’s Gravesande on the application of mathematics in physics and philosophy

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1. Introduction

One of the most important books in the emergence of ‘Newtonian physics’ in the eighteenth century was the *Physices elementa mathematica, experimentis confirmata; sive introductio ad philosophiam Newtonianam*. The first edition of this book was published in 1719; revised, updated, and widely popular versions of it would appear in 1725 and 1742.[1] The last of these was published shortly after the death of its author, the Dutch philosopher, physicist, and mathematician Willem Jacob ’s Gravesande (1688-1742). As the title of the book indicates and subsequent statements by ’s Gravesande confirm, he himself considered the book to be about ‘mathematical physics’ and to follow Isaac Newton’s methodology. This became very clear for instance from the preface ’s Gravesande added to the second edition of his book, where he discussed how he disagreed with Newton on the measure of force but not on the way of doing their business:

> Although I have moved away from the Newtonian opinion in many things […] I have never doubted in any way to still maintain the title an *Introduction to Newtonian Philosophy*, and to give this title to the second edition of this book it-

1 See De Pater 1988, 152 for a list of editions; the book, its abbreviated versions, and their various translations together went through more than 20 print runs in the eighteenth century.

> self. […] He who reasons only from the phenomena in physics, having rejected all feigned hypotheses, and, as much as he is able, follows this method chastely, tries to follow in those footsteps of Newton and rightly professes to follow the Newtonian philosophy; not on the contrary he who vows to the words of the master.[2]

As Steffen Ducheyne has shown in a study published in 2014, historians as yet have not sufficiently questioned to what extent 's Gravesande was justifiably claiming to follow the methodology set out by Isaac Newton. In this article I will expand on the work done by Ducheyne, who in the same study has explored some components of the methodology of 's Gravesande's physics, focusing especially on the question whether 's Gravesande had taken over certain elements particular to Newton's methodology. Ducheyne has shown that, with respect to most of these issues, this was not the case and has claimed that 's Gravesande «was selective in his endorsement of Newton's epistemology» and that «his methodological ideas were quite different from and occasionally even incongruent with Newton's views on the matter».[3]

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2 'S GRAVESANDE 1725(1), second and third of the unnumbered pages of the *Monitum de hac Secunde Editione*: «Quamvis in multis […] a NEWTONIANA recesserim sententia, non tamen titulum *Introductionis ad Philosophiam* Newtonianam servare, & huic secundae editioni ipsum inscribere, ullo modo dubitavi. […] Qui tantum ex Phaenomenis, omni ficta rejecta hypothesi, in Physices ratiocinantur, &, quantum in ipso est, caste hanc methodum sequitur, ille NEWTONI vestigiis insistere conatur, & merito NEWTONIANAM se sectari Philosophiam profitetur; non autem ille, qui in verba jurat magistri». I am grateful to Urte Brauckmann of the Library of the Max Planck Institute for the History of Science for sending me a digital copy of this particular edition of 's Gravesande's book. Throughout, translations are mine, unless indicated otherwise.

3 DUCHEYNE 2014(1), 47 and DUCHEYNE 2014(2), 112 respectively. DUCHEYNE 2014(2), 97-98 contains references to literature that considers 's Gravesande as a follower of Newton's methodology. Many of 's Gravesande's own statements of his adherence to Newton's methods can be found in *ibid.*, 100-104. Qua methodology, Ducheyne has mostly focused on Newton's use of the *regulae philosophandi*, his quest in the *Principia* for mathematical relations that are both necessary and sufficient, and the use Newton made of successive approximations in that same book. Ducheyne's conclusion is that «none of these salient features of Newton's methodology and physico-mathematics were emphasized by 's Gravesande in his *Physices elementa*», see DUCHEYNE 2014(2), 104.

As becomes clear from both Ducheyne's work and that of previous historians who have summarized 's Gravesande's assertions on these matters, 's Gravesande's statements on what he himself called the Newtonian methodology did not amount to a full-fledged methodological programme. Instead, what the literature tells us is that 's Gravesande was mostly concerned with stressing the need for a combination of experimental and mathematical methods, a combination which he claimed no one before Newton had actually followed or had even proposed as the method to be followed.[4] Although this shows that 's Gravesande was without a doubt inspired by Newton's natural philosophy, this minimalistic description does not tell us much about the way 's Gravesande performed his own 'physics'.[5]

Consequently, the situation at present is that we know that 's Gravesande cannot simply be described as a 'methodological Newtonian' but that we do not have an alternative to replace this view with. Given 's Gravesande's pivotal role in eighteenth-century physics, this situation is of course unsatisfactory. It is a well-known fact that the status, contents, and relations between the so-called fields of natural philosophy, physics, and the mathematical sciences underwent significant changes in the eighteenth century.[6] Since 's Gravesande was one of the most widely read contemporary authors in the amalgam of these fields, it is of great interest to understand how 's

4 As is indicated by for instance RUESTOW 1973, 121; DE PATER 1994, 262; DE PATER, 1995, 222; SCHUURMAN 2004, 141-142; SCHLIESSER 2011; 115, 119; JORINK, ZUIDERVAART 2012, 34-35; see 'S GRAVESANDE 1723, last page of the unnumbered preface «ad lectorem», for the claim that Newton was the first to propose this method of reasoning from the phenomena while rejecting hypotheses.

5 Besides Ducheyne, only MAAS 2012 has recently tried to point out particular characteristics of 's Gravesande's physics other than his alleged Newtonianism. Maas has discussed in particular 's Gravesande's attempts to limit human interference and individual imagination, see MAAS 2012, 117, 131-132.

6 For a recent discussion of these disciplinary changes, see HEILBRON 2011.

Gravesande aimed to contribute to them and what he took these fields to be about. Yet, as becomes clear from Ducheyne's conclusions, we cannot simply assume that what 's Gravesande himself claimed to do connects unproblematically to what he did in reality. Studying his epistemology, methodology, and the relations between the two both in precept and in practice is therefore particularly useful to discussions of what eighteenth-century physics consisted of.

In this article, I will make a modest contribution to a more productive understanding of these issues by focusing on one particular part of 's Gravesande's methodology, namely the application of mathematics in philosophy and physics. As becomes already clear from the title of 's Gravesande's book, he considered mathematics to be central to physics. As we will see throughout, 's Gravesande also repeatedly stressed the importance of what he called mathematical methods and mathematical evidence to philosophizing in general, in particular with respect to finding certain knowledge. This, together with the fact that mathematics was of key importance in Newton's natural philosophy, turns the question of whether and how 's Gravesande actually applied mathematics in his own scholarly work into one of particular relevance.

