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John Maynard Keynes and Ludwig von Mises on Probability

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I. Introduction

The complex issues relating to the interpretation and meaning of different concepts of probability and to the legitimate scope of their useful application in the social sciences and in economics belong to the more controversial topics within the sub-field of economic methodology. Several of the most influential economists have expounded outspoken views about the matter. Thus it is probably no exaggeration to assert that John Maynard Keynes's second-best-known book—after his *The General Theory of Employment, Interest, and Money*—is his *A Treatise on Probability*. Ludwig von Mises's views about probability have been no less influential within the context of the Austrian School and even beyond. In this respect some commentators have claimed that Ludwig von Mises basically embraced the frequency interpretation of probability of his brother Richard von Mises,[1] thus

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[1] See, for instance, Hoppe (2006), who assumes that Ludwig von Mises is a representative of the frequency interpretation of probability. Whether or not this author's views on probability are defensible, it is not quite correct to impute these same views to Ludwig von Mises.

Moreover we are unable to detect any essential or exclusive connection between Keynes's economics and Keynes's views on probability; therefore a rejection of Keynesian economics—see e.g., Hoppe (1992)—need not entail a rejection of Keynes's views on probability. Attempts to forge a supposedly essential connection between a particular philosophical suggesting that Ludwig von Mises's views on probability are no less antagonistic to those of John Maynard Keynes than his views on economic theory and public policy. This latter view will here be challenged. While it is not contended that any historical evidence points to any direct historical influence between the views on probability of these two authors, it will be argued that in some relevant respects Ludwig von Mises's views with respect to the meaning and interpretation of probability exhibit a closer conceptual affinity with the views of John Maynard Keynes about probability than with the views concerning probability of his brother Richard von Mises.

As regards the views about probability of Ludwig von Mises, it is undeniably true that these display considerable nuance and that they can be considered as being of a *sui generis* variety. Even if Ludwig von Mises's views on probability exhibit a closer conceptual affinity with Keynes's philosophy of probability than with the frequency interpretation espoused by his brother Richard von Mises, an important difference between the views of Ludwig von Mises and those of John Maynard Keynes in this respect will nevertheless be acknowledged.

II. THE *SUMMA DIVISIO* IN THE PHILOSOPHY OF PROBABILITY: EPISTEMIC VERSUS OBJECTIVE INTERPRETATIONS OF PROBABILITY

Interpretations of probability are commonly divided into (1) *epistemological* (or *epistemic*) and (2) *objective*. Epistemological interpretations of probability take probability to be concerned with the knowledge (or belief) of human beings. On this approach, any

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(ideological) or economic Worldview on the one hand and a particular interpretation of probability on the other, are not new.

Thus, as is also pointed out in Lad (1983), the objective interpretation of probability seems to have been rather influential in Marxist-Leninist philosophy and in Soviet thought under the influence of the mode of thinking of the Russian probabilist B.V. Gnedenko, who wrote about the subjective characterization of probability that "(t)he final outcome of consistently using such a purely subjectivistic interpretation of probability is inevitably subjective idealism" (2005[1962], 25; also quoted in Lad [1983, 286]). Against this interpretation, Lad (1983) argues that an operational subjective construction à la de Finetti is free of Gnedenko's charges and fits Marxist philosophical presuppositions better.

probability assignment describes a degree of knowledge, a degree of rational belief, a degree of belief, or something of this sort. The approaches of both Ludwig von Mises and John Maynard Keynes belong to this category. Objective interpretations of probability, by contrast, take probability to be a feature of the objective material world, which has nothing to do with human knowledge or belief. The theory of Richard von Mises belongs to this category.[2]

Despite the fact that Ludwig von Mises himself clearly embraced what must be considered an epistemic view regarding the interpretation of probability, the objectivist view has been propounded by several Austrian economists, especially among those belonging to the praxeological camp. These authors apparently take it for granted that Ludwig von Mises had simply adopted the philosophy of probability of his brother Richard von Mises. Thus in a characteristic passage of *Man, Economy, and State* M.N. Rothbard wrote:

> "The contrast between risk and uncertainty has been brilliantly analyzed by Ludwig von Mises. Mises has shown that they can be subsumed under the more general categories of "class probability" and "case probability." "Class probability" is the only scientific use of the term "probability," and is the only form of probability subject to numerical expression."[3]

In the two footnotes accompanying this passage M.N. Rothbard refers both to Ludwig von Mises's discussion in *Human Action*, and to Richard von Mises's *Probability, Statistics, and Truth*, thus conflating the views of the two brothers.[4]

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[2] The logical, subjective and intersubjective interpretations are all epistemological. The frequency and propensity interpretations are objective. For a survey and discussion of the different interpretations, see Gillies (2000).

[3] Rothbard (2004, 553).

[4] Rothbard's interpretation is questionable for at least two reasons. First, Ludwig von Mises embraces an epistemic interpretation of his concept of numerical class probability whereas Richard von Mises's interpretation of the concept of frequency probability is objective. Second, whereas for Richard von Mises there is indeed only one scientific use of the term probability, from the perspective of Ludwig von Mises *both* the concept of class probability *and* the concept of case probability are scientifically legitimate. See further. For other references by Prof. Rothbard to Richard von Mises's theory, see in particular Rothbard (1997), 24n, 24-27, 122n, 229n.

Views like the ones expressed by M.N. Rothbard are often, if not always, accompanied, and rather consistently, by a rejection of quantitative methods for the conduct of applied research in economics. Again M.N. Rothbard tells the story of how he came to decide to leave the world of statistics in rather dramatic terms:

> "After taking all the undergraduate courses in statistics, I enrolled in a graduate course in mathematical statistics at Columbia with the eminent Harold Hotelling, one of the founders of modern mathematical economics. After listening to several lectures of Hotelling, I experienced an epiphany: the sudden realization that the entire "science" of statistical inference rests on one crucial assumption, and that that assumption is utterly groundless. I walked out of the Hotelling course, and out of the world of statistics, never to return."[5]

According to Professor Rothbard the questionable assumption is the following:

> "In the science of statistics, the way we move from our known samples to the unknown population is to make one crucial assumption: that the samples will, in any and all cases, whether we are dealing with height or unemployment or who is going to vote for this or that candidate, be distributed around the population figure according to the so-called "normal curve."[6]

Statements like these have been both severely criticized and misunderstood. Thus David Ramsey Steele, in his review of Justin Raimondo's *An Enemy of the State: The Life of Murray N. Rothbard* writes:

> "If the young Rothbard really had found something that refuted all statistical theory, this would be a momentous discovery, and a great consolation to tobacco producers. But, 60 years on, the edifice of statistics has not registered any tremors.
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> In the Rothbard-Raimondo account, statisticians accept the bell curve because of a single example, the distribution of hits around the bull's eye on a target. In fact, statisticians don't view the bell curve as sacrosanct. Since a great many phenomena are, as a matter of fact, so close to normally distributed that the assumption of normal distribution will yield correct predictions, normal distribution can be treated as an empirical generalization and a useful instrument.
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> Alternatively, normal distribution can be strictly derived by the Central Limit Theorem, which shows that where some variable is influenced

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[5] Rothbard (1995, 38).

[6] Rothbard (1995, 38).

> by a large number of unrelated random variables, that variable will be normally distributed. This result holds subject to certain conditions, which are very widely, but not universally, encountered. Statisticians are open to the possibility of non-normal distributions where these conditions don't apply. It doesn't seem likely that Rothbard successfully debunked all of statistics around 1942."[7]

This interpretation of Rothbard's position is certainly questionable. It doesn't seem likely after all that Rothbard was intent upon questioning the *mathematical validity* of the Central Limit Theorem or of any other theorem of formal probability calculus. It may still remain true, however, that in contexts where random collectives do not exist (that is, contexts characterized by lack of independent repetitions), as will often be the case in economics, objective probabilities cannot be used. Given that Rothbard embraced an objective, frequency interpretation of numerical probability, his rejection of statistics is a defensible and logically consistent corollary. Moreover the rejection of the use of objective probabilities in economics is in agreement with the conclusions of some of the most recent research about these matters, and with general arguments for interpreting probabilities in economics as epistemological rather than objective.[8]

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[7] See Steele (2000). The Central Limit Theorem (in the classical sense) is the generic name of a class of theorems which give, in precise mathematical terms, conditions under which the distribution function of a suitably standardized sum of independent random variables is approximately normal. This theorem is one of the most remarkable results in all of mathematics. For an introduction to the Central Limit Theorem from a historical perspective, see also W. J. Adams (1974).

