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Small amendment arguments: how they work and what they do and do not show

Martin van Hees[1] · **Akshath Jitendranath**[2] · **Roland Iwan Luttens**[3]

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Abstract

The small improvement argument has been said to establish that the standard weak preference or value relation can be incomplete. We first show that the argument is one of three possible 'small amendment arguments', each of which would yield the same conclusion. Generalizing the analysis thus, we subsequently present a strong and a weak version of small amendment arguments and derive the exact rationality conditions under which they reveal incompleteness. The results show that the arguments (in any of their variants) need not reveal a problem for the possibility of rational choice. In fact, it can be argued that they only reveal such a problem if the underlying relation is complete rather than incomplete.

Keywords Small improvement argument · Rational choice · Incompleteness of preferences

1 Introduction

Consider two distinct alternatives, x and y, about which you think that neither is strictly better than the other. Next, let x^+ be a marginally improved version of x. Although the improvement is only a marginal one, it is an improvement nonetheless. It is therefore strictly better than x. But now suppose you do not take x^+ to be strictly

✉ Roland Iwan Luttens
r.i.luttens@vu.nl

Martin van Hees
m.van.hees@vu.nl

Akshath Jitendranath
akshath.jitendranath@psemail.eu

1 Department of Philosophy, Vrije Universiteit Amsterdam, De Boelelaan 1105, 1081 HV Amsterdam, The Netherlands

2 Paris School of Economics, Campus Jourdan, 48 boulevard Jourdan, 75014 Paris, France

3 Department of Economics, Vrije Universiteit Amsterdam, De Boelelaan 1105, 1081 HV Amsterdam, The Netherlands

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better than *y*. Then, an influential argument states, you cannot hold that *x* and *y* are equally good with respect to each other. This implies that *x* and *y* cannot be ranked, that is, the axiological relation 'at least as good as' is incomplete. Applying the same reasoning to cases in which the relation between the alternatives describe an agent's preferences, the conclusion is that the weak preference relation is an incomplete one. The implication of incompleteness—whether it is of the axiological relation of goodness ('is at least as good as') or of subjective preferences ('is weakly preferred to')—is the 'claim to fame' of this argument, which is called the small improvement argument.

While it has its antecedents in a remark made by Leonard Savage (1954, p. 17) , the argument has received a second wind in the philosophical literature where it has been central to discussions of rational decision making and incomplete rankings,[1] Indeed, it would be hard to think of a more influential argument—at least among philosophers—that also shows that a binary preference or value relation is incomplete.[2]

Raz (1986, pp. 325–335) uses the axiological interpretation of the binary relation and considers any scenario in which there are two valuable options *x* and *y* such that (a) neither is better than the other, and (b) there can be a third alternative that is better than one but not the other, as establishing incomparability of the elements *x* and *y*.[3] That is, the value of *x* and *y* cannot be compared *vis à vis* each other and a 'gap' in one's assessment of the various alternatives results. Also employing an axiological interpretation, Chang (2002, 2012, 2017) has argued, with considerable influence, that the small improvement argument shows that 'better than' or 'equally good as' do not exhaust the conceptual space of possible value relations. She argues that a different value relation obtains between *x* and *y* rather than the absence of one. Indeed, Chang presents this argument as the first step in a two-step argument to establish the claim that *x* is not strictly better than, strictly worse than, or equally as good as *y*, but is *'on a par'* with *y*.

In the first part of the exposition—Sect. 2—we generalize and characterize the small improvement argument. Our generalization consists in extending the line of reasoning used in the argument to what we shall call 'small amendment' situations. More specifically, we examine choice situations in which the small differences can have a different structure. The upshot is that all such situations, when taken in

[1] See, *inter alia* de Sousa (1974), Raz (1986, pp. 332–335), Sinnott-Armstrong (2004, pp. 66ff), Chang 1997, 2002, 2012, 2017), Regan (1997), Qizilbash (2002), Gert (2004), Wasserman (2004), Hsieh (2005, 2007); Carlson 2006, 2011), Peterson (2006), Espinoza (2008), Rabinowicz (2008, 2012), Boot (2009), Gustafsson (2010), Gustafsson (2013), Anderson (2015), Andreou (2015) and Flanigan and Halstead (2018). It also makes an appearance in other areas of practical philosophy, like the analysis of the money-pump argument Gustafsson (2016) or the investigation of the existence of supererogatory acts (Muñoz, 2020).