So far, little has been written on this account: Ducheyne's study confirms that 's Gravesande considered mathematics central to physics, but Ducheyne, following his remark that 's Gravesande did not «go into the details on […] how mathematics and experimentation are to be integrated exactly», has not elaborated much on how 's Gravesande would go about this in practice.[7] John L. Heilbron, on the other hand, has recently even claimed,

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7 Ducheyne 2014(2), 102.

without providing much evidence, that 's Gravesande's books, «[d]espite their titles, […] contain very little mathematics».[8] Even if this were indeed the case and there were a serious tension between what 's Gravesande said and what he did, the question why he stressed the role of mathematics would still remain of substantial importance.

The only study that has treated the role of mathematics in 's Gravesande's work to some length is the dissertation of Giambattista Gori published in 1972, *La fondazione dell'esperienza in 's Gravesande*. Unfortunately, recent scholarship has not paid sufficient attention to Gori's work; language issues are at least partly to blame for this. Interestingly enough, Gori has pointed out that there is an apparent leap in 's Gravesande's work. In the latter's more philosophical works, mathematics had according to Gori a «generic task» whereas in 's Gravesande's physics, mathematics played the more obvious role of demonstrating relations quantitatively.[9] Gori has argued that we need to «find a unitary answer to the problem of mathematics in 's Gravesande» in order to close the gap between 's Gravesande's philosophical reflections and his scientific content.[10] Although Gori has softened this tension by pointing out that mathematics played a subsidiary role in physics as well—as a true experimentalist, 's Gravesande had strong doubts about finding direct correlations between mathematics and reality—Gori has not really provided

8 HEILBRON 2011, 176. Heilbron extends this claim to 's Gravesande's younger colleague and eventual successor Petrus van Musschenbroek; a comparison between Van Musschenbroek and 's Gravesande on this point might be of great interest but falls outside the scope of this article. See also VANPAEMEL 2003, 210: «'s Gravesande […] gradually made the mathematical proofs disappear in the successive editions of his *Physices elementa mathematica*». Quite the opposite is true, as will become abundantly clear in the next sections: mathematical proofs multiplied in successive editions of the book.

9 GORI 1972, 178-203, in particular 190-192.

10 *Ibid.*, 192. «Tuttavia rimane l'esigenza di trovare una risposta unitaria al problema della matematica in 'sGravesande».

the unitary answer he was looking for himself.[11] Here, instead, I will show that we can narrow the gap between the two different applications of mathematics in ’s Gravesande by regarding both from a methodological perspective.

In what follows, I will explore these different uses of mathematics in the philosophy and physics of ’s Gravesande in detail. A preliminary section summarizes his more programmatic utterances about the methodological use of mathematics. After that, I first address ’s Gravesande’s applications of these methodological statements in the actual practice of his physics; in doing so, I challenge Heilbron’s conclusion about the importance of mathematics in ’s Gravesande’s *Physices elementa mathematica*. The last section of this article explicates how ’s Gravesande’s application of mathematics to philosophy in general was part of one overarching epistemological goal, namely that of finding certain and true knowledge in physics as well as in other scholarly disciplines. As such, this article provides us with a more articulated view on what ’s Gravesande regarded as the subject of *physica* and the proper aims and practices of this discipline.

2. ’s Gravesande on the applications of mathematics

The claim that the study of mathematics was of great use to philosophers can be found consistently throughout ’s Gravesande public lectures, the prefaces of his books, and the methodological chapters of those books. He provided two main arguments for this usefulness. As we will see, the first

11 *Ibid.*, 196-203. Gori does not explicitly come back to the question of mathematics in the section in which he raises it, and neither does he return to the role of mathematics afterwards.

was that knowledge of mathematics was a necessity for philosophers who wanted to do physics. Besides that, according to ’s Gravesande, mathematics was particularly useful for learning «the art of reasoning». Mathematics was the quintessential example of a field that could teach us the logic and rigour needed in order to find true propositions, he claimed in an oration held at Leiden University in 1734:

> It remains for me to say a few things about mathematics […] Not only is it the key to physics, and many of its parts treat of pure physics: but where one deals with the art of reasoning, and of the rules of revealing the truth in any science whatsoever, mathematics has an astonishing use: therefore it is not to be separated from philosophy.[12]

‘Philosophy’ was defined by ’s Gravesande in the same oration literally as the love of wisdom. It included all scholarly pursuits—different disciplines such as physics, history, and logic, to which ’s Gravesande referred as «sciences»—but was not restricted only to the scholarly disciplines themselves. Therefore, the second use of mathematics would apply basically to all of humanity.[13] On its first use, that in physics, ’s Gravesande argued that in fields such as mechanics, astronomy, and hydrostatics we dealt primarily with motion. Consequently, we would need to make use of quantities because motion was best expressed as such:

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12 ’s Gravesande 1734, I, 45: «Superest ut pauca de Mathesi addam […] Non tantum est clavis Physices, & multae illius partes mera Physica tractant: sed ubi de Arte ratiocinandi, & regulis detegendi Veri in scientia quacumque, agitur, mirum usum Mathesis habet: quare a Philosophia separanda haec non est».

13 *Ibid.*, 26: «Philosophia, ut ipsum nomen indicat, est Sapientiae amor […] Hominibus omnibus necessaria est, ad omnes extendit, & in omni vitae statu, & ultra illas disciplinas sese expandit quae vulgo ad ipsam referuntur». I will come back to ’s Gravesande’s views on disciplinary demarcations below.

> All things in physics are accomplished by motion; because no change can be made to bodies, or at least be perceived by us, except that which is made by motion or produces motion […] But motion itself is a quantity; it can be increased and diminished; whatever therefore attends to [motion], that is, all in physics, ought to be treated mathematically.[14]

Since mathematics was according to ʼs Gravesande the field of philosophy that treated of quantities, we evidently would need mathematics in order to do any serious work in mechanics, astronomy and hydrostatics as well. In turn these disciplines, and therefore mathematics, would give us useful knowledge in such practicalities as navigation, water management, and time-keeping.[15]

Both of these uses of mathematics appeared already in the oration ʼs Gravesande held when he accepted the chair of mathematics and astronomy at Leiden University in 1717,[16] when he was only in his late twenties. It is noteworthy that the topic of this inaugural lecture was exactly the use of mathematics and that ʼs Gravesande was initially appointed to a chair of mathematics and astronomy in Leiden. This of course meant that defending his discipline and pointing to its potential uses was of interest to ʼs Gravesande for more than purely intellectual reasons; therefore, we should not jump

14 ʼs Gravesande 1717, 15: «Motu omnia in Physica peraguntur; nulla enim mutatio in corporibus fieri potest, aut saltem a nobis sentiri, nisi quae motu fit aut motum producit […] Motus autem est quantitas; augeri & minui potest; quidquid ergo ad illum spectat, id est, tota Physica, Mathematice tractari debet». See also ʼs Gravesande 1720, sixth page of the unnumbered *Praefatio*: «Agitur ubique [in Physicis] de motuum collatione, id est, de quantitatum comparatione, circa quam qui demonstrationibus Mathematicis in ratiociniis non progrediatur, si non in errorem, saltem in incertas conclusiones incidet». Although the title page gives the year 1720, the first volume appeared in fall 1719; see also the *Privilegie* on the first page of the book, dated 8 November 1719.

