1. Applying Mathematics to Nature*

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1.1 Introduction: Mathematization?

> Philosophy is written in this grand book, the universe, which stands continually open to our gaze. But the book cannot be understood unless one first learns to comprehend the language and read the letters in which it is composed. It is written in the language of mathematics, and its characters are triangles, circles, and other geometric figures without which it is humanly impossible to understand a single word of it; without these, one wanders about in a dark labyrinth.[1]

Galileo's famous quote is often taken to express one of the central characteristics of the Scientific Revolution: the ideal of a thoroughgoing mathematization of (the study of) nature. Yet it is easy to misconstrue the meaning of Galileo's metaphor by ignoring the specific context in which it was introduced. Galileo only characterized mathematics as a language to mock the Aristotelians' presumed dependence on human books as the ultimate authority—if there is an authoritative book that is to be read, it can only be nature itself, and its language cannot be of human origin.[2] One must also reconstruct the specifically mathematical background that made it appealing for a thinker like Galileo to invoke these striking images in this polemical context before reading strong metaphysical commitments into the metaphor.[3]

Stories about the Scientific Revolution usually focus on how ideas about physics and the natural world were profoundly modified in the early modern period, but they often silently assume an ahistorical notion of mathematics that hides the way in which the category of

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[1] (Galilei 1957, 237–38)

[2] See (Biagioli 2003) for an analysis of the polemical background to Galileo's use of the metaphor. See also (Palmerino 2016).

[3] See (Hatfield 1990).

mathematics itself was also being reshaped in the process. There are different, though not unrelated ways in which the historicity of mathematics can be taken into account. One important line of corrections to earlier stories has involved paying close attention to the constraints imposed by the use of particular mathematical techniques and norms that are no longer visible from the perspective of present-day mathematics.[4] Another question concerns the notion of application itself: what did it mean to "apply" mathematics to the empirical world? This chapter will explore a way to understand the issue of application from an early modern mathematical perspective, and will use this analysis to offer a fresh perspective on the idea of the mathematization of physics.[5]

Unpacking the notion of application implies that we should try to understand how one learned to "recognize" Galileo's circles and triangles in the empirical world. Many historical analyses have drawn attention to the importance of the Aristotelian notion of subalternate or middle sciences.[6] These sciences were supposed to result from a two-way operation. First, pure mathematics is established by abstracting away all natural attributes from physical substances, so that only their quantitative attributes remain. Second, mathematical truths ascertained in this abstract setting can be predicated of natural phenomena by superadding specific physical attributes to them, so that, e.g., in the case of optics, we are no longer dealing with straight lines, but with straight lines in light. This results in the following picture of application. Circles and triangles are typically seen as the abstracted shapes of physical objects; mathematical truths established about them can be applied back to the empirical world if we know on natural philosophical grounds that specific phenomena exhibit some mathematical attributes. E.g., if we know on natural philosophical grounds that light moves in straight lines, we can use mathematical knowledge to demonstrate properties of optical phenomena.

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[4] See the chapter by Guicciardini in this volume.

[5] (Roux 2010) warns against monolithic views on a development that actually had many forms. This chapter will identify one particularly important form, without the presumption that this would exhaust the field of options. (Gorham, et al. 2016) is a recent volume that treats the issue of mathematization from a variety of angles.

[6] (Machamer 1978; Lennox 1986) were influential in linking Galileo's work to this notion. (Laird 1983; Laird 1987) analyzes scholastic discussions. (Distelzweig 2013) offers a concise treatment of Aristotle's own treatment. See also (Mueller 1990) for different interpretations of Aristotelian abstraction.

Rather than start from this philosophically motivated picture, Section 2 will sketch how sixteenth-century mathematicians approached the mathematization of often very mundane empirical situations through a set of concrete practices. Focusing on application as a practical issue will bring two related aspects to light: the relation between the empirical world and the mathematical domain was primarily mediated by an operation of construction rather than abstraction; and since this has important implications for the representational role of mathematical diagrams, it will also allow us to better see how mathematicians could hope to characterize an open-ended list of empirical phenomena in geometrical terms.

Galileo's metaphor invites a further question. Reading a book involves more than recognizing its characters; it also requires identifying narrative structures that allow the reader to make sense of the words and sentences formed. But what was the appropriate reading strategy for the book of nature—to what genre did it belong? As we will see, the mathematization of the study of nature did not just depend on the replacement of one language with another, it was also (and maybe primarily) a shift in reading strategy—one that focused on problem-solving, as was suggested by the practice of mathematics. Section 3 will try to gauge the impact that an explicitly mathematical way of approaching the empirical world could have on philosophical ideals and goals.

Section 2 will deliberately avoid connecting mathematical practice with philosophical debates of the period. This is in the first place a methodological choice to better bring out the specific nature of the practical operations that were central in applying mathematics to empirical phenomena, and that often get obscured by philosophical preoccupations. Both the thesis that Platonic metaphysics was the key to understanding Galileo's mathematization of nature and the thesis that this grew naturally out of Aristotelian developments have been defended with vigor in historical scholarship.[7] The use of these philosophical categories to frame the question of mathematization tends to divert attention from the crucial focus of mathematical practice itself: problem-solving through construction. There were other sixteenth century philosophical developments that were closer in spirit to this practice, such as the Ramist movement and the rediscovery of Proclus's philosophy of mathematics.[8] These reflections on what it meant to engage in mathematical reasoning were definitely an important part of the

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[7] (Koyré 1978) is the classic reference for the Platonic interpretation. (Wallace 1984) has been an influential plea for the Aristotelian reading.

