future observations in a quantum measurement situation, i.e., observations they will make, together with other nearby observers who have split, along with the measuring device and the laboratory. We now have an account:

Uncertainty without alternatives:

> *Uncertainty about the future is the cognitive state of assigning partial degrees of belief to multiple futures; whether those futures are thought of as alternative possibilities or coexistent actualities is an arbitrary choice because the occurrence of a future does not entail that the probability of its occurrence is 1.*

If it is useful to our understanding of physics to employ the concept of fission rather than that of stochasticity, then we are free to do so. To be certain that all outcomes will occur entails that each will occur. Therefore, we can be certain that any particular outcome will occur whilst believing that the objective probability of that outcome is 1/6. Assuming the Principal Principle, the observer assigns a degree of belief of 1/6 to the future observation of that outcome, by observers who they and their laboratory colleagues will be.

How can the real-world quantum die split in such a way that the objective probability of each of its immediate future temporal counterparts is 1/6? By being an infinite set that partitions into subsets with a probability measure. The cosmological thought experiment provides the framework for a metaphysics for quantum fission that incorporates a modification of Quine's definition of individuals as being self-membered singletons identified with their hierarchies of unit sets:

Concrete sets:

> *Any physical object is a set of Quineian individuals, which is identified with its hierarchy of unit sets. It has all the properties that its elements share, other than those logically excluded, such as the number of elements and value-definiteness.*

3.1. From Metaphysics to Physics

The cosmological thought experiment invokes an infinite set of elemental parallel stochastic universes populated by Quineian individuals. However, the whole point of Everett's idea was to *replace* stochasticity with fission. For Everettian physics, the elemental universes must have deterministic, linear histories with branches emerging as the set partitions. Pilot wave theory provides possible candidate elemental universes [10]. Interacting worlds theory also provides candidate universes with a purely particle ontology [16–18], though it may be replaceable by a field ontology [19]. However, both the pilot wave and interacting worlds theories are restricted to non-relativistic quantum mechanics and involve nonlocality in the sense that there can be causal connections between spacelike-separated events.

An often-vaunted advantage of many-worlds theory is that it does not face those problems. When conceived of, following Everett, as a *pure wave* theory, all of the physics used by physicists can be recovered, so the story goes. In defense of many-worlds theory as a pure wave theory, Wallace has recommended a *mathematics-first* approach to the ontology of quantum mechanics, which excludes microscopic local beäbles as objects bearing properties [20]. The project of ontic structural realism, which he supports, is an interesting one, but I suggest that it is better suited to a pre-spacetime ontology than to that of quantum mechanics, where stuff happens *in* spacetime.

As Louis de Broglie once remarked, a Schrödinger wave is supposedly in configuration space but lacks configurations [21] (p. 381). There are currently other attempts to fix that by introducing local beäbles to many-worlds theory [22,23]. What I have been describing is a metaphysical framework that is independent of whatever physics may actually be involved. Assuming a particle ontology, just for the sake of illustration, this framework has it that any macroscopic object in our environment is a set of objects that are macroscopically isomorphic but which differ in their microscopic particle configurations. There is a sense in which we inhabit configuration space. Objects in our environment have a spatial extension, and they are extended in configuration space too, as are our bodies. In effect, the unitary interpretation of mind is a *consequence* of assuming that objects in our environment are extended in configuration space.

As explained in Section 2.2, recall that the unitary interpretation of mind is the idea that multiple *doppelgängers* instance a single observer, not multiple observers in qualitatively identical mental states. If your body is understood to be extended in configuration space, in the sense of being a set of bodies that are only anisomorphic at the level of microscopic configurations, then your mental state, now, is instanced by a multiplicity of *doppelgängers*. You are legion, to adapt a biblical phrase.

In light of this, think about Vaidman's quantum die again. It is an apparatus in a quantum optics lab that is a set of labs including all possible configurations of particles consistent with the Born rule. That is the reason why the set partitions in the same way as a set of stochastic dice would. However, is it an infinite set? Earlier, I argued that the subset measures of branches could be identified with objective probabilities since the hypothetical set of stochastic dice in the cosmological setup was presumed to be infinite. Is the set of all possible particle configurations infinite? That would depend on whether space is continuous. Can the branch subset measures still be identified with objective probabilities if the set of quantum die is finite? Perhaps not, or perhaps an effective law of large numbers is good enough for very large samples. In any case, given the cosmological setup, if there is a finite number of configurations, there can be a countably infinite set of each configuration until such time as we have evidence that space is finite.

According to this framework, an unstable particle in our environment would be a set of particles constantly partitioning into a decay subset of increasing measure. An observer with a detector would be constantly splitting into an observer not seeing decay and observers seeing decays at later and later times. The probability of observing decay within a given period would depend on the rate of change of the decay subset measure for that type of particle, i.e., its propensity to decay. We are thus free to hypothesize that the quantum die is a very large or infinite set of isomorphic dice that will partition in the same way as a corresponding set of stochastic dice would. Therefore, the subset measures of the downstream dice will be 1/6 relative to the upstream die.

For another illustration of the idea that objects in our environment are extended in configuration space, consider a free electron at any given moment. It is a set of elemental electrons that are in different corresponding positions and have different corresponding momenta in the elemental universes. The term *elemental* here is strictly set-theoretic. Again, our universe is being construed as a set of universes and any object in an observer's environment is a set of objects. A free electron in our universe is a set of elemental electrons that are on different trajectories in the universes that are elements of our universe. That is why the electron has an indefinite position and momentum in our non-elemental universe, where objects have a definite position and momentum only if their elements have corresponding positions and momenta in the elemental universes.

The introduction of particles as local beäbles in the way I have just described, as being the set-theoretic elements of particles in our environment, effectively preserves the full structure of the wavefunction and avoids the drawbacks of the pilot wave and interacting worlds theories, as I shall now explain.

3.2. The World as a Wavefunction

Consider the wavefunction of a free electron understood in terms of set-theoretic metaphysics. For the pure wave theory, any region of space is assigned a quantum amplitude and the absolute square is taken to give the probability of finding the electron there if a position measurement is made. There is no account of how an electron can be "spread out" in this way, hence Wallace's appeal to a thingless ontology. However, in the set-theoretic metaphysics, the absolute square of amplitude for a spatial region yields a subset measure for the single free electron, which is a set of elemental electrons. Each elemental electron in that subset is at an elemental location that is an element of a location within the given spatial region. There is thus a *fully concrete* interpretation of the electron's wavefunction within the given region. It is not in any sense counterfactual. Every location in that region is a set of elemental locations where elemental electrons may be *actually located*.

It is often said that the paradox of superposition is dealt with by the many-worlds theory by understanding superpositions as being composed of definite states on different branches. Thus, Schrödinger's cat is dead on some branches and alive on others (sometimes put as dead in some worlds and alive in others). However, Everettian theory has only ever given an account of *macroscopic* superpositions in this way. Mystery still surrounds the concept of microscopic superpositions; hence, again, the motivation for defending a pure wave theory in terms of an ontology that does not involve objects. The set-theoretic metaphysics resolves this problem by construing microscopic superpositions as also being constituted by multiple definite states. Again, the free electron becomes an extended object, extended in configuration space. It does so by being a set of electrons, each of which is on a different trajectory in a universe that is a set-theoretic element of the observer's universe.

However, that only provides metaphysics for a momentary snapshot of the electron's wavefunction. There needs to be the dynamics of unitary evolution too; where is that to come from? It strikes me that the most plausible option here is to adopt the interacting worlds theory. The individual elemental universes that contain the set-theoretic elements of the observer's electron interact in such a way as to generate the unitary dynamics. Here there is scope for new physics in order to understand how universes separated in configuration space interact. The possibility of such new physics has already been suggested by interacting worlds theorists, but what must be stressed here is the radically different perspective that the set-theoretic metaphysics brings to the interacting worlds theory.

All the difference is in how the observer is situated. For extant interacting worlds theory, the observer is situated within an individual world, which corresponds to what I have been calling an elemental universe. For the set-theoretic metaphysics, the observer is situated in the *set* of interacting universes; objects in the observer's environment, including their body, are sets. The observer's universe becomes a set of interacting universes.

In a sense, the observer spans the set of interacting universes. They span the universes in the sense that the mental states of an observer are instanced by a multitude of brains in a multitude of *doppelgängers*. Each of those brains is a set-theoretic element of the brain to which the observer indexically refers by a tap to the skull. The observer's mental states are instanced by a multitude of brains rather in the way that a single novel is instanced by a multitude of books.

Extant interacting worlds theory involves causal nonlocality because particle trajectories in the observer's world are mutually interactive at spacelike separation by virtue of the interactions between worlds. By construing our universe as a set of interacting universes rather than an element of the set, this problem is avoided. The long-recognized causal locality of the many-worlds theory is preserved, as we shall see.

4. Being Indefinite

Consider an observer who rolls a quantum die blindfolded. According to Vaidman, the observer will fission into six successors, each on an Everettian branch where the outcomes are different. According to the set-theoretic metaphysics, the body of the observer will partition into six subsets and each subset will have elements that are *doppelgängers* in the presence of elements of one of the six outcomes. The partitioning of the observer's body will be caused by slight physical effects propagating from the six different post-roll dice, even if those effects are very slight indeed, such as gravitational differences. However, the observer themself will not fission because the *doppelgängers* are not different enough to instance distinct perceptual states. The observer does not fission because their perceptual mechanism is screened by the blindfold. Post-measurement and pre-observation, there will be a single successor whose body is the set of all the *doppelgängers* in the six subsets. The environment of that single successor will contain a die with subsets that are six dice displaying different numbers. In other words, the die in the vicinity of the post-measurement, pre-observation successor will be in a *macroscopically indefinite* state.

Now consider a terrestrial observer watching the roll of a quantum die on Mars through a powerful telescope. Post-roll, there will be no causal influence on the observer's body for several minutes, and thus there will be no consequent partitioning of the observer's body. When light from the roll of the die reaches the observer's eyes, their body will partition into six subsets and then, after retinal states have been processed, there will be six sets of *doppelgängers* instancing elements of different perceptual states and the observer will have fissioned. During the intervening few minutes, the quantum die will have been in a macroscopically indefinite state relative to the terrestrial observer, but not, of course, relative to a Martian observer.

Given the set-theoretic metaphysics, an observer cannot fission into observers seeing different outcomes until the observer's body partitions into subsets which are bodies instancing different cognitive states. Note that this has nothing to do with *consciousness*, it has to do with *mental content*. It is well established that we can perceive states of the world around us whilst not being conscious of those perceptions. Two distinct observers may be in identical conscious states and yet act differently because of different unconscious perceptions.

Therefore, necessarily, quantum measurements with multiple outcomes that occur at spacelike separation from an observer are in macroscopically indefinite states relative to that observer. As has generally been recognized for the many-worlds theory, this is enough to scotch the idea that the observation of correlations between spacelike-separated measurements on entangled particles entails nonlocal causation. That conclusion only follows if measurement outcomes necessarily have single definite outcomes.

However, the set-theoretic metaphysics construes the observer's universe as a set of elemental universes and within the elemental universes there seems to be nonlocality because hypothetical spacelike-separated measurements would always have single definite outcomes. So, is nonlocality involved in the set-theoretic metaphysics after all?

No, because the apparent nonlocality at the elemental level is not really nonlocality at all. It would be if observers inhabited the individual elemental universes but the whole idea is that they do not. Observers inhabit sets of elemental universes and, at that level, nonlocality is absent for the reason I have just given. Elemental nonlocality is not nonlocality because elemental locations are not locations. For the set-theoretic metaphysics, there is no reason to suppose that there is causal influence between events at spacelike-separated *locations*, which are locations in the observer's spacetime, which is a set of elemental spacetimes. This will become clearer with an analysis of EPR–Bell experiments, and what is needed by way of preparation for that is a set-theoretic characterization of spin and entanglement.

4.1. Spin

Spin poses a further challenge to set-theoretic metaphysics. We have to take a step back. The universe is being construed as a set of elemental universes. An electron only has a location if all its elemental electrons are at corresponding elemental locations. For the sake of argument, consider an electron to be a point particle. In this case, it is at a spatial point only if all its elements are at corresponding elemental points.

The correspondence can be thought of in the following way. For an observer at a certain time, the universe exhibits a definite distribution of objects in space on the surface of the past light cone. The observer's universe at a time is to be construed as the set of universes containing all possible configurations of particles consistent with that definite distribution of objects. A particle only has a position in the observer's universe if its elements are all at the same position relative to isomorphic distributions of macroscopic objects in each elemental universe.

The set-theoretic metaphysics interprets objects with indefinite properties as sets of objects with definite properties; therefore, when it comes to spin, *elemental* electrons cannot have indefinite spin relative to any axis. An *elemental* electron must have a definite spin, i.e., *up* or *down*, relative to some axis, period. Just as an *environmental* free electron has an indefinite position and momentum whilst the electrons that are its *elements* follow trajectories, likewise, an *environmental* electron has indefinite spin relative to all axes but one whilst the electrons that are its *elements* simply have definite spin relative to a single axis. I shall continue to italicize these terms to avoid confusion. The spin of an *elemental* electron cannot be *measured*. Measurement is something we do in our universe, but not in the *elemental* universes that are its set-theoretic elements. Bearing that in mind, here is an attempt to provide a set-theoretic metaphysics for spin.

In the spirit of string theory, let an *elemental* point be baton-like, having an orientation. In this case, an *environmental* point in the observer's universe will have an orientation too, following the concrete sets rule, since all its elements have orientations. Let an observer's *environmental* point be a set of *elemental* points with all possible orientations. In this case, the *environmental* point will have an *indefinite* orientation. We are in the habit of thinking of spatial points in our environment as lacking orientation but that is to be replaced by the idea that a spatial point has an indefinite orientation because it is a set of *elemental* points with different orientations. This is like the earlier idea that a free electron in our environment does not lack a trajectory but rather has an indefinite trajectory since the electrons that are its elements are not on corresponding trajectories in each *elemental* universe.

As a point particle, an *elemental* electron can be supposed to have an orientation too. Let any *elemental* electron have an orientation that is exclusively either parallel or orthogonal to the orientation of the *elemental* point that it occupies. We can adopt the convention that an *elemental* electron that is parallel is spin-up and an *elemental* electron that is orthogonal is spin-down. An *environmental* electron that is x-spin-up can then be construed in the following way. All its *elemental* electrons that are at *elemental* points oriented parallel to the x-axis are spin-up. A little formalism may help.

Let e_E be an *environmental* electron and e_e an *elemental* electron. Likewise, let p_E be an *environmental*, spatial point and p_e an *elemental* point. Every *elemental* point has an orientation; therefore, for an *elemental* point oriented parallel to the x-axis, we can write xp_e. Every *elemental* electron is at an *elemental* point ($e_e@p_e$) and is either oriented parallel or orthogonal relative to that point, with parallel being spin-up and orthogonal being spin-down. Therefore, we can write $e_e@_{up}xp_e$ for an *elemental* x-spin-up electron and $e_e@_{down}xp_e$ for an *elemental* x-spin-down electron. An x-spin-up *environmental* electron is defined thus:

$$e_E \text{ (x-spin-up) iff } \forall e_e\ [(e_e \in e_E)\&(e_e@xp_e)] \rightarrow [e_e@_{up}xp_e]$$

An x-spin-up *environmental* electron measured on the z-axis has equal probabilities for being measured spin-up and spin-down. Given the earlier analysis of objective probability in terms of subset measure, this implies that the *environmental* x-spin-up electron has a subset of *elemental* electrons that are at *elemental* points parallel to the z-axis and, of that subset, the spin-up and spin-down elemental electrons are of equal measure. In other words, the measures of $\{e_e@_{up}zp_e\}$ and $\{e_e@_{down}zp_e\}$ on $\{e_e@zp_e\}$ are equal.

As a consequence, an observer measuring an x-spin-up electron on the z-axis will fission into observers whose bodies are of equal measure, one observing an *environmental* electron that is z-spin-up and the other observing an *environmental* electron that is z-spin-down. For the post-measurement z-spin-up *environmental* electron, all its *elemental* electrons that are at *elemental* points parallel to the z-axis are spin-up; correspondingly for the z-spin-down electron in the other post-measurement environment.

What does it mean for the post-measurement observers to have bodies of equal measure? Recall the cosmological thought experiment with an infinite set of hypothetically stochastic universes. Now think in terms of an equal-chance measurement being made in each universe, i.e., a quantum coin flip. The set of universes will partition into two subsets of equal probability measure where different outcomes occur. For this setup, if the unitary interpretation of mind is adopted, there is a single observer at the outset whose body is a set of bodies (*doppelgängers*) that partitions into two subsets of equal measure, which are the bodies of the post-coin-flip observers. Recall also that in Section 3.1, the set of hypothetical stochastic universes was replaced by a set of pilot wave or interacting worlds universes, which would partition in the same way as a set of stochastic universes would, i.e., the branch subset measures would take the same values. The reason for this would be that the set of hidden-variable universes would include all possible configurations consistent with the Born rule (corresponding to the assumption of particular initial conditions in the pilot wave theory). To put it another way, the universal wavefunction is being interpreted as a set of hidden-variable universes that include all possible configurations, and thus, Everettian branching is construed as the partitioning of a set where the subset measures just *are* the outcome probabilities. Again, the perspective being taken is that of the unitary interpretation of mind, where the fissioning of the observer arises because of the partitioning of the observer's body.

To return to spin, let an *environmental* x-spin-up electron have subsets of *elemental* electrons at *elemental* points parallel to all possible orientations. For any orientation ô, the subset of *elemental* electrons at *elemental* points parallel to ô has two subsets, namely, $\{e_e@_{up}\hat{o}p_e\}$ and $\{e_e@_{down}\hat{o}p_e\}$, which are *non-elemental* spin-up and spin-down electrons, since any set of *elemental* electrons is an electron. They become the post-measurement *environmental* electrons if a spin measurement is made on the ô orientation. The measures of those subset electrons relative to $\{e_e@\hat{o}p_e\}$ are the probabilities for observing spin-up and spin-down at that orientation.

This provides a characterization of spin for the set-theoretic metaphysics. However, before we can apply it to the analysis of EPR–Bell experiments we need a set-theoretic characterization of entanglement.

4.2. Entanglement

A pair of electrons in a singlet state has zero net spin because they have opposite spins. Emitted from a source and collimated, the wavefunction propagates as a sphere with peaked amplitudes in opposite directions. The wave propagates in configuration space but the set-theoretic metaphysics provides, at any given moment, a characterization of the wave as a distribution in 3D space. Both the *environmental* electrons are sets of *elemental* electrons. At any region of *environmental* space at a certain time (the space in the environment of an observer), there will be subsets of the *elemental* electrons of each of the two *environmental* electrons, which are situated at *elemental* points that are set-theoretic elements of the *environmental* points in the given *environmental* region. This is what I meant earlier when I set that the set-theoretic metaphysics completely recovers the structure of a wavefunction. Here we see an instantaneous reconstruction. The dynamics, which provide the phase aspect of the wave, might be recovered via a many-interacting-worlds theory or by replacing the hypothetical set of stochastic universes with an appropriate set of pilot wave universes with a dual particle–wave ontology.

Consider congruent local spacetime regions of measurement, namely, A and B, that are equidistant from the source and spacelike-separated. Both *environmental* regions are sets of *elemental* regions containing electrons that are elements of each of the two entangled electrons. For both A and B, some *elemental* points will be the location of one of the elements of one of the two entangled electrons, assuming that no two *elemental* electrons can be located at the same *elemental* point. In each *environmental* region, for every *elemental* point that is the location of an element of one of the entangled electrons, there will be another *elemental* point that is the location of an element of the other electron. Since electrons lack haecceity, there is no sense in which elemental electrons can be permutated.

Furthermore, for each of the two *environmental* regions, there will be *elemental* points of all orientations, which are the locations of electrons that are elements of the two entangled electrons. Also, for every orientation, there will be two *non-elemental* electrons, which are subsets of *elemental* electrons of equal measure. One of those *non-elemental* electrons will be spin-up and the other will be spin-down. Both the entangled electrons will be equally present in both regions, so to speak, where the presence of an electron in a region is construed as it having subsets of elements that are located at points that are elements of points in that region. Therefore, the two entangled *environmental* electrons are separable because they are two distinct objects. They are two distinct sets of *elemental* electrons, with no elements in common. This analysis is contrary to what was claimed in [10] (Section 3).

Being entangled, the two *environmental* electrons are causally linked. If one of the electrons is measured spin-up in region A the other must be measured spin-down in region B and vice versa. To see why this does not violate causal locality, we now need to think about the EPR–Bell setup.

4.3. EPR–Bell

We are to consider Alice and Bob who inhabit regions A and B. When Alice makes her spin measurement on the x-axis, she fissions into Alice_{UP} and $\text{Alice}_{\text{DOWN}}$, whose bodies occupy the local regions A_{UP} and A_{DOWN}.. The set of the points that are elements of the points in A is the fusion of the two distinct subsets that are the elements of points in A_{UP} and $A_{\text{DOWN.}}$ The fissioning of Alice's body involves the fissioning of spacetime itself. Prior to the measurement, Alice inhabited an *environmental* region that was a set of *elemental* regions, each in an *elemental* universe. Post-measurement Alice_{UP}'s and $\text{Alice}_{\text{DOWN}}$'s bodies inhabited two distinct *environmental* regions that contained *elemental* points in two distinct subsets.

What distinguishes A_{UP} and A_{DOWN} is that they contain two different *environmental* electrons and different elements of the macroscopic superposition, which is a future temporal counterpart of Alice's body. A_{UP} contains all the elements of Alice_{UP}'s body and none of the elements of $\text{Alice}_{\text{DOWN}}$'s and vice versa. In Bob's absolute elsewhere, Alice's body is in a superposition and Alice_{UP} and $\text{Alice}_{\text{DOWN}}$ occupy distinct branches, i.e., distinct subsets of region A.

Note that Alice's measurement need not change anything in the structure of region B. Keeping things simple to begin with, let Bob make his measurement on the x-axis too. He fissions into Bob_{UP} and Bob_{DOWN} in regions B_{UP} and B_{DOWN}. The key point here is that, because of the entanglement, these two subsets of region B differ from A_{UP} and A_{DOWN} whilst regions A and B are isomorphic. *Necessarily*, Alice_{UP} cannot have measured the *same* electron as Bob_{UP}, and $\text{Alice}_{\text{DOWN}}$ cannot have measured the same electron as Bob_{DOWN}. This is a consequence of the two environmental electrons having been in causal contact at their origin.

Now Bob's successor (immediate future temporal counterpart) is in superposition relative to Alice_{UP} and to $\text{Alice}_{\text{DOWN}}$ and Alice's successor is in superposition relative to B_{UP} and to B_{DOWN}. These four observers' results cannot come into causal contact sooner than half the light-time between regions A and B. To see why, consider Clotilde, halfway along a light path between regions A and B and watching Alice and Bob. When Clotilde sees the results of Alice's and Bob's measurements, she fissions into $\text{Clotilde}_{\text{AliceUP+BobDOWN}}$ and $\text{Clotilde}_{\text{AliceDOWN+BobUP}}$. As Cai Waegell and Kelvin McQueen put it, "A world containing a Bob and an Alice is only created when the wavefront from Alice's measurement meets the wavefront from Bob's measurement" [24] (Section 6). However, it is unclear why they use the term "wavefront"; it is rather a matter of the past lightcones of Alice's and Bob's future temporal counterparts coming to overlap.

Things get a bit more complicated if Bob makes his measurement on a different axis from Alice. Alice measuring spin-up on the x-axis entails that $\text{Clotilde}_{\text{AliceUP}}$ must see Bob_{DOWN} *if Bob measures on the x-axis*. However, as we saw in Section 4.1, the structure of the region where Bob's successor would measure x-spin-down is such that if the measurement had been made on a different axis, the results spin-up and spin-down would have probabilities determined by the subset measures of elements of the electron not measured by $\text{Alice}_{\text{UP.}}$ Those *elemental* electrons would be the ones located at *elemental* points oriented parallel to the axis chosen by Bob. Therefore, a series of measurements would have to be made on a succession of singlet states for Clotilde's future temporal counterparts to gather statistical evidence confirming the predicted probabilities.

5. Beyond Idealization

With the set-theoretic metaphysics in place, consider a non-idealized version of Vaidman's quantum die. Apart from the six equiprobable outcomes, there will be a plethora of extremely low-amplitude outcomes. Outcomes where "quantum accidents" occur, such as your smartphone transforming into a simulacrum of a salamander rather than displaying one of six numbers. These sorts of future events were also conceivable in classical physics, as the result of highly improbable particle trajectories consistent with current observations. However, in the context of the fission interpretation of the many worlds theory, all such bizarre events exist in the multiple futures of an observer. Vaidman does not take them into account because such events have, as he would put it, a very low *measure of existence* [25]. I have effectively argued that Vaidman's measure of existence can be strictly identified with objective probability. Therefore, bizarre futures should be left out of the account when rolling a quantum die because they have ridiculously low probabilities. There is nothing new in this idea.

However, the idea that all these bizarre futures *actually exist* is not necessarily anodyne. Pause for thought is called for in view of scenarios such as Huw Price's *Legless at Bondi* [26] (p. 382). More briefly, suppose that you are ill and are offered treatment that involves quantum processes with multiple outcomes. There is a high probability that you will be cured but a low probability that you will end up much worse off. In a conventional context, you take the risk, even if a little anxiously. In the fission context, you can be sure that the cured person will know that someone else is suffering because of the decision you took. Is it consolation enough to know that the suffering person will also have been the person who took the decision? It is not obvious that a fission interpretation of many worlds is free from moral conundrums, but then why should we expect such a profound change of worldview not to have consequences for the ways we choose to act?

6. Parting Lines

I have argued that Everett's key idea was to replace the concept of a stochastic process with that of a dendritic process, which is the idea that quantum phenomena induce the splitting of observers and their environments. This ostensibly raises problems that cannot be resolved by physics alone because assumptions rooted in folk metaphysics stand in the way. Observers cannot make predictions and test them unless they persist, but how can an observer persist through fissioning into multiple observers? Sider's stage theory solves this problem, but it did not become available until 1996 and remains neglected in the philosophical literature on persistence.

How can an observer be uncertain about future observations whilst believing that all outcomes occur? The folk metaphysics of possibility and actuality stands in the way but logic does not. That the objective probability of all outcomes occurring is 1 does *not* entail that the objective probability of each outcome's occurrence is 1. In that case, uncertainty can be understood as assigning partial degrees of belief to multiple futures without those futures needing to be alternative possibilities, as has always been thought.

How can objective probability be a property of multiple actual outcomes? The proposal that is described and further developed here involves the hypothesis that individual objects in an observer's environment can be construed as sets with many elements that are macroscopically isomorphic and microscopically anisomorphic because they are constituted by different configurations of local beäbles. Quantum processes induce the partitioning of those sets into macroscopically distinct subsets whose measures are the objective probabilities of outcomes. As a consequence, a single observer's body is a set of *doppelgängers*, so the idea that there can be multiple copies of *observers*, which is widely held amongst many-worlds theorists, must be rejected. A future-looking account of objective probability is provided by the idea that a single observer, whose body is a set of *doppelgängers* fissions into multiple observers whose bodies are subsets of *doppelgängers* with probability measures. According to stage theory, the pre-measurement observer bears the relation *will be* to each of the post-measurement observers and is uncertain about the future because the observer assigns degrees of belief to future observations equal to their probability measures. There is no question as to *which* post-measurement observer the pre-measurement observer will be; they will be *each* of them.

This set-theoretic metaphysics provides a framework for a version of the many-worlds interpretation that involves locality, separability, and Everettian fission, rather than divergence. It provides an account of probability that does not appeal to self-location uncertainty and an account of microscopic reality that includes local beäbles. It leaves work to be done on the physics of those beäbles and how they participate in branching processes.

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quantum reports

Article

The Ontology of the Many-Worlds Theory

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Abstract: It is shown that the wavefunction describes our observations using the postulate that relates position to the distribution $|\Psi|^2$. This finding implies that a primary ontology is unnecessary. However, what is real is not directly represented by the wavefunction but by the gauge invariants. In light of the presented ontology, Spacetime State Realism becomes not a fundamental ontology but derived.

Keywords: many-worlds interpretation; interpretations of quantum mechanics; wavefunction; ontology

1. Introduction

Schrödinger's articles in 1926 defining the wave mechanics version of quantum mechanics offered a less abstract and computationally more tractable formulation of quantum mechanics than the matrix mechanics initiated by Heisenberg. However, the wavefunction of many-particle systems depended on N positions in 3-space, which defied a straightforward understanding. At the Solvay conference in 1927, Schrödinger [1] hoped that it would be possible to reformulate the theory to avoid functions of several positions in 3-space, which has not yet been found. Bohr argued in 1927 [2]:

> . . . there can be no question of an immediate connection with our ordinary conceptions because the "geometrical" problem represented by the wave equation is associated with the so-called co-ordinate space, the number of dimensions which is equal to the number of degrees of freedom of the system, and, hence, in general, greater than the number of dimensions of ordinary space.

The hydrogen atom can illustrate the need for the non-relativistic quantum state to be something other than a function of a single point in space. The wavefunction for a free hydrogen atom in the ground state is a product of a center of mass function and a function of the proton and electron relative motion. Both factors are necessary to describe the physics of this system. For example, if a third particle scatters off the hydrogen atom, both factors are necessary to describe the process fully.

Bohr concluded that quantum mechanics did not constitute a description of an existing reality, but nothing more could be stated about what was going on. However, given the enormous success of quantum mechanical calculations, we should consider that the wavefunction closely mimics what is really going on. Bohmian mechanics [3–5], Everett's relative state interpretation [6–8], and wavefunction collapse theories [9] attempt to give a realistic quantum mechanical description of the physical phenomena in which the wavefunction is an integral part of the story or the whole story.

In the context of the mentioned realistic interpretations of quantum mechanics, it is still under debate what kind of entity the wavefunction is. The problem that the founders of QM faced remains. The number of real variables is $3N$ for N particles, but the physical space we experience is three-dimensional. Maudlin [10,11] has been skeptical that a function of so many variables can give the full account of what is going on in 3-space. Albert [12] has taken seriously that the dimension of the domain implies that the dimension of physical space is $3N$. The wavefunction becomes a field in that space. He argues that

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the Hamiltonian implies a 3-dimensional emergent structure corresponding to the 3-space we experience. This is also the view Ney has taken [13], but she differs in precisely how the three-dimensional structure is extracted. The alternative is to take the 3-space as fundamental and to accept that QM introduces a physical quantity that depends on several points in the 3-space. This has been advocated by Lewis [14], Ney [15], and Chen [16].

Section 2 discusses that the wavefunction is a function of several points in space. How such a wavefunction can describe what we experience is elaborated on in Section 3. Section 4 presents the reality the wavefunction implies. Wallace and Timpson have previously suggested what can be understood as the reality of the wavefunction, which is discussed in Section 5. Section 6 summarizes the findings.

2. The Configuration Space for *N* Point Particles and the Wavefunction

N points give the configuration of N classical point particles located somewhere in 3-space. The space of all possible configurations, the configuration space, is the set of all possible configurations of N points, which will here be denoted by $\mathrm{C}(N)$. The symbol $\overline{\mathbf{x}}$ denotes an element in $\mathrm{C}(N)$. Figure 1 shows an element of C(5). Even though $3N$ real numbers give a configuration, it is not correct to state that $\mathrm{C}(N)$ equals $\mathbb{R}^{3N}$. $\mathrm{C}(N)$ contains a structure that is missing in $\mathbb{R}^{3N}$. For example, the coordinates for all points are relative to the same coordinate system in 3-space, and there is a distance measure between points

$$r = \sqrt{(x_i - x_j)^2 + (y_i - y_j)^2 + (z_i - z_j)^2}, \tag{1}$$

where i and j are particle indicies. This kind of distance is available in $\mathrm{C}(N)$ but not in $\mathbb{R}^{3N}$. Simply, $\mathrm{C}(N) \neq \mathbb{R}^{3N}$. One might abstract away those features of $\mathrm{C}(N)$, that is to ignore them, and replace $\mathrm{C}(N)$ by $\mathbb{R}^{3N}$, but only when those features are irrelevant. (If there were one bowl with five apples and another with five oranges, you might use the notion fruits, which is an abstract notion relative to apples and oranges, and say that five apples =(fruits) five oranges. In doing so, we ignore the difference between apples and oranges. In some circumstances, this omission would be fine; in others, less so).

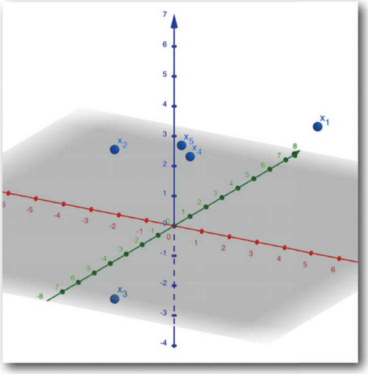

Figure 1. $\overline{\mathbf{x}} = \mathbf{x}_1, \mathbf{x}_2, \mathbf{x}_3, \mathbf{x}_4, \mathbf{x}_5 \in \mathrm{C}(5)$.

When physicists perform calculations of quantum many-particle systems, the wave function, $\Psi(\overline{\mathbf{x}}, t)$, is a mapping $\Psi : \mathrm{C}(N) \rightarrow \mathbb{C}^P$, where P is the dimension of the combined spin space and $\mathbb{C}$ is the complex numbers. (As $\mathbf{E}(\mathbf{x}, t_0)$ gives the configuration of the electric field in space at time t_0, $\Psi(\overline{\mathbf{x}}, t_0)$ is the configuration of the quantum state on $\mathrm{C}(N)$. Hence, we can view $\mathrm{C}(N)$ as the space on which the quantum state is configured, its configuration space.) This definition of the wavefunction is used, together with the standard method for comparing with measurements, to achieve great success. From this fact about how quantum mechanics is actually applied, it is surprising that so many physicists and philosophers of physics take the domain of the wavefunction to be the unstructured $\mathbb{R}^{3N}$ rather than

C(N). For example, Maudlin [10] writes, "The wavefunction is something that evolves in a very, very, very, very high-dimensional space", and then continues that "there is no low dimensional space at all" as a description of the domain of the wavefunction.

In the quest to find the foundations of QM, the actually used theory is to be analyzed. The straightforward understanding of the world that QM describes is that 3-space is fundamental and that the wavefunction is a function of N points in 3-space. Such a mathematical entity has been denoted "poly-wave" or "polyadic field" by Forrest [17] and in the context of pilot wave theory "multi-field" by Hubert and Romano [18]. From QFT, we are used to N-point functions (correlations) $\langle|\phi_1(x_1)\phi_2(x_2), \ldots, \phi_N(x_N)|\rangle$, where $\phi_i(x_i)$ is some local operator at some position and time $x_i = (\mathbf{x}_i, t_i)$. From such N-point functions, we can arrive at the non-relativistic quantum mechanics from QFT. This relation shows that we should take the 3-space as fundamental. However, the degrees of freedom a physical system possesses are a fundamental property of that system, so the space of the $3N$ degrees of freedom is fundamental in a universe of N particles. Henceforth, functions of N points in space will generally be called N-point functions.

Albert and Ney assume that the $3N$ dimensional space is fundamental. They use arguments about the dynamics or invariances of the systems to argue that 3-space emerges. Interactions depend on the Expression (1). Its symmetries are the symmetries of the 3D space that emerges, and r is the distance measure that makes it into a Euclidean space. The arguments also imply that the wavefunction variables are divided into triples that correspond to a point in this space. One might argue that the implied 3D space is a nomic structure as a fundamental feature of the interactions implies its existence. The fact that this 3D space is present for any value of N implies that 3D space is more fundamental than $3N$ space. That Albert and Ney posited the 3N space to be fundamental does not exclude that their investigations will lead to the 3D space also being fundamental and even more so than the 3N space. We end up here in that the wavefunction variables correspond to N points in a 3D Euclidian space, which we have to identify with the ordinary 3-space we observe.

To conclude, the wavefunction domain has the properties of C(N): either we posit it or we discover it (The notion of emergence is misleading here. If we start from one space being fundamental and then find the existence of another fundamental space, we should not think of the second space as emergent. We simply discovered something we were unaware of).

3. The Wavefunction Description of What Is Going on in 3-Space

We have established that the wavefunction domain is a set of geometrical figures in 3-space, N points in 3-space, to a complex linear space. This structure proves that the wavefunction is a structure in 3-space [19]. It remains to be shown that it can explain what we observe.

Everett's vision that the unitarily evolving universal wavefunction can describe every aspect of the physical world needs a statement about the physical significance of the value of Ψ. In particular, the wavefunction has to contain information about where everything is located. As the wavefunction is a distributed object, the position of a particle (or particles) can only be given by a distribution. As the theory does not contain any point-like entities, this distribution is not a probability distribution of the point the particle(s) location. We have to abolish the traditional view of the location of particles as being points. Positions in Everett's Quantum Mechanics (EQM) necessarily have to be a distributed quantity. Arve [20] formulated in a postulate (EQM1) what that distribution should be. The position $\bar{\mathbf{x}}$ and spin value a of the particles is given by the distribution

$$\rho(\bar{\mathbf{x}}, a) = |\Psi_a(\bar{\mathbf{x}})|^2, \tag{2}$$

which is called the presence distribution. It has also been called "measure of existence" [21] and "partial instantiation" [13], which seems to possess a meaning similar to presence. Greaves [22] has argued that ρ is the "caring measure," which fails to give a general understanding of the quantity, as it is only relevant for agents making decisions. Position, existence, and instance are notions that we are not used to having a gradual character in the sense suggested, but Everett's vision implies that we accept that at least one of these notions to be a distributed quantity given by $\rho(\overline{\mathbf{x}}, a)$. In [20], the quantity

$$P = \sum_a \int_{V_1} \cdots \int_{V_N} \rho(\overline{\mathbf{x}}, a) d^{3N}x \tag{3}$$

is called the presence in $V_1 \times \ldots \times V_N$. The article proved that an observer should have the same expected relative frequencies as if the Born was applicable because we should expect to find ourselves in a situation associated with high presence (In turn, this implies that a rational agent should make decisions as if the Born rule is true).

Consider a scattering described by two initial wave packets that collide. One of them, the target, initially has zero group velocity, while the projectile has a finite and known group velocity. After the collision, the combined system is entangled. Assume that an array of detectors is set up at a macroscopic distance away from the collision region at positions covering the angles where the projectile or target will have an appreciable presence. Due to the agreement with the Born rule, we know that EQM describes the frequencies with which detectors measure the projectile and target system at different angles, including the correlations between projectile and target. We have a description of how the combined wavefunction of the target and the projectile evolve in 3-space in agreement with observations. This description contains correlations between the projectile and the target. The description cannot be separated into one description of where the projectile is located and another description of where the target is located. We can calculate what is called the marginals in the context of probabilities to get the distribution of one of the systems, corresponding to measuring only one of the particles. In contrast, the entire presence distribution is necessary to get the correct theory for coincidence experiments.

For an atom, a molecule, or any other bound system free from external forces, the wavefunction is a superposition of states of the type (A subsystem of the world should strictly be given by density matrix. The columns of the matrix are then such superpositions)

$$\psi^{(i)}_{CM}(\mathbf{x}_{CM})\psi_i(\overline{\mathbf{x}}_i). \tag{4}$$

The center of mass wavefunction $\psi^{(i)}_{CM}(\mathbf{x}_{CM})$ can be of any shape that we can consider for a free non-relativistic point particle, and its absolute squared gives the position distribution of the center of mass. The intrinsic state $\psi_i(\overline{\mathbf{x}}_i)$ absolute squared gives the position distribution of the parts relative to the center of mass. From the intrinsic states, we can get the excitation spectra and all matrix elements related to the coupling to an external probe. In atoms with several electrons, there are correlations corresponding to entanglement between the electrons.

The fact that the domain of the wavefunction is C(N) implies that it describes something in 3-space. From the considerations above, the wavefunction clearly describes what is going on in a scattering event and all the structures of atoms and molecules that we can have precise knowledge about according to QM. That the wavefunction is a function of not just one but several points in 3-space made Bohr claim that it does not describe what is going on. Our experience from QM calculations of various physical systems combined with the postulate EQM1 proves the ability of the wavefunction viewed as a N-point function to describe what is going on in 3-space.

So far, we have assumed the existence of macroscopic objects like detectors, tables, and other objects. These are nothing but large generalized molecular structures where atoms have relatively well-defined relative positions. That macroscopic objects have well-defined positions relative to each other is guaranteed by decoherence, which is present under normal circumstances.

Decoherence is also vital in splitting a world into several branches in an experiment where the wavefunction of the measured system contains many values of the measured quantity. In this quantum mechanical description of the measurement process and in many other situations where entanglement is a prominent feature, it is vital to view the wavefunction as a function of several points in 3-space.

4. Proposed Ontology

As argued in [20], the ontology ought to be gauge invariant as the gauge choice has no physical consequences. This result only depends on that the Hamiltonian is a kinetic energy term and a potential energy, which is a function of positions and spin.

A gauge change amounts to adding to the vector potential of particle type l the field $\Delta\mathbf{A}_l(\mathbf{x})$. Here, all particle types have their gauge fields, also neutral particles. For charged particles the gauge field includes the value of the charge. The product of the charge q_l and magnetic field $\mathbf{B}$ are given by $q_l\mathbf{B} = \nabla \wedge \mathbf{A}_l$. For neutral particles $\nabla \wedge \mathbf{A}_l = 0$. The gauge change is culs free, $\nabla \wedge \Delta\mathbf{A}_l = 0$ and changes the wavefunction,

$$\Psi(\bar{\mathbf{x}}) \rightarrow \exp\left(\frac{-i}{\hbar}\sum_{k=1}^{N}\int^{x_k}\Delta\mathbf{A}_{l_k}(\mathbf{x}'_k)dx'_k\right)\Psi(\bar{\mathbf{x}}). \tag{5}$$

The gauge-independent quantities

$$\rho(\bar{\mathbf{x}}) = \sum_a |\Psi_a(\bar{\mathbf{x}})|^2, \;\; \mathbf{j}_k(\bar{\mathbf{x}}) = \frac{1}{m}\mathrm{Re}\sum_a\left[\Psi_a^*(\bar{\mathbf{x}})\left(\frac{\hbar}{i}\nabla_k + \mathbf{A}_{l_k}(\mathbf{x}_k)\right)\Psi_a(\bar{\mathbf{x}})\right], \; k = 1,\ldots,N, \tag{6}$$

and the total spin state Hilbert space ray, $S(\bar{\mathbf{x}})$, is the ontology related to the wavefunction. Given the vector fields, a global phase choice, and $\mathbf{A}_l(\mathbf{x})$, the wavefunction can be derived from these gauge invariant quantities. Note that only if the N point functions ρ, $\bar{\mathbf{j}}$, fulfill a certain condition [23] can they correspond to a wavefunction. But when they are derived from a wavefunction, ρ, $\bar{\mathbf{j}}$, S, together with the set of gauge fields $\{\mathbf{A}_l\}$ they give the wavefunction uniquely except for the global phase choice. Vaidman has denoted the quantity $\rho(\bar{\mathbf{x}})$ by "measure of existence", which suits its ontological character. The spin quantity $S(\bar{\mathbf{x}})$ is gauge invariant and, as far as is known, does not contain any superfluous degrees of freedom.

That the interactions are local favors strongly that the Schrödinger equation is written in the spatial basis. Thus the quantum state is naturally represented by the wavefunction $\Psi(\bar{\mathbf{x}})$ due to the locality of the interactions. That feature also implies the gauge invariance and that the ontology contains the quantities given here. What would be the ontology if the interactions were non-local is something we need not be concerned with because we have no understanding of what such a world would be like, nor do we have any good reason to study such a world.

It is often stated that the fundamental understanding of the quantum state is that it is a vector in Hilbert space. Without further specifications, a statement to this end, the quantum state is empty of physical significance, as the abstract Hilbert space is a purely abstract mathematical entity like the natural numbers. A specific number, e.g., 5, says nothing about the physical world without a context that tells what the number stands for. However, there are concrete Hilbert spaces, specifically $L_2[C(N) \rightarrow \mathbb{C}^P]$. This Hilbert space is the correct concrete one to which the wavefunction belongs. For a Hilbert space representation to describe some features of our world, there has to be one basis directly related to the fundamental description, and for any other basis state, we have to express it in terms of this fundamental basis. Rewriting any state or relation on a non-fundamental basis is nothing but a mathematical transform. As such, it can be beneficial for various considerations. When it comes to quantum mechanics, a warning is prudent. Any such transform is gauge-dependent. If the gauge is changed, the form of the transformed expressions should change to preserve its physical meaning.

5. Spacetime State Realism

Wallace and Timpson [24] have offered an alternative to wavefunction realism which they call Spacetime State Realism (SSR). Their proposal is put forward as an alternative to wavefunction realism, which they have criticized.

They focus on giving the quantum state an understanding in terms of subsystems localized in 3-space. The density matrix gives the quantum state in a spatial region Δ, which is obtained by using a Hilbert space basis divided into states with support inside and support outside Δ. This construction leads to a varying particle number inside the region. In the case of QFT, they let (the action of) local operators restricted to the region define the state in the region.

The authors give a minimal argument about how they thought this could be done. Inside Δ, the wavefunction was considered a superposition of the products of single-particle states, with support only inside the region Δ. For the single-particle wavefunctions to have support inside a region in space, they have to be functions of a position in 3-space, which implies that the universal wavefunction is a function of many points in 3-space. However, the authors never comment on how the wavefunction is related to 3-space. For SSR to give the features that the authors advocated to be its advantage over $\mathbb{R}^{3N}$ wavefunction reality, the wavefunction has to be an entity in 3-space. However, from Section 3, it is clear that wavefunction is an entity in 3-space when it is taken to be a function of positions in 3-space. The present analysis has closed a gap in the argumentation for SSR.

Wallace and Timpson argued that the ontology ought not to be one big system with no subsystem decomposition because we would have only a single property bearer which “would lack sufficient articulation to give the physical meaning of what was presented”. This assertion is not warranted. Taking the universal wavefunction, or rather the position distribution $\rho(\overline{\mathbf{x}})$, the current $\overline{\mathbf{j}}(\overline{\mathbf{x}})$, and the spin state $S(\overline{\mathbf{x}})$ as the fundamental ontology, there would be derived local ontological features in terms of the density matrices that SSR is based on.

Wallace and Timpson recognize that the main drawback of SSR is that it separates into local regions, though the wavefunction is non-separable. This feature is a grave problem that disqualifies a set of local density matrices from being the fundamental ontology as it cannot represent all physical features. SSR leaves out the entanglements of entities in different regions of their space division. A fundamental ontology must be able to represent all physical relations and effects.

Wallace and Timpson described a version of SSR for QFT, which focuses on the algebra of local operators. As the algebra is only local, the algebraic relations between operators at widely different positions and their expectation values were not included. Thus, the entanglements of entities at different spatial regions are omitted here as well. This version is equally unfit to constitute the fundamental ontology as the non-relativistic case. Additionally, Swanson [25] has pointed out technical difficulties in the approach to QFT SSR.

One of the points of criticism against the $\mathbb{R}^{3N}$ wavefunction realism was that relativistic QFT gives a very different picture in which particles are emergent and not fundamental. This criticism implies that any version of wavefunction realism where the particle number N is fundamental is mistaken about what is fundamental. However, it is a legitimate investigation to find out what is real within a theory like non-relativistic QM that describes so much of the world around us. Relativistic QFT can be seen to be more fundamental, but it is hardly the ultimate theory of the physical world. It is indeed vital to investigate the theories we have. The principle that the ontology should be given by the gauge invariant entities, as advocated in Section 4, will probably also produce a good understanding of the QFT ontology.

Maudlin has criticized SSR with that the density matrix will contain information from the many worlds created since the Big Bang, which is present in the region. Maudlin argues that the density matrix will essentially be a continuous distribution containing no discernible information. In particular, this is an argument against the possibility of dividing the density matrix into a sum of quasi-classical worlds, which Wallace and Timpson claimed to be possible.

The most severe criticism against SSR is that it never explains how any aspect of the physical reality is connected to the amplitudes that enter the construction of the density matrices. For example, this could be achieved by statements similar to EQM1, but Wallace and Timpson failed to see its necessity. It can be added that the same criticism applies to the argumentation for Everett's ideas found in Wallace's book [8]. No interpretation of the wavefunction amplitude is given so that the patterns in the amplitude can be interpreted in terms of physical objects.

Albert's Narration Paradox

David Albert [26] has found a consistency problem for the non-relativistic QM description of the following scenarios. The discussion of Albert, called a narratability problem, will follow the presentation in [24]. Two spin-1/2 particles at a distance from each other and the spins form together a spin singlet. In the first scenario, nothing happens. In the second scenario, both spins are flipped simultaneously such that $|\uparrow\rangle \rightarrow |\downarrow\rangle$ and $|\downarrow\rangle \rightarrow -|\uparrow\rangle$. Then the spin singlet state is unchanged afterward,

$$|\uparrow\rangle|\downarrow\rangle - |\downarrow\rangle|\uparrow\rangle \rightarrow -|\downarrow\rangle|\uparrow\rangle + |\uparrow\rangle|\downarrow\rangle. \tag{7}$$

In both scenarios, the spin state of the combined system is always in a spin singlet. However, from the point of a moving frame, the changes of the two spins will not be simultaneous. The state between the two changes might then become

$$|\downarrow\rangle|\downarrow\rangle + |\uparrow\rangle|\uparrow\rangle. \tag{8}$$

In the original frame of reference, there was no period in which the state was in a spin triplet which we have in the moving frame. Wallace and Timpson state that the sequence of states in the moving frame, $\Psi'(t)$, is not a mere redescription of the state sequence in the original frame $\Psi(t)$. They further conclude that the sequence of states demonstrates that $\Psi(t)$ cannot be regarded as fully describing the properties of the system.

There are a couple of problems with the description and the conclusions. That systems might seem qualitatively different in frames moving with respect to each other is well-known to seem paradoxical, but we have to accept the consistency of the theory. For example, in one frame, a train might, for a moment, be entirely inside a tunnel, while in another frame, it is never the case. The difference between the frames in the train "paradox" can easily be resolved. In both frames, consider the events that the back of the train enters the tunnel and the front of the train coming out of the tunnel. For the effects of the triplet state to become apparent, consider simultaneous measurements in the moving frame of the two spins. The result of such measurements in the basic direction will demonstrate that the spins have equal direction. In the non-moving frame, these measurements will happen at different times. One will be before the spin flips and the other one afterward. No surprise that the spins will be measured to have the same direction as is the case for the triplet state. There is no more of a problem or a paradox here than in the case of the train and the tunnel. A shortcoming of the spin scenarios is that it takes some time for the spins to change direction. There also needs to be an apparatus to flip the spins, which should be included in the quantum description. In Albert's version, two additional particles are involved in flipping the spins. This more complicated situation requires a lengthy discussion which we will not embark on here.

The conclusions that Wallace and Timpson made are not warranted. The descriptions that the $\Psi(t)$ or $\Psi'(t)$ give are in as much agreement as is necessary and allowed by the theory of relativity.

6. Summary

The wavefunction is a function of N points in 3-space; the domain is $\mathrm{C}(N)$. This domain implies that the wavefunction describes things happening in 3-space. By examining a couple of example systems, it was shown that the wavefunction could describe our observations. For that end, the postulate EQM1 is necessary. Only gauge invariant quantities can be ontic. The sufficient and minimal ontic components are the presence $\rho(\overline{\mathbf{x}})$, the current $\overline{\mathbf{j}}(\overline{\mathbf{x}})$, and the total spin state $S(\overline{\mathbf{x}})$. Bohr's pessimism about the possibility that the wavefunction describes "our ordinary conceptions" has been proven unwarranted. The success of describing our observations of physical systems and experiments with only the wavefunction gauge invariants demonstrates that a primitive ontology is not necessary.

The previously proposed SSR is problematic. Its authors' arguments against wavefunction realism were directed against the version in which the wavefunction domain was taken to be $\mathbb{R}^{3N}$, for which the ordinary 3-space is not clearly present. However, SSR is founded on the view that the wavefunction domain is $\mathrm{C}(N)$, but Wallace and Timpson never discussed the possibility that it defines what is real. Instead, they defined the density matrix for the subsystem being a region in space to be what is real. Then the information about entanglement with the world outside the region is lost, which renders the ontology incomplete. The most devastating problem of SSR is that the wavefunction is not given any physical significance, rendering the density matrix meaningless.

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Article

Everett's Interpretation and Convivial Solipsism

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Abstract: I show how the quantum paradoxes occurring when we adopt a standard realist framework (or a framework in which the collapse implies a physical change of the state of the system) vanish if we abandon the idea that a measurement is related (directly or indirectly) to a physical change of state. In Convivial Solipsism, similarly to Everett's interpretation, there is no collapse of the wave function. However, contrary to Everett's interpretation, there is only one world. This also allows us to get rid of any non-locality and to provide a solution to the Wigner's friend problem and its more recent versions.

Keywords: measurement problem; Convivial Solipsism; Everett's interpretation; QBism; perspectival interpretation; Realism; entanglement; non-locality; EPR experiment; Wigner's friend

1. Introduction

Entanglement is probably the most intriguing feature of quantum mechanics. Two systems having interacted seem linked in a non-separable way, forming one unique system with the strange property that in the case of full entanglement, knowing everything that can be known about the whole system does not give any information about its subparts. Knowing that two spin one-half particles are in a singlet state gives a complete description of the system of the two particles, but nothing is known about each individual particle. This is counterintuitive since in classical physics knowing everything about a composed system means knowing everything about its parts. Bernard d'Espagnat called non-separability [1] the fact that two entangled systems constitute one unique system inside which the subparts cannot be considered individually before a measurement of one of them separates them. However, the most striking consequence of entanglement is probably that it seems to imply non-locality. The famous Einstein–Podolsky–Rosen (EPR) argument [2] shows that a measurement of one particle of a system in a singlet state allows us to discover instantaneously the value of an analogous measurement of the other particle whatever the distance between the particles. Given Bell's inequality [3], forbidding that the result be determined before the measurement through possible hidden variables, it seems that a spooky action at a distance (as Einstein said) forces instantaneously the state of the second particle to change in conformity with the result obtained on the first one. This action is constitutive of what is called non-locality.

There is a huge literature on the subject both by people defending the fact that non-locality must be accepted and by people trying to propose new interpretations allowing us to get rid of it. Now, if a physicist wants to argue in favour of one or another of these two positions, he has to explicitly say which one between the many existing interpretations of quantum mechanics (Copenhagen interpretation, Everett interpretation, GRW theory, de Broglie Bohm theory, relational interpretation, QBism, or any other) he chooses as the framework for his reasoning. An important demarcation between two kinds of interpretation is linked to the choice between thinking that the state vector of a system represents a real physical state (ψ-ontic interpretation) or simply refers to our knowledge (ψ-epistemic interpretation) [4]. In the first case, the collapse of the wave function following

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measurement is a real physical change of the state of the system while in the second case, it is only an update of the knowledge of the observer. It is obvious that in the latter case, the collapse is not a physical action on the system, and the proponents of this position contend that this proves that there is no non-locality. Nevertheless, as we will see, this needs a deeper examination. On the contrary, if the collapse is viewed as a real physical change in the state of the system and it is impossible to consider that this change is anterior to the first measurement, then it seems that non-locality is unavoidable.

These questions are closely linked to the measurement problem. What exactly is a measurement? The answer depends of course on the interpretation chosen. In the majority interpretation which follows Bohr and the Copenhagen school, the state vector represents the real physical state of the system, and the collapse is a real physical change in the system. This realist position leads to the conclusion that non-locality is a feature that we must accept (as strange as it is). Nevertheless, it is well known that this position faces the measurement problem and cannot provide any good solution for it.

In this paper, I will first present the difficulties arising in the context of an EPR situation inside a realist framework. I will then analyze some other interpretations and will explain why they do not seem fully satisfactory. This discussion will be restricted to the framework of non-modified quantum formalism. So, I will neither examine the case of the hidden variables theory of de Broglie–Bohm [5,6] nor the spontaneous collapse theories such as the GRW theory [7,8], which introduces a nonlinear stochastic term in the Schrödinger equation to take the collapse into account. Everett's interpretation [9–11] will be treated in parallel with the interpretation I propose, Convivial Solipsism (ConSol), since in both theories there is no collapse. ConSol [12–16], although modifying the usual concept of reality, gives a full account of the measurement problem through a different way to understand entanglement and explains why non-locality is an illusion.

2. The Measurement Viewed as Something Happening in the Reality

The vast majority of interpretations rest on an implicit assumption: the world is a kind of theatre inside which all the events take place. We can think of the picture of the universe given by general relativity. Space-time is pre-existing and everything that happens happens inside it. Energy and matter can affect the geometry of space-time, but space-time is the arena inside which energy and matter are situated. An event is a change in some property (position, momentum, type of particle …) belonging to a system and taking place at a certain time and a certain location inside a given reference frame. We can witness such an event or not. This makes no difference. General relativity does not need any observer and describes the dynamics of the universe with or without any person who watches what happens. This picture is compatible with a fully realist position: the universe exists independently of any observer and everything that takes place inside it happens really as it is described by the theory (The theory in question is not mandatorily the current theory we have now but points to the theories towards which science progresses). We must take at face value what the theory says. If under certain conditions the theory predicts an event, we can be sure that this event happened (or will happen) and that it would have happened exactly the same way even if nobody had been there to observe it. Observers play a passive role limited to witnessing what happens independently of them. Facts happen and are facts for the universe; they are absolute facts for all observers.

Of course, this framework is adopted by the standard realist position, but it is shared as well by many "anti-realist" positions. The reason is that Realism comes in three steps:

- Metaphysical Realism (MR) contends that there is an external reality independent of any observer or of the knowledge that any observer could have about it.
- The thesis of Intelligibility of Reality (IR) says that independent reality is composed of entities that are in principle describable and understandable.
- Epistemic Realism (ER) ascribes to science the role of describing and explaining intelligible reality and claims that our good theories give an adequate description of Reality that corresponds to the picture given by them.

The consequence is that the progress that science makes represents true discoveries about the world that are not mere inventions or conventions. Scientific Realism (SR) is the conjunction of these three theses (See Zwirn [12] (pp. 281–283)). Van Fraassen [17] describes Scientific Realism by saying that our theories are not metaphors but are literally true: "If a theory speaks of electrons, then that means that electrons are really existing". In the same spirit but with humour, Rescher [18] says, "To accept a theory about the little green men on Mars is accepting the fact that there really are little green men on Mars". In the framework of Scientific Realism, the concept of truth is the correspondence theory (See Dummett [19]). A sentence is true according to something external that does not depend on our mind or our language or our capacity to verify it but corresponds to the actual state of affairs since facts are absolute.

It is clear that Epistemic Realism assumes Metaphysical Realism and at least a weak form of the thesis of Intelligibility of Reality: ER $\supset$ MR + IR. Scientific Realism is the most currently adopted point of view. It is fully compatible with the whole of classical physics (classical mechanics, thermodynamics, electromagnetism, and general relativity) and is deeply set up in our mind as a very intuitive and natural position.

However, in quantum mechanics, Scientific Realism raises many issues, and this is the reason why different points of view have been offered to interpret quantum formalism.