[8] See Gillies (2000, 187 ff.). The main reason why objective probabilities cannot be validly introduced in economics is not too difficult to grasp and can be related to the impossibility of introducing a satisfactory notion of independent repetitions of conditions and of random and homogeneous samples. In a typical experimental situation in physics, a sequence of independent repetitions of the experiment is perfectly possible. The experiment can be performed in the same laboratory on different days, or in different laboratories on the same day etc., and these repetitions will typically be independent. The conditions necessary for the introduction of objective probabilities are satisfied. It might seem as if there exists a certain structural similarity between a typical situation in economics and the typical experimental situation in physics. The two cases nevertheless

It is worth pointing out that for quite some time the objectivist view had also been rather influential in certain Marxist-Leninist circles. Whereas the objectivist view had indeed been dominant in statistical theory and practice throughout most of the previous century, it was in particular in certain Soviet writings that attempts had been made to provide the objectivist view with supposedly Marxist-Leninist philosophical underpinnings, and to dismiss the subjective characterization of probability as inevitably leading to subjective idealism.[9]

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differ in important respects. Could we not conceivably use observations of the behavior and performance of economic systems as samples of independent repetitions of conditions similar to the ones present in the typical experiment in physics? The different samples could be taken from either (1) data related to the same economic system at different times, or (2) data related to different economic systems at a similar stage of development. One author who recently re-examined these questions aptly summarizes his answer to this question as follows: "In the first case, if the samples refer to 'snapshots' of the economy which are too close together in time, it is hard to maintain that the more recent performance is not influenced by that of the previous periods; thus the independence of the samples cannot be maintained. If the samples relate to historical periods far enough from each other to render the assumption of independence plausible, one is unlikely to get homogeneous samples; thus invalidating the 'experiment'. In the second case the use of a sample of cross-section data would still not give independence as economic systems tend to be integrated in terms of trade and production, and particularly as the flow of information from one country is likely to affect the behavior of agents in others." See Gillies (2000, 192). This view with respect to the interpretation of probability is thus apparently dictated by the fundamentally different nature of the phenomena under study in the realm of human action, when compared with physical phenomena. Acting individuals in a market economy are very different from, say, the molecules of a gas. Since an economic system is composed of acting individuals, who have thoughts and beliefs, an independent repetition of any situation becomes difficult if not impossible.

[9] In this respect attention can be drawn to the influence of B.V. Gnedenko, author of the often revised and reprinted *Theory of Probability* containing an objective characterization of chance and at once the most complete statement of the Soviet Marxist understanding of probability. See also Footnote 1 above and the discussion in Lad (1983).

The critical issue we want to examine here, however, is whether the precepts of praxeological methodology and epistemology indeed entail an exclusive commitment to the objectivist viewpoint. An examination of Ludwig von Mises's viewpoint in this respect has not convinced us that this is actually the case.

In fact, and as mentioned briefly already, Ludwig von Mises's views with respect to the interpretation of probability, are more akin to Keynes's views than to the philosophy of probability of his brother Richard von Mises. In order to substantiate this view, we will compare Ludwig von Mises's position concerning this matter with the positions both of John Maynard Keynes and of Richard von Mises. The two main approaches to the interpretation of probability theory which will be considered here are thus the frequency interpretation, as developed systematically by Richard von Mises, and the logical interpretation, as developed systematically by John Maynard Keynes.[10]

In the third and fourth sections hereafter I present a general characterization of the views on probability of these two authors. In section V I argue that the thesis that Ludwig von Mises embraced the objective frequency interpretation of probability of his brother Richard von Mises is disputable in view of a number of Ludwig von Mises's own statements with respect to this subject matter.

In the sixth section I examine further whether and in what respects Ludwig von Mises's views on probability indeed exhibit a conceptual affinity with John Maynard Keynes's interpretation of probability. In the seventh section an important difference between the respective views about probability of Ludwig von Mises and of John Maynard Keynes is highlighted.

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[10] These correspond by and large—although not exactly—to Carnap's two concepts of probability: probability as used in logic (degree of confirmation) on the one hand, and probability as used in statistical and physical science (relative frequency), on the other. See Carnap (1945). Keynes's views on probability are contained in Keynes (2004 [1921]); for our analysis of Richard von Mises's views we will use Richard von Mises (1981 [1957]) and (1964).

III. RICHARD VON MISES'S OBJECTIVE APPROACH TO PROBABILITY: THE FREQUENCY INTERPRETATION

The principal goal of Richard von Mises was to make probability theory a science similar to other sciences. According to the frequency view probability theory is considered a science of the same order as, say, geometry or theoretical mechanics. He criticizes the view that probability can be derived from ignorance:

> "It has been asserted—and this is no overstatement—that whereas other sciences draw their conclusions from what we know, the science of probability derives its most important results from what we do not know."[11]

Probability should be based on facts, not their absence. The frequency theory relates a probability directly to the real world via the observed objective facts (or the data), in particular repetitive events.

As Richard von Mises wrote:

> "By means of the methods of abstraction and idealization (…) a system of basic concepts is created upon which a logical structure can then be erected. Owing to the original relation between the basic concepts and the observed primary phenomena, this theoretical structure permits us to draw conclusions concerning the world of reality."[12]

In the logical approach to be examined in the next section, probability theory is seen as a branch of logic, as an extension of deductive logic to the inductive case. In contrast to this view, the frequency approach sees probability theory as a mathematical science, such as mechanics, but dealing with a different range of observable phenomena. Probability should thus not be interpreted in an epistemological sense. It is not lack of knowledge (uncertainty) which provides the foundation of probability theory, but experience with large numbers of events.

A probability theory which does not introduce from the very beginning a connection between probability and relative frequency is not able to contribute anything to the study of

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[11] Richard von Mises (1981 [1957], 30).

[12] Richard von Mises (1981 [1957], v).

reality.[13] A key question raised by this view relates to how mathematical sciences relate to the empirical material with which they are concerned. Since Richard von Mises was an empiricist, the starting point for him was always some observable phenomenon such as an empirical collective. In fact, according to the random frequency definition it is possible to speak about probabilities only in reference to a properly defined *collective*. Probability has a real meaning only as probability in a given collective. The basis of Richard von Mises's theory of probability is thus the concept of a collective. The rational concept of probability, as opposed to probability as used in everyday speech, acquires a precise meaning only if the collective to which it applies is defined exactly in every case. Essentially a collective consists of a sequence of observations which can be continued indefinitely. Each observation ends with the recording of a certain attribute. The relative frequency with which a specified attribute occurs in the sequence of observations has a limiting value, which remains unchanged if a partial sequence is formed from the original one by an arbitrary place selection.[14]

To deal with such phenomena, we obtain by abstraction or idealization some mathematical concepts, such as, in this instance, the concept of mathematical collective. We next establish on the basis of observation some empirical laws which the phenomena under study obey. Then again by abstraction or idealization we

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[13] Richard von Mises (1981 [1957], 63).

[14] On the concept of collective, see also Mises (1964, 11-15). As explained further, a collective is a mass phenomenon or an unlimited sequence of observations fulfilling two conditions, the convergence condition and the randomness condition. According to Richard von Mises, many types of repeatable experiment generate collectives, or at any rate would do so if they could be continued indefinitely. The task of statistics is to identify which experiments have this collective-generating property and to elicit the associated probability distributions over their class of possible outcomes. The task of probability calculus in mathematical statistics consists in investigating whether a given system of statistical data forms a collective, or whether it can be reduced to collectives. Such a reduction provides a condensed, systematic description of the statistical data that may properly be considered an "explanation" of these data. See Richard von Mises (1981 [1957], 222).

obtain from these empirical laws the axioms of our mathematical theory. Once the mathematical theory has been set up in this way, we can deduce consequences from it by logic, and these provide predictions and explanations of further observable phenomena.