[2] To be sure, it is not the only argument in the literature that aspires to show this. There are at least seven arguments that share this aspiration. See Chang (1997)—section III in particular—for a critical *tour d'horizon* of these arguments.

[3] Raz uses the term 'incommensurability'. Since incommensurability has also been used to understand a situation where there is no numerical representation of the value of alternatives, we prefer to use 'incomparability' to indicate the incompleteness of the ranking.

conjunction with certain rationality conditions, seem to form a case for incompleteness of the weak preference or value relation. Then, in Sect. 3, we turn to the implications of our analysis. We argue that the small amendment arguments may be less problematic for the possibility of rational choice than they may suggest.

2 How small amendment arguments work

2.1 From small improvements to small amendments

To begin, we present a general framework for the analysis of arguments involving small improvements to alternatives. The framework matters, for two distinct reasons. First, the reference to 'small' changes to alternatives presupposes that we can make a distinction between alternatives that are almost identical to each other and ones that differ substantially, and we make this explicit in our framework. Secondly, we will not only focus on small improvements, but also consider small changes that are taken to be worsenings ('small deteriorations') or to which an individual is indifferent ('small neutral variants').

We let X denote the set of alternatives under consideration. R is the weak preference or value relation and is a reflexive binary relation over X; P and I denote its asymmetric and symmetric parts, respectively. We write $x \bowtie y$ if neither xRy nor yRx holds. We assume that some reflexive and symmetric similarity relation $\sim$ is given that describes which elements in X are similar to each other.[4] In what follows, we simply say that x and y are *similar* rather than writing $x \sim y$ or $y \sim x$. An element x^* is a *small amendment* of x if, and only if, x^* and x are similar, comparable (xRx^* or x^*Rx) *and* distinct ($x^* \neq x$).[5]

With these clarifications of our framework made, we note that an improvement is only one way in which a given alternative can be amended. A small change to an alternative can also mean a deterioration, or it can be a neutral variant that does not affect the overall quality. This raises the question of whether the exact nature of the amendment—be it improvement, deterioration, or neutral—matters for the conclusion that some pair of alternatives cannot be ranked by the weak preference or value relation R. And if it does not matter, would the exact restriction on R that is necessary and sufficient for this conclusion to follow remain the same in all three cases? In order to address these questions, we extend the basic idea underlying the small improvement argument to cover the other types of small amendments as well. Let X, R and $\sim$ be given.

[4] The relation $\sim$, and the idea of similar alternatives, can be derived from an analysis of X in a spatial framework, say by holding that two elements are similar to each other if, and only if, the distance between them does not exceed a certain threshold. Spatial frameworks have been used in the analysis of vagueness, which may be relevant to interpreting the conclusion of the small improvement argument. See *inter alia* Gärdenfors (2000), Douven et al. (2013), Decock and Douven (2014) and Hampton (2007) .

[5] Comparability of x and x^* is an element of the small amendment argument. Since the analysis here concerns the validity of the argument, we make the very same assumption.

Definition 1 Small Amendment Situation (SAS): An SAS (of type $\alpha \in \{1, 2, 3\}$) is a set $\{x, x^*, y\}$ consisting of three distinct elements of X, such that:

1. x^* is a small amendment of x,
2. neither xPy nor yPx, and

   (a) $\alpha = 1$: x^*Px and not x^*Py;
   (b) $\alpha = 2$: xPx^* and not yPx^*;
   (c) $\alpha = 3$: x^*Ix, and x^*Py or yPx^*.