[8] For Ramus, see (Goulding 2010; Pantin 2019). For Proclus, see (De Pace 1993; Claessens 2009).

background against which sense could be made of the idea to extend mathematics beyond its traditional domain to topics that had traditionally belonged to natural philosophy; a background that also included reconfigurations of elements from Platonic and Aristotelian philosophy. But it is important to lay bare the aspects of the mathematical practice itself that allowed such extension and that gave a particular direction to the resulting options for reconceiving natural philosophy. This is the angle from which the topic will be analyzed here.

1.2 The Practice of Mathematics in the Sixteenth Century

Let us begin with some triangles and circles. In 1571, the English mathematician Thomas Digges published *Pantometria,* a text on practical geometry written by his father, Leonard, before his death in 1559. It contained an entirely traditional treatment of the measurement of lengths, surfaces, and volumes. Consider the twenty-third chapter of the first book, which explains how to determine an unknown distance with the use of a geometrical square. What is the distance between locations A and B (in Figure 1)? The surveyor first uses his geometrical square (seen in operation at C) to construct a right angle upon the line of sight AB, and then picks out an arbitrary third location C lying somewhere along the perpendicular line. By directly measuring the distance between A and C and the angle ACB, the distance AB (and CB) can be readily determined.

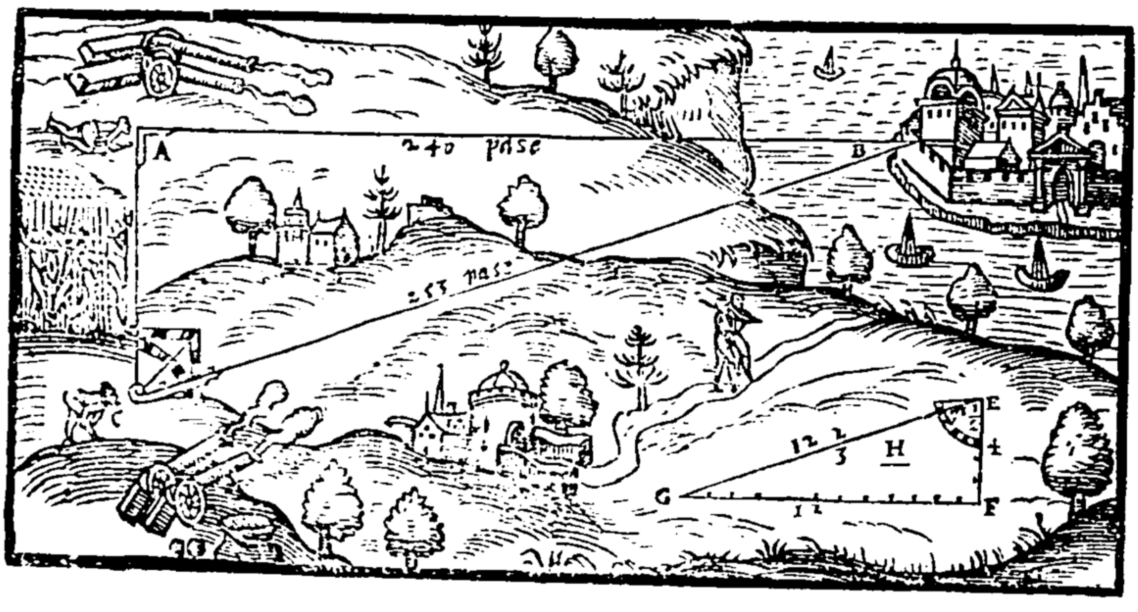

Figure 1. Determining an unknown distance.[9]

[9] (Digges and Digges 1571, ch. 23)

The first thing to be noticed about this procedure is that the triangle *ABC* and the circular arc that measures the angle are not simply "abstracted" from the situation. They cut across all empirically given shapes and boundaries in the landscape. Instead, the triangle is actively constructed by the instrumentally mediated operations of the surveyor, who is pictured in different poses that highlight his active engagement. In fact, the second triangle in the lower-right corner of the figure is one that the surveyor is supposed to materially construct on a piece of paper using ruler and compass. By inspecting this scale drawing, he can superimpose the geometrically established relationships on the landscape.

So what is the triangle in the figure doing if it is not representing any concrete shape? It solves a practical problem by showing how different measurable quantities are systematically related to each other. If you know any three quantities through direct measurement, you can immediately determine a fourth, unknown quantity. One can think of the triangle as geometrically encoding this relation; constructing the geometrical figure is a way of manipulating the accessible information. Seen from this perspective, "applying" mathematics to empirical situations is a matter of constructing geometrical diagrams to solve problems.

Digges's triangle, of course, provides only an elementary example. Sixteenth-century mathematicians spent considerable time and energy on developing mathematical instruments that could perform more complicated tasks, such as sundials (often of amazingly intricate design), quadrants, and astrolabes.[10] Most of these instruments can be understood as operating along similar lines. They gave their user the means both to effect a direct measurement (e.g., of the sun's height above the horizon) and to extract more information from its result (such as the time of the day). The background knowledge required for this latter step was engraved on the instrument and often required further manipulations by its user, who moved parts that helped translate the inputted measurement into the output value. Circles and triangles were only a part of the repertoire of figures engraved on the instruments, which encoded, e.g., the sun's altitude at each time of the day for all different dates of the year at a given latitude.