15 See ʼs Gravesande 1717, 16-17 for his argument for the necessity of mathematics in mechanics and hydrostatics. Later parts of this text deal extensively with astronomy; see *ibid.*, 2 for potential applications of mathematical physics in navigation and time-keeping.

16 *Ibid.*, 2-3.

to conclusions about the aims of his 1717 lecture too soon. As I have pointed out elsewhere, most of ’s Gravesande’s earlier work was in mathematics, for instance in statistics and in the mathematical study of perspective. ’s Gravesande would only later become officially affiliated with the studies of physics and philosophy, the disciplines in which he would become most famous.[17]

However, that we should not merely interpret his discussion of the use of mathematics as defensive rhetoric becomes clear from the fact that the two uses of mathematics would continue to be central to ’s Gravesande’s discourse throughout his entire career. The oration of 1734 cited above was in fact held because of his appointment to the more prestigious chair of philosophy, still at Leiden University, which he would combine with his chair in mathematics and astronomy.[18] After this promotion, ’s Gravesande would start to teach additional courses in philosophy proper, but, as we have seen, the uses of mathematics remained important to him. Mathematics also figured as the privileged way of finding true propositions in his *Introductio ad philosophiam* of 1736, a book devoted predominantly to metaphysics and logic.[19]

Consequently, it seems evident that ’s Gravesande’s pleading for the use of mathematics was sincere. Yet, he was not an uncritical devotee of mathematics: he also addressed the limits of its applications. Wherever he would argue that mathematics was key to doing physics, ’s Gravesande also claimed that one could not draw physical conclusions on the basis of mathematics alone. As I will discuss more extensively in the last part of this article, ’s Gravesande’s argument was that abstract mathematics pertained only to

17 Van Besouw 2016.
18 See the title page of ’s Gravesande 1734.
19 ’s Gravesande 1736, see for instance 139, point 465; and 233, point 663.

our ideas, and not directly to the reality outside of us.[20]

Because of this, 's Gravesande also made quite negative remarks about pursuing mathematics just for its own sake on a number of occasions. In the preface of a book intended for the teaching of the basics of mathematics, he for instance first repeated his claim that this study could help us to train our mind, but after that added that he who would study mathematics without considering its applications «does not learn the proper mathematical method, but creates a disposition suitable only for reasonings about quantity».[21] According to his friend and biographer Jean Allamand, 's Gravesande used mathematics first and foremost as a means to get utile results for society and even disdained those «calculators» whose inquiries led only to pure speculation and not to any use for the other sciences or humanity in general.[22]

To sum up, 's Gravesande claimed that mathematics was useful predominantly or even exclusively because of its possible applications. He discussed two main lines of those applications: that of teaching the way of reasoning needed for finding true knowledge on the one hand, and its practical uses in physics and in technological endeavours on the other hand. Clearly then, 's Gravesande regarded mathematics mainly as a methodological tool, a

20 This argument is present in virtually all of 's Gravesande's methodological discussions. Perhaps the most influential locus where he makes this point is the preface of the first edition of the *Physices elementa mathematica*. See 's Gravesande 1720, seventh page of the unnumbered *Praefatio*.

21 's Gravesande 1727, *Praefatio*: «Qui enim dum Mathesi animum applicat, relique negligit, non proprie methodum Mathematicam addiscit, sed solis ratiociniis circa quantitatem ingenium aptum facit».

22 Allamand 1774, xxiii: «Il méprisoit ces Calculateurs de profession, qui passent leur vie à la recherche de vérités de pure spéculation, dont la découverte n'est d'aucune utilité soit pour les autres sciences, soit pour les besoins de la vie». See for instance also the preface to 's Gravesande 1711, his first book, dedicated to the mathematics of perspectives. In this preface, 's Gravesande argues that he wants to find a middle ground between mathematical theory and application of that theory, explicitly in order to be of use to the painters that need to apply the theory of perspective.

catalyst for the development of philosophy in general and physics in particular.

3. Mathematical physics

The last of these two uses of mathematics, its application to physical problems, seems relatively straightforward. Yet, we have already seen that, according to Heilbron, there is very little mathematics in the *Physices elementa mathematica*. Although most historians argue for the more nuanced position that there is in fact an important mathematical component in the book, it has often been stressed that the most important characteristic of 's Gravesande's physics was its experimental nature. Following a single letter written by 's Gravesande to Newton in 1718, we often read that an important aim of the former's book was to make Newton's *Principia* accessible to those without mathematical training.[23] Indeed, in the letter we read that 's Gravesande had to demonstrate Newton's conclusions by experiment because his audience in general would not really understand mathematics:

> [A]s I talk to people who have made very little progress in mathematics I have been obliged to have several machines constructed to convey the force of propositions whose demonstrations they had not understood.[24]

Although the machines mentioned in the letter indeed figure prominently in his book, we should begin by taking into account that 's Gravesande's letter to Newton talks about the former's lessons rather than about his book, which

23 See for instance Nyden 2014, 213, 218; Jorink, Zuidervaart 2012, 36.
24 Willem Jacob 's Gravesande to Isaac Newton, 13/24 June 1718. This letter is printed in Hall 1982, 26.

still had to be written in 1718. Still, the full title of the book, *The basic mathematical principles of physics, confirmed by experiments; or, an introduction to Newtonian philosophy*,[25] does indeed indicate that ’s Gravesande ‘confirmed’ his physics via experiments. Moreover, anyone who will leaf through the first edition published in 1719 will agree that it indeed contains little mathematics. In general, only elementary operations on proportions between physical entities such as forces, velocities, and distances can be found in this edition. There are some places where ’s Gravesande touched upon non-trivial mathematics, but the sole place where more than basic mathematical skills were required from the reader seems to be his chapter on the rainbow. In that chapter, which counts seven pages out of a total of roughly 400, multiple refractions and reflections led ’s Gravesande to compute among other things the proportions between different arcs.[26]

Clearly, the question of what was mathematical about ’s Gravesande’s actual physics remains to be answered. It seems that the conclusion that there was little mathematics in ’s Gravesande’s *Physices elementa mathematica* is warranted, and it becomes clear where Heilbron’s argument comes from. However, as will become evident here, ’s Gravesande’s omission of heavy math-

25 The contemporary English translations by J. T. Desaguliers, of which the first edition was published in 1719 and the sixth and last in 1747, has the straightforward title *The mathematical elements of natural philosophy* […]. I wish to avoid such a translation of the Latin word ‘elementa’ with the English ‘element’ because of the latter’s connotation of being some small part of a greater whole. The Latin ‘elementa’ does not obviously have this connotation and it is not the case that ’s Gravesande tried to give just some of the mathematical principles of physics.