Plainly, any SAS of type 1 corresponds to x^* being a small improvement, and thus conforms with the case as it is commonly discussed. It is of type 2 when it concerns a small deterioration, and of type 3 if it involves a small neutral variant. A small improvement is thus only one of three distinct types of amendment situations. Whereas it is clear that the third type is different from the other two, it may be less obvious that types 1 and 2 are really different. After all, if some x^* is a small improvement of x, then x is a small deterioration of x^*. Yet this does not mean that a permutation of the two elements means that every small improvement situation also is a small deterioration situation. The reason for this is that the relation with the third element, y, may differ. Assume, for instance, that $\{x^*, x, y\}$ is a small improvement situation of type 1 in which transitivity is violated: yPx^* holds. It would then not be a small deterioration situation in the sense defined. It would only be so when neither x^*Py nor yPx^* holds.

As an axiological illustration for SASs of type 2, take a variation of Joseph Raz's example of a person facing the choice between two successful careers: one as a lawyer and one as a clarinettist (Raz, 1986, p. 332). Neither career is strictly better than the other, and to adjudicate whether or not they are equally as good as each other we introduce a legal career which is less successful than the original legal career. If the musical career is not strictly better than the small deterioration of the legal career, then it is a small deterioration situation. Furthermore, if this is so because, say, the burdens of success entail that the less successful legal career is considered to be better than the musical career, then, even though one legal career is slightly better than the other, it fails to form a small improvement situation.[6]

To illustrate a type 3 SAS, consider a music lover with genuinely catholic taste. Suppose she is comparing, say, the 1966 Bayreuther performance of the

[6] There is also a literature in experimental and behavioural economics that studies choice behaviour in the presence of a SAS of type 2 to show what has variously been called the attraction effect, asymmetric dominance effect, or decoy effect, which is typically summarised as follows: adding a small deterioration (the decoy) x^* of x to a menu $\{x, y\}$ increases the probability of choosing x over y. This effect was originally observed by Huber et al. (1982) and has since led to a large empirical literature. See Castillo (2020) for a recent overview. Because this effect also involves violations of some standard consistency requirements of choice theory, it has served as an inspiration for a literature in economic theory to propose models of choice behaviour that account for it. See, among others, Barbos (2010), Cherepanov et al. (2013), De Clippel and Eliaz (2012), Gerasimou (2016a, b), Masatlioglu et al. (2012), and Ok et al. (2015). We thank an anonymous referee for alerting us to this literature.

opera *Tristan und Isolde* with Bob Dylan's first Amsterdam concert of 2009. She does not have a strict preference for one over the other—so is either indifferent or cannot rank them *vis à vis* each other. Moreover, she is indifferent between Dylan's first and second 2009 Amsterdam concert. Yet she does strictly prefer the second Amsterdam concert to the opera performance: on that second night Dylan sang *The Man in the Long Black Coat*, which he did not do on the first Amsterdam performance. In light of the hours of anguish that Wagner's protagonists are going through and the feelings of bleakness and despair evoked by just one song, she judges the Dylan concert to be superior to the opera.

To indicate the nature of a small amendment of x we sometimes write x^+, x^- and $\hat{x}$, for the three cases respectively. We can now make the general idea of a small amendment argument precise.

Definition 2 Small Amendment Argument (SAA): For any SAS $\{x, x^*, y\}$: $x \bowtie y$.

Crucial according to Raz's reasoning in the career example is that since the musical career is not strictly better than the marginally worse legal career, the two original career options are incomparable. Turning to the Dylan-Wagner example, we see that the music lover considers the two Dylan concerts to be equally enjoyable. Because she has a strict preference of the second Dylan concert over the opera, but not between the first and the opera, the SAA would entail that she cannot compare the first Amsterdam 2009 Dylan concert and the 1966 performance of *Tristan und Isolde*.

The question, then, is under what conditions would this case, as well as the other two cases, establish incompleteness? To answer it, we introduce the following well-known consistency conditions. For all x, y, z, we say that R satisfies *PI-transitivity* if xPy and yIz implies that xPz; *IP-transitivity* if xIy and yPz implies that xPz; and *II-transitivity* if xIy and yIz implies that xIz. Following Savage (1954, p. 17), Gustafsson and Espinoza (2010, p. 755) have argued that PI-transitivity is a 'core premise' of the small improvement argument. The following proposition formalizes this insight, but also shows that it is not crucial for each small amendment argument.