The geometrical engravings and moving parts of the more complicated instruments played a role very closely related to that of what was known as a *theorica.*[11] The term originated in astronomy, where a *theorica* was a geometrical model that allowed one to track the observed motion of a planet (which in turn could be embodied in a material instrument called an

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[10] See (Bennett 2003) for an important analysis of the function of mathematical instruments.

[11] See (Bennett 2003, 142–43; Johnston 2004).

equatorium), but it was also applied in other fields that used geometrical constructions to determine relations between measurable quantities. Their status as models can be called "instrumental" in a more substantial sense than the one often intended in traditional narratives concerning the history of astronomy. They usually were not supposed to be straightforward "realistic" depictions of how things are, but neither were they to be judged merely by their predictive successes. Their predictive role could have been taken over by purely arithmetical or tabular methods, but their geometrical character was crucial since it allowed further conceptual and practical exploration of systematic dependencies. To put it differently, geometrical constructions were essential tools for both thinking about and actually effecting new measurements. They allowed users to immediately see the relations between a wide range of possible observations (e.g., between latitudes, dates, and times), and this spurred mathematicians to develop ever new ways of representing some of these relationships and embody them in new material instruments.[12]

There was an important continuity between "practical" and "pure" mathematics. This is illustrated in the treatise on Platonic solids that Thomas Digges appended to the edition of his father's text. This exercise in pure mathematics can again be understood as an exploration of systematic dependencies, explicitly intended to solve geometrical problems. Digges's Euclidean-style geometry was similarly grounded in constructive activity, even if not necessarily carried out materially, but typically supposed to take place in the imagination (based on "postulated" constructions of straight line and circles).[13] As Digges's preface made clear, the main difference he saw between his own pure and his father's applied mathematics was merely the goal to which the exploration of dependencies was put—in the one case the satisfaction of pure curiosity, in the other the achievement of practical ends.[14]

Given this continuity, Euclid's *Elements* could be seen (and was often explicitly presented) as a toolbox for practical mathematics. The attempt to extend problem-solving techniques to a field of practical operations was often described as the search for a "reduction to

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[12] See (Bennett 2012; Kremer 2016) for examples.

[13] To avoid possible misunderstanding: this claim does not rule out mathematical realism, as I am describing the kind of cognitive activity underlying mathematical thinking, not the ontological status of presumed mathematical entities. On the central place of construction in early modern geometry, see the classic study (Bos 2001).

[14] See (Johnston 2006) for further analysis.

art."[15] Euclidean geometry itself can be considered a successful exemplar of this ideal. By identifying a few elementary operations (drawing a straight line and a circle), and investigating their mutual relationships, Greek mathematicians had been able to show how any spatial measurement problem could, in principle, be solved by a limited number of constructive steps. A general understanding of the nature of these constructions allowed any practitioner to quickly and efficiently reach his goals, in a way that guaranteed the correctness of his results—turning the practice of measurement into a true art. But, if one wanted to move beyond the domain of spatial measurement, one needed further guidance to effect the appropriate constructions. In the case of astronomy, e.g., one had to assume that specific geometrical constructions could encode the properties of astronomical motion that were of interest. Here, I briefly consider two other domains in which sixteenth-century mathematicians were actively seeking ways to extend their problem-solving techniques.

First, let us consider mechanics, the field of practical operations involving the weighing and moving of heavy bodies. The publication of the Latin translation of Archimedes' treatise on the *Equilibrium of Plane Figures* in 1544 was an important event, but it is important to stress that this text remained completely silent on its relevance to any practical challenges. This relation was only elaborated in Guidobaldo del Monte's *Mechanicorum Liber* in 1577 and Simon Stevin's *Weeghconst* (*The Art of Weighing*) and *Weeghdaet* (*The Practice of Weighing*) in 1586.[16] Stevin's treatise was explicitly introduced as the first successful reduction to art of mechanics (a relation that was also signaled by the titles of its parts). Guidobaldo's was modeled on the presentation of mechanics in the eighth book of Pappus's *Mathematical Collection* (originally written in the fourth century AD, and printed in Latin translation in 1588 under Guidobaldo's supervision), which had characterized mechanics as devoted to the solution of the general problem, "to move a given weight with a given force" (*datum pondus data potentia movere*). Both Guidobaldo and Stevin showed how this problem could be constructively solved by building a Euclidean-style framework that identified as the elementary operation the suspension of any body in indifferent equilibrium, the possibility of which could be expressed by ascribing to each body a unique center of gravity (given as the first postulate of Guidobaldo's work and as the first practical operation of Stevin's *Weeghdaet*—in both cases occupying the place of the Euclidean construction of a straight line). This operation allowed the solution of the general problem, since it entailed the proportional law of the lever and, in the case of Stevin, that of the inclined plane.

[15] See (Vérin 2008) for an essential introduction.

[16] See (Bertoloni Meli 2006; Van Dyck 2006; Van Dyck forthcoming).