Applying this scheme to the case of probability theory, there are, according to Richard von Mises, two empirical laws which are observed to hold for empirical collectives. The first of these can be named the *Law of Stability of Statistical Frequencies*; it refers to the increasing stability of statistical frequencies and is designated by Richard von Mises as "the 'primary phenomenon' (*Urphänomen*) of the theory of probability."[15]

As Mises explains:

> "It is essential for the theory of probability that experience has shown that in the game of dice, as in all the other mass phenomena which we have mentioned, the relative frequencies of certain attributes become more and more stable as the number of observations is increased."[16]

The first law of empirical collectives was fairly well known before Richard von Mises. The second law, however, is original to him and it relates to a decisive feature of a collective. This feature of the empirical collective is its lack of order, that is, its *randomness*.

Richard von Mises's ingenious idea is that we should relate randomness to the failure of gambling systems.

As he wrote:

> "The authors of such systems have all, sooner or later, had the sad experience of finding out that no system is able to improve their chances of winning in the long run, i.e., to affect the relative frequencies with which different colours or numbers appear in a sequence selected from the total sequence of the game."[17]

In other words, not only do the relative frequencies stabilize around particular values, but these values remain the same if we choose, according to some rule, a subsequence of our original (finite) sequence. This second empirical law can be called the *Law of Excluded Gambling Systems*.

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[15] Richard von Mises (1981 [1957], 14). In fact, the expression "Stability of Statistical Frequencies" is Keynes'; see Keynes (2004 [1921], 336).

[16] Richard von Mises (1981 [1957], 12).

[17] Richard von Mises (1981 [1957], 25).

The next step in Richard von Mises's programme is to obtain the axioms of the mathematical theory by abstraction (or idealization) from these empirical laws. The first axiom can be easily obtained from the Law of Stability of Statistical Frequencies:

Axiom of convergence:

Let A be an arbitrary attribute of a collective C which is obtained *m* times in *n* trials, then

$\lim_{n \to \infty} m(A)/n$ exists. The probability of A in C [P(A/C)] is now defined as $\lim_{n \to \infty} m(A)/n$. This is the famous limiting frequency definition of probability.

One of the main objections to this theory is that it is too narrow, for there are many important situations where we use probability but in which nothing like an empirical collective can be defined. In particular this definition is too narrow in the context of economics. This was the viewpoint of important economists such as Ludwig von Mises, John Maynard Keynes and John Hicks.

Nevertheless Richard von Mises considers this alleged disadvantage to be a strong point in favour of his theory. We can, according to Richard von Mises, *start* with the imprecise concepts of ordinary language but when we are constructing a scientific theory we must replace these by more precise concepts. Thus we can of course start with the vague ordinary language concept of probability, but for scientific purposes it must be made precise by a definition. This is done by the limiting frequency definition of probability. This definition excludes some ordinary language uses of probability for which a collective cannot be defined, but this is no bad thing. On the contrary, it is positively beneficial to exclude some vague uses of probability which are unsuitable for mathematical treatment. Summing up this line of argument, he writes:

> " 'The probability of winning a battle,' for instance, has no place in our theory of probability, because we cannot think of a collective to which it belongs. The theory of probability cannot be applied to this problem any more than the physical concept of work can be applied to the calculation of the 'work' done by an actor in reciting his part in a play."[18]

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[18] See Richard von Mises (1981 [1957], 15). Regarding his positivist ideas Richard von Mises was much influenced by E. Mach whom he greatly admired. See Richard von Mises (ibid., 225) where he writes: "The point

The limiting frequency definition of probability is supposed to be an operational definition of a theoretical concept (probability) in terms of an observable concept (frequency). It could be claimed, however, that it fails to provide a connection between observation and theory because of the use of limits in an infinite sequence. It is well known that two sequences can agree at the first *n* places for any finite *n* however large and yet converge to quite different limits. A similar objection relates to the question of whether the representation of a finite empirical collective by an infinite mathematical collective is legitimate.

Richard von Mises's answer to this difficulty is that such representations of the finite by the infinite occur everywhere in mathematical physics, and that his aim is only to present probability theory in a fashion which is as rigorous as the rest of mathematical physics. In mechanics, for example, we have point particles to represent bodies with a size, infinitely thin lines to represent lines with a finite thickness, and so on. Richard von Mises argues that he is trying to present probability theory as a mathematical science like mechanics, but it is unreasonable to expect him to make it more rigorous than mechanics. As he wrote:

> "… the results of a theory based on the notion of the infinite collective can be applied to finite sequences of observations in a way which is not logically definable, but is nevertheless sufficiently exact in practice. The relation of theory to observation is in this case essentially the same as in all other physical sciences."[19]

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of view represented in this book corresponds essentially to Mach's ideas." See in this connection also Richard von Mises (1951, *passim*).

[19] See Richard von Mises (1981 [1957], 85). The practical difficulty arises from the fact that a collective is defined for an infinite sequence. A collective is an idealization. Strictly speaking, no relative-frequency probability statement says anything about any finite event, group of events or series. In other words, any calculated frequency is perfectly consistent with any probability attribution from zero to one. Combined with the injunction that there is no such thing as a probability of a "singular" event, it would appear that any definitive empirical attribution of numerical probabilities is a chimera. A statement about the limit of a sequence of trials hypothetically continued to infinity contains by itself absolutely no information about any initial segment of that sequence. Any initial segment of a collective—and we are, of course, only ever capable of observing initial segments—can be replaced with any arbitrary sequence

To complete Richard von Mises's programme, it must be examined how the second mathematical axiom—the axiom of randomness—can be obtained from the empirical *Law of Excluded Gambling Systems*. It turns out that the formulation of the axiom of randomness does involve some rather considerable mathematical difficulties. Even if these were eventually overcome, the quite

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of the same length without affecting any of the limits in the collective. Richard von Mises acknowledges that "[i]t might thus appear that our theory could never be tested experimentally." (ibid. 84) His probabilistic solution to this problem is a pragmatic one. The empirical validity of the theory does not depend on a logical solution, but is determined by a practical decision. This decision should be based on previous experience of successful applications of probability theory, where practical studies have shown that frequency limits are approached comparatively rapidly. Moreover the idealization of the collective is comparable with other well-known idealizations in science, such as the determination of a specific weight (perfect measurement being impossible), the existence of a point in Euclidean space, or the concept of velocity.

The velocity of an accelerating object at a moment in time is the ratio of the change in distance to the change in time, ds/dt. Supposing the motion is not uniform, as in the case of a freely falling body whose velocity increases as it falls, to obtain the velocity we calculate the "instantaneous rate of change" of the distance with respect to the time by taking the limit as follows:

$$\text{i.e., } v = \lim_{dt \Rightarrow 0} ds/dt.$$

It is impossible to verify that this limit exists. It does not follow, however, that the concept of velocity is non-operational. This criticism would duplicate the criticism of probability as the limit of a sequence, but it would not be considered a serious objection, because the definition of velocity as a limit has proven itself to be applicable to many different instances of motion, in just the same way the frequency theory has been successfully applied to many instances. The relation of theory to observation in the latter case is essentially the same as in all other physical sciences. It is reminded here that Ludwig von Mises's definition of class probability, which is discussed further, is finitist in the sense that it dispenses entirely with any reference to the concept of a limit. In that limited sense it can be considered that Ludwig von Mises's definition of class probability constitutes an improvement upon the definition of a collective offered by Richard von Mises.

subtle mathematical developments which finally gave Richard von Mises's theory a rigorous mathematical foundation, are of little relevance in the present context. The main idea is reminded here, however:

Randomness condition:

The fixed limits to which the relative frequencies of particular attributes within a collective tend are not affected by any place selection, that is, by choosing an infinite sub-sequence whose elements are a function of previous outcomes. That is, if we calculate the relative frequency of some attribute not in the original sequence, but in a partial set, selected according to some fixed rule, then we require that the relative frequency so calculated should tend to the same limit as it does in the original set. In this respect Richard von Mises made the following stipulation:

> "The only essential condition is that the question whether or not a certain member of the original sequence belongs to the selected partial sequence should be settled *independently of the result* of the corresponding observation, i.e., before anything is known about this result."[20]

An important implication of Richard von Mises's frequency theory is that, when dealing with unique events, statistical or stochastic methods will be essentially useless. Where collectives do not exist, probability theory and the calculations based on it will add nothing to our knowledge concerning the world of reality. Only where previous experience has established that events can be considered as belonging to a collective, can statistical methods play a role. The calculations of insurance companies for instance demonstrate that stochastic methods play a legitimate role in certain kinds of business decisions, namely when dealing with events belonging to a collective. The theory of probability starts with certain given frequencies and derives new ones by means of calculations carried out according to certain established rules. In other words, each probability calculation is based on the knowledge of certain relative frequencies in long sequences of observations,

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[20] See Richard von Mises (1981 [1957], 25). As indicated already, the fulfillment of the second condition, insensitivity to place selection, is also described by Richard von Mises as the Principle of the Impossibility of a (successful) Gambling System. (ibid.)

and its result is always the prediction of another relative frequency, which can be tested by a new sequence of observations. The task of the theory of probability is thus to derive new collectives and their distributions from given distributions in one or more initial collectives.[21]

Richard von Mises's limiting frequency definition of probability was clearly intended to limit the scope of the mathematical theory of probability, and, in fact, of the scientific concept of probability.[22] We can only, he claims, introduce probabilities in a scientific sense—which here also means: in a mathematical or

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[21] The derivation of a new collective from the initial ones consists in the application of one or several of the four fundamental operations of selection, mixing, partition and combination. See Richard von Mises (1981 [1957], Second Lecture; 1964, 15-35).

As regards the frequentist solution to the problem of inference given by Richard von Mises, it consists of a combination of the frequency concept of a collective with Bayes's theorem, a result known as the 'Second Law of Large Numbers'. (ibid. 125) Bayes's formula shows a relationship between prior and posterior probability functions. If knowledge of the prior distribution does exist, there is no conceptual problem with the application of Bayes's theorem. Often the prior probability function will not be known, however, and it is then an important part of probability theory to know what influence the prior probability function has in the calculation of the posterior distribution. In general the following will hold: no substantial inference can be drawn from a small number of observations if nothing is known *a priori*, that is, preliminary to the experiments, about the object of experimentation. If the prior distribution is not known, and the number of observations, say rolls of a die, is small, then the posterior distribution will not allow to draw any conclusions accurately. On the other hand, a large number of observations limits the importance of knowing the prior distribution. As long as the number of experiments is small, the influence of the initial distribution predominates; however, as the number of experiments increases, this influence decreases more and more.

Often the prior distribution will not be known. The actor will then have to guess at a distribution, sample the population, and then revise his guess according to Bayes's formula. This means that actions of an individual will also be guided by the accuracy of his or her guess.

[22] As he wrote: "Our probability theory has nothing to do with questions such as: `Is there a probability of Germany being at some time in the future involved in a war with Liberia?'" See Richard von Mises (1981 [1957], 9).

quantitative sense—where there is a large set of uniform events, and he urges us to observe his maxim: "First the collective—then the probability."[23, 24]

Despite controversy it can be expected that the frequency theory of probability will remain significant for the conduct of natural science.[25]

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[23] Richard von Mises (1981 [1957], 18)

[24] Richard von Mises thus advocated a monist view of probability, that is, he asserts that there is only one concept of probability that is of scientific importance, in contradistinction to his brother Ludwig von Mises who espoused a dualist view of probability.

[25] For recent testimony of this fact, see e.g., Khrennikov (1999). This author argues that certain problems in the foundations of quantum mechanics—such as the Einstein-Podolsky-Rosen paradox—are connected with the foundations of probability theory and thus have a purely mathematical origin. In particular, the pathological (or non-classical) behaviour of "quantum probabilities"—in particular Bell's inequality—is a consequence of the formal use of Kolmogorov's probability model. This author uses the ensemble and frequency interpretations as the two fundamental interpretations of probability and arrives at surprising results. Bell's inequality cannot be used as an argument for non-locality or non-reality. Historically, and although it has been argued that the philosophical background of subjective probability strongly resembles that underlying quantum mechanics (see Galavotti 1995), it is frequentism that became the "received view" of probability and seems to have been tacitly assumed also by the upholders of the Copenhagen interpretation of quantum mechanics (although the attribution of probabilities to the single case was generally admitted). In this context attention has often been drawn to Heisenberg's viewpoint according to which "(t)he probability function combines objective and subjective elements. It contains statements about possibilities or better tendencies ("potential" in Aristotelian philosophy), and these statements are completely objective, they do not depend on any observer; and it contains statements about our knowledge of the system, which of course are subjective in so far as they may be different for different observers. In ideal cases the subjective element in the probability function may be practically negligible as compared with the objective one. The physicists then speak of a 'pure case.'" See Heisenberg (1958 [1990], 41), and also the discussion in Galavotti (1995).

IV. John Maynard Keynes's Epistemic Approach to Probability: The Logical Interpretation

The logical interpretation of probability considers probability as the degree of a partial entailment. Keynes's *Treatise* is concerned with the general theory of arguments from premises leading to conclusions which are reasonable but not certain. Let e be the premises and h the conclusion of an argument. Keynes holds that the familiar relation 'e implies h' is the limiting case of a more general (probability) relation 'e partially implies h.' Keynes's aim in the *Treatise* is to systematize statements involving such relations of partial implication. The logical theory uses the word "probability" primarily in relation to the truth of sentences, or propositions.

It aims at assigning truth values other than zero or one to propositions. In this process, that part of our knowledge which we obtain directly, supplies the premises of that part which we obtain indirectly or by argument. From these premises we seek to justify some degree of rational belief about all sorts of conclusions. We do this by perceiving certain logical relations between the premises and the conclusions. The kind of rational belief which we *infer* in this manner is termed *probable* (or in the limit *certain*), and the logical relations, by the perception of which it is obtained, we term *relations of probability*.[26]

Comparisons are possible between two probabilities, only when they and certainty all lie on the same ordered series. Probabilities which are not of the same order cannot be compared. Only when numerical measurement of probabilities is possible, which is only occasionally possible and which is thus a matter for special enquiry in each case, algebraical operations such as addition and arithmetical multiplication, can be performed. The numbers zero and one figure as extreme cases. A probability of zero indicates impossibility, a probability equal to one indicates the truth of a proposition.

The idea of a logic of probability which should be the art of reasoning from inconclusive evidence was systematically developed by John Maynard Keynes although hints towards this

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[26] See Keynes (2004 [1921], 111). Keynes mostly takes the empiricist line that knowledge acquired by direct acquaintance constitutes true and certain knowledge. Knowledge by argument, in contrast, proceeds through direct knowledge of relations of the form 'e implies h' or 'e partially implies h'.

approach had been expressed at least since Leibniz. Keynes regards probability theory, like economics, as a branch of logic. Although Richard von Mises calls Keynes "a persistent subjectivist,"[27] Keynes makes it clear at the beginning of his book that his theory is, in an important sense, an objective one. For Keynes probability was degree of *rational* belief *not* simply degree of belief. The relevant passage is worth being quoted in its entirety:

> "The terms *certain* and *probable* describe the various degrees of rational belief about a proposition which different amounts of knowledge authorise us to entertain. All propositions are true or false, but the knowledge we have of them depends on our circumstances; and while it is often convenient to speak of propositions as certain or probable, this expresses strictly a relationship in which they stand to a *corpus* of knowledge, actual or hypothetical, and not a characteristic of the propositions in themselves. A proposition is capable at the same time of varying degrees of this relationship, depending upon the knowledge to which it is related, so that it is without significance to call a proposition probable unless we specify the knowledge to which we are relating it.
>
> To this extent, therefore, probability may be called subjective. But in the sense important to logic, probability is not subjective. It is not, that is to say, subject to human caprice. A proposition is not probable because we think it so. When once the facts are given which determine our knowledge, what is probable or improbable in these circumstances has been fixed objectively, and is independent of our opinion. The Theory of Probability is logical, therefore, because it is concerned with the degree of belief which it is *rational* to entertain in given conditions, and not merely with the actual beliefs of particular individuals, which may or may not be rational." [28]

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[27] Richard von Mises (1981 [1957], 94).