26 See ’s Gravesande 1721, 92-98 and plate 18. Besides this chapter, dense mathematical language can be found only on a singular page, namely *ibid.*, 162, on gravitational interaction between the Moon and the Earth. The reference is here to the second volume of the first edition. As this volume is numbered separately from the first and the date on the title page differs between the volumes as well, I will refer to the specific volume throughout and will distinguish the two in the bibliography.

ematics was grounded in educational considerations rather than methodological ones. If we look deeper, we will see that mathematics did in fact play an important methodological role in the book. Moreover, from the second edition of the *Physices elementa mathematica* on, this role reveals itself much more openly. Since such differences between the editions have not received sufficient attention in previous studies, I will discuss their various contents in some detail. It will become apparent that the book did much more than just making Newton's physics «accessible to those without advanced mathematics». This section ends with one example of 's Gravesande's application of mathematics to an experiment in physics. From that example, we can draw some conclusions about what role mathematics played in 's Gravesande's physics in general.

In the same note to the reader in the second edition in which 's Gravesande argued that one should follow Newton's method rather than Newton's opinions, he also pointed to the main difference between the two editions of his work, this being that he had taken the trouble in the second to make his book more valuable for those with extended mathematical training:

> So that [the first edition of] the book would be useful especially to beginners, I left everything difficult untouched, I often indicated that propositions, to which I only referred, had been proved by geometers. However, so that this second edition would be of use, and to readers more versatile in mathematics, I have added mathematical demonstrations to all such propositions, in *scholia* annexed to those chapters wherever they have been indicated.[27]

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27 'S GRAVESANDE 1725(1), first of the unnumbered pages of the *Monitum de hac Secunde Editione*: «Cumque, ut tironibus praecipue liber hicce utilis esset, difficiliora omnia intacta relinquerem, saepe propositiones indicavi, de quibus tantum monuji, has a Geometris probari. Ut autem secunda haec editio, & lectoribus magis in Mathematicis versatis, usui esset, propositiones tales omnes, in capite quicunque indicatas, Mathematice demonstratas in scholiis, capitibus subjunctis, adjeci».

And this is indeed what one finds in the second edition, as well as in the third, of the *Physices elementa mathematica*. The second edition of 1725 contains roughly 150 pages more than the first, the third edition of 1742 adds more than 500 pages to the second one. The typesetting remains the same throughout all editions apart from the *scholia* which are «printed in minor character so that those other readers [not versatile in mathematics] are not disturbed».[28] That many of the additions to the second concern 'difficult things' left out for didactic reasons in the first edition becomes clear as well from the 'supplement' version of the 1725 edition. This supplement was created for those who already owned the 1720 edition. Roughly half of its 174 pages concern things which are clearly mathematical in nature; the rest of the supplement relates mostly to 's Gravesande's own work in the *vis viva* controversy: as is well known, 's Gravesande changed from the 'Newtonian' to the 'Leibnizian' measure of force in 1722.[29] The supplement contains the experiments which led 's Gravesande to change his initial position as well as the implications this change had on his discussion of such topics as composite motions and the mechanics of fluids.[30]

Some examples of actual mathematical problems addressed in the *scholia* of the 1725 edition might serve to prove that 's Gravesande did not add merely trivial mathematics to his physics: in the opening chapters on general philosophical ideas of bodies, the concept of divisibility and its infinite application to extension led him to introduce the logarithmic spiral and to discuss different classes of infinities; these discussions served mainly to show the possibility of an infinite contained in a finite.[31] To his considerations of

28 *Ibid.*: «Et ne haec lectores alios turbarent, ipsa minore charactere imprimi curavi».
29 This story is best told in HANKINS 1965, 286-291 and ILTIS 1973, 358-363.
30 'S GRAVESANDE 1725(2).
31 'S GRAVESANDE 1725(1), 9-12 and plate I. The status of the infinite was something to

pendulums, which were of course used for experiments both on collision and on free fall as well as for their use in controlling time, 's Gravesande added *scholia* on the properties of cycloids and showed how to determine the centre of oscillation.[32] These *scholia* did not merely constitute an exhibition of mathematical knowledge. In the first edition, 's Gravesande had warned his readers that his demonstration of the fact that a pendulum performs its respective oscillations in equal times held only under certain conditions. This passage was no longer needed in the second edition because the *scholia* would explain the properties of oscillating pendulums in mathematical detail.[33] Moreover, in the first edition 's Gravesande had to state in the same chapter that certain things had been «demonstrated further by geometers»; in the second edition, his readers no longer had to take him for his word on this as he actually provided the proofs himself in the new *scholia*.[34]

Likewise, for his chapter on central forces, which was largely concerned with experiments performed with an elaborate instrument developed by himself, 's Gravesande added no less than twelve pages fully filled with *scholia* running from the determination of circular motions to the determination of ellipses, and then via accelerated elliptical motions to the «computation of the movements of the apsides in curves very little different from the circle».[35]

which 's Gravesande returned on different occasions. See for instance 's Gravesande 1717, 13, and references in Gori 1972, 190-191. These discussions must be understood in the context of ongoing debates on the status and foundations of the concepts of the infinite and the infinitesimal. Discussion of these issues in the late seventeenth century can be found throughout Mancosu 1996.

32 's Gravesande 1725(1), 71-75 and plate XI;

33 Compare 's Gravesande 1720, 45 with 's Gravesande 1725(1), 67.

34 's Gravesande 1720, 46: «Demonstratur ulterius a Geometris […]», compare the same passage in the second edition, 's Gravesande 1725(1), 68: «Ulterius in primo scholio demonstramus […]».

35 's Gravesande 1725(1), 98-109 and plate XV, see p. 108: «De computatione motuum apsidum in curvis parum cum circulo differentibus».

These *scholia* obviously had important applications in astronomy as set out by Newton in the *Principia*.

Naturally, it is in this last case unlikely that ʼs Gravesande added much new to the already existing mathematical treatments. We can assume that his discussion of central forces relied on the revolutionary treatments of Christiaan Huygens and Newton of 1673 and 1687 respectively, even though all references in these twelve pages are either to Euclid's Elements or to La Hire's 1673 *Nouvelle méthode en géométrie pour les sections des superficies coniques et cylindriques*.[36] As becomes clear from reading these particular *scholia*, however, these references are not intended to credit La Hire and Euclid for the mathematical results in the *scholia* but rather to point to demonstrations for particular inferential steps ʼs Gravesande took. His *scholia* were, as noted above, themselves often mathematical demonstrations of propositions he used elsewhere and these additional references simply served to complete the mathematical proofs. ʼs Gravesande nowhere claimed that he himself was the first to provide these demonstrations.