Proposition 1 *The SAA is valid for some SAS* $\{x, x^*, y\}$ *if, and only if, the restriction of R to* $\{x, x^*, y\}$ *is PI-transitive* (*when* $x^* = x^+$), *IP-transitive* ($x^* = x^-$) *or II-transitive* ($x^* = \hat{x}$).

Proof Let $\{x, x^*, y\}$ be an SAS.

$\Rightarrow$: If the SAA is valid for $\{x, x^*, y\}$, we have $x \bowtie y$, (a) x^+Px, and not x^+Py, or (b) xPx^- and not yPx^-, or (c) $xI\hat{x}$, and $\hat{x}Py$ or $yP\hat{x}$. In these scenarios the conditions of PI-, IP- and II-transitivity are trivially fulfilled, respectively.

$\Leftarrow$: Assume xIy. If R is PI-transitive, then x^+Py. If R is IP-transitivity, we obtain yPx^-; and if it satisfies II-transivity, we get $yI\hat{x}$. Together, the scenarios contradict the definition of an SAS and thus we must reject the assumption that xIy. Since neither xPy nor yPx, we conclude that $x \bowtie y$. □

Obviously, X may contain multiple SASs of different types. The following is then immediate.[7]

Corollary 1 *For any SAS, the SAA is valid if R is transitive.*

Before we proceed, it is of some interest to consider the case of an SAS $\{x, x^*, y\}$ that is of Type 1 *and of* Type 2 if we permute x with x^*. By definition of the two types of SAS, this means that none of xPy, yPx, x^*Py, yPx^* holds. Since we do have x^*Px, it follows that *II*-transitivity here also establishes incompleteness of R: either x and y are incomparable or y and x^* are. Hence, in the special case of an SAS being of both Type 1 and 2, any of the three rationality requirements entails that R is incomplete. The difference is that with II-transitivity we do not know where to locate the incompleteness exactly.

2.2 A weaker version

The observation that we may sometimes know that there is incompleteness though not exactly where is of interest in its own right. It suggests a further extension of the analysis. That is, if we 'merely' want to show that *some* incompleteness may arise, we can use the following weakening of the argument.

Definition 3 Weak Small Amendment Argument (WSAA): The restriction of R to any SAS is incomplete.

We take the WSAA to be of distinct significance. With the exception of contributions that aim to show the existence of a fourth value relation—and here Ruth Chang's work is of obvious importance—the philosophical interest in incomplete binary relations does not depend on specifying which two alternatives remain unranked by a binary relation. For instance, in the context of moral or political choice, positions that are under attack on the grounds of incompleteness—like consequentialism, cost-benefit analysis, ideal theories of justice, and so on—do not require demonstrating which pair of alternatives are unranked. Indeed, the criticisms of these views only require showing, as the WSAA shows, that a binary relation is incomplete.[8]

To check, then, under what conditions this weaker argument would be valid, we use the notion of Suzumura consistency (SC).[9] Given a set A and relation R over A,

[7] For all x, y, z, R is transitive if, and only if, xRy and yRz implies xRz.

[8] Amartya Sen's oeuvre has been particularly influential in emphasising the importance of incomplete relations in contexts as varied as the measurement of inequality and poverty, rational decision making, measuring freedom and capabilities, consequentialism and utilitarianism, and theories of justice. See the papers collected in Sen (2004). But see also Levi (1986) for a critique of the association of rational choice with going for the best on the basis of incompleteness. See Nussbaum (2000) for a critique of cost-benefit analysis on the basis of incompleteness.

[9] Suzumura consistency was first introduced in Suzumura (1976) . For an introduction to SC, see Bossert (2008). A book-length discussion of the condition is Bossert and Suzumura (2010).

let R^* be the transitive closure of R over A.[10] A binary relation R over A (with transitive closure R^*) satisfies SC if, and only if, for all $x, y \in A$: if xR^*y, then not yPx. SC is another distinct weakening of transitivity, but one that is especially important in the context of analysing an argument that establishes incompleteness. This is because it is a consistency condition that does not assume completeness. For in their non-trivial applications, that is, when the antecedent is true, consistency conditions like transitivity and its weakenings that have been considered thus far—*PI*, *IP*, and *II* transitivity—entail completeness of the binary relation R over the triple. And, in fact, the difference between SC and transitivity disappears when the ranking is complete (and reflexive).