It is possible to accept Metaphysical Realism without fully accepting Scientific Realism. This is the case of the Copenhagen interpretation, which does not deny that there is an independent reality but states, according to Bohr and Heisenberg, that we must speak only of the results of measurements that are performed at a macroscopic level and not of what happens at the microscopic level between two measurements. Quantum mechanics can predict what the possible outcomes are if the observer makes such and such measurements but says nothing about "what the system itself does" between two measurements. For example, quantum formalism can give the probability that a particle will be observed at a certain position x_2 after having been observed at a position x_1 but says nothing about the trajectory that the particle follows between x_1 and x_2. In a certain sense, the Copenhagen interpretation gives up the goal to describe precisely microscopic reality. As Bohr [20] says:

> "In our description of nature the purpose is not to disclose the real essence of phenomena but only to track down as far as possible relations between the multifold aspects of our experience."

Positions which state that the goal of the theory is not to describe reality but only to predict the results of our experiments are grouped together under the name of instrumentalism. Instrumentalist positions are numerous and differ by degrees from the ones claiming that the question of knowing whether there is a reality or not is meaningless to those accepting that there is indeed a reality but that it is outside of the scope of science to describe it. Pragmatism is a close point of view with some nuances from instrumentalism. The reduction in the wave function happening after a measurement is nevertheless often interpreted as meaning that the system switches to a new physical state. A measurement has a real physical impact, changing the state of the system or witnessing a real change that happened.

The ontic interpretations view the state vector as describing the real physical state of the system. The epistemic interpretations posit that the state vector is not representing the physical state of the system but only the knowledge that the observer has of it. Hence, the reduction in the wave function is interpreted as a mere update of knowledge after the measurement since the observer has learnt new information. However, this raises the question: "knowledge about what?", "information about what?". As Brukner [21] rightly says:

> "The distinction between a realist interpretation of a quantum state that is psi-ontic and one that is psi-epistemic is only relevant to supporters of the first approach."

The psi-epistemic approach seems similar to the case of an update in knowledge in a classical case when for example an observer watches a die after it has been rolled and discovers the face on which it fell. However, in this classical case, the face on which the dice stops is perfectly determined before the observer has a look at it, and the observer only witnesses what happened independently of her. It is different in quantum mechanics, which shows that when the measurement of a property is performed, it is in general not possible to assume that the property had a definite value before the measurement. The value is determined only after the measurement. So, even if the collapse of the state vector only represents an update of the knowledge of the observer, we must assume that the result obtains only at the moment of the measurement. That means that there is a physical change during the measurement even if the state vector is not supposed to represent the physical state of the system.

In summary, almost all interpretations, whether they take the state vector to be ontic or epistemic, see the collapse as something related to a change in the physical state of the system (Of course, this does not apply to Everett's interpretation or ConSol in which there is no collapse). This is obvious in the case of ontic interpretations. The state vector is representing the real physical state of the system and of course, the collapse is a physical event changing the state of the system, which switches from an initial state that is a superposition of eigenstates of the observable that is measured to a definite eigenstate (I assume here that the corresponding eigenvalue has not degenerated but that is not important for this point). In the case of epistemic interpretations, the collapse is an update of the observer's knowledge and not a physical action. Nevertheless, this very knowledge is "about" the system. If the knowledge is updated, it is because the observer has learnt something new about the system. So, unless the assumption is that the state of the system before the measurement was such that the value that is measured was already the value that is measured, which, as we said above, is excluded in general in quantum mechanics (a measurement is not simply a record of a pre-existing value), that means that for whatever reason the state of the system has changed (This is the reason why it is an updating of knowledge and not a revising (according to the standard difference made in the belief revision theory [22–24]). The collapse is then the very action to take into account the change of the system after the measurement in the observer's knowledge. So, we see that in both types of interpretations, the collapse is directly or indirectly "about" a physical change in the system.

Let us define the collapse as the fact that the observer observes one definite value. This raises first the question of which process allows the system to adopt a definite value for the property that is measured. This is the problem often called the "big" problem: *What makes a measurement a measurement?* Some interpretations explicitly say that this is an empirical fact that they do not want to explain. That is the case of Healey's pragmatism [25–27]. Some others such as QBism stay fuzzier about this question (This is what QBists call "participatory realism" [28] after Wheeler. See also Zwirn [29]). However, the fact is that in all these interpretations, the measurement problem is solved because the measurement is not causing the change in the state of the system but is only witnessing this change that happens independently of any description by formalism. So, there is no more contradiction between two physical laws, the Schrödinger equation describing the change of the state when there is no measurement, and the reduction describing the change in the state when a measurement (i.e., an update of knowledge) is made since formalism does not describe the physical state of the system but only the knowledge we have of it. However, unless we discard the question as meaningless, this raises the issue of understanding how the system evolves from a state in which it is impossible to attribute a definite value to a property toward a state where this value is defined, and the observer can watch it and update the vector state that represents the knowledge she has. So the question of what constitutes a measurement is not solved. We are not in classical physics, and it is not possible to think that the value is always determined. So what makes the value determined?

The most important point that I want to emphasize is the following: whether or not an explanation of the fact that the system adopts a definite value for the property that is measured is given, the point of view that is adopted means that the system is seen as something real that evolves. Its state changes (even if this is not described by formalism): in all the interpretations, a measurement notices a change in the physical world. During a measurement, something physical happens (or has happened) to change the state of the system. The only difference between the realist interpretation and the epistemic interpretation is that in the epistemic point of view, the collapse of the state vector is not directly related to the physical state of the system but to the fact that the observer witnesses the new state of the system and updates her knowledge. Nevertheless, it is implicit that even in this latter case, the state of the system must have changed.

As we said at the beginning of this paragraph, the world is a kind of theatre inside which all the events, including changes in the physical state of systems, take place. These changes can either be directly caused by the measurement (ontic interpretations) or can happen indirectly for non-considered reasons and are simply witnessed (epistemic interpretations). The important fact is that even in epistemic interpretations, a measurement must always be accompanied by a physical change in the state of the system.

A second question is the status of this change. Is it absolute (i.e., true for all the observers) or relative to one observer?

In a very interesting talk, Leifer [30] gives a sort of taxonomy of the interpretations that he calls "Copenhagenish" and that shares some resemblance between them and the Copenhagen school. He gives four principles that define Copenhagenish interpretations:

(1) Outcomes are unique for a given observer
(2) The quantum state is epistemic (information, knowledge, beliefs)
(3) Quantum theory is universal
(4) Quantum theory is complete (i.e., it does not need to be supplemented by hidden variables)

In this category he puts QBism, Healey's pragmatism, Bruckner's position, Bub's and Pitowski's information-theoretic interpretation [31] and Relational Quantum Mechanics [32]. These interpretations fall into two categories: objective ones and perspectival ones. The objective ones consider that the results obtained after a measurement, what observers observe, are facts about the universe. This is what I described above using the picture of the theatre. The perspectival ones consider that what is true depends on the observer. So, the result an observer gets is a result for her but not necessarily for another observer. As Leifer says: "*what is true depends on where you are sitting*". Healey's interpretation and Bub's and Pitowski's interpretation are objective while QBism and Relational Quantum Mechanics are perspectival (But see the very good comparison between QBism and Relational Quantum Mechanics made by Pienaar [33,34]).

The interesting point is that he shows that due to what he calls the Bell/Wigner mashup no-go theorem, Copenhagenish interpretations should be perspectival.

Convivial Solipsism belongs clearly to this family and is, as we will see, probably the most perspectival of all these interpretations.

3. Interpreting the Measurement of Entangled Systems

If, as assumed in objective interpretations, there is a real change in the state of the system after a measurement, then some strange consequences happen which are left aside too often. I have already detailed these problems [12–16] and will just summarize them here.

Let us consider the EPR experiment [2], where two half-spin particles A and B in a singlet state are measured by two spatially separated experimenters Alice and Bob:

$$|\psi\rangle = \frac{1}{\sqrt{2}}\left[|+\rangle^A|-\rangle^B - |-\rangle^A|+\rangle^B\right]$$

Assume that Alice completes her measurement first and finds a result. Then, Bob will surely find the opposite outcome. If Alice's measurement causes a collapse that is a real physical change. we must conclude that it causes instantaneously a collapse of the state vector of B, hence a physical change of B. As we know, thinking that the result is determined from the moment the particles separate is not allowed since Bell's inequality [2] forbids local hidden variables. So, there seems to be an instantaneous change in the state of B caused by the change in the state of A. However, if the two measurements are space-like separated, no one can be said to be before the other in an absolute way. For two observers moving in the opposite direction, Alice's measurement will be the first for one of them while Bob's measurement will be the first for the other. So, which one of the two measurements causes the result of the other? This seems to be violating special relativity. It is often said that it is not the case because it is here a question of mere correlation between two results and that correlation is not causality. Causality would violate special relativity but not correlation. However, this is not an acceptable reason since in statistics the precise reason for the difference between correlation and causality is the fact that a common cause can be invoked, which is here forbidden by Bell's inequality. Another way to accommodate the strangeness of the situation is to notice that it is not possible to use entangled particles to communicate at will a message faster than light. That is true but such a pair of measurements nevertheless brings a kind of information faster than light. Indeed, if Alice gets "+", she will instantaneously know that B has obtained "$-$". It is true that neither Alice nor Bob can use this process to communicate to the other something particular they have in mind, nevertheless, the information that Alice obtained "+" and Bob obtained "$-$" has been transmitted instantaneously. To illustrate the strangeness of the situation, assume that Alice (on Earth) and Bob (somewhere inside Andromeda Galaxy) have synchronized their watches and that they have agreed to throw a die each day at noon. If the two dice were falling each time exactly on the same face that would really be considered astonishing even if it is not possible to communicate that way. However, this is exactly what happens in the EPR context if we assume that a measurement causes a physical change in the system. Moreover, this is a way to synchronize different actions instantaneously between distant points. Assume that Alice and Bob have agreed on the fact that if the spin is "+" for Alice she will drink a cup of tea and if it is "$-$" she will drink a glass of wine (and the same for Bob). They will be able to perform exactly the same action at the very same instant even if they are at a distance of 1 billion light-years, and nothing has been decided before. Of course, Alice cannot decide by herself what she is going to do and then send the information to Bob but she is able to tell Bob instantaneously what she is doing. It is even possible to imagine more sophisticated protocols relying on several measurements to synchronize Alice's and Bob's actions among a set of possible ones.

Then, one sees that there is a problem in interpreting the collapse as a real change in the physical state of the system (This is only a part of the measurement problem, which comes essentially from the fact that inside the quantum formalism one cannot define rigorously what a measure is). Some physicists sweep things under the carpet and say that this problem cannot be properly discussed in non-relativistic quantum mechanics. Anyway, no satisfying explanation is given inside the objective interpretations, and the question of why, how, and when the collapse occurs is usually neglected.

4. Everett Interpretation

If we are to take seriously the idea that the universal wave function evolves in a unitary way and that there is no reduction, then we have to explain what the ontology of the world is and explain why we see a classical world that does not correspond to the superposition of results that the wave function represents. Everett's goal was an attempt to give an account of that.

Unfortunately, the proponents of Everett's interpretation are stuck with a classical view implying that the only existing entities can be classical worlds similar to our usual macroscopic world (even though they can differ from our own world by different results of experiments). Following Vaidman's [35] description:

> "The "world" in my MWI [Many Worlds Interpretation] is not a physical entity. It is a term defined by us (sentient beings), which helps to connect our experience with the ontology of the theory, the universal wave function. My definition is: A world is the totality of macroscopic objects: stars, cities, people, grains of sand, etc., in a definite classically described state."

This leads them to interpret the superposed wave function as many classical worlds as there are branches describing determinate results. They are looking for an ontology of worlds that are similar to our world (i.e., classical) and they cannot imagine that very different worlds could exist because they are stuck with the idea that "what exists" cannot be totally different from "what we see". So, if we take a simple example, let us consider the wave function of a particle in a superposition of two positions:

$$\Psi = 1/\sqrt{2}\, [x_1 + x_2]$$

The way proponents of Everett are led to interpret this state is that it describes two worlds, one where the particle is in x_1 and the other where the particle is in x_2. However, this interpretation is only due to their inability to imagine that the world could *really* be such that this superposition describes a world no less legitimate than a world where the particle has a defined position. So let us take seriously the idea that the superposed wave function describes a unique world that is really in this state. The question is then to explain why we see a determinate value of the position. Convivial Solipsism explains that what our consciousness sees is limited to classical things even if the world itself is not classical (See below (Section 5) for what I mean by "classical" and why we can only see classical things). Convivial Solipsism makes a clear distinction between what the world is and what we see from it. In this case, the artificial split in as many worlds as there are possible results is eliminated because it is no more needed. This solves also the puzzling questions attached to Everett's interpretation: When is the world supposed to split? Is it when a measurement is made? However, in this case, what is a measurement? Does that need the involvement of an observer? If not, is the world splitting every time there is an interaction between two systems? None of these questions has a clear answer, and the different supporters of Everett can even supply different answers.

Another big issue in this interpretation is the status of probabilities. Since all possible results happen, the very concept of probability disappears. In the universe made of all the possible worlds, there is no place for probabilities. Nevertheless, it is necessary to explain not only why we (our actual we) have the feeling that only one result happens (the reason is that each observer splits into as many observers as there are possible results) but also why the results we get seem to follow a probabilistic law in agreement with the probabilities given by the usual Born rule. There have been many attempts to try to justify the Born rule through decision theory, preferences, and so on . . . [36,37]. These attempts are not at all satisfying. As Vaidman [35] rightly says: "The postulate of the unitary evolution of the universal wave function alone is not enough".

At least, Vaidman has a coherent position when he says that this has to be a separate postulate added to the basic Everett interpretation:

> "What is the probability of self-location in a particular world? I claim that it has to be postulated in addition to the postulate of unitary evolution of the universal wave function and a postulate of the correspondence between the three-dimensional wave function of an observer within a branch and the experience of the observer. The postulate is that the probability of self-location is proportional to the "measure of existence", which is a counterpart of the Born rule of the collapse theories."

This postulate has exactly the same status as the Born rule in standard quantum mechanics or the probabilistic postulate I use inside the hanging-on mechanism of ConSol (see below). For me, this is the only way to give meaning to probabilities inside Everett's interpretation and I have nothing to say against it apart from the name "measure of existence" which I find meaningless. I also reject the "behavior principle" [38] that teaches us that one should care about one's descendants according to the measures of the existence of their worlds. It is also used by those who try to justify probabilities inside this framework, but I fail to succeed in giving any meaningful sense to it.

Vaidman [35] says that this interpretation is the only one allowing one to escape non-locality. However, this is not true since many others (QBism, ConSol, relational interpretation) do the same. More surprising is the argument he gives for justifying this claim. Usually, what we have to explain (or at least what we want to understand) is what we can observe, what is part of our actual experience. We are surprised when we observe something that comes into conflict with what we are accustomed to experiencing or with what our most confirmed theories predict. Then, a good explanation must concern our world, the world in which we live and not a virtual world that we cannot grasp (even if this virtual world is presented as embracing our usual world). If something that shocks us is predicted to happen in our usual world, we must either explain how that happens or why that never happens. Strangely enough, the way Vaidman tries to use the MWI interpretation to get rid of non-locality is the reverse:

> "A believer in the MWI witnesses the same change, but it represents the superluminal change only in her world, not in the physical universe which includes all worlds together, the world with probability 0 and the world with probability 1. Thus, only the MWI avoids action at a distance in the physical universe."

So, Vaidman speaks as if the fact that superluminal change in our usual world was not problematic since it does not happen in the domain of "all the worlds". However, no observer has any access to the domain of all worlds. What any observer can witness is by definition stuck inside one unique world. What is shocking is that a superluminal change could happen in the world where the observer lives. A good argument would be exactly the reverse: showing that such a superluminal change cannot happen in our world even though it could happen in the domain of all worlds.

Everett's interpretation seduces cosmologists because the universe is not a system that it is possible to consider from the outside, and so they want to get rid of the problem of having to involve an observer. However, strangely enough, the supporters of Everett's interpretation who want to derive the Born rule make large use of the decision theory and rationality to argue in favour of the fact that probabilities naturally emerge from formalism (I consider these attempts as largely not relevant and at least as unsatisfying). This is proof that Everett's interpretation cannot be defended as a theory that is totally independent of human observers (I have given more details elsewhere on the reasons why I do not agree with the Everett interpretation [13]).

5. Convivial Solipsism (ConSol)

Convivial Solipsism and its consequences have been presented in many articles [12–16,29]. I will here restrict myself to presenting the core philosophical ideas and will not enter into the mathematical details for which the interested reader is referred to the previous articles. Inside ConSol there is no collapse. As in Everett's interpretation, the unitarity of the evolution of the system is never broken. However, to understand how it is possible that observers nevertheless see definite results and not superposed ones, it is necessary to explain more precisely what reality is inside the framework of ConSol.

There are two levels of reality. The first level is what I call empirical reality. This is the underlying level, and it is described by a global and entangled wave function which encompasses all the systems that we want to study. The evolution of this wave function is unitary and given by the Schrödinger equation. There is no collapse, and this global wave function stays entangled forever. On this point, this is very similar to the no-collapse Everett's interpretation. The empirical reality contains the potentialities of all the systems we are considering. However, actually, we are unable to perceive the empirical reality as it is. Let us say that our brain (or our mind) is not equipped for that. When we look at the empirical reality, we can only perceive it partially. An analogy (very limited) could be to think of a fully colour-blind person who can see only shades of grey when she sees a coloured picture. She is unable to perceive the richness of the different colours. Another more interesting analogy is given by the spinning dancer, a kinetic, bistable, animated optical illusion (See: https://www.youtube.com/watch?v=BZevSglezAE, accessed on 28 February 2023). This is a video where some see the dancer spinning clockwise and others counterclockwise. Asking the question "which is the real direction?" is a meaningless question. The video is just a set of moving pixels that are neither rotating in one nor the other direction. It is our brain that interprets this set of moving pixels as a dancer rotating in a definite direction. Much in the same way, the empirical reality is constituted by entangled systems whose properties are not defined. It is our brain which selects one of the components of the superposition to consider that this is the reality. So when we look at a system described in the empirical reality by a superposed wave function (i.e., a wave function that is a linear combination of eigenstates of the considered observable), we see a definite value. This selection of one component is what is called the "hanging-on mechanism" in ConSol. This is similar to the fact that we see the dancer rotating in one definite direction.

Let us be more precise. According to the standard von Neumann description of the measurement of a system in a superposed state of an observable, the apparatus interacts with the system, and both become entangled. Going a step further and including the observer looking at the apparatus should lead to the fact that the observer becomes entangled too. Of course, this never happens and the reduction postulate is used to explain why the observer sees only one result. However, there is no clear rule to state either where the reduction occurs (Heisenberg/von Neumann cut) or when the reduction postulate should be used. This is the famous measurement problem. ConSol solution is that when the observer looks at the entangled big system (apparatus plus micro-system), she is unable to see the richness of the entangled state and she perceives only one of the components of the superposition: the observer hangs on to one component of the superposition. However, nothing happens either to the micro-system or to the apparatus and both stay entangled in the empirical reality. The perception of one component obeys the Born rule and the choice of the component is made probabilistically according to the coefficients of the superposition. Once a component has been chosen, the observer's perception is stuck to the branch of the entangled wave function that is linked to this component for all subsequent measurements (That is a complementary requirement of the hanging-on mechanism). Everything happens for this observer as if a projection had been done, but the way she perceives the entangled state (which has not changed at all in the empirical reality) is only in her mind and is restricted to one component.

It is pointless to wonder why only what we call classical states correspond to definite results. Asking this question assumes that there is an absolute definition of what a determinate result is, which is wrong. The question must be considered the reverse way. It is what we are accustomed to perceiving that we call definite results and classical states. Such a denomination is a posteriori and it is what we cannot directly perceive that we call superposed results. Aliens differently mentally oriented with brains differently designed could perhaps perceive as "classical for them" the states that we call superposed states.

What is important to understand is that in Convivial Solipsism a measurement is neither a physical action changing the state of the system nor an update of knowledge about the system. It is the adoption of a point of view allowing us to perceive the empirical reality in one of the possible ways (We will not here analyse here in detail this difficulty, but the possible ways are determined by the preferred basis that is chosen through the decoherence mechanism) and giving birth to the second level that I call the phenomenological reality, which is what we usually call reality. We live in our phenomenological reality. We have no direct access to the empirical reality that we cannot perceive as it is but which gives rise through our observations to our phenomenological reality that we usually take as the only "reality".

ConSol is a radical position in that it implies that the empirical reality that is described by the entangled wave function is to be taken seriously even at the macroscopic level. That means that in empirical reality, microscopic systems are entangled, but macroscopic ones, including measurement apparatus and observers, are also. However, we cannot witness this entanglement, which is revealed only through the choice of one component when we observe a superposed state to create our phenomenal reality. On this point I agree with Vaidman [35] when he says:

> "We, as agents capable of experiencing only a single world, have an illusion of randomness".

This point of view is private (i.e., accessible only to one observer). For one observer, the other observers are analogous to physical systems (or measurement apparatuses), which can be in superposed states. Communicating with other observers is exactly similar to looking at the needle of an apparatus. It is a measurement. So asking an observer who made a measurement on a system which result she obtained is the same as looking at the result given by a measurement apparatus. Before getting the answer, the other observer is entangled with the apparatus and the micro-system. It is only when she gets an answer that for the first observer, the system, the apparatus, and the other observer seem to be in a definite state. An observer has no means to share her "real" point of view with another observer. The consequence is that each observer builds her own phenomenological reality to which no other observer has any access. In the empirical reality, everything (including the other observer) stays entangled. In addition, taking into account the fact that we have no access to the other observer's point of view, it is meaningless to ask what the other observer "really" saw. This question is outside the scope of the phenomenological reality of the first observer. The only thing that can be said is that for the first observer, everything happens exactly as if the second observer has seen the result that the first observer hears when she asks the question.

So asking the question "what did the second observer really see?" or "is it possible that the second observer saw something different than what the first observer hears when she asks the question 'what did you see?'" is meaningless. In ConSol, a sentence is always relative to the observer who pronounces it. An observer cannot speak of the "real" perceptions of another observer since she has no access to her private perceptions. Similarly, it is forbidden to speak simultaneously of the perceptions of two observers (from a third-person point of view). Sentences such as "Alice saw the result 'a', and Bob saw the result 'b'" are forbidden. Then, questions that could come naturally such as "is it possible that Alice hears Bob saying he saw the result 'a' while in reality, he saw the result 'b'?" are forbidden as meaningless.

To use the words of Leifer, ConSol is perspectival in the maximum sense. There is no absolute truth, no global point of view shared by different observers. Everything is relative to one unique observer.

6. The Dissolution of the Problems

What has been presented above amounts to making a move towards a phenomenological approach to quantum mechanics. Actually, many quantum paradoxes arise when a comparison is made between several (at least two) observers' results. This is the case for the question of non-locality, and this is also the case for the well-known Wigner's friend [39] problem or its more recent version by Frauchiger and Renner [40] involving four observers, or Bruckner's version of it [41]. As Bitbol and de la Tremblay say in a recent paper [42]:

> "The dissolution of this family of "paradoxes" is based on the remark that "Bob's answer is created for Alice only when it enters her experience". As long as one compares the outcomes and predictions of agents from some "God's eye standpoint", discrepancies between them can (artificially) occur. And as long

> as experimental outcomes are dealt with as intrinsically occurring macroscopic events, or macroscopic traces of former events, comparing them from "God's eye standpoint" is a permanent temptation. However, if outcomes and predictions are compared in the only place where they can be at the end of the day, namely in the experience of a single agent at a single moment, any contradiction fades away, and even the need for mysterious actions (or passions) at a distance disappears. We can conclude from these remarks that, far from being the whim of some maverick physicists, the strict transcendental reduction to pure experience, the uncompromising adhesion to the first-person standpoint, is indispensable to make full sense of quantum mechanics by making its "paradoxes and mysteries" vanish at one stroke."

This quotation is given by these authors to support the QBist interpretation of quantum mechanics, but it applies perfectly well to Convivial Solipsism too since, despite many differences, the two interpretations share some similarities. In particular, both are perspectival.

Many so-called quantum paradoxes arise when one considers that the result of a measurement is objective, is an absolute fact. Actually, these paradoxes arise when a comparison is made between the results obtained by different observers. Such a comparison is natural in a realist framework where getting the result of a measurement by an observer is supposed to reveal a real event happening in a reality that is shared by all the observers.

Let us come back first to the EPR experiment. In a realist framework where the collapse is a real change in the state of the particle, the reasoning is as follows:

Alice measures the spin along Oz of A and Bob the spin along Oz of B. Alice makes her measurement and finds a certain result, say "+". Then, she knows that if she asks Bob which result he found after he did his measurement she will invariably get "−" even if Bob's measurement was space-like separated from Alice's one. Since we know that before the two measurements, neither the spin of A nor the spin of B was defined (this is forbidden by Bell's inequality), Alice must conclude that the measurement of A caused the value of A's spin and hence the value of B's spin instantaneously. Of course, if the two measurements are space-like separated, it is impossible to say in an absolute way which measurement has been made first, and Bob can also think that it is his measurement of B that caused the value of B's spin, and hence the value of A's spin, instantaneously. In this case, as we mentioned in Section 3, this is very strange because it becomes difficult to say that one of them is the cause of both results. This is one of the problems of trying to understand this experiment in a realist context. However, let us examine more carefully the way Alice draws the conclusion that her measurement of A caused instantaneously the determination of the spin of B. It is not from direct observation of the spin of B immediately after she measured the spin of A that Alice knows the spin of B. She knows it only after having communicated with Bob in a way which necessarily respects the speed of light limitation. However, in a realist context where the reality is the same for all the observers, Alice can naturally think that even though she received Bob's answer later, the result Bob reports has been determined at the very time Bob made his measurement. Hence, this is proof for her that the spin of B was determined as soon as she made her measurement of A whatever the distance between A and B was. This is non-locality.

In ConSol, the way to interpret these results is different. The sentence "Alice can naturally think that even though she received Bob's answer later, the result Bob reports has been determined at the very time Bob made his measurement" is no more true. We remind ourselves that for Alice, Bob is a mere physical system and that when Bob takes a measurement nothing more than an entanglement between Bob, the apparatus, and the system happens for Alice. So, the counterfactual reasoning allowing her to infer that the spin of B has really been "−" immediately after the moment she made her measurement is no longer correct. What Convivial Solipsism states is that, in agreement with the hanging-on mechanism, when Alice made her measurement, her awareness hung on to the branch "+" of the entangled wave function. However, nothing happened at the physical level either to A or to B. Asking Bob about his result is equivalent to measuring Bob. When she does that, in her future light cone, the hanging-on mechanism says that Alice can only get a result given by the branch she is hung on. That means that she can get nothing else but "−". However, that does not mean anymore that the physical state of B was "−" as soon as she took her first measurement. Alice's measurement is only the fact that her awareness hangs on to one branch of the entangled global wave function while there is no physical change. There is no non-locality anymore.

The famous Wigner's friend paradox dissolves in the same way. The usual way to present it is to consider two observers, Alice outside a laboratory and Bob inside the laboratory. Bob performs a measurement of the spin along one direction of a spin one-half particle in a superposed state of spin along this direction. He can find either "+" or "−". After the measurement, the system {Bob + particle} is in a definite state: let us say {Bob having obtained "+" and spin of the particle "+"}. However, Alice, who took no measurement and who only knows that Bob interacted with the particle, must use the Schrödinger equation and for her, the state of the system {Bob + particle} is a linear combination of {Bob having obtained "+" and spin of the particle "+"} and {Bob having obtained "−" and spin of the particle "−"}. The two points of view seem equally valid, hence the paradox. In ConSol, there is no paradox since the collapse has meaning only relative to one observer (through the hanging-on mechanism). So, when Bob makes his measurement he gets a result and in his own phenomenal reality, he sees a definite value of the spin. Hence, he must use the collapsed state. However, for Alice that is not a measurement. So she is right to use the superposed state until she asks Bob (which is a measurement for Alice) about the result he obtained. It is only after that that she gets a definite result and can use the collapsed state. So for Alice, the result Bob obtained remains undetermined until she asks Bob even if Bob is assumed to have made his measurement a long time ago. However, as soon as she gets an answer from Bob, the fact that Bob obtained this result becomes true at the very moment Bob did his measurement. This is a perfect example of what I describe in [16] where I show that past events are not necessarily determined. Past events can stay undetermined for an observer if they belong to a branch that is not linked to a branch that has already been selected by previous use of the hanging-on mechanism. They become defined as soon as a measurement selects one branch that is related to the branch they belong to. In addition, when such a measurement is made they become defined from the moment they are supposed to have happened in the past. I recall that asking if the result Alice hears when she asks Bob is the same as the result Bob obtained is not allowed in this context. That would be comparing results from a third-person point of view, which is precisely what is forbidden. More simply, it is enough to recall that the state vectors (and the observables) are relative to each observer to see that the paradox cannot occur.

The more sophisticated version by Frauchiger and Renner [40] involves four agents, two pairs of Wigner and his friend. The goal of the argument was to show that using quantum theory for modelling agents who are themselves using quantum theory leads to inconsistencies. The procedure is complicated but following it allows us to show that in certain cases, an agent can be certain both of a proposition and of its negation, which is inconsistent. To derive this conclusion, Frauchiger and Renner assume three basic hypotheses that seem natural in a standard realist context. The conclusion they reach allows them to say that at least one of these hypotheses must be abandoned. This is precisely what Consol does. Their rule C is that if an agent A has established "I am certain that another agent B is certain at time t that the result is α", then agent A must conclude "I am certain that the result is α at time t". This rule is not respected in ConSol since an agent cannot have any access to what another agent obtained when she did her measurement. Comparing the result that two agents obtained is forbidden. So, no inconsistency of this type can occur in ConSol.

The Frauchiger and Renner argument is interesting because it explicitly points toward the hypothesis that is the cause of paradoxes in quantum mechanics. The authors state the three assumptions they need and then arrive at the conclusion that to avoid an inconsistency it is necessary to abandon at least one of them. The two other hypotheses are difficult to relax. Rule S roughly states that an agent cannot prove simultaneously that a result α occurs and does not occur. This rule is purely logical. The first rule Q simply means that if according to the Born rule, one agent predicts that a result α has a probability of 1 at time t, then she can be certain that the result is α at time t. This is an immediate consequence of the Born rule (this consequence is not accepted by QBism). Rule C is more subtle and is exactly what is denied in ConSol. We cannot know what another observer knows. Hence, knowing something because we know that somebody else knows it is totally forbidden. In a certain sense, this argument leads naturally to ConSol.

This kind of no-go theorem is what leads Leifer to conclude that Copenhagenish interpretations have to be perspectival, which is the case of QBism, Relational Quantum Mechanics, Bruckner's position and of course ConSol. That seems to exclude Healey's pragmatism and Bub's and Pitowski's information-theoretic interpretation. That seems also to exclude Everett's interpretation since it is not perspectival. Inside each world, facts are absolute and real. Facts are absolute facts for the world in which the part of the split observer lives and in this world all the observers can share the same reality. Of course, it could be possible to object that Everett's interpretation is not Copenhagenish since it fails to respect the first principle of Copenhagenish interpretations: outcomes are unique for a given observer. However, actually, it depends on what we consider to be a given observer. Once we are inside one world everything happens as if this world was unique for the observer who lives inside. Then, it is perfectly possible to apply the same reasoning as the one that leads to Leifer's conclusion for strict Copenhagenish interpretations and to see that Everett's interpretation is found wanting.

7. Conclusions

Everett's interpretation and Convivial Solipsism are the only two interpretations which take unitary evolution seriously and where there is no collapse. However, in my opinion, Everett's interpretation is plagued with many issues (even if we set aside the problem of probabilities and adopt Vaidman's solution for postulating a kind of Born rule). If we take into account the questions raised by the various recent versions of Wigner's friend, it seems that a correct interpretation should be perspectival (at least a little). What I have said above, referring to Leifer's talk, is just a sketch of an argumentation aiming at proving that non-perspectival interpretations are disqualified. Of course, it remains to be analyzed if Everett's interpretation could really fit inside Leifer's taxonomy as he did not mention it. A more detailed analysis is necessary and that will be part of a forthcoming work. However, ConSol is solving many paradoxes without facing embarrassing issues and is perspectival in the maximal sense. Of course, this last feature can seem a big price to pay. We need to abandon our usual picture of the world. Reality is entirely relative to each observer, and there exists no absolute reality that could be shared by all observers. Despite the provocative name I gave to it, Convivial Solipsism is not at all a solipsistic interpretation. It allows for the existence of all the observers and does not pretend that the reality of an observer is created by her brain. It relies on an empirical reality from which each observer builds her own phenomenological reality. In this sense, it is a kind of realist interpretation even if the concept of reality is profoundly different from the usual one.

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quantum reports

Opinion

The Everything-Is-a-Quantum-Wave Interpretation of Quantum Physics

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Abstract: In this paper, I would like to outline what I think is the most natural interpretation of quantum mechanics. By natural, I simply mean that it requires the least amount of excess baggage and that it is universal in the sense that it can be consistently applied to all the observed phenomena, including the universe as a whole. I call it the "Everything is a Quantum Wave" Interpretation (EQWI) because I think this is a more appropriate name than the Many Worlds Interpretation (MWI). The paper explains why this is so.

Keywords: quantum; waves; many worlds interpretation

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1. Introduction

Let me dive straight into explaining what I have in mind. According to quantum physics, everything is actually made up of waves, but these are quantum waves (or q-waves for short), meaning that the entities that are doing the waving are what Dirac called q-numbers (as opposed to the ordinary c-numbers, "c" being classical). Mathematically, this entails having a set of (generally non-commuting) operators specified at every point in space and at every instance of time. These operators satisfy one wave equation or another, in other words they causally propagate at some finite speed (light or otherwise). This q-wave picture emerged through the work of Heisenberg [1], Jordan [2], von Neumann [3,4], Mott [5], Darwin [6,7], Schrödinger [8], Everett [9] (all standing on the shoulders of Hamilton), and many others [10–14] who have all more or less reached the same conclusion.

Let me explain a bit more how everything is a q-wave and why this presents us with the best picture of reality at present. First, there was a problem. Remember that before quantum physics, we had two fundamental entities in the world, waves and particles; however, quantum physics unified the two notions into one, leading to the well-known wave–particle dualism. However, if, according to quantum physics, particles are waves, the key phenomenon to explain in the 1920s was the observation of the alpha-particle decay in a cloud chamber. This experiment seemed to present a paradox for quantum physics.

An alpha-particle is a Helium nucleus (two protons and two neutrons) and it sometimes gets ejected in the nuclear decay of a larger nucleus. A cloud chamber was a great invention in which to observe such particles (worthy of several Nobel Prizes), though nowadays you can make one in 15 min in your own house with the usual kitchen utensils (there are many YouTube videos on this). The idea, as the name (cloud chamber) suggests, is to have a particle travel through a gas that can readily be ionized by collisions with the particle (thereby creating a cloud). As the particle collides with the gas molecules, it ionizes them in succession. Ionisation attracts neighbouring gas which condenses around the ionized molecules. Therefore, the travelling and colliding particle leaves a track of condensed vapour in its wake. To date, so good, but the problem was that the tracks are always straight lines. If, as quantum physics suggests, everything is a wave, why do we get straight lines from alpha-particles? Why not concentric circles, just like waves spreading in a pond when we throw a stone in it?

Heisenberg was the first person to explain this using their Uncertainty Principle. The emitted alpha-particle, he said, starts out as a wave, but the first molecule it hits localizes it to a small region of space (roughly the size of that molecule). In other words, this collision acts like a measurement of the position of the alpha-particle. However, the more accurately the position is determined (i.e., the more strongly the alpha particle is localized), the less determined is its momentum and, so, subsequently, the alpha-particle starts to spread as it travels, just like a wave would. However, the next collision comes pretty soon as the gas in the chamber is dense. This again focuses the particle's position reducing its position uncertainty. From there onwards, the rapid chain of collisions with the gas acts like a sequence of measuring devices that do not allow the alpha particle to spread out like a wave. It therefore leaves a straight track just as a particle would!

Mathematically, this is very simple to understand through Heisenberg's "matrix mechanics". The basic classical formula for particles motion with no forces acting on it is $x(t) = x(0) + p/mt$. Quantum mechanically, x and p (but not t and m) are operators (this was Heisenberg's route to quantum physics: keep the classical dynamics, but reinterpret some quantities algebraically differently). Taking the commutator with $x(0)$ on both sides yields:

$$[x(t), x(0)] = [x(0), x(0)] + \frac{[p, x(0)]}{m} t \,. \tag{1}$$

Recalling that $[p, x(0)] = i\hbar$, leads us to conclude that:

$$[x(t), x(0)] = \frac{i\hbar}{m} t \,. \tag{2}$$

We note that the later position commutes less and less with the initial position under free evolution. This implies that

$$\Delta x(t) = \frac{\hbar}{m \Delta x(0)} t \,. \tag{3}$$

In other words, the trajectory of a free particle spreads out with time. In this sense, particles in quantum mechanics behave just like waves in classical physics; they diffract (and interfere). However, if the particle suddenly becomes localised through interaction with the gas, the spreading then restarts and so long as the time between the collisions is not too large (so that $\Delta x(t) \approx \Delta x(0)$), this process clearly leads to a straight trajectory. In the optical wave parlance, this is like having a laser beam of light broadening as it propagates but then encountering a sequence of slits, each of which re-focuses it and narrows it down to its original size.

2. The Core Argument

It is a magical explanation of how particle-like behaviour arises from waves, and it seems to make sense. However, in 1929 Mott went even further. He actually set the scene for the Many Worlds Interpretation of quantum mechanics (which, as I advocate here, should actually be called EQWI). Mott said that a single particle simultaneously traverses all the tracks in all physically allowed directions, meaning that all possible trajectories exist in a superposition and at the same time. Even though a single alpha-particle takes all the paths simultaneously and in all the directions, when we look at it, we can only see one of these trajectories. Darwin actually realised this even before Mott [15] as is clear from the following statement [6]: "so without pretending to have mastered the details, we can understand how it is possible for the ψ function, so to speak, not to know in what direction the track is to be, but yet to insist that it should be a straight line. The decision as to actual track can be postponed until the wave reaches the uncovered part, where the observations are made".

It is simpler to model this in the Schrödinger picture of quantum physics, which is how Mott himself (as well as Darwin) approached the problem. The total wave-function (un-normalised) for the alpha particle and the gas is given by

$$|\Psi\rangle = \sum_n |\alpha_n\rangle|\xi_n\rangle \tag{4}$$

where α_m is the m-th trajectory of the alpha particle and ξ_m is the state of the excited atoms of the gas along that trajectory. The fact that when one atom is excited the probability is high for the next excited atom to lie on a straight line connecting the two follows from Huygens' principle (which applies to wave-functions in quantum physics, and operators in quantum field theory, the same way that it applies to waves in classical optics—this is because the equations that quantum waves obey are the same as classical waves, it is just that—in quantum physics—the entities that obey them are q- instead of c-numbers).

However, and this is the crux of the matter, making an observation can also be described with quantum waves, so everything is unified and consistent. The reason for the fact that we only see one trajectory at a time is that even though everything is a q-wave within which things exist at the same time, when we interact with this q-wave we can only reveal some of its aspects, one at a time (this is where Heisenberg's Uncertainty comes from). We ourselves are also a collection of q-waves and it is when our q-waves correlate with the q-waves of the alpha-particle that c-numbers emerge. These correlations between q-waves are called quantum entanglement and so the classical world owes its own existence to quantum entanglement. The entangled state in Equation (4) clearly illustrates this point.

This kind of logic works at all levels and there is never any need to introduce ad hoc assumptions, such as that of a "spontaneous collapse" (which, in addition, also leads to irreversible dynamics, contrary to quantum physics). In quantum physics, even a collision between two particles is actually described as an interaction between two q-waves. This constitutes our most accurate description of nature, called quantum field theory. A particle in this theory is just one stable configuration of the underlying q-wave (or, a single excitation of the quantum field, in a more formal language of quantum field theory).

In fact, quantum field theory is the ultimate expression of the view that everything is a quantum wave. The alpha particle experiment does not need the full quantum field theory since all the particles involved are stable throughout and we need not consider their creation and annihilation. We could have completed the analysis with the full quantum field theory formalism, which would entail treating the wave-function as a field operator, but this would just have been an unnecessary overkill (actually, the whole of quantum field theory could also be performed in the Schrödinger picture, in which case the states of fields become functionals; this fact, however, does not change the logic of my argument). Mathematically, the treatment is no more complicated than solving the Schrödinger equation in the first place.

The bottom line is that reality emerges from interactions of q-waves with other q-waves. There is also no need to introduce a special classical measurement apparatus, or conscious observers or anything like that. Schrödinger, in lectures given towards the end of their life [8], clearly spelt out the same picture of quantum physics according to which everything is a q-wave. He advocated this view not only because it avoids the confusion arising from the dualistic wave–particle language (since particles are of secondary importance, being as they are specific excitations of q-waves) but also because it contains no collapses of the wave-function, no abrupt discontinuities due to measurements and no quantum jumps (as I said, the quantum wave interpretation has the least amount of excess baggage; Schrödinger was particularly keen to avoid quantum jumps, about which he said that if they turned out to be true he had wished he was a plumber and not a physicist).

Everett usually receives the credit for promoting the picture in which the whole universe is quantum and measurements are just entanglements between different quantum systems, however, as I have argued, many other physicists reached the same conclusion well before them (as the famous cat thought experiment testifies to, Schrödinger's did so some 20 odd years before Everett). Everett emphasized the relative nature of quantum observations, meaning that relative to my state of being happy, the state of the cat is alive, while—at the same time—there is another simultaneously existing branch (you can also call it a path or a track or what-have-you) of the quantum state in which the cat is dead and I am sad. These two branches are orthogonal, but they could—at least in principle—be interfered, which is how we test their simultaneous existence. Without this interference, each branch has their own "classical" reality and one would never know that they existed in a superposition unless one was able to perform interference on them.

In the modern jargon, when one system maximally entangles to another, both systems lose coherences in their respective bases that become correlated to each other. This loss of coherence is known as decoherence. Decoherence is not another phenomenon that needs to be added to quantum physics in order to explain the emergence of classicality. It is already contained within quantum physics and emerges naturally whenever there is interaction.

So why EQWI instead of MWI? Precisely because the state of the universe where we can talk about the worlds is just a limiting, special case of EWQI. The worlds only emerge fully when we have fully orthogonal states of observers (i.e., the quantum systems performing measurements, and measurements are—in this interpretation—just entangling unitaries with other systems). Otherwise the classical reality is only approximate. To a high degree of accuracy, each alpha particle tract is orthogonal to every other one, which means that you can think of them as different worlds. This state is analogous to the Fraunhoffer, or far-field limit in wave optics. At the other end is the Fresnel, or near-field limit, and it would correspond to the quantum state that does not allow us to talk about the separate worlds since different branches have a high degree of overlap. However, we know that they exist at the same time because all these paths could—at least in principle—be brought together to interfere. The possibility of being able to interfere different worlds is crucial to this view and leads to the fact that the "unobserved outcomes can affect future measurements" as Deutsch's version of Schrödinger's cat experiment has taught us. I have written about this elsewhere [16–18], and recommend it to the interested reader (see also [19,20]).

This way of thinking about quantum physics, namely that everything is a quantum wave, a quantum field whose relevant q-numbers can be specified at every point in space and at every instance of time, automatically inherits one important feature of classical field theory. Quantum fields too (just like classical fields) can be constructed so as not to allow action at a distance, i.e., using fields enables us to keep the principle that all interactions are local in space (i.e., no interaction takes place instantaneously at a distance). In this sense, the EQWI is as local as Maxwell's electrodynamics, even though the elements of reality in quantum physics, the q-numbers, are very different from their classical counterparts.

It is sometimes said that this quantum wave-like view of reality is incapable of explaining the origin of probabilities since everything is always seen and phrased only at the level of amplitudes. However, this is not true and both the single-shot notion of probability (such as in the notion of the "degree of belief") as well as the frequentist one (such as is obtain in a ensemble of identically prepared quantum systems) can be derived from the quantum waves. The point being that different probabilities are emergent, derived notions when one takes the quantum waves as the primary entities. In this sense, even a single "click" in a photodetector is an extraordinarily complex phenomenon if one wants to reduce it to the interactions between quantum fields (which can be performed, though, in practice, there is hardly even a reason to do so). The current exposition is clearly not the place to go into these details and the interested reader is referred to [21] and references therein.

3. Conclusions

Finally, the everything is a q-wave interpretation is uniquely quantum, but I would like to conclude by explaining how its existence owes everything to Hamilton's version of classical physics. Hamilton died well before the birth of quantum physics, so how could he have anticipated all this? This is because Hamilton discovered an ingenious way of doing Newtonian physics that ultimately paved the way to quantum physics.

Hamilton thought of particles moving in straight lines as rays of light moving in a uniform medium. When a force acted on a particle to change its direction of motion, this was for Hamilton analogous to light entering a denser medium and refracting (bending). Hamiltonian mechanics therefore uses the methodology of waves (things like wavefronts, rays, refractive indices, etc.) to describe the mechanics of particles (things like trajectories, forces, accelerations, etc.). If we could resurrect him, Hamilton would have no problem understanding the alpha-particle tracks in a cloud chamber. In fact the relationship between their wave and particle mechanics is the classical analogue of the relationship between EQWI and MWI. The "only" thing he was missing at the time were the q-numbers. There simply was no need for them (i.e., no experimental evidence, such as a particle in a superposition of different locations, to force us to use them) prior to the twentieth century. Otherwise, Hamilton would have probably written down the Schrödinger equation some fifty years before Schrödinger. All of the challenges that we are facing when trying to understand quantum physics are related to the fact that the fundamental entities are q-numbers, and that, unlike the c-numbers, they do not correspond to individual measurement outcomes. The classical world of c-numbers is a consequence of quantum entanglement.

It is also in the spirit of Hamilton that we can phrase quantum dynamics in a timeless way. Namely, we could just use the wave-function of the universe written in 3-space and think of different elements in this superposition as different times [22]. One might call this picture of "different universes being different times" the ultimate expression of the EQWI. However, no matter how we choose to represent the evolution of quantum waves, the interpretation advocated here remains valid for all of them.

I hope I have convinced you that this is the most natural picture of the universe we have at present. I don't for one second believe that it is our final picture. What lies beyond is, of course, wide open, and we have to wait for the next theory of physics to be able to talk about its interpretation. The next theory of physics will have to contain quantum physics as a special limiting case which means that the q-waves are here to stay with us and whatever notion extends and replaces them in the new theory will be at least as weird, but more likely much weirder than the operators we have at present.

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Article

The MWI and Distributive Justice

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Abstract: Everettians generally argue that their view recommends just the same rational choices as orthodoxy. In this note, however, we will show that Everettians should advocate non-standard choices in one specific kind of situation, namely situations where different people have unequal claims to an indivisible good.

Keywords: many worlds; distributive justice; Everett; quantum mechanics; rational choice

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1. Introduction

Defenders of the Everettian view typically hold that their strange metaphysical commitments make no difference to how we should live our lives. Everettians should proceed as normal when they make decisions. They should aim to maximise expected utility across all physically possible futures, weighted by those futures' probabilities, or intensities, or caring measures, or anyway squared wave function amplitudes. It does not matter what the quantities are called, argue Everettians, given that they have reason to attach the same numbers to envisaged futures as everybody else.

This might well hold true in general. In this note, however, we will show that Everettianism dictates unorthodox behaviour in one specific kind of situation, namely situations where different people have unequal claims to an indivisible good. In such situations, we shall argue, Everettian agents should act differently to non-Everettians.

In what follows, we shall assume the "fission programme" version of Everettianism [1]. In this version, which was originally adopted by Everett himself, and is endorsed by perhaps the majority of his followers, any quantum "collapse" is followed by the macroscopic objects involved, including any observers, "splitting" in a way that results in actual "branches" for all outcomes with a non-zero probability.

2. Normal Probabilities and Utilities

Before proceeding, let us put one issue to one side. Huw Price argued that a concern with fairness should lead Everettians to reject the Born rule that dictates the standard probabilities for outcomes in chancy quantum situations. If all the possible selves who experience such outcomes are equally real, argues Price, you should not favour the interests of the high-probability ones over the low-probability ones, but should treat them all equally. It would be unconscionable, argued Price, for an Everettian to deliberately engender a real successor who dies miserably in a plane crash, just to allow another successor to enjoy a few days on a sunny beach [2].

David Wallace responded that Price's underlying reaction might well be a rational response to Everettianism, but that it is no argument against Everettians respecting the Born rule. Much of Price's concern, said Wallace, can be met by reducing the positive utility attached to a few days' holiday, by comparison with the negative utility attached to an untimely death, and perhaps the Everettian realization that the death is sure to be real on some branch of reality should indeed encourage such a change of attitude. However, these utility adjustments can leave the Born probabilities as they are. Moreover, highlighted Wallace, no assignment of probabilities to chancy outcomes is going to ensure equal treatment of all successors, unless agents avoid all risky choices, and he goes on to present further arguments that no systematic deviations from the Born rule can be coherently sustained [3].

We think that Wallace is right about all this, but we need not assume this here. Our point is quite different from Price's. In the contexts we shall consider, all the probabilities and utilities at issue will be ones assumed both by orthodoxy and by Everettians. We shall show that, even so, in these cases, Everettian metaphysics advises different choices to orthodoxy.

3. Unequal Claims

Suppose Ann and Bill both suffer from a painful disease that lasts a year. Ann's suffering is worse. Let us say her degree of pain over a week is six, to Bill's four. (As will become clear, our general form of argument will not depend on the precise numbers).

Each week, just one dose of a drug that counters the disease is available. Ann can be rendered free of the pain for a week, or Bill can, but not both.

What is the best policy? One option (let us call it *All-Weeks-Ann*) would be to give Ann the drug every week. Clearly, that would maximise the cross-individual aggregate pain relief. After all, if Bill and not Ann were to receive the drug for any week, we would only alleviate four degrees of pain rather than six.

Still, nearly everybody will regard *All-Weeks-Ann* as less than ideal. It is unfair to Bill. He deserves some consideration too. He should have, at least, some weeks free of pain. So, some *Mixed-Weeks* option would seem better than *All-Weeks-Ann*. For example, we could give Ann the drug on just 60% of the weeks, and Bill on the other 40%. (Again, the precise number does not matter. Let us assume, for the sake of the argument, that the optimal mixed strategy is the 60-40 one).

There are issues in this situation about the commensurability of pain relief and fairness. However, let us not complicate things. Our points are most easily made if we assume some common utility scale for both.

As far as aggregate pain relief goes, *All-Weeks-Ann* outscores our assumed optimal *Mixed-Weeks* by 312 to 270.4 units of value over the 52 weeks of the year (*All-Ann* scores 52 × 6 = 312 for pain relief over the year. As to *Mixed-Weeks*, on 60% of the 52 weeks, it delivers six units of pain relief, and on 40% it delivers four. In total, that is (0.6 × 52 × 6) + (0.4 × 52 × 4) = 270.4).

However, we are assuming that *Mixed-Weeks* will outscore *All-Weeks-Ann* by more than that on fairness. To give it a number, let us suppose that the extra fairness of *Mixed-Weeks* adds 100 units of value. Then, all things considered, *Mixed-Weeks* outscores *All-Weeks-Ann* by 58.4 units of value.

4. An Indivisible Good

Now we turn to a variant case—a once-off one-week problem. The pain will only last one week for both Ann and Bill, and there is just one dose of the drug available for that week.

Clearly, the maximum pain relief will be delivered if we simply administering the drug straight to Ann—*This-Week-Ann*. That will deliver six units of pain relief, rather than the four that we would obtain by administering it to Bill.

Even so, some theorists think that *This-Week-Ann* would be unfair too. They say that, even in the once-off case, we should mimic the *Mixed-Weeks* strategy by holding a *lottery*. We should draw straws—the *Lottery-Mixing* option, let us call it—with the result that Ann has only a 60% chance of receiving the drug, and Bill has a 40% chance. That would be much fairer, they say, and this fairness would, as before, offset the prospect that we might fail to maximise the aggregate pain relief. (See, for example [4]).

In truth, however, the case for *Lottery-Mixing* over *This-Week-Ann* is highly uncompelling (at least as long as we assume orthodoxy over Everettian metaphysics). As far as pain relief goes, *Lottery-Mixing* lags behind *This-Week-Ann* by an expected 5.2 units to 6.

(There is a 40% chance that we will only alleviate Bill's four degrees of pain rather than Ann's six). What is supposed to compensate this pain-relieving inferiority to make us end up preferring *Lottery-Mixing*?

Well, we could say that, in addition to the issue of actual pain relief, on which *Lottery-Mixing* clearly lags behind, there is the point that *Lottery-Mixing* does definitely give Bill a 40% chance of pain relief. Additionally, you might think that this is where the extra fairness lies. At least Bill's chance of pain relief is proportional to his entitlement, and that is what makes *Lottery-Mixing* better than *This-Week-Ann* overall.

Still, what good are chances in themselves, when abstracted from their realizations? I make a bet that gives me a 90% chance of winning £100. However, the stupid horse fails to come home. Has anything good happened to me? Surely not. I wanted money, not a high chance of money. The chance without the money will not put food on the table. Chances, as such, butter no parsnips.

By the same coin, we should reject the idea that the chances being shared appropriately between Bill and Ann can compensate for *Lottery-Mixing*'s failure to maximize pain relief. It is not similar to the extended week-by-week *Mixed-Weeks* strategy. In that case, the overall loss of aggregate pain relief was balanced by the extra fairness Bill receiving some real pain relief alongside Ann.

However, in *Lottery-Mixing*, the supposed extra fairness dividend is bogus. Giving Bill a 40% chance of pain relief does not in itself add anything worthwhile to *Lottery-Mixing*'s negative impact on aggregate pain relief. As stated before, nobody cares about chances as such, but only actual outcomes. Additionally, *Lottery-Mixing* is clearly behind on that score. (Cf [5]).

(Do not be distracted by the attractiveness of a lottery in a once-off case where the entitlements are 50-50. In that situation a lottery will all least have the advantage of ensuring the drug is allocated impartially and not on some improper basis. However, this rationale falls away as soon as the pain symmetry is broken. There is nothing improperly partial about giving the drug straight to Ann because her need is greater).

5. Everettian Fairness

However, let us now bring in Everettian metaphysics. Now, the case for *Lottery-Mixing* looks quite different. Assume we have Ann and Bill in the once-off one-week situation of the preceding section. And now we hold a genuinely quantum lottery which gives Ann a 60% chance of the drug and Bill 40%. This will mean a future with two branches, on each of which both Ann and Bill have successors. On the first 60% branch, Ann's successor receives the drug and Bill's does not; on the other 40% branch, Bill's successor receives the drug and Ann's does not.

Now we have genuine fairness. We have managed to spread the actual pain relief over Ann's and Bill's successors, in proportion to their relative entitlements. *Lottery-Mixing* will still mean that the total amount of pain relief on weighted average across the branches is less than we would obtain from *This-Week-Ann* (5.2 to 6, as before). However, this loss of aggregate pain relief can now be compensated by some genuine extra fairness, just as it was for the *Mixed-Weeks* strategy in the year-long case (We originally offered this argument in favour of Everettian lotteries in Rowe and Papineau 2022 [6]).

Think of it in the following way. On orthodoxy, a lottery assigns the good proportionately over two *possible* futures (Ann wins, Bill wins). However, only one of these futures is actual, and that is the one where we want our actions to produce the best result. This means administering the drug to Ann, whatever any lottery says. If we end up administering the drug to Bill in the actual future because he won the lottery, we have simply done the wrong thing. Imagine then saying to Ann, in explanation of why we did not help her, "But don't forget the lottery fairly gave you a 60% chance of the drug". Ann will quite rightly respond, "What good are mere chances to anybody? Sure, there is a *possible* Ann who benefits from that chance. But why bring her into it? Morality is about real people, not possible people, and you could have ensured the best for real people simply enough by giving the drug straight to me".

On the Everettian view, by contrast, both the possible futures are actual. A lottery assigns the drug proportionately over actual worlds, not merely possible ones. Additionally, this should change our attitude. In effect, Everettianism renders the once-off drug divisible, after all. We can share it across the future, just as we did in the year-long case. True, we are not now administering it to Bill rather than Ann in different weeks of the year, but rather in different versions of this coming week. However, the upshot is the same. We have genuinely spread the pain relief across both Bill and Ann, and so, achieved the kind of fairness that can compensate for loss of aggregate pain relief.

6. Conclusions

It should be clear that the analysis we have given is of general significance. Everettian rational choices will diverge from orthodoxy across a wide range of situations. Our issue arises, in any case, where different people have unequal claims to an indivisible good. On orthodox metaphysics, there is a compelling argument that, in such cases, the good should simply be administered to the person with the greatest claim. However, Everettianism implies that, in such cases, allocating the goods by an appropriately weighted lottery will always be fairer.

We should note that it is not to be taken for granted that, in all such cases, the extra fairness delivered by an Everettian lottery will outweigh the diminution of aggregate benefit. In our examples, we assumed that this will be so. However, in real life, this will depend on the details of the unequal claims and of the lottery. Still, it is uncontentious that extra fairness does sometimes compensate for a loss of aggregate benefit, as is shown by the way orthodox metaphysics regards the just distribution of divisible goods. Everettianism significantly expands the class of cases in which such compensation is available.

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Article

The Open Systems View and the Everett Interpretation

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Abstract: It is argued that those who defend the Everett, or 'many-worlds', interpretation of quantum mechanics should embrace what we call the general quantum theory of open systems (GT) as the proper framework in which to conduct foundational and philosophical investigations in quantum physics. GT is a wider dynamical framework than its alternative, standard quantum theory (ST). This is true even though GT makes no modifications to the quantum formalism. GT rather takes a different view, what we call the open systems view, of the formalism; i.e., in GT, the dynamics of systems whose physical states are fundamentally represented by density operators are represented as fundamentally open as specified by an in general non-unitary dynamical map. This includes, in principle, the dynamics of the universe as a whole. We argue that the more general dynamics describable in GT can be physically motivated, that there is as much prima facie empirical support for GT as there is for ST, and that GT could be fully in the spirit of the Everett interpretation—that there might, in short, be little reason for an Everettian not to embrace the more general theoretical landscape that GT allows one to explore.

Keywords: open systems view; Everett interpretation; general quantum theory of open systems; many worlds

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1. Introduction

The so-called 'measurement problem' of quantum theory, as it is commonly framed, runs as follows. (See, e.g., [1], p. 63. Note that this is not the only way to approach the question of how to make sense of measurement in quantum theory. Another approach, which allows one to make finer distinctions among the various interpretations of the formalism, identifies two separate problems: (a) the problem of accounting for the, according to quantum theory, objective indeterminacy associated with a given measurement context, and (b) the problem of accounting for the mutual incompatibilities that exist between the various possible measurement contexts associated with a given dynamical system. For more on this, see [2], pp. 10–12, 223–225. For our purposes, it will be sufficient to frame the measurement problem in the more traditional form). On the one hand, in the absence of a measurement, the dynamics of a given quantum system are unitary according to the theory. On the other hand, given a measurement, the apparent 'collapse' of the vector representing a system's dynamical state, in accordance with the Born rule, is, in general, non-unitary. Positing, via the Born rule, a special measurement dynamics over and above the standard unitary dynamics is widely regarded as ad hoc, however. After all, from a physical point of view, measurement interactions are just dynamical interactions like any other. In a sense, this is true even according to quantum theory since a quantum description of a measurement interaction can be given in unitary terms to any desired level of detail given an appropriate placement of the so-called 'Heisenberg cut', on one side of which lies our quantum description of a measurement interaction, and on the other side of which lies our classical description of our observation of its result. Stated in these terms, the measurement problem is that of closing this gap, either by proposing a new theory to explain the connection between them or by showing that, upon reflection, it is already fully explained by quantum theory. The Everett, or 'many-worlds', interpretation of quantum theory chooses the second option. (Note that Everett is not the only interpretation that chooses not to supplement the theory. For examples of other contemporary interpretations of this kind, see [2–9]. For further discussion of such interpretations in the context of open systems, see [10]).

