ARNOUD VAN THIEL

ANALYTICAL DIALECTIC AND BASIC PHYSICS

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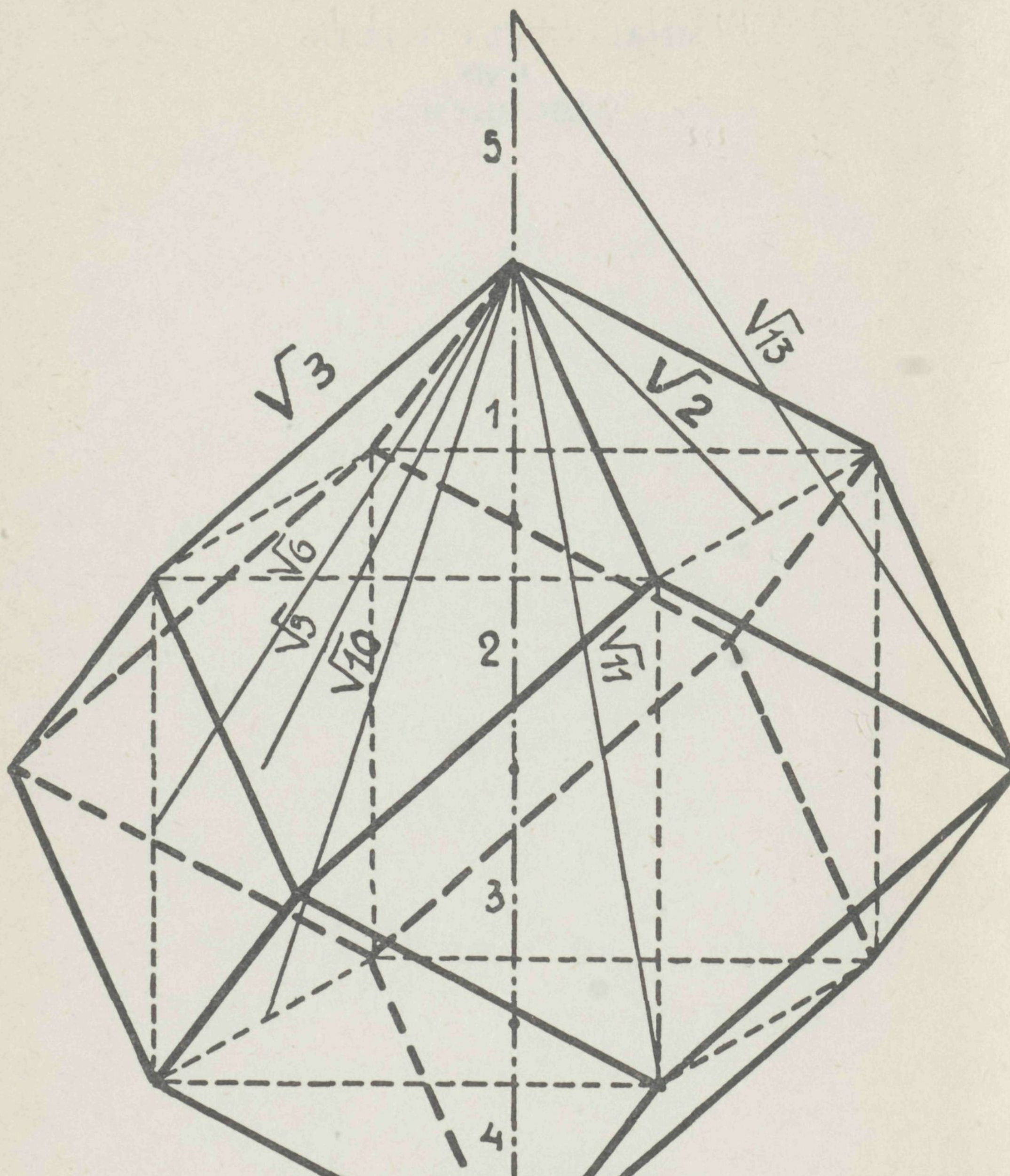
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ARNOUD VAN THIEL

ANALYTICAL DIALECTIC AND BASIC PHYSICS

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ANALYTICAL DIALECTIC AND BASIC PHYSICS

An ideal shines in front of us, far ahead perhaps but irresistible, that the whole of our knowledge of the physical world may be unified into a single science which will perhaps be expressed in terms of geometrical or quasi-geometrical conceptions.

Sir Arthur Eddington

The Nature of the Physical World

Chapter I

ANALYTICAL DIALECTIC

Awareness is individual. A link between awarenesses can only consist in their being acts of one and the same power of being aware. Awareness of such a link is, therefore, awareness of a power of being aware, in such a way that this power of being aware is aware of itself.

All awareness of a power of being aware is its activity, but besides this activity there is the inactive power of being aware, in so far as it is not active. In this way, the awareness of A is accompanied by non A, which is the power of being aware, the activity of which is A; an active power of being aware is dialectical. In awareness the absence of awareness is lacking, therefore non non A = A.

The A — non A scheme provides the power of being aware with the notion of difference which consists of a certain awareness and of its absence elsewhere. There is also familiarity with the notion of similarity since one and the same power of being aware is both active and inactive. In so far as the basis is the same, there is similarity.

Thus as soon as a link between awarenesses is assumed there is also familiarity with difference and similarity. The power of being aware is able to distinguish and to constate similarity.

Self-awareness has three basic forms. In the first the pure power of being aware is the object; in the second the awareness of the first basic form is the object and in the third the awareness of the second basic form is the object. These three forms of awareness differ fundamentally since in the object of the first there is no awareness, no activity at all, while in the object of the second there is no awareness of awareness. An actuality cannot be derived analytically from a possibility nor can an actuality be so derived from another actuality.

Owing to the dialectical basis, the scheme of the first three basic forms of self-awareness may be called

a, non a
b, non b
c, non c

Any further forms of self-awareness add nothing fundamentally different to the object, since they only add awareness of awareness and thus do not differ fundamentally from the third basic form.

The object of the fourth form of self-awareness contains the three basic forms. For each basic form there are two others. Since they are simultaneously present, each may be chosen first, second or third.

The power of being aware must be alive, since otherwise it would not be capable of activity; it must also be free, since otherwise there would be an unending chain of self-awarenesses of which there is no sign. It can also cease to be aware.

To exist means to be able to be object of awareness. Since it is possible to be aware of oneself and to be aware of this awareness both the power of being aware and its activities exist. The self-awareness of the first basic form is symmetrical; subject and object are in equilibrum and are given simultaneously. It is therefore possible to choose both the being subject and the being object first or last. If being subject is chosen first the being object comes last and contrariwise. Each is both beginning and end of the relation between them.

If in the first basic form the object is replaced by the world or part of it, there still are three fundamentally different forms of awareness.

Chapter II

SELF-AWARENESS AND MATHEMATICS

Continued self-awareness makes the notion of number familar. Each phase of this self-awareness may be taken as an ordinal number to which the act of self-awareness is tacitly to be added. It also makes familiar the cardinal numbers which may be obtained from the objects of continued self-awareness in so far as they contain awareness. The self-awareness in the first basic form contains none, and makes familiar the notion of nought. To each cardinal number tacitly awareness should be added. Because of the dialectical basis A — non A, there are both positive and negative numbers. Addition of the two kinds produces nought, the pure power of being aware of which both A and non A are states.

Each phase of self-awareness is taken separately so that one act of self-awareness or any other is under consideration. If, however, the various separate awarenesses of a phase are being considered simultaneously, the sum of the infinite series of continued self-awareness is the number e, formed by taking one entity, viz. the power of being aware, and relating it to its own acts of self-awareness which are represented by the series

$$1 + \frac{1}{1} + \frac{1}{1 \times 2} + \frac{1}{1 \times 2 \times 3} + \frac{1}{4!} \cdots$$

ad infinitum. The 1 above the line represents the power of being aware, the numbers underneath represent the self-awarenesses seen simultaneously and therefore multipliable since relations exist between them all.

The number e is thus seen to be the perfect expression of infinitely continued self-awareness of a given power of being aware.

Time also becomes familiar through continued self-awareness. Like it, time flows in one direction and each phase except the first necessarily follows another, while the relation between two phases is asymmetrical, one necessarily following the other.

By concentrating on the second phase of self-awareness, the first is seen to be past and the third, future.

The notions even and odd also become familiar. To the unit another unit may be added, which gives 2 as the prototype of an even number. Since unity is not yet counted, since this requires continued self-awareness, adding another unit to the first two gives the first odd number, viz. 3. In both the component units may be substitued by prime numbers, which themselves are units in a certain sense.

A geometrical point is an adequate representation of an object of awareness, since it is obtained by taking the smallest possible part of the whole of an object of awareness. In the scheme A—non A taken as an object of awareness the point representing A cannot coincide with that representing non A, since A is lacking in non A, and vice versa. Two points are therefore needed to represent the dialectical basis of awareness. Since both points are states of one and the same power of being aware, their connection may adequately be represented by a straight line connecting them. The distance between the two points may be chosen arbitrarily.

Each of the three basic forms of self-awareness may be represented by such a pair of points, and a connecting straight line. The lines must, however, be perpendicular to each other since they differ fundamentally. Perpendicularity expresses fundamental difference since the lines then only have in common that they represent states of one and the same power of being aware. This power of being aware may also be represented by a point and the various states may then be grouped around this point as a centre, so that a system of co-ordinates is obtained in which on one side there are found the three awarenesses a, b and c and on the other side the states non a, non b and non c. The fact that there are only three basic forms of self-awareness which itself is based on the fact that there are only awareness and the power of being aware, is thus seen to account for the tridimensionality of space, which is the form of continued self-awareness such as it is in the object of the fourth form of self-awareness wherein the three forms are present simultaneously. Since in the continued self-awareness the object never coincides with the subject, two points are needed to represent the first phase to which further equal distances may be added in a straight line for each phase of the continued self-awareness. Time, therefore, may be represented by a single straight line.