In fact, quite the opposite was true. In the first edition of his book, he had argued that, since he published only the 'basic principles' of physics, references to other works were unnecessary as most was already known.[37] Although the second edition clearly contained more than only basic principles, ʼs Gravesande apparently did not change his mind about referring to other works. It was only in the third edition that he provided a bibliographical

36 *Ibid*.

37 ʼS GRAVESANDE 1720, tenth page of the unnumbered *Praefatio*: «He who draws up the basic principles of a science, does not offer as much new material to the learned world; and for that reason, I have considered it useless to remind [the reader] where what is treated here would be found»; «Qui scientiae elementa conscribit, non quid novi, quantum ad materiam, Orbi Litterato pollicetur; ideoque inutile duxi monere ubi reperiantur quea hic traduntur».

essay so as to credit others and to refer his readers to the places where they could find more.[38] Yet, these references cannot be found in the body of the text itself, and therefore more work would be required to determine on whom 's Gravesande built in the *scholia* to the *Physices elementa mathematica* and to what extent he contributed to mathematics itself. To answer this question, it would be necessary to study particular mathematical *scholia* in detail. This would however fall outside of the scope of this study. Rather than claiming that 's Gravesande published new mathematical results, my aim here is to show that he was in fact applying advanced mathematics in his physics. That 's Gravesande was a gifted mathematician has been proved before, in particular with regard to his *Essai de perspective* published in 1711.[39]

The subject of this *Essai* was the mathematical study of perspective, which was of course related to optics. We can infer from this that 's Gravesande also knew how to apply mathematics to other fields than mechanics, to which all of the above mentioned examples relate. In the second edition of the *Physices elementa mathematica* of 1725, however, we can still only find *scholia* related to either mechanics itself or to fluid mechanics. The reason for this is that the first edition of the book was divided into two volumes, the first concerned with mechanics and the study of fluids, almost exclusively fluid mechanics, whereas the second volume dealt with astronomy and the study

38 'S GRAVESANDE 1742, vol. I, *Praefatio hujus tertiae editionis*, xv-xxxv, see in particular xviii: «In praecedentibus editionibus non indicavi ubi habeantur illa, quae ex aliis desumsi, qoud a multis improbari percepi». It becomes clear from the respective prefaces of his work that 's Gravesande himself cared little about intellectual credit. I plan to discuss this elsewhere in more detail.

39 'S GRAVESANDE 1711. Many aspects of this book have been studied by ANDERSEN 2007, 328-360; see also the shorter treatment in CANTOR 1908, 594-597. For the importance of this work to 's Gravesande's early career, see VAN BESOUW 2016, 238-242. Some aspect of 's Gravesande's mathematics have been discussed in SHOESMITH 1987, as well. An overview of 's Gravesande's mathematics would be extremely useful but does not exist as yet.

of light—its bulk being optics but some chapters on fire and electricity were included as well. In the 1725 edition, only the first of these volumes was updated,[40] and its new mathematical *scholia* consequently related exclusively to mechanics and fluids. With regard to the latter, ’s Gravesande in 1725 for instance treated the deceleration of bodies moving in fluids and from this also came to discuss logarithms. With these mathematical treatments, he was able to differentiate between two types of deceleration, one where equal decreases of velocity took place in equal times, and another where the decrease of velocity was proportional to the square of that velocity.[41]

There is one single instance of an application of mathematics in this 1725 edition that starts to cross the boundaries of fluid mechanics. In the last part of the first volume, still in the half of that volume dedicated to fluids, ’s Gravesande discussed the properties of air, which he called ‘an elastic fluid’, as well as the density of air and the propagation of sounds. Many of the arguments made in these last chapters concerned experiments made with air pumps.[42] In earlier classifications, these chapters could have been described as belonging to the field of pneumatics rather than to that of fluids. Two *scholia* were added to this part on air in the second edition of the *Physices elementa mathematica*. For our purposes, the first is the most interesting, as ’s Gravesande there set out to show that the particles of air would move according to the same mathematical law as a pendulum vibrating in a

40 ALLAMAND 1774, xxx, states that «les changemens faits au second [Tome] étoient peu considérables» in the 1725 edition, but as far as I can see there were no changes whatsoever. All of the editions of the 1725 editions I have been able to locate either do not include the second volume at all or contain a copy of the original second volume as it was printed first in 1721. This is confirmed by the list of DE PATER 1988, 152.

41 ’S GRAVESANDE 1725(1), 283-296 and plate XXXVII. These two types of deceleration are closely connected to the *vis viva* controversy discussed above and therefore relate to ’s Gravesande’s new discussion of mechanics as well.

42 ’S GRAVESANDE 1720, 158-188.

cycloid,[43] thus connecting the study of air with the better known case of the mechanics of free fall.

Many more mathematical demonstrations were added in the *scholia* of the last edition of the book, published in 1742. This edition was published again in two volumes, but both of these single volumes are larger in themselves than the two volumes of the first edition taken together. As in that first edition, the first volume of the 1742 print concerns mechanics and fluid mechanics, but the parts on the mechanics of air are shifted to the second volume, where they are combined into «Liber IV»[44] with the parts on fire, now taken out of the earlier parts on optics. Together, the discussions of air and fire, as 's Gravesande called them, rather than pneumatics and heat, take up 127 pages, but no *scholia* are added to these parts besides the two already existing in the 1725 second edition. Besides these parts, however, new *scholia* can be found anywhere in the 1742 edition, both in the first volume on mechanics and fluids as well as in the parts on optics and astronomy in the second volume. Interestingly, the computations on the rainbow, singled out above as the only case of significant mathematical content in the 1720 edition, are no longer found in the main text of the third edition, but are instead put in an additional *scholium* to the chapter.[45]

Clearly, mathematics played an important part in the *Physices elementa mathematica*, in contrast to what has sometimes been assumed. But what was the exact role of such mathematical additions to 's Gravesande's practice? A more detailed example might help to answer this question. In one of his most

43 'S GRAVESANDE 1725(1), 342-344 and plate XLVII.

44 See the list of contents, 'S GRAVESANDE 1742, vol. I, lxx-lxxi. «Liber IV» of course translates as «book IV», but I will avoid this translation in order not to create confusion.

45 *Ibid.* II, 918-920; compare with 'S GRAVESANDE 1721, 93-97.

famous experiments, discussed at length in the last edition of his work, 's Gravesande dropped copper balls into a tray filled with clay in order to determine the 'force' the balls would acquire during their fall. With this experiment, he set out to demonstrate that this acquired force would be proportional to the height of the fall and therefore, via the relations known from Galileo's work, to the square of the velocity of the ball. This would show that the force acquired in free fall was a concept that needed to be kept apart from the concept of momentum, which was proportional to the velocity itself. By making this distinction 's Gravesande attempted to solve the *vis viva* controversy mentioned above. According to 's Gravesande, the force of the balls could be found by the effect they had on the clay while coming to rest, that is, by the volume of the cavity they made in the clay.