To show how these laws could solve concretely given challenges, the treatises again offered sustained exploration of the systematic dependencies they implied, in this case between given weights, forces, and their geometrical dispositions. The elementary operation thus allowed the mathematician to construct—conceptually and materially—different kinds of systems that were guaranteed to be in equilibrium, and which could be put into motion by adding a small amount of force. (In theory, the smallest amount should suffice; in practice, friction and other impediments had to be overcome, requiring a force that varied with material circumstances.)

The second example is a field in which mathematicians had the same ambitions as in mechanics, but not the same success. The inclusion of cannons in the figures illustrating Digges's *Pantometria* (as in Figure 1) was not accidental, since the surveying exercises described played an important role in warfare. This naturally raised the question whether, given that the distance to a target could be geometrically determined, one could also determine how to aim the cannon to guarantee accuracy. In the second edition of the *Pantometria*, published in 1591, Thomas Digges included a short new treatise devoted to this question, following up on the 1579 *Stratioticos*, a text on military science, again partly based on his father's work, which had also included ballistics.[17] He repeatedly formulated his ambition as the reduction of the proportional relations characterizing shots to "a *Theorike* certain," but he also had to admit that he had not yet been able to "reduce that art to such perfection" as could content him.[18] The admitted imperfection of his own work did not stop Digges from criticizing his predecessors, though, central among which was the Italian mathematician Niccolò Tartaglia.

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[17] See (Johnston 1994, ch. 2; France 2014, ch. 2).

[18] (Digges and Digges 1591, 169, 191)

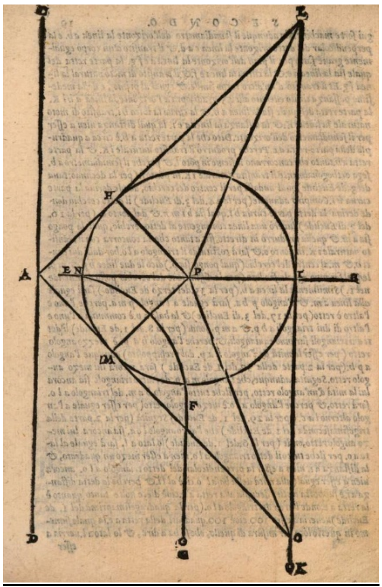

Figure 2. Tartaglia's *theorica*.[19]

Tartaglia's *theorica* combined a "violent" straight-line motion along the direction in which the cannon is aimed; a circular middle part resulting from the modification of the straight motion by the bullet's weight; and a "natural" motion straight downward at the end, when only the body's weight remains operative. (See Figure 2 for the complete diagram, which also encodes more fine-grained information concerning the distances reached.) Modern commentators often express surprise at Tartaglia's neglect of the curvature at the beginning of the motion, but Digges was happy to ignore this, as well—as a *theorica*, the geometrical construction represented empirically measurable properties of the shots (mainly range in relation to angle), it need not offer a realistic depiction of the shape (although that could be one way to guarantee correct results).[20] Digges's main criticism had to do with the circular part. The *Stratioticos* suggested that this could be rather a conic section, which would change from elliptical to parabolical to hyperbolical, depending on the angle of projection. In the second edition of the *Pantometria*, he assumed it is a "helical" line, constructed in a way similar to Archimedes' spiral. (We can understand this along the following lines, although Digges is not clear: the "natural" motion downwards continually changes the direction of the "violent" motion, resulting in a continuous rotation of its line of motion, yet without changing the violent speed that needs to be superposed on the rotation.)[21] The complexity of this construction he likened to

[19] (Tartaglia 1537, bk. II, prop. 9)

[20] See (Büttner, et al. 2003).

[21] This possible reconstruction of Digges's intention is missed in (Johnston 1994; France 2014).

that of astronomical epicycles, eccentric deferents, and equant points. Digges clearly thought that the complexity of the problem required him to use tools beyond the strictly Euclidean toolbox taken from other works of antique geometry, such as Apollonius's *Conics* or Archimedes' *On Spirals*.

Digges was convinced that Tartaglia's *theorica* had to be misguided since it wrongly implied that the angle of maximum range should be 45 degrees, which was "an Error knowne even to the first Practitioners."[22] With hindsight it is of course tempting to see this as a mistake, but Digges was actually right in claiming that in practice shots reached their maximum range at an angle somewhere between 40 and 45 degrees—and he saw the task of the mathematician as finding a geometrical construction that could fit every bit of relevant practical knowledge, which also led him to call for further practical experiments.[23] While both authors appealed to the terminology of "natural" and "violent" motion, philosophical ideas played no direct role in Digges's criticism of Tartaglia. The distinction between the kinds of motion could be grounded in experience, and it was intuitively attractive to conceptualize the trajectory as "somehow" composed out of their combination. But the operationalization of this idea was almost completely guided by the combination of the practical knowledge at hand and the geometrical tools available, rather than by explicitly philosophical ideas— a pattern that would be often repeated in the century to follow.