If the ends of the lines forming the space-scheme for the three basic forms of self-awareness are connected by straight lines, a regular octahedron is obtained. Since the three axes are simultaneous in the object of the fourth form of continued self-awareness, each may be considered first, second, or last. Each line can therefore precede or follow two others and precede one and follow another. This is adequately represented by the cube which is reciprocal to the octahedron.

For each edge of the cube there are two others which are only potentially, not actually under consideration. These two other edges may be combined into an octahedron with axes of twice the length of the cube edge.

The middles of the edges of this octahedron can be taken to be the combination of pairs of vertices. They are equal to the middles of the cube edges, representing what each pair of cube vertices has in common, and form a semi-regular cubocthedron. By combining the octahedron and the cube, so that the middles coincide, a semi-regular rhombic dodecahedron is obtained.

This body which, like the cube, the truncated octahedron and the hexagonal prism, has the property of equipartitioning space, is the adequate representation of continued self-awareness in its three basic forms. Its properties and their importance for basic physics will be dealt with in the next chapter. Since the regular bodies and their expansion imply group-theoretical operations, these may also be derived from continued self-awareness.

Chapter III

SELF-AWARENESS AND PHYSICS

Analysis of a rhombic dodecahedron to be called R12, shows that it has many aspects which are also to be found in basic physics. This would seem to indicate that the laws of physics like those of mathematics, may be found by analysing continued self-awareness of which R12 is the geometrical representation.
The first basic form shall forthwith be called — X and its absence + x, the second basic form — Y and its absence + y and the third basic form — Z and its absence + z. The vertices of the octahedron which together with the cube forms R 12, shall be so named.
Altogether 12 combinations of two units of a length of half a cube edge are possible. They are to be found in the axes of the small octahedron representing continued self-awareness. Since the first basic form precedes the second and the third and the second the third, and awareness precedes its absence, — X precedes — Y and — Z and — Y, — Z and + x precedes + y and + z and + y, + z. Similarly — X + x precedes — Y + y and — Z + z and — Y + y, — Z + z.
A pair of these units forms a unit square, the first of which is — X — Y.
As soon, however, as the unit square + x + z is formed, all the twelve unit squares are present, therefore — X — Y is the initial stage, + x + z the final; all other unit squares are intermediate stages. The unit squares may be obtained by combining the earlier with the later or vice versa. Since it is contended that — X — Y stands for electron and + x + z for proton, the spin of these particles is the squaring order in

which the units composing them are combined. The intermediate stages naturally have a spin.

The natural order is the more problabe one since only two conditions are required, while for the opposite order the natural order must be thought first and then reversed.

The stability of electron and proton thus is a property derived from the structure of R 12. Unit squares of opposite squaring orders or spins form squares which are completely accounted for and explain the stability of these structures, the unit to be added being to the left of the unit with which it is to be combined, while in the opposite case the unit is added at the right side. This must be so because the forming of unit squares is a special thought.

As soon as — X — Y and + x + z are given, they imply — Y + z, i.e. the neutron, the anti-particle of which is + y — Z. These unit squares may also be formed independently, but the fact that they are also implied accounts for the greater number of neutrons in the universe. Clearly the neutron is not exactly neutral since — Y is neutralized by + y which is implied in + z. There is therefore an excess of positivity of + z less + y.

Similarly electron — X — Y and proton + x + z do not exactly neutralize each other since the electron would be neutralized by + x + y, the positron, so that there is an excess of positivity of + z less + y. This might account for the so called electrical universe and its expansion.

It will now be shown that the so called quantum numbers define geometrical aspects of R 12, and lead to a geometrical concept, long postulated, of quantum physics.

The distance from the vertex of R 12 belonging to the octahedron axes to the nearest centre of a cube square is one unit; that to the centre of R 12 is two units; that to the centre of the most distant cube square

is three units and that to the opposite vertex of the octahedron four units. Squaring the units belonging to a pair of axes gives
$2 \times (1^2 + 2^2 + 3^2 + 4^2) = 32$
This represents the principal quantum numbers.
The quantum numbers l and m are supplied by the sidelines in R 12, except the unit which is the same in both cases. The edge of the rhombs is $\sqrt{3}$ units which squared supply $2 \times 3 = 6$ unit squares. The time from the vertex to the centre of the side-squares of the cube has a length of $\sqrt{5}$ units. They supply $2 \times 5 = 10$ unit squares.
The distance from this centre to the middle of the edges of the square is one unit and therefore supplies $2 \times 1 = 2$ unit squares.
From this middle to the vertex, the distance is $\sqrt{6}$ units which supply $2 \times 6 = 12$ unit squares.
Adding up the unit squares obtained gives $2 + 6 + 10 + 2 + 12 = 32$ unit squares. This is the same as the number obtained from the axes. 2 is the total for the first unit on the axes; $2 + 6 = 8$ is the total for two units on the axes; $2 + 6 + 10 = 18$ for three and $2 + 6 + 10 + 2 + 12 = 32$ for the complete axes of four units.
After this it is possible to add a unit at each end of an axis so that a total of 5 and then 6 is obtained. These units may be found in an octahedron with axes of three cube edges of which the cube vertices form the centres of the faces. This octahedron is available since there are three basic forms of self-awareness and therefore each can be first, second or third.
The line connecting a cube vertex with the middle of the most distant cube edge is 3 units long. It may therefore supply $2 \times 9 = 18$ unit squares which added to the 32 already obtained give $50 = 2 \times 5^2$. The line connecting the vertex with the most distant cube vertex is $\sqrt{11}$ units long. From these, $2 \times 11 = 22$ unit squares may be obtained. Added to the 50 they give $72 = 2 \times 6^2$ unit squares.

The final result is:

K	$= 2$	$= 2 \times 1^2$
L	$= 2 + 6$	$= 2 \times 2^2$
M	$= 2 + 6 + 10$	$= 2 \times 3^2$
N	$= 2 + 6 + 10 + 2 + 12$	$= 2 \times 4^2$
O	$= 2 + 6 + 10 + 2 + 12 + 18$	$= 2 \times 5^2$
P	$= 2 + 6 + 10 + 2 + 12 + 18 + 22$	$= 2 \times 6^2$
	S P D F G H	

7^2 may also be obtained by adding unit squares from the $\sqrt{11}$ line to those of the $\sqrt{2}$ line connecting the vertex with the nearest middle of a cube edge.

In this way the results which quantum theory introduces ad hoc have been obtained in a purely geometrical manner.

The forming of the squares is an act of thought since in R 12 the side-lines are not perpendicular to each other.

It will be noted that in R 12 there is no natural line of a length of $\sqrt{7}$ units, so that 7 has to be obtained by adding the results of the squaring of two pairs of lines. This difference is important and accounts for various irregularities, including those in the periodic system. Since these results are the same as those of quantum theory they are indicated by the same symbols, F indicating the sum of 2 and 12.

The so called magic numbers of nuclear physics may also easily been obtained from R 12 and are characteristic of this semi-regular body.

2 is the number obtained by squaring the unit in the two squaring orders.

8 is the number of unit squares based on half the axes.

14 is the result of adding up the unit squares based on the unit, two units and three units for each squaring order. For the two squaring orders 28, another magic number, is obtained. 50 is the result of the

addition to these 28 unit squares of the 22 unit squares obtained by squaring the $\sqrt{11}$ lines.

82 is obtained by adding up the unit squares for all the units in the axes, i.e. $2 + 8 + 18 + 32 = 60$ and then the 22. 60 unit squares are obtainable from the axes, but if first the 18 unit squares from the lines of 3 units are added and then the 22, a total of 100 unit squares is obtained. Since it is possible to find 26 additional unit squares within R 12 the magic number 126 is obtainable. 26 is the sum of the unit squares based on $\sqrt{2}$ and $\sqrt{11}$ lines. There is also an independent $\sqrt{13}$ line, viz. that which connects a vertex of the larger octahedron with one of the smaller, if not situated on the same axis.

146 is obtainable by adding the 20 unit squares originating in the lines connecting the R 12 vertex to the middle of the most distant cube edge $\sqrt{10}$ units long, to the 126 obtained earlier. This number may also be considered magical since it is the number of neutrons in stable uranium. All these numbers are obtained by squaring whole lines in R 12 in both squaring orders, so that wholly accounted for squares are formed. This causes their stability which, therefore, has a purely geometrical reason.

The electron, — X — Y is plane-like since it only required two dimensions, but proton and neutron require the introduction of the third axis and are therefore space-like. Moreover, they have the + z unit in common, and may therefore be constructed by using three units.

They are seen to be more closely connected than electron and proton and may be considered the nucleus of the atom. Maybe the fact that by rotation the proton can be changed into a neutron and vice versa accounts for isospin.

It is possible to form elements out of the unit squares available, be they particles or anti-particles.

In this connection only — X — Y, + x + z and — Y + z, viz. electron, proton and neutron will be considered.

The first element is hydrogen which consists of unit squares:

(— X — Y) and (+ x + z) for one spin = squaring order and: (— Y — X) and (+ z + x) for the other.

The next element is deuterium for which one neutron — Y + z is added to the former. Since there is an implied neutron available, deuterium may be changed into tritium which contains two neutrons. Two hydrogen atoms of equal or opposite spin (i.e. squaring order) may be combined into a hydrogen molecule.

When a hydrogen molecule is formed the two unit squares are not superimposed, but remain separate so that next to the actual units there are the possible ones. This combination of actuality and possibility constitutes the magnetic moment, since the essence of magnetism is such a combination, as will be shown later.