To make his experiment yield results that could be generalized into a relation between force and velocity, 's Gravesande made different trials with balls of known weights and velocities. He performed the experiment with balls of equal volume but with weights in ratios 1, 2, and 3 to each other, and had a machine built with which he could vary the heights of fall. These different heights stood in fixed ratios of 1, 2, 3, and 4 to each other. These simple relations enabled 's Gravesande to give the theoretically expected proportions between the cavities with ease: given that the force would be as the weight multiplied by the height of the fall, the force, and therefore the cavity it made in the clay, of a ball with weight 3 and height 2 would be expected to be six times as large as that of a ball with weight 1 and height 1. 's Gravesande compared these theoretical values with the volumes of the cavities he found in the actual experiments, and concluded that these experimental values showed the same proportions and thereby confirmed his theoretical measure of force.[46]

Thus, 's Gravesande used simple mathematical proportions in order to compare the theoretical values for the forces with values he could find in a tightly controlled experiment. This way, he was able to generalize the quantitative results of his experiment into a more complex relation between different physical concepts, those being mass, height, velocity, and force in this case. Yet, this is not the only way in which mathematics was involved in this experiment. When 's Gravesande compared the theoretical value with the volume of the cavity yielded by the experiment, he could not measure this volume directly. Instead, he measured the diameter of the cavity and referred to a table (cf. Table 1, Appendix) he provided in a *scholium* in order to find the volume from that diameter. As far as I am aware, 's Gravesande nowhere discussed how to compute these volumes out of the diameter.[47] Nevertheless, this was not a particularly straightforward computation. The values 's Gravesande actually compared were those of the third column of the table. These values are the volumes of the cavity in proportion to the volume of the

46 'S GRAVESANDE 1742, vol. I, 243-244, see also 235-237 and plate XXXII. My understanding of 's Gravesande's experiment, and in particular of the difficulties involved in drawing out the results, has benefited enormously from discussions with Tiemen Cocquyt and Ad Maas, as well as from their reenactment of the experiment: «The truth in a layer of clay: A replication of 's Gravesande's vis viva experiment» during the international workshop «Early eighteenth-century experimental philosophy in the Dutch Republic», 7 July 2014, Royal Flemish Academy of Belgium for Science and the Arts, Brussels. Cocquyt and Maas are preparing a study of this experiment for publication. ILTIS 1973, in particular 359-362, and COSTABEL 1964 are prominent older interpretations of the experiment. Iltis offers a discussion of the context in which 's Gravesande first performed these experiment as well as a short description of her own repetition of one of his experiments. Costabel describes the experiment in detail and focuses on the experimental difficulties. Although his discussion is useful, the accuracy of Costabel's conclusions suffers from the vehemence of his anti-positivism.

47 For the table see 'S GRAVESANDE 1742, vol. I, 246-247. Three more scholia follow directly after this one, but these are mostly concerned with finding the time in which the cavity is made after impact. 's Gravesande found different curves to express this time for different figures; see in particular *ibid.*, 252-254.

hemisphere of the ball dropped down; this volume of the hemisphere is set to the value 1,000 in the table. To find the number in the third column, one first needs to calculate the height of the cavity with the formula:

$$h = R - \sqrt{R^2 - r^2}$$

with R the radius of the ball itself and r the radius of the cavity, i.e. half its diameter. This relation follows easily from elementary geometry but in order to compute the values of h, one still has to approximate the value of the square root. Furthermore, in order to find the volume of the cavity and state this in proportion to the volume of the ball itself one needs to make use of the following two formulas:

$$V_{cav} = \frac{\pi h}{6}(3r^2 + h^2)\, ;\, V_{hemisphere} = \frac{2}{3}\pi R^3$$

With V_{cav} the volume of the cavity and $V_{hemisphere}$ the volume of half the ball. To find the values of the third column, one then has to find the proportions between these volumes. Inserting the above formula for h and multiplying by 1,000 gives the following recipe to find S, the proportional volumes of the cavities that ’s Gravesande actually compared and that are stated in the third column of table 1:

$$S = \frac{10^3}{2R^3}\left(R - \sqrt{R^2 - r^2}\right)\left(R^2 + r^2 - \sqrt{R^2 - r^2}\right)$$

Although none of these operations demands advanced mathematics, it seems reasonable to suppose that one needs to be at least comfortable with basic geometry. Similarly, finding dozens of numerical values of S, as ’s Graves-ande did in his *scholium*, would have costed a significant amount of time for one who did not deal frequently with such computational issues.[48]

In this particular case, ’s Gravesande used mathematics first of all in setting up his experiments in such a way that one could get useful but simple quantitative relations between the different entities encountered in the experiments. From these experiments and their simple results, he would then try to find more general relations by applying somewhat more difficult geometry and algebra. In the other examples discussed in this section, we have seen that he also applied more complex mathematics in order to discuss such things as the oscillations of pendulums. This same description of pendulums was furthermore used in a later chapter to describe the propagation of sounds. Thus, it seems that we should understand ’s Gravesande’s claim to perform mathematical physics most of all in the sense that he first worked with carefully set up quantitative measurements, and second that he used mathematical relations to derive more abstract generalizations from these controlled experiments. Evidently, mathematics played a significant methodological role in ’s Gravesande’s experimental physics.

4. Mathematical reasoning and mathematical evidence

Having said this about the use of mathematics in his physics, the question of what ’s Gravesande exactly meant with the other application of mathematics needs to be addressed as well. What about mathematics made it so useful to philosophy and the art of reasoning in general? To answer this question, we should begin to ask what it meant to think at all, according to ’s

48 In my case, I wish to thank Nigel Vinckier for his help in dealing with the mathematics involved. Getting the numerical values is of course unproblematic with modern technology; those of ’s Gravesande’s table generally seem to be correct. This indicates that his approximation of the square roots was performed with sufficient precision.