[28] See Keynes (2004 [1921], 3-4). It is widely held that Keynes yielded to Ramsey's (1988) critical arguments and that he abandoned the idea that rational beliefs are founded on logical relations of partial implication and accepted instead that they are closer to our perceptions and our memories than to formal logic. As Runde (1994) points out, Keynes's theory of comparative probability emerges unscathed. On the one hand Ramsey's theory embodies strong implicit presuppositions of its own and is in certain respects a considerably more idealistic construction than Keynes's. On the other hand, Keynes's emphasis is on incompleteness and on the fact that numerically definite probabilities can only be determined in situations which approximate games of chance.

It is important to acknowledge the point for point disagreement which exists between the theories of Richard von Mises and John Maynard Keynes.[29] For Richard von Mises probability is a branch of empirical science; for Keynes it is an extension of deductive logic. Von Mises defined probability as limiting frequency; Keynes as degree of rational belief. For von Mises the axioms of probability are obtained by abstraction from two empirical laws; for the other they are perceived by direct logical intuition. On one point there seems to be some agreement. Neither thinks that all probabilities have a numerical value, but the attitude of the two authors to this situation is very different. For Richard von Mises only probabilities defined within an empirical collective can be evaluated and only these probabilities have any scientific interest. The remaining uses of probability are examples of a crude pre-scientific concept towards which he takes a dismissive attitude. For Keynes on the other hand all probabilities are essentially on a par. They all obey the same formal rules and play the same role in our thinking. Certain special features of the situation allow us to assign numerical values in some cases, though not in general. Through the acknowledgement that frequency probability does not cover all we mean by probability, Keynes's position is thus also closer to that of other economists such as Ludwig von Mises and John Hicks. Finally the position of statistics is different in the two accounts. For von Mises it is a study of how to apply probability theory in practice, similar to applied mechanics. For Keynes statistical inference is a special kind of inductive inference and statistics is a branch of the theory of induction.

The most striking differences between John Maynard Keynes and Richard von Mises are thus:

—according to Richard von Mises, the theory of probability belongs to the empirical sciences, based on limiting frequencies, while Keynes regards it as a branch of logic, based on degrees of rational belief; and

—Richard von Mises's axioms are idealizations of empirical laws, Keynes's axioms follow from the intuition of logic.

It is a quite remarkable fact that the practical significance of these differences in principles does not prevent the two authors from reaching nearly complete agreement on almost all of the mathematical theorems of probability, as well as on the potentially

[29] See also Gillies (1973, 14-5).

successful fields of application of statistics. Thus their complete disagreement on all the philosophical issues is accompanied by complete agreement on the mathematical side. Moreover an essentially similar conclusion can be drawn as regards the potential scope of successful application of numerical probability concepts.

Thus in Part V of the *Treatise* in the context of his discussion of statistical inference, Keynes has the great merit of noticing that the applicability of some of the essential parts of the classical doctrine assumes independence or irrelevance.[30]

Keynes also suggested renaming the law of large numbers the *Law of Stability of Statistical Frequencies*, which provides a clear summary of its meaning:

> "But the 'Law of Great Numbers' is not at all a good name for the principle which underlies Statistical Induction. The 'Stability of Statistical Frequencies' would be a much better name for it. The former suggests, as perhaps Poisson intended to suggest, but what is certainly false, that every class of event shows statistical regularity of occurrence if only one takes a sufficient number of instances of it. It also encourages the method of procedure, by which it is thought legitimate to take any observed degree of frequency or association, which is shown in a fairly numerous set of statistics, and to assume with insufficient investigation that, because the statistics are *numerous*, the observed degree of frequency is therefore *stable*. Observation shows that some statistical frequencies are, within narrower or wider limits, stable. But stable frequencies are not very common, and cannot be assumed lightly." [31]

According to the frequency view the successful application of probability theory, in particular for purposes of statistical inference, is conditioned by the fulfillment of a particular presupposition: in a particular domain of reality, one or more collectives exist as a matter

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[30] As Keynes writes: "It is assumed, first, that a knowledge of what has occurred at some of the trials would not affect the probability of what may occur at any of the others; and it is assumed, secondly, that these probabilities are all *equal à priori*. It is assumed, that is to say, that the probability of the event's occurrence at the *r*th trial is equal *à priori* to its probability at the *n*th trial, and, further, that it is unaffected by a knowledge of what may actually have occurred at the *n*th trial." (2004 [1921], 344).

As Karl Popper points out, the theory of independence or irrelevance is equivalent to the law of the excluded gambling system. See Popper (1983, 299).

[31] Keynes (2004 [1921], 336).

of fact. This means that adequate applications of the laws of large numbers rest on a supposition of homogeneity with respect to the phenomena which are subjected to study.

Quite remarkably Keynes, when examining the validity and conditions of applicability of Bernoulli's Theorem and its Inversion, arrives at similar conclusions.

As he wrote:

> "If we *knew* that our material world could be likened to a game of chance, we might expect to infer chances from frequencies, with the same sort of confidence as that with which we infer frequencies from chances." [32]

These reservations are similar to those expressed by several Austrian economists. For instance Ludwig von Mises clearly doubts whether the empirical *Law of Stability of Statistical Frequencies* is operative in social reality:

> "However, what the statistics of human actions really show is not regularity but irregularity. The number of crimes, suicides, and acts of forgetfulness (…) varies from year to year. These yearly changes are as a rule small, and over a perio d of years they often—but not always—show a definite trend toward either increase or decrease. These statistics are indicative of historical change, not of regularity in the sense which is attached to this term in the natural sciences."[33]

V. Richard von Mises versus Ludwig von Mises, with Respect to Probability

In this section a certain amount of evidence is presented which is drawn from Ludwig von Mises's writings and which is difficult to square with the thesis that Ludwig von Mises embraced what is

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[32] Keynes (2004 [1921], 384-5) Significantly, Keynes also wrote in connection with the application of Bernoulli's formula: "In cases where the use of this formula is valid, important inferences can be drawn; and it will be shown that, when the conditions for objective chance are approximately satisfied, it is probable that the conditions for the application of Bernoulli's formula will be approximately satisfied also." (ibid. 290).

[33] See Ludwig von Mises (1969 [1957], 84-5). See also (1978 [1962], 56) where Mises wrote: "There is no such thing as statistical laws." According to this view, statistics is rather a sub-discipline, or an auxiliary discipline, of historiography.

basically the frequency interpretation of probability of his brother Richard von Mises.

It is remarkable that some of Ludwig von Mises's most revealing statements about the nature and meaning of the concept of probability relate to a context which is alien to economic science proper. If there is one field of scientific enquiry where the nature and interpretation of the probability calculus have been the subject of much and reiterated debate, it is the domain of quantum mechanics and the philosophy of quantum mechanics. We have already noted at the end of section III that, controversy notwithstanding, the frequency interpretation remains highly significant for the conduct of natural science. Here we turn our attention more particularly to a comparison of Ludwig von Mises's concept of class probability with Richard von Mises's concept of frequency probability.