Proposition 2 *If R is Suzumura consistent, then WSAA is valid for any SAS. Conversely, if WSAA is valid for some SAS, then the restriction of R to that SAS is Suzumura consistent.*

Proof Let $\{x, x^*, y\}$ be a small amendment situation. We only prove the result for the scenario in which x^* is a small improvement of x, that is, $x^* = x^+$.

(A): Let R satisfy SC. By definition of an SAS, either xIy or $x \bowtie y$. If $x \bowtie y$ we are done. If xIy, then x^+Px and SC entail not-xR^*x^+. Since xRy, we cannot have yRx^+, otherwise xR^*x^+. From not-x^+Py and not-yRx^+ follows $x^+ \bowtie y$.

(B): Assume R is incomplete. First, consider the scenarios in which $x \bowtie y$ and yRx^+. If (a) yIx^+, then $P = \{(x^+, x)\}$ and $R^* = \{(x^+, x), (x^+, y), (y, x^+), (y, x), (x, x), (x^+, x^+), (y, y)\}$. If (b) yPx^+, $P = \{(x^+, x), (y, x^+)\}$ and $R^* = \{(x^+, x), (y, x^+), (y, x), (x, x), (x^+, x^+), (y, y)\}$. In both cases, SC is satisfied. Secondly, consider scenarios in which $x^+ \bowtie y$. Then either $P = \{(x^+, x)\}$ and (a) $R^* = \{(x^+, x), (x, y), (y, x), (x, x), (x^+, x^+), (y, y)\}$, when xIy or (b) $R^* = \{(x^+, x), (x, x), (x^+, x^+), (y, y)\}$, when $x \bowtie y$. In both cases SC is satisfied. □

With this, we conclude our discussion of how small amendment arguments work. We turn now to scrutinizing their implications.

3 What small amendment arguments show and do not show

Now, because small amendment arguments presumably show that a binary relation is incomplete, one of its main upshots is that it seems to threaten the possibility of making what is conventionally understood to be a rationally justified choice (Chang 2009, p. 10). This has motivated alternative—non-standard—responses, like Chang's fourth value relation. Our objective is to show that we do not necessarily require such alternatives.

To see why, note that one way of interpreting an SAA is to view it as describing an inference made by an external observer. Assume that the observer knows that the

[10] That is, for all $x, y \in A$, xR^*y if, and only if, there are $x_1, \dots, x_k \in A$ such that $x_1 R \dots Rx_k$.

agent is in an SAS $\{x, x^*, y\}$ of type α ($\alpha \in \{1, 2, 3\}$). The observer does *not* know whether the agent's ranking over the set of three elements is complete or not but may know further parts of the relation on $\{x, x^*, y\}$. We call $\mathcal{R}_\alpha$ the observer's information—the partial rankings over $\{x, x^*, y\}$ that describes the information available to a particular observer of a particular SAS of type α. For instance, if in addition to the knowledge given by the SAS itself, the observer also knows that x^*Iy, then the information set $\mathcal{R}_1$ consists of exactly two rankings: (1) x^*Px, x^*Iy, xIy, and (2) x^*Px, x^*Iy, $x \bowtie y$.

Viewing $\mathcal{R}_\alpha$ as the set of all possibilities that the observer of an SAS $\{x, x^*, y\}$ of type α ($\alpha \in \{1, 2, 3\}$) cannot preclude, we can use the following definition.

Definition 4 Given some $\mathcal{R}_\alpha$, the agent's ranking in the SAS 'can' have property ϕ, if, and only if, some $R \in \mathcal{R}_\alpha$ has the property.

With this terminology, we can now present—as a corollary—the following.