Gravesande: what was his description of the process of reasoning itself? ’s Gravesande spent ample time to explain exactly this in several of his works, and in particular in his last book, the *Introductio ad philosophiam*, which was used to teach logic and metaphysics. There, too, he claimed that mathematics was central to reasoning, as he argued that «by well ordered exercise in the mathematical sciences, the higher faculties of memorizing are made more perfect».[49]

The gist of his ideas on this topic was however already contained in the oration on mathematics ’s Gravesande gave in 1717, and it is illuminating to follow the argument presented there in some detail. According to ’s Gravesande, if we wanted to reason well we should first of all train our minds to do so. Like all other skills, reasoning could be improved by practice. Furthermore, he claimed that the art of reasoning was the same in all sciences. It would begin with comparing different ideas in our mind and judging whether these ideas would be similar or different. In all our reasonings, he asserted, we would therefore constantly make judgements about the agreement of two ideas.[50] This would be straightforward as long as ideas could be compared easily. The best thing about numbers, ’s Gravesande explained, was that comparing them posed no problems whatsoever:

> Of the idea of sums of numbers, for instance four and five, if they would be compared with the idea of the number nine, we will perceive with a single look that these do not to differ between them.[51]

49 ’s Gravesande 1736, 266: «In Mathematicis scientiis, exercitatione bene ordinata, perfectiores fiunt superius memoratae facultates».

50 ’s Gravesande 1717, 3-4.

51 *Ibid.*, 5: «Idea numerorum summae, ex. gr. *quator* & *quinque*, si conferatur cum idea numeri *novem*, unico intuitu videmus ideas has inter se non differre».

Yet, in most other cases such a straightforward comparisons could not be made. In these cases, ’s Gravesande maintained, we need to introduce «middle ideas». He continued to argue that we need to be very precise in making trains of thoughts and that we need to train our mind in finding the right middle ideas to connect the ideas that we want to compare; that we should not invoke things which are irrelevant; and that we should have evident foundations for our thoughts.[52] Most interestingly, however, was that ’s Gravesande claimed that even though it was the study of logic that would teach us the rules of reasoning, it was «mathesis [that] renders [them] truly familiar by continuous use, and by that study, the indispensable force of the mind of reasoning well is strengthened».[53] This, he repeated over and over, was the case because the object of mathematics was quantity and we understood the relations between quantities so well that we could not err in seeing whether quantities agreed with each other or not. Consequently, practicing our skill in reasoning would best be done in mathematics.[54] In a second oration held in 1724, ’s Gravesande made his point about the strength of mathematical reasoning even stronger:

> I would say that anybody, [even if this person] is not well versed in the mathematical disciplines, but is only a beginner, and somebody at the first steps, would perceive that these sciences lay claim to the particular method to prove

52 *Ibid.*, 5-6. According to SCHUURMAN 2004, 131-132, ’s Gravesande’s discussion of ideas and faculties of understanding, as set out most elaborately in the *Introductio ad philosophiam*, largely follows the tradition of the ‘logic of ideas’ developed by Descartes, Arnauld, Malebranche, and Locke; yet Schuurman insists that ’s Gravesande’s discussion of moral and mathematical evidence, to which I will turn now, contains many new insights.

53 *Ibid.*, 6: «Logica regulas in ratiocinando observandas exponit, Mathesis vero continuo usu familiares reddit, & in illius studio, mentis vis ad bene ratiocinandum necessaria corroboratur».

54 *Ibid.*, 7: «ut mens attenta nunquam erret in pronuntiando de illarum [quantitatum] convenientia aut dissensione».

> the truth; and that mathematical demonstrations are accompanied by evidence, which overcomes stubbornness that is invincible by all other means.[55]

Here again, we see that 's Gravesande stressed the methodological importance of mathematics. In this second oration, which had the title *De evidentia*, 's Gravesande's elaborated on the issue by distinguishing two types of evidence, mathematical evidence and moral evidence. Although this distinction has attracted quite some attention in recent years, the focus has generally been on moral evidence rather than on mathematical evidence because of the former's critical role in 's Gravesande's legitimation of empirical knowledge.[56] I will come back to moral evidence shortly, but our focus here should be on the role of mathematical evidence in reasoning. In 1724, 's Gravesande argued again that if we have two ideas in our mind, we will necessarily see whether they are similar or not:

> Therefore, these things are opposed: for the mind to perceive ideas, and it not to perceive a true comparison that is given between the ideas; from this [the mind] will be self-conscious of this perception, and it will have the persuasion that no doubt about this comparison is able to survive.[57]

The first example he provided for such a comparison was again one taken from elementary arithmetic, now between the number seven and the sum of

55 'S GRAVESANDE 1734, vol. II, 3. «Nemo non dicam in Mathematicis disciplinis versatus, sed in hisce scientiis tiro, & quidem in primo limine, non percepit, peculiarem probandae veritatis Methodum sibi vindicare sciencias hasce; Mathematicasque demonstrationes Evidentiâ concomiitari, pertinaciam omni alio modo invictam superante».

56 As becomes clear from the titles of GORI 1991 and DE PATER 1995. More recent treatments follow this pattern, see SCHUURMAN 2004, 141-146 and DUCHEYNE 2014(1), 41-43.

57 'S GRAVESANDE 1734 II, 6: «Pugnantia ergo haec sunt, Mentem percipere ideas, & hanc non percipere veram quae datur inter ideas comparationem; & eo ipso hujus perceptionis sibi conscia erit, persuasumque habebit dubium nullum circa hanc comparationem superesse posse».

the numbers three and four. He concluded that the immediate perception we have of such comparisons is the «foundation of mathematical evidence» and that this evidence itself was its own criterion of truth.[58] Right after this, 's Gravesande repeated that mathematics had «the privilege not to err» and recapitulated in three points «the object of *mathesis* and the method of mathematics».[59] In reverse order, these three points were that mathematicians proceeded from the simple to the more compound by the use of middle ideas; that mathematics treated of quantities, the easiest ideas to compare;[60] and, first in 's Gravesande's list, that:

> Mathesis treats of ideas, and of ideas only; and the mathematician, qua mathematician, attends not in the least to whether the ideas about which is reasoned correspond or not to any thing that exists.[61]

His point here was that mathematics would not treat of what we would call concrete things, but concerned only the abstract. 's Gravesande clearly made a distinction between what was in our mind, namely ideas, and that which was not, the concrete. If we wanted to reason about something concrete, as we would do for instance in physics, we first had to form an idea of it, and ideas were of course located in our mind. This formation of ideas of concrete things, the passage from the concrete to our mind, however, was according to

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58 *Ibid.*, 7: «veri desideratum criterium ipsam esse Evidentiam», 's Gravesande explicitly proposes this as the answer to a sceptical demand for a criterion of truth. See Gori 1991, 21 for discussion.

59 *Ibid.*, 8: «haud difficulter probabimus quare Mathesis sibi non satis aestimandum vindicet privilegium *non errare:* cujus ut pateat justus titulus, quaedam de Matheseos objecto, Mathematicorumque methodo breviter memoranda erunt».

60 *Ibid.*, 8-9.

61 *Ibid.*, 8: «Versatur Mathesis circa ideas, & circa ideas tantum; minimeque curat Mathematicus, qua Mathematicus, utrum ideae de quibus ratiocinantur cum ulla re quae est congruant an non».