When, however, the unit squares are superimposed so that all the four sides are actual, then possibility is changed into actuality, and there is no magnetic moment. Binding the unit squares by superimposition is a far stronger bond than if they remain separate though connected. The first case of such a superimposition is the helium atom. Its electronic square is complete and so are the squares for proton and neutron, which moreover, share a + z unit. This accounts for the particular stability of the alpha-particle. Far less units are needed to build the helium nucleus since the actual units of one square serve as possible units for the other, so that many units which have to be stated for the hydrogen molecule become superfluous. This is the reason why combining hydrogen atoms into helium releases so much energy.

With the formation of helium the K-series is exhausted. Apparently implied neutrons are not available after the square has been formed. As for helium, implied neutrons must be formed out of protons and electrons so long as they are partly actual, partly possible. This may explain why there is no stable nucleus He 5.

The formation of elements by thought may be continued by means of the higher series, taking first the shorter and then the longer ones. In any case this can be done for the construction of the electronic part. Since the nucleus and more especially the proton, belong to the side of R 12 which is opposite to that of the electron, the longer lines are exhausted first in forming the nucleus.

There seems, however, to be a predilection for first adding unit squares corresponding to a single unit. Similarly, while exhausting the longer series, the unit is used first in the two squaring orders and then all the remaining unit squares are first taken in one squaring order and then in the other. It is possible, in most cases, to use both independent and implied neutrons, and thus to construct the various isotopes.

The periodic system of elements arises from the fact that in the series the same unit squares return in the repetition of the same lines. Construction of the elements continues without complications so long as there are unit squares available by squaring a pair of lines.

The series K, L and M are then completely exhausted and so are the normal unit squares of series N. This is why the element 46 Palladium resembles an inert gas. Its squares are completed in both squaring orders. After this the situation changes. Palladium has 23 unit squares in one squaring order and 23 in the opposite squaring order, which gives a certain distinction to the number 23 and may throw light on the properties of 23 Vanadium. Similarly, since the 23 may be formed out of atoms in which electron and proton are of the same squaring order or of an opposite one, there are in reality four possibilities, for a total of $4 \times 23 = 92$. In this case the stability of Uranium and the instability of the higher elements may be the consequence of the geometrical fact that after exhausting the sidelines which are squared in pairs, another procedure has to be found to obtain the 14 unit squares called F.

The unit and the $\sqrt{6}$ line cannot by themselves be added, since their direction is different. This addition can only be done in a triangle with sides of $\sqrt{5}$, 1 and $\sqrt{6}$ units. The $\sqrt{5}$ line is thus given a new function and must be restated while it cannot be thought without the preceding unit and the $\sqrt{3}$ line. For this reason a further 2 + 6 unit squares become available, which may be added to the 46 of Palladium and bring the total to 54, the element called Xenon, an inert gas. The 10 unit squares obtainable from the $\sqrt{5}$ line must be proportionally divided between the unit and the $\sqrt{6}$ line of the triangle. Their ratio is approximately 3 : 7. Since the unit follows the $\sqrt{5}$ line, the three unit squares apportioned to it must first be added, which brings the total to 57, Lanthanum. After this, the 2 + 12 unit squares may be added. These form the group of rare earths, or lanthanids, whose position in the periodic system is so peculiar. The total has then become 71, after which the remaining 7 unit squares may be added to bring the total to 78, Platinum.

The procedure may be repeated; again the 2 + 6 are introduced, bringing the total to 86, Radon, then the 3, the 2 + 12 and the 7.

It seems improbable that ever higher elements may be constructed in this way. Presumably 182, the sum of the 6 double squares, is the limit, since these correspond with the three basic forms of self-awareness and there are no more. It wil be noticed that the well known similarity between the lanthanids and actinids is accounted for by assembling them according to the same process.

Chemical valency is also based on R 12 and more especially on the fact that the sidelines in R 12 are mostly an irrational number of units long. This accounts for quantisation in general. In order to arrive at a whole and positive number of unit squares, these lines of irrational length must be squared and there are no intermediate stages. For instance, in order to explain the four-valency of 6 Carbon, it should be remembered that 6 is not a number representing a final stage of the K-and L-series. Four unit squares have to be added to 6 in order to reach the final stage of completeness. For 7 Nitrogen the number is 3 and therefore Nitrogen is tri-valent. For 8 Oxygen, 2 unit squares have to be added, and so its valency is 2.

There is, however, another way of accounting for valency, similarly based on the need to achieve completely squared lines. This is done by shedding unit squares. In this way 6 Carbon can arrive at the $2 + 2$ lines by shedding 2 unit squares and is thus also bi-valent. For Nitrogen, 3 results in both cases, but if the two units of the L-series are shedded, only the complete K-series is left, and Nitrogen is found to be five-valent. Shedding 6 unit squares reduces 92 Uranium to 86 Radon and thus Uranium is six-valent. As expounded above, another reason for valency is the difference in squaring order. Since the two squaring order are available in the elements comprising a complete set of square units, the inert gases are monatomic. If unit squares are formed out of the axes i.e. if the principal quantum numbers are considered, every unit square is one out of the square of the quantum number. Their difference lies in the line to which they belong and this would account for the formula $\frac{1}{m^2} - \frac{1}{n^2}$, which describes the ordinary hydrogen spectrum. Unit squares may be added by lining them up but also by combining and adding up the units into whole lines. In the latter case each addition is squared together with the original unit, and a great number of unit squares arises, since the mixed unit squares also count. This would seem to account for the multiple spectra. Because an electron consists of two squared units, a number of Z electrons gives Z^2 pure and mixed unit squares. Thus high numbers are reached easily for all kinds of unit squares. This accounts for the Moseley-rule, among others, by which the elements are lined up according to Z^2 if Z is the number of positive unit squares in the nucleus.

In this way it is possible to account for the proton-electron rate, viz. 1836 : 1.

The electron is supposed to be at the top of R 12, and the proton at the bottom. Moreover the Y-axis has to be changed into the + z — axis. If the electron has mass 1 (i.e. one unit square found at the beginning of the axes) the proton has to be located at the end. In order to reach the electron the whole R 12-axis, consisting of 4 units, has to be percurred. This amounts to adding up all the unit squares formed in the course of moving from bottom to top. This means that $32 + 18 + 8 + 2 = 60$ unit squares are available, resulting in $60^2 = 3600$ unit squares, 1800 for each squaring order. Apart from this, the $\sqrt{6}$ lines have to be accounted for, since this line is space-like and represents the rotation from the electron-axes to the proton-axes. They account for $\sqrt{6} \times \sqrt{6} = 6$ unit squares, which because of the Z^2 formula, supply 36 unit squares. Added to the 1800 previously found, they give 1836 as the number of unit squares involved in the relation between electron and proton. The neutron value is obtained by adding one unit square to the proton mass, which results in 1837.

That this way of calculating the proton mass is the right one, is proved by the fact that the proton dissolves into two pions. These have a mass of 272 electron masses. Clearly this number is obtained by taking first the R 12-axes in one direction and then in the other. Their squares are 16 unit squares and supply 256 pure and mixed unit squares. To these the unit squares obtainable from the axis-rotation, (i.e. those based on the $\sqrt{3}$ lines and the unit) must be added. They total $3 + 1$ and therefore 16 for each squaring order, which added to the 256 previously found, give 272. In order to obtain the proton, the axes have to be percurred in both directions and thus there is a negative and a positive pion. The neutral pion with mass 265 may be obtained by adding only the $3 \times 3 = 9$ unit squares, and the value may actually be 266 which would mean that the $\sqrt{3}$ lines and the unit are accounted for separately.

The K-meson has a mass of approximately 960 electron masses. It is based on the fact that the 32 unit squares leading from the pions to the protons are now taken in combination. The square of 32 is 1024, but since the axis-rotation has already been allowed for in the pions, the $3 + 1$ unit squares per squaring order $= 8$, have to be eliminated. This reduces the total by $8 \times 8 = 64$ unit squares, leaving 960.

The fact that the K-meson disintegrates into two* pions seems to support this way of calculating its mass.

The mass of the mu-meson 207 is also characteristic of R 12. Since there are no $\sqrt{7}$ lines in R 12, the situation is abnormal in the N-series. 7 unit squares for each squaring order are obtained in a special way. Of the $16 \times 16 = 256$ unit squares, $7 \times 7 = 49$ are purely based on these abnormal lines, leaving 207 more or less normal ones.

After the 32 unit squares based on the axes have been obtained, 18 more are added, bringing the total to 50, which supply $50 \times 50 = 2500$ pure and mixed unit squares. Of these $18 \times 18 = 324$ are to be ascribed exclusively to the lines of 3 units; the remainder, $2500 - 324 = 2176$ have another origin. This would seem to supply the value for the Lambda-hyperon. If the 18 unit squares are obtained from the K-, L-and M-series there is an intermediate stage in which $2 + 6 + 5 = 13$ square units are available. $13 \times 13 = 169$ are therefore in a special position leaving $2500 - 169 = 2331$ in a different position. This

* If the inversed squaring order is stated first there are three pions since the natural order is then stated twice; this throws light on the thêta-tau puzzle.

might be the basis of the value of the Sigma-hyperon. Since the 50 are less than the 60 on which the proton is based, the Lambda-hyperon may be said to be inside the proton. Naturally the K-meson is also inside, and the proton may thus disintegrate into a K- and a Lambda-particle. The last stage in the analysis of the structure is to reach 6^2 for two squaring orders. The result is that 72 is obtained which supplies 5184 unit squares, 2592 for each squaring order, i.e. the value of the Xi-hyperon.

All these numbers are in good agreement with experimental results. The calculated values cannot absolutely agree since the structure is taken to start from geometrical points, while in practice there is no such thing. Therefore all the results are somewhat blurred.

The calculations can be made for particles and anti-particles.