In the form of quantum theory presented in most textbooks on the topic—what we will be calling *standard quantum theory* (ST)—the physical state of a system, $\mathcal{S}$, at a time t is represented by a normalized state vector, $|\psi(t)\rangle$, one element in a Hilbert space, $\mathcal{H}_{\mathcal{S}}$, representing $\mathcal{S}$'s possible states. $\mathcal{S}$'s dynamical evolution is given by

$$|\psi(t)\rangle = U(t)\,|\psi(0)\rangle, \tag{1}$$

where the time evolution operator $U(t) = \exp(-\mathrm{i}\,H\,t)$, with H being a Hermitian operator representing the system's Hamiltonian whose eigenvalues are the possible energy values of the system.

ST is not itself a particular physical theory. It is an abstract theoretical framework within which various particular relativistic and non-relativistic quantum theories may be expressed [2,10–12]. The particular form of the dynamics of a given class of systems is the purview of the particular quantum theory that pertains to that class. However, there are a number of things that can be said about every dynamical model of ST irrespective of the particular quantum theory that model has been formulated in. Here, we note the following two. (I) It follows from the above assumptions that $U(t)$ is a unitary map on the state space of $\mathcal{S}$. (II) S is a closed system. This follows from the fact that H, which describes the possible energy values of the system, includes no terms representing its interaction with an environment. We will call any theoretical framework, such as ST, in which all phenomena are modeled as closed systems a framework formulated in accordance with the *closed systems view* [10].

ST is a highly successful theoretical framework that has been used to study all sorts of phenomena, even though many of these—so-called 'open systems phenomena'—cannot be modeled in a simple way as closed systems. Lasers, which are pumped by an external energy source, are an example. The spontaneous emission of a photon by an atom is another. In cases such as these, one models the phenomena in terms of a dynamical coupling between the system of interest and an idealized representation of its environment, $\mathcal{E}$, such that the combined state of $\mathcal{S} + \mathcal{E}$, which will be entangled in general (although one usually assumes that $\mathcal{S}$ and $\mathcal{E}$ are initially uncoupled or only weakly coupled), is given by the state vector $|\Psi^{\mathcal{S}+\mathcal{E}}\rangle$.

It is not possible to represent the state of a subsystem, such as $\mathcal{S}$, of an entangled system using a state vector in ST. However, there is a probabilistic generalization of the state vector,

$$\rho = \sum_i p_i |\psi_i\rangle\langle\psi_i|, \tag{2}$$

where ρ is called the *density operator* associated with the system and the p_i are interpreted as probabilities with $\sum_i p_i = 1$ that can be used to represent $\mathcal{S}$'s probabilistic state, though it will not, in this case, be possible to interpret Equation (2) literally, i.e., as representing that $\mathcal{S}$ is in the 'pure state' $|\psi_i\rangle\langle\psi_i|$ corresponding to the state vector $|\psi_i\rangle$ with probability p_i (since a subsystem of an entangled system can never be in a pure state). For the purposes of providing a general probabilistic description of the dynamical phenomena that we associate with $\mathcal{S}$, however, the density operator completely suffices in the sense that every probability measure over yes-or-no questions concerning the observable quantities associated with any system is representable by means of a density operator acting on that system's state space [13]. (Note that Gleason's theorem [13] represents measurements as projections and is valid for Hilbert spaces of dimension ≥ 3. See Busch [14] for a generalization).

Given that $\mathcal{S}$'s initial (probabilistic) state is described by the density operator $\rho^{\mathcal{S}}(0)$, to derive the (generalized) dynamics of $\mathcal{S}$ up to some time t, one first calculates the dynamical evolution of the combined state, $|\Psi^{\mathcal{S}+\mathcal{E}}\rangle$, from the initial time until t (in the case of non-relativistic quantum theory using the Schrödinger equation, for instance). One then takes the partial trace over the combined state with respect to $\mathcal{E}$, yielding a reduced density operator, $\rho^{\mathcal{S}}(t)$, representing $\mathcal{S}$'s final (probabilistic) state. The above procedure may also be effectively and conveniently represented as an in general non-unitary dynamical map, Λ, mapping density operators in the probabilistic state space of $\mathcal{S}$ to other density operators, though it must not be forgotten that such non-unitary evolution is always only effective in ST. Since a system's dynamics is always modeled as generated by its associated Hamiltonian and is unitary, it follows that non-unitary dynamics must always be taken to be a mere manifestation of an underlying unitary dynamics, in this case of the larger system $\mathcal{S} + \mathcal{E}$. (Note that Cuffaro and Hartmann [10], from whom we have taken the terminology "ST" and "GT" (to be introduced in the next section), discuss a number of different senses of fundamentality relevant to the concepts of a given theoretical framework, as well as between frameworks. What we mean by fundamental here is what they call OntFund-O, the relation of 'ontic fundamentality' within a given framework. A given concept is fundamental, in that sense, in a given framework if and only if instances of the concept are not described in the framework as determined by anything else).

2. The General Quantum Theory of Open Systems

ST is not the only theoretical framework that can be formulated using the quantum-mechanical formalism. There is an alternative, *the general quantum theory of open systems* (GT), that takes a different view—the *open systems view*—of the formalism. On the open systems view, the dynamics of interacting systems, not the dynamics of closed systems, are taken to be fundamental, so that the influence of a system's environment is not represented in terms of a dynamical coupling between two parts of one closed system, but fundamentally via the equations that govern the dynamics of the system of interest. Accordingly, in GT, a physical system, $\mathcal{S}$, is fundamentally represented in terms of an in general non-unitarily evolving density operator whose dynamics is governed by a linear (in the sense of preserving mixtures), positive, trace-preserving dynamical map, Λ, acting on $\mathcal{S}$'s state space. (Note that GT is, essentially, the framework originally laid out in [15,16] and subsequently elaborated upon in, for instance, [17,18], as well as in the works of other authors. For more on the properties of not completely positive maps than we will be discussing below, see, for instance, [19]).

Although both employ the same quantum formalism, GT is a wider dynamical framework than ST. (Note that GT is a theoretical framework that allows for the modeling of more general dynamic evolutions of a system than ST. However, which dynamics one chooses depends on the problem at hand and on the physical principles one wishes to assume in the case under study. Dynamical collapse theories, for example, assume that collapse occurs in position space, and the resulting dynamics can be derived from a suitable Lindblad equation. Collapse theories are of course controversial and they run into problems with a relativistic extension. However an advocate for GT is not obliged to accept collapse theories. It is merely possible to formulate them within this framework). The reason is that, although GT requires that Λ be a positive map on the valid states of $\mathcal{S}$, unlike in ST, it is not required in GT that Λ be completely positive in a sense we will presently explicate. The standard argument for imposing what is called the 'principle of complete positivity' is that we should require the effect of a given map, Λ, to be a valid dynamical evolution for every possible initial state of $\mathcal{S}$ irrespective of the existence of a 'witness' system, $\mathcal{W}_n$, of a given dimensionality n that is not interacting with $\mathcal{S}$ (though it may have in the past). The trouble, according to this argument, with a positive but not completely positive map, Λ, on $\mathcal{S}$'s state space is that extending Λ to include the, let us assume, trivial dynamics of the witness, $\Lambda \otimes I_n$, will, in general, result in a final state that yields negative probabilities for the outcomes of certain measurements on $\mathcal{S} + \mathcal{W}_n$. Indeed, requiring that a map, Λ, be completely positive on its full state space is equivalent to requiring that $\Lambda \otimes I_n$ be a positive map for all n, and the principle is usually defined explicitly in this way (see, e.g., [20], p. 86). The standard argument is less than compelling, however (note that our reply to the standard argument is drawn mainly from [18]). Note first that a prediction of negative probabilities for certain measurements is only possible if $\mathcal{S}$ is initially entangled with $\mathcal{W}_n$. However, in that case $\mathcal{W}_n$ should really be considered to be a part of $\mathcal{S}$'s environment, $\mathcal{E}$, and it can be shown that, in fact, there actually is no completely positive map that can describe the dynamics of $\mathcal{S}$ when it is initially entangled with $\mathcal{E}$ ([17], pp. 13–14; note that [17]'s result generalizes an earlier result for two-dimensional systems proved by Pechukas [21]). We should not trouble ourselves about this too much, however, for if $\mathcal{S}$ is a subsystem of an entangled system, then certain states of $\mathcal{S}$ will be impossible; this includes, for instance, pure states, and generally any state that is not a valid partial trace over the entangled state of the overall system. With this in mind, one can then define a 'not completely positive map' that is completely positive vis á vis the states that are not ruled out by a given setup, while for states that are ruled out, we allow that the map may not even be positive on $\mathcal{S}$, let alone its trivial extension positive on $\mathcal{S} + \mathcal{W}_n$ (for discussion, see [22]).

In the context of ST, one can give a better argument for imposing complete positivity as a dynamical principle, namely that complete positivity is assumed in the derivation of Stinespring's dilation theorem [23], which asserts that corresponding to a given $\rho^{\mathcal{S}}$, there is a unique (up to unitary equivalence) pure state $|\Psi^{\mathcal{S}+\mathcal{A}}\rangle$ of a larger system $\mathcal{S} + \mathcal{A}$ (where $\mathcal{A}$ is called the 'ancilla' subsystem), whose dynamics is unitary, and from which we can derive the in general non-unitary dynamics of $\mathcal{S}$. In other words, the procedure for deriving the effective dynamics of an open system that we discussed in the last section, i.e., the procedure of first considering the dynamics of the closed system $\mathcal{S} + \mathcal{E}$ and then taking the partial trace over its final state with respect to $\mathcal{E}$, is predicated upon the assumption of complete positivity. Since ST is formulated in accordance with the closed systems view, every system is modeled as a part of some closed system by definition, from which it follows that complete positivity must be imposed as a fundamental physical principle. As Raggio and Primas [24] put it:

> A system-theoretic description of an open system has to be considered as phenomenological; *the requirement that it should be derivable from the fundamental automorphic dynamics of a closed system* implies that the dynamical map of an open system has to be completely positive (p. 435, our emphasis).

Models formulated in GT are under no such restriction. In a framework in which the dynamics of interacting systems are represented as fundamental, there is no need to be able to derive the dynamics of a system in this way. We can, in principle, describe the dynamics of any system, even the universe as a whole, *as if* it were initially a subsystem of an entangled system.

3. The Open Systems View and the Everett Interpretation

Unitarity is clearly deemed to be important by many Everettians. Simon Saunders, for instance, in the context of a discussion of the metaphysics of personal identity, writes that one of the drawbacks of adopting the rule that 'persons' and 'things' correspond to 'branch parts' is that

> [i]nvoking it seems to compromise a chief selling point of the Everett interpretation, which is that many-worlds follows from the unitary dynamics, with no added principles or special assumptions. This is what puts the Everett interpretation in a class of its own when it comes to the quantum realism problem: there are plenty of avenues for obtaining (at least non-relativistic) one-world theories if we are prepared to violate this precept. ([25], p. 193).

We will not comment on the efforts of Everettians to make sense of the metaphysics of personal identity, nor on the wider context of Saunders' discussion: chance. (Note that although we will not comment on these issues, we do think it would be interesting for

Everettians to reconsider them in the light of a framework in which fundamental non-unitary dynamics is not a priori ruled out. For instance, if the only arguments against certain characterizations of personal identity (such as the rule that 'persons' and 'things' correspond to 'branch parts') or of chance are that they are in tension with quantum theory's restriction against fundamental non-unitary evolution, then we believe that Everettians should rethink those arguments. These are not the only issues that may be clarified in a framework in which the dynamics of density operators are taken as basic. Chen [26–28], for instance, discusses the issue of the arrow of time. See also Maroney [29] and Robertson [30] for related discussions of how taking the density matrix seriously as a real representation of the state of a system helps to clarify the physical basis of the Gibbs entropy and its role in providing a statistical-mechanical underpinning for the second law of thermodynamics). We merely note that modifying the quantum formalism is only one way to allow for fundamentally non-unitary dynamics. Another way is to recognize that there is more than one way to view the formalism, and that we are only required to regard non-unitary evolution as non-fundamental in a framework that has been formulated in accordance with the closed systems view. If, instead, we take the dynamics of open systems—ultimately all that we really have empirical access to in any case—to be fundamental, then it is clear that such dynamics may, in general, be non-unitary according to quantum theory.

The density operator representing the state of a subsystem of an entangled system is an objective description of the degrees of freedom of the subsystem being modeled, and in that sense, it is no less a description of something real in the world than the state of the entangled system as a whole [31]. There is, further, nothing conceptually incoherent about describing even the state of the universe as a whole in these terms on the Everett interpretation; this is a description of the universe that David Wallace, for instance, explicitly entertains, at least as a real possibility (see [32], section 10.5; note that the backdrop to Wallace's discussion in [32] is the 'black hole information loss paradox'; Wallace's opinion about whether the paradox is evidence for fundamental non-unitarity has shifted over the years; see, e.g., [33]). For our part, we think that although it is clear that, empirically, ST is a highly successful—arguably physics' most successful—theoretical framework, there are nevertheless reasons coming from both non-quantum physics as well as from ST's own applications to motivate asking the question of whether it really is as expressive as it needs to be to make sense of the physical world.

Although standard cosmological models based on the FLRW solutions to the Einstein field equation describe a closed universe, they are well-known to be based on strong idealisations introduced with little other reason than that they simplify the relevant mathematics ([34], section 1.1). The scale factor, for instance, represents nothing physically significant in itself [35]. Hawking's proposal [36] to model the quantum description of a black hole in terms of a linear map on the space of density operators is, to be sure, controversial (for discussion, see [33,37,38]). However, at least part of the motivation for wanting to reject it seems to amount to nothing more than that the proposal runs counter to ST (see, e.g., [37], pp. 32–34; cf. [33], p. 219), according to which non-unitary dynamics simply cannot be fundamental. Hawking's particular proposals aside, we are rather inclined to take the opposite view and to ask the question of whether a closed system description of the universe is really apt (see also [39,40]).

Against this, it will, of course, be argued that no contradictions have been demonstrated between any of the phenomena currently known and their theoretical descriptions in ST, and since ST is highly empirically successful, we should, therefore, seek to conform whatever picture of nature is suggested by these phenomena to it rather than the other way around. Even putting to one side the question of what fundamental unitary evolution is supposed to mean in the context of the quantization of gravity [41–43], however, this objection misconstrues what the empirical success of ST actually rests upon, namely its applications to systems that are empirically accessible to us. These are the subsystems of the universe. Since neither gravity [44] nor entanglement [45–47] can be shielded, strictly speaking, the subsystems of the universe are all open systems (of course, the mere fact that a given system happens to be a subsystem of an entangled system does not imply that its dynamics are non-unitary; our point here is only that such subsystems are open systems, and that the dynamics of open systems are non-unitary in general). It is true, of course, that in many cases, one can treat a given system of interest as effectively isolated. Even when one cannot do so, one can still, in many cases, model the dynamics of the system, $\mathcal{S}$, by first modeling the dynamics of the larger dynamically coupled closed system, $\mathcal{S} + \mathcal{E}$, and then abstracting away from $\mathcal{E}$'s degrees of freedom. However, we should not forget that it is the dynamics of $\mathcal{S}$, not the dynamics of $\mathcal{S} + \mathcal{E}$, that we should consider ourselves to have successfully described when we do this (as $\mathcal{E}$ will, as we mentioned in Section 1, typically be highly idealized). The basis of the empirical success of ST, in other words, arguably lies in the way that it effectively describes the dynamics of open systems. (Note that this argument is our gloss on a point suggested in discussion by Wayne Myrvold). Since open systems are represented in ST by density operators that evolve non-unitarily in general, there seems to us to be a clear prima facie empirical motivation to, without necessarily modifying the quantum-mechanical formalism, formulate a more general theoretical framework in which that is how the fundamental dynamics of systems are most generally described. (Note that one might object to this that we have not considered whether the simplicity (or other virtues) of the physical laws used in an explanation might play a role in their confirmation. This is an interesting issue, which we discuss in more detail in [10], section 4.3, with the upshot being that we can find no way to cash out what we there call the relation of 'explanatory fundamentality' in terms of theories that are 'simpler' or 'better' than other theories in a way that gives the right answer to the question, for instance, of whether general relativity or Newtonian gravity is more fundamental. We do not, of course, claim that our arguments are the final word on these rather complicated matters, which is why we qualify the empirical motivation we describe above as only prima facie).

4. Conclusions

We have argued that the more general dynamics describable in the theoretical framework of GT can be physically motivated, that there is as much prima facie empirical support for GT as there is for ST, and that GT may be as much in the spirit of the Everett interpretation as ST is. There might, in short, be little reason for an Everettian not to embrace GT and the more general theoretical landscape that it allows one to explore as the proper conceptual space with which to engage in foundational and philosophical investigations (and speculations) regarding the quantum world.

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quantum reports

Article

The Relation between Wavefunction and 3D Space Implies Many Worlds with Local Beables and Probabilities

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Abstract: We show that the quantum wavefunctional can be seen as a set of classical fields on the 3D space aggregated by a measure. We obtain a complete description of the wavefunctional in terms of classical local beables. With this correspondence, classical explanations of the macro level and of probabilities transfer almost directly to the quantum. A key difference is that, in quantum theory, the classical states coexist in parallel, so the probabilities come from self-location uncertainty. We show that these states are distributed according to the Born rule. The coexistence of classical states implies that there are many worlds, even if we assume the collapse postulate. This leads automatically to a new version of the many-worlds interpretation in which the major objections are addressed naturally. We show that background-free quantum gravity provides additional support for this proposal and suggests why branching happens toward the future.

Keywords: wavefunction; 3D space; many-worlds interpretation; Born rule; branch counting; wavefunctional formulation of quantum field theory; quantum gravity; background-independence

1. Introduction

This article explores the relationship between the wavefunction and 3D space in quantum mechanics. This relation should be clarified because even in nonrelativistic quantum mechanics (NRQM), the wavefunction is not a function defined on 3D space but on the higher dimensional configuration space. Apparently, the situation does not seem to improve in more sophisticated theories, such as quantum field theory (QFT) or quantum gravity (QG). We will see that the answer to this question touches several foundational questions in quantum mechanics and suggests that a version of the many-worlds interpretation gives the answers.

It is important to understand the wavefunction in terms of fundamental entities having a clear 3D space ontology, i.e., entities that are *in* or *on* 3D space. J.S. Bell calls such entities local beables [1]. We will work with quantum fields in the wavefunctional formulation of quantum field theory. Because the configuration space consists of fields instead of positions, the wavefunction is replaced by a wavefunctional. In Sections 2 and 3, we will see how the wavefunctional has a natural interpretation as many classical fields on 3D space aggregated by a measure. This answers the following

Question 1. *Can the wavefunction encode local beables or be described in terms of them?*

We use this in Section 3 to propose answers to the following related question:

Question 2. *What is the ontology of the wavefunction?*

The answer is "a set of classical fields aggregated by a measure". The phases become absorbed in the $\mathrm{U}(1)$ gauges of the classical fields, so this also addresses the question:

Question 3. *Why is the wavefunction a complex function?*

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The classical fields determine a basis of the Hilbert space. Because they fully consist of local beables, we call the states from the resulting basis ontic states. The ontic states are compatible with the macrostates in which the universe is observed to be. Then, building on Sections 2 and 3, Section 4 deals with the questions:

Question 4. *How do 3D objects in space arise from the wavefunction?*

Question 5. *Why does the world look classical at the macroscopic level?*

Most wavefunctionals describe macroscopic superpositions. Therefore, to answer Question 5, it is important to understand which of the wavefunctionals do not describe such superpositions. In classical physics, this problem does not exist precisely because all classical entities are local beables. This indicates that local beables should give the answer in quantum theory too. We argue that it does: microstates have to belong to a basis (determined by the classical fields) whose states will be called ontic.

In classical physics, since ultimately the results of experiments are examined at the macro level, the fact that a macrostate corresponds to more possible microstates is the key to explain how probabilities arise. This is a problem in quantum theory:

Question 6. *How do probabilities arise in quantum theory?*

The key difference is that in quantum theory, the sample space seems to depend on the experiment. In Section 5, we will see that, in quantum theory, the relation between wavefunctional and 3D space leads to a unique sample space for all experiments if we understand that, ultimately, all observations are macroscopic. Therefore, the answer is similar to the classical one, but thinking that subsystems are separate systems obfuscates this because the ontic basis only exists for the total system, not for subsystems.

We will see that, while in classical physics, probabilities describe the agent's ignorance of the actual microstate of the system, in quantum theory, they represent the ignorance of the agent's self-location in one of many microstates. This leads to a derivation of the Born rule and the meaning of probabilities by "counting" the ontic states per macrostate.

In Section 6, it is shown that if we assume wavefunction collapse, probabilities encounter severe difficulties. Whether we assume wavefunction collapse or not, multiple ontic states have to exist simultaneously. This suggests as the natural interpretation a version of the many-worlds interpretation (MWI) [2–4] that results from this analysis. This addresses

Question 7. *How should we interpret quantum mechanics?*

This version of MWI includes probabilities in the classical sense due to the distribution of microstates per macrostate rather than by simply interpreting the squared norm of the state vector as a probability, as it is often proposed. In Section 7, our derivation of the Born rule is compared with other possible ways to count microstates or worlds.

In Section 8, we will see that strong additional support for these findings comes from background-free quantum gravity (which includes most approaches to QG). In the background-free approaches, most linear combinations of states with different 3D geometry cannot represent superpositions. This leads to the dissociation of the state into states with different classical geometries, practically forcing upon us a new version of MWI.

When applied to the Big Bang, this dissociation effect suggests an answer to the time asymmetry of the branching structure problem of the MWI (Section 9):

Question 8. *Why does branching happen toward the future and not also toward the past?*

These results address the major objections against the MWI, in a very conservative and classical-like manner. The big picture resulting from this analysis will be discussed in Section 10.

Several technical details were relegated in Appendices A– C to simplify the article.

2. The Wavefunctional and the 3D Space

Let Σ be the 3D space, which is usually a manifold. If we ignore the curvature due to gravity, we can assume that $\Sigma = \mathbb{R}^3$, but this works for any 3D manifold and even for discrete structures.

Intuitively, we expect that an object is in 3D space if it can be seen as consisting of parts, each of them having a definite position in 3D space. For example, a function or a field defined on a space can be recovered from its values at different positions.

Strictly speaking, a point or set of points from 3D space is *in* 3D space. A classical field ϕ is *on* 3D space, in the sense that ϕ is a function on the 3D space Σ, $\phi : \Sigma \to \mathcal{S}$, where $\mathcal{S}$ is a set in which the field is valued. For example, $\mathcal{S}$ can be $\mathbb{R}$ or $\mathbb{C}$ for real or complex scalar fields, $\mathbb{R}^3$ for real vectors, etc. More generally, a field is a section in a fiber bundle over Σ. For example, the field $\phi : \Sigma \to \mathcal{S}$ is a section of the trivial bundle $\Sigma \times \mathcal{S} \overset{\pi_1}{\mapsto} \Sigma$, where π_1 is the projection on Σ. The field ϕ is a section in the sense that $\pi_1 \circ \phi = 1_\Sigma$, where 1_Σ is the identity map of Σ.

In Appendix A, it is explained that the wavefunction is, in fact, an object of 3D space geometry, and that it can even be faithfully represented as infinitely many fields on Σ that have a local Hamiltonian evolution. However, the representation that will be used in this article comes directly and naturally from quantum field theory (QFT).

In the Schrödinger wavefunctional formulation of QFT, the configuration space $\mathcal{C}$ consists of classical fields [5]. Therefore, instead of a wavefunction, one uses a function of functions or fields, a wavefunctional $\Psi[\phi]$, $\Psi : \mathcal{C} \to \mathbb{C}$.

There are more types of classical fields to be quantized, which can be scalar, spinor, vector, or tensor fields. They can also have internal degrees of freedom, corresponding to the internal spaces of gauge symmetries. Let $\phi = (\phi_1, \ldots, \phi_n)$ contain all the components of all these fields. The operators $\widehat{\phi}_j(x)$ act by multiplication with $\phi_j(x)$. Their canonical conjugates are the functional derivatives $\widehat{\pi}_j(y) := -i\hbar\delta/\delta\phi_j(y)$. They satisfy the canonical commutation relations if they are bosonic and the canonical anticommutation relations if they are fermionic, in which case they are Grassmann numbers. We assume that the manifold $\mathcal{C}$ is endowed with a measure μ (see Appendix B for a discussion of its existence).

The Hilbert space $\mathcal{H}$ consists of the μ-measurable functionals $\Psi : \mathcal{C} \to \mathbb{C}$ that are square-integrable with respect to the measure μ,

$$\mathcal{H} := L^2(\mathcal{C}, \mu, \mathbb{C}). \tag{1}$$

The state vectors labeled by $\phi \in \mathcal{C}$ form an orthogonal basis $(|\phi\rangle)_{\phi\in\mathcal{C}}$ so that, for any compact-supported continuous functional $\Psi : \mathcal{C} \to \mathbb{C}$,

$$\int_{\mathcal{C}} \langle\phi|\phi'\rangle \Psi[\phi'] \mathcal{D}\mu(\phi') = \Psi[\phi]. \tag{2}$$

The time evolution of the universe is governed by the Schrödinger equation:

Postulate 1 (Unitary evolution)**.** *The state of the universe can be represented by a unit vector* $|\Psi(t)\rangle \in \mathcal{H}$*, whose evolution is described by the equation*

$$|\Psi(t)\rangle = \widehat{U}_{t,t_0}|\Psi(t_0)\rangle. \tag{3}$$

Here, the unitary evolution operator $\widehat{U}_{t,t_0} := e^{-\frac{i}{\hbar}(t-t_0)\widehat{\mathsf{H}}}$ between the times t_0 and t is determined by the time-independent selfadjoint operator $\widehat{\mathsf{H}}$, called the Hamiltonian.

The Hamiltonian operator acts locally in 3D space [5]. The wavefunctional formulation allows the recovery of the usual formulation of QFT in terms of operator-valued distributions and of the Fock representation [5].

The wavefunctional Ψ can be understood naturally as consisting of a number $|\mathcal{C}|$ (usually infinite) of fields on 3D space of the form (ϕ, c_ϕ), where $\phi \in \mathcal{C}$, $\phi : \Sigma \to \mathbb{C}$ is a classical field from $\mathcal{C}$, $c_\phi := \Psi[\phi]$ is constant in space, and $|\mathcal{C}|$ is the cardinal of $\mathcal{C}$. Then, Ψ is equivalent to a classical field $\Psi : \Sigma \to \mathbb{C}^{2|\mathcal{C}|}$ on the 3D space Σ,

$$\Psi(x) = \big(\phi(x), c_\phi\big)_{\phi \in \mathcal{C}}. \tag{4}$$

This representation follows directly from the wavefunctional formulation. In the next section, we will see how the phase of c_ϕ can be absorbed in ϕ and that this allows us to interpret the microstate of the universe as a classical field with a given gauge. We will see that Ψ can be understood as a densitized set of gauge classical fields. For this reason, we call the basis $(\phi)_{\phi \in \mathcal{C}}$ the ontic basis.

3. The Wavefunction'S Ontology: A Densitized Set of Classical Worlds

Let us write down the wavefunctional Ψ in polar form with $r[\phi] \geq 0$,

$$\Psi = \int_{\mathcal{C}} r[\phi] e^{i\theta[\phi]} |\phi\rangle \mathcal{D}\mu[\phi]. \tag{5}$$

We assume that there is a global U(1) gauge symmetry so that at least one of the classical fields ϕ_j, $j \in \{1, \ldots, n\}$ transforms nontrivially under global U(1) gauge transformations. We know that this is true in our universe because there are always electromagnetic potentials and Dirac fields. Although U(1) acts differently on different types of fields, for simplicity, we denote by $\phi \mapsto e^{i\theta}\phi$ the gauge transformation of the classical field ϕ. Then, $\phi \neq e^{i\theta}\phi$ for any θ that is not an integer multiple of 2π.

A global gauge transformation of a classical field ϕ results in a physically equivalent field $e^{i\theta}\phi$. On the other hand, a multiplication of the vector $|\phi\rangle$ with $e^{i\theta}$ results in a physically equivalent vector $e^{i\theta}|\phi\rangle$. Then, without loss of consistency, we can identify

$$e^{i\theta[\phi]}|\phi\rangle = |e^{i\theta[\phi]}\phi\rangle. \tag{6}$$

In other words, a phase change in $|\phi\rangle$ is made equivalent to a U(1) gauge transformation of ϕ. This is physically consistent because the physical state remains unchanged under these transformations. The commutative diagram (7) summarizes this.

$$\begin{tikzcd}
\phi_\gamma \arrow[r, mapsto, "\text{gauge transformation}"] \arrow[d, "\text{quantization}"] & e^{i\theta}\phi_\gamma \arrow[d, "\text{quantization}"] \\
|\phi_\gamma\rangle \arrow[r, mapsto, "\text{phase transformation}"] & e^{i\theta}|\phi_\gamma\rangle = |e^{i\theta}\phi_\gamma\rangle
\end{tikzcd} \tag{7}$$

Since $\mathrm{U}(1) \cong \mathrm{SO}(2, \mathbb{R})$, the complex numbers $e^{i\theta[\phi]}$ from Equation (5) can be interpreted as real gauge transformations, answering Question 3.

Since the configuration space $\mathcal{C}$ was constructed by fixing a gauge, a gauge transformation leads to a different configuration space $\widetilde{\mathcal{C}}$ and a different ontic basis $(\widetilde{\phi})_{\phi \in \widetilde{\mathcal{C}}}$,

$$\widetilde{\phi} := e^{i\theta[\phi]}\phi \text{ and } \widetilde{\mathcal{C}} := \{\widetilde{\phi} | \phi \in \mathcal{C}\}. \tag{8}$$

However, Equation (6) shows that the resulting Hilbert space is independent of the gauge coefficients $\theta[\phi]$ from Equation (8) that define the configuration space $\widetilde{\mathcal{C}}$, $\mathcal{H} = L^2(\widetilde{\mathcal{C}}, \mu, \mathbb{C})$.

On the other hand, from Equations (6) and (8), Ψ in the form from Equation (5) becomes a real functional in the basis $(\widetilde{\phi})_{\phi\in\widetilde{\mathcal{C}}}$ because the phases are absorbed,

$$\Psi = \int_{\widetilde{\mathcal{C}}} r[\widetilde{\phi}]|\widetilde{\phi}\rangle \mathcal{D}\mu[\widetilde{\phi}]. \tag{9}$$

Since $r[\phi]$ has to be a μ-measurable function on $\mathcal{C}$, there is a measure $\widetilde{\mu}$ so that

$$\mathcal{D}\widetilde{\mu} = r[\phi]\mathcal{D}\mu[\phi]. \tag{10}$$

Then, from Equations (6) and (10), Equation (5) becomes

$$\Psi = \int_{\widetilde{\mathcal{C}}} |\widetilde{\phi}\rangle \mathcal{D}\widetilde{\mu}[\widetilde{\phi}]. \tag{11}$$

Equation (8) explains complex numbers in quantum theory, addressing Question 3. Representation (11) addresses Question 2, by suggesting the following ontology of the wavefunctional: it consists of ontic states combined according to a density.

4. The World Appears Classical at the Macroscopic Level

At the macroscopic level, the observers have imperfect "resolution" so that states that are microscopically different cannot be distinguished. We assume that this defines an equivalence relation of states. Classical macrostates are equivalence classes of classical states from the configuration space $\mathcal{C}$, and they form a (disjoint) partition of $\mathcal{C}$,

$$\mathcal{C} = \bigsqcup_{\alpha\in\mathcal{A}} \mathcal{C}_\alpha. \tag{12}$$

This induces a direct sum decomposition of the Hilbert space $\mathcal{H}$ defined in Equation (1),

$$\mathcal{H} = \bigoplus_{\alpha\in\mathcal{A}} \mathcal{H}_\alpha, \mathcal{H}_\alpha := L^2(\mathcal{C}_\alpha, \mu, \mathbb{C}). \tag{13}$$

Definition 1. *In the following, the subspace $\mathcal{H}_\alpha$ will represent macrostates. The states represented by vectors from macrostates will be called quasiclassical states. Projectors $\widehat{\mathsf{P}}_\alpha$ on subspaces representing macrostates, so that $\mathcal{H}_\alpha = \widehat{\mathsf{P}}_\alpha\mathcal{H}$, will be called macroprojectors.*

Postulate 2 (Macroclassicality)**.** *(i) If the state of the universe is $|\Psi\rangle$, the world is observed to be in a macrostate $\widehat{\mathsf{P}}_\alpha\mathcal{H}$ for which $\widehat{\mathsf{P}}_\alpha|\Psi\rangle \neq 0$. (ii) Subsequent observations are consistent with the state of the universe being $\widehat{\mathsf{P}}_\alpha|\Psi\rangle / |\widehat{\mathsf{P}}_\alpha|\Psi\rangle|$ at that time.*

If Postulate 2 seems too complicated, it is because it carefully avoids assuming more than can be observed. In particular, it avoids presuming whether the wavefunction collapses or not. For quantum measurements, it avoids assuming too much about the state of the "observed subsystem", because what we actually observe is a macrostate in which the pointer observable has a definite state.

It is useful to detail how Postulate 2 applies to quantum measurements. Let $\mathcal{H}_S$ be the Hilbert space of the observed system. Let $\widehat{\mathsf{A}}$ be a Hermitian operator on $\mathcal{H}_S$, representing the observable of interest, with eigenbasis $(\psi_1^{\mathsf{A}}, \ldots, \psi_n^{\mathsf{A}})$. To indicate the result of a measurement, the measuring device contains a pointer, which is readable at the macroscopic level and can be found in one of the eigenstates $(\zeta_0^{\mathsf{A}}, \zeta_1^{\mathsf{A}}, \ldots, \zeta_n^{\mathsf{A}})$ of the pointer observable $\widehat{\mathsf{Z}}^{\mathsf{A}}$. Let ζ_0^{A} represent the "ready" state of the pointer, and $|\psi\rangle$ the state of the observed system before the measurement. If the measurement of $\widehat{\mathsf{A}}$ takes place between t_0 and t_1, Equation (3) leads to a linear combination involving pointer states,

$$|\Psi(t_1)\rangle = \widehat{U}_{t_1,t_0}|\psi\rangle \otimes |\zeta_0^{\mathsf{A}}\rangle \otimes \ldots = \sum_j \langle\psi_j^{\mathsf{A}}|\psi\rangle|\psi_j^{\mathsf{A}}\rangle \otimes |\zeta_j^{\mathsf{A}}\rangle \otimes \ldots \tag{14}$$

Since the pointer eigenstates are macroscopically distinguishable, the states $|\psi_j^{\mathsf{A}}\rangle \otimes |\zeta_j^{\mathsf{A}}\rangle \otimes \ldots$ are quasiclassical and correspond to distinct macrostates. Therefore, a state containing the pointer in an eigenstate of $\widehat{\mathsf{Z}}^{\mathsf{A}}$ is quasiclassical, as stated in Postulate 2.

Postulate 2 accommodates the possibility of different measurement setups that we would normally consider incompatible. The measuring devices that perform different measurements are macroscopically distinct, so the macrostates corresponding to different measurement results are orthogonal. The incompatibility is between the observables associated with the observed subsystem, but the possible macrostates from which we normally infer the state of the observed subsystem are orthogonal.

The possible resulting states of the universe are not determined by the eigenstates of the observed system nor by those of the pointer of the measuring device. A pointer is a macroscopic object, and it corresponds to the macrostates of the universe, but each macrostate consists of a continuum of microstates from $(|\phi\rangle)_{\phi\in\mathcal{C}}$. The world should be in a definite ontic state. This suggests the following

Postulate 3 (Microstates)**.** *Only the ontic states* $(|\phi\rangle)_{\phi\in\mathcal{C}}$ *can be microstates.*

At first sight, there is a tension between Postulate 2, which says that the future observations are consistent with the state being $\widehat{\mathsf{P}}_\alpha|\Psi\rangle/|\widehat{\mathsf{P}}_\alpha|\Psi\rangle|$, and Postulate 3, which says that microstates can only be from $(|\phi\rangle)_{\phi\in\mathcal{C}}$. However, what Postulate 3 says is that each macrostate consists of microstates that are ontic states, $\widehat{\mathsf{P}}_\alpha = \int_{\mathcal{C}_\alpha} |\phi\rangle\langle\phi|\mathcal{D}\mu[\phi]$. This is consistent with Postulate 2, since $\widehat{\mathsf{P}}_\alpha|\Psi\rangle = \int_{\mathcal{C}_\alpha} \Psi[\phi]\mathcal{D}\mu[\phi]$.

Postulate 3 is consistent with Postulate 2, because the classical states $|\phi\rangle$ are also quasiclassical, since each ϕ belongs to a unique macrostate $\widetilde{\mathcal{C}}_\alpha$. It also clarifies Postulate 2: the world looks classical because its microstates are classical ontic states. Since the ontic states consist of objects in 3D space, this addresses Questions 4 and 5.

In standard quantum mechanics (SQM), the Projection Postulate was introduced to explain why we observe only one of the states $|\psi_j^{\mathsf{A}}\rangle \otimes |\zeta_j^{\mathsf{A}}\rangle \otimes \ldots$. The Projection Postulate was given in terms of quantum measurements [6,7]. Here, we replaced the Projection Postulate with Postulate 2, which

- is more general, including measurements as particular cases,
- avoids presuming whether the wavefunction collapses or not,
- relates the macrostates to microstates of the form $|\phi\rangle$, where $\phi \in \mathcal{C}$ have clear relations with 3D space.

The probabilities are given by the Born rule:

Rule 1 (Born rule)**.** *If the state of the universe is represented by* $|\Psi\rangle$*, the probability that an observation of the world finds it in the macrostate* $\widehat{\mathsf{P}}_\alpha\mathcal{H}$ *is*

$$P_\alpha = \langle\Psi|\widehat{\mathsf{P}}_\alpha|\Psi\rangle. \tag{15}$$

From Equation (11) $\widehat{\mathsf{P}}_\alpha|\Psi\rangle = \int_{\widetilde{\mathcal{C}}_\alpha} |\widetilde{\phi}\rangle\mathcal{D}\widetilde{\mu}[\widetilde{\phi}]$, therefore,

$$\left|\int_{\widetilde{\mathcal{C}}_\alpha} |\widetilde{\phi}\rangle\mathcal{D}\widetilde{\mu}[\widetilde{\phi}]\right|^2 = \langle\Psi|\widehat{\mathsf{P}}_\alpha|\Psi\rangle. \tag{16}$$

This is not yet a proof of the Born rule. In SQM, the Born rule is postulated, but in Section 5, we will derive it based on the relation between Postulates 2 and 3.

5. Naive Counting Gives the Born Rule in the Continuous Limit

Suppose Alice asks Bob to participate in the following experiment. Alice instructs Bob to wait until a bell rings and as soon as the bell rings, to push a button. The button stops a stopwatch, and Bob, without reading it, has to guess whether the stopwatch indicates an even or an odd number for the millisecond.

A way to interpret the probability that Bob assigns to the event is that the state of the universe contains the state of the stopwatch, including its property that the millisecond is an even or an odd number. Bob does not know the state of the world, but he can attribute the probability 1/2 to the event that the millisecond is even. This subjective probability is based on the incomplete knowledge of the state of the system.

Another interpretation is that Bob is a succession of infinitely many instances, one for each moment of time. There is an instance of Bob which stops the stopwatch as a result of (a previous instance of Bob) hearing the bell ringing. Then, (a subsequent instance of) Bob can interpret the probability as representing the odds that his instance that pressed the button was located along the time axis in an interval labeled by an even or an odd number representing the millisecond. This is the self-location probability of Bob in time.

In the example with the stopwatch, both the subjective view and the self-location view are valid. However, an adept of presentism may prefer the subjective view, while an adept of eternalism may prefer the self-location view of probability.

Now consider an experiment in which Alice sends Bob a qubit in the state $1/\sqrt{2}$ $(|0\rangle + |1\rangle)$, asking him to determine whether the qubit's state is $|0\rangle$ or $|1\rangle$. The probability that Bob determines that the qubit is in the state $|1\rangle$ is 1/2. However, the subjective view applies if the wavefunction collapses, while if both worlds exist, the probability comes from Bob's ignorance of whether he is the Bob instance in the world where the result is $|0\rangle$ or the one in which the result is $|1\rangle$, so the self-location view applies.

Now, let $(|\phi_k\rangle)_{k\in\{1,\ldots,n\}}$ be orthonormal eigenvectors of the operator $\widehat{\mathsf{A}}$ representing the observable, and $\mathcal{H}_S$ the observed system's Hilbert space. Or $(|\phi_k\rangle)_{k\in\{1,\ldots,n\}}$ can be an orthogonal system of quasiclassical states, and $\widehat{\mathsf{A}}$ a macroscopic observable. Then, if

$$|\psi\rangle = \frac{1}{\sqrt{n}}\sum_{k=1}^{n}|\phi_k\rangle \tag{17}$$

is the state vector of the observed system, and $\widehat{\mathsf{P}}_j$ is the projector of the eigenspace corresponding to the eigenvalue λ_j, the Born rule coincides with counting states:

$$\langle\psi|\widehat{\mathsf{P}}_j|\psi\rangle = \frac{1}{n}\sum_{|\phi_k\rangle\in\widehat{\mathsf{P}}_j\mathcal{H}_S}\langle\phi_k|\phi_k\rangle = \frac{n_j}{n}, \tag{18}$$

where n_j is the number of the eigenbasis vectors $|\phi_k\rangle$ that are eigenvectors for λ_j.

However, this "naive state counting" does not give the right probabilities because it coincides with the Born rule only in this special situation. In general, the coefficients in Equation (17) are distinct complex numbers, and counting them will give a different probability from the Born rule. For this reason, in the standard versions of MWI it was proposed to interpret self-location uncertainty as being given by the squared amplitude and not simply by counting [8], and even that this should be postulated [9].

However, the worlds are not determined by the vectors $|\phi_k\rangle$. What is naive about the "naive self-location view" is to count the eigenstates of the observed system or of the pointer state as worlds in which the observer can be located. The full ontic states should be counted, and an agent should be in a definite ontic state. Self-location should be about the possible ontic states of the universe, which are $(|\phi\rangle)_{\phi\in\mathcal{C}}$ (Postulate 3).

Moreover, while counting states works only for states of the form (17), in the continuous limit, it works for all states $|\Psi\rangle \in \mathcal{H}$. However, counting should be applied to the whole system, not to its parts (Postulate 2), and only to ontic states (Postulate 3).

Theorem 1. *The Born rule is obtained as the continuous limit of counting ontic states.*

Proof. The macroprojectors consistent with Postulate 3 have the form

$$\widehat{\mathsf{P}}_\alpha = \int_{\mathcal{C}_\alpha}|\phi\rangle\langle\phi|\mathcal{D}\mu[\phi], \tag{19}$$

where the set $\mathcal{C}_\alpha \subset \mathcal{C}$ is μ-measurable. For any unit vector $|\Psi\rangle \in \mathcal{C}$, there is an infinite sequence $(\mathcal{P}_n)_{n\in\mathbb{N}}$ of sets of projectors with the following properties:

(i) Each projector from $\mathcal{P}_n$ has the form $\widehat{\mathsf{P}}_{n,k} = \int_{D_{n,k}} |\phi\rangle\langle\phi|\mathcal{D}\mu[\phi]$, where $\mathcal{C} = \bigsqcup_{k=1}^{2^n} D_{n,k}$ is a partition of $\mathcal{C}$ into measurable subsets so that $\int_{D_{n,k}} r^2[\phi]\mathcal{D}\mu[\phi] = 1/2^n$.

(ii) For each n, $\mathcal{P}_{n+1}$ refines $\mathcal{P}_n$, i.e., projectors from $\mathcal{P}_n$ are sums of those from $\mathcal{P}_{n+1}$.

(iii) The measure of the sets $D_{n,k}$ included in $\mathcal{C}_\alpha$ converges to the measure of $\mathcal{C}_\alpha$.

Then, from (i) and (ii), for each n, $|\Psi\rangle$ decomposes as $|\Psi\rangle = 1/\sqrt{2^n}\sum_{k=1}^{2^n}|n,k\rangle$, where $|n,k\rangle := \sqrt{2^n}\widehat{\mathsf{P}}_{n,k}|\Psi\rangle$ are orthogonal unit vectors. From (iii), the sequence $(\mathcal{P}_n)_{n\in\mathbb{N}}$ converges to a refinement of the set of macro-projectors $(\widehat{\mathsf{P}}_\alpha)_{\alpha\in\mathcal{A}}$. Hence, the continuous limit of a counting as in (18) gives the Born rule. For more details, see [10]. □

Then, due to Postulate 3, the Born rule is obtained as a probability measure over the ontic states. This is possible because $\widetilde{\mathcal{C}}$ becomes a sample space, $(\widetilde{\mathcal{C}}_\alpha)_{\alpha\in\mathcal{A}}$ an event space, and $P : (\widetilde{\mathcal{C}}_\alpha)_{\alpha\in\mathcal{A}} \to [0,1]$, $P(\widetilde{\mathcal{C}}_\alpha) = \int_{\widetilde{\mathcal{C}}_\alpha} r^2[\widetilde{\phi}]\mathcal{D}\mu$ a probability function. Therefore, $\left(\widetilde{\widetilde{\mathcal{C}}}, (\widetilde{\mathcal{C}}_\alpha)_{\alpha\in\mathcal{A}}, \widetilde{\mathcal{C}}_\alpha \mapsto \int_{\widetilde{\mathcal{C}}_\alpha} r^2[\widetilde{\phi}]\mathcal{D}\mu\right)$ becomes a classical probability space. At any instant in time, the probability density $|\Psi[\phi]|^2$ on $\widetilde{\mathcal{C}}$ can be interpreted similarly to the probability density on the phase space from classical physics. If only one microstate exists, but it is unknown, the probability is subjective. If more microstates can coexist simultaneously, it can be interpreted as self-location probability. This answers Question 6.

6. Wavefunction Collapse Is Inconsistent with Our Derivation of the Born Rule

It may seem that we can interpret Equation (16) probabilistically in two different ways and get the Born rule (15). The subjective view applies if there is only one world whose microstate is unknown to the agent, and the wavefunction collapses to be consistent with Postulate 2. The self-location uncertainty view applies if there are many worlds, but the agent does not know in which of them they are located.

Now we will see that SQM, which assumes wavefunction collapse, is inconsistent with Postulate 3 and, therefore, with our derivation of the Born rule. In SQM, $|\Psi(t)\rangle$ is a microstate at all times. Whenever it evolves into a linear combination over more macrostates it collapses to one of them to ensure consistency with Postulate 2.

However, if there is only one world that collapses to avoid macroscopic superpositions, it should be allowed to be in states that do not belong to the same basis. To see this, let us look again at Equation (14). It assumes that at t_0

$$|\Psi(t_0)\rangle = |\psi\rangle \otimes |\zeta_0^{\mathsf{A}}\rangle \otimes \ldots. \tag{20}$$

The vector $|\psi\rangle \in \mathcal{H}_S$ can be any unit vector from $\mathcal{H}_S$. Let $|\psi'\rangle \in \mathcal{H}_S$ be another unit vector. Then, the total state vector is $|\Psi'(t_0)\rangle = |\psi'\rangle \otimes |\zeta_0^{\mathsf{A}}\rangle \otimes \ldots$. In particular, there are vectors $|\psi\rangle, |\psi'\rangle \in \mathcal{H}_S$ so that, at t_0, $\langle\psi|\psi'\rangle \neq 0$, which implies $\langle\Psi(t_0)|\Psi(t_0)'\rangle \neq 0$. The states $|\Psi(t_0)\rangle$ and $|\Psi'(t_0)\rangle$ are distinct microstates of the same macrostate in which the pointer state is $|\zeta_0^{\mathsf{A}}\rangle$. Since, in SQM, the world is allowed to be in any of them, and they are not orthogonal, the world is not restricted to be only in the states from an orthogonal basis. This contradicts Postulate 3, so the derivation of the Born rule from Theorem 1 does not seem to apply to SQM. The following proposition shows this.

Proposition 1. *If any state from a macrostate should be counted as a world, the proof of Theorem 1 cannot be used to derive the Born rule.*

The proof is given in Appendix C.

If, to keep Postulate 3, we assume that there is only a single world that is always in an ontic state, Postulate 2 will be satisfied without invoking the wavefunction collapse. However, this would be a single-world unitary theory [11–13], and this is possible only if the initial conditions are very strongly fine-tuned [14], violating Bell's statistical independence assumption [1]. Even if this would mean something like superdeterminism, conspiracy, retrocausality, or global consistency [15], it is a possibility.

We can try a modified version of Postulate 3: "Linear combinations of ontic states can exist as long as they belong to the same macrostate. When they belong to more macrostates, collapse is invoked so that the resulting microstate is from $(|\phi\rangle)_{\phi\in\mathcal{C}}$." However, when the collapse is invoked for a measurement of S, and a measurement of a different subsystem S' follows immediately, the subsystem S' can also be in any state at the same time when the collapse is invoked for system S. This contradicts the modified version of Postulate 3. We can try to modify it more: "Linear combinations of ontic states can exist as long as they belong to the same macrostate. When they belong to more macrostates, collapse is invoked, but all ontic states in the macrostate that remains after the collapse are preserved." This works, but it requires the self-location interpretation of probabilities, and it would be a version of MWI where some of the worlds disappear, and the remaining ones are macroscopically indistinguishable, an ad hoc strategy. Since after recording the results of the measurements, the worlds from different macrostates no longer interfere anyway, why postulate the disappearance of some of them? It follows that the only consistent and natural way to satisfy the conditions required by the proof of Theorem 1 is the MWI. This suggests an answer to Question 7.

7. What Should Be Counted as a World?

The question "what should be counted as a world?" has two meanings:

Meaning 1. What kinds of unit vectors in the Hilbert space count as worlds?

Meaning 2. What components of the wavefunction should be counted when we calculate the probabilities?

However, the answer to both these questions is the same, Postulate 3.

However, since linear combinations of ontic vectors $|\phi\rangle$ from the same $\mathcal{H}_\alpha$ also belong to $\mathcal{H}_\alpha$, they are quasiclassical, and maybe they should be counted as worlds too. This happens, for example, if we try to prove the Born rule by finding a finite number of orthonormal vectors for the macrostates that add up to $|\Psi\rangle$, as in Equation (18), and counting them, as in [2,16]. If the basis $(|\phi_k\rangle)_{k\in\{1,\dots,n\}}$ from Equation (18) depends on $|\Psi\rangle$, this implies that we have to interpret all such possible orthogonal systems as consisting of words. Proposition 1 shows that this leads to overcounting, and it cannot give the Born rule. However, Theorem 1 shows that in the continuous case, if we use the same basis, in agreement with Postulate 3, this works. Therefore, Theorem 1 can be understood as the continuous limit of the proposal from [16], necessarily amended with Postulate 3.

Can Postulate 3 be avoided by defining the worlds differently?

The worlds cannot be the macrostates because this will give the naive branch counting according to which all outcomes with nonvanishing amplitude have the same probability.

Can the worlds be the nonvanishing components $\widehat{\mathsf{P}}_\alpha|\Psi\rangle$ of $|\Psi\rangle$? It seems that they cannot be, for the same naive branch-counting argument. However, we can reinterpret probability in a decision-theoretic way as in [17,18], or as a measure of existence as in [19], or other arguments that the size of $\widehat{\mathsf{P}}_\alpha|\Psi\rangle$ matters so that its square is the probability. It can be argued that Theorem 1 offers an alternative to these new interpretations of probability. It can also be argued that Theorem 1 is consistent with them, and it only shows that they can be understood as a coarse-graining of a more conservative probability, that of self-location in the ontic states.

8. The 3D Geometry as the Preferred Basis

Several important approaches to quantum gravity are background-free. We will see that background freedom brings strong evidence for the existence of an ontic basis, as in Postulate 3, but based on 3D space geometry.

Canonical quantum gravity, as formulated in [20] is based on quantizing Einstein's equation expressed in $3+1$ dimensions $\Sigma\times\mathbb{R}$ as in [21]. Since after quantization, time seems to disappear, the time-evolving wavefunction is decoded from the Wheeler-de Witt constraint equation by using the Page–Wootters formalism [22]. The result is a wavefunctional formulation, in which the configuration space of classical fields includes the components of the metric tensor on the 3D space Σ. The theory is invariant to diffeomorphisms, similar to gauge invariance. This makes it background-free.

The classical configuration space consists of fields $\phi = (\gamma_{ab}, \phi_1, \ldots, \phi_n) \in \mathcal{C}$, where $a, b \in \{1, 2, 3\}$, $\gamma = (\gamma_{ab})$ contains the components of the 3D metric, and $|\varphi\rangle$ represents the matter fields on Σ and any other fields that may be needed by the theory. Let $\mathcal{C}_S$ be the configuration space of 3D metrics up to diffeomorphisms, and $\mathcal{C}_M$ the configuration space of matter fields, so that $\mathcal{C} = \mathcal{C}_S \times \mathcal{C}_M$.

A state vector with classical geometry has the form $|\Psi\rangle = |\gamma\rangle|\varphi\rangle$, where $|\varphi\rangle$ is a general quantum state of matter. Because of the invariance to diffeomorphisms, there is no correspondence between the points of (Σ, γ_1) and those of (Σ, γ_2), except in the special case when they are isometric. For any linear combination of states with classical geometries, there are infinitely many sets of field operators $(\widehat{\phi}_j(x), \widehat{\pi}_j(y))_j$ that satisfy the canonical (anti)commutation relations. They depend on the relative diffeomorphisms of the 3D spaces of the states in the linear combination. It is possible to fix such a set of field operators, but this would make the theory background-dependent. This is why, in background-free quantum gravity, even though the vector $c_1|\gamma_1\rangle|\varphi_1\rangle + c_2|\gamma_2\rangle|\varphi_2\rangle$ exists in $\mathcal{H}$, in general, it represents dissociated states with distinct geometries and not a superposition of two states on Σ.

This dissociation becomes even more evident if the theory of quantum gravity has a discrete 3D space or spacetime because, in this case, the underlying graphs or hypergraphs of the states in a linear combination can be nonisomorphic, so a correspondence between their points is not even possible. Examples of background-free approaches to quantum gravity in which space or spacetime is discrete include causal sets [23], Regge calculus [24], causal dynamical triangulations [25], the spin network formulation of loop quantum gravity [26,27], etc. In these approaches, the 3D space Σ or the spacetime is a graph or a hypergraph with values attached to their vertices and (hyper-)edges to encode the metric, curvature, or spins, depending on the approach. All these approaches can be described in the Schrödinger formulation. The classical fields $\phi \in \mathcal{C}$ have to include the possible configurations of Σ. In the discrete approaches, graphs or hypergraphs representing Σ are not assumed to be embedded in a 3D manifold. Therefore, they are background-free, in the sense that only the intrinsic properties of Σ matter [28].

The problem of superpositions of states with different classical geometries was discussed, for example, in [29,30]. However, maybe this is not a bug but a feature of background-free quantum gravity. We claim that this dissociation leads to a new version of MWI [31].

Observation 1. *Due to the background freedom, linear combinations* $c_1|\gamma_1\rangle|\varphi_1\rangle + c_2|\gamma_2\rangle|\varphi_2\rangle$ *cannot be interpreted, in general, as superpositions.*

A state $|\Psi\rangle = |\gamma\rangle|\varphi\rangle$ with classical geometry immediately evolves into a linear combination of states with distinct geometries. This means that the basis $(|\gamma\rangle)_{\gamma \in \mathcal{C}_S}$ determines an absolute branching structure. The wavefunctional evolves on the configuration space, and its branches can interfere again. Therefore, dissociated states can reassociate. When dissociation corresponds to differences recorded at the macro level, it becomes irreversible and macroscopic branching occurs. These macroscopic differences may coincide with those due to usual branching in the MWI or may lead to additional observable effects at the macro level. This remains to be explored. As in the case of branching in Everett's interpretation, this irreversibility is not due to unitary evolution, which is reversible but to initial conditions of the universe similar to those responsible for the Second Law of Thermodynamics [4]. We will return to the problem of time asymmetry of the branching structure in Section 9.

We do not know yet if background freedom is a feature of our universe and to what extent the dissociation of the state into states with different classical geometries prevents superposition. Probably states with different geometries that are isometric on some regions of 3D space allow for local superpositions and interference in those regions. In any case, this problem is open, and future theoretical and experimental investigations can hopefully tell us more about it. The deviations from regular quantum mechanics may be accessible to empirical testing, and experiments may corroborate or refute background freedom.

The existence of dissociation into states with different classical 3D geometries due to the absence of superpositions would make a much stronger case for the existence of ontic states. In this case, the 3D space metric of the ontic states has to be classical, so they are of the form $|\gamma\rangle|\phi\rangle$.

Whether or not quantum gravity has to be background-free in this way remains to be seen. Even if it were background-dependent, the states with classical 3D space form a special basis, consistent with our experience and with all the experiments conducted so far. Therefore, they deserve to be considered ontic states.

9. The 3D Geometry and the Branching Structure

To prevent violations of the Born rule in the MWI, distinct worlds should not interfere again. Branching has to occur only toward the future. It is often believed that decoherence answers Question 8, but unitary evolution is time-symmetric, so the initial conditions should break this symmetry to ensure branching only toward the future. There are strong reasons to believe that the low entropy of the initial state of the universe, postulated to explain the Second Law of Thermodynamics, also explains branching asymmetry [4]. However, we do not have a satisfactory answer for the initial low entropy either.

However, quantum gravity reveals a strong connection between the branching asymmetry and the cosmological arrow of time, i.e., the Big Bang followed by the expansion of the universe.

The Big Bang singularity consists of the fact that the 3D space metric vanishes as $t \to 0$ [32]. It is often believed that classical general relativity breaks down at singularities. However, there is a formulation of general relativity whose equations do not break down for a large class of singularities. Its equations are equivalent to Einstein's outside singularities but remain finite at singularities [33]. Such "benign" singularities require that the matter fields are constant in the directions in which the metric tensor is degenerate. This means that, since $\gamma_{ab} \to 0$ in all directions as $t \to 0$, the matter fields have to become constant on the 3D space Σ. The set of possible classical fields consistent with this condition is described by a very small number of parameters. The wavefunctional is, therefore, constrained initially to a small subspace of the Hilbert space, a single macrostate of very low entropy. The wavefunctional gradually expands and spreads over more and more, larger and larger macrostates.

This explanation makes sense even if our quantum-gravitational universe is not background-free. However, since at the Big Bang singularity, there is a unique 3D space geometry $\gamma_{ab} = 0$, the state is fully associated. Since background freedom implies that Ψ dissociates as it evolves, it seems to give a stronger reason for the time asymmetry of the branching structure than the background-dependent theories.