1.3 Mathematizing Physics in the First Half of the Seventeenth Century

1.3.1 Mathematizing Physics/Physicalizing Mathematics

The preceding has suggested geometry was not necessarily understood as a descriptive theory depicting an "abstract" reality, and that many mathematicians would more naturally see it as an art that allowed one to solve diverse problems. Petrus Ramus captured this spirit when he defined geometry as "the art of measuring well."[24] The introduction of quantitative measure in other empirical domains could be achieved by similarly reducing them to art. The lavish, elaborate instruments (often richly and symbolically decorated) that came out of the mathematicians' workshops and were eagerly collected by the powerful stand as testimony to

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[22] (Digges and Digges 1590, 358)

[23] See (Büttner 2017) for an important analysis of the complicated relation to practical knowledge of early modern ballistics.

[24] (Ramus 1569, 1)

the fascination that could be triggered by this practice.[25] Its presence and cultural status would have an important impact on ideals about how to do natural philosophy, but this impact could take many forms. Let us start with the most familiar case: astronomy.

It is no accident Stevin and Digges were among the first committed Copernicans.[26] The main advantages of the Copernican system could be considered "instrumental" in the sense discussed above. Even if it was predictively equivalent to the family of Ptolemaic models (given the observational possibilities of the time), it encoded more systematic dependencies between the planetary motions in one geometrical model of the world-system, and this allowed the determination of relative planetary distances that had been unmeasurable.[27] The crucial step that these mathematicians took was to treat this superior instrumentality as a criterion of truth. Both Digges and Stevin were themselves too much and too proudly mathematicians to care much about the implications this had on natural philosophy, both with respect to determining legitimate criteria of truth, and with respect to the consequences a moving earth would have on ideas about motion and causality.[28] But both kinds of question would be taken up by others, and not only with respect to astronomy.

To start with the second implication, the Copernican case can be seen as an occasion for what has been usefully called the "physicalization of mathematics."[29] All accepted mathematical theories (including Euclidean geometry as a theory of spatial measurement) depended on some explicit or implicit assumptions about empirical properties of bodies (e.g., the circular motion of planets), which could be taken up in relation to what were traditionally considered to be natural philosophical questions—but which now had to be investigated with the constraint imposed by the independently assumed truth of the mathematical theories. This implied a reversal with

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[25] See, e.g., (Korey 2007) for a description and analysis of one such collection.

[26] See (Johnston 1994) for Digges and (Vermij 2002, ch. 4) for Stevin.

[27] See (Evans 1998, 410–13) for a clear and concise presentation. See also the chapter by Omodeo and Regier in this volume.

[28] As with the other authors treated below, the diverse factors that could explain their different stances will not be discussed here; but it is striking to what extent Digges and Stevin for the most part confidently ignored philosophy as a meaningful practice. It is also interesting to note that both mathematicians strongly believed that Copernicus could be improved upon by gathering further observations.

[29] See (Schuster 2012; Schuster 2013).

respect to what was traditionally understood as subalternation, where mathematics was only used to derive further consequences that followed if one could assume on prior natural philosophical grounds that some empirical phenomenon could be mathematically characterized.[30] A telling example of the opposite move of "physicalization" is Descartes's youthful interpretation of Stevin's hydrostatic paradox.[31] Stevin had offered a mathematical treatment of the pressures exerted on the base of a vessel, in which the only properties that mattered were the weight of the fluid and the geometrical properties of the container. Descartes literally tried to fill out Stevin's geometrical diagrams by offering a corpuscular theory of matter in which pressure was interpreted as arising out of the particles' "tendency to motion" propagated through the medium. Most importantly, the laws guiding this propagation were conceived such that the mathematically established result could be recovered. To put it differently: natural philosophical notions were now *guided* by mathematically established truth.

This brings us to the first implication. My analysis has suggested that the establishment of mathematical truth depended on criteria that were directly related to superior problem-solving ability. The introduction of this kind of norm into natural philosophy should perhaps be considered the primary meaning of what we can call the "mathematization of physics." Understood this way, mathematization was a process in which (practical) mathematical construction took over (at least part of) the role of the Aristotelian scheme for causal explanation. This process need not have started from prior metaphysical beliefs about the ultimate nature of reality, as Koyré famously contended, but it was rather naturally suggested by the successes and broader appeal of mathematical practice, which allowed it to serve as a new ideal for knowledge more broadly.[32] This move had two immediate consequences: it invited the

[30] See (Laird 1987; Distelzweig 2013). (Laird 1997) already noted that the debates on subalternation could not have been of much interest to someone like Galileo, since they simply assumed what would have been the most pressing issue for him—how to characterize empirical phenomena in mathematical terms in the first place. See also (Biener 2004; Van Dyck 2013).

[31] See (Schuster 2012). Other examples could easily be added. To name just some of the most influential ones: Kepler's work on optics (Dupré 2012) and Gilbert's on magnetism (Bennett 2003; Johnston 2004). The philosophical elaboration of the mathematical theories' presuppositions needed not necessarily have revolutionary ambitions. Guidobaldo del Monte, e.g., spent some time on showing how the science of mechanics (and more specifically the central role of bodies' centers of gravity) could be given a place within a broadly Aristotelian conception of the world (Van Dyck 2006; Van Dyck 2013).

[32] This was of course only "rather naturally suggested" for authors and readers with a specific background, while many other thinkers could only consider it an obvious metaphysical blunder. The important point

reinterpretation of questions traditionally belonging to the domain of natural philosophy as new problems to be constructively solved, greatly expanding the domain of mathematical treatments (often necessitating the development of new mathematical tools); and it occasioned the development of philosophical discourses aimed at legitimating this reinterpretation (which could, but need not appeal to explicitly metaphysical arguments). This latter step was often closely related to the physicalization of mathematics, as the discourses could offer conceptual resources for interpreting the assumptions underlying the constructions as natural philosophical principles.