The writings of Ludwig von Mises contain many important insights with respect to the philosophy of the sciences and it is not quite surprising that he had an outspoken opinion about the matter. In *Theory and History*, in a section entitled *Determinism and Statistics*, he expressed his view with respect to quantum mechanics as follows:

> "Quantum mechanics deals with the fact that we do not know how an atom will behave in an individual instance. But we know what patterns of behavior can possibly occur and the proportion in which these patterns really occur. While the perfect form of a causal law is: A "produces" B, there is also a less perfect form: A "produces" C in *n*% of all cases, D in *m*% of all cases, and so on. Perhaps it will at a later day be possible to dissolve this A of the less perfect form into a number of disparate elements to each of which a definite "effect" will be assigned according to the perfect form. But whether this will happen or not is of no relevance for the problem of determinism. The imperfect law too is a causal law, although it discloses shortcomings in our knowledge. And because it is a display of a peculiar type both of knowledge and of ignorance, it opens a field for the employment of the calculus of probability." [34]

Mises then provides the well-known definition of his concept of class probability:

> "We know, with regard to a definite problem, all about the behavior of the whole class of events, we know that A will produce definite effects

[34] Ludwig von Mises (1969 [1957], 87-8).

in a know proportion; but all we know about the individual A's is that they are members of the A class. The mathematical formulation of this mixture of knowledge and ignorance is: We know the probability of the various effects that can possibly be "produced" by an individual A." [35]

Significantly Ludwig von Mises is also explicitly critical of the mainstream indeterminist interpretation of quantum mechanics since he pursues:

"What the neo-indeterminist school of physics fails to see is that the proposition: A produces B in *n*% of the cases and C in the rest of the cases is, epistemologically, not different from the proposition: A always produces B. The former proposition differs from the latter only in combining in its notion of A two elements, X and Y, which the perfect form of a causal law would have to distinguish. But no question of contingency is raised." [36]

In *Human Action* Ludwig von Mises raised similar concerns when he wrote:

"The treatment accorded to the problem of causality in the last decades has been, due to a confusion brought about by some eminent physicists, rather unsatisfactory. (…)

There are changes whose causes are, at least for the present time, unknown to us. Sometimes we succeed in acquiring a partial knowledge so that we are able to say: in 70 per cent of all cases *A* results in *B*, in the remaining cases in *C*, or even in *D*, *E*, *F*, and so on. In order to substitute for this fragmentary information more precise information it would be necessary to break up *A* into its elements. As long as this is not achieved, we must acquiesce in a statistical law."[37]

These passages are important and interesting because they clearly illustrate the fact that in the context of the well-known historical debate between physicists who believed that quantum

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[35] Ludwig von Mises (1969 [1957], 88).

[36] Ludwig von Mises (1969 [1957], 88).

[37] Ludwig von Mises (1998, 22) In *The Ultimate Foundation of Economic Science* Ludwig von Mises also wrote: "There is always in science some ultimate given. For contemporary physics the behavior of the atoms appears as such an ultimate given. The physicists are today at a loss to reduce certain atomic processes to their causes. One does not detract from the marvelous achievements of physics by establishing the fact that this state of affairs is what is commonly called ignorance."(1978 [1962], 23).

mechanics is incomplete and who were tempted to assume that "God does not play dice," on the one hand, and the physicists who, on the contrary, believed that the fundamental laws of nature are irreducibly probabilistic, on the other hand, Ludwig von Mises takes sides with the former.[38] Ludwig von Mises clearly associates the use of the probability calculus with partial knowledge, that is, with ignorance and the imperfections of our knowledge, and not with the existence of any *contingency in re*. Similarly Einstein believed, from the very beginning, that quantum theory lacked some key ingredients and that, in a very significant sense, it was "incomplete." He compared it with the theory of light before the advent of light quanta. Quantum theory, he believed, was perhaps a "correct theory of statistical laws," but it provided "an inadequate conception of individual elementary processes."[39]

Thus Ludwig von Mises's concept of class probability, in contradistinction to the frequency concept of his brother Richard von Mises, contains a reference to the deficiency of our knowledge, that is, to the idea that any probability assignment describes only a state of knowledge. A statement is probable if our knowledge

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[38] In particular quantum theory is irreducibly probabilistic. Unlike classical probabilities, quantum probabilities do *not* reflect our ignorance of the intricate details of some underlying physical reality. In particular Einstein disliked the element of chance implied by quantum theory. In a letter to Max Born, dated 4 December 1926, he wrote: "Quantum mechanics is very impressive. But an inner voice tells me that it is not yet the real thing. The theory produces a good deal but hardly brings us closer to the secret of the Old One. I am at all events convinced that *He* does not play dice." Quoted in Baggott (2004, 34). Reference can in this context also be made to the confrontation between Einstein and Bohr over the interpretation of quantum theory, and to subsequent debates along similar lines, and which have often been portrayed in the past as a direct conflict between realism and positivism. For a good survey and discussion of these issues see also Baggott (2004). The issue for Einstein indeed seems to have been realism rather than determinism. Ludwig von Mises is apparently on the realist side. For a sophisticated analysis of Einstein's views in this respect, see also Fine (1986); Einstein's remark about the dice-playing God ("…*ob der liebe Gott würfelt*") is also related in Bohr (1949, 218); see also Fine (1986, 29).

[39] Einstein, Albert, letter to Sommerfeld, Arnold, dated 9 November 1927. Quoted in Fine, A. (1986), p. 29.

concerning its content is deficient.[40] According to this view the use of statistical laws signals partial knowledge and fragmentary information. There do not exist any statistical laws in an objective, physical sense.

As Popper reminds us too, the widely-held view that whenever probability enters our considerations, this is due to our imperfect knowledge, is reminiscent of subjective interpretations of the probability calculus.[41] The objective frequency interpretation does not have this connotation.

According to the mainstream view with respect to this matter, (nearly all) the probabilities appearing in theoretical quantum mechanics are indeed *objective* probabilities. That is to say, they inhere in the world and do not simply reflect the degrees of belief, or the degrees of knowledge, of an observer.[42]

These remarks are sufficient to establish the fact that Ludwig von Mises's interpretation of numerical probability theory, and in particular his interpretation of the concept of class probability, is in a fundamental sense distinct from that of his brother Richard von Mises. Indeed, according to Richard von Mises, the point of view that statistical theories are merely temporary explanations, in contrast to the final deterministic ones which alone satisfy our desire for causality, is nothing but a prejudice which is bound to disappear with increased understanding.[43]

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[40] Ludwig von Mises (1998, 107).

[41] Popper (1983, 295).

[42] See Hughes (1992, 218). The possible exceptions occur when a system is in a mixed state. Under the ignorance interpretation of a given mixture, a subjective probability is assigned to each of the pure states represented in it, and each of these in turn assigns objective probabilities to events. Not all mixtures *can* be given the ignorance interpretation, however. For a discussion of pure and mixed states, see also van Fraassen (1991, ch. 7). The interpretation of quantum states is a matter of much debate. For in-depth discussions of these and related matters, see in particular also Willem M. de Muynck (2002 *passim*).

[43] See Richard von Mises (1981 [1957], 223). As Richard von Mises writes: "The assumption that a statistical theory in macrophysics is compatible with a deterministic theory in microphysics is contrary to the conception of probability expressed in these lectures. Modern quantum mechanics or wave mechanics appears to be a purely statistical theory; its fundamental

The contrast between the views of Ludwig von Mises and of Richard von Mises in this respect can also be related to the fact that Ludwig von Mises's worldview, in contradistinction to that of his brother Richard von Mises, apparently exhibited some leaning towards metaphysical determinism.[44]

It is true that the contrast between Ludwig von Mises's concept of class probability and Richard von Mises's notion of a collective remains somewhat concealed and thus runs the risk of going unnoticed because of the fact that on a few occasions Ludwig von Mises uses terminology which is reminiscent of the idea of "frequency."

In *Human Action* for instance Ludwig von Mises explicitly and unambiguously characterizes the notion of class probability as a variant of frequency probability.[45]

Nevertheless this terminological issue cannot invalidate our thesis that, all things considered, Ludwig von Mises's philosophy of probability exhibits a closer affinity with an epistemological view—such as Keynes's logical theory—than with the frequency view of his brother Richard von Mises. The

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equations state relations between probability distributions." (ibid. 223) The incompatibility with the views expressed by his brother Ludwig von Mises in this respect cannot be clearer. Therefore we do not share the view of an author who explains the absence of any reference in Ludwig von Mises's *Human Action* to Richard von Mises's frequency interpretation with reference to a supposed "estrangement" between the two brothers. See Hoppe (2006, 13). Clearly the two brothers disagreed on philosophical grounds.