10. Conclusions

We have seen that the wavefunctional formulation of quantum field theory comes implicitly with a natural interpretation of Ψ in 3D space. This has implications for several different problems in quantum mechanics. The central implication is that it provides an ontology in terms of local beables. This ontology requires a preferred basis, the ontic basis. Since we can only directly observe the macrostates, the ontology of the ontic microstates justifies counting them as possible states in which the system is, just like in classical physics. However, unlike classical physics, in quantum mechanics, a state can evolve into a linear combination of microstates. The local beable ontology of the wavefunctional suggests interpreting these linear combinations as multiple ontic states coexisting in parallel. Since a macrostate is an equivalence class of microstates, probabilities arise by taking into account the possible microstates in each macrostate. It turns out that this probability satisfies the Born rule.

If there were a single ontic world, this probability would be subjective, representing the uncertainty about the microstate. However, we have seen that, even in the standard interpretation of quantum mechanics, multiple ontic states have to coexist in parallel. Therefore, the probability should be about the self-location of the agent in one of the microstates. It follows that a new version of MWI is unavoidable in this framework. In this version of MWI, because the ontic states are orthogonal, the agent can exist only in an ontic state, and the macrostates can consist of a different amount of microstates, probabilities appear from the agent's self-location uncertainty about the microstate.

If background freedom is a feature of quantum gravity, it implies that the wavefunctional dissociates into states with distinct but classical 3D geometries. This gives strong additional support to the big picture described above. In addition, quantum gravity suggests that the Big Bang singularity may explain the time asymmetry of the branching structure because at the Big Bang singularity, the state is not dissociated, all of its components having the same geometry $\gamma_{ab} = 0$ and constant fields. As the universe evolves, it spreads over more and more macrostates, so the wavefunctional branches more and more.

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Appendix A. The Wavefunction as an Object in 3D Space

In NRQM, the wavefunction for n particles is defined on the configuration space Σ^n, and it can be expressed as n functions on Σ only in the absence of entanglement.

However, in NRQM, the wavefunction is also an object of Euclidean geometry. A figure consisting of triangles and other polygons is an object of Euclidean geometry. This remains true if we label its vertices with complex numbers. $\Psi(x_1, \ldots, x_n)$ is equivalent to infinitely many figures consisting of n points in $\mathbb{R}^3$, each such figure $(x_1, \ldots, x_n)$ being labeled with the complex number $\Psi(x_1, \ldots, x_n)$. We can also interpret labeled figures as unlabeled figures in a complex line bundle over 3D space [34].

The wavefunction is an object of Euclidean geometry also, according to Klein's Erlangen program [34,35]. Moreover, if we apply Klein's ideas to quantum theory and require the Hilbert space to be a representation of the Galilei group or the Poincaré group, as Wigner and Bargmann did, we get that the wavefunction is an object of spacetime, the classification of the types of particles by spin and rest mass, and the free evolution equations as in quantum theory [36–38]. For more details, see [34].

Moreover, it is also possible to represent the wavefunction as a vector field with infinitely many components on Σ. In [39], it was shown that the usual tensor product of functions defined on 3D space can be represented as a direct sum by using an additional global gauge symmetry. By direct sums between these vector bundles subject to gauge equivalence, the full tensor product Hilbert space can be represented as a vector field. Since the resulting representation is redundant, the redundancy is removed by using an even larger global gauge symmetry. Then, this global gauge symmetry can be made local by introducing a flat connection for its group. This allows the field representing Ψ to be locally separable in the sense that it can be changed in an open subset A of Σ without affecting its values outside of A. The Hamiltonian is local, and the field evolves locally as long as no wavefunction collapse is assumed to take place.

This representation also applies to quantum field theory in the Fock representation. It is a faithful representation of Ψ, which can, therefore, be seen as consisting of local beables. However, this representation is artificial and was given in [39] only as a proof of concept. The natural representation is given in Sections 2 and 3.

Appendix B. The Existence of a Measure on the Configuration Space of Classical Fields

If the configuration space of classical fields $\mathcal{C}$ were an infinite-dimensional manifold, no analog of the Lebesgue measure could be defined on it (although other measures are possible [40]). However, there are indications that the dimension of $\mathcal{C}$ is finite: the fields are constrained by equations, the gauge degrees of freedom need to be factored out, the entropy bound indicates that the Hilbert space has a finite number of dimensions in bounded regions of space [41,42], and the arrow of time requires severe additional constraints [43]. Therefore, we will assume that the manifold $\mathcal{C}$ is finite-dimensional if this is what it takes for it to be compatible with a measure μ.

Appendix C. Possible Worlds Should Form a Basis

Proof of Proposition 1. For every n, let $|\Psi\rangle = 1/\sqrt{n}\sum_{k=1}^{n}|n,k\rangle$ be a decomposition of $|\Psi\rangle$ in orthonormal vectors, so that, as $n \to \infty$, $N_{n,\alpha}/n$ converges to $\langle\Psi|\widehat{\mathsf{P}}_{\alpha}|\Psi\rangle$, where $N_{n,\alpha} = \{k \in \{1,\ldots,n\}||n,k\rangle \in \mathcal{H}_{\alpha}\}$. Let $S_{n,\alpha}$ be the set of vectors obtained from $|n,k\rangle$ by all unitary transformations of $\mathcal{H}_{\alpha}$ that preserve $\widehat{\mathsf{P}}_{\alpha}|\Psi\rangle$. Unitary symmetry implies that any vector from $S_{n,\alpha}$ belongs to orthogonal systems similar to $\{|n,k\rangle|k \in N_{n,\alpha}\}$. Therefore, by the hypothesis of Proposition 1, they should be counted as worlds. Let $\mathfrak{p}(\mathcal{S})$ denote the probability measure of a set $\mathcal{S} \subseteq \mathcal{H}$ of state vectors counting as worlds. Let $\alpha \neq \beta \in \mathcal{A}$ so that $|\widehat{\mathsf{P}}_{\alpha}|\Psi\rangle| = |\widehat{\mathsf{P}}_{\beta}|\Psi\rangle| \neq 0$. Due to unitary symmetry, there is a unitary transformation $\widehat{\mathsf{S}}$ that maps the line $\mathbb{C}\widehat{\mathsf{P}}_{\beta}|\Psi\rangle \subset \mathcal{H}_{\beta}$ to the line $\mathbb{C}\widehat{\mathsf{P}}_{\alpha}|\Psi\rangle \subset \mathcal{H}_{\alpha}$, so that either $\widehat{\mathsf{S}}\mathcal{H}_{\beta} = \mathcal{H}_{\alpha}$, or $\widehat{\mathsf{S}}\mathcal{H}_{\beta} \subsetneq \mathcal{H}_{\alpha}$, or $\mathcal{H}_{\alpha} \subsetneq \widehat{\mathsf{S}}\mathcal{H}_{\beta}$. The symmetry requires that $\mathfrak{p}(\widehat{\mathsf{S}}\mathcal{H}_{\beta}) = \mathfrak{p}(\mathcal{H}_{\alpha})$. It also allows the existence of infinitely many such transformations. Let $\widehat{\mathsf{S}}'$ be another one with the same properties so that $\widehat{\mathsf{S}}'\mathcal{H}_{\beta} \neq \widehat{\mathsf{S}}\mathcal{H}_{\beta}$. Since $\widehat{\mathsf{S}}\mathcal{H}_{\beta} \cap \widehat{\mathsf{S}}'\mathcal{H}_{\beta}$ is a strict subspace of $\widehat{\mathsf{S}}\mathcal{H}_{\beta}$, $\mathfrak{p}(\widehat{\mathsf{S}}\mathcal{H}_{\beta} \cap \widehat{\mathsf{S}}'\mathcal{H}_{\beta}) = 0$, and $\mathfrak{p}(\widehat{\mathsf{S}}'\mathcal{H}_{\beta}) = \mathfrak{p}(\widehat{\mathsf{S}}'\mathcal{H}_{\beta} \setminus \widehat{\mathsf{S}}\mathcal{H}_{\beta}) = \mathfrak{p}(\widehat{\mathsf{S}}\mathcal{H}_{\beta} \setminus \widehat{\mathsf{S}}'\mathcal{H}_{\beta}) = \mathfrak{p}(\widehat{\mathsf{S}}\mathcal{H}_{\beta})$. Therefore, $\mathfrak{p}(\mathcal{H}_{\alpha}) > \mathfrak{p}(\widehat{\mathsf{S}}\mathcal{H}_{\beta}) + \mathfrak{p}(\widehat{\mathsf{S}}'\mathcal{H}_{\beta}) > \mathfrak{p}(\widehat{\mathsf{S}}\mathcal{H}_{\beta}) = \mathfrak{p}(\mathcal{H}_{\beta})$. However, according to the Born rule, $\mathfrak{p}(\mathcal{H}_{\alpha}) = \mathfrak{p}(\mathcal{H}_{\beta})$. It follows that the Born rule is satisfied only if $\widehat{\mathsf{S}}\mathcal{H}_{\beta} = \mathcal{H}_{\alpha}$ for every $\alpha \neq \beta \in \mathcal{A}$. However, now we will show that, for $|\widehat{\mathsf{P}}_{\alpha}|\Psi\rangle| > |\widehat{\mathsf{P}}_{\beta}|\Psi\rangle|$, this contradicts the Born rule. The angle $\omega_{n,\alpha}$ between $|n,k'\rangle$ and $1/\sqrt{n}\sum_{k\in N_{n,\alpha}}|n,k\rangle$ when $k' \in N_{n,\alpha}$ satisfies $\cos\omega_{n,\alpha} = |\langle n,k'|1/\sqrt{nN_{n,\alpha}}\sum_{k\in N_{n,\alpha}}|n,k\rangle| = 1/\sqrt{nN_{n,\alpha}}$. Therefore, as $n \to \infty$, $\omega_{n,\alpha} \to \pi/2$, for all α. It follows that in the limit $n \to \infty$, $\mathfrak{p}(S_{n,\alpha})/\mathfrak{p}(S_{n,\beta}) = 1$. Therefore, counting all vectors from the sets $S_{n,\alpha}$ as worlds contradicts the Born rule. □

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Teleportation Revealed

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Abstract: Quantum teleportation is the name of a problem: How can the real-valued parameters encoding the state at Alice's location make their way to Bob's location via shared entanglement and only two bits of classical communication? Without an explanation, teleportation appears to be a conjuring trick. Investigating the phenomenon with Schrödinger states and reduced density matrices shall always leave loose ends because they are not local and complete descriptions of quantum systems. Upon demonstrating that the Heisenberg picture admits a local and complete description, Deutsch and Hayden rendered its explanatory power manifest by revealing the trick behind teleportation, namely, by providing an entirely local account. Their analysis is re-exposed and further developed.

Keywords: quantum teleportation; locality; unitary quantum theory

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1. Introduction

In the context of the recent Nobel Prize of physics, the *Scientific American* published [1] an article titled "The Universe Is Not Locally Real, and the Physics Nobel Prize Winners Proved It". I could dedicate my piece to the fact that we do not "prove" such claims in science or to the fact that the universe is real. Instead, I will address the question of locality, which creates tremendous confusion in the community of quantum foundations. The key message that I want to advocate is that *quantum systems can be described in a local and complete way, and we should do so.*

Since Bell [2], the term locality, more often seen negated, has come to mean compatibility with an underlying explanation by local hidden variables. However, local hidden variables are only one way in which locality can be instantiated, whose full generality is captured by *Einstein's locality* [3]: "the real factual situation of system A is independent of what is done with the system B, which is spatially separated from the former." Scientific theories are tentative descriptions of the real factual situation; thus, Einstein's locality can be lifted (and slightly generalized) into a criterion for theories: *The description of system A is independent of what is performed with system B, which is dynamically isolated from the former.*

If Alice and Bob share an entangled pair of particles in a pure state, the reduced density matrices provide a local mode of description. Indeed, an action by Bob on his quantum system shall alter its density matrix, but Alice's remains unchanged. However, reduced density matrices are not a *complete* mode of description, a sufficient definition of which is that *the distribution of any joint measurement can be computed from the individual descriptions*. The object that encompasses the distribution of any joint measurement is the global state vector, which cannot be retrieved from the reduced density matrices since too much information has been traced out.

The global state vector can also serve as a mode of description in which both Alice and Bob take it as the description of their own system. It is complete; however, it is not local, for if Bob makes a local change to his quantum system, it alters its description, which is fine, but it also alters the description of Alice's system. We seem to be stuck in a dichotomy: quantum systems are described either locally or completely, and the appropriate description is chosen based on the problem at hand.

However, this dichotomy was proven false in 2000 by Deutsch and Hayden [4], who showed that the Heisenberg picture admits *descriptors*, which fulfill both Einstein's locality and completeness. To insist, Bob's action on his system alters its descriptor but leaves the descriptor of Alice's system invariant; moreover, the global state vector can be recovered from the pair of descriptors. Note that the existence of such a local mode of description in quantum theory makes the theory local: the existence of non-local ways in which the theory can be expressed is irrelevant. (Terminology-wise, that the wave function is not a local and complete description has been referred to as its *nonseparability* [5,6]. Descriptors are a separable account, and therefore, quantum theory is separable.)

In the more recent literature, there are two other approaches to what can be called quantum locality: the proposals of Raymond-Robichaud. *Evolution matrices* [7] are framed within the quantum formalism. *Noumenal states* [8], on the other hand, take a more general approach, as they apply to a class of theories for which operations form a group. Since quantum theory qualifies, noumenal states can be instantiated in quantum theory. I proved [9] all these modes of description to be formally equivalent, and investigated some of the consequences of embracing these modes of description as an account of reality.

We have been inundated with numerous alternative approaches to quantum theory, and it may appear that I am advocating yet another. However, that is not what it is. The proposal by Deutsch and Hayden hinges on previous work by Gottesman on the Heisenberg representation of quantum computers [10], i.e., the Heisenberg picture of quantum theory applied to networks of qubits. This picture is the way in which the theory was discovered [11] in 1925, with the usual dynamical variables being promoted to dynamical operators. Consequently, what I shall present here is not about interpretation; it is about the mathematical formalism of quantum theory. It is the Heisenberg picture of unitary quantum theory.

If one finds that locality and completeness are not good enough reasons to adopt Heisenberg-picture descriptors as our tentative best account of quantum systems, then how they can be put to work might be more persuasive. To demonstrate the explanatory power of Heisenberg-picture descriptors, I delve into a problem; the problem of teleportation [12]. *How can the real-valued parameters encoding the state at Alice's location make their way to Bob's location via shared entanglement and only two bits of classical communication?*

In this piece, I re-expose and further develop Deutsch and Hayden's solution. If teleportation felt like a magic trick, they unveiled it. After an overview of teleportation in the Schrödinger picture (Section 2) and of Heisenberg-picture descriptors (Section 3), teleportation is revisited in the light of descriptors (Section 4). Then, it is argued that more than two real-valued parameters are "teleported," as descriptors also encompass counterfactual descriptive elements (Section 5). A conclusion (Section 6) and a discussion (Section 7) follow.

2. The Usual Take on Teleportation

The teleportation protocol—whose textbook appearance is displayed in Figure 1—starts by preparing a pair of entangled qubits in the $|\Phi^+\rangle$ state, which is then shared between Alice and Bob. (The four Bell states are $|\Phi^\pm\rangle = \frac{|00\rangle \pm |11\rangle}{\sqrt{2}}$ and $|\Psi^\pm\rangle = \frac{|01\rangle \pm |10\rangle}{\sqrt{2}}$.) Then, or in the meantime, Alice prepares a qubit in the state to be teleported, $|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$. Alice might put her personal diary into α and β, as there is plenty of room in α and β; being complex numbers, they can encode infinitely many bits. Since all qubits involved are initiated in their $|0\rangle$ state, Alice's preparation is seen as an operation U, which takes $|0\rangle$ to $|\psi\rangle$. She then performs a Bell measurement between her shared entangled state and her prepared qubit. It yields two classical bits of output that she communicates to Bob over a classical communication channel, such as a telephone. Bob then manipulates his qubit in accordance with the two bits that he receives. He will, or not, apply the X gate; and he will, or not, apply the Z gate. After Bob's processing of his system, its corresponding state is $\alpha|0\rangle + \beta|1\rangle$. The very fact that a phenomenon is called "teleportation" underlines its puzzlement:

How do α and β make their way from Alice to Bob?

Let me insist that α and β, in principle, contain infinitely many bits, but only two classical bits are communicated.

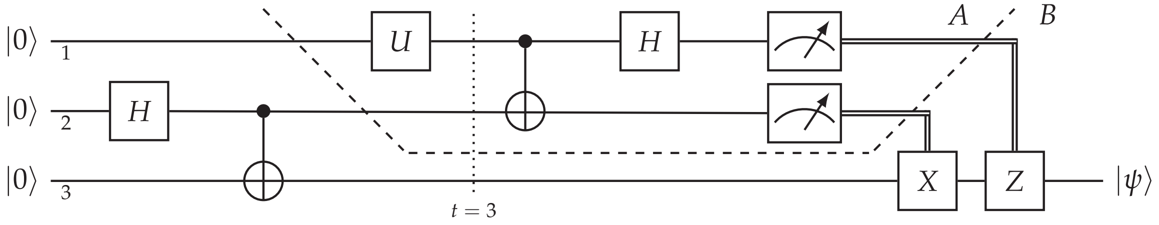


Figure 1. Teleportation in a yet-to-be-defined quantum–classical dualistic theory.

To review the computation in the Schrödinger picture, it is expositive to re-express the global state after $|\Phi^+\rangle$ and $|\psi\rangle$ have been prepared. Calling this time $t = 3$ and disregarding normalization,

$$\begin{aligned}
|\Psi_3\rangle &= |\psi\rangle\otimes|\Phi^+\rangle \\
&= (\alpha|0\rangle + \beta|1\rangle) \otimes (|00\rangle+|11\rangle) \\
&= \alpha|000\rangle + \beta|100\rangle + \alpha|011\rangle + \beta|111\rangle \\
&= \alpha\big(|\Phi^+\rangle+|\Phi^-\rangle\big)|0\rangle + \beta\big(|\Psi^+\rangle-|\Psi^-\rangle\big)|0\rangle + \alpha\big(|\Psi^+\rangle+|\Psi^-\rangle\big)|1\rangle + \beta\big(|\Phi^+\rangle-|\Phi^-\rangle\big)|1\rangle \\
&= |\Phi^+\rangle(\alpha|0\rangle + \beta|1\rangle)+|\Phi^-\rangle(\alpha|0\rangle - \beta|1\rangle)+|\Psi^+\rangle(\beta|0\rangle + \alpha|1\rangle)+|\Psi^-\rangle(-\beta|0\rangle + \alpha|1\rangle) \\
&= |\Phi^+\rangle|\psi\rangle+|\Phi^-\rangle Z|\psi\rangle+|\Psi^+\rangle X|\psi\rangle+|\Psi^-\rangle XZ|\psi\rangle.
\end{aligned}$$

Expressing $|\Psi_3\rangle$ in such a way helps to verify the rest of the protocol; the Bell measurement distinguishes the four possible terms and provides the information about which correction needs to be performed on Bob's system to recover $|\psi\rangle$. Verifying that the protocol achieves the purported functionality does not amount to explaining how it works. If the flexibility in the way $|\Psi_3\rangle$ can be expressed is useful for the verification of the protocol, it is disastrous to provide an explanation of the transmission of α and β, for the mathematical equality between an expression describing α and β at Alice's location and an expression displaying them to be at Bob's location annihilates the hopes to locate the information accurately in the state vector.

Importantly, embracing unitary quantum theory does not lead to much progress. In fact, records of the measurement outcome can be appended as displayed in Figure 2. In this quantum setting, the state vector evolves according to

$$\begin{aligned}
|\Psi_3\rangle &= |00\rangle\big(|\Phi^+\rangle|\psi\rangle+|\Phi^-\rangle Z|\psi\rangle+|\Psi^+\rangle X|\psi\rangle+|\Psi^-\rangle ZX|\psi\rangle\big) \\
|\Psi_5\rangle &= |00\rangle(|00\rangle|\psi\rangle+|10\rangle Z|\psi\rangle+|01\rangle X|\psi\rangle+|11\rangle ZX|\psi\rangle) \\
|\Psi_7\rangle &= |00\rangle|00\rangle|\psi\rangle+|10\rangle|10\rangle Z|\psi\rangle+|01\rangle|01\rangle X|\psi\rangle+|11\rangle|11\rangle ZX|\psi\rangle \\
|\Psi_9\rangle &= (|00\rangle|00\rangle+|10\rangle|10\rangle+|01\rangle|01\rangle+|11\rangle|11\rangle)|\psi\rangle. \qquad (1)
\end{aligned}$$

On the question of the apparent instantaneous transfer in teleportation, what inhibits progress is the Schrödinger picture, for full unitarity does not change the fact that in $|\Psi_3\rangle$, α and β can freely jump around the state vector. Therefore, on Schrödinger's stage, the local transfer remains unseeable, and one might be fooled by the suggestion that α and β really are teleported. Heisenberg takes us backstage.

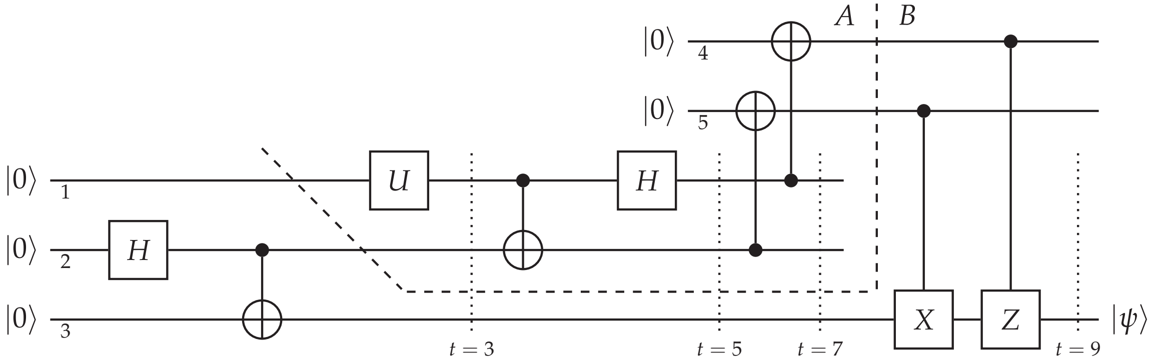


Figure 2. Teleportation in unitary quantum theory.

3. Heisenberg-Picture Descriptors

In this section, I present an overview of how descriptors work. For a thorough exposition, see [13]. A reader who prefers avoiding this section—perhaps due to its more elaborate mathematical content—can have a glance at Equation (2), accept the action of gates on descriptors as given by Equations (5), (6) and (8), and proceed to the following section.

In the Heisenberg picture, observables evolve in time while the state remains constant and conveniently set to $|0\rangle^{\otimes n}$ in the context of quantum computational networks. In front of the uncountable number of observables that evolve in time, the tempting thought is to give up on Heisenberg-picture descriptions. Fortunately, the algebra of observables admits a *generating set*, i.e., a set of operators that multiplicatively generate a basis of all operators. This generating set can be chosen such that only a few operators act non-trivially on each system. These operators are then encompassed into a single object, the *descriptor* of the system, which evolves in time and can be used to calculate any time-evolved observable of that system. Similarly, any time-evolved observable that pertains to a collection of systems can be obtained from the descriptors of those systems. To connect with the more familiar language of the Schrödinger picture, the descriptors corresponding to a collection of systems permit the reconstruction of the density matrix of this collection of systems. In particular, the global density matrix can be obtained from all individual descriptors.

3.1. Qubit Descriptors

Consider a network of n qubits, whose ith qubit is denoted $\mathfrak{Q}_i$. At time 0, the descriptor of $\mathfrak{Q}_i$ can be expressed as the pair of operators acting on $(\mathbb{C}^2)^{\otimes n}$

$$\boldsymbol{q}_i(0) = (q_{ix}(0), q_{iz}(0)) = \left(\mathbb{1}^{i-1} \otimes \sigma_x \otimes \mathbb{1}^{n-i}\,,\, \mathbb{1}^{i-1} \otimes \sigma_z \otimes \mathbb{1}^{n-i}\right), \tag{2}$$

where σ_x and σ_z are Pauli matrices, and $\mathbb{1}^k$ is the identity on $(\mathbb{C}^2)^{\otimes k}$. A third descriptor component, $q_{iy}(0)$, can be obtained as $iq_{ix}(0)q_{iz}(0)$. Descriptors evolve as observables do; namely, if U denotes the evolution operator of what happens to the whole network between time 0 and time t, then

$$\boldsymbol{q}_i(t) = U^\dagger \boldsymbol{q}_i(0) U, \tag{3}$$

where the U acts on both components of $\boldsymbol{q}_i(0)$. Time evolution preserves the algebra of descriptors, which, in the context of qubits, is the Pauli algebra,

$$\begin{aligned}
[q_{iw}(t), q_{jw'}(t)] &= 0 && (i \neq j \text{ and } \forall w, w') \\
q_{ix}(t) q_{iy}(t) &= i q_{iz}(t) && (\text{and cyclic permutations}) \\
q_{iw}(t)^2 &= \mathbb{1} && (\forall w).
\end{aligned}$$

Suppose that between the discrete times $t-1$ and t, only one gate is performed, whose matrix representation on the whole network is denoted G_t. Therefore, $U = G_t V$, where V consists of all gates from time 0 to $t-1$. The evolution of descriptors can also be expressed in a step-by-step fashion,

$$\boldsymbol{q}_i(t) = \mathsf{U}^{\dagger}_{G_t}(\boldsymbol{q}(t-1))\boldsymbol{q}_i(t-1)\mathsf{U}_{G_t}(\boldsymbol{q}(t-1)), \tag{4}$$

where $\boldsymbol{q}(\cdot) = (\boldsymbol{q}_1(\cdot), \dots, \boldsymbol{q}_n(\cdot))$ is the $2n$-component object that encodes the descriptor of each qubit at the corresponding time and $\mathsf{U}_{G_t}(\cdot)$ is a fixed operator-valued function of some components of its argument. The function satisfies the defining equation $\mathsf{U}_{G_t}(\boldsymbol{q}(0)) = G_t$, which is guaranteed to exist by the generative ability of the components of $\boldsymbol{q}(0)$. More precisely, any linear operator G_t can be expressed as a polynomial in the $2n$ matrices $q_{1x}(0)$, $q_{1z}(0), \dots, q_{nx}(0), q_{nz}(0)$, and $\mathsf{U}_{G_t}(\boldsymbol{q}(0))$ is one such polynomial. The expressions (3) and (4) for the evolution of $\boldsymbol{q}_i(t)$ can be recognized equivalent:

$$\begin{aligned} V^{\dagger}G_t^{\dagger}\boldsymbol{q}_i(0)G_t V &= V^{\dagger}\mathsf{U}^{\dagger}_{G_t}(\boldsymbol{q}(0))VV^{\dagger}\boldsymbol{q}_i(0)VV^{\dagger}\mathsf{U}_{G_t}(\boldsymbol{q}(0))V \\ &= \mathsf{U}^{\dagger}_{G_t}\Big(V^{\dagger}\boldsymbol{q}(0)V\Big)V^{\dagger}\boldsymbol{q}_i(0)V\,\mathsf{U}_{G_t}\Big(V^{\dagger}\boldsymbol{q}(0)V\Big) \\ &= \mathsf{U}^{\dagger}_{G_t}(\boldsymbol{q}(t-1))\boldsymbol{q}_i(t-1)\mathsf{U}_{G_t}(\boldsymbol{q}(t-1)). \end{aligned}$$

The second equality follows because in each term of the polynomial $\mathsf{U}^{\dagger}_{G_t}\big(V^{\dagger}\boldsymbol{q}(0)V\big)$, products of components that are surrounded by $V^{\dagger}$ and V will have their inner $V^{\dagger}$s and Vs cancelled, leaving only the outer ones, which can be factorized outside of the polynomial to retrieve the first line.

3.2. Locality and Completeness

The *locality* of the descriptors is due to the fact that if the gate G_t acts only on qubits of the subset $I \subset \{1, 2, \dots, n\}$, then its functional representation U_{G_t} shall only depend on components of $\boldsymbol{q}_k(t-1)$, for $k \in I$. Therefore, for any $j \notin I$, the descriptor $\boldsymbol{q}_j(t-1)$ commutes with $\mathsf{U}_{G_t}(\boldsymbol{q}(t-1))$, and thus remains unchanged between times $t-1$ and t: the description of system A is independent of what is performed with system B, which is dynamically isolated from the former. Einstein's criterion, generalized as above with dynamical isolation instead of spatial separation, stresses that the locality of the mode of description is inherited from the locality of interactions. If, in spacetime, the interactions are constrained by a lightcone structure, the descriptors inherit the constraint, and spatially separated systems shall be described independently of what is performed on the other.

When the constant reference vector $|0\rangle^{\otimes n} \equiv |\mathbf{0}\rangle$ is also taken into account, the descriptors are *complete*. The expectation value $\langle\mathbf{0}|\mathcal{O}(t)|\mathbf{0}\rangle$ of any observable $\mathcal{O}(t)$ that concerns only qubits of I can be determined by the descriptors $\{\boldsymbol{q}_k(t)\}_{k\in I}$. This can be seen more clearly at time 0, where an observable on the qubits of a subset $I \subset \{1, 2, \dots, n\}$ is, like a gate G_t, a linear operator that acts non-trivially *only* on the qubits of I. Again, any such operator can be generated additively and multiplicatively by the components of $\boldsymbol{q}_k(0)$, with $k \in I$. So, there exists a polynomial $f_{\mathcal{O}}$ such that $\mathcal{O}(0) = f_{\mathcal{O}}(\{\boldsymbol{q}_k(0)\}_{k\in I})$. Therefore,

$$\mathcal{O}(t) = U^{\dagger}\mathcal{O}(0)U = f_{\mathcal{O}}(\{U^{\dagger}\boldsymbol{q}_k(0)U\}_{k\in I}) = f_{\mathcal{O}}(\{\boldsymbol{q}_k(t)\}_{k\in I}).$$

(Again, in a term that consists of a product of various components, the inner $U^{\dagger}$s and Us cancel out, and the outer $U^{\dagger}$s and Us can be factored out of the polynomial.) Since the elements of the density matrix can be computed as the expectation value of an appropriate operator, the density matrix $\rho_I(t)$ of the joint subsystems in I can be obtained from $\{\boldsymbol{q}_k(t)\}_{k\in I}$. In what follows, we shall only be interested in computing the reduced density matrix of one qubit from its descriptor. The reduced density matrix $\rho_k(t) = \mathrm{tr}_{\overline{\mathfrak{Q}}_k}(U|\mathbf{0}\rangle\langle\mathbf{0}|U^\dagger)$ of qubit $\mathfrak{Q}_k$ at time t can be expressed in the Pauli basis, like any 2×2 hermitian matrix of trace one, as

$$\rho_k(t) = \frac{1}{2}\left(\mathbb{1} + \sum_{w\in\{x,y,z\}} p_w(t)\sigma_w\right).$$

From the trace relations of Pauli matrices, the components $p_w(t)$ are

$$p_{kw}(t) = \mathrm{tr}(\rho_k(t)\sigma_w) = \mathrm{tr}\left(U|\mathbf{0}\rangle\langle\mathbf{0}|U^\dagger(\mathbb{1}^{k-1}\otimes\sigma_w\otimes\mathbb{1}^{n-k})\right) = \langle\mathbf{0}|q_{kw}(t)|\mathbf{0}\rangle.$$

The second equality comes from that $\rho^A \mapsto \rho^A \otimes \mathbb{1}^B$ is, as a super-operator, the adjoint of $\rho^{AB} \mapsto \mathrm{tr}_B(\rho^{AB})$, and the rightmost equality follows from the cyclicity of the trace. That $q_{ky}(t)$ is not tracked in time is no problem since it can be computed as $iq_{kx}(t)q_{kz}(t)$.

3.3. The Action of Gates

For concrete calculations in a network that admits a fixed set of gates, it is convenient to find the action that each gate has on the descriptors. The matrix representation H_i of the Hadamard gate applied to $\mathfrak{Q}_i$ (and the identity elsewhere) can be expressed as

$$H_i = \frac{q_{ix}(0) + q_{iz}(0)}{\sqrt{2}},$$

which defines its functional representation $\mathsf{U}_{H_i}(\boldsymbol{q}(\cdot))$. From Equation (4), and using the algebra of operators at time $t-1$ to simplify the right-hand side, one finds $\boldsymbol{q}_i(t) = (q_{iz}(t-1), q_{ix}(t-1))$, or more elaborately,

$$(q_{ix}(t-1), q_{iz}(t-1)) \equiv \boldsymbol{q}_i(t-1) \overset{H_i}{\to} \boldsymbol{q}_i(t) = (q_{iz}(t-1), q_{ix}(t-1)).$$

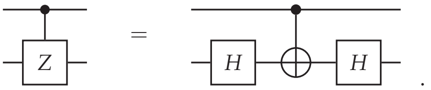
Therefore, the Hadamard gate switches the components of the descriptor on which it acts (regardless of how these components are expressed in terms of Pauli operators at time $t-1$). Abstracting away the time at which the gate occurs, the action of H_i is specified by

$$H_i: \ (q_{ix}, q_{iz}) \to (q_{iz}, q_{ix}). \tag{5}$$

In a like manner, the action of the Cnot can be found to be

$$\text{Cnot}: \left\{\begin{matrix}(q_{cx} & , q_{cz}) \\ (q_{tx} & , q_{tz})\end{matrix}\right\} \to \left\{\begin{matrix}(q_{cx}q_{tx} & , q_{cz} \quad) \\ (q_{tx} & , q_{cz}q_{tz})\end{matrix}\right\}, \tag{6}$$

where the label c refers to the *c*ontrol qubit and the label t, to the *t*arget qubit. The z-component of the control is copied onto the z-component of the target, while the x-component of the target is copied onto the x-component of the control (regardless of what those components are at that time). The action of the controlled-Z gate can be found from



In the teleportation protocol, Alice's preparation consists of a generic one-qubit gate U, which maps $|0\rangle$ to $|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$. Such a generic transformation of SU(2) can be expressed as the exponentiation of a generator, $U = e^{-i\frac{\phi}{2}\hat{\phi}\cdot\vec{\sigma}}$, or alternatively, it can be parametrized with Euler angles as

$$U = e^{i\varphi_3\sigma_z}e^{i\varphi_2\sigma_x}e^{i\varphi_1\sigma_z} = \begin{pmatrix} e^{i(\varphi_1+\varphi_3)}\cos\varphi_2 & ie^{-i(\varphi_1-\varphi_3)}\sin\varphi_2 \\ ie^{i(\varphi_1-\varphi_3)}\sin\varphi_2 & e^{-i(\varphi_1+\varphi_3)}\cos\varphi_2 \end{pmatrix}. \tag{7}$$

Note that α and β are parametrized, as in the first column of U, with respect to $\vec{\varphi} = (\varphi_1, \varphi_2, \varphi_3)$. Since Alice's preparation involves an action only on $\mathfrak{Q}_1$, it can be expressed as in Equation (7) where the components of $\boldsymbol{q}_1(0)$ replace σ_x and σ_z. This thus defines the functional representation of $\mathsf{U}_U(\boldsymbol{q}(\cdot))$ from which the action can be computed,

$$U: \boldsymbol{q}_1 = (q_{1x}, q_{1z}) \to (\mathcal{D}_x^{\vec{\varphi}}(\boldsymbol{q}_1), \mathcal{D}_z^{\vec{\varphi}}(\boldsymbol{q}_1)). \tag{8}$$

Sparing the detailed expressions, $\mathcal{D}_x^{\vec{\varphi}}(\boldsymbol{q}_1)$ and $\mathcal{D}_z^{\vec{\varphi}}(\boldsymbol{q}_1)$ denote two functions that depend on the operators q_{1x}, q_{1z} and on the parameters $\vec{\varphi}$ (and therefore on α and β).

4. The Heisenberg Picture of Teleportation

Teleportation is now revisited in the Heisenberg picture. Descriptors are *such* a local description of quantum systems, that a computation in a network can be carried out by writing the descriptors directly on each qubit wire, as in Figure 3.

The correspondence with the famous result in the Schrödinger picture can be verified by computing the reduced density matrix of $\mathfrak{Q}_3$ at time 9, $\rho_3(9)$. It is expressed by

$$\rho_3(9) = \frac{\mathbb{1}}{2} + \frac{1}{2}\sum_{w\in\{x,y,z\}} \langle\mathbf{0}|q_{3w}(9)|\mathbf{0}\rangle\,\sigma_w,$$

where $|\mathbf{0}\rangle \equiv |0\rangle^{\otimes 5}$ the fixed Heisenberg state. Because

$$\boldsymbol{q}_3(9) = (\mathcal{D}_x^{\vec{\varphi}}(\boldsymbol{q}_1)q_{2z}q_{4z}, \mathcal{D}_z^{\vec{\varphi}}(\boldsymbol{q}_1)q_{3z}q_{5z}) \quad \text{and} \quad \boldsymbol{q}_1(3) = (\mathcal{D}_x^{\vec{\varphi}}(\boldsymbol{q}_1), \mathcal{D}_z^{\vec{\varphi}}(\boldsymbol{q}_1))$$

only differ by operators that have eigenvalues 1 with respect to $|\mathbf{0}\rangle$,

$$\langle\mathbf{0}|q_{3w}(9)|\mathbf{0}\rangle = \langle\mathbf{0}|q_{1w}(3)|\mathbf{0}\rangle. \tag{9}$$

Therefore, $\rho_3(9) = \rho_1(3) = |\psi\rangle\langle\psi|$; the state vector that corresponds to Bob's descriptor after his correction is $|\psi\rangle$.

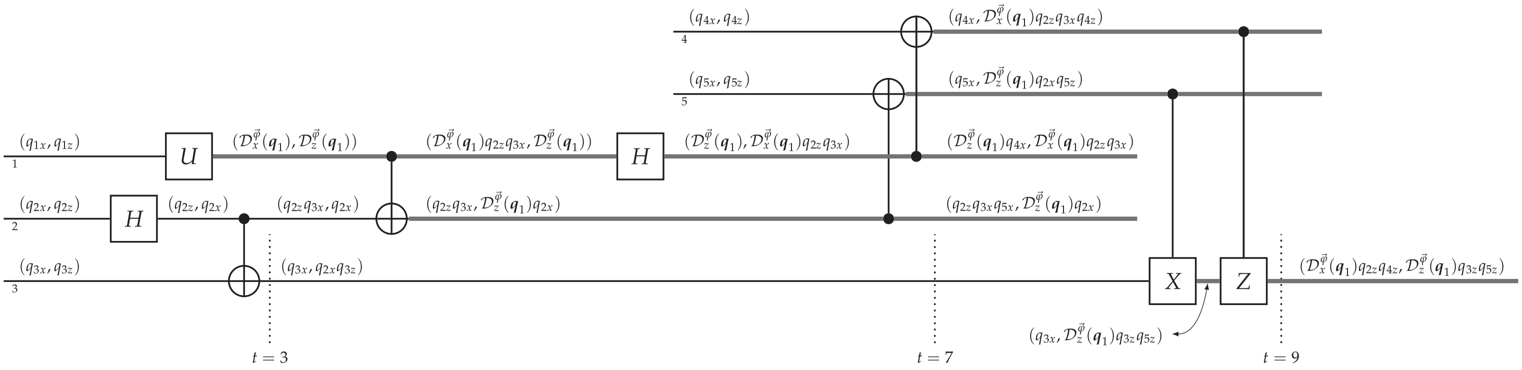


Figure 3. The descriptors in quantum teleportation. The computation starts with all qubits properly initialized to $\boldsymbol{q}_i(0)$, which, for conciseness, is denoted without time dependence as (q_{ix}, q_{iz}). When entering a gate, the components of the input are shuffled into the output in accordance with the action of the gate, which is prescribed by Equations (5), (6) and (8). The information flow of the parameters α and β, encoded in $\vec{\varphi}$, is highlighted in dark grey, with the wires thickened. It spreads locally in the network through the interactions, and, as can be seen, *the "classical" bits are responsible for carrying the parameters encoding Alice's system over to Bob's location*.

4.1. Locally Inaccessible Information

To articulate the localization of information, Deutsch and Hayden did not require a quantitative notion of information. Instead, they coined and worked with the following criteria:

(i) A system $\mathfrak{S}$ is deemed to contain information about a parameter θ if (though not necessarily only if) the probability of some outcome of some measurement on $\mathfrak{S}$ alone depends on θ;

(ii) A system $\mathfrak{S}$ is deemed to contain no information about θ if there exists a complete description of $\mathfrak{S}$ that satisfies Einstein's criterion and is independent of θ.

Following these criteria, Deutsch and Hayden realized that there is such a thing as *locally inaccessible* information, namely, information that is present in a system but does not affect the probability of any outcome of any possible measurement on that system alone. Notably, in the communication channel used in teleportation, α and β are locally inaccessible. Indeed, the collection of systems $\mathfrak{Q}_3$, $\mathfrak{Q}_4$ and $\mathfrak{Q}_5$ at time 7 contains information about α and β since, as the rest of the protocol shows, the parameters can crop up in the probability distributions of some measurement brought about only by those systems. However, by (ii), α and β do not reside in $\mathfrak{Q}_3$, for $\boldsymbol{q}_3(7)$ is independent of them. Therefore, α and β are located in $\mathfrak{Q}_4$ and $\mathfrak{Q}_5$, notwithstanding their associated density matrix proportional to the identity. Being locally inaccessible, the information about α and β that is carried in $\mathfrak{Q}_4$ and $\mathfrak{Q}_5$ remains unaffected by measurements, and thus it remains unaffected by decoherence. This observation has prompted Deutsch and Hayden to realize the tradeoff between local accessibility and robustness to decoherence in information transfer.

4.2. On the Classicality of the Bits

The discoverers of teleportation pointed out in the first sentence of their abstract that the process relies on "purely classical information". However, as a potential critique might have it, the use of the quantum bits $\mathfrak{Q}_4$ and $\mathfrak{Q}_5$ as a communication channel contradicts the requirement that classical bits are to be utilized. Not only does this, *prima facie*, undermine the very purpose of teleportation by seemingly having a quantum channel already in place, but it appears to be a flagrant category mistake: classical and quantum bits are of a fundamentally different kind, one might argue.

The claim of "purely classical information" is the crucial element in the conjuring trick's setup. Let us not be fooled by it, for *there is no such thing, fundamentally, as purely classical information*: either quantum theory holds universally, or it does not, but in the latter case, an explanatory theory about a boundary of its domain of validity is required. Everett's proposal [14] was that although measurement interactions seem to impose a boundary on the domain of unitary quantum theory, they do not. The key to unraveling teleportation is to accept that, by the same token, classical information also does not push against the domain of unitary quantum theory; rather, it is absorbed by it. For a unified theory, the primary concern is to explain "purely classical information" in terms of quantum systems [15,16] and not vice versa. Whatever it is that we view as purely classical information is instantiated in physical systems, which, after all, satisfy quantum theory.

Yet, it can be argued that, indeed, the two bits are classical, according to explanations of what "classical" can mean *within* quantum theory. First, as illustrated in Figure 4, a nearby environment can be modeled to decohere the two qubits on the basis that has been selected by the measurement interaction. The environment contains at least the logical space of two qubits, whose descriptors $\boldsymbol{q}_E$ and $\boldsymbol{q}_{E'}$ are given by some generic representation of the Pauli algebra, i.e., they need not be initialized as in Equation (2). When the environment is affected by the records $\mathfrak{Q}_4$ and $\mathfrak{Q}_5$, the records are also affected: the x-components of the environment reach the x-components of the records. However, these operators do not make it any further towards $\mathfrak{Q}_3$ because the interactions that follow involve the records as control qubits, and so only pass on their z-components. Therefore, a decohering environment does not prevent teleportation; the transfer is *robust to decoherence*, a distinguishing property of classical communication.

Second, I suggested that the classical communication channel could be thought to be a telephone, but in the protocol displayed in Figure 3, the records $\mathfrak{Q}_4$ and $\mathfrak{Q}_5$ are physically brought to Bob's location. In usual classical communication channels, precise (quantum) systems are seldom sent from one location to another; rather, the information is transmitted through a *chain reaction* that occurs in a collection of quantum systems. Therefore, a telephone, or more generally, the relevant degrees of freedom in classical communication, can be modeled by a chain reaction that involves the prepared systems $\mathfrak{Q}_{4'}$, $\mathfrak{Q}_{5'}$, $\mathfrak{Q}_{4''}$, $\mathfrak{Q}_{5''}$, and generically many others. See Figure 4 for the calculation, which yields a final descriptor with dependencies on $q_{4'z}, q_{4''z}, q_{5'z}, q_{5''z}$. If the telephone line is properly initialized with descriptors of the form (2), the operators appended to the final descriptor do not affect the expectation values in (9) because they have eigenvalue 1 with respect to $|\mathbf{0}\rangle$; the Schrödinger state corresponding to $\mathfrak{Q}_3$ after the process remains $|\psi\rangle$. *Thus, not only is the information transfer robust to decoherence, but it is realizable in a chain reaction that resembles classical communication.*

Note, moreover, that decoherence can occur everywhere in the telephone line without inhibiting the transfer of $\mathcal{D}_x^{\vec{\varphi}}(\boldsymbol{q}_1)$ and $\mathcal{D}_z^{\vec{\varphi}}(\boldsymbol{q}_1)$ on Bob's final descriptor. As a final remark: where is Bob? Is he not supposed to receive the communication and act on $\mathfrak{Q}_3$ in accordance with the received bits? Yes, but this is taken into account in the generically many other systems involved in the communication line. No special assumptions are required to include Bob in Figure 4, only his ability to manipulate systems mechanically and decoherently, like any other parts of the communication line.

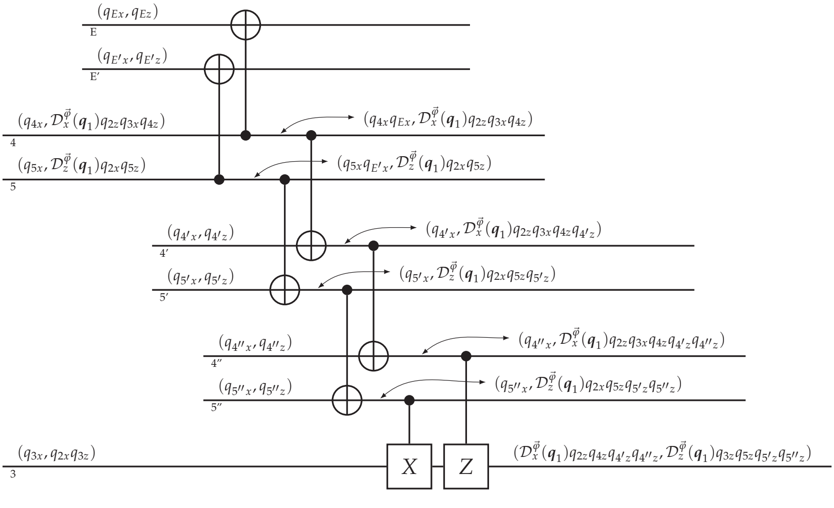


Figure 4. A telephone.

4.3. *Explaining the Information Transfer*

So, how do α and β make their way from Alice's location to Bob's? The Copenhagen tradition would have it due to the collapse of the state vector, which has prompted many to view the transfer as instantaneous, through action at a distance. Jozsa suggested that "nonlocal influences" allow them to "fly across the entanglement" [17]. For Penrose, the entangled pair has a channel that "proceeds into the past" [18] and into the future again. In the Bohmian theory, the "transfer is mediated by the nonlocal quantum potential" [19]. Vaidman suggests that "the nonlocality of Everett's world is the basis of the teleportation of quantum information" [20]. (As Popescu mentioned [21], these worlds should be called *"Lev's worlds"*, for they extend arbitrarily far in space and are constructed from so-called "macroscopic objects" in a "definite classically described state" [22], but no signs of such concepts at the fundamental level can be found in Everett's writings.) Most proponents of unitary quantum theory in the Schrödinger picture do not resign to such extravagant conjectures, for they do not attempt to amend quantum theory. Braunstein argued that "the quantum information is 'hidden' within the correlations between the system and the environment while being wholly absent from any of the individual subsystems" [23], and Timpson wrote that "global rather than local properties are being used to carry information" [24]. These proposals are confined by the unfulfillment of Einstein's criterion of locality by the global state vector, which prevents the localization of the parameters in the Schrödinger picture. However, they can be localized in the Heisenberg picture. According to Deutsch and Hayden, the information about α and β is transported "simply, prosaically, in the qubits $\mathfrak{Q}_4$ and $\mathfrak{Q}_5$ as they traveled from A to B".

5. Counterfactual Elements of Reality

Despite being expressed in terms of two complex numbers, the state prepared at Alice's location, $\alpha|0\rangle + \beta|1\rangle$, contains only two free (real) parameters due to the constraints imposed by normalization and the irrelevance of the phase factor. The latter can be considered by demanding that $\alpha = |\alpha|$, which also fixes $|\beta|$ through normalization, so viewed in this way, the second free parameter is the phase of β, $\arg(\beta)$. Alice's preparation U, however, is a generic one-qubit gate, which, as parameterized by $\vec{\varphi}$ in Equation (7), contains three free parameters. (In fact, a generic element of $\mathrm{U}(2)$ contains four free parameters. However, since $U \in \mathrm{SU}(2)$, $\det U = 1$, which almost amounts to quotiating the global phase of the unitary operator. "Almost", because there remains a $\mathbb{Z}_2$ ambiguity due to a possible factor $-\mathbb{1}$, which leaves the determinant equal to 1 while being a non-trivial global phase. However, the possibility for this factor can be avoided by suitable constraints on the domain of $\vec{\varphi}$.) Since U can be expressed as

$$
\begin{aligned}
U &= e^{i(\varphi_1+\varphi_3)}\begin{pmatrix} \cos\varphi_2 & e^{-2i(\varphi_1+\varphi_3)} i e^{2i\varphi_3}\sin\varphi_2 \\ i e^{-2i\varphi_3}\sin\varphi_2 & e^{-2i(\varphi_1+\varphi_3)}\cos\varphi_2 \end{pmatrix} \\
&\equiv (\eta^*)^{1/2}\begin{pmatrix} \alpha & -\eta\beta^* \\ \beta & \eta\alpha \end{pmatrix},
\end{aligned}
$$

where $\alpha \in \mathbb{R}^+$ and β are the amplitudes of the prepared state, and the extra parameter η, of the unit norm, labels a one-parameter family of states that are legitimate images of $|1\rangle$ under U: each of the $\eta(\beta^*|0\rangle + \alpha|1\rangle)$ are orthogonal to $\alpha|0\rangle + \beta|1\rangle$. Therefore, the descriptor's components $\mathcal{D}_x^{\vec{\varphi}}(\boldsymbol{q}_1)$ and $\mathcal{D}_z^{\vec{\varphi}}(\boldsymbol{q}_1)$ that depend on $\vec{\varphi}$ can alternatively be thought to depend on $|\alpha|$, $\arg(\beta)$ and η. Since the descriptor of $\mathfrak{Q}_3$ at time 9 also carries the dependency on η, a question arises: *has η also reached to Bob's location?*

The Instrumentalist Temptation

Deutsch and Hayden's criteria remain silent on the question of the localization of η. Indeed, even if all systems are collected at some time after the preparation, there exists no measurement on the network as a whole whose distribution of outcomes would depend on η. Therefore, η fails to fulfill criterion (i), even if $\mathfrak{S}$ is taken to be the network as a whole. A tempting view is to dismiss the existence of all that is oblivious from experiments, which embodies the instrumentalist attitude, namely, the consideration of scientific theories as mere tools for predictions. In spite of Deutsch and Hayden's warning "(though not necessarily only if)", which insists that criterion (i) is only sufficient, instrumentalism would also demand it be necessary in a strong sense; namely, it might demand that if the distribution of some measurement outcomes on $\mathfrak{S}$ alone is independent of θ, then θ is not a descriptive element of $\mathfrak{S}$. Not only would η be deemed to be absent from the system, but α and β, too, could not be thought to be localized in $\mathfrak{Q}_4$ and $\mathfrak{Q}_5$ as they are transferred. The Heisenberg-picture description would vanish, for what remains after the instrumentalist's mutilation would be informationally equivalent to the global state, and Raymond-Robichaud ([7], Section 4) showed that any attempt to build a local and complete description of quantum systems from the state vector alone must fail. (In

Timpson's terminology [25], the instrumentalist's mutilation amounts to shifting from the ontological to the conservative interpretation of Heisenberg-picture descriptors. In Raymond-Robichaud's [7], from noumenal to phenomenal states.)

In his thesis, Everett criticizes instrumentalism [14]:

> It is necessary to say a few words about a view which is sometimes expressed, the idea that a physical theory should contain no elements which do not directly correspond to observables. This position seems to be founded on the notion that the only purpose of a theory is to serve as a summary of known data, and overlooks the second major purpose, the discovery of totally new phenomena. The major motivation for this viewpoint appears to be the desire to construct perfectly "safe" theories which will never be open to contradiction. Strict adherence to such a philosophy would probably seriously stifle the progress of physics.

To embrace the full power of Heisenberg-picture descriptors is not merely to view them as another way to think of quantum theory, which may be convenient in some cases—for instance, to make sense of teleportation. Nor is it a tool whose sole purpose is to make predictions. It is to consider them as an account of reality. The reality captured by the descriptors is larger than that captured by the universal state vector [9,26,27]. In particular, it has room for η. The descriptive elements which, like η, lie in Heisenberg-picture descriptions but not in the Schrödinger state are globally inaccessible (not just locally). They reside in the multiverse, yet, in some unobservable sector, for only the sector which is singled out by the Heisenberg state is amenable to observations. The unobservable sector encompasses η and, more generally, all that resides in the global unitary operator that embodies the dynamics and yet is beyond the "column" selected by the Heisenberg state. All of it is *counterfactual descriptive elements of reality*; it accounts for what would be accessible had some prior operation been performed. In the teleportation setting, η would have cropped up in the distributions had $\mathfrak{Q}_1$ been rotated anyhow except around the Z-axis before being prepared with U.

6. Conclusions

The solution by Deutsch and Hayden to the problem of teleportation provides a probe into the classical realm, which signals that it is much deeper than expected. In fact, it is even deeper than expected from the Schrödinger picture of unitary quantum theory. Anyone who takes for granted that communication between Alice and Bob involves "purely classical information" is fooled by teleportation. The classical realm is quantum; a classical communication channel is one that is robust to decoherence and realizable in a chain reaction in quantum systems.

Explaining classical communication from some interaction within quantum systems might seem radical at first glance. However, the opposite is true. If one posits that quantum theory does not universally hold, then one must explain where its boundary resides and why. The proposal here simply follows Everett's program to take the quantum theory seriously and, in the absence of the need to introduce a boundary to its domain of applicability, consider it universal.

Unitarity does not fully clarify the explanation of teleportation in the Schrödinger picture. The explanation presented here is only possible in the Heisenberg picture of unitary quantum theory. Those accustomed to unitary quantum theory (i.e., Everettian quantum theory) shall see arguments for adopting and further developing the Heisenberg picture. However, those who are still agnostic about how to "interpret" quantum theory—namely, still deciding whether unitary quantum theory needs to be truncated, merged with another theory, or completed in some way—will see in the proposed explanation of teleportation arguments for both the Heisenberg picture *and* for unitary quantum theory, as their conjunction solves the problem of the locality of information transfer in teleportation. Progress can be assessed by the problems that are solved. When I explained the teleportation protocol in the Heisenberg picture to Gilles Brassard, one of the discoverers of the phenomenon, he right away told me that it was the most satisfactory elucidation of teleportation that he had ever heard. I hope that this piece can have a similar effect on you.

Teleportation is not the only phenomenon whose apparent non-locality has been puzzling. Heisenberg-picture descriptors also make locality manifest in superdense coding [13] and in the Aharanov–Bohm effect [28].

I shall conclude with a brief reflection on Lev's problem, *why is Everettian quantum theory not in the consensus?* Deutsch wrote [29]

> Some people may enjoy conjuring tricks without ever wanting to know how they work. Similarly, during the twentieth century, most philosophers, and many scientists, took the view that science is incapable of discovering anything about reality. Starting from empiricism, they drew the inevitable conclusion (which would nevertheless have horrified the early empiricists) that science cannot validly do more than predict the outcomes of observations, and that it should never purport to describe the reality that brings those outcomes about. This is known as instrumentalism.

The prevalence of instrumentalism might be a part of the explanation as to why Everettian quantum theory is not in the consensus. The denial of taking a theory seriously as a tentative account of reality also denies a proper investigation of its consequences. Moreover, satisfaction with mere predictions entails satisfaction with conjuring tricks: why should we strive for an explanation of teleportation when we already have a theory that predicts Bob's observations after the protocol?

7. Discussion

Lev Vaidman: If I understand correctly, the story, your story, is about the universe. When we talk about teleportation, we talk about our world. And in the many world's interpretation, there is a part that concerns the whole universe. It is the part of the MWI where there is no collapse. There is no collapse here, no question. But there are no many worlds. Many worlds is when I perform my measurement, I split the world. In teleportation, in every world, α and β jump on Bob's qubit, and they jump at the moment of the measurement. So there is no other explanation within the world.

CAB: In the teleportation protocol, the records of the measurement eventually affect—and get entangled with—many other record-like systems, as well as many systems in the environment. In the Schrodinger picture, this leads to a wave function with four highly entangled terms, which, for all practical purposes, can no longer interact with one another via quantum interference: each term becomes autonomous. What is more, in each term, there are relative properties between systems, which give a consistent account of what resembles a quasi-classical single "world". This is the quantum theory of Everett, the unitarily evolving universal wave function, with important analyses further developed by Zeh, Zurek, Gell-Mann, Hartle, Saunders, Wallace and others.

Is it also yours? It appears to me that you grant fundamental importance to notions such as "macroscopic objects" in a "definite classically described state" from which you define your "worlds" (as the totality of all such objects) [22]. The importance that you grant those words is also manifest in your attempt at explaining teleportation within those worlds as if the universal wave function was not a fundamental description but just a convenient way to stack all those worlds together. Moreover, defining worlds via intuitive appeals to classicality and macroscopicity leads you close to a collapse theory; and in both cases, one is forced to suggest that in teleportation, α and β jump to Bob's qubit.

To come back to descriptors, they also admit a decomposition into a sum of *relative descriptors*, which, like in the Schrödinger picture, account for relative properties between systems. Yet, descriptors are foliated locally. See [30].

Andrew Jordan: Let me make a critical comment. You made the claim that the bits that are transmitted between Alice and Bob by the telephone are really secretly quantum bits, and

I must object to that because I think that if you claim that, then the logical consequence is that there is no such thing as classical information theory. I think you have to give up on classical information theory as a thing that exists. You say that, really, everything is quantum information theory. But there are classical bits. And we are communicating with a classical channel, and so there are classical channels. So how do you respond to that criticism?

CAB: Classical information theory can still exist; however, not fundamentally. In fact, we know well how it is instantiated as a subcase of quantum information theory: a decohered state has a diagonal reduced density matrix whose numbers form a distribution, so we speak of its Shannon information. However, this misses my point. I took it as a premise that the world is quantum. A telephone is made of quantum systems. And yes, it looks like it is classical to me, but that is the program launched by Everett, namely, to understand how the quantum theory can explain the emergence of the classical.

Simon Saunders: Rather similar question: I do not quite get it. The classical channel is really just telling Bob what the outcome of Alice's Bell measurement is. There is very little information there, whereas α and β encode potentially vast amounts of information. So I just don't quite get that. Can you elaborate?