These characterizations are of course very schematic, and need to be fleshed out by showing how they allow us to make sense of the work of different authors—not only what they have in common, but also bringing to light the different options made possible by the shared ideal of articulating a new physics based on the mathematical arts. The next sections will very briefly discuss three historically influential examples, without claiming that this would exhaust the field of options.

1.3.2 Galileo: Mathematizing the Phenomena

Soon after his appointment as professor of mathematics at the University of Pisa in 1589, the young Galileo composed a treatise on natural philosophy (commonly called *De Motu Antiquiora*), which he never published but which survives in a few manuscript drafts.[33] The text stridently opposed the method that Galileo claimed to have learned from "his mathematicians" to that used by the traditional philosophers.[34] The result can somewhat provocatively be characterized as an attempted reduction to art of the philosophical science of local motion. One of the manuscript notes consists of a list of some of the most important questions treated in the debate on motion among Pisan philosophers,[35] and we can read the treatise as trying to show that these could all

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remains that metaphysical pictures can be challenged from perspectives other than those provided by articulated alternatives. See (Hatfield 1990).

[33] See (Fredette 2001).

[34] (Galilei 1890-1909, 1:285)

[35] (Galilei 1890-1909, 1:418)

be constructively solved using Archimedean mathematical tools.[36] Hydrostatics offered Galileo a scheme of systematic dependencies between weights and volumes of bodies and of the medium in which they move that allowed him to determine under which circumstances bodies move up or down. In a crucial extrapolation, he suggested that these dependencies could also solve a further problem (the formulation of which brings to mind Pappus's central problem of mechanics): "to give the speed of motion given the weights of a body and the medium" (*data gravitate mobilis et medii, datur velocitas motus*).[37] His suggestion was to set the velocity equal to the hydrostatic force, which could already be determined. We can see Galileo tentatively attempting a construction based on the models already at his disposal, again not dissimilar from the way that Tartaglia and Digges were exploring possible *theoricae* for projectile motion.

The preceding description could misleadingly suggest that Galileo's text was presented as a treatise in mathematics, but the mathematical constructions actually formed the backbone for an exercise in "physicalization." Galileo used the constructions to rethink the crucial natural philosophical concepts implicated in the formulation of the original questions, which also guaranteed that the constructive solutions could be presented as "answers" to these philosophical questions. The opening of the treatise accordingly tied the meaning of "heaviness" and "lightness" to conditions of measurement—in a way that allowed the problems formulated in terms of heaviness to be in principle solvable. This, in turn, suggested some options to reconceptualize the notions of "natural" and "violent" motion, options that were progressively worked out in the different drafts, and which also lead to the suggestion to categorize horizontal motion as neither natural nor violent but "neutral."[38]

In the decade following this first attempt, Galileo would reshape both his mathematical constructions and the accompanying philosophical interpretations, based on new empirical findings triggered by some of the problems treated in his early treatise. In a first experiment, Galileo found that the trajectory of a projectile is approximately parabolical.[39] In a second

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[36] See (Camerota and Helbing 2000) for the Pisan context. It is very suggestive to compare Galileo's list with the comparable list of more practically oriented questions concerning ballistics that Thomas Digges had included in his *Stratioticos* as a preliminary step to the reduction to art.

[37] (Galilei 1890-1909, 1:418)

[38] (Galilei 1890-1909, 1:300)

[39] See (Renn, et al. 2000) for the experiment and its relation to the theory of *De Motu Antiquiora*. It is important to notice that Galileo could find this out by taking the question out of the realm of ballistics

experiment, he found that a simple pendulum is approximately isochronous. The relevance of this latter phenomenon was probably brought to his attention by his analysis of motion on inclined planes, already included in a chapter of *De Motu Antiquiora*, and it further invited the search for a mathematical construction of this empirical finding based on an approximation of circular motion by motion over a sequence of inclined planes. The latter search could be further guided by the results of a third experiment, in which Galileo famously established the times-squared law of free fall by rolling balls down inclined planes.[40]

This last finding had at least two important consequences. First, it set a new fundamental problem to be solved: how to geometrically model accelerated motion? This would bring Galileo into uncharted mathematical terrain, since the toolbox of antique mathematics could not provide him with the instruments he needed to represent the systematic relations between distances, speeds, and times he was in the process of uncovering.[41] Second, it invited a further philosophical reconceptualization of motion. Galileo now had to find a way to present accelerated motion as natural, and this gave rise to an even more far-reaching consequence once related to the first experimental finding. If distances were as the squares of times in free fall, then the parabolic shape of the trajectory of a projectile could be directly constructed from the composition of free fall with a horizontal component that was uniform. Assuming the latter allowed Galileo to answer a question that had already arisen in *De Motu Antiquiora*, but which had remained unanswered: whether a "neutral" motion once started would persist or end.[42] The positive answer to this question would form the kernel of a new approach to the analysis of motion, as it can be seen to play a role related to what would become the inertial principle. (It also allowed Galileo to partly meet the challenge of physicalizing the Copernican hypothesis.)[43]

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proper: rather than experimenting with actual artillery (as Digges claimed to have done), Galileo and his collaborator Guidobaldo del Monte simply threw an inked ball along a plane that was tilted with respect to the horizon and inspected the trajectory drawn by the ball itself. Arguably, it is only the natural philosophical context (in which bodies in violent motion were typically assumed to be thrown) that could have suggested the relevance of this kind of situation for the more general question. See (Büttner 2017) for the very limited relevance of Galileo's finding for the practice of artillery.