Since there is a formula:

$(-X - Y) + (+x + z) = (-Y + z) +$ neutrino, it is possible that the neutrino is a unit seen in both directions, but of course it is not plane-like. This would account for its neutrality and for the fact that it has no mass. On the other hand there are many nearly neutral unit squares, among which might be the neutrino and the anti-neutrino.*

A strictly group-theoretical analysis of the rhombic dodecahedron might allow results to be obtained in a systematic way.

There appears to be a numerical agreement between the so called missing elements and R 12. The first of these is 43 Technetium, while R 12 has 43 internal diagonals.

When R 12 is placed within the largest octahedron its 12 cube edges and 6 prolongations become internal lines, bringing their total to 61. This is the atomic number of the missing element Promethium. Similarly the former octahedron edges of R 12 have become internal

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* $e^- + e^+ = \upsilon + \bar{\upsilon}$ or $(-X - Y) + (+x + y) = (+x - Y) + (-X + y)$

lines, bringing their total to 85, the atomic number of the missing element Astatine. The former octahedron edges have equally become internal lines, so that their total is now 97, the atomic number of Berkelium. The absence of 85 Astatine and of 87 Francium is easily explained by the fact that Francium supplies one unit square of each type to Astatine, so that it becomes 86 Radon and both elements have disappeared as such. For the other missing elements it is more difficult to see any reason why they should disappear, except for the fact that any disintegration of the complete R 12 immediately reduces the number of its internal lines so that the disappearance of these numbers is characteristic of R 12 and therefore of the periodic system based on it. As has been shown, the absence of the transuranic elements may also be ascribed to the structure of R 12. Since R 12 equipartitions space, the conditions proper to R 12 obtain anywhere in space. The absolute length of the unit has been chosen arbitrarily, but for reasons of symmetry all distances should be put on the same basis. A smallest length, however, is unavoidable, since the A and non A points cannot coincide. The cube in R 12 contains the laws of gravitation and electromagnetism as formulated by Sir Arthur Eddington in his New Pathways in Science and elsewhere. Eddington obtains his formulas by carrying out operations of a certain type on a sequence of letters A, B, C, D. One of the operations is to turn them all upside down. Repetition of this operation restores the original position. It is easy to see that this corresponds with
A, non A; non non A = A
The eight cube vertices shall be called:

XYZ = A	xyz = $\underline{A}$
Xyz = B	xYZ = $\underline{B}$
xYz = C	XyZ = $\underline{C}$
xyZ = D	XYz = $\underline{D}$

Each pair is connected by a main diagonal of the cube, and underlined letters are those which are upright in the first column, and upside down in the second, and conversely.

Eddington begins by turning upside down two letters (but not the first) in the series A, B, C, D. This operation is called a D-operation. There are four such operations altogether, if the ommission of the operation is counted as one operation.

After this operation has been carried out c.q. been omitted, a second operation of interchanging the order of the letters is executed. Again including the case where the order remains unchanged, there are four pairs of these operations in which either the first and the second and the third and the fourth or the first and the third and the second and the fourth or the first and the fourth and the second and the third have been interchanged. This operation is called an S-operation. The results of these operations and their combinations may be found in appendix I. The interchanging is seen to be the connection of two vertices, and considering the resulting line in a particular direction. Each pair of S-operations appears to compare two cube edges or two cube face diagonals. If there is no interchanging, the letters are considered as states which may be compared.

Attaching the number 1 or 2 or 3 to X, Y and Z or x, y, and z indicates that the line concerned is first, second or third choice. The question then arises as to in what respect the various lines differ. In the cube each pair of vertices on an edge have two of the three letters in common. This pair of letters may be taken to be concentrated in the middle of the cube edge concerned.

Each cube edge may be considered to be accompanied by these pairs of letters, and their differences may be seen to be due to the pairs of letters by which they are thus accompanied. The differences originate in the fact that the edges are chosen first, second or third.

In comparing the diagonals, two such pairs of accompanying middles are taken together.
Similarly the states represented by vertices differ as to the pairs of letters they contain.
The SD-operations may thus be seen to be an analysis of the cube. The distance from a vertex to the nearest middle of a cube edge is half a cube edge; that to the second nearest middle is $\frac{1}{2}\sqrt{5}$ times a cube edge, and that to the furthest middle is 3/2 times a cube edge. The nearest middle means that the cube edge on which it is situated is chosen first; the second nearest means that the edge concerned was chosen in second place, and the furthest middle means that the cube edge concerned was chosen in the third place.
The second nearest middle includes one letter standing for an absence of awareness, and the most distant middle contains two such absences. In the line of a length of $\frac{1}{2}\sqrt{5}$ cube edges $\frac{1}{2}$ a cube edge represents the nearest middle; the remainder, i.e. $\frac{1}{2}\sqrt{5} - \frac{1}{2}$ cube edge, represents the second nearest middle. This is the length of the larger part of a cube edge divided according to extreme and mean ratio, to be called g.
The distance between a cube vertex and the most distant middle of an edge is 3/2 cube edges, of which $\frac{1}{2}\sqrt{5}$ may be attributed to the first and second nearest middles. The remainder $3/2 - \frac{1}{2}\sqrt{5}$, i.e. the smaller part of the cube edge divided according to extreme and mean ratio, to be called s, is proper to the middle of the cube edge consisting of two absences of awareness. In triangles in which g and s are at right angles to a cube edge, the hypotenuse stands for the complements of g and s.
It is possible to construct in a cube a pair of regular dodecahedrons which are perpendicular to each other in such a way that they equipartition each others edges, s long. Similarly two similarly situated regular

icosahedra with an edge length of g, and main diagonals of a length of the hypotenuse in the rectangular g triangle, may be constructed.
The regular dodecahedron and icosahedron may be compounded into a semi-regular rhombic triacontahedron.
The regular dodecahedron has 100 internal diagonals, 10 of which are main diagonals, complement of s long, and 30 edges which belong to two pentagons. They may therefore be counted twice.
The regular icosahedron has 36 diagonals 6 of which are main diagonals, and 30 edges which belong to two triangles, and may therefore be counted twice.
As stated by Eddington, the results of the combined SD-operations are, in their first stage, 16 in number. Continuing these operations on the results of the first operations, provides $16 \times 16 = 256$ arrangements of letters, 100 of which are of one type in 10 different kinds, 36 of which are of another type in 6 different kinds, 60 of which are of still another type in 10 kinds and 60 of which are of a fourth type in 6 kinds.
The first 100 are claimed to be the gravitational coefficients, the first 60 the electromagnetic coefficients and the remaining 60 and 36 their complements obtained by replacing them by their opposites. Clearly there is a connection between the Eddington formulae and the geometrical structures mentioned above. It is contended that the gravitational coefficients only describe the regular dodecahedron with edges of a length of s, and that the electromagnetic coefficients describe the regular icosahedron with edges of a length of g. The rhombic triacontahedron is thus seen to be the geometrical representation of the unitary field theory. For the gravitational coefficients, diagonals or vertices are compared, while for the electromagnetic coefficients, edges are compared.
This comparison shows how much g or s is contained in a line and ab-

sent in another. The main diagonals may supply the 100 resp. 36 differences, but is it also possible to take the shorter internal diagonals, since these are implied in the main diagonals.
The stellated regular bodies show the systems of diagonals to be formed. Since it is possible to analyse any cube in this manner, the coefficients are invariant, and since the cube equipartitions space they are valid everywhere, being just geometrical aspects of the cube.
Owing to the fact that the rhombic dodecahedron contains a cube, there is also a link between atomic physics expressed by R 12 and gravitation and electromagnetism.
The fine structure constant 1/137 may be accounted for by taking the 100 + 36 diagonals and forming them into a whole consisting of 136 parts. The whole itself is then 1/137 of the total of whole and parts. Eddington's operations only compare the lines taken in one direction. This is admissible since the accompanying combinations are the same for both directions. However, only the operations selected contain rotation in their gravitational results. This is because only in that selection is the natural order of the awarenesses and their complements observed. Appendix I gives the relevant cycli. The electromagnetic coefficients compare, in three cases, lines seen in the same direction and, in three cases, in an opposite direction. The first three seem to be the electrical coefficients, the last three the magnetic ones. In the last case an awareness is combined with its absence, so that an actuality is combined with a possibility. This accounts for the particular character of magnetism.
The number 2^{256} is easily obtained from R 12.
In this way the whole of physics turns out to have a geometrical basis which is contained in R 12 even if this body should not be considered to be an adequate representation of self-awareness but of observation only.

Chapter IV

CONCLUSION

The human, subjective power of being aware, by reflection on its own activity, appears to be able to obtain results similar to those of experimental physics. It is therefore reasonable to assume that the objectively valid laws of nature are the effect of an objective power of being aware carrying out an identical analysis of its own self-awareness in its three basic forms. Nature then would consist of objective thoughts.

The geometrical structures which are claimed to be the representation of the threefold self-awareness, do not in themselves supply these laws and entities. In atomic physics the unit squares must be formed by thought, the elements must be assembled by special acts of thought, and their disintegration in various ways is again the cessation of a previous thought assembling unit squares. Similarly the comparisons between edges, diagonals, and vertices, are special acts of thought. Since these acts of thought cease as soon as they are completed, the objective power of being aware is continuously repeating them, i.e. creating them anew. Because of equipartition all conditions expounded obtain everywhere in space. The repetition of the assembling act may be effected in different places not too distant from each other, so that the semblance of motion arises. Since there is no permanence in the assembling acts, there cannot be motion but only change. The objective power of being aware is the cause of everything; it is the all pervading aether which has escaped detection because it is spiritual, not material.