CAB: The channel is indeed telling Bob what the outcome of Alice's Bell measurement is. But it is not *just* doing that. Wouldn't you grant that any sort of classical channel that we can imagine is ultimately made of quantum systems? *This is not an irrelevant fact when we are trying to solve the capacity problem of teleportation.* The quantum systems involved in the communication line transfer α and β in a way that is locally inaccessible, resilient to decoherence and realizable in a chain reaction.

Eric Curiel: I have a quite general question about the approach. How should I understand entanglement entropy in this picture? It plays a very fundamental role from condensed matter physics to black hole thermodynamics. How should I understand what seems to be the manifest non-locality of quantum mechanics that makes the efficacy of Von Neumann entanglement entropy possible?

CAB: Since density matrices can be recovered from descriptors and the constant Heisenberg state, so can Von Neumann entropy. But one way to understand entanglement between systems that is more in line with the Heisenberg picture is that no observable of a subsystem has a definite outcome, while some observables on the joint system do. For instance, the preparation of $|\Phi^+\rangle$ on $\mathfrak{Q}_2$ and $\mathfrak{Q}_3$ yields the following descriptors (see Figure 3):

$$\boldsymbol{q}_2(3) = (q_{2z}(0)q_{3x}(0), q_{2x}(0)) \qquad \text{and} \qquad \boldsymbol{q}_3(3) = (q_{3x}(0), q_{2x}(0)q_{3z}(0)).$$

None of the observables corresponding to the descriptor components have a definite outcome since their expectation value is 0 while their spectrum is ± 1. This also holds for $q_{2y}(3)$ and $q_{3y}(3)$, and so for any observable obtained as a linear expansion of the descriptor components. However, $q_{2x}(3)q_{3x}(3)$, $-q_{2y}(3)q_{3y}(3)$ and $q_{2z}(3)q_{3z}(3)$ have a definite outcome: 1.

Tim Maudlin: I have two comments of a different character. One is just coming back to this telephone. There is only one information theory; it is Shannon information. You can apply it to bits, which by definition have only two possible states, you can apply it to spin 1/2 particles that have infinitely many possible states, given by your α and β. It doesn't change information theory at all. In this protocol, all that is required to implement the protocol are two bits. That is all that is required. You may say, "Oh, but I have to send a quantum system physically because physics is quantum mechanics." It doesn't matter if Alice sends a note classically; of course, it has more than two states, right? She can write in cursive, or she can write this way, so what? The point is that the protocol merely demands that you resolve between four possibilities. That requires two bits of information. Period end of the story.

The other comment is: the reason they gave that Nobel prize was for tests of violation of Bell's inequality at spacelike separation. That's the reason they say it shows nonlocality. Quantum teleportation is puzzling, but one thing it sure doesn't do is violate Bell's inequality at spacelike separation. So to say, even if it were true, "I have a local understanding of teleportation" would not at all have any influence on the reason they say those experiments are so important.

CAB: The two comments are not of a different character: they answer one another. The primality of Shannon's information in one's mind makes one uncritical not only of its use in teleportation but also of the way in which the assumptions are coined in Bell's theorem, namely, in terms of classical probability distributions. For whom the very use of classical probability distributions is not considered to be an assumption made by Bell, then indeed, the violation of Bell's inequality at spacelike separation challenges locality. Otherwise, the violation simply dismisses the hypothesis that quantum theory can be underlain by classical probability distributions.

David Wallace: I want to go back to the Deutsch–Hayden claim that once I have a local formulation of the theory, then the theory is local. The worry is that there are relatively clear cheap ways of making a theory local. I'm not claiming this is a cheap way. But there are cheap ways. For instance, I can just attach a copy of the state of the universe to every local system, I can say whatever my wildly nonlocal description is, in my new theory, the state of the system is the ordered pair of the state of the old theory and the state of the universe. It is horrendously expensive; call that a monadology move, for Leibniz's fans. That framework is formally going to be local, but clearly, it is not telling us that the theory is interestingly local. I don't think that the framework of descriptors has this character, although there are bits of it that sometimes worry me. But I just want to flag that a bit more needs to be done to clarify that a theory is local just because it has a local formalism. I think we have to avoid making moves of that kind.

CAB: What you suggest does not fulfill Einstein's criterion because if a state of the whole universe is included in the description of each localized system, then if Bob performs an operation on his system, it affects Alice's description.

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Local Quantum Theory with Fluids in Space-Time

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Abstract: In 1948, Schwinger developed a local Lorentz-covariant formulation of relativistic quantum electrodynamics in space-time which is fundamentally inconsistent with any delocalized interpretation of quantum mechanics. An interpretation compatible with Schwinger's theory is presented, which reproduces all of the standard empirical predictions of conventional delocalized quantum theory in configuration space. This is an explicit, unambiguous, and Lorentz-covariant "local hidden variable theory" in space-time, whose existence proves definitively that such theories are possible. This does not conflict with Bell's theorem because it is a local many-worlds theory. Each physical system is characterized by a *wave-field*, which is a set of indexed piece-wise single-particle wavefunctions in space-time, each with its own coefficient, along with a memory which contains the separate local Hilbert-space quantum state at each event in space-time. Each single-particle wavefunction of a fundamental system describes the motion of a portion of a conserved fluid in space-time, with the fluid decomposing into many classical point particles, each following a world-line and recording a local memory. Local interactions between two systems take the form of local boundary conditions between the differently indexed pieces of those systems' wave-fields, with new indexes encoding each orthogonal outcome of the interaction. The general machinery is introduced, including the local mechanisms for entanglement and interference. The experience of collapse, Born rule probability, and environmental decoherence are discussed, and a number of illustrative examples are given.

Keywords: many worlds; local quantum theory; local hidden variable theory; relativistic quantum theory; Bell's theorem

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1. Introduction

Despite insubstantial but influential claims from the early days of quantum theory, Bohm proved in 1952 [1] that it is possible to give a straightforward realist interpretation of quantum mechanics with particles in space-time. However, in that theory the underlying physics occurs in a higher-dimensional configuration space, resulting in explicitly nonlocal dynamics in space-time. In this article we lay out the general framework for a local realist collapse-free theory of quantum mechanics, and work through the simplest examples, with all dynamics occurring explicitly in space-time. This realizes an unachieved goal of Einstein, Schrödinger, and Lorentz, who were never satisfied with the configuration space treatment, precisely because it introduced fundamental nonlocality [2]. The new model makes identical empirical predictions to standard quantum theory, and can serve as a full replacement. This model is consistent with the Lorentz covariant Heisenberg-Schrödinger model proposed by Schwinger in 1948 [3], and restores the equivalence between the local Heisenberg and Schrödinger pictures. However, we now know from Bell's theorem [4–8] that if we wish to maintain independence of measurement settings, then this is unavoidably a theory of many local worlds [9].

It is important to emphasize here the breadth of Schwinger's accomplishment. In deriving quantum electrodynamics (QED) in parallel to Feynamn, he obtained a new Lorentz covariant state vector, defined on a single space-like hypersurface (which can be the 'present' hypersurface in at most one specific frame), with information at each point in the surface restricted to that point's past light cone (the events on a space-like hypersurface are space-like separated in all frames, and have the same past events in their respective past light cones). He also obtained the localized Schrödinger-like dynamics that shows how this state evolves locally to the next parallel space-like hypersurface, and obtained a space-time invariant local interaction unitary for QED. This treatment is at the heart of modern particle physics, but these state vectors are completely inconsistent with the configuration-space wavefunctions in prevalent use throughout modern quantum foundations and information theory. To be very clear, the Lorentz covariant state vectors of the most precisely verified theory of modern physics are defined on space-like hypersurfaces in space-time, where each event on such a surface contains separate physical information, which can only pertain to past events within its past light cone. *The apparent nonlocality of conventional quantum theory is a mathematical artifact of projecting all of the space-like separated information from an entire Schwinger state, defined on a given 'present' hypersurface, into a single nonseparable delocalized state.* Furthermore, there is generally not enough information in such a delocalized quantum state to reconstruct the corresponding Schwinger state. Thus, the entangled wavefunctions of Copenhagen and spontaneous collapse theories, or the universal wavefunctions of Wheeler-DeWitt, Everett, Bohm, and others, are all delocalized approximations of a fundamentally local state in QED. This fact is not at all obvious, because one needs a proper local many worlds interpretation with explicit local hidden variables in space-time to make sense of empirical observations.

The present model is an attempt to interpret the empirical data from table-top quantum experiments, rather than high energy particle collisions, using the QED structure, by reconstructing the information content of Schwinger's space-time state vector using more familiar single-particle spatial wavefunctions. This turns out to be the natural theoretical framework for refining the local Schrödinger picture of the Parallel Lives interpretation of quantum mechanics [10,11], and should also be consistent with (but not identical to) the local Heisenberg picture frameworks that have been developed elsewhere [12–22]. I recommend perusing the detailed examples in the Supplemental Information as you read this article to help develop a clear idea of the local hidden variable treatment for Wigner's friend, Mach-Zehnder Interferometers, Wheeler's Delayed Choice, the Delayed Choice Quantum Eraser, and Quantum Teleportation. This model is a major paradigm shift from standard quantum theory, and while many familiar mathematical objects are still present, they have been put together in an entirely different way. The detailed examples should help to develop intuition for the new paradigm, and to put aside intuition from standard quantum theory which does not apply here.

In the present model, all (quantum) systems are comprised by pseudo-classical fluids in a single objective locally-Minkowski space-time and the classical particles in these fluids follow world-lines through that space-time. To give an explicit example, the probability density $|\psi(\vec{x})|^2$ for a standard single-electron wavefunction is re-interpreted as the local density of a literal fluid in space-time, and the conservation of probability current is re-interpreted as conservation of fluid current. Thus, a single electron with a spatially distributed wavefunction is interpreted as an entire fluid made of a countably infinite number of fluid particles, each of which is like a classical point particle on a world-line. There are many worlds ***only*** in the sense that there are many world-lines for the many such particles in space-time, and each particle experiences a unique perspective from its location in space-time. According to relativity theory, all empirical experiences necessarily follow from these unique local perspectives, and are fully restricted to an observer's past light cone. That is, for each observer, the 'world' is the image of the surface of that observer's past light cone. This definition makes the set of events in a 'world' Lorentz invariant. There are no global 'worlds' in this theory - there is only the one global space-time, containing many particles on world-lines, each with its own past light cone and 'world', as shown in Figure 1. To be very explicit, even though their resolutions to the measurement problem are similar, the local space-time model presented here is fundamentally different from the many-worlds theory of Everett [23,24], which is delocalized in configuration space, and describes global worlds in a particular Lorentz frame. There is no space-like hypersurface that is observed by even one, let alone many observers, and it is a mistake to define global worlds on these hypersurfaces. This is corroborated by the fact that one cannot Lorentz transform the delocalized wavefunctions defined for these surfaces between different inertial frames (a Lorentz transform is a mapping that can act only on a 4-vector or field tensor at a single event, and the descriptions of these 4-vectors and fields are inherently separable event-by-event). All empirical data pertains to events within the observer's past light cone, and that data defines the world for that observer. Importantly, no observer's world contains the results of two space-like separated measurement events until signals have arrived to the observer from both of those events (i.e., once the observer has seen the empirical data).

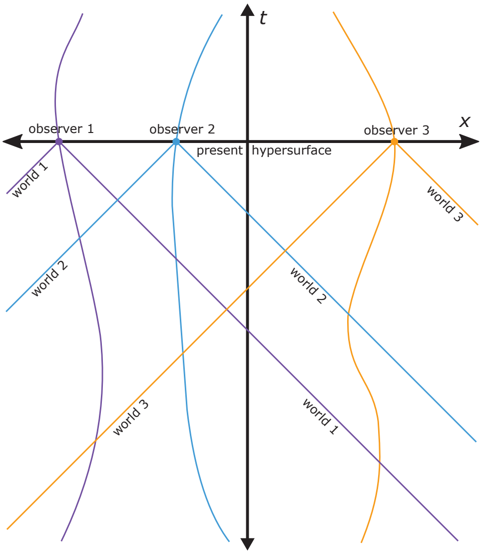

Figure 1. The distinct perspectives of observers on different world-lines in special relativity already provide a clear notion of many worlds. Each fluid particle has its own world in exactly this sense, and the worlds of two particles can only coincide if and when they are both present at the same event. No one observes a space-like hypersurface, so it is a mistake to define global worlds on those surfaces.

It should be noted that there are some similarities between the present model and the work of Madelung [25], and also various works on many-interacting-worlds [26–31] for a single quantum particles, but the details of the present model are distinct.

We will not be working with the individual trajectories of the classical particles in the fluids here, since we do not yet know how to choose a unique solution. The decomposition of the conserved single-particle quantum probability current $\vec{j}(\vec{x})$ into fluid streamlines where the velocity field is given by $\vec{v}(\vec{x}) = \vec{j}(\vec{x})/|\psi(\vec{x})|^2$ serves as the simplest proof-of-concept example of a viable set of trajectories - but this is not the only possible set of trajectories consistent with $\vec{j}(\vec{x})$ and $|\psi(\vec{x})|^2$.

Here we interpret the single-particle Schrödinger/Dirac equations to be continuum fluid equations consistent with coarse-graining over the trajectories of the individual particles comprising the fluid - which is also to say taking the smoothed local average over their (unknown) velocity distribution. The behavior for multiple quantum particles is completely different than in the standard treatment, which is the main focus of this article. The empirical experience of collapse and many of its consequences are explained later, but for now, the right intuition is that each fundamental quantum system comprises a conserved fluid in space-time - and it helps to keep in mind that the fluid is composed of classical particles on world-lines.

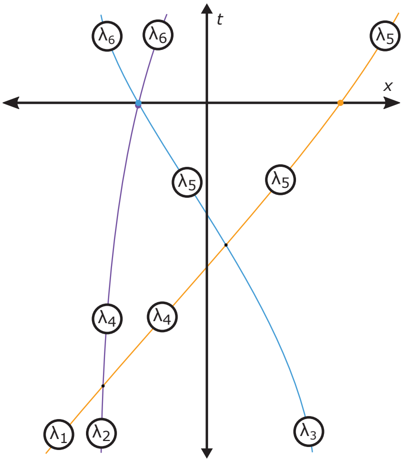

Figure 2. A local ballistic model is an unambiguous local hidden variable theory wherein all causal information is carried along world-lines in point-like packets. When two or more such packets meet at an event in space-time, their information undergoes a joint evolution resulting in new information that all of the packets may carry away (e.g., λ_4 results from the joint evolution of λ_1 and λ_2, etc.).

This is a *local ballistic* model of the universe, meaning all interactions are local scattering events between ballistic classical particles, and there are no nonlocal or long-range interactions or objects of any kind (i.e., all long-range effects are mediated by force-carrying particles on world-lines which undergo local collisions). In the most general local ballistic model, classical particles can carry an internal memory containing an arbitrary amount of information, and when two particles interact locally at an event in space-time, their two memories undergo a joint evolution, as shown in Figure 2. In the coarse-grained fluid picture, the set of scattering rules for such local collisions should ultimately come from the Standard Model Lagrangian, and these take the form of boundary conditions between different packets of fluid, while the memories become local properties of the continuum fluid packets. It should be emphasized that the information in the internal memories cannot be directly measured, which makes it fundamentally different from various types of practical physical memory, so unlike measurable physical memories, there is no energy density associated with these internal memories - just as there is no energy density associated with the information content of the wavefunction in standard quantum theory.

A single quantum system may comprise a superposition of many different indexed single-particle wavefunctions, each evolving independently of the others in space-time, in the absence of an interaction with another system. We can think of the indexes that delineate the different wavefunctions of a given system as belonging to its local memory, along with a separable local copy of the entangled state from which the indexes are drawn (in this model, an entangled state is just a piece of information that lives at a single event in space-time, not a delocalized nonseparable object spread across multiple events). For each system, it is the local scattering interactions with other fluid particles of the same system, with the same indexes, that produces the collective Schrödinger/Dirac wave evolution in the fluid, and interactions with other systems can result in local entangled states with more distinct indexes, and thus more distinct wavefunctions for each system.

We call the collective description of all indexed packets of a quantum system in space-time a *wave-field*. As we will show later, the wave-field for a single fundamental system is expressed as a piece-wise multi-valued wavefunction in space-time, where each indexed value evolves independently according to the single-particle Schrödinger/Dirac equation. The pieces are separated in space-time by interaction-based boundary conditions, which are the locations where the fluid particles scatter and their internal memory states synchronize and evolve. The synchronization of the internal memory states generally increases the respective numbers of orthogonal terms, and thus also increases the number of indexed wavefunctions on the other side of the boundary. The wave-field of a system is a separable mathematical description for that system alone - even if it is entangled with other systems. The set of all wave-fields on a given space-like hypersurface should be consistent with the covariant state introduced by Schwinger.

In the non-relativistic limit, we can use Bohm's eikonal form of a set of indexed single-particle wavefunction $\psi_i = R_i e^{iS_i/\hbar}$ to elucidate the fluid picture, where R_i^2 is the fluid density, S_i is Hamilton's principal function, and $\vec{\nabla} S_i/m$ is the local average velocity field of the particles in the fluid. Then R_i and S_i evolve according to the coupled continuity equation and Hamilton-Jacobi equation, which motivates the fluid picture much more clearly than the Schr/"odinger evolution. For a superposition state with multiple orthogonal terms indexed by i, the coefficient $a_i = |a_i| e^{i\phi_{a_i}}$ of each terms give the total proportion $|a_i|^2$ and global phase ϕ_{a_i} of the corresponding packet of fluid. These are the phases and proportions of the total fluid in the total wave-field, which are relevant for interference. As we will see, it is still essential that each particle in the fluid carries its own copy of the entire local state in its memory, in order to properly define the local transfer matrices at interactions. The relativistic treatment is conceptually identical, with the fluid particles moving along world-lines.

Macroscopic systems are truly composed of many fundamental single-particle systems, each with its own fluid and set of single-particle wavefunctions, but in many cases the correct intuition can be obtained by approximating the macroscopic system as a single fluid, whose particles are different copies of the whole system. This allows us to neglect the fine details of the internal local interactions between the fundamental constituents of that system.

For macroscopic systems, there is a clear preferred basis, and the indexes correspond to the experience of one outcome or another during an interaction (measurement), which ultimately gives rise to the Born rule. A system does not directly experience its own internal memory - only its index (external memory). In practice, these macroscopic external memory records are permanent (although it is possible in theory to project those systems into noncommuting bases, which would overwrite that memory).

Finally, to get some physical intuition for this model, it helps to think of each indexed wave packet as an isolated drop of fluid floating through space. This is a very nonclassical fluid, which behaves more like a gas than a liquid, allowing significant compression and rarefaction as it moves. This facilitates longitudinal waves passing through the drop, which produces familiar wave behavior. Unlike a classical gas, these waves can create zeros in the fluid density (the nodes of a stationary state, for example) - so the local scattering rules for the particles in the fluid must also be quite nonclassical. Despite this, fluid particles never cross these zeros, and the motion of the entire fluid can always be decomposed into their world-lines.

Quantum tunneling through a finite barrier highlights the nonclassicality of the fluid. As a pulse is incident upon a barrier, the interference with the reflected wave may cause temporary zeros to form in front of the barrier, and the fluid to form a series of compressed and rarefied regions, which quickly vanish as the reflected pulse moves away. Part of the packet also penetrates inside the barrier, and the probability current there is nonzero, so the fluid particles' world-lines are literally passing through the barrier and continuing on the other side - and clearly with a nonzero tunneling time.

As we will see, the complexity of the picture grows with the dimension of the usual Hilbert space, which only obscures some of the relevant features, and this is why we focus most of our attention on examples with 2-level systems. This article begins with analysis of states of one, two, and three spins in space-time, including Von Neumann measurement and Born's rule. We then give a full demonstration of the local treatment of an experimental test of Bell's theorem, treating Alice and Bob as spins. Spatial entanglement and the Stern–Gerlach experiment are also discussed. We conclude with some discussion of the historical context of this model as well as its potential future applications. The Supplemental Information contains discussion of a number of other important experiments.

2. Internal and External Memory

The internal memory of a particle in the fluid of a given system has a simple general form, using standard quantum language, but it must be emphasized that this is a fully separated piece of information contained at a single point in space-time, and carried with the particle on its world-line.

First, the memory contains a product state of the initial state for its own system, and the initial states for the first interaction with any other system in its past-interaction-cone (even if these systems did not directly interact with the present one). Second, it contains a list of pair-wise unitary coupling operations from interactions between two systems, along with a list of local single-particle (kinetic) unitary evolution operators, all in temporal order for causally connected operations (in the special relativity sense). Together, these pieces of information give a standard Hilbert-space quantum state,

$$\Big(\prod_i U_i\Big) \bigotimes_j |\phi_j\rangle^j, \tag{1}$$

where the index i includes all single-particle unitaries and pair-wise interaction unitaries U_i in the past-interaction cone, and the index j includes the initial state of every system j in the past-interaction cone (superscripts label different systems here and throughout this paper). This form does not necessarily imply that the initial state of the universe was a product state, since it could have begun with entangled local memories of this same form.

The set of indexed single-particle wavefunctions in any product basis can be trivially extracted from this state, which is also to say that the external memory can be trivially extracted from the internal memory. Each term in the superposed state represents a separate spatial wavefunction of the system, each with its own complex coefficient, and the term itself becomes the index of that wavefunction (this treatment differs from previous discussions [11] where the external-memory-history of each particle was tracked. This turns out to needlessly complicate the formalism, so we have removed it from the present treatment).

Whenever two systems interact at an event, their past-interaction cones become identical by definition, and thus their internal memories synchronize, merging their two prior lists into a new shared set, $\{U_i\}$ and $\{|\phi_j\rangle^j\}$. For systems that are entangled within their respective (local) internal memories due to past interactions, this synchronization causes entanglement correlations to be obeyed if/when those systems interact in the future. That is essentially the complete local hidden variable theory, but many of the finer details of how this reproduces the empirical predictions of standard quantum theory are not obvious. This is a many-local-worlds theory, where each different external memory of a given macroscopic system is a different outcome experienced by the fluid particles of that system, all in one objective space-time. The worlds of two fluid particles coincide if and only if they are at the same event in space-time and they share the same external memory.

It must be emphasized that this construction can be transparently applied to any experimental analysis done using standard quantum theory and local unitary operations, and it produces the corresponding local hidden variable model of that experiment. This applies to experimental tests of Bell's theorem, delayed-choice quantum erasure, weak measurement, quantum teleportation, Wigner's friend experiments, tests of indefinite causal order, among many others for which a local hidden variable model is not obvious. We work through the essential examples here and in the Supplemental Information.

When a system undergoes a local (measurement) interaction with another system, its fluid is divided up into proportions given by the Born rule using the reduced density matrix of the synchronized internal memory state, and the external memories of each sub-part of the fluid are the different outcomes. The other interacting system has the same synchronized entangled state in its internal memory, and its fluid is thus divided up into matching proportions, with consistent external memories.

The fluid particles with different external memories have experienced different outcomes (with Born rules proportions, which also results in Born rule ensemble statistics), even though they are all still in the same space-time. Again, this constitutes many worlds only in this sense of fluid particles on many world-lines each with its own external memory. It is also empirically evident that the different fluid particles of the same systems never see or experience each other in this way (they are 'hidden' from one another), and are thus oblivious to this division of their fluid.

Macroscopic external memories of measurement outcomes are never erased in practice, so if the two macroscopic systems meet (the identity interaction) whose memories are already entangled, then the proportions of the fluids with each possible pairing of external memories in the macroscopic preferred basis are given by the Born rule for that synchronized entangled state. When this matching occurs, each product-state term in the internal memory state corresponds to a different empirical outcome for the meeting of two macroscopic systems.

Entanglement correlations are never realized at space-like separation. Instead they are realized if and when the internal memories are locally synchronized and fluids with different external memories are matched up. Prior to this matching, the overall distribution in space-time is given by the tensor product of the different reduced density matrices, and is thus completely uncorrelated.

3. Macroscopic Preferred Bases and Relative Collapse

A macroscopic preferred basis typically means that the environment is entangled with the system, and encodes a memory record of different outcomes that is never erased in practice. Orthogonal degrees of freedom in the terms of the internal memory state prevent interference between those terms during a local unitary interaction, and thus macroscopic systems never undergo interference effects wherein fluid particles with different external memories are mixed together as their external memories are erased and rewritten. For a macroscopic system, the fluid of copies with a given external memory may be matched and subdivided, but the macroscopic external memory record of a given copy never changes. When we think of a quantum system as having 'collapsed', this really only means that the system is entangled with the environment in some macroscopic preferred basis.

In contrast, a microscopic system can be defined as one which is not entangled with the environment in this way, which means that there is no preferred basis for observers and all of the relevant degrees of freedom can be manipulated during an experiment, so that any pair of terms can be made to interfere, and we would think of this state as remaining 'uncollapsed' during these manipulations. This means that microscopic external memories are routinely erased and rewritten, allowing interference effects. Because of this erasure/rewriting process, there is no restriction on the matching between a given copy's prior external memory, and its new external memory, provided that the final proportions match the final internal state. Previous versions of this model [11] attempted to track the external memory history of each copy, and to develop rules for the proportion of each former memory that is reset to each new memory during the interaction. While such histories must exist in this model, there can never be any macroscopic empirical evidence of them, nor of the erasure/rewriting process, so these rules cannot be uniquely determined. In the end, there is really no reason to discuss the external memories of microscopic systems in one basis or another, since they are empirically inaccessible by definition.

To make this more explicit, consider a case where a detector for a 2-level quantum system has clicked, and thus the outcomes $|0\rangle^t$ and $|1\rangle^t$ for the 2-level system has been amplified to the macroscopic scale as detector outcome states $|0\rangle^d$ and $|1\rangle^d$ and propagated through the environment such that the internal memory of an arbitrary system in the environment, including an observer, now contains a superposition of these terms in its internal memory state. The amplification has implicitly created a macroscopic preferred basis, since we never observe a superposition of two different detector clicks at the macroscopic level. To erase/rewrite this memory would require manipulating all records of the detector clicks throughout the entire macroscopic environment, which never happens in practice. This means that the internal memory of every system in the environment is entangled with the 2-level system, with this preferred basis, and thus for those copies of a macroscopic system with $|0\rangle^d$ in their external memory, the system has 'collapsed' to state $|0\rangle^t$, since they will never meet the fluid particles of that system which had external memory with $|1\rangle^t$. Likewise, the 2-level system has 'collapsed' to $|1\rangle^t$ relative to copies of macroscopic systems with $|1\rangle^d$ in their external memory.

On the other hand, if we let two quantum systems with states $|\psi\rangle^1$ and $|\phi\rangle^2$ interact via unitary U while remaining isolated from the environment (as in a quantum computer), then there are no orthogonal macroscopic states in internal memory to prevent interference between the different orthogonal terms, and thus this state is 'uncollapsed' relative to copies of a macroscopic observer. A subsequent interaction unitary V is free to erase and rewrite the external memories of fluid particles of both systems *exactly because* there is no macroscopic empirical record of these external memories. For example, if U is followed by U^{-1}, then the original product state $|\psi\rangle^1|\phi\rangle^2$ is restored.

4. Spins

In this model, the wave-field of an isolated Pauli spinor comprises a superposition of two indexed single-particle wavefunctions in space-time, each of which is represented by a fluid density $R_i^2(\mathbf{x},t)$ and a principal function $S_i(\mathbf{x},t)$ with velocity field $\vec{\nabla}S_i(\mathbf{x},t)/m$. If two spins are entangled, then each spin comprises up to four wavefunctions in space-time. For three entangled spins, each comprises up to eight wavefunctions in space-time – one for each orthogonal term in the local internal memory state. We can work in any product basis without loss of generality, so we use the binary basis for all Pauli spinors, meaning $i = 0, 1$, corresponding to spin states $|0\rangle$ and $|1\rangle$.

For a single spin (system 1, denoted by the superscript), the two wavefunctions $a\psi_0^1(x)$ and $b\psi_1^1(x)$ correspond to the spin states $|0\rangle^1$ and $|1\rangle^1$, respectively, and it is the sum of these two probability densities that is normalized in space ($|a|^2 + |b|^2 = 1$, $\int_{-\infty}^{\infty} |\psi_i^1(x)|^2 dx = 1$). The point is, if $\psi_0^1(x) = \psi_1^1(x) = \psi^1(x)$, then the spin-position Hilbert space product state $(a|0\rangle^1 + b|1\rangle^1)\psi^1(x_1)$ in standard quantum theory is replaced in the new theory by the pair of fluid packets in 3-space $\{a\psi_0^1(x), b\psi_1^1(x)\}$. Note that if the spin and position are entangled, the description is more complicated.

We can change the spin basis used for the representation, which results in new coefficients, and a new division into two different fluids. Regardless of the basis, the different fluids undergo independent local evolution, from which it is clear that the basis is not physically relevant for the evolution.

The wave packets themselves are constructed in the local Fock basis, but we use the shorthand notation $\psi(x) \equiv \int \psi(x) a_x^\dagger |0\rangle dx$ throughout this text.

4.1. Two Spins

If two spins (initially in state $|0\rangle$) have interacted via a unitary U^{12} (a pure spin-spin coupling, that does not entangle the spatial degrees of freedom), then to find the spatial wavefunctions of each system, we construct the 2-spin Hilbert space state $U^{12}|0\rangle^1|0\rangle^2 = \sum_{i,j=0}^{1} a_{ij}|i\rangle^1|j\rangle^2$, and then treat the spin states as indexes that delineate separate spatial wavefunctions of the present system. For example, the four local states of system 1 are,

$$a_{00}|0\rangle^1_{|0\rangle^2}, \quad a_{01}|0\rangle^1_{|1\rangle^2}, \quad a_{10}|1\rangle^1_{|0\rangle^2}, \quad a_{11}|1\rangle^1_{|1\rangle^2}, \tag{2}$$

where the subscript contains an external memory state for other systems, and the four corresponding wavefunctions are

$$\begin{aligned} &a_{00}\psi(x)^1_{0,|0\rangle^2}, \quad a_{01}\psi(x)^1_{0,|1\rangle^2}, \\ &a_{10}\psi(x)^1_{1,|0\rangle^2}, \quad a_{11}\psi(x)^1_{1,|1\rangle^2}, \end{aligned} \tag{3}$$

where the first subscript is the spin external memory of the present system, and it is the sum of these four probability densities that is normalized in space ($|a_{00}|^2 + |a_{01}|^2 + |a_{10}|^2 + |a_{11}|^2 = 1$). Likewise for system 2 the local states are,

$$a_{00}|0\rangle^2_{|0\rangle^1}, \quad a_{01}|1\rangle^2_{|0\rangle^1}, \quad a_{10}|0\rangle^2_{|1\rangle^1}, \quad a_{11}|1\rangle^2_{|1\rangle^1}, \tag{4}$$

and the four corresponding wavefunctions are

$$\begin{gathered} a_{00}\psi(x)^2_{0,|0\rangle^1}, \quad a_{01}\psi(x)^2_{1,|0\rangle^1}, \\ a_{10}\psi(x)^2_{0,|1\rangle^1}, \quad a_{11}\psi(x)^2_{1,|1\rangle^1}. \end{gathered} \tag{5}$$

In the absence of an interaction, each of these spatial wavefunctions evolves independently according to the single-particle Schrödinger/Dirac equation. Local interactions take the form of spatial boundary conditions that connect the different spatial wavefunctions.

After the local interaction, each of them carries a copy of the information $U^{12}|0\rangle^1|0\rangle^2$ in its local memory. This process of collecting local interaction unitaries and initial states along a single world-line is the essence of the local Heisenberg treatment used in relativistic quantum field theory. It is important to emphasize that each of these copies is separate and independent from the others, and each copy encodes local information at a single point in space (a single fluid particle). Whenever two systems locally interact, the memories of the two systems first synchronize before the new interaction unitary is applied, so that they now share all unitary operations and initial states from both of their past local interaction cones. Then the new interaction unitary is added to both of them, resulting in an equal number of indexed spatial wavefunctions for each system, with matching coefficients.

Taking the simple case that the spin and spatial wavefunctions are not entangled, we have $\psi(x)^1_{0,|0\rangle^2} = \psi(x)^1_{0,|1\rangle^2} = \psi(x)^1_{1,|0\rangle^2} = \psi(x)^1_{1,|1\rangle^2} = \psi(x)^1$ and $\psi(x)^2_{0,|0\rangle^1} = \psi(x)^2_{1,|0\rangle^1} = \psi(x)^2_{0,|1\rangle^1} = \psi(x)^2_{1,|1\rangle^1} = \psi(x)^2$, and the standard quantum state of two entangled spins with separable position states in Hilbert/configuration space,

$$\psi(x_1)^1 \otimes \psi(x_2)^2 \otimes \sum_{i,j=0}^{1} a_{ij}|i\rangle^1|j\rangle^2, \tag{6}$$

is replaced in the local theory by the above set of eight fluid packets in 3-space, and the many local copies of the Hilbert-space state $U^{12}|0\rangle^1|0\rangle^2$ they carry in memory.

Note that the local memories of each spin carry all of the same information about this interaction as the entangled Hilbert space state of conventional quantum theory, and at the fine-grained scale, every fluid particle of each system carries this information on its world-line as well. The main point here is that these eight fluid packets evolving in space-time contain all of the information needed to produce the correct empirical probabilities and entanglement correlations for these systems. Because all interactions are local, we can completely replace delocalized wavefunctions in higher-dimensional spaces with the wave-field in space-time, and obtain all of the original empirical predictions. This is our local hidden variable theory.

By way of notation in the present formalism, we will use superscripts to indicate which system a spatial wavefunction belongs to, rather than subcripts on the coordinates in a single configuration space wavefunction for all systems. As shown above, all internal degrees of freedom (like spin) now correspond to additional indexed spatial wavefunctions of a given system, and entanglement with other systems results in additional spatial wavefunctions for both systems.

4.2. Three or More Spins

Now, suppose that system 1 interacts locally with system 3, while system 2 is not involved, and does not change in any way. The interaction unitary is V^{13} and the initial state of system 3 is $|0\rangle^3$, and system 3 carries no other relevant memory. First, the two systems synchronize memory to $U^{12}|0\rangle^1|0\rangle^2|0\rangle^3$, so system 3 splits into four indexed wavefunction, whose coefficients match those of systems 1. Then V^{13} is added to both memories, resulting in state,

$$\begin{aligned} V^{13}U^{12}|0\rangle^1|0\rangle^2|0\rangle^3 &= \sum_{i,j,k,l=0}^{1} a_{ij}b_{ikl}|k\rangle^1|l\rangle^3|j\rangle^2 \\ &= \sum_{j,k,l=0}^{1} c_{jkl}|k\rangle^1|l\rangle^3|j\rangle^2. \end{aligned} \tag{7}$$

From this, we see that there are eight local spin states for each system,

$$\left\{c_{jkl}|k\rangle^1_{|l\rangle^3|j\rangle^2}\right\}, \quad \left\{c_{jkl}|l\rangle^3_{|k\rangle^1|j\rangle^2}\right\}, \tag{8}$$

where in either case the other two systems are treated as indexes, and thus eight spatial wavefunctions for each system as well,

$$\left\{c_{jkl}\psi(x)^1_{k,|l\rangle^3|j\rangle^2}\right\}, \quad \left\{c_{jkl}\psi(x)^3_{l,|k\rangle^1|j\rangle^2}\right\}. \tag{9}$$

Now, systems 2 and 3 have not interacted, but there are now entangled within the local memory of system 3, which will effect what happens if they interact in the future. Let us consider that case next.

Systems 2 and 3 now interact via unitary W^{23}. First, their memories are synchronized to $V^{13}U^{12}|0\rangle^1|0\rangle^2|0\rangle^3$, which splits the four indexed wavefunctions of system 2 into eight, such that the entanglement correlations between systems 2 and 3 become physically manifest. Note that if two interacting systems already share some unitaries or initial states in memory, they will necessarily match, and so the two memories can be simply be merged, as in this case (before the interaction, system 2 had $U^{12}|0\rangle^1|0\rangle^2$).

Now the new interaction unitary is added to both memories, resulting in

$$W^{23}V^{13}U^{12}|0\rangle^1|0\rangle^2|0\rangle^3 = \sum_{j,k,l=0}^{1} d_{jkl}|k\rangle^1|l\rangle^3|j\rangle^2, \tag{10}$$

and eight new wavefunctions for systems 2 and 3. The eight wavefunctions of system 1 are not involved in this interaction, and are unchanged. This series of interactions are shown in Figure 3, along with the internal memory being carried by each system. Hopefully the general picture for larger numbers of spins is clear at this point. The Supplemental Information contains the detailed treatment of several important experiments.

Note that after any of the interactions in Figure 3, the proportion of fluid with each orthogonal external memory in any basis is given by the Born rule, and exactly matches standard quantum theory for the same initial states and unitary interactions. Over an ensemble of experimental trials, empirical probabilities emerge which match the proportions of fluid with each orthogonal external memory in the macroscopic preferred basis, and this exactly reproduces the probabilities predicted by standard delocalized quantum theory.

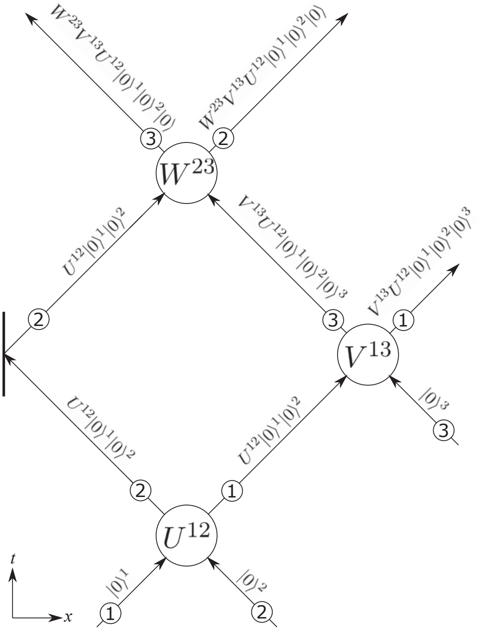

Figure 3. An illustration of the local hidden variables of the present model, showing three systems undergoing a series of local spin-spin interactions in space-time. The memories of systems 1 and 2 synchronize when they meet, and the interaction unitary U^{12} is added to both. Then the memories of systems 1 and 3 synchronize when they meet, and the interaction unitary V^{13} is added to both. The memory of system 2 is unaffected by this space-like separated interaction. Finally, the memories of systems 2 and 3 are synchronized when they meet, the interaction unitary W^{23} is added to both, and the expected entanglement correlations between those systems are obeyed. The internal states can be expanded in any product basis to give the local set of single-particle wavefunctions in space-time indexed by the external memories in that basis, where the interaction unitaries define boundary conditions connecting the pre-interaction fluids to the post-interaction fluids. If there is a macroscopic preferred basis, then expanding the internal memory in that basis gives the set of different external memories experienced by different copies of macroscopic observers in space-time. If one considers a Schwinger state on any space-like hypersurface that cuts across this diagram, it is easy to see what information is encoded at each event on that surface, and to verify that this information only pertains to that event's past light cone. It is also easy to see that the Schwinger state on the 'present' surface contains too much information to be reconstructed from a standard delocalized quantum state.

4.3. Local Entanglement

What remains is to show how two systems interact locally and become entangled in this way. We will begin with the simplest possible example, which is also quite illustrative. We expect the general theory to contain only one type of coupling potential, and this is of the form $\delta(\vec{x}_1 - \vec{x}_2)V$, where V is a general space-time-independent potential. This says that when two systems meet at an event in space-time, the potential V produces the local scattering between them, via the unitary $U = e^{-iV/\hbar}$. The new states are written into the memories of the fluid particles as this happens, causing them to separate into more distinct fluids than before. This general potential should be uniform throughout space-time, and encompass all possible scattering events between all types of quantum systems. In other words, all Standard Model particle interactions should be encoded in V. The specific formalism for deriving a local unitary operation that acts only at a single event is highly nontrivial, and subject to ongoing research.

Because this model does not support long-range interactions, it is relatively complicated to recover Coulomb-potential based interactions between charges, which are mediated by massless force carriers. To demonstrate the general mechanism, we have restricted ourselves to a *gedanken* experiment with just two quantum systems in space-time, where the only coupling potential is a spin–spin interaction – thus $U = e^{-iV/\hbar}$ is some 4×4 2-spin matrix.With this potential, the spatial wavefunctions never change or become entangled with the spins. Thus, if the two systems are incident upon one another, the fluid packets pass through the interaction without being deflected or deformed, but the spin states interact locally as this occurs, causing the fluid to acquire new indexes that separate it into more distinct packets than before - each moving independently but identically. These local interactions are where the internal memory states become entangled, so it is still appropriate to say that two systems became entangled during this interaction, even though this model has no delocalized entangled state in Hilbert/configuration space.

We now consider such an entangling interaction for two spins that begin in a separable state, each with two wavefunctions in space-time, $a_1\psi_0^1(x,t)$ and $b_1\psi_1^1(x,t)$, and $a_2\psi_0^2(x,t)$ and $b_2\psi_1^2(x,t)$, respectively. They can only interact locally, and thus the only reason they have not interacted is that they have no overlapping support. In fact, there must be a boundary point x_{12} that separates their supports. For this simple one-dimensional example, we will begin with spin 1 located fully to the left of x_{12} and propagating towards spin 2, which is fully to the right of x_{12}. In this example, once their supports begin to overlap, the spins will directly interact via a 4-dimensional unitary U (strictly speaking, it need not be unitary so long as it is norm-preserving), which maps the two pre-interaction wavefunctions of each system into its four post-interaction wavefunctions. Note that for spin 1 the two pre-interaction wavefunctions are only supported at $x \leq x_{12}$, while the four post-interaction wavefunctions are only supported at $x \geq x_{12}$ (since the wave-packets continue to propagate with the same momentum). From here, it is clear that U simply defines the boundary conditions that connect these six wavefunctions at x_{12} (four post-interactions wavefunctions on one side to two pre-interaction wavefunctions on the other side), and the fully normalized piece-wise wave-field $|\Psi(x,t)\rangle^s$ of each system includes contributions from all six, and all twelve for both systems. The situation is shown in Figure 4.

4.4. The Interaction Boundary

The next important detail we need to examine is the actual location x_{12} between the systems, which is not a fixed boundary at all, but rather a dynamic one that moves in time depending on the shapes of the two incident systems' wavefunctions (in 3D this is a dynamic boundary surface). The boundary is defined by a special rule that applies to all entanglement couplings in this model - the fluid flux of the two systems across the boundary must be equal and opposite. For any two normalized wavefunctions $\psi(x)$ and $\phi(x)$, there is always a boundary point where,

$$\int_{-\infty}^{x_{12}} |\psi^1(x)|^2 dx = \int_{x_{12}}^{\infty} |\psi^2(x)|^2 dx, \tag{11}$$

and

$$\int_{-\infty}^{x_{12}} |\psi^2(x)|^2 dx = \int_{x_{12}}^{\infty} |\psi^1(x)|^2 dx. \tag{12}$$

The initial value of the boundary can be found in this way (ideally when the two packets are well-separated), and then it moves according to,

$$\dot{x}_{12}(t) = \frac{j^1(x_{12},t) + j^2(x_{12},t)}{|\psi^1(x_{12},t)|^2 + |\psi^2(x_{12},t)|^2}, \tag{13}$$

where j^s is the current density of each fluid. This equal-and-opposite flux condition guarantees that an equal amount of fluid from each system is always crossing the boundary in a given time.

This condition is required to guarantee that the a_{00} in $a_{00}\psi^1_{0_{0^1 0^2}}$ is the same as in $a_{00}\psi^2_{0^1 0^2}$, the a_{01} in $a_{01}\psi^1_{0,|1\rangle^2}$ is the same as the a_{01} in $a_{01}\psi^2_{|0\rangle^1}$, etc. As they cross the interaction boundary, the fluid particles of both systems acquire all of the 2-spin entanglement information. Importantly, these are independent copies of the coefficients, in different memory records, and local interventions on one copy have no nonlocal effect on other copies.

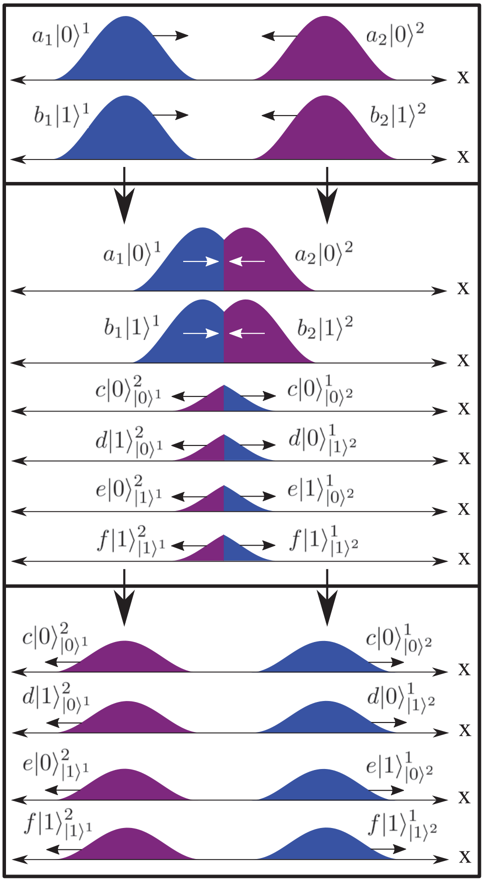

Figure 4. Three frames showing the local interaction process as two particles in one dimension pass through each other, with only their spins interacting. The spatial density $|\psi(x)_i|^2$ of each fluid pulse is shown, each indexed by past interactions, for the particular case that $|a_1|^2 = |b_1|^2 = |a_2|^2 = |b_2|^2 = 1/2$ and $|c|^2 = |d|^2 = |e|^2 = |f|^2 = 1/4$. The piece-wise wave-field $|\Psi(x,t)\rangle^s$ of each system formally includes all six wavefunctions as separated at the dynamic boundary $x_{12}(t)$ (stationary in this example) which all occupy the same space-time. Also consider this example in a boosted Lorentz frame, where the boundary is moving such that the fluid fluxes of the two systems are equal and opposite.

In 3D, there is no longer a unique boundary surface for two normalized functions, so an initial boundary must be assumed. The motion of this boundary is then defined locally such that as two systems move together, an equal amount of fluid from each crosses per unit time. At the fine-grained scale, the boundary corresponds to the locations where fluid particles of the two systems are locally scattering and synchronizing memory.

A boundary like this exists between every pair of systems. It goes beyond the scope of the present article, but once the spatial degrees of freedom of a system are entangled, there can generally be different interaction boundaries for different wavefunctions of a given pair of systems.

Finally, the hard boundary presented here may be a proof of concept, rather than a physically correct rule. A possible generalization is that when two fluids meet, only some fraction of them interacts, and the remainders simply continue in their pre-interaction states. This would effectively smear the boundary between the pre- and post-interaction wavefunctions, but it would also mean that two fluids can never fully switch into their post-interaction states.

While the mathematics of dynamical boundaries is quite complicated, we expect that the exact details can be cleanly extracted from Schwinger's theory in the future. The exact details of the boundary are not important in many practical situations anyway, so for the purpose of this article, we will simply assume the existence of a hard boundary.

4.5. Boundary Conditions

To obtain the boundary condition at x_{12}, we consider the action of a general norm-preserving transformation matrix on a general product state of two spins. The actual choice of basis for this analysis is completely arbitrary. The matrix can be expanded as

$$U^{12} = \sum_{i,j,k,l\in[0,1]} u_{ijkl}|i\rangle^1|j\rangle^2\langle k|^1\langle l|^2, \tag{14}$$

and then

$$\begin{aligned} &U^{12}\big(a_1a_2|0\rangle^1|0\rangle^2 + a_1b_2|0\rangle^1|1\rangle^2 \\ &+ b_1a_2|1\rangle^1|0\rangle^2 + b_1b_2|1\rangle^1|1\rangle^2\big) = \\ &\big(u_{0000}a_1a_2 + u_{0001}a_1b_2 + u_{0010}b_1a_2 + u_{0011}b_1b_2\big)|0\rangle^1|0\rangle^2 \\ &+ \big(u_{0100}a_1a_2 + u_{0101}a_1b_2 + u_{0110}b_1a_2 + u_{0111}b_1b_2\big)|0\rangle^1|1\rangle^2 \\ &+ \big(u_{1000}a_1a_2 + u_{1001}a_1b_2 + u_{1010}b_1a_2 + u_{1011}b_1b_2\big)|1\rangle^1|0\rangle^2 \\ &+ \big(u_{1100}a_1a_2 + u_{1101}a_1b_2 + u_{1110}b_1a_2 + u_{1111}b_1b_2\big)|1\rangle^1|1\rangle^2 \\ &= c|0\rangle^1|0\rangle^2 + d|0\rangle^1|1\rangle^2 + e|1\rangle^1|0\rangle^2 + f|1\rangle^1|1\rangle^2 \end{aligned} \tag{15}$$

This allows us to define the two 4×2 transfer matrices T_1 and T_2 that map the two pre-interaction wavefunctions of each system onto its four post-interaction wavefunctions,

$$\begin{aligned} T_1^U &= U^{12}\big(a_2|0\rangle^2 + b_2|1\rangle^2\big) \\ &= \sum_{i,j,k\in[0,1]} (u_{ijk0}a_2 + u_{ijk1}b_2)|i\rangle^1|j\rangle^2\langle k|^1 \\ &= \begin{bmatrix} u_{0000}a_2 + u_{0001}b_2 & u_{0010}a_2 + u_{0011}b_2 \\ u_{0100}a_2 + u_{0101}b_2 & u_{0110}a_2 + u_{0111}b_2 \\ u_{1000}a_2 + u_{1001}b_2 & u_{1010}a_2 + u_{1011}b_2 \\ u_{1100}a_2 + u_{1101}b_2 & u_{1110}a_2 + u_{1111}b_2 \end{bmatrix}, \end{aligned}$$

and

$$T_2^U = U^{12}\big(a_1|0\rangle^1 + b_1|1\rangle^1\big)$$

$$= \sum_{i,j,l \in [0,1]} (u_{ij0l}a_1 + u_{ij1l}b_1)|i\rangle^1|j\rangle^2\langle l|^2$$

$$= \begin{bmatrix} u_{0000}a_1 + u_{0010}b_1 & u_{0001}a_1 + u_{0011}b_1 \\ u_{0100}a_1 + u_{0110}b_1 & u_{0101}a_1 + u_{0111}b_1 \\ u_{1000}a_1 + u_{1010}b_1 & u_{1001}a_1 + u_{1011}b_1 \\ u_{1100}a_1 + u_{1110}b_1 & u_{1101}a_1 + u_{1111}b_1 \end{bmatrix}.$$

Because $U^\dagger = U^{-1}$, we also have

$$\left(T_s^U\right)^\dagger T_s^U = \hat{I}_s, \tag{16}$$

where $\hat{I}_s$ is the identity for system s alone. Finally, if U is expanded into outer products then the Ts can be expressed using the subscripts and without using matrices (see the Bell Test example below).

It is clear that the local state of the other spin appears in each transfer matrix, which makes perfect sense given that this is a local interaction between the two spins.

We can read off the coupled boundary conditions for the four post-interaction wavefunctions of each system as,

$$\begin{bmatrix} \tilde{\psi}^1_{0,|0\rangle^2}(x_{12}(t),t) \\ \tilde{\psi}^1_{0,|1\rangle^2}(x_{12}(t),t) \\ \tilde{\psi}^1_{1,|0\rangle^2}(x_{12}(t),t) \\ \tilde{\psi}^1_{1,|1\rangle^2}(x_{12}(t),t) \end{bmatrix} = T_1^U \begin{bmatrix} \tilde{\psi}^1_0(x_{12}(t),t) \\ \tilde{\psi}^1_1(x_{12}(t),t) \end{bmatrix}, \tag{17}$$

and

$$\begin{bmatrix} \tilde{\psi}^2_{0,|0\rangle^1}(x_{12}(t),t) \\ \tilde{\psi}^2_{0,|1\rangle^1}(x_{12}(t),t) \\ \tilde{\psi}^2_{1,|0\rangle^1}(x_{12}(t),t) \\ \tilde{\psi}^2_{1,|0\rangle^1}(x_{12}(t),t) \end{bmatrix} = T_2^U \begin{bmatrix} \tilde{\psi}^2_0(x_{12}(t),t) \\ \tilde{\psi}^2_1(x_{12}(t),t) \end{bmatrix}, \tag{18}$$

and for the pre-interaction wavefunctions as,

$$\begin{bmatrix} \tilde{\psi}^1_0(x_{12}(t),t) \\ \tilde{\psi}^1_1(x_{12}(t),t) \end{bmatrix} = (T_1^U)^\dagger \begin{bmatrix} \tilde{\psi}^1_{0,|0\rangle^2}(x_{12}(t),t) \\ \tilde{\psi}^1_{0,|1\rangle^2}(x_{12}(t),t) \\ \tilde{\psi}^1_{1,|0\rangle^2}(x_{12}(t),t) \\ \tilde{\psi}^1_{1,|1\rangle^2}(x_{12}(t),t) \end{bmatrix}, \tag{19}$$

and

$$\begin{bmatrix} \tilde{\psi}^2_0(x_{12}(t),t) \\ \tilde{\psi}^2_1(x_{12}(t),t) \end{bmatrix} = (T_2^U)^\dagger \begin{bmatrix} \tilde{\psi}^2_{0,|0\rangle^1}(x_{12}(t),t) \\ \tilde{\psi}^2_{0,|1\rangle^1}(x_{12}(t),t) \\ \tilde{\psi}^2_{1,|0\rangle^1}(x_{12}(t),t) \\ \tilde{\psi}^2_{1,|1\rangle^1}(x_{12}(t),t) \end{bmatrix}, \tag{20}$$

where the $\tilde{\psi}$ are general un-normalized individual wavefunctions for each index.

These reduce back to simple mappings between the spin coefficients, since all of the normalized packets are identical, so the transfer matrices really only produce the coefficients c, d, e, and f, and show how a_s and b_s define them.

The piece-wise multivalued wave-fields of each system are,

$$|\Psi(x,t)\rangle^1 = \begin{cases} a_1\psi_0^1(x,t) & \\ b_1\psi_1^1(x,t) & x \leq x_{12}(t) \\ & \\ c\psi^1_{0,|0\rangle^2}(x,t) & \\ d\psi^1_{0,|1\rangle^2}(x,t) & \\ e\psi^1_{1,|0\rangle^2}(x,t) & x > x_{12}(t) \\ f\psi^1_{1,|1\rangle^2}(x,t) & \end{cases} \tag{21}$$

and

$$|\Psi(x,t)\rangle^2 = \begin{cases} a_2\psi_0^2(x,t) & \\ b_2\psi_1^2(x,t) & x > x_{12}(t) \\ & \\ c\psi^2_{0,|0\rangle^1}(x,t) & \\ d\psi^2_{1,|0\rangle^1}(x,t) & \\ e\psi^2_{0,|1\rangle^1}(x,t) & x \leq x_{12}(t) \\ f\psi^2_{1,|1\rangle^1}(x,t) & \end{cases} \tag{22}$$

Since all six spatial wavefunctions of each system are identical and normalized, as are the coefficients in each region ($|a_s|^2 + |b_s|^2 = 1$ and $|c|^2 + |d|^2 + |e|^2 + |f|^2 = 1$), we can verify that the wave-field of each system describes a conserved fluid distribution in space-time.

4.6. Von Neumann Measurement and the Born Rule

We can simplify this example to illustrate the role of local entanglement during the measurement process and the experience of collapse with Born rule probability [32] in the new fluid picture.

For the Von Neumman measurement [33], we keep the initial state of spin 1, expressed as $(a_1|0\rangle^1 + b_1|1\rangle^1)\psi^1(x_1)$ in the conventional theory and $\{a_1\psi_0^1(x), b_1\psi_1^1(x)\}$ in the present theory, but set the initial state of spin 2 to the 'ready' state $|0\rangle^2\psi^2(x_2)$, meaning the we have only one spatial wavefunction $\psi_0^2(x)$ for spin 2 in this basis ($a_2 = 1$, $b_2 = 0$).

For a projective measurement, the unitary is then $U^{12} = \text{CNOT}$ [34], with spin 1 as the control qubit, which produces the standard entangled state,

$$\left(a_1|0\rangle^1|0\rangle^2 + b_1|1\rangle^1|1\rangle^2\right)\psi^1(x_1)\psi^2(x_2) \tag{23}$$

in the conventional theory. In the present theory, this means each system is carrying the local memory state $U^{12}(a_1|0\rangle^1 + b_1|1\rangle^1)|0\rangle^2$ and the corresponding set of four spatial wavefunctions

$$\begin{aligned} &a_1\psi^1_{0,|0\rangle^2}(x), \quad b_1\psi^1_{1,|1\rangle^2}(x), \\ &a_1\psi^2_{0,|0\rangle^1}(x), \quad b_1\psi^2_{1,|1\rangle^1}(x), \end{aligned} \tag{24}$$

with a fraction $|a_1|^2$ of the particles in the spin 2 fluid recording the outcome $|0\rangle^1$ into their external memories, and fraction $|b_2|^2$ recording $|1\rangle^1$ (see Figure 5). The external memories of each particle in the fluid also define the experience of the particle, and thus from the perspective of each particle in the fluid of spin 2, spin 1 seems to collapse into one of its eigenstates or the other. Furthermore, in a large ensemble of identically prepared runs, spin 2 will experience $|0\rangle^1$ with relative probability $|a_1|^2$ and $|1\rangle^1$ with relative probability $|b_1|^2$, thus satisfying the Born rule.

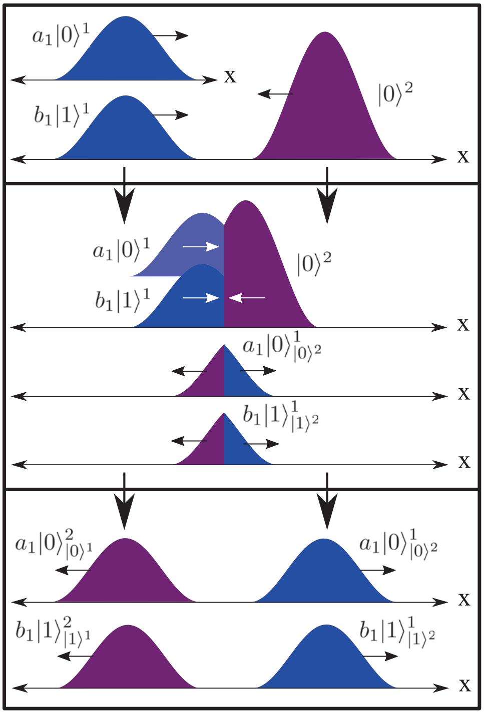

Figure 5. Three frames showing the local interaction process as two particles in one dimension pass through each other, with only their spins interacting. The interaction is a Von Neumann measurement of the binary basis, where system 2 is the pointer, which starts in 'ready' state $|0\rangle^2$. The spatial density $|\psi(x)_i|^2$ of each fluid pulse is shown, for the particular case that $|a_1|^2 = |b_1|^2$, each indexed by past interactions.