[40] See (Wisan 1974; Büttner 2019).

[41] See (Damerow, et al. 2004) for an analysis of Galileo's attempts to come to terms with this problem.

[42] See (Van Dyck 2018) for further analysis.

[43] See (Roux 2006) for a careful treatment of the pre-history of the inertial principle.

We again find the same move: a natural philosophical concept was given empirical content by tying its meaning to the requirements of mathematical construction. But Galileo was well aware that his conceptual apparatus was still incomplete, since he was not able to fully identify the concepts that could play the role that the center of gravity had played for mechanics, expressing the relevant empirical property that could satisfactorily ground the mathematical superstructure.[44]

When Galileo presented the final results of his investigations in *Two New Sciences* in 1638, the constructions were explicitly presented as forming an elaborate and self-contained mathematical treatise on motion in Latin. This was still embedded in a broader philosophical discourse, in the form of an Italian dialogue, but the nature of the discourse had shifted significantly compared to *De Motu Antiquiora*. It was no longer intimately tied to a philosophical agenda that had been set by Aristotelian natural philosophy. Motion as the treatise's topic was simply presented as an "ancient subject" about which much had been written.[45] Some of the discussions of *De Motu Antiquiora* were taken up again, but they were integrated into a more exploratory discussion of many different topics that all shared a relation to mathematical problems, an approach that was explicitly justified by a reference to "the richness joined with great liberality of nature"[46] that allowed ever new discoveries to be made—a process for which mathematics with its constructive capacity for infinite invention based on the study of systematic dependencies was eminently suited.[47]

The discovery of mathematical regularities characterizing acceleration and pendular motion had brought Galileo to a position that saw mathematizability of phenomena as the main criterion for inclusion in natural philosophy. Philosophical enquiry became an open-ended search into the richness of nature, with no *a priori* guarantee that everything could be treated mathematically—but with the norms of success straightforwardly defined by mathematical practice, where the material instrumental infrastructure of the latter now included pendula and inclined planes besides quadrants and balances.

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[44] See (Wisan 1974; Damerow, et al. 2004; Büttner 2019) for the unfinished nature of Galileo's mathematical framework for dealing with the phenomena of motion.

[45] (Galilei 1890-1909, 8:190, 266–67)

[46] (Galilei 1890-1909, 8:140)

[47] (Galilei 1890-1909, 8:267)

1.3.3 Descartes: Mathematizing the World

While Galileo's endeavor started from an approach to mathematics informed by the eighth book of Pappus's *Mathematical Collection* (as further developed by Guidobaldo del Monte), Descartes's wider program can be seen to start from Pappus's fourth and seventh books, with their discussions of the appropriate means of geometrical construction and the analytic method in mathematics. As has been well documented, the young Descartes was fascinated by the prospects of an absolutely general problem-solving scheme involving a central place for mathematical instruments.[48] Significantly, he expected this scheme to be directly relevant to natural philosophical problems, which should thus be constructively solved.

In the decade after the first letters and notes (from 1619), Descartes kept returning to mathematical and natural philosophical problems. His work on optics was initially in line with other approaches to applied mathematics at the time. By analyzing the systematic dependencies encoded in a geometrical model that had been constructed taking into account empirical observations, Descartes was able to establish the law of refraction.[49] At the same time, he was keen on reinterpreting these constructions in natural philosophical terms as "following" from hypotheses concerning the nature of light (whereas, in a familiar move, these hypotheses could only be operationalized by first assuming the geometrical constructions). This physicalizing step was further foregrounded and given an explicitly metaphysical foundation by the end of the 1620s, when Descartes started developing metaphysical ideas implying a limitation on possible modes of constructing explanations of all natural phenomena.[50] Interestingly, this was paralleled by a similar development in his work on pure mathematics, where he also came to a position implying a principled distinction between legitimate and illegitimate modes of constructing curves for solving geometrical problems that went beyond the strictly Euclidean canon.[51]

Descartes's focus on absolute universality set his work clearly apart from that of Galileo. At the latest by 1633, when he finished work on *Le Monde*, Descartes believed he had been able to metaphysically determine a fixed basis of operations that would suffice for the constructive

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[48] See (Bos 2001; Sasaki 2003) for a general introduction. See also the chapter by Guicciardini in this volume.

[49] See (Schuster 2012; Heeffer 2017) for possible reconstructions.

[50] See (Garber 1992) for a classic study of Descartes's "metaphysical physics."