The objective power of being aware may start paying attention to particular series of its thoughts, and this may be how the subjective power of being aware is created. This subjective power is only aware of a small part of what is going on in the universe; the remainder, however, may be imagined since the subjective power is able to find all aspects by reflection on its own self-awareness. It may, for instance, be impossible ever to see electrons, though they really exist as objective thoughts. Thoughts and mental operations on thought, are as real as the power of being aware which thinks or performs them, but they are not as permanent and are never necessary, but only contingent.

According to the views expounded, it would not appear to be impossible for man to totally comprehend nature, since the structure of objective and subjective thoughts is identical.

Such harmony is in immortal souls

Appendix I

D-operations on A, B, C, D replace by their complements:

D_a: 3rd and 4th
D_b: 2nd and 4th
D_c: 2nd and 3rd
D_d: — —

S-operations on the results of D-operations interchange:

S_a: 1st and 2nd, 3rd and 4th
S_b: 1st and 3rd, 2nd and 4th
S_c: 1st and 4th, 2nd and 3rd
S_d: — — — —

The results of the initial SD-operations are for:

S_aD_a: $BA—\underline{D}\underline{C} = Xyz - XYZ — XYz - XyZ$
$= y_{2,1}Y_{2,1};\ z_{2,1}Z_{2,1} - Y_2y_2;\ z_1Z_1$
S_bD_a: $\underline{C}A—\underline{D}B = XyZ - XYZ — XYz - Xyz = y_1Y_1 - Y_2y_2$
S_cD_a: $\underline{D}A—\underline{C}B = XYz - XYZ — XyZ - Xyz = z_1Z_1 - Z_2z_2$
S_dD_a: $A, B, \underline{C}, \underline{D} = XYZ, Xyz, XyZ, XYz$

S_aD_b: $\underline{B}A—\underline{D}C = xYZ - XYZ — XYz - xYz = x_1X_1 - X_2x_2$
S_bD_b: $CA—\underline{D}\underline{B} = xYz - XYZ — XYz - xYZ$
$= x_{2,1}X_{2,1};\ z_{2,1}Z_{2,1} - X_2x_2;\ z_1Z_1$
S_cD_b: $\underline{D}A—C\underline{B} = XYz - XYZ — xYz - xYZ = z_1Z_1 - z_2Z_2$
S_dD_b: $A, \underline{B}, C, \underline{D} = XYZ, xYZ, xYz, XYz$

S_aD_c: $\underline{B}A—D\underline{C} = xYZ - XYZ — xyZ - XyZ = x_1X_1 - x_2X_2$
S_bD_c: $\underline{C}A—D\underline{B} = XyZ - XYZ — xyZ - xYZ = y_1Y_1 - y_2Y_2$
S_cD_c: $DA—\underline{C}\underline{B} = xyZ - XYZ — XyZ - xYZ$
$= x_{2,1}X_{2,1};\ y_{2,1}Y_{2,1} - X_2x_2;\ y_1Y_1$

S_dD_c: A, $\underline{B}$, $\underline{C}$, D = XYZ, xYZ, XyZ, xyZ

S_aD_d: BA—DC = Xyz - XYZ — xyZ - xYz

= $y_{2,1}Y_{2,1}$; $z_{2,1}Z_{2,1}$ - y_2Y_2; Z_2z_2 (Z_3z_3)

S_bD_d: CA—DB = xYz - XYZ — xyZ - Xyz

= $x_{2,1}X_{2,1}$; $z_{2,1}Z_{2,1}$ - x_2X_2; Z_2z_2 (Z_3z_3)

S_cD_d: DA—CB = xyZ - XYZ — xYz - Xyz

= $x_{2,1}X_{2,1}$; $y_{2,1}Y_{2,1}$ - x_2X_2; Y_2y_2 (Y_3y_3)

S_dD_d: A, B, C, D, — XYZ, Xyz, xYz, xyZ

The continued SD-operations sometimes result in:

$\underline{B}\underline{A}$—DC = xYZ - xyz — xyZ - xYz

= $Y_{2,3}y_{2,3}$; $Z_{2,3}z_{2,3}$ - y_2Y_2; Z_3z_3

C$\underline{A}$—D$\underline{B}$ = xYz - xyz — xyZ - xYZ = Y_3y_3 - y_2Y_2

D$\underline{A}$—C$\underline{B}$ = xyZ - xyz — xYz - xYZ = Z_3z_3 - z_2Z_2

$\underline{A}$, $\underline{B}$, C, D, = xyz, xYZ, xYz, xyZ

B$\underline{A}$—D$\underline{C}$ = Xyz - xyz — xyZ - XyZ = X_3x_3 - x_2X_2

$\underline{C}\underline{A}$—DB = XyZ - xyz — xyZ - Xyz

= $X_{2,3}x_{2,3}$; $Z_{2,3}z_{2,3}$ - x_2X_2; Z_3z_3

D$\underline{A}$—$\underline{C}$B = xyZ - xyz — XyZ - Xyz = Z_3z_3 - Z_2z_2

$\underline{A}$, B, $\underline{C}$, D = xyz, Xyz, XyZ, xyZ

B$\underline{A}$—$\underline{D}$C = Xyz - xyz — XYz - xYz = X_3x_3 - X_2x_2

C$\underline{A}$—$\underline{D}$B = xYz - xyz — XYz - Xyz = Y_3y_3 - Y_2y_2

$\underline{D}\underline{A}$—CB = XYz - xyz — xYz - Xyz

= $X_{2,3}x_{2,3}$; $Y_{2,3}y_{2,3}$ - x_2X_2; Y_3y_3

$\underline{A}$, B, C, $\underline{D}$ = xyz, Xyz, xYz, XYz

$\underline{B}\underline{A}$—$\underline{D}\underline{C}$ = xYZ - xyz — XYz - XyZ

= $Y_{2,3}y_{2,3}$; $Z_{2,3}z_{2,3}$ - Y_2y_2; z_1Z_1

$\underline{C}\underline{A}\text{—}\underline{D}\underline{B} = XyZ \text{ - } xyz \text{ — } XYz \text{ - } xYZ$

$= X_{2,3}x_{2,3};\ Z_{2,3}z_{2,3} \text{ - } X_2x_2;\ z_1Z_1$

$\underline{D}\underline{A}\text{—}\underline{C}\underline{B} = XYz \text{ - } xyz \text{ — } XyZ \text{ - } xYZ$

$= X_{2,3}x_{2,3};\ Y_{2,3}y_{2,3} \text{ - } X_2x_2;\ y_1Y_1$

$\underline{A}, \underline{B}, \underline{C}, \underline{D} = xyz, xYZ, XyZ, XYz$

60 times, in 6 different kinds of 10 times each, difference is attributed to g:

$\underline{B}A\text{—}\underline{D}C = xYZ \text{ - } XYZ \text{ — } XYz \text{ - } xYz = x_1X_1 \text{ — } X_2x_2 \quad Yz$

$\underline{B}A\text{—}D\underline{C} = xYZ \text{ - } xYZ \text{ — } xyZ \text{ - } XyZ = x_1X_1 \text{ — } x_2X_2 \quad yZ$

$\underline{C}A\text{—}\underline{D}B = XyZ \text{ - } XYZ \text{ — } XYz \text{ - } Xyz = y_1Y_1 \text{ — } Y_2y_2 \quad Xz$

$\underline{C}A\text{—}D\underline{B} = XyZ \text{ - } XYZ \text{ — } xyZ \text{ - } xYZ = y_1Y_1 \text{ — } y_2Y_2 \quad xZ$

$\underline{D}A\text{—}\underline{C}B = XYz \text{ - } XYZ \text{ — } XyZ \text{ - } Xyz = z_1Z_1 \text{ — } Z_2z_2 \quad Xy$

$\underline{D}A\text{—}C\underline{B} = XYz \text{ - } XYZ \text{ — } xYz \text{ - } xYZ = z_1Z_1 \text{ — } z_2Z_2 \quad xY$

36 times, in 6 different kinds of 6 times each, difference is attributed to complement of g:

$D\underline{C}\text{—}B\underline{A} = xyZ \text{ - } XyZ \text{ — } Xyz \text{ - } xyz = x_2X_2 \text{ — } X_3x_3 \quad yZ$

$\underline{D}C\text{—}B\underline{A} = XYz \text{ - } xYz \text{ — } Xyz \text{ - } xyz = X_2x_2 \text{ — } X_3x_3 \quad Yz$

$D\underline{B}\text{—}C\underline{A} = xyZ \text{ - } xYZ \text{ — } xYz \text{ - } xyz = y_2Y_2 \text{ — } Y_3y_3 \quad xZ$

$\underline{D}B\text{—}C\underline{A} = XYz \text{ - } Xyz \text{ — } xYz \text{ - } xyz = Y_2y_2 \text{ — } Y_3y_3 \quad Xz$

C$\underline{B}$—D$\underline{A}$ = xYz - xYZ — xyZ - xyz = z_2Z_2 — Z_3z_3 xY
$\underline{C}$B—D$\underline{A}$ = XyZ - Xyz — xyZ - xyz = Z_2z_2 — Z_3z_3 Xy

60 times, in 10 different kinds, of 6 times each, difference is attributed to s:

$\underline{B}\underline{A}$—DC = xYZ - xyz — xyZ - xYz
= $Y_{2,3}y_{2,3}$; $Z_{2,3}z_{2,3}$ - y_2Y_2; Z_3z_3
$\underline{A}$ — $\underline{B}$, C, D = xyz — xYZ, xYz, xyZ
$\underline{C}\underline{A}$—DB = XyZ - xyz — xyZ - Xyz
= $X_{2,3}x_{2,3}$; $Z_{2,3}z_{2,3}$ - x_2X_2; Z_3z_3
$\underline{A}$ — B, $\underline{C}$, D = xyz — Xyz, XyZ, xyZ
$\underline{D}\underline{A}$—CB = XYz - xyz — xYz - Xyz
= $X_{2,3}x_{2,3}$; $Y_{2,3}y_{2,3}$ - x_2X_2; Y_3y_3
$\underline{A}$ — B, C, $\underline{D}$ = xyz — Xyz, xYz, XYz
$\underline{B}\underline{A}$—$\underline{D}\underline{C}$ = xYZ - xyz — XYz - XyZ
= $Y_{2,3}y_{2,3}$; $Z_{2,3}z_{2,3}$ - Y_2y_2; z_1Z_1
$\underline{C}\underline{A}$—$\underline{D}\underline{B}$ = XyZ - xyz — XYz - xYZ
= $X_{2,3}x_{2,3}$; $Z_{2,3}z_{2,3}$ - X_2x_2; z_1Z_1
$\underline{D}\underline{A}$—$\underline{C}\underline{B}$ = XYz - xyz — XyZ - xYZ
= $X_{2,3}x_{2,3}$; $Y_{2,3}y_{2,3}$ -X_2x_2; y_1Y_1
$\underline{A}$ — $\underline{B}$, $\underline{C}$, $\underline{D}$ = xyz — xYZ, XyZ, XYz

100 times, in 10 different kinds of 10 times each, difference is attributed to complement of s:

BA—$\underline{D}\underline{C}$ = Xyz - XYZ — XYz - XyZ
= $y_{2,1}Y_{2,1}$; $z_{2,1}Z_{2,1}$ — Y_2y_2; z_1Z_1
A — B, $\underline{C}$, $\underline{D}$ = XYZ — Xyz, XyZ, XYz
CA—$\underline{D}\underline{B}$ = xYz - XYZ — XYz - xYZ
= $x_{2,1}X_{2,1}$; $z_{2,1}Z_{2,1}$ — X_2x_2; z_1Z_1

A — $\underline{B}$, C, $\underline{D}$ = XYZ — xYZ, xYz, XYz
DA—$\underline{C}\underline{B}$ = xyZ - XYZ — XyZ - xYZ
= $x_{2,1}X_{2,1}$; $y_{2,1}Y_{2,1}$ — X_2x_2; y_1Y_1
A — $\underline{B}$, $\underline{C}$, D = XYZ — xYZ, XyZ, xyZ
BA—DC = Xyz - XYZ — xyZ - xYz
= $y_{2,1}Y_{2,1}$; $z_{2,1}Z_{2,1}$ — y_2Y_2; Z_3z_3
CA—DB = xYz - XYZ — xyZ - Xyz
= $x_{2,1}X_{2,1}$; $z_{2,1}Z_{2,1}$ — x_2X_2; Z_3z_3
DA—CB = xyZ - XYZ — xYz - Xyz
= $x_{2,1}X_{2,1}$; $y_{2,1}Y_{2,1}$ — x_2X_2; Y_3y_3
A — B, C, D = XYZ — Xyz, xYz, xyZ

s causes difference for:

$\underline{B}\underline{A}$—DC = xYZ - xyz — xyZ - xYz
xz ; xy xy owing to xz
$\underline{C}\underline{A}$—DB = xYz - xyz — xyZ - Xyz
yz ; xy xy owing to yz
$\underline{D}\underline{A}$—CB = XYz - xyz — xYz - Xyz
yz ; xz xz owing to yz
$\underline{B}\underline{A}$—$\underline{D}\underline{C}$ = xYZ - xyz — XYz - XyZ
xz ; xy - - owing to xz; xy
$\underline{C}\underline{A}$—$\underline{D}\underline{B}$ = XyZ - xyz — XYz - xYZ
yz ; xy - - owing to yz; xy
$\underline{D}\underline{A}$—$\underline{C}\underline{B}$ = XYz - xyz — XyZ - xYZ
yz ; xz - - owing to yz; xz
$\underline{A}$ — $\underline{B}$ = xyz — Xyz
xy;xz;yz yz owing to xy; xz
$\underline{A}$ — $\underline{C}$ = xyz — xYz
xy;xz;yz xz owing to xy; yz
$\underline{A}$ — $\underline{D}$ = xyz — xyZ
xy;xz;yz xy owing to xz; yz

$\underline{A}$ — $\underline{B}$, $\underline{C}$, $\underline{D}$ = xyz — xYZ, XyZ, XYz
xy;xz;yz - - - owing to xy; xz; yz

The complements of s cause difference for:

BA—$\underline{D}\underline{C}$ = Xyz - XYZ — XYz - XyZ
XZ ; XY XY owing to XZ
CA—$\underline{D}\underline{B}$ = xYz - XYZ — XYz - xYZ
YZ ; XY XY owing to YZ
DA—$\underline{C}\underline{B}$ = xyZ - XYZ — XyZ - xYZ
XZ ; YZ XZ owing to YZ
BA—DC = Xyz - XYZ — xyZ - xYz
XZ ; XY - owing to XZ; XY
CA—DB = xYz - XYZ — xyZ - Xyz
YZ ; XY - owing to YZ; XY
DA—CB = xyZ - XYZ — xYz - Xyz
XZ ; YZ - owing to XZ; YZ

A — $\underline{B}$ = XYZ — xYZ
XY;XZ;YZ YZ owing to XY; XZ
A — $\underline{C}$ = XYZ — XyZ
XY;XZ;YZ XZ owing to XY; YZ
A — $\underline{D}$ = XYZ — XYz
XY;XZ;YZ XY owing to XZ; YZ
A — B, C, D = XYZ — Xyz, xYz, xyZ
XY;XZ;YZ - - - owing to XY; XZ, YZ

The cycles are:

XyZ - xYZ CB

X_2x_2; y_1Y_1

followed by

XYz - xyz DA

$X_{2,3}x_{2,3}$; $Y_{2,3}y_{2,3}$

followed by

xYz - Xyz CB

x_2X_2; Y_3y_3

followed by

xyZ - XYZ DA

$x_{2,1}X_{2,1}$; $y_{2,1}Y_{2,1}$

followed by

XyZ - xYZ CB

X_2x_2; y_1Y_1

and so on

XYz - xYZ DB

X_2x_2; z_1Z_1

followed by

xyZ - xyz CA

$X_{2,3}x_{2,3}$; $Z_{2,3}z_{2,3}$

followed by

xyZ - Xyz DB

x_2X_2; Z_3z_3

followed by

xYz - XYZ CA

$x_{2,1}X_{2,1}$; $z_{2,1}Z_{2,1}$

followed by

XYz - xYZ $\underline{DB}$

X_2x_2; z_1Z_1 and so on

XYz - XyZ $\underline{DC}$

Y_2y_2; z_1Z_1

followed by

xYZ - xyz $\underline{BA}$

$Y_{2,3}y_{2,3}$; $Z_{2,3}z_{2,3}$

followed by

xyZ - xYz DC

y_2Y_2; Z_3z_3

followed by

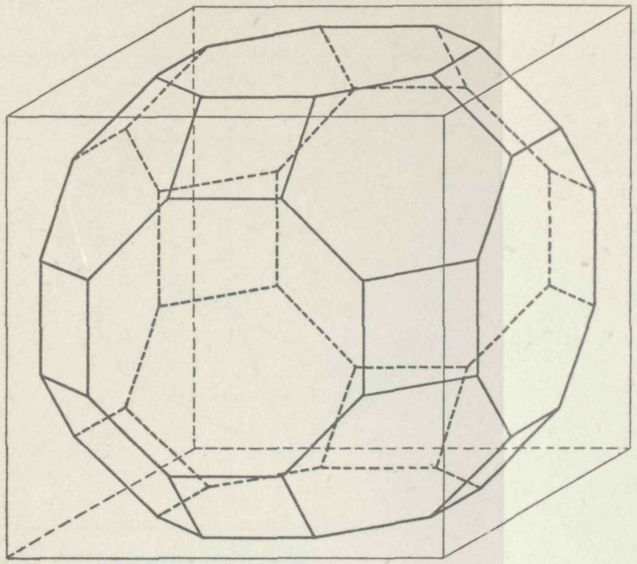
Xyz - XYZ DA

$y_{2,1}Y_{2,1}$; $z_{2,1}Z_{2,1}$

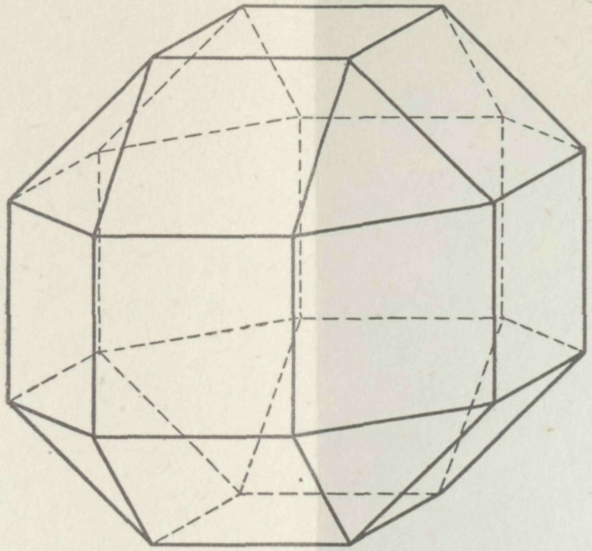
followed by

XYz - XyZ $\underline{DC}$

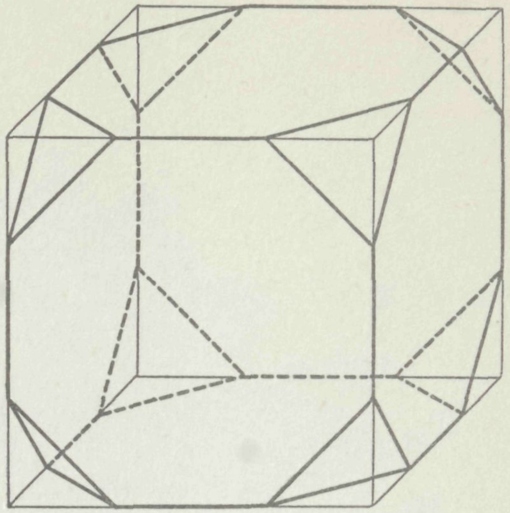
Y_2y_2; z_1Z_1 and so on



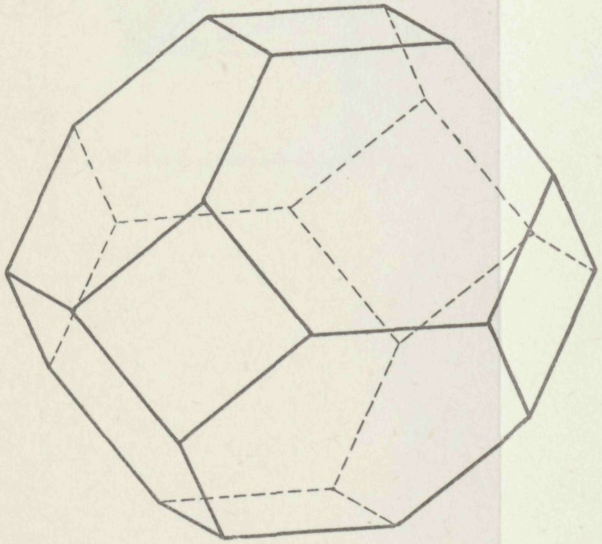
Great rhombicuboctahedron



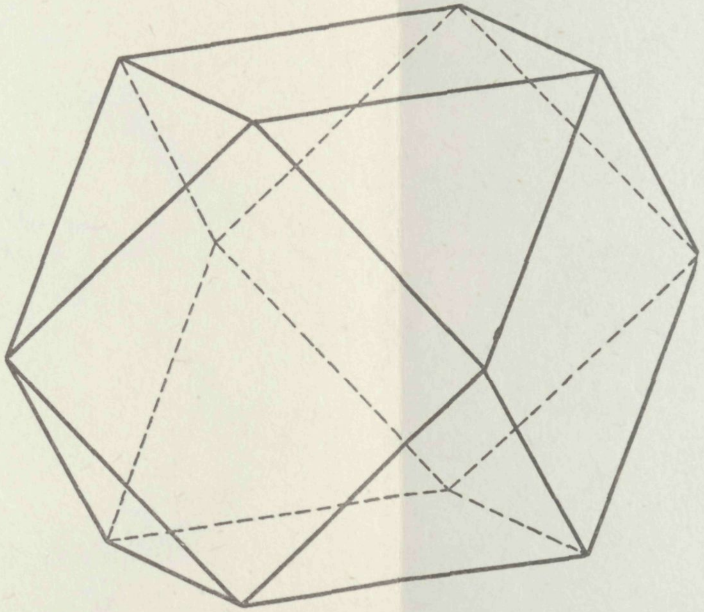
Small rhombicuboctahedron



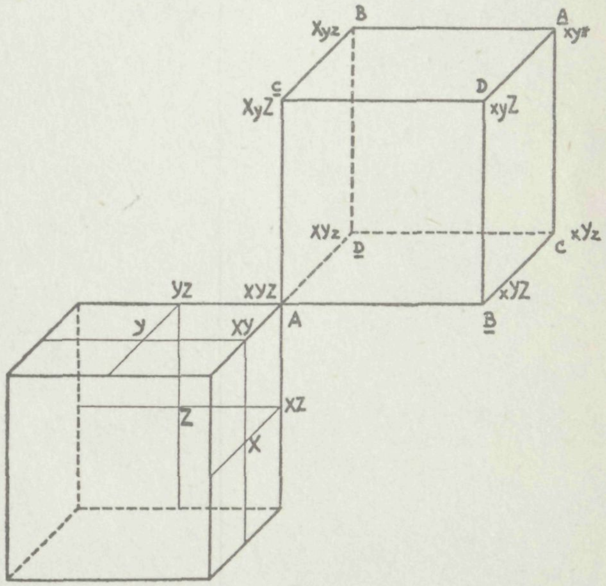
Truncated cube