[44] See e.g., Ludwig von Mises (1978 [1962], 115). Turning back to quantum mechanics, it may be noted that the American-born physicist David Bohm has formulated in the 1950s an alternative interpretation of quantum mechanics that is fully deterministic (although non-local). The very idea of probability enters into this theory as some kind of an epistemic idea, just as it enters into classical statistical mechanics. Despite all the advantages of Bohm's theory, an almost universal refusal even to consider it, and an almost universal allegiance to the standard formulation of quantum mechanics has persisted in physics throughout most of the past 50 years. For a summary introduction to Bohm's approach, see David Z Albert (1994).

[45] Ludwig von Mises (1998, 107).

conclusion at which we have thus arrived is nuanced. On the one hand Ludwig von Mises clearly relates the idea of probability to the state of knowledge of the knowing subject. This is true both of class probability and of case probability. A statement is probable if our knowledge concerning its content is deficient. This view is shared by all adepts of an epistemological interpretation of the concept of probability, including John Maynard Keynes. Richard von Mises, to the contrary, very explicitly rejects the idea that the concept of probability refers to a state of partial or deficient knowledge. On the other hand, Ludwig von Mises clearly recognizes that the meaning of probability is different according to the field of knowledge in which it is used or according to the kind of phenomena to which it is applied. He thus embraces a dualist view in the philosophy of probability.[46] But in this respect his view is again clearly different from and opposed to that of his brother Richard von Mises who obviously embraces a monist theory of probability.

Moreover, from the perspective of the logical theory of probability too, the concept of probability *sometimes* refers to relative frequency. Contemporary adepts of the idea of *probability theory as extended logic* are confident that their approach can encompass frequentist methods, but merely as only one specialized application of probability theory.[47]

Apparently this was also Keynes's view since he wrote that "the theory of this Treatise is the generalised theory, comprehending within it such applications of the idea of statistical truth-frequency as have validity."[48]

In other words, on this view the problems that can be solved by frequentist probability theory form a subclass of those that are amenable to probability as logic; probability theory as logic, however, can also be applied consistently in many problems that do not fit into the frequentist preconceptions.

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[46] Accordingly probability *sometimes* involves a reference to the notion of relative frequency, but relative frequency is not the general defining characteristic of the scientific concept of probability according to Ludwig von Mises.

[47] See Jaynes (2003, *passim*).

[48] Keynes (1921 [2004], 104).

It would be premature to conclude that such concerns about the meaning of probability as are raised by Ludwig von Mises have now become obsolete and unambiguously belong to the history of the philosophy of probability. As one adept of the logical interpretation of probability explained recently:

> "Probabilities in present quantum theory express the incompleteness of human knowledge just as truly as did those in classical statistical mechanics; only its origin is different.
>
> In classical statistical mechanics, probability distributions represented our ignorance of the true microscopic coordinates—ignorance that was avoidable in principle but unavoidable in practice, but which did not prevent us from predicting reproducible phenomena, just because those phenomena are independent of the microscopic details.
>
> In current quantum theory, probabilities express our ignorance due to our failure to search for the real causes of physical phenomena; and, worse, our failure even to think seriously about the problem. This ignorance may be unavoidable in practice, but in our present state of knowledge we do not know whether it is unavoidable in principle; the 'central dogma' simply asserts this, and draws the conclusion that belief in causes, and searching for them, is philosophically naïve. If everybody accepted this and abided by it, no further advances in understanding of physical law would ever be made; indeed, no such advance has been made since the 1927 Solvay Congress in which this mentality became solidified into physics. But it seems to us that this attitude places a premium on stupidity; to lack the ingenuity to think of a rational physical explanation is to support the supernatural view."[49]

Again a disagreement about the meaning of probability at the philosophical level need not preclude an approximate consensus regarding the legitimate scope of application of numerical probability theory. It is certainly doubtful whether the criterion of convergence and the conditions for the availability of a collective are ever satisfied in economic or econometric applications. Probabilities in economics are not the kind of physical entities that

[49] See Jaynes (2003, 328-9). In particular, this author's views contrast sharply with those of Popper. With respect to the situation in physics, Popper, who argues for the compatibility of indeterminism with realism and objectivism, has gone so far as to blame the determinist interpretation of classical physics, or rather, what he characterizes as some unconscious determinist prejudice with respect to classical physics, for the subjective theory of probability and its consequence, the invasion of mysticism, irrationalism etc., into physics. See Popper (1982, *passim*).

Richard von Mises seems to have had in mind in constructing his theory.

The empirical foundation for probability in this sense, that is to say for objective frequency probability, will typically be lacking. Richard von Mises himself seems to have suggested that the frequentist conception is not applicable to the moral sciences owing to the absence of events meeting the conditions of a collective. As he wrote:

> "The unlimited extension of the validity of the exact sciences was a characteristic feature of the exaggerated rationalism of the eighteenth century. We do not intend to commit the same mistake."[50]

On this point Ludwig von Mises and Richard von Mises seem to have agreed.

VI. MORE ABOUT LUDWIG VON MISES AND JOHN MAYNARD KEYNES, WITH RESPECT TO PROBABILITY

Attention has already been drawn to the fact that both Ludwig von Mises and John Maynard Keynes embrace an epistemological rather than an objective interpretation of probabilities. Both of these authors also point to certain limits of the applicability of numerical probability, and in particular of the laws of large numbers. These authors' respective views on probability have another important characteristic in common, however. Both authors recognize and acknowledge the epistemological and scientific legitimacy of qualitative, non-measurable probabilities.

With respect to the question of whether a numerical measurement of probabilities is always possible, John Maynard Keynes was critical of the tendency to interpret probabilities as being, in general, numerically measurable. Thus he wrote:

> "The attention, out of proportion to their real importance, which has been paid, on account of the opportunities of mathematical manipulation which they afford, to the limited class of numerical probabilities, seems to be a part explanation of the belief, which it is the principal object of this chapter to prove erroneous, that *all* probabilities must belong to it."[51]

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[50] See Richard von Mises (1981 [1957], 9).

[51] Keynes (2004 [1921], 37).

In similar vein Ludwig von Mises wrote:

> "The problem of probable inference is much bigger than those problems which constitute the field of the calculus of probability. Only preoccupation with the mathematical treatment could result in the prejudice that probability always means frequency."[52]

Ludwig von Mises, who distinguishes between two kinds of probability—class probability, which by and large corresponds to frequency probability, and case probability—accorded the second meaning of probability important scientific status.

In Ludwig von Mises's words:

> "Case probability means: We know, with regard to a particular event, some of the factors which determine its outcome; but there are other determining factors about which we know nothing." [53]

Here too, however, the idea of probability relates to the general idea of partial or imperfect knowledge; in this respect, and only in this respect, case probability is indeed similar to class probability:

> "Case probability has nothing in common with class probability but the incompleteness of our knowledge. In every other regard the two are entirely different."[54]

Keynes, while he does not adopt the terms case and class probability, believes, like Ludwig von Mises, that frequency probability does not encompass all we mean by probability. Clearly the random frequency definition of probability is too narrow to encompass what we mean when we use the term probability. We *do* say of unique events that they are more or less probable. Many decisions that people make daily are based on probability statements that have no frequency interpretation.

In Chapter VIII of *A Treatise on Probability*, while discussing Venn's elaboration of the frequency theory, he wrote:

> "It is the obvious, as well as the correct, criticism of such a theory, that the identification of probability with statistical frequency is a very grave departure from the established use of words; for it clearly

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[52] Ludwig von Mises (1998, 107).

[53] Ludwig von Mises (1998, 110).

[54] Ludwig von Mises (1998, 110).

excludes a great number of judgments which are generally believed to deal with probability."[55]

While the frequency theory of probability is concerned with a cardinally measurable degree of probability, case probability is not open to any kind of numerical evaluation according to Ludwig von Mises.[56]

According to this view, case probability focuses on individual events which as a rule are not part of a sequence, and case probability is not measurable in any but an ordinal sense; there is no cardinal measure of case probability.