Corollary 2 *For any $\mathcal{R}_\alpha$: the agent cannot have a ranking that is complete as well as PI-transitive (when $\alpha = 1$), IP-transitive ($\alpha = 2$) or II-transitive (when $\alpha = 3$). Hence, regardless of the value of α, the ranking cannot be an ordering, i.e., cannot be complete and transitive.*

With that said, does the fact that an agent is facing an SAS preclude the possibility of making an optimal choice?[11] To answer this question, we weaken SC to acyclicity, that is, to the requirement that for any $x_1, \dots x_n \in X$: if $x_1Px_2, \dots, x_{n-1}Px_n$, then it is not the case that x_nPx_1.

Proposition 3 *For any $\mathcal{R}_\alpha$: the agent's ranking in the SAS in question can be complete and acyclic.*

The result directly follows from the definitions. Since the agent does not know whether the ranking over $\{x, x^*, y\}$ is complete or not, there is always one ranking in $\mathcal{R}_\alpha$ the restriction of which to $\{x, x^*, y\}$ is complete. That ranking cannot have a P-cycle over the three alternatives in an SAS, since by definition of any type of SAS we have neither xPy nor yPx.

This insight, straightforward as it is, is the first ground on which our criticism of small-amendment arguments stands.[12] It shows us that any small amendment situation is compatible with R being complete and a-cyclic over the elements involved. Of course, our argument does not say that R *is* complete but only that it *can* be so.

[11] An alternative x in X is optimal if and only if it is weakly preferred to all other alternatives in X. It is said to be maximal if there is no other alternative that is strictly preferred to it.

[12] It is formally more challenging to establish the conditions under which the result can be generalised to situations where an individual may also face multiple and possibly partly overlapping SASs. Given the purposes of this paper, we do not pursue that issue here.

But it is exactly that possibility that allows us to reject the SAA as an argument that purports to establish incompleteness. Furthermore, an SAS does not commit us to abandon optimality as the standard for rational choice in these cases. This is because a complete and acyclic relation is necessary and sufficient for an optimal alternative to exist in a given situation—optimization then is possible.

We do not claim that an agent with a complete and a-cyclic ranking is always able to make a rational choice. That is, we do not argue that optimization is the right standard for rational choice. Money pump arguments, for example, indicate that it may not be so: even if there are optimal elements an agent may be vulnerable to exploitation. But here the analysis of the weak small amendment argument—our second proposition—has an interesting implication. Bradley (2015, p. 34) has argued that there is 'good reason to think of Suzumura consistency as being the appropriate consistency condition for incomplete preferences'. It is necessary and sufficient for the possibility of 'strong maximality', a standard of rational choice that can be said to be in between optimization and maximization. It for instance entails the absence of money pumps since it rules out cycles with at least one strict preference. So, and this then is a second reason for not being too sombre, even if a small amendment argument does indeed reveal the incompleteness of the underlying value relation, this does not yet establish a problem of rational choice.

4 Conclusion

To recapitulate, we began by explaining how small amendment arguments work. This basically consisted in showing the close relationship that exists between the small amendment argument (SAA) and the weak small amendment argument (WSAA) on the one hand, and transitivity and Suzumura consistency (SC) on the other. Subsequently, we examined what happens when we weaken the rationality requirement to acyclicity. We argued, firstly, that small-amendment situations do not establish incompleteness and that they are compatible with the possibility of making a rationally justified choice if such choice is defined in terms of optimization. A complete and acyclic relation is necessary and sufficient for an optimal alternative to exist in a given situation. Since the small-amendment argument does not entail that the ranking in question is incomplete (or cyclic), it does not entail the impossibility of optimization in those cases either. Furthermore, drawing on Bradley's account of the notion of strong maximality as a standard of rational choice, we argued that even when a small amendment argument establishes incompleteness of the value relation, it need not mean that no rational choice can be made.

We can now infer the following general conclusion about small amendment situations. They show that either the underlying ranking is complete and acyclic and possibly raising a problem for rational choice, or that it is incomplete but Suzumura-consistent. If Suzumura-consistency is seen as sufficient for rational choice, the somewhat surprising upshot of our analysis is that the small amendment arguments may only reveal a problem for rational choice if the underlying relation is complete rather than incomplete.

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