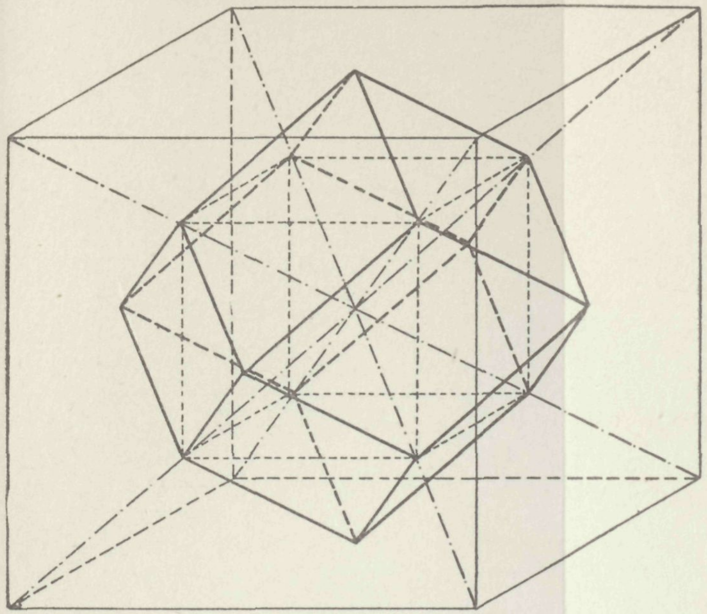
Truncated octahedron



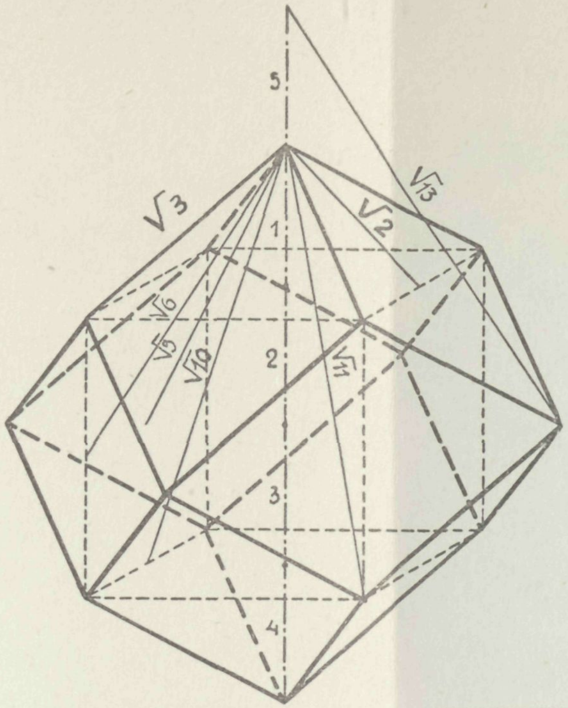
Cuboctahedron



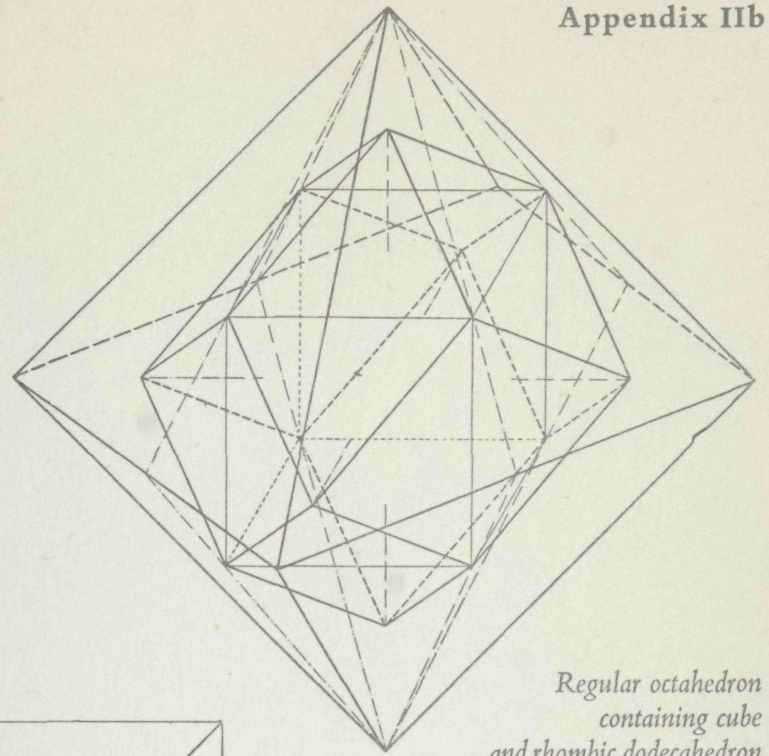
Drawings by J. J. J. PIJNENBURG



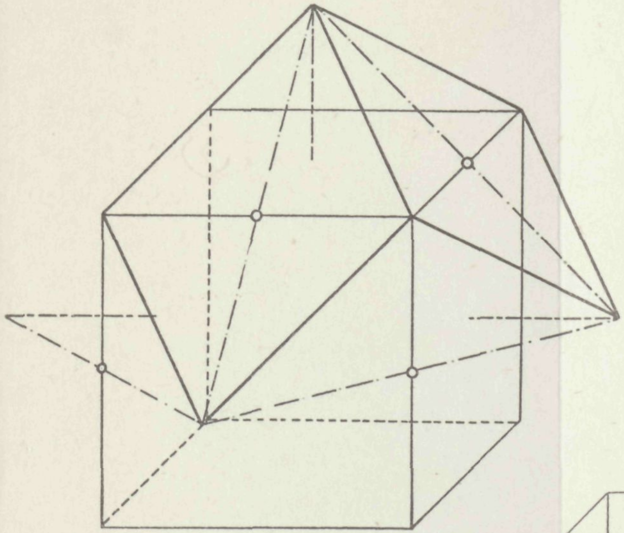
Rhombic dodecahedron with cubes

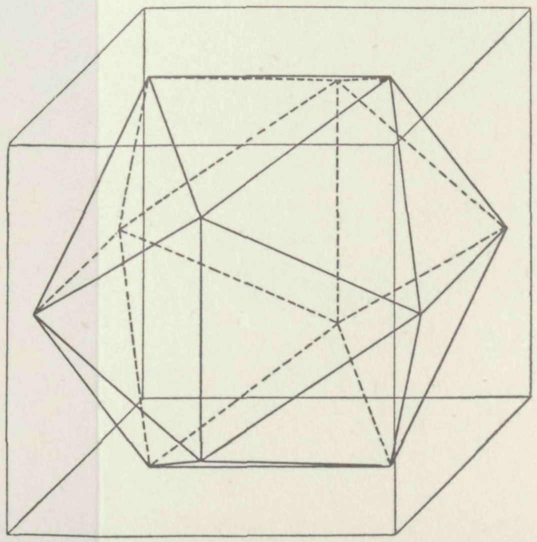


Rhombic dodecahedron

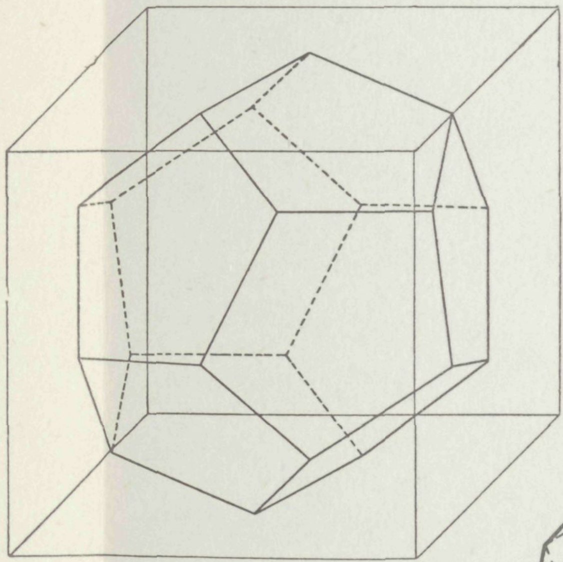


Regular octahedron containing cube and rhombic dodecahedron

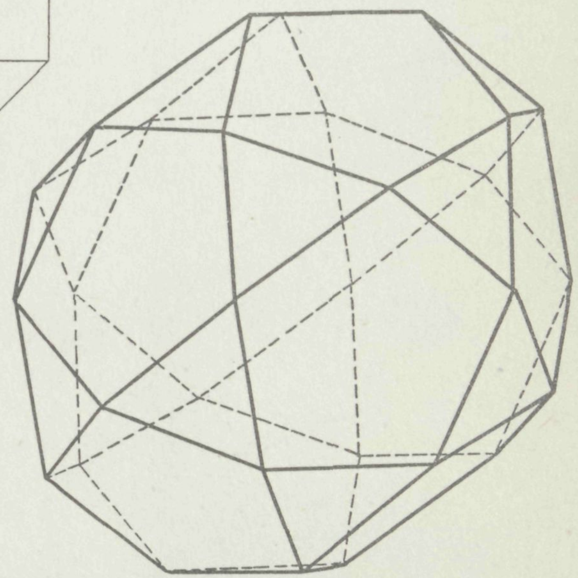




Regular icosahedron



Regular dodecahedron



Icosidodecahedron

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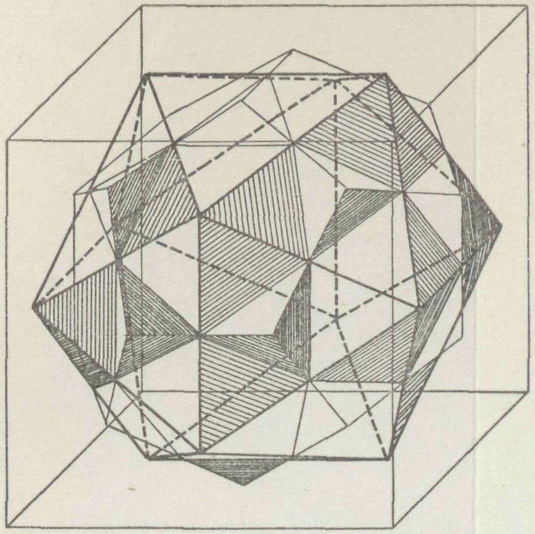
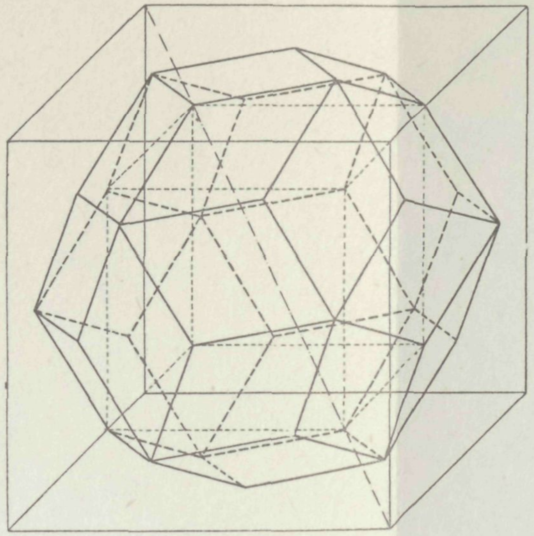
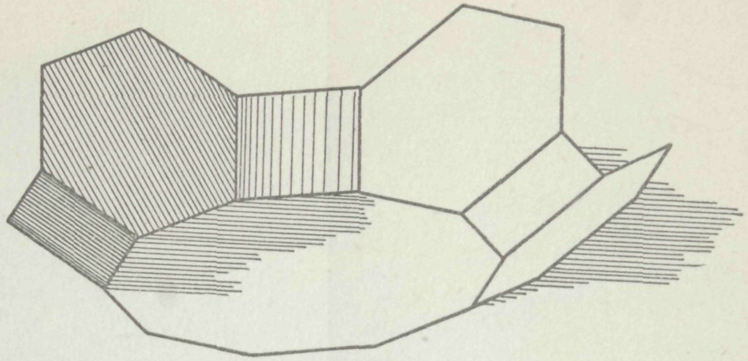


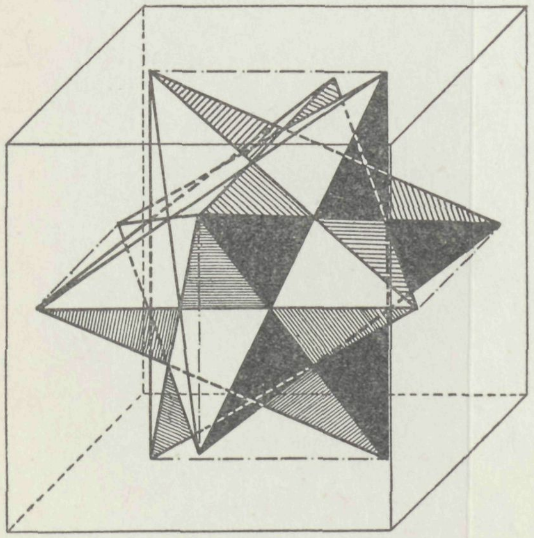
Regular icosa- and dodecahedron within cube. Perpendicular equipartition of edges



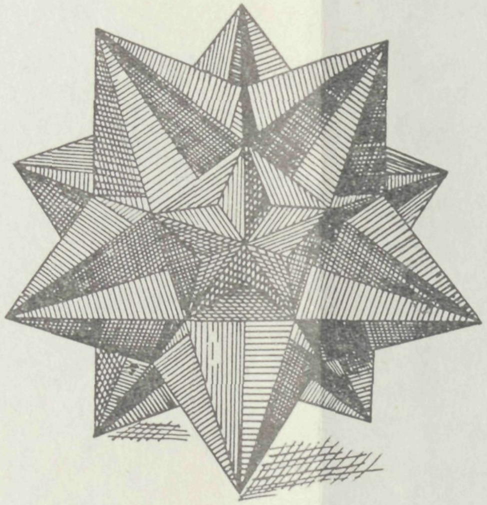
Rhombic triacontahedron



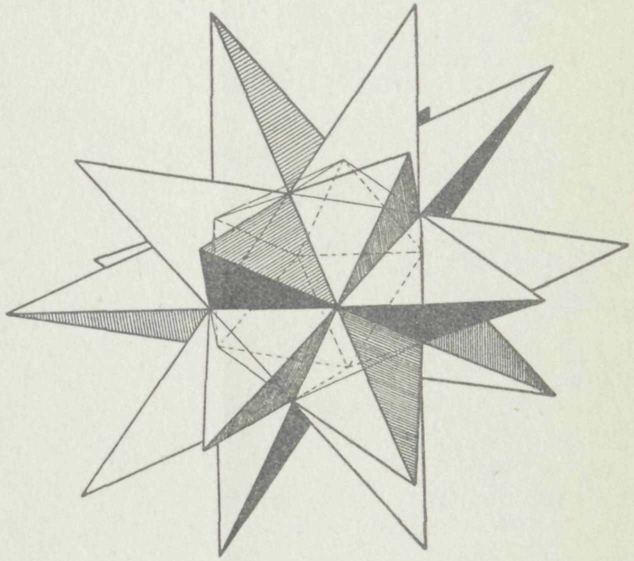
Part of great rhombicosidodecahedron



Small stellated dodecahedron



Great stellated icosahedron



Great stellated dodecahedron

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