What is commonly considered as a numerical evaluation of case probability, Mises argues, exhibits, when more closely scrutinized, a different character, viz. that of a *metaphor*.[57] When we proceed to a numerical evaluation of case probability, this amounts to an attempt to elucidate a complicated state of affairs by resorting to an *analogy* borrowed from the calculus of probability. As it happens, this mathematical discipline is more popular than the analysis of the epistemological nature of *understanding*. As has been pointed out already, a distinctive feature of Keynes's view too is that not all probabilities are numerically measurable, and in many instances, they cannot even be ranked on an ordinal scale.[58]

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[55] Keynes (2004 [1921], 95).

[56] Keynes (2004 [1921], 95).

[57] See Ludwig von Mises (1998, 114). Ludwig von Mises's view regarding this matter is thus distinct from the view of Bayesians such as Howson and Urbach who argue that choices of personal fair betting quotients can provide a basis for making numerical assessments of uncertainty. See Howson and Urbach (2006, 51 ff.).

[58] In the *Treatise* Keynes illustrates this point with the famous example of the "beauty contest." (2004 [1921], 25 ff.)

Keynes explains how one of the candidates of the contest sued the organizers of the *Daily Express* for not having had a reasonable opportunity to compete. Readers of the newspaper determined one part of the nomination. The final decision depended on an expert, who had to sample the top fifty of the ladies chosen by the readers. The candidate complained in front of the Court of Justice, that she had not obtained an opportunity to make an appointment with this expert. Keynes argues that the chance of winning the

Keynes's views on the applicability of large number statistics to singular propositions are in this respect somewhat similar to those espoused by Ludwig von Mises. Keynes was clear on why one might adopt case probability judgments even where large number statistics are available:

> "In some cases, moreover, where general statistics are available, the numerical probability which might be derived from them is inapplicable because of the presence of additional knowledge with regard to the particular case."[59]

VII. THE DISTINCTIVENESS OF LUDWIG VON MISES'S POSITION IN THE PHILOSOPHY OF PROBABILITY

Acknowledging certain similarities between Ludwig von Mises's and John Maynard Keynes's respective positions in the philosophy of probability should not blind us to the fact that their respective views also exhibit some important differences. The most important of these relates to the fact that Ludwig von Mises advocates a pluralist, and in particular a dualist view of probability. According to a pluralist view of probability, there exist several different, though possibly interconnected, notions of probability which apply in different contexts, or with respect to different kinds of phenomena. Ludwig von Mises's dualist position

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contest could have been measured numerically, if only the response of the readers (who sent in their appraisals and thus provided an unambiguous ranking of the candidates) had mattered. The subjective taste of the single expert could not be evaluated in a similar way. Hence, a rational basis for evaluating the chances of the unfortunate lady was lacking.

Keynes concludes:

> "Whether or not such a thing is theoretically conceivable, no exercise of the practical judgment is possible, by which a numerical value can actually be given to the probability of every argument. So far from our being able to measure them, it is not even clear that we are always able to place them in an order of magnitude. Nor has any theoretical rule for their evaluation ever been suggested." (ibid. 27-8)

[59] See Keynes (2004 [1921], 29). In similar vein, Hoppe (2006), analyzing the meaning of Ludwig von Mises's concept of case probability, points out that the method of *Verstehen* can be characterized as a method of place selection, or a method of individualization.

in the philosophy of probability is an aspect of his more general methodological dualism, which is based on a recognition of certain fundamental ontological, epistemological and methodological differences between the natural sciences on the one hand and the sciences of human action on the other, and between the natures of their respective subject matters. Moreover, in the particular case of Ludwig von Mises, his dualism in the philosophy of probability coincides with the distinction between measurable, numerical probability on the one hand and non-measurable, non-numerical probability on the other, that is, with the distinction between class probability and case probability.[60, 61]

Ludwig von Mises's solution to the problem of defining the concept of probability remains, no less than Keynes's, original and highly relevant. Where others have pleaded in favour of the introduction of operationalist procedures in the social sciences, as an alternative way of making the qualitative quantitative,[62] Ludwig von Mises's concept of case probability remains radically non-numerical, geared to the needs of historical and entrepreneurial understanding.

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[60] It is *not* the case that according to Ludwig von Mises's dualist (two-concept) view with respect to probability, the different concepts of probability are conceived of *as different interpretations of the same mathematical calculus*, or as applications of the same mathematical calculus to different sets of phenomena, as is the case according to certain other dualist views of probability. The distinction between class probability and case probability is ultimately based upon the different kind of cognitive accessibility of human actors in contrast to non-communicative entities. See Hoppe (2006).

[61] Ludwig von Mises's view with respect to the meaning of probability may thus seem to occupy a truly unique place in the philosophy of probability. Another economist who adopted a nuanced viewpoint in this connection is John Hicks. This author wrote: "I have myself come to the view that the frequency theory, though it is thoroughly at home in many of the natural sciences, is not wide enough for economics." (1979, 105) Hicks is contrasting two interpretations of probability—the frequency and the logical. The framework used here is wider since we distinguish objective theories of probability from epistemological theories.

[62] See Gillies (2000, 200 ff.).

VIII. CONCLUSION

While certain fundamental differences between the natural and the social sciences and the consequent need for a nuanced solution to the problem of finding an adequate definition of the concept of probability have been recognized by various authors and schools of thought, the solutions to this problem offered by both Ludwig von Mises and John Maynard Keyes remain both interesting from a theoretical perspective and useful from a more practical viewpoint.

We have been entitled to conclude that Ludwig von Mises's views concerning the interpretation of the concept of probability, as they can be ascertained from certain passages of his writings, are in some respects more akin to the logical interpretation of probability as developed by John Maynard Keynes than to the frequency view as developed by his brother Richard von Mises. Summarizing, it can be acknowledged that this conclusion is supported by the fact that the views of Ludwig von Mises and of John Maynard Keynes about the interpretation of probability—that is, their philosophy of probability—have two important characteristics in common which are not shared by the probability theory of Richard von Mises.

First, both Ludwig von Mises and John Maynard Keynes adopt an epistemological (or epistemic) interpretation of probability, whereas Richard von Mises clearly embraces an objective theory of probability. The viewpoints of Ludwig von Mises and John Maynard Keynes, in so far as they amount to an argument for interpreting probabilities in economics as epistemological rather than objective, are thus in agreement with the conclusions of recent research. *Second*, both Ludwig von Mises and John Maynard Keynes, in their respective ways, acknowledge the existence and the epistemological and scientific legitimacy of non-measurable (or non-numerical) probabilities, besides the usual measurable probabilities having a definite numerical value in the interval [0, 1]. Although Richard von Mises did acknowledge that there was an ordinary language or common sense notion of probability which was not covered by his frequency theory, he asserts that there is only one concept of probability that is of scientific importance. In other words, according to this view there is, in a *scientific* approach to the subject matter, no room for a purely qualitative notion of probability.

While some authors have gone so far as to question the adequacy of the orthodox frequency theory even for the physical sciences, there is a somewhat greater amount of consensus in favour of the conclusions (1) that in any case an objective interpretation of probability such as the orthodox frequency theory is not wide enough for economics, and (2) that in economics a qualitative non-numerical concept of probability is both needed and scientifically legitimate. Both of the aforementioned characteristics have much relevance for the conduct of social science in general and of economics in particular.

An important difference between the views of Ludwig von Mises and those of John Maynard Keynes in this respect has nevertheless been acknowledged. Whereas Keynes advocated a monist view of probability and claimed that his interpretation of probability applies to all uses of the concept, Ludwig von Mises, in accordance with his methodological dualism, embraced a dualist view, recognizing more emphatically the existence of important differences between the natural sciences on the one hand and the social sciences, including economics, on the other. The particular solution offered by Ludwig von Mises thus remains highly distinctive and sophisticated, even if in comparison with the Keynesian approach, it has until present received somewhat less attention.[63]

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[63] Those contemporary Austrian economists who acknowledge the usefulness of modern data analysis methods for the conduct of applied research in economics can be confident that the now more and more widespread practice of interpreting probabilities as merely epistemological is in general agreement with Ludwig von Mises's approach to probability. Moreover, it is neither clear nor obvious why a recognition of the usefulness of modern data analysis methods would have to amount to a denial of the essential importance of the method of understanding or *Verstehen*.

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