To round out the example, the two transfer matrices are

$$T_1^U = U^{12}|0\rangle^2 = \begin{bmatrix} 1 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 1 \end{bmatrix},$$
$$T_2^U = U^{12}\left(a_1|0\rangle^1 + b_1|1\rangle^1\right) = \begin{bmatrix} a_1 & 0 \\ 0 & a_1 \\ 0 & b_1 \\ b_1 & 0 \end{bmatrix}, \tag{25}$$

and we have,

$$T_1^U\left(a_1|0\rangle^1 + b_1|1\rangle^1\right) = \left(a_1|0\rangle^1_{|0\rangle^2} + b_1|1\rangle^1_{|1\rangle^2}\right), \tag{26}$$

and

$$T_2^U|0\rangle^2 = \left(a_1|0\rangle^2_{|0\rangle^1} + b_1|1\rangle^2_{|1\rangle^1}\right), \tag{27}$$

which shows why we only have two nonzero wavefunctions for each system instead of four.

We can use these relations to define the boundary conditions at x_{12} as the packets pass through one another,

$$
\begin{gathered}
\tilde{\psi}^1_{0,|0\rangle^2}(x_{12}(t),t) = \tilde{\psi}^1_0(x_{12}(t),t), \\
\tilde{\psi}^1_{1,|1\rangle^2}(x_{12}(t),t) = \tilde{\psi}^1_1(x_{12}(t),t), \\
\tilde{\psi}^1_{1,|1\rangle^2}(x_{12}(t),t) = \tilde{\psi}^1_{0,|0\rangle^2}(x_{12}(t),t) = 0, \\
\tilde{\psi}^2_{0,|0\rangle^1}(x_{12}(t),t) = a_1\tilde{\psi}^2_0(x_{12}(t),t), \\
\tilde{\psi}^2_{1,|1\rangle^1}(x_{12}(t),t) = b_1\tilde{\psi}^2_0(x_{12}(t),t), \\
\tilde{\psi}^2_{1,|0\rangle^1}(x_{12}(t),t) = \tilde{\psi}^2_{0,|1\rangle^1}(x_{12}(t),t) = 0, \\
\tilde{\psi}^2_0(x_{12}(t),t) \\
= a_1^*\tilde{\psi}^2_{0,|0\rangle^1}(x_{12}(t),t) + b_1^*\tilde{\psi}^2_{1,|1\rangle^1}(x_{12}(t),t), \\
\tilde{\psi}^2_1(x_{12}(t),t) \\
= a_1^*\tilde{\psi}^2_{0,|1\rangle^1}(x_{12}(t),t) + b_1^*\tilde{\psi}^2_{1,|0\rangle^1}(x_{12}(t),t),
\end{gathered}
\tag{28}
$$

along with the spatial derivative of these expressions evaluated at the boundary.

Each of the spatial wavefunctions $\psi(x,t)$ evolves under its own single-particle Schrödinger/Dirac equation, and all interactions occur via the boundary conditions, which automatically produce the correct splitting into more wave packets.

In summary, by treating all spin-spin interaction unitaries as local boundary conditions, and otherwise allowing each of the indexed fluids in space-time to evolve independently, we obtain the correct multiparticle quantum dynamics (without any delocalized Hilbert/configuration space evolution).

4.7. Synchronization and General Transfer Matrices

We have considered an interaction between two systems which were not already entangled with any other systems, and so no synchronization was necessary prior to applying the interaction unitary. However, in general, the synchronization process will extend the unitary operation being applied to each system. The transfer matrices come from the overall unitary operation, and so they are not generally 4 × 2. Note that the synchronization unitary is applied even if the interaction unitary is identity. Nevertheless, the construction above gives the right idea for how to construct the transfer matrices and the corresponding boundary conditions for the general case.

As an example of synchronization, let us return to the interaction between systems 1 and 3 in Section 4.1. The unitary synchronization operation S, when system 1 interacts interacts with system 3 results in updating the memory of system 3 from $|0\rangle^3$ to $U^{12}|0\rangle^1|0\rangle^2|0\rangle^3$, and thus the synchronization matrix is $S_3 = U^{12}I^3|0\rangle^1|0\rangle^2$, where the identity I^3 has been added to emphasize that this is an operation on system 3. In this case, all the operation does is introduce additional indexes that separate system 3 into four identical spatial wavefunctions. Applying $S_1 = I^{12}|0\rangle^3$ to the memory of system 1 does not change number of distinct indexes, so there are still four spatial wavefunctions.

After the synchronization, the interaction V^{13} is added to the memory of both systems, resulting in the 8×2 transfer matrix $T_3^{VS} = V^{13}S_3 = V^{13}U^{12}|0\rangle^1|0\rangle^2$ for system 3. This defines the boundary condition between the two initial wavefunctions, and the eight resulting wavefunctions of system 3. For system 1, the 8×4 transfer matrix is $T_1^{VS} = V^{13}S_1 = V^{13}I^2|0\rangle^3$, which defines the boundary conditions between the four initial and eight resulting wavefunctions of system 1.

When using the single-particle unitary approximation, the transfer matrix is simply identical to the unitary, with identity matrices tacked on for other systems in the local memory.

As we can see, branching is never global in the new model. New branchings arise from the creation of new indexes during local interactions, and each branching spreads via synchronization from the systems where it originates to any other systems they interact with, and then to other systems that those interact with, and so on, in a chain that eventually applies that branching to the entire environment. This is the mechanism of decoherence in the new model, and explains the emergence of classical macroscopic experiences in dense and frequently-interacting systems.

This also explains how thought experiments like Schrödinger's cat [35], and Wigner's friend [36,37] are resolved as sequences of local interactions where branching is spread from one system to the next (see the Supplemental Information).

It is also worth noting that this mechanism produces the correct empirical experience of collapse after a measurement, because if the same two systems interact again via identity, or the measurement is repeated using a newly prepared device, the indexes all stay unchanged, meaning each observer sees the same outcome as before. If this were not the case, then the observer would in general experience violation of natural conservation laws of the dynamic quantities (energy, momentum, etc.) involved in the interaction. Even if the other system has participated in another interaction before being measured again, the results will still be consistent with the collapsed state having undergone that operation, as expected when preparing a state, and applying a unitary to it.

5. Bell Test

As a final example of the model, and also to demonstrate the full local treatment of entanglement, we go through a simple gedanken example of a test of Bell's theorem. This is the Mermin-Wigner test [38,39], where Alice and Bob each choose to measure their spin in one of three equally spaced directions in the zx-plane of the Bloch sphere.

We will begin with systems 1 and 2 having locally interacted to form the anticorrelated Bell state of two spins, $\frac{1}{\sqrt{2}}\big(|0\rangle^1|1\rangle^2 - |1\rangle^1|0\rangle^2\big)\psi^1(x_1)\psi^2(x_2)$ in the old quantum theory, and in the present theory, both systems carry the local states $U^{12}|0\rangle^1|0\rangle^2 = \frac{1}{\sqrt{2}}\big(|0\rangle^1|1\rangle^2 - |1\rangle^1|0\rangle^2\big)$ in their memory, and the four corresponding spatial wavefunctions are:

$$\begin{aligned} &\tfrac{1}{\sqrt{2}}\psi^1_{0,|1\rangle^2}(x), \quad -\tfrac{1}{\sqrt{2}}\psi^1_{1,|0\rangle^2}(x), \\ -&\tfrac{1}{\sqrt{2}}\psi^2_{0,|1\rangle^1}(x), \quad \tfrac{1}{\sqrt{2}}\psi^2_{1,|0\rangle^1}(x). \end{aligned} \tag{29}$$

By symmetry, there are only two distinct types of measurement - those with parallel settings and those with nonparallel settings, so we only need to consider one example of each type. In both cases, Alice measures system 1 in the binary basis, while in Case 1, Bob measures system 2 in the binary basis, and in Case 2, Bob measures in the basis,

$$\begin{aligned} |\phi^+\rangle^2 &= \tfrac{1}{2}\big(|0\rangle^2 + \sqrt{3}|1\rangle^2\big), \\ |\phi^-\rangle^2 &= \tfrac{1}{2}\big(\sqrt{3}|0\rangle^2 - |1\rangle^2\big). \end{aligned} \tag{30}$$

We treat Alice and Bob as 2-level systems in this analysis, for simplicity, and because it gets the right point across. To complete the experiment and obtain the entanglement correlations, Alice and Bob meet and interact via the identity.

5.1. Case 1

Alice begins in state $|0\rangle^A$ with a single spatial wavefunction $\psi_0^A(x)$ and her measurement is a $V^{1A} = \text{CNOT}$ gate with system 1 as the control and Alice as the target. The local state carried in memory synchronizes and updates to .

$$V^{1A}U^{12}|0\rangle^1|0\rangle^2|0\rangle^A = \frac{1}{\sqrt{2}}\left(|0\rangle^1|1\rangle^2|0\rangle^A - |1\rangle^1|0\rangle^2|1\rangle^A\right), \tag{31}$$

resulting in the four spatial wavefunctions,

$$\begin{aligned} &\tfrac{1}{\sqrt{2}}\psi^1_{0,|0\rangle^A|1\rangle^2}(x), \quad -\tfrac{1}{\sqrt{2}}\psi^1_{1,|1\rangle^A|0\rangle^2}(x), \\ &\tfrac{1}{\sqrt{2}}\psi^A_{0,|0\rangle^1|1\rangle^2}(x), \quad -\tfrac{1}{\sqrt{2}}\psi^A_{1,|1\rangle^1|0\rangle^2}(x). \end{aligned} \tag{32}$$

The situation is symmetric for Bob and system 2, with local state,

$$\begin{aligned} &W^{2B}U^{12}|0\rangle^1|0\rangle^2|0\rangle^B \\ &= \frac{1}{\sqrt{2}}\left(|0\rangle^1|1\rangle^2|1\rangle^B - |1\rangle^1|0\rangle^2|0\rangle^B\right), \end{aligned} \tag{33}$$

resulting in the four wavefunctions,

$$\begin{aligned} &-\tfrac{1}{\sqrt{2}}\psi^2_{0,|0\rangle^B|1\rangle^1}(x), \quad \tfrac{1}{\sqrt{2}}\psi^2_{1,|1\rangle^B|0\rangle^1}(x), \\ &-\tfrac{1}{\sqrt{2}}\psi^B_{0,|1\rangle^1|0\rangle^2}(x), \quad \tfrac{1}{\sqrt{2}}\psi^B_{1,|0\rangle^1|1\rangle^2}(x). \end{aligned} \tag{34}$$

Now, Alice and Bob meet (the identity transformation), and their local memories synchronize to

$$\begin{aligned} &V^{1A}W^{2B}U^{12}|0\rangle^1|0\rangle^2|0\rangle^A|0\rangle^B \\ &= \frac{1}{\sqrt{2}}\left(|0\rangle^1|1\rangle^2|0\rangle^A|1\rangle^B - |1\rangle^1|0\rangle^2|1\rangle^A|0\rangle^B\right), \end{aligned} \tag{35}$$

which results in the four wavefunctions,

$$\begin{aligned} &\tfrac{1}{\sqrt{2}}\psi^A_{0,|0\rangle^1|1\rangle^2|1\rangle^B}(x), \quad -\tfrac{1}{\sqrt{2}}\psi^A_{1,|1\rangle^1|0\rangle^2|0\rangle^B}(x), \\ &-\tfrac{1}{\sqrt{2}}\psi^B_{0,|1\rangle^1|0\rangle^2|1\rangle^A}(x), \quad \tfrac{1}{\sqrt{2}}\psi^B_{1,|0\rangle^1|1\rangle^2|0\rangle^A}(x). \end{aligned} \tag{36}$$

We can now see that any fluid particle of Alice that experienced system 1 in state $|0\rangle^1$ ($|1\rangle^1$) also meets a fluid particle of Bob that experienced system 2 in state $|1\rangle^2$ ($|0\rangle^2$), and thus from the perspectives of all Alices and Bobs, the correct entanglement correlations for the Bell state have been obeyed. The steps are shown in Figure 6.

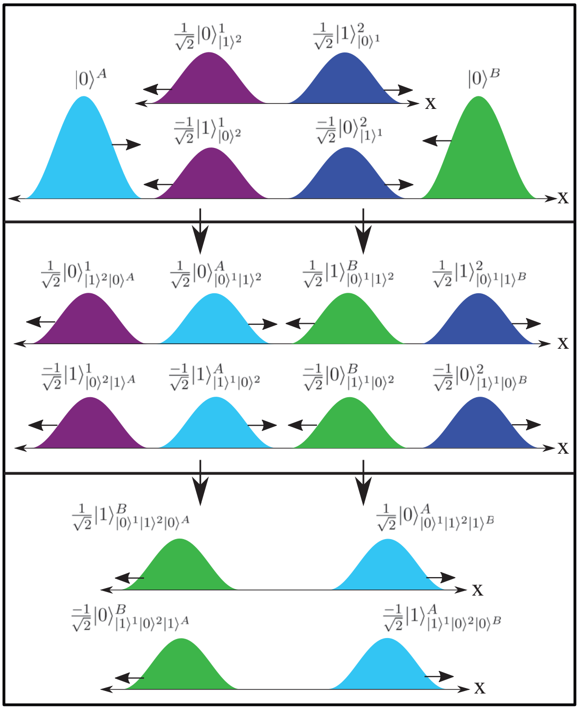

Figure 6. Three frames showing the steps of the Mermin–Wigner Bell test, for the case that Alice and Bob measure the same setting. The **top** frame shows the Bell state being sent to Alice and Bob, the **middle** frame is after Alice's and Bob's measurements are completed, and the **bottom** frame is after Alice and Bob meet to share their results. In the experience of each Alice and Bob, the proper entanglement correlations have been obeyed.

5.2. Case 2

The situation for Alice's measurement of system 1 is the same as in Case 1. For Bob's measurement, with the same ready state $|0\rangle^B$, a viable unitary is,

$$\begin{aligned} W^{2B} &= |\phi^+\rangle^2|0\rangle^B\langle\phi^+|^2\langle 0|^B + |\phi^-\rangle^2|1\rangle^B\langle\phi^-|^2\langle 0|^B \\ &+ |\phi^+\rangle^2|1\rangle^B\langle\phi^+|^2\langle 1|^B + |\phi^-\rangle^2|0\rangle^B\langle\phi^-|^2\langle 1|^B. \end{aligned} \tag{37}$$

Thus, when Bob measures system 2, the local state carried in the memory of the two systems synchronizes and updates to,

$$W^{2B}U^{12}|0\rangle^1|0\rangle^2|0\rangle^B \tag{38}$$

$$\begin{aligned} &= \sqrt{\frac{3}{8}}|0\rangle^1|\phi^+\rangle^2|0\rangle^B - \sqrt{\frac{1}{8}}|1\rangle^1|\phi^+\rangle^2|0\rangle^B \\ &- \sqrt{\frac{1}{8}}|0\rangle^1|\phi^-\rangle^2|1\rangle^B - \sqrt{\frac{3}{8}}|1\rangle^1|\phi^-\rangle^2|1\rangle^B, \end{aligned}$$

resulting in the eight wavefunctions,

$$\begin{aligned}
&\sqrt{\tfrac{3}{8}}\psi^{B}_{0,|0\rangle^1|\phi^+\rangle^2}(x), && -\sqrt{\tfrac{1}{8}}\psi^{B}_{0,|1\rangle^1|\phi^+\rangle^2}(x),\\
-&\sqrt{\tfrac{1}{8}}\psi^{B}_{1,|0\rangle^1|\phi^-\rangle^2}(x), && -\sqrt{\tfrac{3}{8}}\psi^{B}_{1,|1\rangle^1|\phi^-\rangle^2}(x),\\
&\sqrt{\tfrac{3}{8}}\psi^{2}_{\phi^+,|0\rangle^1|0\rangle^B}(x), && -\sqrt{\tfrac{1}{8}}\psi^{2}_{\phi^+,|1\rangle^1|0\rangle^B}(x),\\
-&\sqrt{\tfrac{1}{8}}\psi^{2}_{\phi^-,|0\rangle^1|1\rangle^B}(x), && -\sqrt{\tfrac{3}{8}}\psi^{2}_{\phi^-,|1\rangle^1|1\rangle^B}(x).
\end{aligned} \tag{39}$$

Now, Alice and Bob meet, and their memories synchronize to the local state,

$$V^{12}W^{2B}U^{12}|0\rangle^1|0\rangle^2|0\rangle^A|0\rangle^B \tag{40}$$

$$\begin{aligned}
&= \sqrt{\frac{3}{8}}|0\rangle^1|\phi^+\rangle^2|0\rangle^A|0\rangle^B - \sqrt{\frac{1}{8}}|1\rangle^1|\phi^+\rangle^2|1\rangle^A|0\rangle^B\\
&- \sqrt{\frac{1}{8}}|0\rangle^1|\phi^-\rangle^2|0\rangle^A|1\rangle^B - \sqrt{\frac{3}{8}}|1\rangle^1|\phi^-\rangle^2|1\rangle^A|1\rangle^B,
\end{aligned}$$

resulting in the eight spatial wavefunctions,

$$\begin{aligned}
&\sqrt{\tfrac{3}{8}}\psi^{A}_{0,|0\rangle^1|\phi^+\rangle^2|0\rangle^B}(x), && -\sqrt{\tfrac{1}{8}}\psi^{A}_{1,|1\rangle^1|\phi^+\rangle^2|0\rangle^B}(x),\\
-&\sqrt{\tfrac{1}{8}}\psi^{A}_{0,|0\rangle^1|\phi^-\rangle^2|1\rangle^B}(x), && -\sqrt{\tfrac{3}{8}}\psi^{A}_{1,|1\rangle^1|\phi^-\rangle^2|1\rangle^B}(x).\\
&\sqrt{\tfrac{3}{8}}\psi^{B}_{0,|0\rangle^1|\phi^+\rangle^2|0\rangle^A}(x), && -\sqrt{\tfrac{1}{8}}\psi^{B}_{0,|1\rangle^1|\phi^+\rangle^2|1\rangle^A}(x),\\
-&\sqrt{\tfrac{1}{8}}\psi^{B}_{1,|0\rangle^1|\phi^-\rangle^2|0\rangle^A}(x), && -\sqrt{\tfrac{3}{8}}\psi^{B}_{1,|1\rangle^1|\phi^-\rangle^2|1\rangle^A}(x).
\end{aligned} \tag{41}$$

It is again clear from these final wavefunctions that the entanglement correlations for the Bell state have been correctly obeyed for the case that the measurement settings were not aligned. The steps are shown in Figure 7.

The two cases together show that the local fluid model exactly reproduces all of the empirical predictions of standard nonlocal quantum mechanics for this Bell test.

It is important to note that the entanglement correlations are not obeyed in any meaningful sense until Alice and Bob meet, their memories synchronize, and their wavefunctions are paired by their indexes. Prior to that, there were Alices in space-time who had experienced either outcome, and also Bobs who had experienced either outcome, but there is no correlation among them, which is clear because their distributions in space-time always match their respective reduced density matrices.

There is also a general lesson here about post-selected ensembles of quantum measurements. All of the measurement outcomes exist as wavefunctions with different indexes, with an overall distribution still given by the reduced density matrix of that system. When a single observer locally collects data from the post-selected system and the other systems of interest, this is where the entanglement correlations associated with the post-selection are realized. The observers who saw the desired post-selection will also see the anticipated entanglement correlations. Examples include spontaneous parametric down conversion of entangled states [40,41], delayed-choice quantum erasure [42], measurements of weak values [43], and the Delft Bell experiment [44].

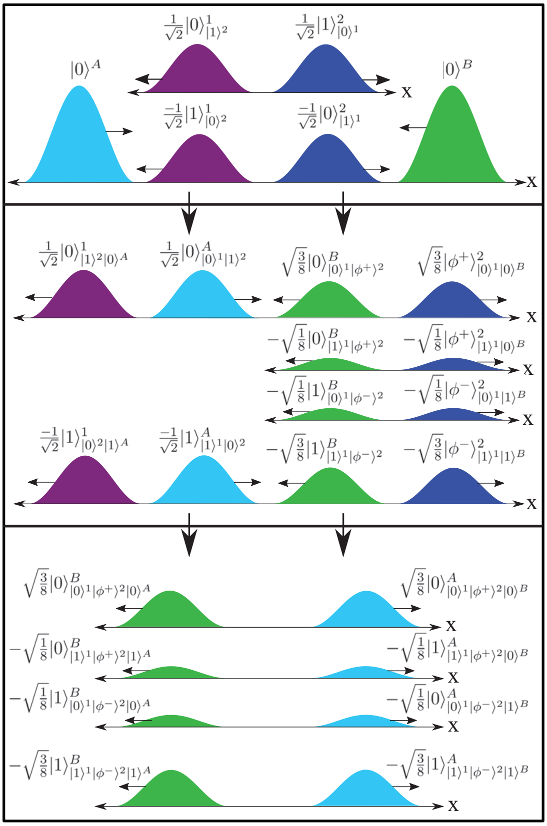

Figure 7. Three frames showing the steps of the Mermin-Wigner Bell test, for the case that Alice and Bob measure the different settings. The **top** frame shows the Bell state being sent to Alice and Bob, the **middle** frame is after Alice's and Bob's measurements are completed, and the **bottom** frame is after Alice and Bob meet to share their results. In the experience of each Alice and Bob, the proper entanglement correlations have been obeyed.

5.3. Demonstrating the Local Hidden Variables

As mentioned in the introduction, the empirical facts of this theory lead to a picture with many-worlds. It is worth emphasizing again that the treatment we have just given is an explicit local hidden variable model of the Bell experiments, that successfully reproduces the entanglement correlations in a single-space-time (*one-world* is an often-unstated assumption of Bell's theorem, which is violated here).

To make this completely undeniable, we consider a quick demonstration using students which makes it clear that all entanglement correlations and Born rule statistics are obeyed, and everything happens on world-lines in a single space-time, with a Lorentz invariant causal structure.

We will have 8 students in one room who play different copies of Alice, and 8 more in another room who play different copies of Bob, with each group receiving one of the qubits from the singlet state. Students playing the same person are understood to not observe each other. The students in each room collectively choose one of the three settings, and then they all measure that same setting. The results is that a random 4 of the students in that room get 'up' and the other four get 'down', consistent with the 50% Born rule probability for the reduced density matrix of the Bell state available in each room. Each student writes their chosen setting and their result on a sign they then carry, but they are still completely separated in their different rooms, and have not communicated in any way.

The students then gather in a single room with absolutely no light, so they cannot see the other students' signs. A referee with night-vision goggles then pairs the students up (one Alice and one Bob), and then sends these pairs out of the room, where the Alice and Bob meet and look at each other's signs. In both cases, the students will always find that the Born rule probabilities from the singlet state were obeyed, regardless of their settings.

In Case 1, the referee does this by pairing the four 'up' Alices with the four 'down' Bobs, and vice versa, so all eight Alices meet a Bob with the opposite spin, as expected for the singlet state.

In Case 2, the referee pairs one 'up' Alice with an 'up' Bob, one 'down' Alice with a 'down' Bob, three 'up' Alices with three 'down' Bobs, and three 'down' Alices with three 'up' Bobs. The fraction of students with each outcome thus reproduce the Born rule statistics, so in a large ensemble of identical trials, the students will experience them as frequentist probabilities.

To help visualize this, we can think of the original Alice wave packet in Figure 6 as containing 8 students, who divide up into 2 groups of 4 when Alice measures qubit 1. There are likewise 8 students in the original Bob packet, which divide up into 2 groups of 4. When Alice and Bob meet, the 4 from each group are paired off as indicated in the cases above, depending on what settings each group chose. Figure 7 for Case 2 can also be broken up using the 16 students.

6. Single-Particle Unitaries and Spatial Superpositions

As discussed above, all single-particle unitary operations on a system really correspond to weak entanglement in a standard two-system interaction. This is made explicit here. The control system may be macroscopic, but we treat it as a single quantum system in an environmentally decohered basis. After the interaction, the entangled state in local memory contains orthogonal terms for the target system and terms that are nearly indistinguishable for the control system. To get the single-particle approximation, the observer (environment) measures the control system in the same decoherent basis it began in, which results in multiple nearly identical wavefunctions, each having undergone approximately the intended single-particle unitary. The single-particle unitary approximation is to ignore the differences between these wavefunctions and treat them as one (dropping the indexes corresponding to the control system).

For two spins, the initial state of the target system is $|0\rangle^t$, and for the control system it is $|0\rangle^c$. After the interaction, the local state carried in each system's memory is $a|0\rangle^c|0\rangle^t + b\big(\cos\epsilon|0\rangle^c + \sin\epsilon|1\rangle^c\big)|1\rangle^t\big)$, for $|\epsilon| \ll 1$.

The six wavefunctions of the two systems are

$$\begin{aligned} &a\psi^t_{0,|0\rangle^c}(x), &&a\psi^c_{0,|0\rangle^t}(x), \\ &b\cos\epsilon\psi^t_{1,|0\rangle^1}(x), &&b\cos\epsilon\psi^c_{0,|1\rangle^t}(x), \\ &b\sin\epsilon\psi^t_{1,|1\rangle^1}(x), &&b\sin\epsilon\psi^c_{1,|0\rangle^t}(x), \end{aligned} \tag{42}$$

The experimenter begins in state $|0\rangle^e$. The experimenter now measures the control system in the binary basis, resulting in local state

$$\begin{aligned} &a|0\rangle^t|0\rangle^c|0\rangle^e + b|1\rangle^t\big(\cos\epsilon|0\rangle^c|0\rangle^e + \sin\epsilon|1\rangle^c|1\rangle^e\big)\big) \\ &\approx \big(a|0\rangle^t + b|1\rangle^t\big)|0\rangle^c|0\rangle^e = U^t|0\rangle^t|0\rangle^c|0\rangle^e \end{aligned} \tag{43}$$

in both systems' memories. The state of the target system has effectively undergone single-system unitary U^t, and the states of the control and experimenter systems are unchanged. Under this approximation, only the memory of the target system is updated from $|0\rangle^t$ to $U^t|0\rangle^t = a|0\rangle^t + b|1\rangle^t$, and such single-system unitaries must be included when memories synchronize during local interactions.

This simple treatment for spin systems can be easily generalized to any pair of systems whose interaction results in a weakly entangled state.

6.1. The Beam Splitter and Einstein's Objection to Nonlocal Collapse

Although the full treatment of entangled infinite-dimensional systems in space-time is quite complicated, we can still get a good idea what is going on for cases where only a finite number of spatial modes need to be considered.

The simplest example is a particle incident on a beam splitter, where we use the single-system unitary approximation. After the beam splitter the internal memory for the superposed particle state on paths I and II is $Ua_{\mathrm{I}}^{\dagger}|0\rangle = \frac{1}{\sqrt{2}}(a_{\mathrm{I}}^{\dagger} + a_{\mathrm{II}}^{\dagger})|0\rangle = \frac{1}{\sqrt{2}}(|1\rangle^{\mathrm{I}}|0\rangle^{\mathrm{II}} + |0\rangle^{\mathrm{I}}|1\rangle^{\mathrm{II}})$ in the Fock basis. Treating the vacuum mode as a local state, this corresponds to the four wavefunctions,

$$\begin{gathered}
\frac{1}{\sqrt{2}}\psi(x,t)^{\mathrm{I}}_{1,|0\rangle^{\mathrm{II}}}, \quad \frac{1}{\sqrt{2}}\psi(x,t)^{\mathrm{I}}_{0,|1\rangle^{\mathrm{II}}}, \\
\frac{1}{\sqrt{2}}\psi(x,t)^{\mathrm{II}}_{1,|0\rangle^{\mathrm{I}}}, \quad \frac{1}{\sqrt{2}}\psi(x,t)^{\mathrm{II}}_{0,|1\rangle^{\mathrm{I}}},
\end{gathered} \tag{44}$$

where it is implicit that $\psi(x,t)^{\mathrm{I}}$ and $\psi(x,t)^{\mathrm{II}}$ are two different wavefunctions, evolving along two different paths. For a given system, the fluid from different paths can mix and interfere locally, consistent with the local evolution of the internal memory state.

As an aside, we can see that for a single-system unitary situation with a continuous path degree of freedom like a single-slit diffraction, the local state in internal memory becomes,

$$\int_{\{x\}} \phi(x) a_x^{\dagger}|0\rangle dx = \int_{\{x\}} \phi(x)|1\rangle^x \bigotimes_j^{\{x\}-x} |0\rangle^j dx \tag{45}$$

where $\{x\}$ is the set of all paths, and the interaction produces some normalized distribution $\int_{\{x\}} |\phi(x)|^2 dx = 1$ over all of the paths. We then have an infinite number of spatial wavefunctions for each specific path x_0,

$$\begin{gathered}
\psi(\mathbf{x},t)^{x_0}_{\left(1,\otimes_j^{\{x\}-x_0}|0\rangle^j\right)} \phi(x_0), \\
\psi(\mathbf{x},t)^{x_0}_{\left(0,|1\rangle^{x_1}\otimes_j^{\{x\}-x_0-x_1}|0\rangle^j\right)} \phi(x_1),
\end{gathered} \tag{46}$$

where x_1 is any path other than x_0, and where each $\psi(\mathbf{x},t)_i^{x_0}$ is a distinct wavefunction that evolves on path x_0, which break down into one case where the particle is on path x_0 (Fock state $|1\rangle^{x_0}$, upper Equation (46)), and infinitely many others where there is vacuum on path x_0 (Fock State $|0\rangle^{x_0}$, lower Equation (46)), because the particle is on path x_1. The fluid on the different paths can mix and interfere if the paths meet locally, just as in the 2-path case. In the spin cases analyzed above, there is only one path, and the vacuum modes have zero amplitude. We won't spend any more time on continuous degrees of freedom here, but this discussion is included to emphasize the generality of the present theory.

Now, returning to the beam splitter, we have a simple tool to demonstrate how the present model resolves Einstein's objection at the 1927 Solvay conference to the instantaneous and nonlocal nature of wavefunction collapse in the emerging quantum theory. We have already explained how the experience of collapse and Born rule probabilities arise for the individual fluid particles along their world-lines, so this is just a matter of applying these principles. The situation is analogous to Case 1 in the Bell test.

Suppose we send a particle through a beam splitter, and then path I leads to Alice's detector, and path II to Bob's space-like separated detector. After the particle is detected, Alice and Bob meet to compare results, and they always find that only one of them has detected the particle. Roughly speaking, Einstein's objection was that in a single objective world where a spatially superposed wavefunction causally mediates between the source and detectors, once Alice detects the particle, something must instantaneously prevent the wavefunction from also triggering Bob's detector, which violates local causality.

In the present local model, half of the fluid goes to Alice and the other half to Bob. Alice branches into two subgroups of fluid; those that detected the particle and those that did not. Bob likewise branches into two subgroups, with the indexes reversed from Alice. When they meet, the Alices who detected the particle have matched indexes with the Bobs who did not, and vice versa, and so they always find that only one of them has detected the particle, as expected.

Once Alice's detector on path I has either fired or not, the local state in her memory updates to $\frac{1}{\sqrt{2}}\big(|1\rangle^{\mathrm{I}}|0\rangle^{\mathrm{II}}|1\rangle^{A} + |0\rangle^{\mathrm{I}}|1\rangle^{\mathrm{II}}|0\rangle^{A}\big)$, where $|1\rangle^{A}$ indicates her detector has fired, and she has the two spatial wavefunctions,

$$\tfrac{1}{\sqrt{2}}\psi(x,t)^{A}_{1,|1\rangle^{\mathrm{I}}|0\rangle^{\mathrm{II}}}, \quad \tfrac{1}{\sqrt{2}}\psi(x,t)^{A}_{0,|0\rangle^{\mathrm{I}}|1\rangle^{\mathrm{II}}}. \tag{47}$$

Likewise for Bob's detector on path II, the local state in memory updates to $\frac{1}{\sqrt{2}}\big(|1\rangle^{\mathrm{I}}|0\rangle^{\mathrm{II}}|0\rangle^{B} + |0\rangle^{\mathrm{I}}|1\rangle^{\mathrm{II}}|1\rangle^{B}\big)$, and his two wavefunctions are,

$$\tfrac{1}{\sqrt{2}}\psi(x,t)^{B}_{0,|1\rangle^{\mathrm{I}}|0\rangle^{\mathrm{II}}}, \quad \tfrac{1}{\sqrt{2}}\psi(x,t)^{B}_{1,|0\rangle^{\mathrm{I}}|1\rangle^{\mathrm{II}}}. \tag{48}$$

Finally, when Alice and Bob meet, the local state carried in both their memories synchronizes to

$$\frac{1}{\sqrt{2}}\big(|1\rangle^{\mathrm{I}}|0\rangle^{\mathrm{II}}|1\rangle^{A}|0\rangle^{B} + |0\rangle^{\mathrm{I}}|1\rangle^{\mathrm{II}}|0\rangle^{A}|1\rangle^{B}\big), \tag{49}$$

and we have the four expected wavefunctions, where in each case, either only Alice or only Bob has detected the particle,

$$\begin{aligned} &\tfrac{1}{\sqrt{2}}\psi(x,t)^{A}_{1,|1\rangle^{\mathrm{I}}|0\rangle^{\mathrm{II}}|0\rangle^{B}}, \quad \tfrac{1}{\sqrt{2}}\psi(x,t)^{A}_{0,|0\rangle^{\mathrm{I}}|1\rangle^{\mathrm{II}}|1\rangle^{B}} \\ &\tfrac{1}{\sqrt{2}}\psi(x,t)^{B}_{0,|1\rangle^{\mathrm{I}}|0\rangle^{\mathrm{II}}|1\rangle^{A}}, \quad \tfrac{1}{\sqrt{2}}\psi(x,t)^{B}_{1,|0\rangle^{\mathrm{I}}|1\rangle^{\mathrm{II}}|0\rangle^{A}}. \end{aligned} \tag{50}$$

6.2. Stern-Gerlach Devices

The true function of a Stern-Gerlach device [45] in the local model involves many force-carrying particles being emitted locally from the magnet and then propagating to the spin and interacting locally with it. Here we approximate that entire process by a single local interaction unitary and boundary condition, using the single-system unitary approximation, and treating just two output modes.

In conventional quantum theory, the incoming state will be $\big(a|0\rangle^{s} + b|1\rangle^{s}\big)|1\rangle^{\mathrm{I}}|0\rangle^{\mathrm{II}}$, and the action of the magnetic field will be to transmit the $|0\rangle^{s}$ state and reflect the $|1\rangle^{s}$ state, causing the path and spin to become entangled, and the state to become, $\big(a|0\rangle^{s}|1\rangle^{\mathrm{I}}|0\rangle^{\mathrm{II}} + b|1\rangle^{s}|0\rangle^{\mathrm{I}}|1\rangle^{\mathrm{II}}\big)$. The process is shown in Figure 8

In the present quantum theory, there are initially two identical spatial wavefunctions, $a\psi^{s}_{0}(x,t)$ and $b\psi^{s}_{1}(x,t)$ which move on path I, and after the interaction they have evolved to,

$$\begin{aligned} &a\psi^{s}_{0,|1\rangle^{\mathrm{I}}|0\rangle^{\mathrm{II}}}(x,t), \quad b\psi^{s}_{1,|0\rangle^{\mathrm{I}}|1\rangle^{\mathrm{II}}}(x,t), \\ &a\psi^{\mathrm{I}}_{1,|0\rangle^{s}|0\rangle^{\mathrm{II}}}(x,t), \quad b\psi^{\mathrm{I}}_{0,|1\rangle^{s}|1\rangle^{\mathrm{II}}}(x,t), \\ &a\psi^{\mathrm{II}}_{0,|1\rangle^{s}|1\rangle^{\mathrm{I}}}(x,t), \quad b\psi^{\mathrm{II}}_{1,|0\rangle^{s}|0\rangle^{\mathrm{I}}}(x,t) \end{aligned} \tag{51}$$

where $\psi_i^{\mathrm{I}_s}(x,t)$ is a wavefunction that propagates along path I as it evolves, and $\psi_i^{\mathrm{II}_s}(x,t)$ is a different wavefunction that propagates along path II.

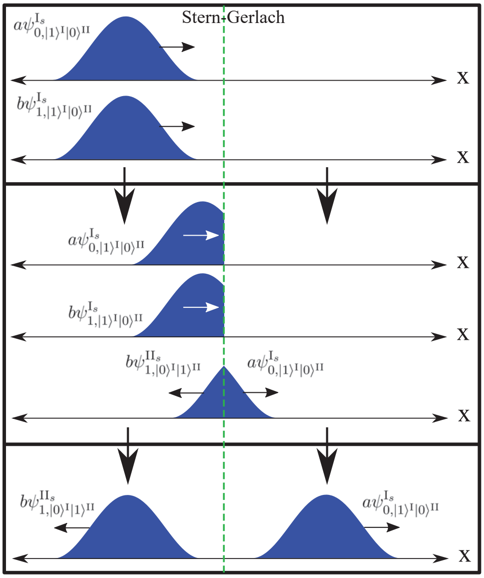

Figure 8. Three frames showing the local entanglement of the spin and path degrees of freedom as particle s passes through a Stern-Gerlach device, approximated as a point (vertical dotted line), which either transmits or reflects the particle. The incoming wavefunction $\psi^{I_s}(x,t)$ is identical for both spin states prior to this entanglement (as in all previous examples in this article). After the interaction, there are two different wavefunctions, $a\psi^{I_s}_{0,|1\rangle^{I}|0\rangle^{II}}(x,t)$ continuing in the same direction and $b\psi^{II_s}_{1,|0\rangle^{I}|1\rangle^{II}}(x,t)$ along a different direction. The interference between the incoming and reflected waves is not shown. The vacuum modes are also omitted for clarity.

7. Conclusions

The local space-time quantum fluid model presented here fully supplants the configuration-space Schrödinger-picture quantum mechanics for multiple particles, and reproduces all of the standard predictions. The relativistic generalization of the fluid model, following Schwinger's covariant formulation, should only contain world-lines as particle trajectories, which results in a local Lorentz invariant causal structure.

We expect this will require a correction to the coarse-grained single-particle Schrödinger equation even for nonrelativistic energies, to prevent superluminal signalling [46,47]. It may also require a correction of the Dirac equation for the same reason, but this is less clear. Either way, these equations are still quite close to the true (unknown) equations of motion that should underlie this model, and if we use them for all of our single-particle evolution, and the same coupling unitaries for our local boundary conditions, then the new model makes identical predictions.

These details notwithstanding, we now have a quantum theory compatible with the local Heisenberg-Schrödinger picture that Schwinger called the 'interaction representation'. This theory is consistent with the local Heisenberg treatment used in relativistic quantum field theory and the Standard Model, while delocalized Hilbert/configuration space treatments are not.

That said, there are clearly many situations where the configuration space wavefunction is a useful tool for calculations, but it is truly only a delocalized approximation of the proper local physics. This calls into question every development in the foundations of quantum mechanics based on this delocalized treatment of entanglement. The fact that this was not better understood in the 1950s seems baffling at first, but when one considers the historical context, the picture starts to becomes clear.

First, Bohr and Heisenberg's *complementarity* had made the pursuit of any realist interpretation of quantum mechanics with a clear narrative unpopular. Ideas like this started to be denigrated as 'philosophy' rather than 'physics', and students were taught to 'shut up and calculate.' It was no longer encouraged for physicists to know what they were talking about, so long as their mathematics led to accurate predictions.

Second, the very successful formalism of quantum field theory that developed in that environment makes use of both past and future boundary conditions, and the mathematics can be interpreted as retrocausal effects propagating from future to past. Notions of propagation from past to future, or even descriptions of what is happening between distant past and future boundary conditions, are heavily obscured in the mathematical and conceptual machinery of these theories - particularly in the path integral formalism. Furthermore, the plane wave solutions of the Dirac and Klein-Gordon equations are delocalized, filling all of space, and local packets are treated as emerging from their interference. The mathematics of the theory are delocalized in both space and time, and there is simply no clear physical narrative of what is going on.

Third, Bell's theorem has had a much more recent impact on the community, and has created the widespread and mistaken impression that any local realist interpretation of quantum mechanics is impossible. What Bell's theorem actually proves is that a local theory must be either superdeterministic, or have multiple copies of each observer, who may experience different outcomes from the measurement, but who each experience just one—exactly like the multiple perspectives of quantum fluid particles on different world-lines in space-time. QED has always been a local many worlds theory of this type, but this fact was obscured by the lack of a proper interpretation.

All told, it is easy to see why the pursuit of a local realist narrative in space-time has not been a high priority in the foundations community, but this appears to have been a colossal mistake. In particular, it has led to the idea that we must abandon the notion of definite causal order, especially at the interface between quantum mechanics and relativity. Very few people seem to understand that there is already a covariant local realist theory hidden away in the Standard Model.

Finally, it may be possible to extend this model to include a local ballistic treatment of quantum gravity in a single space-time with a fixed shape. This is not to say that we can provide a complete theory at present, but the treatment for gravitons as local force-carriers of gravity should be fundamentally similar to the treatment of photons as local force-carries of electromagnetism, but with an aspect that affects the rates of local clocks. This model, with its many local perspectives in space-time, might then untangle the issues of causal structure, and allow quantum theory and relativity theory to be fully integrated. Even if gravity actually does affect the shape of space-time, we could still have a quantum theory of fluid particles on world-lines in different branched space-times, which should still have a definite causal structure.

Supplementary Materials: The following supporting information can be downloaded at: https://www.mdpi.com/article/10.3390/quantum5010011/s1. Supplemental Information: Local quantum theory with fluids in space-time. References [48–54] are cited in the supplementary materials.

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quantum reports

Review

Defending Many Worlds via Case Discrimination: An Attempt to Showcase the Conceptual Incoherence of Anti-Realist Interpretations and Relational Quantum Mechanics

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Abstract: In this work, an alternative attempt to motivate the Many-Worlds Interpretation (MWI) is undertaken. The usual way of arguing for MWI mostly revolves around how it might solve the measurement problem in a more straightforward and concise manner than rival interpretations. However, here an effort is made to defend MWI in an indirect manner, namely via repeated case discrimination and a process of 'conceptual elimination'. That is, it will be argued that its major rivals, with QBism and Relational Quantum-Mechanics being among the most noteworthy ones, either face conceptual incoherence *or* conceptually collapse into a variant of MWI. Finally, it is argued that hidden-variable theories face severe challenges when being applied to Quantum Field Theory such that appropriate modifications may lead back to MWI, thereby purportedly leaving MWI as the only viable option.

Keywords: foundations of physics; interpretations of quantum mechanics; QBism; relational quantum mechanics; Informational Monism; Ontic Structural Realism; many worlds; inference to the best explanation; case discrimination; counterfactuals

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1. Introduction

More than nine decades after its conception, what quantum mechanics is actually telling us, or, for that matter, what the actual ontological implications of the formalism are, is still hotly debated.

While its empirical adequacy has been experimentally confirmed countless times up until today, the theory being implemented into virtually all contemporary electronics and digital technology, its metaphysical implications and its fundamental ontology remain to be debated; this is without apparent progress or consensus in terms of which direction or approach can be regarded as the most promising. Despite, or rather, *because* of this standstill, critics of such quantum-mechanical interpretational disputes tend to object something along the following lines: 'what the formalism *means* is a nonsensical question since it eventually cumulates into nothing but endless debates over semantics without any visible progress, whereas technical progress—continuing its implementation into technical applications and working out the details—eventually will, albeit indirectly, resolve the riddle in near or distant future by revealing new structure'.

However, here I strongly oppose this view: not only do I believe that ontological commitments in fact *do* affect the fruitfulness of future empirical findings and potential breakthroughs, but also, that from a meta-physical viewpoint, we already are in a position to reasonably subject the various interpretations to a high-level conceptual analysis and thereby to identify the most truthful interpretation in an *epistemological coherentist* fashion: that is, while each piece of argumentation in and by itself may only have moderate persuasive force, the mutual enforcement of all linked pieces may lead to an exponential increase in credibility. In other words, I shall argue that in fact, all the relevant meta-theoretical pieces already have been gathered in order to heuristically arrive at a definite decision in terms of *one* veritable interpretation. However, so far, rarely one single work has collected, discussed and juxtaposed an amount of arguments which seems reasonable for such decision. My humble contribution shall hence be to provide, in this spirit, a brief, but hopefully adequate, big-picture overview; - this is one which however, as I shall argue, already speaks for itself in terms of favoring the one interpretation that regards the formalism to be empirically adequate *and* complete.

2. Methods

Let me begin with some preliminary remarks: firstly, in this review, I shall claim that the Many Worlds Interpretation (MWI), and the decoherence-based MWI (dbMWI) in particular, will be the only interpretation to leave this conceptual 'Mexican standoff' as an alive and winning party. However, I will do so indirectly, that is, by mainly analyzing the competitors and highlighting the problems *they* face and how the respective dissolutions might ultimately lead to a conceptual collapse into MWI. Thus, up until a final section in which the better part of this work culminates, I will not go deeper into the details of MWI itself. Secondly, although for the purpose of this paper and for reasons of simplicity it may be of higher priority that all variants of MWI stand united against rival interpretations, I still take dbMWI to be the most straightforward and concise version of MWI (as I will discuss in the final subsections). Hence, if not explicitly stated otherwise, from now on, MWI refers to dbMWI.

Numerous different interpretations of Quantum-Mechanics (QM), a number impossible to keep track of, seem to have matured into a noteworthy state, an affair which is an overkill that seemingly defies any attempt at conceptual disentanglement (Ref. [1] provides a table of 13 different quantum mechanical interpretations/theories and their respective features and properties. This can be taken as a good overview for introductory purposes, although, obviously, some new and recent developments are not even considered). However here, despite, or rather *because* of this confusion, I want to demonstrate the possibility of defending MWI in the spirit of *reductio ad absurdum*, that is, by showing that any rival interpretation or theory might lead to some or other form of conceptual incoherence or challenge, the dissolution of which leads back to MWI eventually. The latter process may also be understood as an attempt to arrive at MWI by an inference to the best explanation, one which also relies heavily on the use of counterfactuals.

However, some reasonable constrictions are still in order, constrictions which hopefully strike the right balance between scope, simplicity and adequacy. In Section 3.1, I will therefore choose a coarse-grained view and restrict myself to the discussion of *anti-realist* interpretations in general. While there are noteworthy fine-grained distinctions between several views belonging to this group, for the aim of this paper, it might nevertheless suffice to regard the default, the Copenhagen Interpretation (CI) that is, but also the QBism and informational 'neo-CI', - due to their recent growth in popularity, as relevant placeholders for *any* anti-realist interpretation. Their phenomenological emphasis of the act of observation, and instrumentalist/pragmatist tradition heritage are good enough demarcating criteria for being paradigmatic examples of *any* anti-realist interpretation (see [2–4] for some further general and introductory discussions of and around CI).

Next, Section 3.2 will be solely dedicated to a discussion of Relational Quantum Mechanics (RQM). This one is a noteworthy exception, for two reasons: firstly, although its founder Carlo Rovelli clearly emphasizes that it should be understood in realist terms, it nevertheless contains elements that likewise attract people with anti-realist leanings. Secondly, as I shall argue, it might actually be closer to MWI than it is generally perceived. The sheer fact that MWI is also regarded as the 'relative state interpretation' [5,6] may however already hint at this. Stronger still, I shall ultimately argue that RQM must collapse into a version of MWI *or* it is conceptually incoherent.

I will claim that, up to this point, anti-realist advocates still have two realist options left, hidden-variables theory or MWI. Hence, in Section 3.3, I shall review two of the most established 'realist rivals' of MWI, which are Bohmian mechanics (BM) [7] and Objective Collapse Theories (OCT) [8], such as the Ghirardi–Rimini–Weber model (GRW) [9]. I will come to the conclusion that in this domain, conceptual difficulties are particularly severe when considering application in quantum field theoretic (QFT) contexts.

Finally, after having purportedly laid bare the conceptually problematic baggage of any major alternative, I will inspect the core of QM which remained untouched throughout the analysis, more closely. I will then briefly review how 'pure' QM plus decoherence straightforwardly leads to MWI and how decoherence may ultimately also provide a solution to the preferred basis and probability problem. Before briefly addressing the sociological reasons for why the physics community seemingly still favors CI, I will come to the conclusion that we already have all the rationale necessary for putting MWI in the consensus.

3. Juxtaposing QM Interpretations via Repeated Case Discriminations

3.1. Anti-Realist Interpretations

3.1.1. The Copenhagen Interpretation

Let us begin with common and rather established arguments against the Copenhagen Interpretation (CI). Despite its challenges, the still most common way to justify (and thereby 'quasi-solving´ the measurement-problem) the apparent incompleteness of the formula is orthodox CI, as is notably found in textbooks. According to CI, the formalism is not incomplete in the nomological sense, but only such that it fails to factor in the causal efficacy of the observer or measurement apparatus. Whether considered as a conscious being or a mere device, it is, for the CI advocate at any rate, a macroscopic *non-QM* entity that is supposed to 'collapse' a probabilistic distribution into a reified physical and substantial, definite outcome. Setting aside its ' . . . *and then a miracle occurs* . . . ' obscurity, and ignoring other general issues regarding two-realm metaphysics or Metaphysical Dualism (MD) for the moment, the conceptual difficulties remain severe; as such, it necessarily follows that the entity responsible for the wavefunction collapse must be in some sense acting *beyond* the micro-physical realm in order to avoid contradiction; in other words, it must be 'non-quantum physical'. This however violates the thesis that macro-physical objects are composed of micro-physical ones (it would itself be in the very superposition it ought to collapse if *not*). It is needless to say that this alleged solution to the measurement problem is beyond a serious contemporary scientific attitude.

What is more, the Heisenberg Cut involved in CI seems rather arbitrary and may be in conflict with the overall goal of science, namely, the search for further levels of unification. Howe justified is the rationale that the laws of quantum mechanics break down at a certain limit? If nothing else, ever more sophisticated experiments have provided evidence to the contrary, i.e., they have demonstrated that there seems to be no absolute upper size limit for interference, which is in consonance with the unificatory mission of science [10]. That being said, while quantum and quasi-classical domains surely are *qualitatively* different, it is still the case that quantum-mechanical (QM) laws' convergence towards the classical limit is a *continuous*, rather than an *abrupt* affair.

Furthermore, one could bring forth a general objection against any interpretation that relies on a so-called '*conscious* observer'-dependence. For a start, even *if* a (either cartesian MD or physicalist) mind of an agent may be responsible for both the nature of the wavefunction and the measurement (updating or collapsing) process, one might still argue that this situation *in-itself* requires a mind-*independent* mechanism that mediates such observer–object correspondence in the first place; indeed, such a (inevitably hidden-variable-involving) mechanism, in turn, *must* then objectively be *in the world*, either as a physical, or perhaps 'meta-physical', structure; however, at any rate, and that is the crucial point I intend to make, it is as something *independent* of the mind. When, at this point, neither accepting full blown Metaphysical Monistic Idealism (MI) nor posing a mind–matter correspondence mechanism is an option, while mental causal efficacy is still assumed as it purportedly is by QBist interpreters of 'Wheeler´s *Participatory Universe*' [11–13], conceptual incoherence seems to arise. A structurally similar argument can also be brought forth against the claim that measurement *devices* are responsible for the collapse; indeed, an objective mind-independent mechanism would also be required for explaining such 'top-down causation', - eventually turning also *this* reading of CI into a hidden variable theory).

Finally, in order to ward off other common, albeit rather handwavy 'solutions' to the measurement problem, I shall quote Dürr and Lazarovici [14], who state the following:

> "*(simply) pointing to Born's rule does not avoid the measurement problem*".

This is so because it alone can explain neither the interference phenomena of the wave-function components, nor why a definite outcome appears at the phenomenological level. Hence, the explanatory options that essentially remain are *either* hidden nomological variables *or* to regard the formalism as physically substantial and *complete*; however, merely pointing to the instrumental or toolkit-like nature of the quantum formalism will not do. In the subsequent subsection, we will rephrase this state of affairs explicitly in terms of truthmakers.

3.1.2. QBism and the Problem of Truthmakers

It hence appears as if anti-realist interpretations in general only sweep the interpretational issue under the carpet, instead of actually solving it. Unless one accepts full blown MI, something which is rarely the case among defenders of *any* anti-realist or instrumentalist interpretation, the anti-realist still owes us an explanation regarding what it is that creates the empirical adequacy of such an instrumentalist wave function. In other words, they seemingly commit the categorical mistake of speaking of representational patterns without ever dissolving the riddle of what *it is that is being represented* or even downright deny the existence of the latter. David Wallace puts it well:

> "*(A)ny viable (...) account of quantum mechanics will owe us another account of what the probabilities encoded in the quantum state are probabilities of, of what the physical features of systems are. Or, put in more pragmatic terms: what the non-quantum features of a system are such that the quantum state is a tool for answering questions about those features*." [2], (p. 12)

Clearly observable structures such as interference patterns in the double-slit experiments might in themselves already rule out an ontological status as *mere calculational device*. Tim Maudlin has a similar take on this when he writes the following:

> "*one might also wonder how any theory that is not ψ-ontic could possibly account for interference phenomena, such as the Double-Slit experiment*". [15]

A fortiori, The unequivocal 'balloon'-like structures that appear when observing hydrogen atoms (with non-vanishing angular momentum), albeit being only a *diachronic* ensemble of particle detections, make it equally plausible to conclude that there is at least *some real* physical structure that causes such distinguishable 'real patterns' [16]. Stronger still, they also causally affect each other, which can be taken as prototypical for 'being physical'. Wallace, likewise, states the following:

> "*In physics jargon, there is interference between the ψ_x and ψ_y parts of the state, so that the x outcome is reinforced and the y outcome is cancelled out; interference phenomena like this are very general, and rule out the possibility of a probabilistic interpretation of the state space*." [17], (p. 5, my emphasis)

In other words, epistemic interpretations alone cannot explain such patterns and interactions and it seems to be equally ill-founded to call them *instrumental constructs*, as much as it would be to call snowflakes *instrumental constructs.* What they have in common is that both phenomena exist in the world and both involve an ensemble of a detectable particle-like structure, whereas the former supposedly only exists as a *temporal* diachronic (as opposed to a synchronic) physical ensemble within spacetime; however, this is a difference that should not be crucial for the point being made. At any rate, there are good reasons to conclude that *physical* truthmakers are what is being represented by the QM formulism. Wallace elaborates this general argument by stating the following:

> "*In any concrete instantiation of quantum theory, the observables over which probability distributions are defined are particle positions, field strengths, collective spins and the like. The only way to say anything non-circularly about an agent's experience in quantum mechanics is to characterize it externally, as an experience of something describable in a more physical language. And then the problems confronted by (such) strategies, (. . .), reappear*." [2], (my emphasis)

This oppositional overture culminates in the following observation: anti-realists and instrumentalists cannot possibly explain quantum effects *outside of the laboratory context*, such as cosmic background radiation, fusion inside of stars, Geiger counters, etc. These clearly violate any attempt to regard quantum states as being 'constructed' within artificial experimental setups and any claims of them merely enjoying an existence dependent on (complementary) measurement arrangements:

> "*Explanations of, say, superconductivity, or the heat capacity of crystals, or the thermodynamic features of the quark-gluon plasma, or the colour of gold, or any of the thousands of concrete applications of quantum theory that form its real empirical base, seem out of reach for QBism, or for pragmatism, at least as they are currently stated*." [2]

As stated above, *if* it is only a 'useful fiction/instrumental device', what is it then that establishes a correspondence to (objective) truthmakers that are clearly beyond human artifacts; in addition: *if* the wavefunction equals an anti-realistic representational placeholder, what is it *in* the physical world that *is represented*, or what is it that the wavefunction corresponds to? QBism in particular intentionally remains vague or silent on these matters. In fact, Wallace claims, and I think rightly so, that by the *parity of (QBist) reasoning*, even objects of classical mechanics better ought to be regarded as non-realist:

> "*(. . .) just as it would be misleading to call classical statistical mechanics non-realist simply because the distribution function does not play a representational role, so would it be misleading to call these approaches to quantum probabilities, as QBist ´s particularly tend to argue, non-realist simply because in those theories the quantum state does not play a representational role either*." [2]

Hence, when applying QBist oeuvre more generally (as actually being undertaken in [12,18]), then one might also claim that it is indeed nonsensical to ask what probabilities in classical mechanics are probabilities *of*. If this strikes one as absurd, then it should also do so in the quantum case.

3.1.3. An Ontology of 'Pure Experience'?

In this spirit, one could polemically (but quite legitimately) ask the QBist, if, when all there is to quantum mechanics are 'calculational devices' in terms of agent-relative experience, this in turn implies that we human agents, who play a rather crucial role according to QBists, actually '*consist*' of calculational devices and/or agent-relative experiences. It is indeed a difficult task to comprehend how a conglomerate of Bayesian patterns (and nothing else!) constitute tables and chairs and, indeed, observers themselves. QBist Bayesian beliefs (and updates thereof), it seems, are in desperate need of truthmakers in order to stay coherent.

The QBist, in response to this challenge, however, seems to be willing to bite the bullet. Indeed, the QBist in fact *does* claim, in some defenses at least, that the world *is* made *purely* of experience (and, presumably, corresponding Bayesian structure thereof). At one place [19], Chris Fuchs seems to flirt with this extreme route by following and quoting William James:

> "*My thesis is that if we start with the supposition that there is only one primal stuff or material in the world, a stuff of which everything is composed, and if we call that stuff 'pure experience', then knowing can easily be explained as a particular sort of relation towards one another into which portions of pure experience may enter. The relation*

> *itself is a part of pure experience; one of its 'terms' becomes the subject or bearer of the knowledge, the knower, the other becomes the object known"* [19], (p. 32)

However, here a follow-up challenge can be brought forth: despite phenomenological leanings to which QBists are obviously committed to, even to such a radical extent as to follow William James' '*Pan-Experientialism*' (PE), which argues that the world is in fact *made* of experience (as the quote from above shows), an unresolved question remains: why is it then that the number of possible experiences seemingly gets *arbitrarily reduced* to only a handful of experiences, when there is no other ontological category (no *non-experience stuff*!) or experience-independent truthmaker that performs the orchestration? This brings us to the following counter-factual claim:

Counterfactual 1.1. *If no reduction or selection happens in this 'possible experience space', then PE collapses into a pan-experientialist variant of MWI (PEMWI).*

The QBist will certainly deny this, but here again, I want to argue that conceptual coherence dictates that the following is, conversely, the case:

Counterfactual 1.2. *If certain experiences are privileged via updating events and/or state reduction, then objective state of affairs, known in terms of hidden-variables, are required to make such a selection reasonable.*

Either way, QBism collapses (into rival objective views) and hence seems conceptually incoherent *in and by itself*.

In fact, it appears then, *even* after implementing MI, MD or PE (each controversial in its own right), QBism either logically implies the existence of *some* form of hidden-variables theory to solve these issues concerning state selection/reduction and/or mind–object correspondences, or, conceptually collapses into a variant of MWI, e.g., PEMWI. However, according to their respective orthodox formulations, any anti-realist interpretation discards any class of hidden-variables *and* MWI likewise. *These assumptions in tandem seem to entail an internal conceptual incoherency in anti-realist views in general.* Hence, the suspicion arises that any stance belonging to this class might be conceptually incoherent.

3.1.4. An Ontology of Pure Information

However, here it is worth noting that some 'reconstructional' information-theoretic approaches to QBism, similar to informational neo-CI views, regard the quantum state as an *information-theoretic* entity [18,20]. This, however, in turn entails an ontic or quasi mind-independent existence such that less, or no priority at all, is assigned to the role of the observer. This alone however does not in any way help with the aforementioned problem of state selection or reduction. Quite to the contrary, a realist take on an 'informational state vector', without any additional hidden variables, seemingly dictates the realness and persistence of the *whole* state-vector. Stronger still, arbitrary reduction may violate the conservation of information, a principle which might naturally accompany all such information-theoretic approaches.