[51] See (Bos 2001) and (Domski 2009), which focusses on the parallel role of motion in both natural philosophy and pure mathematics.

explanation of everything that happened in nature, and this allowed him to resolutely take his distance from more piecemeal empirical and mathematical exploration as the guide towards the selection of both the mathematical basis and the phenomena to be constructed. This especially comes to light in Descartes's critical reaction to Galileo's law of fall.[52] Descartes was convinced that this mathematical regularity could only be considered an approximate characterization, valid under limited and contingent circumstances and with no wider relevance, since it could not be squared with a construction of acceleration as due to consecutive pushes by rotating subtle matter that should explain gravity (an empirical property of bodies simply assumed by Galileo). Prioritizing a metaphysical norm that had initially been suggested by mathematical practice, Descartes was now overruling a mathematical characterization of phenomena as not part of proper physics. The metaphysical nature of matter as pure extension and of change as local motion, with collision the only form of interaction, constrained all possible models, rather than independently established mathematical models constraining philosophical pictures of nature. The empirical world had been made co-extensive with the subject-matter of mathematics, but in the same move the practice of physics had become metaphysical.

1.3.4 Mersenne: Mathematizing the Scientific Community

It is no coincidence that Descartes's comments on Galileo's work had been solicited by Marin Mersenne. The French polymath's personal and correspondence network played a crucial role in constituting the beginning of an institutional space for the new approaches in mathematics and philosophy mainly being developed outside the universities.[53] Beside Descartes, it included mathematicians and philosophers like Isaac Beeckman, Pierre Gassendi, Gilles Person de Roberval, Etienne and Blaise Pascal, Pierre de Fermat, Claude Mydorge, Girard Desargues, Jean de Beaugrand, Jean-François Niceron, Honoré Fabri, Thomas Hobbes, Giovanni Battista Baliani, Bonaventura Cavalieri, Evangelista Toricelli, and Christiaan Huygens, offering an amazing cross-section of the mid-century European mathematical landscape. Through this network, application of mathematics started to become a more communal process. Mersenne had seen that constructing a community could be integral to the construction of solutions to particular and general problems.

Mersenne's letters and publications (among which were translations and summaries of Galileo) were important in reporting ideas, techniques, results, and empirical data. They allowed

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[52] See (Palmerino 1999, 282–95) for discussion.

[53] For the characterization of the network as an institution, see (Goldstein 2013). See also (Grosslight).

the construction of a compendium of physico-mathematical problems (to use a term favored by Mersenne) deemed important in the period: free fall and motion on inclined planes, pendular motion (including that of rigid bodies), ballistics, simple machines, music, optics and catoptrics, hydrostatics and pneumatics (with an important place for the alleged existence of a void), hydrodynamics, the force of percussion, the strength of materials, etc.[54] Mersenne's role was much more active than that of a mere informational nexus, though. He continually challenged his correspondents with new problems, and as the exchange with Descartes on Galileo's work illustrates, these challenges typically also included requests to critically assess the work of other mathematicians. Mersenne's own relatively uncommitted stance on many matters made him ideally suited to play the role of mediator (and vice versa—the way he saw his role probably drove his stance).[55]

The community constructed by Mersenne was not held together by common ideas about the mathematization of physics or by widely accepted results. The application of mathematics to new problems forced everyone involved to rethink the proper place of experience and philosophical interpretation or supplementation, and to try out new mathematical techniques that could in turn be contested. Almost nobody agreed on the answers, but everyone recognized what was at stake—and it is primarily around this recognition that the fledgling disciplinary community was being built.

1.4 Conclusion: Physics as Problem-Solving

When Isaac Newton published his *Mathematical Principles of Natural Philosophy* in 1687, he could straightforwardly claim in his Preface that "the whole difficulty of philosophy seems to be to discover the forces of nature from the phenomena of motions and then to demonstrate the other phenomena from these forces."[56] The fundamental natural philosophical challenge had become a problem (the formulation of which again distantly recalls Pappus) that was to be mathematically solved. Galileo's metaphor had depicted the universe not only as a book but also as a labyrinth, Newton proposed his new concept of force as Ariadne's thread with which to construct clear paths that made nature legible. A new ideal of knowledge was taking hold

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[54] For a discussion of most of these problems see (Bertoloni Meli 2006), a work on the history of seventeenth-century mechanics that seems to have been written very much in Mersenne's spirit.

[55] On Mersenne's stance, see (Dear 1988; Garber 2004). This did not mean that Mersenne did not take position on important issues. See (Palmerino 2010).

[56] (Newton 1999, 382)

according to which knowing nature implied being in a position to gradually find out more by constructively exploiting the information at one's disposal and making it fruitful for further exploration.

The practice of "mathematical physics" (as it would become known) also triggered important developments in pure mathematics that extended and transformed Descartes's analysis of the conditions under which geometrical problems could be solved. By the end of the century, the calculus had established itself as a general analytic scheme that could, in turn, be justified by its use in solving important physical problems.[57] Solving differential equations would become the core of a whole new approach to the study of systematic dependencies that far outstripped what could be expressed by geometrical proportions. This created a mathematical setting where many of the problems already raised by Mersenne, such as the rotation of rigid bodies, the vibrations of a taut string, or the behavior of jets of waters, found novel solutions that moved physics decidedly away from Newtonian methods and concepts.[58] Yet how to philosophically interpret these later successes remained an open problem.

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[57] See (Mahoney 1990) for an analysis of the "bootstrapping" process through which physical and mathematical problem-solving mutually shaped and supported each other.

[58] See the chapters by Stan and Biener and Hepburn in this volume.

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