This plot thickens when it is considered that, many MWI advocates, such as David Wallace and Simon Saunders, have Ontic Structural Realist (OSR) leanings for reasons that go beyond the scope of this work (here it should suffice to say, that this appears to many as the 'best of both worlds' in the scientific realist vs. anti-realist debate, see e.g., [21–24] for introductions to and discussions of OSR). However, further conceptual support for the claim of natural affinity between OSR and MWI might come from the heritage of OSR, namely Mathematical Structuralism (MS). According to MS, any mathematical object, a number say, is merely a node in a structure that exists only due to other nodes (and vice versa). It is therefore an affair of mutual ontological dependence, one that requires that all nodes are ontologically on a par (the irreducible, 'primal difference' of binary numbers may be a prototypical example–see e.g., [21] discussing the work of Floridi for details on this point). Likewise, if we transfer this concept to quantum mechanical structures, all states or state-vector components must be interpreted as structural nodes that are ontologically on the same footing. *Conservation of information, together with a stance of capturing information in terms of structural principles of ontological interdependence, seem then to dictate the realness and interdependence of all state vector components* (due to their binary nature, this seems to be of particular concern for qbits). Hence, it follows that any state is equally substantial as any other, exactly what MWI presupposes (as an aside: OSR might foster the plausibility of MWI for yet another reason; if it is granted that elementary particles are nothing but structural relations, then the quest after "what breathes fire into certain equations" - paraphrasing Stephen Hawking´s infamous question - becomes non-sensical. Following the lines of Max Tegmark [25], who in turn got inspired by Wigner´s 'Unreasonable Effectiveness of Mathematics' [26], there is no true distinction between mathematical and wordly structure: both stand in an isomorphic relationship. Similarly, and consistent with the lines of thought of MWI proponents, it would be pointless to ask what breaths fire into the *one* particular quantum state to be actualized. All of them, since all of them are equally real in structure, *are* actualized.)

Though this arguments are certainly controversial, in terms of the underlying *ontology* at the very least, neo-CI, information-theoretic approaches to QBism and OSR come close to being indistinguishable (a point that becomes relevant below). Particularly, Information-Theoretic Structural Realism (ITSR), as defended by Ladyman and Ross [21], may be apt for such a deflationary argument. However, there are more reasons for a deflation, which will become obvious in a moment. The crucial take away message of this section is however the following: if the underlying ontology is regarded as informational – if the state vector is regarded as informational without postulating further principles, structures or mechanisms regarding *state reduction*, then the principle of conservation of information, if nothing else, prohibits, some *arbitrary* state reduction. *Then, despite of being labeled QBist or neo-CI, a MWI treatment of the state vector follows necessarily.*

3.1.5. A Trilemma for the Anti-Realist

The whole situation can be formulated and reduced to a trilemma of three mutually incompatible counterfactuals, whereas prima facie, only one of them can be true:

Counterfactual 2.1. *If quantum mechanics is only a mind-dependent calculational device in terms of subjective Bayesian patterns and experiences, and since no further class of structural or non-structural entities are involved as constituent elements in the world ´s furniture, then full blown PE has necessarily to be accepted, such that reality is purely experiential and the whole possibility space of experiences is ontologically on a par.*

Argument I.

AI.1. QM entities are mind-dependent Bayesian structures

$$(\mathrm{QMx} \rightarrow \mathrm{Bx}) \tag{1}$$

AI.2. All physical objects are QM

$$(\forall \mathrm{x}\ \mathrm{QMx}) \tag{2}$$

AI.C. *Conclusion*: the physical world utterly consists of Bayesian structures. (*AI.1. & AI.2.*)

$$(\forall \mathrm{x}\ \mathrm{Bx}) \tag{3}$$

Now, after adding QBist 'PE assumptions', it can furthermore be argued:

AI.3. No Bayesian structure/experience is *privileged or in any way selected without additional ontology or principles*, i.e., there is no distinction between manifest/unmanifest experience.

$$(\forall \mathrm{x}\ \nexists \mathrm{y}\ \mathrm{Bx} \wedge \mathrm{My} \wedge \neg \mathrm{By}) \tag{4}$$

AI.C2. *Conclusion*: all Bayesian belief- or 'experiential' structures, that is, all components of the state-vector according to QBism, exist on a par and comprise the sole ontological furniture of reality.

Let us label this stance accordingly *Pan-Experientialist MWI* (PEMWI).

Counterfactual 2.2. *If quantum mechanical states are informational and real (mind independent) entities, and since no further class of structural or non-structural entity is involved as a constituent element in the world´s furniture, then some form of Informational Monism or OSR follows. This position, however, might, without adding a dualistic mind or hidden variables that cause state selection, similarly collapse into a version of MWI. Furthermore, arbitrary state reduction would violate the principle of conservation of information.*

Argument II.

AII.1. QM is all informational/structural.

$$(\mathrm{QMx} \rightarrow \mathrm{Ix}) \tag{5}$$

AII.2. The physical world is comprised of no further (physical or mental) structural or non-structural entities, and no element of this structure is ontologically privileged.

$$(\forall \mathrm{x}\ \nexists \mathrm{y}\ \mathrm{Ix} \wedge \mathrm{My} \wedge \neg \mathrm{Iy}) \tag{6}$$

AII.C. *Conclusion*: QM is informational and can be taken as 'OSRist MWI' (OSR–MWI).

Counterfactual 2.3. *If quantum mechanics is a calculational device that merely represents the mind-independent structure incompletely, then some hidden-variable theory necessarily must be taken as true.*

Argument III.

A.III.1. QM is mind-dependent representational structure.

$$(\mathrm{QMx} \rightarrow \mathrm{Rx}) \tag{7}$$

A.III.2. QM is an *incomplete* representation, such that next to the represented structure, further ontic-structural or non-structural entities necessarily exist.

$$(\forall \mathrm{x}\ \exists \mathrm{y}\ \mathrm{Rx} \wedge \mathrm{Hy} \wedge \neg \mathrm{Ry}) \tag{8}$$

A.III.C. *Conclusion*: Other mind-independent structures, i.e., hidden-variables, exist.

Hence, it seems the proposition can be made that any anti-realist position *either collapses into a realist hidden-variable view or into a version of MWI, or is conceptually incoherent.* From a purely logical point of view, there seems to be no further option left.

3.1.6. Anti-Realist Interpretations: Preliminary Conclusion

The suggestion being made in this section is that even epistemic interpretations either require some ontological explanans in order to explain either observer–object links, hidden variables causing a state selection/reduction, or, an ontological explanans of which the wavefunction is a *complete* representation of; however, this in turn implies that an ontological or realist interpretation sneaks in through the backdoor at any rate.

As hinted above, however, an apparent loophole might be the proposal that quantum information *itself* plays the role of the truthmaker, such that ontological fundamental status is assigned *to the representational structure itself* (Counterfactual 2.2). However, then, the view culminates into one involving quasi objective truthmakers: if one follows this information-theoretic approach to its logical extreme, one might end up with a form of OSR or ITSR. Furthermore, as argued before, in this scenario also, removing some state vector components of the information-theoretic structure in 'updating events' while preserving others, seems ad hoc. In contrast, an ontological egalitarian co-existence of all state-vector components appears to be more natural within such 'Informational Monism'. A fortiori, an arbitrary state reduction would violate the *conservation of information*, a principle that might be all the more relevant given the presupposition of a *pure* information-theoretic ontology.

By the same token, if *everything* is regarded as informational, then *any* distinction between *possible* and *actualized* states cannot plausible be drawn without adding any additional hidden variables as selecting principles. Without this additional step, any eigenstate of an informational Ψ is equally 'real' as any other and stands on the same ontological footing, without any 'asymmetry of realness' whatsoever. An ontology follows in which no part of a wavefunction is in any way ontologically privileged. The distinction of MWI then seemingly becomes weaker, if not downright deflated.

As a consequence, such prima facie 'anti-realist' informational interpretations might not only collapse into OSR, but ultimately also into MWI. Let us label the resulting view OSR–MWI. Furthermore, as mentioned above, given that defenders of MWI very often rely one OSR ontology to begin with, OSR–MWI may turn out to be also indistinguishable from 'ordinary' MWI.

However, there is more. Given that QBism suggests Bayesian beliefs being coupled to pure experiences which are also taken to be informational, the distinction between AI.C2 and AII.C also becomes vague if not negligible. As a consequence, if reality is indeed a pure 'experience space' (as Chris Fuchs suggests), then mind-dependence also becomes vacuous: if a *whole range* of experiences exist (which follows necessarily when equating pure experience with the quantum-state and nothing more), multiple corresponding observers exist as well, given that they are themselves to be taken as *subsets* of an all-encompassing 'set' of experience (recall the William James quote above). What then follows is a mind-independent *range of experiences with varying 'amplitudes'* that, like before, without any selection or reduction principles, are on a par in terms of reality (principles or entities which would, recall, be desperately needed to create a distinction between 'potential' and 'actualized' experience).

In other words, both the 'object' *and* 'subject' pole are then, according to this view, encoded in experience, such that the former is not in any way transcending the latter. And not only is it then encoded in experience, but multiple copies of it are encoded, due to the ruled out, *in*-egalitarian existence of state-vector components.

Hence, when formulating experience in purely structural or information-theoretic terms, (which is, as the QBist herself seems to be willing to accept as the only reasonable way to capture experience in scientific terms, the distinction between AI.C2 and AII.C evaporates completely.

It is time to wrap up the preliminary results gained so far:

i. Ad. Counterfactual 2.1.: PE entails that reality is comprised entirely of Bayesian belief structures coupled to pure experience such that no structure is ontologically privileged or in some way selected.
ii. Ad. Counterfactual 2.2.: Information theoretic 'anti-realist' (e.g., informational QBist or neo-CI) interpretations have ultimately to be understood in terms of OSR such that they themselves in fact comprise an *ontic* structure of reality. Without adding mind-induced collapses or further hidden variables for state reduction or selection, thus having an egalitarian outlook at all possible eigenstates, these views collapse into OSR–MWI. The principle of conservation of information furthermore prohibits any arbitrary state-reduction.
iii. Ad. AI.C2 and AII.C: The difference between a Bayesian structure (without additional ontology) and an information-theoretic ontology becomes negligible. Hence OSR–MWI~PEMWI

iv. Given that many defenders of MWI very often have OSR leanings, it might ultimately also hold that OSR–MWI~PEMWI~MWI.

v. Ad. Counterfactual 2.3.: Bayesian belief structures either require objective truthmakers and mind-independent mechanisms for state reduction or selection, which are themselves hidden variables, or, are structurally complete. If the former, hidden variables exist; if the latter, then it is as elaborated above, i.e., QBism → PEMWI~OSR–MWI~MWI.

Hence, a first preliminary result of this analysis is that, either way, a realist interpretation (either hidden variables or MWI) is necessary to solve this interpretational issue, a conclusion which, if correct, already critically narrows the field of possible interpretations.

Similarly to what has been discussed by Dürr and Lazarovici [14] (Section 2.2 of ref. [14]), any attempt to solve the dilemma of the measurement problem basically leaves us, after having discarded conventional anti-realist options as conceptually incoherent, with two remaining options: Either the formulation is

(a) *empirically adequate, but in some sense, nomologically incomplete or*
(b) *empirically adequate and nomological complete.*

Hence, remaining agnostic about option (b) (completeness) for the moment, we should have a closer look at (a), assuming hidden variables.

However, before investigating the potential of hidden-variable theories, being this subsection´s conclusion only remaining alternative to MWI, let us first make a detour and inspect Carlo Rovelli´s Relational Quantum Mechanics (RQM) more closely. Due to regarding relations between the observer and observed system as crucial, it seems natural to discuss it right after QBism. However, it has a special status, for it neither falls within the domain of anti-realist interpretations, hidden-variable theories nor, presumably, within MWI. However, in the subsequent subsection, I will argue, similarly to the results gained in this subsection, that the latter better ought to be the case.

3.2. Rovelli´s Relational Interpretation—MWI in Disguise?

In this subsection, I want to take a closer look at RQM. In order to provide a short introduction, I will, for reasons of adequacy and efficiency, directly quote Carlo Rovelli´s own words:

> "RQM is based on an ontology given by physical systems described by physical variables, as in classical mechanics. The difference with classical mechanics is that (a) *variables take value only at interactions* and (b) *the values they take are only* relative *to the (other) system affected by the interaction*. Here "relative" is in the same sense in which velocity is a property of a system *relative* to another system in classical mechanics (*as opposed to "subjective"*). The world is therefore described by RQM as an evolving network of sparse relative *events*, described by punctual relative values of physical variables." [27], (emphasis & addition in brackets added)

Furthermore, Laudisa and Rovelli emphasize the following:

> "*the interference observed by a system S′ is not erased by the actualization of variables relative to a different system S″*". [27]

At this point already, I must object: are not such dissimilar states, observed by S′ and S″, as proposed by Rovelli, necessarily *disparate realities* and hence, *many worlds in disguise*? In this subsection, I thus want to argue for the following dilemma:

Counterfactual 3. *If the contrary is true, that is, two different observers agree on a quantum state such that its observed value is observer-independent, then it cannot be intrinsically relational.*

In other words, I want to make the following case discrimination, which can be taken as two horns of the same dilemma for defenders of RQM:

(a) either they are giving up the relational aspect of RQM, or,
(b) they too are bound to accept MWI.

If (a), then the observed state is non-relational and hence the view might be reducible to some variant of objective-collapse hidden-variable theory such that state reductions happen unaffected by the relations of the physical interactions between the observer and observed, thereby making the name-giving notion '*relational*' redundant. A recent work by Lawrence, Markiewicz and Zukowski [28] seems to support this first horn of the dilemma by stating that (i.) RQM is not merely an interpretation but a new theory and (ii.) that hidden variables occur in RQM: "*relative facts asserted by RQM are either a void concept, or form a direct contradiction with quantum mechanical predictions*", given that "*if an effectively complementary measurement is done by [S", after measurement by S′ on S has been performed, variables] are algebraically equivalent to non-contextual hidden variables*" [28], (my addition).

At any rate, this is not how RQM is treated and it would render the notion itself inconsequent and even contradictory. RQM clearly states that two different observers, or interacting subsystems, perceive different experimental outcomes when interacting with a third subsystem. After all, Rovelli himself coined the following phrase:

> "*Different observers can give different accounts of the same set of events*". [27]

Hence, by reductio, if (b), then one seems to be bound to—in order to really and uncompromisingly account for the relational aspect of quantum measurement—also be committed to a (parallel) multiplicity of observed states and hence to a multiplicity of 'branches' (relative to observers). This clarified definition of RQM, as a consequence, might ultimately be phraseable in the following way:

> *The particular conjunction of variables an agent observes exists relative to the sum of all her physical interactions, i.e., to the whole web of physical relations involved. The latter, in turn, selects definite values for each quantum system that an interaction has occurred with. However, this sum total is not unique given that different 'webs of variable values' occur for different interacting systems. Furthermore, given that such web of objective physical interactions behave similar to a spreading entanglement (Rovelli uses the phrase of "an evolving network of sparse relative* events"*), it seems ultimately conceptually no different from a decoherence-induced emergent branch as according to MWI.*

In a sense then RQM might be nothing over and above a reverse-engineered notion of decoherence-based branching. What is more, given that I myself am the object of other observations/interactions, multiple versions of myself necessarily exist if the RQM is consistent and true; these are then alternate versions to which my *particular* self-observation has no access to.

Even Rovelli himself admits the following at one point:

> "*Understood in this manner the quantum state is always and only a* relative state *in the sense of Everett. In this sense RQM is "Everettian"; it is so in a different sense than the Many Worlds interpretations, which are based on a realistic interpretation of the universal wave function, rejected in RQM.*". [27]

The last statement requires further elaboration which leads to another fine-grained distinction: if Rovelli chooses to discard only a *universal* but not *local* realist wavefunctions, RQM requires something akin to objective collapse, hence also the inclusion of hidden variables which would lead to same incoherent state of affairs as discussed above.

If on the other hand, this 'denial' of the universal wavefunction is to be understood as challenging the realist status of the wavefunction *per se*, then also, formerly discussed challenges re-emerge, for this would contradict Rovelli´s own introductory notes, in which he clearly emphasizes that RQM should *not* be understood as relational in the 'anti-realist' or 'mind-dependent' sense, but as similar to *relationalism* in special relativity: it is *objectively* relational, namely relative to *frames of reference*, which however implies that a whole spectrum of possible values exist in the realist sense. The only remaining logical option for a mind independent relative state interpretation, seems then to be one that involves an egalitarian outlook or co-existence of all states.

Simon Saunders [29] likewise defends a relative state interpretation, but contrary to Rovelli, *his* interpretation of relational states does not deny the implication of 'parallel realism'. Saunders compares this to the tenseless block universe, using the latter as analogy. For a start, inasmuch as the notion of the *now* is indexical, different *nows* do exist. Similarly, a relative state does not neglect the co-existence of different *actualities*. Or, Saunder's states the following in his own words:

> "*(...) the interpretation had better not introduce a notion of privilege for one domain over another, in anything more than an interest-relative sense. Again, there is an analogy in the case of time: if we are to provide an interpretation of the "flow" of time, consistent with a relational account of tense, then it had better not lead to any "absolute" significance of one space-time foliation over another; and conversely, if we do have the latter, then there is no sense to the appeal to anthropocentric factors*." [29]

According to the relative state interpretation defended by Saunders, the locally observed state of the wavefunction can be regarded as something akin to an 'actuality slice', with its locally observed values depending on the observer´s own relative state with respect to that of the quantum state to be measured, which, in turn, can be translated to the particular web of relational interactions that one is entangled with, according to RQM. Therefore, when Rovelli talks of relative states, he has to bear in mind that when different interacting subsystems allow *different* physical variables to emerge relative to one particular observed subsystem, then this is almost synonymous to the existence of a multitude of 'actuality slices', each of them relative to observer -frames of references, - 'carved' out of a 'block *multiverse'.* Different slices (which can, as we saw, in this context interchangeably be used with branches) then correspond to different, but equally real, webs of relational interactions and variables thereof. Interestingly, Carlo Rovelli himself acknowledges this, while presumably not the global consequence it entails:

"[T]he state of the cat *with respect to* the external world does *not* collapse when *a part* of the cat interact *with another*." [27].

In other words, it seems that RQM entails the fact that we must acknowledge that there is a different actuality relative to the cat than that relative to other potential observers in its surrounding world. Since such differences can be as radical as being alive vs. being dead, RQM seems to be bound to accept different genuine *realities.* When it does follow from RQM that the cat is dead for Wigner, but not for his friend, then it unavoidably states a multiplicity of realities, or branches, for that matter.

In his delightful book '*Helgoland*' [30], Rovelli however explicitly tries to circumferent this consequence: "*Prima facie, RQM may seem to imply a form of perspective solipsism, as the values of variables realized in the perspective of some system S′ are not necessarily the same as those realized with respect to another system S″. This is however not the case, as follows directly from quantum theory itself. The key is to observe that any physical comparison is itself a quantum interaction. Suppose the variable E of S is measured by S′ and stored into the variable Z of S′. This means that the interaction has created a correlation between E and Z. In turn, this means that a third system measuring E and Z will certainly find consistent values. That is: the perspectives of S′ and S″ agree on this regard, and this can be checked in a physical interaction.*".

Rovelli labeled this preservation of intersubjectivity 'Cross-Perspective Link'. A recent detailed critique of this purported "fix" can however be found in [31] by Lahti and Pellonpää.

They respond that "*this is a new independent assumption and it appears to be incompatible with the preceding ideas trying to exhibit the assumption (of RQM)*" and "*without assuming that the postulate of cross-perspective links holds also in this case, the conclusion that we all 'see the same world' is still unjustified.*" [31].

Hence, it seemingly follows that *either* RQM is not truly relational if it tries to recover a one-world picture: a particular relational value selected by S (perhaps via self-interaction) then holds *globally*. However, this is not then truly selected 'relationally', but in a manner of objective (agreeable) collapse, thus as a non-relational hidden variable for the whole world. Or, *if* it remains to stays true to the original RQM assumptions, 'one world' cannot possibly follow.

The crucial point which must, I think, be stressed over and over again, however, is this: if Rovelli denies the realism of other possible relational states, then his whole view appears to collapse into a non-relational one and hence becomes redundant at best and incoherent at worst.

When putting the discussion of this section in an argument-conclusion form, we obtain the following:

Argument IV.

AIV.1. According to RQM, physical structure is a substantial and realized structure of *interrelated* physical properties ('facts') and their respective definite variables.

AIV.2. Due to the nature of RQM, more than one coherent network of physical definite variables (due to density matrices) of interacting objects exist; *outcomes would not be relational, but would be guided by hidden variables otherwise*.

AIV.3. Any such '*evolving network of sparse, relative events*' is equivalent to a branch of decoherence-based MWI as physical interactions are, for all practical purposes, restricted to non-decohered domains. In addition, according to both views, in principle, all permutations of variable values and correlations are possible, which is synonymous with the existence of multiple branches.

AIV.4. *If* one such network is physically privileged, RQM would no longer be *relational* [28], but would involve hidden variables instead. RQM would be conceptually or even logically inconsistent ($R \land \neg R$ -contradiction).

AIV.C *Conclusion*: RQM collapses into MWI.

Proof Sketch.

$$(\mathrm{RQM} \rightarrow \mathrm{MWI}) \lor (\mathrm{RQM} \rightarrow \mathrm{HV}) \tag{9}$$

$$(\mathrm{RQM} \rightarrow \mathrm{HV}) \rightarrow (\mathrm{RQM} \land \neg\, \mathrm{RQM}) \text{ (Contradiction)} \tag{10}$$

$$\mathrm{RQM} \rightarrow \mathrm{MWI}. \quad \square \tag{11}$$

After having discarded RQM as either conceptually incoherent or as MWI-in-disguise, let us now, in this final round of conceptual analysis, inspect one remaining alternative group of interpretations, namely hidden-variable theories; this is the option that we also identified as one of the two remaining viable alternatives to Anti-Realist Interpretations in Section 3.1.

3.3. Bohmian Mechanics (BM) and Objective-Collapse Theories (OCT)

3.3.1. Introducing BM and OCT

Both Bohmian Mechanics (BM) [7] and Objective Collapse Theories (OCT) [8] share the common theme of postulating one way or another an objective and real mechanism that constitutes localized particle dynamics (see e.g., [32] for discussion). Both theories only differ in the way they try to accomplish this. In the case of the latter, however, the 'collapse of the wave-function' is regarded as a real occurring, physical substantial, quasi mechanical process, one that can be regarded as a combination of deterministic and stochastic hidden variables. Objective collapse theories (OCT), such as the Ghirardi–Rimini–Weber (GRW) Theory, which is its most discussed example in the literature, take collapses as physical events, from which highly confined particle-like patterns emerge. However, how this works out in detail depends on a further sub-distinction (see e.g., [9,14]): GRWf uses a so-called *flash ontology*. Here, particles are understood as instantaneous 'flashes' and the wavefunction represents the probability of where they occur stochastically. In contrast, GRWm regards the wavefunction as a physical substantial distribution in spacetime, - much like a continuum, representing matter density. Thus, in this picture, the collapse can quite vividly be understood as a *contraction* of a physical medium. However, either way, the ontic existence of the wavefunction seems to be required, either in the form of mass density or as something that provides physical grounds for flashes. Wallace likewise claims the following: "*For collapse theorists, the wavefunction is a physical entity*." [33].

In contrast to both, BM basically can be taken as an even stronger attempt to bring back a classical way of thinking and tries to accomplish this via hidden-variables qua hidden particle *trajectories*. In an attempt to both 'save' the (classical) wave *and* (classical) particle phenomena and by that, allegedly choosing a concept "free of paradoxes", the idea emerged that there exists *both* a physical real particle *and* a wave, whereby the former kind of "surfs" on the latter, which in the theory, is labeled accordingly as the '*pilot wave*'. Hence, similar to classical ideas we are used to, there is a discrete particle following the real trajectories; the former, however, in our current understanding, is the *only* thing to be measured. Then, there is no such thing as a wavefunction collapse: indeed, the wave neither collapses nor is measured directly, but is only *indirectly* observed, by affecting, for instance, the probability distribution of where a photon or electron hits a screen.

3.3.2. General Challenges

The most severe objections this subgroup of theories is facing might be that they focus on and revolve too much around ordinary waves and particles 'living' in a (often) non-relativistic spacetime. However, the wavefunction as a extended spatial object is only a special case of a much more broad and abstract concept, according to which the wavefunction exists in Hilbertspace; this is something that might turn out to be the greatest weakness of both approaches: both BM and GRW seem to *require* a physical, spatially extended form of the wavefunction in spacetime (or within configuration space in a derivative sense); this is a semi-classical way of thinking that seemingly rests on outdated beliefs, more than anything else. Further still, they are mainly discussed in non-relativistic contexts, as put by Wallace:

> "*(...) the way Bohmian mechanics, and GRW theory, are normally discussed in philosophy of physics (especially in more metaphysical contexts) is sharply at odds with the relatively humble role non-relativistic particle mechanics plays in real quantum theory. The only way I know to make sense of (most of) this literature is to interpret it as discussing non-relativistic quantum particle mechanics under the fiction that it is a fundamental and universal theory.*". [2], (p. 43)

However, in [14], (p. 115) on the other hand, it is stated that at least GRWf indeed "*can be generalized to relativistic spacetime without violating any principles of relativity*" and similar attempts exist for BM.

Still, Wallace legitimately claims that such theories, which aim at *modifying* the QM formalism (such as BM and GRW), always only account for *one application* of quantum theory *but rarely for all of them*, while MWI´s wide-ranging applicability extends to practically all QM tools and applied contexts.

Further received objections, concerning BM in particular, are related to the assessment that a real physical wave acts on a particle, *but not vice versa*. It might seem reasonable to detect non-linearity or some form of feedback loop between both entities. However, such non-linear effects so far have not been observed [7] (though it might be fair to admit that the confirmation of such deviation from linear dynamics is technically challenging at best and empirically unverifiable at worst, given that the effects may be subtle and hard to separate from environmental noise). In addition, if the wavefunction is a physically *real* object in the classical sense, then some measurable interaction between it and the surrounding matter should likewise be detectable, very much like measurable nonlinear disturbances of a classical field or fluid, which, again, is not what has so far been detected.

A highly relevant objection against BM, mostly stemming from MWI defenders, is the following: if the pilot wave acts only as 'particle carrier' at one particular trajectory or spatial eigenstate, then what happens to the rest of the non-collapsing (enormous and supposedly universal) wave function? [7] This becomes particularly relevant in the macro realm: does BM propose whole distinct world-like branches that are simply 'empty'? That is why David Deutsch coined the phrase that BM is

> "*parallel-universe theories in a state of chronic denial*". [34], (p. 225)

Due to environmental-induced decoherence, which is also relevant to such real 'carrier' wave which can be understood as a quasi-spacetime-state realist wavefunction, the totality of empty components would then constitute a structure similar to 'empty' branching worlds; this is, however, only in terms of wave structure, while being devoid of particle 'content'. Simon Saunders elaborates this at [35]. In order to avoid this, the Bohmian would be at pains to add an OCT-like process, thus dropping 'empty' waves from the guidance equation as some sort of *quasi state-reduction*. Given that ordinary BM is already technical challenging, this would blow up the required calculational apparatus beyond reasonable dimensions.

Interestingly, the same objection can also be brought forth against OCTs: here, as well, given that the Schrödinger wave is realistically interpreted in most versions, the problem of 'empty' parts (i.e., free of 'contractions' or 'flashes') of the wavefunction persists. For instance, GRWm involves stochastic processes that transform a spread wavefunction in space into a localized Gaussian shape (by multiplying it with a Gaussian function), something which involves the formulation of a non-linear version of the Schrödinger equation. However, this process then still exhibit non-vanishing (relative) amplitudes outside of the collapsing region, known as the so called "*problem of tails*" (see [8,35]). Given that OCT wavefunctions are very often taken as physically substantial, such tails should not only be measurable in principle, but, due to their realist status, as before, should also seemingly lead to 'empty branches' [35] (here, replacing the continuous with a spatially discretized, i.e., cellular automata-like formulation of GRW or BM, might perhaps be legitimate attempt to get rid of such non-vanishing amplitudes and empty-branches. However, this might still be of no help to the challenges to be discussed in the next subsection).

However, one may suspect some further interpretational issues: first of all, how is such an objective collapse (or 'flash') in GRWf qualitatively conceptualized to begin with? What is the nature of such a 'flash' and how is it in turn ontologically related to the wavefunction and its amplitude? Or, in the case of GRWm, should we really imagine it, in a loose analogy, to be similar to a field or field-like medium that 'contracts' eventually?

In '*On the Common Structure of Bohmian Mechanics and the Ghirardi–Rimini–Weber Theory*' [9], exactly this is suggested:

> "*GRWm is a theory about the behavior of a field m(·,t) on three-dimensional space. The microscopic description of reality provided by the matter density field m(·,t) is not particle-like but instead continuous, in contrast to the particle ontology of BM. This is reminiscent of Schrodinger's early view of the wave ¨function as representing a continuous matter field. But while Schrodinger was obliged to abandon his early view because of the tendency of the wave function to spread, the spontaneous wave function collapses built into the GRW theory tend to localize the wave function, thus counteracting this tendency and overcoming the problem*."

However, here in particular, the question will arise as to what sort of law establishes such repeated alternations between the contraction and expansion of mass density. Although thermodynamics usually do not play a significant role in microphysical happenings, such a reversal of the spreading event is seemingly at odds with the second law of thermodynamics. One might also object: in the case of interference patterns such as in the double-slit experiment, how would such a physical interfering wave pattern, consisting of *alternating* amplitudes, *contract as whole*, and what is it that makes the locus of such contraction more likely in a region with high amplitude? One might respond that the question is ill-formed as it is simply the theory. However, given that dissatisfaction with the ad hoc nature of the original collapse postulation is what motivated the conception of hidden-variable theories in the first place, one might, by parity of reasoning, also demand a sufficient reason for this state of affairs. Besides, in the very same double-slit setup, what is it that conserves the effective particle number and quantization? Why would further contractions not suddenly occur within the same wavefunction, given their stochastic nature, such that one electron turns into several? Why does the process preserve particle characteristics *precisely*? All of this seems to be conceptually opaque.

Similarly, in the case of GRWf, it seems implausible that flashes are rigorously quantized in a way that saves the observed well-defined particle properties. In such a stochastic process of random 'lighting ups', randomness *also* in terms of particle properties, rather than conserved symmetries, might be expectable. Wallace likewise states that

> "even in the non-relativistic domain [GRW] is not fully satisfactory: manifestly, the collapse mechanism does not preserve the symmetries of the wavefunction, and so *it is not compatible with the existence of identical particles*". [33], (p. 42, my emphasis)

3.3.3. QFT and Particle Indiscernibility as Master Arguments against BM and OCTs

The problems culminate when finally taking Quantum Field Theory (QFT) into account. Wallace states that challenges concerning OCT/GRW and BM become especially severe when one tries to reconcile both BM and OCTs with the standard model of particle physics or QFT:

> "*(I)t is much harder than is generally recognized to construct a quantum-field-theory version of Bohmian mechanics or GRW theory and so confidence that such a theory even exists i[s] premature, because most of the features of nonrelativistic quantum theory appealed-to by metaphysicians of quantum mechanics are emergent approximations at best in QFT*." [2], (p. 19)

This should be taken as the most severe blow against OCT and BM interpretation brought forth so far. For both BM and GRW, it is part and parcel to assume particles as something to be understood in the classical, *spatially confined*, sense, or at least in its proximity: if they are not regarded as localized, clear-cut separated entities, then they are at least regarded as localized 'bumps' in a field. However, modern findings in QFT seem to shatter not only the former, but even the latter weaker notion of particles [36]. Dürr and Lazarovici [14] state the following:

> "*one might think that a field configuration would represent a particle configuration in some way, e.g., by distinguished "bump configurations" in the field. But that does not work out*."

The hope might remain that such a quasi-classical picture of field configurations might at least work out for bosons:

> "*Fermions are not conducive to a naïve field ontology, while bosons are*" [14]

A hope that eventually becomes shattered too:

> "*(...) bosons are commonly viewed as particles. For example, think of photons, the quanta of electromagnetic fields. A first thought might once again be that we should "see" the bosons in the field configuration as "bumps" in the field. But that is also more or less impossible*." [14]

This might indeed be the most devastating blow that both BM and OCTs can receive, since being interpretations that remain loyal to picturing particles as localized entities is what motivated their conception in the first place. Wallace puts it in the following way:

> "*so a fundamental ontology based on the positions of particles looks forlorn in quantum field theory*." [37], (p. 5)

Hence, what particles *are* is then presumably nothing over and above Fourier modes or 'excitations of quantum fields', something which can, by and large, be regarded as a received view within the physics community. This, in turn, leads to yet another argument, namely for the ultimately structural nature of particles. Ladyman and Ross claim the following:

> "*OSR agrees with Cassirer that the field is nothing but structure*." [21], (p. 153)

In the same vein, Simon Saunders [36], (p. 305) holds that

> "*coincidences of field values, and complexes of relations among them—(...) is a world understood in terms of structural descriptions, a world as graph, not a collection of things that evolve in time*."

And that

> "*in strongly interacting high-energy physics, it is doubtful that objects as individuated (using the Principle of Indiscernibles) by the invariant properties and relations definable in quantum field theory will be quanta at all*."

Such a state of affairs however would render a well localized particle ontology as BM and OCT require highly implausible, if not impossible.

Due to q-numbered (or operator valued) fields and the second quantization of QFT, even the particle number (and thereby particle *existence*) is equivocal in terms of being in a superposition. This, prima facie, violates the picture of a discrete and stable localized particle within a Bohmian guidance wave.

It is, however, fair to say that there *are* attempts to reconcile QFT and BM, such as the one given in '*A persistent particle ontology for QFT in terms of the Dirac Sea*' [38].

However, three prima facie objections can be launched against this particular approach:

1. The conception only works for fermions—bosons, are still regarded as field-like all space-pervading entities.

2. In [38], it is is stated that "*particles are primitive objects in the sense that they only have a position in space. All the other parameters including mass, charge, spin, etc. are not additional elements of the ontology characterizing the particles, but dynamical parameters employed to describe the evolution of the particle positions.*" It seems, then, that all the relevant physical characteristics of the fermions, save position, are to be found in their anti-symmetric wavefunction. A similar observation is made by Saunders and phrased in the following question:

> "*don't supposedly intrinsic properties of Bohmian particles like charge or mass (both gravitational and inertial mass) act, in experimental contexts, as if associated with the pilot wave rather than the particles?*" [35]

However, prioritizing the wavefunction that way seems to eradicate the difference between a pilot-wave and a spacetime state realist take of the wavefunction [39], and may render the particle trajectory for all practical and empirical purposes physically superfluous.

3. This 'Bohmian Dirac Sea' model only works reasonably well for high-energy cut-offs: it is doubtful that one can recover unequivocal Bohmian trajectories without them. Rather, spacetime regions are then to be taken as tightly 'occupied' such that, again, the distinction between a pilot- and an ontic (non-guidance) wavefunction seemingly becomes small (conversely: a MWI *lattice*–QFT approach makes the quantum fields 'grainy' such that, as a result, both a Bohmian and Everettian lattice–QFT model may become indistinguishable at a certain limit).

At any rate, when considering the fact that well localized particles and unequivocal particle numbers at a certain spacetime region are themselves non-fundamental, then no spatial component of a pilot wave can truly be regarded as precisely empty. It seems then, when factoring in QFT, BM becomes not just 'MWI in chronic denial', but rather MWI "in disguise" or indeed, MWI *itself*. That being - not just *any* version of MWI, but an explicitly decoherence-based MWI, given that what then *does* allow for the emergence of localized particles is then, presumably also, decoherence (this may perhaps also amount to a version of BM in which 'all possible initial conditions' are simultaneously realized).

The same state of affairs also seems to be conceptually challenging for any OCT, as such collapses are usually taken to account for unequivocal particle localizations and, thereby, numbers. The latter, however, as above, are better be regarded as an approximation and special case rather than the norm, -no serious interpretation or theory should solely focus on limiting case scenarios. For instance, when, in GRWf, well-localized flashes qua particles are neither well-defined nor well-localized but rather themselves subject to the 2nd quantization, then, here as well, flashes become, due to operator-valued fields, not clearly distinguishable. Here, it also then seems that, as in the case of BM, -if all position eigenstates are not precisely empty, the distinction from a MWI-compatible spacetime state realist wavefunction [39] seems to become small, if not negligible (another option for recovering compatibility might be to assume OCT or GRW collapses to be identical with vacuum fluctuations. However, then also the distinction to MWI evaporates, for then only at low QFT energies *and only after* decoherence phenomenologically distinguishable particles arise from a resulting n-particle QM).

Finally, the following remark could be made about *any* interpretation or modification of QM that assumes some or other hidden determinism: indeed, the whole argument of this subsection can be generalized such as to affect other classes of modal interpretations or hidden-variable theories like Superdeterminism [40], Retrocausality [41], or 'Contextual Collapse' [42] interpretations: if applying any of these hidden determinates to QFT, it seems that, similar to the critical remarks on [38], it will be still necessary to regard *all* spatial components of the wavefunction as physically substantial, thus leading into spacetime state realism and thus to decoherence-based branching eventually (however, investigating this further has to wait for another occasion).

The result gained in this subsection can be summarized in the following argument–conclusion form (for the sake of the argument and reasons of simplicity, I restrict myself to spacetime state realism. However, the argument might work just as well when assuming Hilbertspace realism such that everything is part of the Hilbertspace, and no part is ontologically privileged):

Argument V.

AV.1. According to BM and OCT, some spacetime regions are ontologically privileged (being occupied by particles). Hence, for all field points in spacetime, *some* are occupied by particles.

$$(\forall x\ \exists y\ Fx \wedge Py) \tag{12}$$

AV.2. According to MWI, *no* spacetime region is ontologically privileged. The universal spacetime state realist wavefunction has a value at every point in spacetime and nothing is not part of the universal spacetime state realist wavefunction.

$$(\forall x\ \neg\exists y\ Ux \wedge Py \wedge\neg Uy) \tag{13}$$

AV.3. According to standard QFT, no spacetime region is ontologically privileged. The Q-fields have, Fock-space formulations notwithstanding, a value at every point in *spacetime* and nothing in spacetime is not part of any Q-field.

$$(\forall x\ \neg\exists y\ Qx \wedge Py \wedge\neg Qy) \tag{14}$$

AV.C. *Conclusion*: while the universal wavefunction is not identical to Q-fields, both are spatial field-like entities and structurally similar.

QFT and MWI ontology hence seem to be more straightforwardly reconcilable than QFT and OCT/BM ontology, particularly when considering a *wave functional interpretation* of the former.

Finally, non-separability equally harms an ontology of classical fields and particles, as the latter rather belongs to an outdated ontology of monadic intrinsic objects and properties (see [43–46] for discussion)

Wallace elaborates this in the following way:

> "*Chris Timpson and I Wallace (. . .) regard this as a major failure of Lewis's doctrine of Humean supervenience, the doctrine that all facts about the world supervene on monadic properties of spacetime points and the spacetime relations between them: in our view, the entanglement between (say) spacetime regions A and B should be understood precisely as encoding certain irreducible relations between A and B.*" [17], (p. 17)

Despite the fact that both BM and OCTs are necessarily taken as non-local, the conclusion is rarely drawn in terms of the consequences for its presupposed particle ontology. Though debatable, it very much seems as if such irreducible relations in and by themselves already contradict classical particle and field ontologies as they are found in the ontological inventory of GRW and BM. Given that non-separability suggests that holistic entities are irreducible, it follows that extended objects such as the wavefunction, or indeed the universal wavefunction, is ultimately not reducible to point-like particles. Hence, when factoring in this assessment, a universal Ψ must, at any rate, be more than a mere *carrier* wave but rather an ontic entity to be *prioritized*; after taking this into account, BM and OCT ontologies may conceptually collapse into an ontic wavefunction ontology that is taken as substantial and *complete*, akin to that of spacetime state realism.

Finally, for obvious reasons, any interpretation that relies on pilot waves or collapses *within* spacetime is unsuited for theories of quantum gravity or 'emergent spacetime', which poses a severe limitation for future research in terms of unification [47].

After having, hopefully in a systematic and heuristically sound way, shown that anti-realist/instrumentalist, RQM and hidden-variable approaches either face conceptual challenges or collapse themselves into the bare formalism (to be taken as complete), we finally want to investigate what remains and why this alone suffices for MWI to be true; this is, the bare QM framework, from which, under the assumption of universal, unrestricted application, environmental-induced decoherence naturally follows, from which a branching structure naturally emerges.

3.4. Decoherence-Based MWI

Mainly following David Wallace´s influential *The Emergent Multiverse* [39], in this subsection, -after having discussed all major rival interpretations above, I take the unmodified framework of QM to be the one thing that has defied any criticism that has so far been launched. This is hence the interpretation that regards the framework as both empirically adequate *and* complete. Interpreting the formalism realistically essentially means *taking it seriously;* as such, interpreting the unitary evolution of the wavefunction realistically without any modifications or adding further ingredients unavoidably leads to the Everett Interpretation or MWI, the inception of which can be traced back to physicist Hugh Everett in the 1950s.

Let us finally inspect more closely what 'taking the formula seriously' implies; it suggests a 'collapse' to one definite outcome, or that a 'state reduction', in fact, never takes place or only *appears* to do so. In MWI, as opposed to CI, there is neither a causally effective observer, nor is there an abrupt quantum-to-classical transition, or Heisenberg-cut; this is such that quantum mechanics might, in an ad hoc manner, be assumed to stop working at the classical level. It is, quite to the contrary and in accordance with the general aim of unifying physics, assumed that, given that everything consists of a QM structure, a macroscopic structure likewise is subordinated to its laws. Hence, without adding additional hidden mechanisms, the unitary universal wavefunction [6] in its entirety, including its initial set of eigenstates, cannot possibly cease to exist. The only reason why they, except for the measured outcome, *appear* to vanish is the mechanism of decoherence [48]. Here is why, in a nutshell:

Since any measurement apparatus (and indeed the environment it presupposes) involves an enormous number of interacting particles, the effective wavefunction to be measured not only consists of the (prepared) particle itself, but also of the enormity of particles that constitute the apparatus and environment (a minor disclaimer: here I am using the term *particle* out of custom and convenience. By now, it should be clear that this does not imply a point-like ontology, however, decoherence is what will ultimately constitute a reasonably well-localized wave packet in a certain measurement setup). Due to an unavoidable spread of ordinary local physical interactions, which is a natural consequence of the measurement process, or indeed, the measurement process *itself*, the whole participating particle conglomerate becomes entangled eventually. Due to its complexity, one particular state to be measured, such as the particle position of x_1, will then be hopelessly out of phase relative to another position x_2: then, from the relative 'point of view' of x_1 (plus the environmental states it is entangled with), other position states (plus their respective entangled environment) *effectively disappear* due to non-interference (for the record, while interference becomes negligibly small, it does not disappear totally. This is also why branching is regarded as a continuous process, and as a consequence, *branch counting* is best understood in a coarse-grained sense).

These alternative (however similarly real and 'unaltered') position eigenstate-outcomes (qua state vector components), in turn, are entangled with yet another set of environmental particle state components. Each entangled 'state conglomerate monstrosity' will then eventually constitute a 'branch' or world, whereas non-interaction between *these* is practically guaranteed due to the effective impossibility of interference.

Wallace elaborates the following:

> "*Notice that it is not merely the linearity of quantum mechanics which allows us to interpret superpositions as instantiating multiple structures. Rather, it is the disappearance of interference terms between the relevant terms in those superpositions*." [39], (p. 68)

Following Wallace, describing this state of affairs in a formal toy model may look like the following:

$$(\alpha\psi_{x1} + \beta\psi_{x2}) \otimes \varphi_0 \rightarrow \alpha\psi_{x1} \otimes \varphi_{x1} + \beta\psi_{x2} \otimes \varphi_{x2}. \tag{15}$$

Here, the pre-measurement state can be taken as superposition of particle position eigenstates x1 and x2, which in turn is coupled with a (still) superposed environmental initial state φ_0 (strictly speaking, such position eigenstates are non-normalizable delta-functions, hence unphysical. However, when putting limited resolutions of detectors into account, one ought to regard them as a mixture of position eigenstates, i.e., a 'gaussian spikes'). As decoherences process in time, each conjunct on the right-hand side of the equation then expresses individually entangled states consisting of effectively two environmental states, φ_{x1} and φ_{x2}; one is entangled with measurement outcome $\alpha\psi_{x1}$, while not interfering with $\beta\psi_{x2}$, and vice versa. For both particle position states ψ_{x1} and ψ_{x2} then exist as an entangled set, whereas each can be taken as a world; however, both are relatively out of phase to the other, thus they no longer interfere.

Following Carroll and Singh [47], a slightly more detailed representation of this state of affairs in bracket notation might be the following: this treats the to-be-measured quantum object, apparatus *and* environment as quasi separate wavefunctions, which, however, become entangled during the measurement process:

$$|\psi> = (\alpha\ |+>_q + \beta\ |->_q) \otimes |0>_a \otimes |0>_e \tag{16}$$

$$(\alpha|+>_q|+>_a + \beta|->_q|->_a) \otimes |0>_e \tag{17}$$

$$\alpha|+>_q|+>_a|+>_e + \beta|->_q|->_a|->_e. \tag{18}$$

In this formal representation of decoherence, the index 'q' stands for the quantum to-be-measured object, 'a' represents the apparatus and 'e' the environment. Therefore, when, for instance, the spin-pointer states of the quantum object, represented by $|+>$ and $|->$, each become entangled with the wavefunction of the apparatus 'a' being in the $|0>$—state (16), then both spin states $|+>$ and $|->$ evolve into a tensor product representing 'q' & 'a' -*coupling*; - 'a' then effectively differentiates itself (borrowing this notion from

Wallace [39]) likewise into a $|+>$ and $|->$ state while the environment still remains in $|0>-$state (17). However, given that entanglement spreads to the environment, it too becomes part of the newly formed wavefunction products for both ($\pm$) states. This final time step (18) then effectively accounts for a world branching taking place. Ultimately, both factorized product states will be hopelessly out of phase; however, without any asymmetry of 'realness', both have to be regarded as equally existent.

What is important to notice here is that, at any rate, it is the relative complexity of such highly entangled many-particle states that causes the practically zero interaction between branches:

> "*So, in the case of chaos, 'worlds'—that is, emergent quasi-classical systems—are constantly splitting from one another. And since the system's state is always a mixture of reasonably localized wave-packets, the failure of classicality which we predicted for isolated chaotic systems will not occur here.*" ([39], p. 84)

While orthodox MWI still has to add distinct worlds 'by hand', so to speak, I take it to be an advantage of dbMWI that no such ad hoc addition has to be made; the bare formalism, plus rather complex tensor products and dynamics, which reasonably follow on from complex matter arrangements within our universe, are all that is needed to let branches emerge.

4. Discussion

As argued throughout this review, a realist interpretation of the bare formalism and structure seems to be what is left 'alive' after carefully inspecting all major interpretations and rival theories; and this alone suffices to let worlds emerge. Hence, while it was shown above that rival interpretations and theories face severe conceptual problems or collapse into a structure isomorphic to MWI, MWI itself seemingly straightforwardly follows, without adding any ad hoc ingredient and when only applying the bare formalism to arbitrarily complex quantum systems, -and the latter are something to be expected when applying it to our messy world. The sheer fact that this alone (bare formulism and decoherence) saves empirical adequacy and appearances is an achievement that cannot be overemphasized. Above that, the '*reductio* strategy' of this paper only leaves MWI 'alive' for its benefits that it neither faces the circularity issues that anti-realist interpretations do, nor the problem that it can only be applied to a restricted domain of quantum physics.

However, this should and will not mask the fact that the perplexing metaphysical consequences it yields are far too much for many to bear, or, in fact, are willing to tolerate. More often than not, it simply ignites an "incredulous stare". Such a reaction is, of course, merely emotional.

It is noteworthy however that even among Everettians there is no absolute agreement in terms of ontology and metaphysical consequences, for instance, when it comes to the aforementioned egalitarian view in terms of realness of individual eigenstates: here, Lev Vaidman [49] argues that $|\alpha^2|$ and $|\beta^2|$ measure *unequal* 'degrees of existence', if $\alpha \neq \beta$. While being an interesting concept, I fear however that 'degrees of existence' is conceptually vague and potentially problematic. Here, an adaption of Lewisian 'indexicality of actuality' [50] might be a promising alternative. Then, 'degrees of existence' might not be absolute but an indexical or relational value depending on the location within the multiverse (however, discussing this goes beyond the scope of this review. At any rate, even if 'degrees of existence' is true, a MWI still follows, as the better part of the state vector, at least in terms of reasonable approximations, is ontologically on a par. Hence, then also, a constrained version of branches, one with somewhat vague 'boundaries', seemingly follows.)

Also, according to the prevailing consensus at least, MWI also faces technical challenges, in particular, the problem of preferred basis or the meaning of probability in this context [51]. However, it might be fair to say that when embracing dbMWI in particular, a straightforward solution to the former seems to be at hand, as decohered quasi-classical states *are* what provide a preferred basis in a non-ad hoc manner, as discussed by Wallace [39] (here it might also be noteworthy that the preferred basis problem does not arise in the original Everett–Wheeler formulation, but does so as an artifact of the 'many worlds' language first published by de Witt and Graham in '*The Many-Worlds Interpretation of Quantum Mechanics*' [6] – though by following Everett´s unpublished notes).

There, Wallace also provides a fully worked out decision-theoretic analysis of probability. It is, however, also fair do admit that decision theory has been criticized as a basis of (QM) probability, but other solutions have been given, for instance, by Sudbury in 'The logic of the future in quantum theory' [52]. Sudbury´s alternative proposal is to understand probabilities in terms of non-classical truth values that are assigned to each potential outcome relative to a particular observer. However, this would turn truth into a relative notion and cannot account for the fact that from a 'view from nowhere' perspective, everything occurs with certainty: indeed, a view-from-nowhere account in terms of truth might be preferable. However, I agree with Sudbury when he states that decision theory may be irrelevant to passive agents and hence cannot possibly fully represent (QM) probabilities in their utmost general applicability. But a more general critique of a decision-theoretic account can be given. Similar to how Wallace himself criticized the lack of objective truthmakers in anti-realist interpretations, one might object that a decision-theoretic account of probabilities likewise requires an objective modal structure by which it is packed. Wallace talks of *branch weights* presumably doing that kind of work [39], but I would rather suggest the use of *local* relative frequencies, similar to Simon Saunders' recent proposal of an '*equi-amplitude rule*' in terms of branch-counting [53]; while *global* frequencies are indeed ill-defined within an 'Emergent Multiverse' in which allegedly *all* states are realized, *local* frequencies might hold relatively to respective decohered branches. For instance, if $|\alpha^2|<|\beta^2|$ holds for equation (17) within a particular branch, then relatively fewer sub-branches will exhibit $|+>_q$ than $|->_q$. In other words, relative frequencies in terms of ratios can be given: "*There may be no true number of the relevant microstates, in each case, but there may yet be true ratios*." [53] (as Saunders argues, instead of Vaidmanian 'degrees of existence', this may rather be an instance of [relative!] *numbers* of existence.). While Wallace argues in [39] that decoherence provides no well-defined notion of branch count, *I* would argue that it provides a *reasonably* well-defined branch count, in the same way it provides *reasonably* well-localized wave packets.

At any rate, the following might be something to be agreed upon and be sufficient for the overall argument of this review: whether ultimately to be captured in decision-theoretic terms, observer-relative non-classical truth values or relative frequencies, probability seems to be related to self-locating uncertainty either way; whether such probabilistic datum is to be ultimately understood as subjective in its nature or not, it does not seem ultimately to be crucial to the integrity of MWI. Thus, contrary to the conceptual issues of its rivals discussed in the main sections of this review, I would suggest that these problems ought to be regarded as second-order challenges, rather than arguments for immediately dismissing MWI as a viable interpretation.

Finally, I dare to make the following claim: poll results notwithstanding, the majority of the physics community in fact prefer (an unmodified) realist interpretation and are only Copenhagen advocates out of custom and convenience, or because they do not deeply question anti-realist assumptions or hidden-variable theories´ (limited) applicability. I dare to say that the majority may already *subconsciously* be 'many-worlders', and did not, mostly due to shut-up-and-calculate advice, rigorously reflect on their *consciously* preferred presuppositions or think them through to their logical endpoint. That is to say, even an advocate of the 'no-interpretation interpretation' may, when taking the formulae as something *substantial*, come to the same conclusion or at least utilizes MWI as a working hypothesis.

Another final argument in MWI´s favor may be that quantum information theory is much more easily reconcilable with MWI and both seem to converge, as argued above, towards one unified interpretation when putting forward OSR as a framework and taking the other conclusions drawn in Section 3.1 into account. In addition, decoherence-research is already heavily used in quantum computing, if only for the reason that *coherence* is what is trying to be technically achieved. Again, it, when being interpreted realistically, quite naturally leads to MWI branches without additional ingredients.

It might also be worth mentioning that both MWI and information-theoretic interpretations are, according to Wallace [17] (p. 9), by now, the two most popular ones among *physicists*. Ironically, attempts to recover interpretations that bring back a classical worldview are, for the most part, coming from philosophers. So much the worse for philosophy.

5. Conclusions

In this paper, the plausibility of two main claims have been argued:

Firstly, that rival interpretations or theories either face limited applicability/conceptual incoherence, or can be reduced to MWI on closer inspection. It has been discussed that there are four options for the anti-realist in general (and QBist in particular) in order to bypass conceptual incoherence: PEMWI, OSR–MWI, MWI and hidden variables. However, the first three views may themselves deflate under the assumption of OSR. Likewise, in Section 3.2, I hope to have demonstrated that under the presupposition of preserved consistency, the distinction between RQM and MWI eventually evaporates.

Secondly, up to this point, hidden-variables theories have seemed to be a viable alternative; however, while in the case of hidden-variables theory the threat may not be as grave as conceptual incoherence, compatibility with QFT might still dictate a modification that ultimately also leads, as argued, to an ontology that strongly suggests 'MWI implications'. Thirdly, *dbMWI* in particular may be regarded as the most straightforward interpretation of QM.

Thus, when adding all these provisional results together, MWI might indeed remain the only viable option. Even if some of the arguments propounded are not regarded as utterly sound, I still hope that the overall framework of this conceptual analysis may inspire others and be utilized for similar but improved future approaches. Finally, even if there is no agreement in terms of the overall conclusions drawn, it may nevertheless be sufficient to demonstrate that MWI is, if nothing else, *ripe* for being included in the consensus.

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Many-Worlds: Why Is It Not the Consensus?

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Abstract: In this paper, I argue that the many-worlds theory, even if it is arguably the mathematically most straightforward realist reading of quantum formalism, even if it is arguably local and deterministic, is not universally regarded as the best realist quantum theory because it provides a type of explanation that is not universally accepted. Since people disagree about what desiderata a satisfactory physical theory should possess, they also disagree about which explanatory schema one should look for in a theory, and this leads different people to different options.

Keywords: interpretations of quantum mechanics; Everettian quantum mechanics; many-worlds theory; primitive ontology; principle and constructive theories

1. Introduction

Quantum mechanics is known to be resistant to a realist understanding, as it is unclear what picture of reality it provides us. Nonetheless, a variety of realist quantum theories have been proposed, and among these, one finds the many-worlds theory, also known as Everettian quantum mechanics. There are various readings of this theory, but they all have in common that all there is in the theory is a quantum state, evolving according to the Schrödinger unitary evolution equation, which produces experimental results distributed according to the Born rule. Other notable realist quantum theories are the spontaneous localization theory, also known as GRW theory, and the pilot-wave theory, or de Broglie–Bohm theory or Bohmian mechanics. Nonetheless, according to many advocates of Everettian quantum mechanics, there should be no debate over which quantum theory is best: the many-worlds theory is the simplest most straightforward interpretation of the quantum formalism, as it does not require any modifications of the mathematics of the theory. Moreover, it is maintained, it is consistent with how physicists use the theory as well as its relativistic extensions. Therefore, one question arises naturally: why is it not the consensus? Why are all people not Everettians?

According to some (Wallace p.c.), Everettian quantum mechanics is the implicit consensus, at least among practicing physicists; when they perform calculations, they use the Born rule, they never write down the guidance' equation for the waves, and they never need to modify the unitary evolution. That is, they implicitly adopt the many-worlds theory. However, *pace* Wallace's optimism, I think this is not exactly how most physicists characterize what they are doing. When informally asked, many of them say that they use standard quantum mechanics, namely the unitary evolution and the collapse rule, rather than unitary evolution alone, and they do not believe that they and their labs are continuously 'splitting' into infinitely many worlds. Indeed, some of them will not even see the point of 'adding' these worlds on top of the empirical adequacy of the standard theory. If the many-worlds theory makes the same predictions of standard quantum mechanics, but also postulates an infinity of unobservable worlds on top of the one we experience, then why should one prefer this theory to standard quantum mechanics? In any case, among philosophers of physics, the situation is certainly very different: many do not find Everettian quantum mechanics satisfactory or the best alternative, and even among those who do, they have their own ways of formulating the view.

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Vaidman [1] argues that the many-worlds theory is not the consensus due to its revolutionary, groundbreaking metaphysics. He writes: “We would like to think that we are the center of the Universe: that the Sun, together with other stars, moves around Earth, that our Galaxy is the center of the Universe, and we are unhappy to accept that there are many parallel copies of us which are apparently not less important”. He also blames technical difficulties that the theory faces, such as the justification of the Born rule or the preferred basis problem. Moreover, he thinks that the fact that some understand the theory as fundamentally not in spacetime obscures “the connection between ontology and our experience”. While I think that these are important issues, I disagree that they are the fundamental reasons why the many-worlds theory is not universally accepted.

In this paper, I argue that there is no consensus because people favoring different theories have profoundly different motivations guiding their search for a satisfactory theory, which leads them to favor specific explanatory structures. The paper is organized as follows. In the next section, I provide a brief overview of standard quantum theory, the measurement problem, and the distinction between principle and constructive theories, as well as between frameworks and interaction theories, which will help me compare the various approaches. To make my case that disagreement is connected with theory desiderata and preferred explanatory types, first, I contrast the information-theoretic (IT) approach of quantum mechanics with the primitive ontology approach. I discuss the former in Section 3, and I argue that the proponents of this approach are satisfied with an empirically adequate theory. This fits well with an explanation in terms of principles, without requiring a microscopic ontology, and this naturally leads them towards standard quantum theory. In Section 4, I analyze the primitive ontology approach, and I argue that the proponents of this view are guided by a constructive understanding. This requires a theory to possess a spatiotemporal microscopic ontology, and the pilot-wave theory is the simplest theory of this kind. Then, in Section 5, I move to Everettian approaches, which aim at reading physics at face value, thereby conceiving of quantum theory as a framework, which could fit different theories. Consequently, they argue that Everettian quantum mechanics is the only option which describes the entire framework, rather than a single theory. In contrast, wavefunction realism, which is discussed in Section 6, is guided by finding a theory with a local and separable ontology. This leads them to think of the various quantum theories as interaction theories, which describe how the basic ontology, provided by the non-spatiotemporal wavefunction, behaves. I summarize and conclude in Section 7.

2. Setting the Stage

Quantum mechanics, as it is found in physics textbooks (‘standard quantum mechanics’ for short), is presented axiomatically, in terms of postulates. The first of them is that possible states of any physical system are described by quantum states, which are rays in a Hilbert space. When written as a function of position, the quantum state is called the wavefunction. In addition, it is postulated that the measurable properties of a physical system (often called ‘observables’) are represented by self-adjoint (Hermitian) operators. There is a preferred observable, the Hamiltonian, which generates the dynamical evolution of the quantum state in terms of the (linear and deterministic) Schrödinger equation (sometimes called unitary dynamics). Since the Schrödinger equation is linear, superpositions of solutions will be solutions, and they will propagate macroscopically, even in situations in which we do not observe any, as in the infamous example of the Schrödinger cat. That is, the theory is, as is, empirically inadequate: it fails to predict what we observe (lack of macroscopic superpositions). One can fix this problem by adding the so-called von Neumann collapse rule, which kicks in when a measurement of a given observable is performed. This rule states that the wavefunction randomly and instantaneously ‘collapses’ into one of the possible solutions of the Schrödinger equation for that measurement situation. That is, the possible values of the observable associated with some operator are given by the eigenvalues of that operator, and the wavefunction collapses into the eigenstate corresponding to that eigenvalue. This is the so-called eigenvalue–eigenstate rule (EER). Finally, it is stipulated that the probability to find a particular measurement result is provided by the Born rule: the probability of obtaining some eigenvalue is given by the scalar product of the initial state and the eigenvector corresponding to that experimental result.

With these postulates, even without specifying whether matter is made of particles or waves or something else, standard quantum theory can account for the known phenomena, and it can make novel predictions.

Classical mechanics instead is very different. As it is taught in physics books, it starts from a stipulated and clear metaphysical hypothesis, namely that everything is composed of point particles, which evolve in time according to Newton's second law, moving in space, which is suitably described by having a three-dimensional Euclidean structure. In contrast with the quantum case, the classical formalism does not require any interpretation, as it is clear what the various mathematical objects correspond to: x is the particle's position, v is its velocity, m is mass, and so on. Furthermore, as every physics student is taught, there are abstract spaces without a physical meaning only to ease computation, like in the case of Lagrangian and Hamiltonian mechanics, and these are not to be taken ontologically seriously.

Nonetheless, dissimilarities aside, both classical and quantum mechanics have enjoyed an enormous amount of success, albeit the type of success they have is very different. For instance, while the predictions of quantum theory have shown an unprecedented amount of precision and accuracy, we no longer have a clear understanding of the underlying microscopic reality. In other words, while standard quantum mechanics enjoys incredible predictive success, classical mechanics also had a type of explanatory power which quantum theory completely lacks. That is, classical mechanics could provide a pictorial image of, say, what water is. Water is a two-hydrogens-one-oxygen molecule, in which the atoms are kept together by a polar bond. On top of that, one could also picture hydrogen and oxygen as small balls with peculiar appendages, which would allow them to fit together, and water could be imagined to be a collection of such composites held together by some sort of loose rubber band. This pictorial understanding could explain, say, why water at a given temperature becomes solid: the composites are stuck in the corner of a hard, geometrical, crystalline structure. In other words, in classical mechanics, one could have explanatory illustrations: one can draw pictures of the physical phenomena to understand them better. Instead, standard quantum theory does not provide any microscopic picture of the reality underlying the phenomena. There are pictures in the textbooks, but one is always warned not to take them too seriously. For instance, the orbitals of the electrons around the nucleus are not like the planetary orbits around the Sun. Rather, the orbitals are to be understood as 'probability clouds'; we are told that they represent a surface where the probability per unit of volume of finding an electron is constant. Similarly, the interference pattern in a two-slit experiment with electrons is not generated, as one would have understood classically, by a physical wave passing through both slits and interfering with itself. Rather, it is a 'probability wave': the interference pattern expresses the probability to find the electron when measured. That is, standard quantum mechanics does not allow us to draw or to picture in our mind what an electron does or how it moves. Nonetheless, it seems too much to say that standard quantum theory does not explain anything. It can explain, but in a different way: standard quantum theory can explain why one observes the two-slit experiment interference pattern, not in the sense that it tells us where the electron has gone and, thus, where it will be detected, but in the sense that the observed distribution of detections is the one predicted by the theory.

It has been argued that we cannot have anything better than this: standard quantum mechanics cannot provide a coherent microscopic picture of reality. That is, someone providing such a description would run into contradictions. For instance, the two-slit experiment performed with entities so far understood as particles, such as electrons, shows that they inexplicably behave like waves because they interfere. Alternatively, the photoelectric effect shows that light, so far understood as a wave, inexplicably behaves as a particle, because the observed intensity of the emitted light is compatible with a particle rather than a wave ontology. But how is it possible? It is because of examples like these that some people became convinced that a coherent microscopic picture of the quantum world was impossible and many more decided to ignore questions about ontology. They instead decided to focus on formulating the theory as to make contact directly with macroscopic observable quantities rather than microscopic unobservable entities. That resulted in the axiomatic quantum theory presented above: the postulates are about measurement results, and they are expressed in terms of 'abstract' entities, rather than in terms of the motion of unobservable microscopic entities, as in the classical theory. As such, the theory has been taken to be incompatible with scientific realism, the view that theories can give us information about the nature of reality beyond the phenomena.

2.1. The Measurement Problem

Some have argued that this is the true quantum revolution: our classical desire of understanding is doomed to fail, so perhaps we should become anti-realist. Nonetheless, one may think this is too harsh. Perhaps we did not think enough about the possibility of making standard quantum mechanics compatible with a realist reading. Indeed, what would be required from a realist quantum theory? It is usually maintained that such a theory would have to solve the so-called measurement problem.

One of the things which seems to make the standard theory unsuitable for a realist reading is the fact that there are two evolution equations, and that they are expressed in terms of measurement: the Schrödinger evolution holds when no measurements are performed, and the collapse rule when a measurement takes place. As noted, the collapse rule is needed to eliminate unobserved macroscopic superpositions produced by the linear evolution, but this is at the expense of promoting measurement processes to a privileged status, as whether a measurement happens or not determines which evolution the wavefunction would follow. This is bad news for the realists, as they would like to think of measurements as merely special types of physical processes. If so, then it is natural to assume that the wavefunction is the fundamental ontology of everything, and its fundamental evolution is given by the Schrödinger equation. As a result, however, there will be 'superpositions of states' at all scales, which we never observe. Therefore, the measurement problem is the problem of dealing with unobserved macroscopic superpositions without postulating a measurement-dependent double dynamics, as the standard theory does.

The measurement problem is sometimes formulated by stating that three claims are incompatible [2]: (1) the wavefunction provides the complete description of any physical system; (2) the wavefunction evolves according to the Schrödinger equation; (3) measurement outcomes are unique (which is to say that there are no unobserved macroscopic superpositions). Solutions of the measurement problem are often portrayed as denying one of these three claims: the pilot-wave theory denies that the description provided by the wavefunction is complete; the GRW theory denies that the wavefunction evolves according to the Schrödinger dynamics; and the many-worlds theory allows for superpositions at all scales. Usually, the solutions of the measurement problem are taken to be the realist quantum theories, namely the quantum theories that realists should look at in their investigations about the nature of the quantum world.

There are too many ways of understanding the metaphysics of quantum mechanics to analyze them all. In any case, one can group them depending on which is their favorite theory. On the one hand, we have primitive ontologists, who favor the pilot-wave theory (or some versions of GRW) and, on the other hand, we have wavefunction realists who favor either the many-worlds theory (or other versions of GRW). Oxford Everattians, championed by Wallace, also favor the many-worlds theory, which is also defended by Vaidman.

In the next sections, I analyze these different approaches, and I argue that people disagree because they require different desiderata for a theory to be successful: they have different motivations, connected to different understandings of explanation, which lead them to favor a given type of theoretical structure, and this translates into naturally favoring different quantum theories over others. I discuss different theoretical structures in the next subsection.

2.2. Constructive Explanations, Principle Theories, Interactions, and Frameworks

Not all physical theories are of the same sort: thermodynamics is different from classical mechanics, which is different from optics, which is different from electromagnetism, and so on. Some have dynamical equations, while some others have principles, some have forces, while some others have constraints, *et cetera*. Therefore, one may think it is not surprising that quantum and classical mechanics are different: they are just another pair in the list. Nonetheless, one could recognize shared features among the various theories. Some theories are what Einstein [3] called constructive theories. For one thing, these theories have a microscopic ontology, which constitute the building blocks of everything else. Constructive theories allow one to understand the phenomena compositionally and dynamically: macroscopic objects are composed of microscopic particles, and the macroscopic behavior is completely specified in terms of the microscopic dynamics. Therefore, the type of explanation these theories provide is bottom-up, rather than top-down. According to Einstein, there is another type of theory, which he dubbed principle theory. Theories of this type, also called kinematic theories, are formulated in terms of principles, which are used as constraints on physically possible processes: they exclude certain processes from physically happening. In this sense, principle theories are top-down: they explain the phenomena identifying constraints the phenomena need to obey to. They are 'kinematic' theories because the explanations they provide do not involve dynamical equations of motion and they do not depend on the interactions the system enters into. Instead, by definition, constructive theories involve dynamical reductions in macroscopic objects in terms of the motion and interactions of their microscopic three-dimensional constituents. Flores [4] argued that this distinction could be expanded in terms of framework theories, which deal with general constraints, and interaction theories, which explicitly invoke interactions. He thought that framework theories are principle theories while interaction theories include a larger set of theories than constructive theories. Furthermore, he connected framework theories with unification and interaction theories with mechanistic explanation (see also [5,6]).

I think that, contrary to Flores, the two distinctions do not capture the same idea, as I discuss in Section 6. In any case, I am going to use them both to characterize the different motivations and explanatory strategies of the various approaches. While the constructive-principle distinction is useful to contrast the primitive ontology approach with the IT approach, the interaction-framework characterization will be helpful in the comparison between wavefunction realism and Everettian quantum mechanics.

Be that as it may, an example of a principle theory is thermodynamics (e.g., "energy is conserved" is a principle), and an example of constructive theory is statistical mechanics, which reduces the behavior of gases to the motion of atoms. Another example of principle theory (which motivated Einstein's distinction in the first place) is the 1905 theory of special relativity (before the introduction of Minkowski spacetime), as it was formulated in terms of two principles: the principle of equivalence of inertial frames for all physical laws and the principle of constancy of the velocity of light. This theory explains relativistic effects (such as length contraction and time dilation) as the physical phenomena compatible with the theory's principles. By contrast, Lorentz's 1909 theory was proposed to explain the same phenomena, but it does it constructively: it derives the relativistic effects from the electromagnetic properties of the ether and its interactions with matter.

One can read standard quantum mechanics as a theory of principles, as the axioms presented above constrain the phenomena. This is arguably one of the reasons why Einstein disliked this theory. In fact, Einstein maintained that "most [theories in physics] are constructive. They attempt to build up a picture of the more complex phenomena out of the materials of a relatively simple formal scheme from which they start out. Thus, the kinetic theory of gases seeks to reduce mechanical, thermal, and diffusional processes to movements of molecules". Moreover: "When we say that we have succeeded in understanding a group of natural processes, we invariably mean that a constructive theory has been found which covers the processes in question" (*ibid.*). As mentioned above, Einstein introduced the distinction to explain why he was not happy about his theory of relativity being a principle theory. He did not accept Lorenz's constructive relativity because Einstein did not think we had the right understanding of the theory of matter involved (an atomistic understanding of the Lorentz contraction was unavailable). According to Einstein, principle theories are provisional in nature. That is, we usually accept principle theories when we do not have other explanations of the phenomena. Einstein believed that physics should look for constructive theories and that we accept principle theories only when we have no other option. Arguably, then, he could have said something similar for quantum theory, as his preference for constructive theories is compatible with his idea that quantum mechanics is incomplete. Moreover, it fits well with his statistical interpretation of quantum theory, as it is a principle theory by constraining the phenomena with suitable rules, which is, however, in need of a constructive explanation in terms of a still-unknown more fundamental theory expressed in terms of 'hidden variables'.

Nonetheless, some have argued that there is nothing wrong with explanation in terms of principles. Let us turn to this approach in the next section.

3. The IT Approach: Standard Quantum Mechanics as a Principle Theory

Some have argued that one can provide a realist understanding of standard quantum mechanics even without solving the measurement problem. For instance, according to the proponents of the IT approach, it is a matter of being careful in choosing the fundamental ontology for standard quantum theory [7]. They claim that it is a dogma that the fundamental ontology of a theory should be microscopic: it is a dogma that measurement should be understood in terms of other, more fundamental, microscopic processes. Rather, if one takes measurement results as primitive and unanalyzable, one can consider them to be standard quantum theory's fundamental macroscopic ontology.

3.1. Motivation: Empirical Adequacy

The main motivation for this approach is empirical adequacy. That is, a satisfactory theory is one which adequately reproduces the phenomena at the macroscopic level. Thus, one should be content with a theory which predicts the measurement outcomes, understood as the fundamental ontology of the theory. As discussed earlier, unitary quantum theory, namely the theory of a Schrödinger evolving wavefunction, is not empirically adequate: it fails to predict that there are no macroscopic superpositions. This problem is solved by standard quantum theory introducing the collapse rule and allowing for a double dynamics, depending on whether a measurement is performed or not. The proponents of the IT approach are not bothered by not having a unique, not measurement-dependent dynamics, because they actually do not understand the two evolution equations as dynamical laws at all. They provide the specification of what we should expect in terms of measurement outcomes. It happens that this specification is conveniently provided in terms of the collapse rule and the Schrödinger equation, but one should not really take these specifications too seriously, ontologically speaking. Other formulations could be possible, and perhaps more convenient. What is important is that they are effective in correctly reproducing the data and in adequately predicting what we should expect to observe. That is, what is important is for the theory to predict what to expect in terms of its principles. In fact, given that compositionality fails, explanation cannot be constructive. Standard quantum theory (with the Schrödinger equation and the collapse rule) provides that, and in virtue of that, they find nothing wrong in having the collapse rule. Consequently, for them, there is no need to solve the measurement problem: there is no need to have a unique dynamics for the microscopic entities because they are not what the theory is about.

3.2. Explanations as Kinematic Top-Down Systematizations

According to the IT approach, standard quantum theory is best understood as a principle theory. As mentioned, principle theories provide a top-down explanation, in the sense that they explain the phenomena identifying constraints they need to obey. More specifically, in the case of standard quantum mechanics, Hilbert space is thought of as "the kinematic framework for the physics of an indeterministic universe, just as Minkowski space-time provides the kinematic framework for the physics of a non-Newtonian, relativistic universe" [7]. This type of explanation is fundamentally different from the one provided by classical mechanics: quantum theory lays out a set of constraints imposed on the empirical data, rather than specifying some microscopic story about how a given result comes about.

In general, supporters of the IT approach maintain that in order to provide a satisfactory explanation, one does not need a deeper, dynamical account: "There is no deeper explanation for the quantum phenomena of interference and entanglement than that provided by the structure of Hilbert space, just as there is no deeper explanation for the relativistic phenomena of Lorentz contraction and time dilation than that provided by the structure of Minkowski space-time" (*ibid.*). As relativity explains the phenomena when it tells us what we should expect in a certain physical situation, so does standard quantum theory. There is no reason and no need to ask for more. In fact, they do not say this, but one could maintain that this approach is to be preferred because principle theory explanations are independent on the detailed assumption about the structure or the constitution of matter.

Be that as it may, also, QBists think of quantum theory as providing constraints on measurement outcomes (see [8] and references therein). However, in contrast, QBists leave open the possibility for a deeper understanding: "What is the stuff of the world? QBism is so far mostly silent on this issue, but not because there is no stuff of the world. The character of the stuff is simply not yet understood well enough. Answering this question is the goal, rather than the premise" [9]. In addition, Pragmatist quantum realism (see [10] and references therein; see also [11]) agrees with the IT approach that standard quantum mechanics is a principle theory: the Born rule assigns phenomena probabilities of happening, and these probabilities express our degrees of belief that a given phenomenon will happen.

It is important to underline that all these proposals characterize themselves as not anti-realist: they do believe that standard quantum theory with the collapse rule tells us something objective about the world. It does not matter that the collapse rule does not specify what measurements are because in these approaches, measurements are unanalyzable primitives. Moreover, since they believe that standard quantum theory is about experimental results, they are realist about measurement outcomes. They exist objectively and mind independently. In other words, these attitudes are realist, in the sense that theories are taken to be objectively informative about the world: the description they provide is independent from us.

However, from a theory, they require very little: they think that it is enough to provide an accurate description at the macroscopic level and an explanation in terms of principles constraining the phenomena. They do not require a unique dynamics for all levels of description: in these approaches, the two equations are seen as principles, rather than dynamical laws. Correspondingly, since their explanation is not constructive, they do not have to require a microscopic ontology. In this approach, as long as the principles make the theory empirically adequate, the theory is amenable to a realist interpretation.

A realist approach similar to the one described here rainforest realism [12]. Rainforest realism is a view, according to which objects, both at the microscopic and the macroscopic level, do not fundamentally exist. All there is at the fundamental level is structure. This is a radical structuralist position in which all objects are eliminated from the fundamental level but are seen non-fundamentally as real patterns, defined by their usefulness. What we call 'particles' are neither fundamental nor composed entities. Rather, they are merely useful fictions: they allow us to conveniently express certain regularities at a given level of description. The same is true for chemical compounds, molecules, tables, chairs, and measurement devices. They are not analyzed in terms of more fundamental entities, because they are not composed entities. Rather, they are understood as effective descriptions. This position is realist: theories talk about the word, and at all levels, many types of entities emerge. This approach fits nicely with the idea that a theory explains the phenomena in terms of principles rather than dynamically. Moreover, in this case, the fact that the collapse rule is vague does not create a problem for rainforest realism because measurement devices and measurement outcomes are patterns, and patterns are vague.

4. Primitive Ontology: The Pilot-Wave Theory as a Constructive Quantum Theory

If the proponents of the IT approach think of standard quantum theory as a principle theory and, in virtue of that, they argue that there is no need for a microscopic description of reality, primitive ontologists instead defend the constructive point of view that principle theories always need a microscopic explanation (see, to start with, [13–16]).

4.1. Motivation: Constructive Explanation

According to primitive ontologists, a satisfactory explanation is a constructive explanation. Here is a quote:

> " ... in the classical framework we have a clear and straightforward scheme of explanation: given the primitive ontology at the microscopic level, one can employ standard methods to determine the properties of familiar macroscopic objects. Since in classical theories this is possible because the theories have a primitive ontology, for any other fundamental physical theory with a primitive ontology, like the quantum theories we just discussed, we could employ an explanatory scheme derived along the lines of the classical one". [16]

We had a constructive account in classical theory, but we do not have one in quantum theory. Quantum and classical mechanics are two proposals for fundamental physical theories, but they have barely anything in common. They both have 'mechanics' as part of their names, but what that amounts to in the two theories is very different, aside from very generally conveying the idea that both theories deal with the motion of physical bodies in terms of forces, potentials, and energy. In classical mechanics, it is arguably clear what matter is made of, how it behaves, and how one recovers the macroscopic behavior we observe from these ingredients: macroscopic objects are composed of microscopic particles, whose position in time and whose mutual interaction are described by Newton's laws. Primitive ontologists think that this precise microscopic picture allows one to account for the observed macroscopic properties and phenomena. For instance, water is liquid at a given temperature and solid at another because the interaction between water's molecules accordingly changes with temperature. Accordingly, they think that we should construct a constructive counterpart to standard quantum theory. A theory in which the quantum phenomena are explained constructively, namely a theory in which macroscopic objects are composed of microscopic entities and their properties, is dynamically understood in suitable scale limits (see below).

As observed earlier in the case of Einstein, traditionally, physicists have looked for constructive theories. Pauli, among others, explicitly favored constructive theories, even if he rejected Lorentz theory:

> "Should one, [...] completely abandon any attempt to explain the Lorentz contraction atomistically? We think the answer to this question should be No. The contraction of a measuring rod is not an elementary but a very complicated process. It would not take place except for the covariance with respect to the Lorentz group of the basic equations of the electron theory, as well as of those laws, as yet unknown to us, which determine the cohesion of the electron itself.

> We can only postulate that this is so, knowing that then the theory will be capable of explaining atomistically the behaviour of moving rods and clocks". [17]

Some have argued that principle theories are not explanatory: "Explanations are about the reality behind the phenomena (be it about their causes or about their nature). Principle theories [. . .] are agnostic about that" [18]. Others have argued that constructive theories are better, as they provide insight about the reality underlying the phenomena:

> "In a theory of principle, one starts from some general, well-confirmed empirical regularities that are raised to the status of postulates (e.g., the impossibility of perpetual motion of the first and the second kind, which became the first and second laws of thermodynamics). With such a theory, one explains the phenomena by showing that they necessarily occur in a world in accordance with the postulates. Whereas theories of principle are about the phenomena, constructive theories aim to get at the underlying reality. In a constructive theory one proposes a (set of) model(s) for some part of physical reality (e.g., the kinetic theory modeling a gas as a swarm of tiny billiard balls bouncing around in a box). One explains the phenomena by showing that the theory provides a model that gives an empirically adequate description of the salient features of reality". [19] (see also [20–22])

I take this to mean that to explain constructively seems deeper: constructive explanations not only account for the phenomena, as they also explain where the phenomena come from. In other words, a principle explanation provides a reason why one should expect a phenomenon to happen, while a constructive explanation also explains why it happens. Constructive theories not only predict the correct results, but they also give you a reason why these predictions come about and thus why you should expect certain results and not others.

What is required for a constructive explanation? The essence of this type of explanation is to be bottom-up. That is, it explains compositionally and dynamically: it is a Lego brick style of explanation, in which there are fundamental entities which build up the rest of the non-fundamental entities. In order to have such an explanation, one needs a fundamental spatiotemporal ontology which is suitably microscopic. This is because both of them are requirements to make sense of this Lego bricks picture: the individual Lego bricks (the fundamental ontology) used to build a castle (the macroscopic phenomena) are in the same space as the castle (spacetime) and they are smaller than the castle (microscopic). As in classical mechanics, with an ontology of particles, one could think of macroscopic objects as composed of microscopic particles and account for their properties in terms of the microscopic dynamics. A quantum constructive explanation would then require such a spatiotemporal microscopic ontology.

Standard quantum theory falls short of constructive explanation in at least two ways: it has two dynamical evolutions, and it has no clear ontology. The IT approach has a macroscopic ontology, but that does not allow for constructive explanation. Moreover, they do not interpret measurements as physical processes, while constructivists should. Since a constructive quantum theory provides us with a microscopic spatiotemporal picture of the world, it should describe measurement processes in terms of such fundamental spatiotemporal microscopic ontology and its (unique) dynamics, valid for all scales. Therefore, the way to go to obtain a constructive quantum theory is to recognize that we should treat standard quantum theory as we treated thermodynamics. They are both principle theories: the quantum recipes describe the phenomena by specifying the statistics of the experimental results, just as thermodynamics provides constraints on macroscopic phenomena. According to the constructivist, one can (and should!) reduce thermodynamics in terms of classical mechanics: if one thinks of gases as collections of microscopic particles, one obtains a deeper explanation of the behavior of the gases. In contrast with the case of thermodynamics, in which we already had the more fundamental microscopic theory, in the quantum case, we still do not have it. Therefore, the constructivist should look for it.

Through this theory, one would be able to understand quantum systems in terms of a more fundamental ontology and to arrive at a deeper understanding of why quantum phenomena happen, rather than merely settling for accounting for what we should expect. In this way, as it would be absurd to use a gas ontology for classical mechanics to reduce thermodynamics, one should not use the wavefunction as the ontology of the reducing constructive quantum theory. Since the wavefunction 'belongs' to the reduced theory, it does not make sense to use it as the ontology for the reducing one. In addition, the wavefunction is not defined in spacetime, which allows for constructive explanation. Rather, it is defined in a high dimensional space, usually called configuration space. Since constructivists require a spatiotemporal fundamental ontology, the obvious choice is the one of particles. For once, they seem more compatible with the empirical evidence of tracks in detectors. Nonetheless, waves can also be a suitable ontology for a constructive approach, but they at least need to be oscillating in (three-dimensional) space, evolving in time. Furthermore, waves need to superimpose to form stable and localized wave packets to reproduce particle-like observed behavior. This, however, would require a nonlinear dynamics, like the one in theories such as GRWm, where the fundamental spatiotemporal ontology is given by a matter density field, defined in terms of a wavefunction evolving according to the GRW (nonlinear stochastic) evolution [23]. Alternatively, one could try to develop de Broglie's double solution program, in which the fundamental ontology is a wave oscillating in three-dimensional space, guided through a nonlinear equation by another wave in configuration space (the wavefunction), which has "only a statistical and subjective meaning" [24] (for a review, see [25]).

Be that as it may, notice that this constructive attitude is the attitude that all realist physicists have always had, even when standard quantum theory was initially proposed. It can arguably be tracked down, for instance, to Lorentz, who objected to Schrödinger that his wavefunction was physically unacceptable because it is a field in configuration space, rather than a three-dimensional field such as electromagnetic fields. He wrote: "If I had to choose now between your wave mechanics and the matrix mechanics, I would give the preference to the former, because of its greater intuitive clarity, so long as one only has to deal with the three coordinates x, y, z. If, however, there are more degrees of freedom, then I cannot interpret the waves and vibrations physically, and I must therefore decide in favor of matrix mechanics" (Lorentz in [26]). Similar concerns were raised by Einstein. In a letter to Lorentz dated 1 May 1926, he writes: "Schrödinger's conception of the quantum rules makes a great impression on me; it seems to me to be a bit of reality, however unclear the sense of waves in n-dimensional q-space remains". Similarly, here is an excerpt from a 18 June 1926 letter that Einstein sent to Paul Ehrenfest: "Schrödinger's works are wonderful– but even so one nevertheless hardly comes closer to a real understanding. The field in a many-dimensional coordinate space does not smell like something real" (both these quotes are taken from [27]). In addition, de Broglie and an early Schrödinger were skeptical about interpreting the wavefunction as a physical field. Schrödinger wrote: "The direct interpretation of this wave function of six variables in three-dimensional space meets, at any rate initially, with difficulties of an abstract nature". Also: "Of course this use of the q-space is to be seen only as a mathematical tool, as it is often applied also in the old mechanics; ultimately [. . .] the process to be described is one in space and time" [28]. Moreover, de Broglie wrote: "Physically, there can be no question of a propagation in a configuration space whose existence is purely abstract: the wave picture of our system must include *N* waves propagating in real space and not a single wave propagating in the configuration space" [29]. Interestingly, even Heisenberg expressed his refusal to accept a theory with no fundamental three-dimensional fields and with no fundamental three-dimensional physical space. He has been reported to have said, very vividly, referring to Schrödinger's work: "Nonsense, [. . .] space is blue and birds fly through it" [30]. This attitude has been inherited by the primitive ontologists who, therefore, propose that any satisfactory theory should have a spatiotemporal suitably microscopic ontology. Accordingly, primitive ontologists think that standard quantum theory as well as all solutions of the measurement problem except the pilot-wave theory are fundamentally incomplete because, otherwise, constructive explanation would fail, given that they all lack a spatiotemporal ontology (this is explicit in [15,16,31,32]). In fact, the many-worlds theory and GRW are usually seen as theories of the wavefunction, which is in configuration space. That is, for primitive ontologists, solving the measurement problem is not enough to generate a satisfactory theory because not all solutions of the measurement problem are amenable to a constructive understanding. For them, there are different satisfactory quantum theories, depending on the choice of the microscopic spatiotemporal ontology. The pilot-wave theory is arguably the simplest among the options: it has the simplest type of ontology (particles) and the simplest evolution equations (linear and deterministic).

Nonetheless, some stochastic nonlinear constructive quantum theories have been recently proposed, arguably because they seem to be more compatible with relativity than the pilot-wave theory: GRWm, which we saw above, and GRWf, which is a theory of a set of spatiotemporal events ('flashes'), are defined in terms of a GRW-evolving wavefunction [33]. These theories each have a relativistic extension, which uses only relativistic spatiotemporal structures (see, respectively, [34,35]) rather than a preferred spatiotemporal foliation, as it happens for relativistic extensions of the pilot-wave theory [36]. For more discussion about the alleged advantage of GRW-type theories, see [37].

In addition to primitive ontologists, others have emphasized the importance of space-time or three-dimensionality for a satisfactory ontology. For instance, Maudlin [38] popularized Bell's idea of local beables: "those which (unlike for example the total energy) can be assigned to some bounded space-time region" [33]. Moreover, Norsen [39] proposed that we should actively look for a theory entirely formulated in terms of spatiotemporal ontologies, without a wavefunction in high dimensional space. This turns out to be technically difficult, but perhaps the essence of this can be saved by understanding the wavefunction as a multi-field, or poly-wave, in three-dimensional space. This multifield is an extension of the concept of field, as it assigns a number to a set of locations, rather than only one location, in three-dimensional space [40–44]. Arguably, these approaches can all be seen in constructive terms: just like classical electromagnetism has, in addition to particles, electric and magnetic fields oscillating in three-dimensional space, in quantum theory, the wavefunction is seen as a suitable field also oscillating in three-dimensional space.

4.2. Explanations as Dynamical Bottom-Up Constructions

In the constructive understanding, once the fundamental spatiotemporal microscopic ontology is specified, compositionality and dynamics can be used to explain the macroscopic phenomena, along the classical lines. In this way, the entities of the microscopic fundamental ontology aggregate into composites: quarks and gluons form protons and neutrons, which form atoms, which form molecules, which, depending on how they interact, form liquids, gases, complex proteins, or crystalline structures, viruses, bacteria, animals, stars, and nebulas. These non-fundamental entities constitute the non-fundamental ontology of high-level sciences. These non-fundamental ontologies have the remarkable feature of being autonomous at certain scales. For instance, one can formulate a theory of chemical elements to explain their behavior as if atoms are effectively the ontology, without the need of specifying their detailed microscopic composition in terms of subatomic particles. That is, atoms may be thought of as the effective ontology of the theory valid at that scale, even if such an ontology could be explained in terms of a more fundamental ontology at a smaller scale, such as protons and neutrons. This is, for instance, what thermodynamics does for gases, hydrodynamics does for fluids, or rigid body dynamics does for solid bodies: gases, fluids, and rigid bodies behave autonomously, independently of their composition, and one has laws to describe how they behave. Nonetheless, the constructivist thinks that identifying the microscopic compositions of these non-fundamental entities can explain why these theories are successful (in addition to why, under certain circumstances, these theories fail). Thermodynamics and the other theories can be understood in terms of particle dynamics. Indeed, the fact that such theories exist is what has made it possible for us to come to know anything at the unobservable scale; if we want to explain regularities at one level in terms of regularities at a lower level, we have to come up with a lower-level theory which does that, even if our observation is at a higher level.

Another interesting feature is that while the fundamental ontology is precisely defined, the non-fundamental effective ontology may be vague: while water is precisely defined as having two hydrogen atoms and one oxygen atom, we do not need to specify how many molecules a tiger has in order to identify her as a tiger. In other words, up to a certain scale, the effective ontology is precisely defined. Instead, at a more macroscopic level, the precision of the definition becomes less relevant and arguably unnecessary in order to be explanatory. Indeed, a macroscopic effective ontology is defined functionally: a tiger is what a tiger does. Presumably, at that level, the tiger effective ontology is explanatory because it is vague rather than precise, and this is presumably because, at the macroscopic level, the type of explanation we adopt is more teleological in nature, in terms of desires and intentions, rather than forces or properties. When we observe an electron turning right in a magnetic field, we explain that behavior in terms of its charge and the direction of the magnetic field. Instead, when we observe a tiger hunting a deer, we explain this behavior in terms of the fact that she is a carnivore and that she needs to eat every such-and-such number of hours. Be that as it may, the point is that high-level sciences have effective ontologies that can be reduced, compositionally and dynamically, to the fundamental microscopic ontology. This is compatible with the fact that high-level sciences are explanatory in virtue of the fact that they use a macroscopic rather than a microscopic language.

5. Everettians: Unitary Quantum Mechanics as a Framework

Usually, the attempts described in Section 3 are taken to be 'not realist enough'. For example, Egg [45] puts forward a set of arguments that some implementations of this type of realism do not deserve to be labelled realist. Others, as anticipated, have complained that what seems wrong is having two evolution equations, for a variety of reasons. First, if one thinks of measurements as physical processes, then there is little justification for a double dynamics. Otherwise, one may complain that the theory is not simple or elegant enough. Indeed, Everett did not like the von Neumann collapse rule, which he found was inconsistent [46]. Therefore, he took the unitary wave dynamics seriously and embraced the consequences.

Let us call Everettians those who follow Everett's steps and favor a pure wave dynamics. There are at least two prominent approaches: Oxford Everettians, championed by David Wallace, and Lev Vaidman's approach. There are differences, but there are also many things in common. Here, I will focus on the commonalities.

5.1. Motivation: Practice in Physics

I think that the driving motivation for Everettians is to make realist sense of quantum theory, as practiced. This is, at least, Wallace's explicit position [47], but I believe this is also what Vaidman thinks (p.c.). If one considers what is involved in a typical physicist's daily job, they never invoke the collapse rule, they never use the stochastic nonlinear GRW evolution, and they never solve the guidance equation for the Bohmian particles. Rather, they use the Schrödinger dynamics, operators as observables, and the Born rule. These practices reveal something objective about the world, so, the Everettians ask: How can we make realist sense of standard quantum mechanics without the collapse rule? That is, how can we take the unitary quantum dynamics at face value? Their response is that we need to embrace superpositions at all levels. That is, the unitary Schrödinger dynamics without the collapse is compatible with realism, as long as one recognizes that the superpositions produced by such a linearly evolving wavefunction describe multiplicity. Vaidman's approach is similar in the sense that he cares about having a precise and simple mathematical formalism, which neither modifies nor adds anything to the formalism of quantum theory, just like it is used by physical practitioners.

> "This, in short, is the Everett interpretation. It consists of two very different parts: a contingent physical postulate, that the state of the Universe is faithfully represented by a unitarily evolving quantum state; and an a priori claim about that quantum state, that if it is interpreted realistically it must be understood as describing a multiplicity of approximately classical, approximately non-interacting regions which look very much like the 'classical world'". [47]

First, decoherence, namely the interactions with the environment, guarantees that there is a preferred way of writing the quantum state as the wavefunction. Then, the superposition terms of the wavefunction are taken to suitably represent 'worlds'. Finally, the various worlds are effectively non-interacting: because of decoherence, the interference terms between the different terms of the superposition are effectively and consistently suppressed. In this way, the quantum state effectively describes an emergent branching structure of non-interfering quasi-classical 'worlds'. Vaidman also sees 'words' as vague entities, which classically and autonomously evolve up to the next splitting time. However, he disagrees with Oxford Everettians about the role of decoherence in this. In any case, these words need to be suitably 'weighted' by the probabilities as defined by the Born rule. These weights are necessary to reproduce quantum predictions and are justified by Oxford Everettians in terms of rationality constraints (see [47,48] and references therein). Otherwise, Vaidman takes the Born rule as an additional principle, which we are justified to assume because it makes the theory empirically adequate.

Wallace's argument to favor Everettian mechanics over the alternative is that this theory better respects the practice of physics before and after quantum theory [49,50]. He maintains that classical mechanics is not a constructive theory after all. To have a constructive theory, one needs to have a single theory, taken to describe the behavior of the fundamental building blocks of the physical world. However, Wallace thinks it is a mistake to think that there is a single classical theory. Usually, when people talk about classical mechanics, they mean point-particles classical mechanics, which I described above. Nonetheless, Wallace thinks this is just one of many other theories, which we should also call classical mechanics. For instance, the dynamics of a spring, the vibrations of a rigid body, the flow of a fluid, and the behavior of fields are all 'classical mechanics' in virtue of having a common formalism. They are all formulated in some sort of phase space with a common mathematical structure, whose elements represent physical systems and in which the Hamiltonian generates the dynamics, and which is such that separability holds: the state of a composite system decomposes into the state of the composites. In other words, Wallace thinks that classical mechanics is a framework, in Flores' sense. Unitary quantum theory is also a framework, a set of formal rules: there is a Hilbert space, whose elements represent physical systems and in which the Hamiltonian generates the dynamics. The main difference with the classical case is that this time separability does not hold. That is, the state of a composite system does not decompose into the state of the composites. Many theories can fit the quantum framework: theories of particles, of fields, and so on. Everettian mechanics is the only quantum theory that describes the whole framework, not specific theories. Consequently, it is misguided to ask what 'the' ontology of Everett is, in general, because it depends on the specific framework-fitting theory we are discussing.

Wallace does not say that, but, given the close connection between principle theories and frameworks, I think that if the Everettians' motivation is a comprehensive understanding of physical practice, then understanding standard quantum mechanics as a principle theory makes it also more compatible with the common understanding of relativity theory as a principle theory. For similar compatibility reasons between quantum and relativity theory, it is important for them to have a local quantum theory. In fact, even if it is not a required ingredient of the quantum framework, locality is, arguably, the spirit of relativity.

In this respect, also for Vaidman the many-worlds theory has the advantage, over the alternatives, of being local, in addition of being deterministic. Locality, or local causality, is the idea that interactions propagate continuously. This is something that physicists always assumed, because otherwise it seems impossible to think of systems as isolated.

Notice that Newton's theory violates locality, as forces between objects act instantaneously. Nonetheless, the intensity of the interaction quickly decreases with the relative distance of the systems in question, so that, for all practical purposes, one can forget about objects that are sufficiently distant from the system under examination. The theory of relativity, however, imposes a new limit to local causality, namely that interaction can travel, at most, at the velocity of light. However, standard quantum mechanics is nonlocal, due to the collapse rule, which instantaneously collapses the quantum state, regardless of the relative distance of the superposition terms. Bell's inequality and its violation arguably show that any theory which reproduces the predictions of quantum theory has to be nonlocal, if we assume that the so-called hypothesis of statistical independence is true (see [51] for a review of Bell's theorem and [51] for a critical review of some ways of avoiding nonlocality denying statistical independence). In a many-worlds picture, one arguably could recover locality within a branch: "A believer in the MWI (Many Worlds Interpretation) witnesses the same change, but it represents the superluminal change only in her world, not in the physical universe which includes all worlds together, the world with probability 0 and the world with probability 1. Thus, only the MWI avoids action at a distance in the physical universe" [1]. Wallace agrees that Everettian mechanics is local: "the quantum state of any region depends only on the quantum state of some cross-section of the past light cone of that region. Disturbances cannot propagate into that light cone" [47].

Be that as it may, perhaps more importantly, the Everettians' desire to understand quantum field theories as relativistic extensions of standard quantum mechanics, together with their understanding of relativity as fundamentally a theory about spacetime, requires them to have a spatiotemporal understanding of the quantum state. Wallace and Timpson [52] propose the so-called spacetime state realism, which puts spacetime back into the quantum picture: "just take the density operator of each subsystem to represent the intrinsic properties which that subsystem instantiates, just as the field values assigned to each spacetime point in electromagnetism represented the (electromagnetic) intrinsic properties which that point instantiated" [47]. Vaidman does not propose such an argument but agrees on the centrality of spacetime. He thinks that while interference experiments have shown that matter is wave-like, as opposed to particle-like, the mathematical description of the phenomena provided by the wavefunction is contingent: other descriptions, in terms of density matrices or similar, may be useful or convenient in other contexts. What is essential instead is that this wave-like object cannot live in three-dimensional space; because of entanglement, it has to live in $3N$ dimensional space. Therefore, I think that Vaidman claim that 'reality is only wavefunction' is merely a slogan to convey that 'reality is wave-like, and such a wave is entangled'. He believes that, in order to explain our experiences, three-dimensional space and some three-dimensional 'properties' (such as the matter density field or the particle density field) have to be extracted from the wavefunction and should be considered as fundamental [1,53,54]. Therefore, in this respect, Vaidman is similar to spacetime state realism, or perhaps to the multi-field interpretation of the wavefunction (even if he would disagree with the latter that there is something special about the description of reality given by the wavefunction aside from describing a wave-like object displaying entanglement). Indeed, his view seems to also share some similarities with the matter density theory ontology proposed by primitive ontologists which has been dubbed Sm [55]. Vaidman leaves unspecified which of these fields should be extracted from the wavefunction to explain our experience, but he is adamant that these are what need to be looked at.

5.2. Explanations as Dynamically Emerging Structures

What type of explanation does the many-worlds approach provide? To respond, let us first discuss their fundamental ontology. They do not have a macroscopic ontology, such as IT or QBism. In fact, it is not needed, because superpositions are dealt with by multiplying the worlds. While IT needs the collapse rule to effectively eliminate unobservable macroscopic superpositions, Everettians embrace them at face value. Worlds and macroscopic objects are seen as emerging from the fundamental ontology of the theory, namely the quantum state. Aside from their differences, Vaidman and Oxford Everettians both agree that their theory needs to be rooted in spacetime. They have this requirement in common with primitive ontologists, but their motivations are very different: primitive ontologists require a spatiotemporal ontology because they wish to preserve compositionality, while Everettians do not care about that but care about relativity. Moreover, they also do not require microscopicality. This is because they do not recover macroscopic phenomena in terms of the dynamics of some microscopic ontology, and they do not think of macroscopic objects as composed of microscopic entities, as primitive ontologists would do. Rather, they understand theories as framework and they use principles to constrain the phenomena: as we have seen above, for instance, they use principles of rationality to account for probabilities. Moreover, they adopt structuralist techniques to define what a world or an object is. This is achieved using what Wallace calls the Dennett's criterion [56]: "A macro-object is a pattern, and the existence of a pattern as a real thing depends on the usefulness—in particular, the explanatory power and predictive reliability—of theories which admit that pattern in their ontology" [47]. In this approach, objects are also seen as emergent patterns: "So a cat is a subsystem of the microphysics structured in cattish ways" (*ibid*.). One could see these patterns as defined in terms of principles, but Wallace prefers to think of them as objective emerging structures: "there are structural facts about many microphysical systems which, although perfectly real and objective (try telling a deer that a nearby tiger is not objectively real), simply cannot be seen if we persist in describing those systems in purely microphysical language. Zoology is of course grounded in cell biology, and cell biology in molecular physics, but the entities of zoology cannot be discarded in favour of the austere ontology of molecular physics alone. Rather, those entities are structures instantiated within the molecular physics, and the task of almost all science is to study structures of this kind" (*ibid*.). Therefore, Wallace thinks these structures are needed to account of the explanatory power of higher-level sciences: biology, say, is explanatory because at that level of description, the explanation is in terms of DNA, seen as an emerging autonomous structure.

Everettians and primitive ontologists agree that biology is explanatory, even if it is not expressed in the fundamental language of physics. They also agree that at a certain scale, such as the level of tigers, the effective ontology may be vague and functionally defined. However, they disagree in their understanding of the higher-level ontology. Everettians propose a top-down approach, starting from the quantum state and then reading off from that the non-fundamental structure (worlds, objects, tigers, DNA, etc.). Instead, as described in Section 4, the primitive ontologist constructive understanding is bottom-up: at certain scales, some collections of the microscopic entities will show autonomous behavior.

In addition, in the Everettian account, similarly to the primitive ontologists and unlike pure principle theories, the dynamics is important: these structures emerge in virtue of decoherence, which is a dynamical process. Therefore, there is a sense in which they justify their structures dynamically: at the fundamental level, one has the quantum state; the dynamics, through decoherence, selects a spatiotemporal ontology; the unitary of the dynamics then allows for the formation of superpositions, and, again, decoherence dynamically ensures that the emerging worlds are effectively non-interacting, and that the emerging structures are effectively autonomous and stable. Therefore, even if the explanation provided by this approach is bottom-up, such as principle theorists, here, principles and structures have a dynamical justification.

When Everettians ask for a realist quantum theory, they want a theory with a spatiotemporal ontology from which physical phenomena dynamically emerge. Therefore, there are two ingredients: a spatiotemporal ontology and a structuralist explanation. These requirements are independent from one another. In fact, one could have a structuralist understanding without a spatiotemporal ontology. In fact, as we have seen, rainforest realists understand everything structurally, including the fundamental ontology. Alternatively, one could maintain that worlds are structurally emerging directly from a high-dimensional quantum state. This is, indeed, how one may decide to implement a structuralist version of wavefunction realism (see next section). Moreover, as primitive ontologists do, one can have a spatiotemporal ontology and a compositional explanation: macroscopic objects are not emerging as structures, but they are literally composed of microscopic fundamental entities.

Having said that, the reason why Everettians want a spatiotemporal ontology is that they care about our best physical theories to cohere with one another, and a realist quantum theory for them would have to be compatible with relativity theory (as noted above). Notice that this spatiotemporal ontology would allow for a constructive explanation, if the ontology was one of particles (like in the pilot-wave theory) or with a wave ontology, with a nonlinear dynamics (like the GRW theory with a spatiotemporal ontology or de Broglie's double solution). However, both options would require a substantial modification of the standard theory, and Everettians do not want to go that way without additional reasons to do so. Therefore, they have to make sense of a linearly evolving wave ontology, because this is what unitary dynamics provides us with. A wave ontology and a linear temporal evolution together generate a many-worlds picture, because waves superimpose due to linearity. This, in turn, requires a non-compositional understanding of macroscopic objects, because waves spread. Instead, a particle ontology would certainly not have given rise to superpositions, and it would have allowed for a compositional understanding of macroscopic objects. Notice that if the waves were to combine to form stable wave-packets, which, from afar, look like particles, then one could have constructive explanation as if we had particles at the fundamental level. Nonetheless, this does not happen: there are superpositions at all levels, and this is why Everettians take the multiplicity of worlds seriously. Therefore, it is because of their choice of a wave ontology that they need a structuralist explanation: given the spatiotemporal fundamental ontology, macroscopic phenomena emerge as dynamical structures.

6. Wavefunction Realism: Quantum Theories as Interaction Theories

We have not exhausted the most preeminent proposals for quantum ontology, as we have not yet talked about wavefunction realism. According to wavefunction realism, all quantum theories, which are solutions of the measurement problem, are to be interpreted as theories of the wavefunction, understood as representing a physical field in configuration space (see, most notably, [57–60]). In this way, wavefunction realism is not an approach designed to favor one quantum theory over another, but it is intended as a general strategy for naturalistic metaphysics: given a theory, what should we conclude about the nature of reality? When considering the solutions of the measurement problem, what is the best way to interpret their formalism? The many-worlds theory and GRW are theories about the evolution of the wavefunction, an object in configuration space, while the pilot-wave theory is a theory in which there is a particle and a wave ontology. Straightforwardly, therefore, one is led to interpret these theories at face value, namely as theories in which the fundamental ontology is given by the wavefunction (or part of the ontology, as in the case of the pilot-wave theory). This is very similar to what the Everettians are doing. Ultimately, however, there is another, stronger motivation for some wavefunction realists, as discussed in the next subsection.

6.1. Motivation: Locality and Separability

Ney [60] argues that the best way to motivate wavefunction realism is to notice that this is the only view with a local and separable ontology. As we have seen, local causality is the idea that interaction travels continuously, so it takes time. The violation of Bell's inequality has arguably shown that all quantum theories are nonlocal (setting aside theories violating statistical independence). Therefore, how can wavefunction realism recover locality? There is a sense in which this nonlocality in spacetime is built in the wavefunction, since it is a function of all particle configurations. Therefore, it seems straightforward that if one were to restore locality for the fundamental ontology, one can do that by thinking that the fundamental ontology is the wavefunction understood as a field in configuration space.

That is, if configuration space is the fundamental space, and the wavefunction is a field in that space, then the wavefunction is local in the fundamental space.

If local causality is a property of the interaction, separability is a property of the ontology of matter: as we have seen before, in classical mechanics, the properties of the whole are given by a suitable combination of the properties of its parts. In quantum theory instead, given entanglement, separability fails. Even introducing multiple branches will not help, in contrast with locality. Nonetheless, one could argue that if the fundamental ontology is the wavefunction in configuration space, the properties of the whole would be determined by the properties of its parts: the wavefunction, the whole, is completely determined by the amplitude and phase in each point in the fundamental space. Separability is important for physical practice because it is connected with compositionality, namely that macroscopic objects are composed of microscopic, more fundamental, entities. However, this reason is not available to wavefunction realists because, as we will see later, their explanation is not fundamentally compositional. Otherwise, it is important because it is consistent with Humean supervenience [60], as in configuration space, all properties are determined locally. To preserve Humean supervenience is desirable because it is simple, as we do not have to postulate any additional (relational or otherwise) fact to account for the phenomena.

Wavefunction realists, therefore, allow for non-spatiotemporal ontology because they care about locality and separability, and in a suitable high-dimensional space, they can recover these features, which are otherwise lost in spacetime. In fact, primitive ontologists and the proponents of the IT approach both have a spatiotemporal ontology, so that separability holds, but there are nonlocal interactions. Instead, even if also Everettian mechanics has a spatiotemporal ontology, given that there are the branches, in each branch, the interaction is arguably local but the object across branches is nonseparable. In contrast, if one allows for the ontology and the interaction to both be in high dimensions, like wavefunction realists suggest, then the theory is both local and separable.

Thus, the many-worlds theory as well as GRW should be understood as theories in which the fundamental ontology is a field in a high-dimensional space. It is unclear which of these two theories one should prefer, but certainly the pilot-wave theory fits less straightforwardly in this framework. In fact, the pilot-wave theory can be understood as a theory in which there is a wave and a particle in the high-dimensional space [57], or as a theory in which there are N particles in spacetime and a wave in configuration space. Either way, this approach seems to create more problems than it solves for the pilot-wave theory: for instance, what is matter made of, wave(s) or particle(s)? Why does the wave act on the particle(s) while the particle(s) does/do not?

6.2. Explanations as Non-Constructive Dynamical Hybrids

How does wavefunction realism explain the phenomena? In this view, neither three-dimensional space nor spacetime are fundamental, as the wavefunction is taken to be a material field in configuration space, which is a high-dimensional space. Thus, respectively, the fundamental physical space is represented by configuration space, not by three-dimensional space or spacetime. Three-dimensional space and the macroscopic objects we experience exist, albeit not fundamentally.

In order to explain what this means, that is, in order to provide an explanation for the phenomena, this approach is set to recover three-dimensional space from their fundamental space; then, to recover a microscopic ontology from the wavefunction; and finally, it needs to account for macroscopic behavior. As far as the first step is concerned, the strategy of wavefunction realists is to show that three-dimensional space suitably emerges as non-fundamental from the fundamental high-dimensional space. This emergence happens because of principles which suitably constrain the phenomena, even if these principles differ from proponent to proponent. For example, Albert uses the principle that the dimensions displayed in the Hamiltonian are privileged [57]. Ney instead uses the principle that dimensions that respect the fundamental symmetries of the dynamics are privileged [60]. A similar approach is Carroll's vector space realism, also called Hilbert space fundamentalism [61], even if perhaps it is not motivated by preserving locality and separability. He proposes, in the framework of Everettian quantum theory, to consider the wavefunction as representing a vector in Hilbert space, rather than a field in configuration space. To recover three-dimensional space, Carroll uses the principle that three-dimensionality, which allow for the simplest decomposition of the whole of Hilbert space into subsystems, should be privileged. That singles out three dimensions, and, in this sense, explains why we should expect to observe a three-dimensional world.

Moreover, these approaches use principles to recover a microscopic three-dimensional ontology from the wavefunction as well. Albert and Loewer [62] propose to modify the so-called EER (eigenvalue–eigenstate rule) of standard quantum mechanics as to define three-dimensional, microscopic particles as suitably emergent. Later, Albert [58] proposed that three-dimensional particles are 'functional shadows' of the high-dimensional wavefunction. The idea is that it is possible first to define functionally what it means to be a three-dimensional object, and then it is possible to show that the wavefunction can play that role. This functional reduction can give rise to microscopic three-dimensional objects, which then can be understood as usual; in particular, as composing macroscopic objects. Ney [60] has a different proposal but, I think, the essence is the same. She wishes to derive three-dimensional microscopic particles as derivative parts of the wavefunction, which is the fundamental whole. In her view, there is a particle when there is a 'bump' in the function described by the squared module of the wavefunction. Understanding particles in this way, a particle location may be indeterminate, as the function describing it may be spread out. Particles so-defined may partially instantiate different locations to different degrees, given by the squared amplitude of the wavefunction in that point. Having defined particles in this way, we can proceed to think of macroscopic objects as suitably composed of the non-fundamental three-dimensional ontology. In the many-worlds theory, this will include the use of decoherence, as proposed by the Everettians.

Therefore, wavefunction realists care more about microscopicality and compositionality than Everettians. They use principles to go from high-dimensional space to spacetime and, then, once they have a microscopic non-fundamental ontology they proceed with the usual compositional and dynamical explanation. That is, wavefunction realists use the dynamics from the non-fundamental three-dimensional microscopic ontology to the non-fundamental three-dimensional macroscopic ontology. Instead, Everettians use principles and structural and functionalist techniques to extract the non-fundamental macroscopic ontology from the fundamental spatiotemporal one. Notice that one could use structuralist or functionalist strategies to go directly from high-dimensional space to macroscopic three-dimensional space. Presumably, this is what priority monism does: according to this view, the entire universe is more fundamental than its components, and the parts still exist, even if in a derivative fashion [63]. Otherwise, relational holism holds that only the 'whole' exists and nothing else [64].

How does wavefunction realism correlate with other views? If the IT and the primitive ontology approach can be understood and contrasted as being the principle and constructive counterparts of one another, wavefunction realism emerges as the view obtained when one thinks of quantum theories as interaction theories, without thinking of them in constructive terms, while one can see Everettians as endorsing a framework approach. In fact, let me suggest the following about the alleged similarity between frameworks and principle theories. Principle theories are formulated in terms of constraints on the phenomena, but the phenomena are, obviously, physical processes. In fact, in the case of thermodynamics, the phenomena involve gases expanding, or heat being dissipated, or work being generated. Instead, frameworks are devoid of direct physical significance: they are empty mathematical structures, which could be interpreted freely. They are like argument forms, which are neither true nor false until one substitutes some sentences in place of the letters. As Wallace emphasizes, many different ontologies and theories can fit into the classical or into the quantum framework. So, simply looking at the framework, one cannot say what the ontology of the theory is, because what we are looking at is not a theory but a framework that needs to be filled by some ontology. Many theories, even with different ontologies, could fit that framework, while this is not true for principle theories. Flores contrasts framework theories with interaction theories, which are the ones that Wallace calls point-particle mechanics. Classical point-particle mechanics describes the dynamics of point particles in three-dimensional space, evolving according to Newton's second law. Quantum particle mechanics is the theory of 'particles' whose motion is specified by the wavefunction (namely, a function from the configuration space of those particles to the complex numbers, satisfying the Schrodinger equation). The physical content of the theory is given by the Born rule, which specifies the probability density, on measurement, of finding the particles in a given configuration at a given time. Therefore, while Everettians can be taken as endorsing Everettian mechanics because it describes the whole quantum framework, I think that wavefunction realists are better seen as thinking of quantum theory as a theory of interaction: it is about some fundamental ontology, which they take to be the wavefunction evolving to some dynamical equation. Notice that thinking of interactions in these terms does not require any need for a spatiotemporal microscopic ontology or any notion of compositional explanation. In fact, this is why wavefunction realists, in contrast with primitive ontologists, think of the wavefunction as a suitable ontology for the theory.

7. The Disagreement in a Table

Table 1 graphically shows where the disagreement rests. I have argued, in this paper, that there is no consensus in the foundations of quantum mechanics on which theory should be preferred by the realist because theory preference is connected to types of explanation, and there is no agreement on which one should be favored. Realists who favor principle explanations will likely find nothing objectionable about standard quantum theory with the collapse rule, as it will provide the necessary principles. Instead, constructivists like primitive ontologists want a microscopic spatiotemporal ontology to explain where these principles are coming from, and this naturally leads them to favor the pilot-wave theory. Instead, Everettians follow the attitude of making sense of physics as it is practiced. This leads them to think of quantum theory as a framework, where the unitary quantum theory is taken at face values, namely as suggesting the existence of superpositions at all levels. Also, this leads them to see objects as dynamically emerging patterns. Finally, wavefunction realists require the theories to be local and separable in the fundamental space. To do so, they allow for ontologies to be non-spatiotemporal, and they allow for a hybrid type of explanation.

Personally, I favor a constructive explanation of the phenomena, and I think that, if one has such inclinations, the pilot-wave theory should be the clear consensus. Nonetheless, even if I did not care about constructive explanations, I would still have some questions for both wavefunction realists and Everettians.

For one thing, how can wavefunction realists justify the importance they give to locality and separability? They claim that they are motivated to choose the ontology which would give them a local and separable ontology, but what they have is a local and separable ontology in the fundamental space, namely high-dimensional space. Why should one care about locality and separability in a space other than spacetime? One could argue that locality and separability are desiderata for a spacetime ontology. For instance, Einstein thought that failure of locality would make physics impossible, as it would be impossible to think of systems as isolated. A similar argument could be put forward for separability. However, this seems no longer important if the locality and separability in question are in some high-dimensional space (for more on the comparison between the explanatory strategies of the primitive ontology approach and wavefunction realism, see [65,66]).

Table 1. Different views have different motivations and require different types of explanations.

View	Motivation	Ontology	Explanation	Theory
IT approach QBism Pragmatists	Empirical adequacy	Macroscopic Ontology	Principle explanation: Macroscopic phenomena are explained in terms of principles constraining the phenomena.	Standard QM (unitary dynamics and collapse rule)
Primitive Ontology	Compositionality and dynamical reduction	Spatiotemporal & microscopic ontology	Constructive, dynamical explanation: macroscopic objects are composed of the fundamental microscopic entities, and macroscopic behavior is explained in terms of the microscopic dynamics.	Pilot-wave theory (GRW-x) x = some spatiotemporal microscopic ontology
Spacetime State Realism	Coherence with physical practice	Spatiotemporal ontology	Structuralist explanation: macroscopic phenomena are accounted for in terms of structures dynamically emerge from a spatiotemporal fundamental ontology. Functionalist explanation: objects are what they are because of what they do.	Many-worlds
Wavefunction Realism	Keep locality and separability	Local & separable (not necessarily in spacetime)	Non-constructive/ dynamical explanation: principles are used to recover the nonfundamental spatiotemporal microscopic ontology from the fundamental high dimensional space, then constructive explanation is used to account for macroscopic objects and their behavior.	Many-worlds (bare GRW)

My main question to the Everettians is that it is unclear to me why one would insist on having a wave ontology, knowing that this inevitably leads to a many-worlds picture. After all, if one wants a spatiotemporal ontology already because of relativity, a particle ontology would also remove the multiplicity of worlds. Their likely reply is that this would amount to a radical departure from quantum mechanics as it is practiced by physicists, while a wave ontology would not. However, why should one care about quantum mechanics as it is used by physicists? One may argue that it is because it is incredibly successful. However, how is that connected with realism? One could argue that success is evidence for truth: the best explanation for a theory's success is that the theory is true. Nonetheless, quantum theory has been developed by instrumentalists, so it was not aimed at providing a fundamental description of reality. So, why should we follow the practices of scientists who only care about empirically adequate macroscopic description? Why should such a theory be the guide for the realists, especially if it leads us to adopt a revisionary metaphysics, which does not seem to have any empirical support? For instance, what is the evidence for the existence of the other words?

Setting these questions aside, this paper aimed to show that there is currently no consensus about which is the best realist quantum theory because there is no consensus about which should be a theory's desiderata. If so, not only is there no consensus now, but likely there will never be one in the future (at least if no new physics comes about).

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