’s Gravesande excluded from the discipline of mathematics. This was the case for a couple of other sciences as well, ’s Gravesande claimed. Among these were logic, ontology, and the foundations of ethics. These sciences did not address anything but ideas in the mind, excluding the world around us from their scope. Because of that, we could simply deduce propositions from self-evident first principles, that is, we could exclusively make use of mathematical evidence.[62] Other sciences, though, such as physics, history, and theology had no first principles that were self-evident and therefore could not rely purely on logical deduction: this is where we find one of the limits of the application of mathematics. Instead of on deduction, ’s Gravesande maintained, these sciences had to be founded on our observations of the world around us. Therefore, they had to make use of what ’s Gravesande referred to as moral evidence rather than mathematical evidence.[63] This way, ’s Gravesande explicitly distinguished between sciences that are, and the sciences that are not based on this so-called mathematical evidence.

If, according to ’s Gravesande, mathematical evidence was gained through comparison of ideas and strict deduction from such comparisons, this entailed that this type of evidence could only be gained from demonstratively true propositions, that is, principles of which the negation would be evidently false.[64] If the elementary rules of arithmetic are taken for granted, this is certainly the case for ’s Gravesande’s cherished example of $4 + 5 = 9$. Its opposite, that is, the negation of this statement, would be $4 + 5 \neq 9$.

62 *Ibid.*, 11-14.
63 *Ibid.*, 17-19.
64 Gori 1991, 21, rightly asserts that, for ’s Gravesande, «mathematics have their criterion of truth in the principle of contradiction», whereas this is not the case for «matters of fact», *ibid*. According to ’s Gravesande, a proposition and its opposite could both be possibly true if we talked about concrete things. Mathematical evidence on the contrary applied where the opposite of a true proposition was necessarily false.

Obviously, the first statement is correct and the second incorrect. Thus, we have mathematical evidence for a proposition if and only if we can demonstrate by deduction that it is correct in this sense.

For this to be possible, one of course needed to start the deduction from self-evidently true principles. One might grant that this is the case in arithmetic, but for a field like ontology, where according to ’s Gravesande mathematical evidence applied as well, this was much less obvious. One particular short example ’s Gravesande provided in the oration of 1724 might help to clarify his thoughts. This particular demonstration made use of two axioms. The first of these was that «there is something now, therefore there has been something from eternity»;[65] if this would not be true, ‘something’ had been created at a particular moment out of ‘nothing’, which ’s Gravesande clearly considered impossible.[66] His second axiom was «cogito ego»; ’s Gravesande clearly regarded both of these as self-evidently true. From the fact that he was thinking ’s Gravesande deduced that he was intelligent. Combining this with the first axiom, he concluded that the first cause also needed to be intelligent for otherwise intelligence would have come out of nothing, which would be impossible. Moreover, this intelligence must infinitely exceed all other intelligences in order to create them: this first intelligent cause of course would be God.[67]

Hence, ’s Gravesande claimed to have mathematical evidence for the existence of God as the first cause. From the fact that God must be infinitely

65 ’S GRAVESANDE 1734, vol. II, 12. «Aliquid nunc est; ergo aliquid ab aeterno fuit».

66 This is a version of the cosmological argument which posits God as the first cause of all things. In a manuscript published only posthumously, ’S GRAVESANDE 1774, in particular 176-179, ’s Gravesande developed this argument more completely. I will discuss this elsewhere in more detail.

67 ’S GRAVESANDE 1734, vol. II, 12.

intelligent, ʼs Gravesande furthermore set out to prove that we could have certain and reliable knowledge in the fields that depend on moral evidence. From God's intelligence, his goodness immediately followed according to ʼs Gravesande.[68] Moreover, since man is in need of knowledge of the world around him, God would contradict his own goodness if he would not have given man the means to acquire such knowledge.[69] Because of this, knowledge of things outside of our minds could be gained if we would carefully handle the three «aids» we had for finding it, these aids being our senses, the testimony of others, and analogical reasoning.[70] Therefore, ʼs Gravesande claimed to have demonstrated with mathematical evidence, the method exemplified by mathematics itself, that those sciences relying on mere moral evidence could give us true knowledge as well:

> [We] see how much the foundations of assent will differ for different circumstances. But even as these different foundations are allowed, and it is allowed that mathematical evidence would not in the least coincide with moral evidence, a different persuasion [for them] nevertheless does not follow from that.[71]

Thus, the type of deductive reasoning learned from mathematics was used by ʼs Gravesande to prove that the sciences that depended on moral evidence, such as physics, could deliver true knowledge if we used our resources well. Physics in particular, however, had another relation with mathematics as

68 *Ibid.*, 13.

69 *Ibid.*, 21-22.

70 *Ibid.*, 20: «Auxilia haec sunt Sensus, Testimonium, & Analogia». This argument is treated at length in for instance GORI 1991; GORI 1972, 228-265; DE PATER 1995, and STRAZZONI 2015, chapter VI

71 *Ibid.*, 24: «Videtis AA. NN. quantum different pro diversis circumstantiis assensionis fundamenta. Sed licet different fundamenta haec, licet Evidentia Mathematica minime cum Morali congruat, non tamen diversa inde sequitur persuasio».

well, as «physics pertains to mixed mathematics»,[72] according to ’s Gravesande. As we have seen, he argued that pure mathematics treated only of ideas. Yet, he also allowed for a «mixed mathematics» that reasoned «about the things themselves», that is, things outside our mind.[73] This type of mathematics clearly was not based on abstract, self-evident first principles, and therefore lacked the first of ’s Gravesande’s three reasons why mathematics would not err. Yet, physics, or mixed mathematics, did of course treat of quantities, the simplest ideas. It also proceeded from these simple ideas to more complex ones, and therefore followed the method of mathematics to a certain extent:

> When, in physics, we have properly cognized the phenomena from the aids of moral evidence, that is, when it is correct for us to hold ideas of these phenomena, [ideas] which agree with the things themselves, the reasonings about these ideas will be mathematically certain, and the conclusions can be applied to the things themselves.[74]

’s Gravesande furthermore explained that, in sciences such as metaphysics, we could depend solely on mathematical evidence because of the fact that they concerned ideas only. Yet, since these sciences did not follow mathematics in treating quantities, they could not define the terms they used as rigorously as mathematics could. Consequently, the axioms of metaphysics were less obvious to interpret and therefore conclusions could not be drawn

72 ’s Gravesande 1720, second page of the unnumbered *Praefatio*: «Ad Mathesim mixtam pertinet Physica». See Ducheyne 2014(2), 101 for ’s Gravesande on mixed mathematics.

73 ’s Gravesande 1734, vol. II, 8-9: «Matheseos partibus in quibus de rebus ipsis agitur […] Mixtam in hoc casu dicimus Mathesim».

74 *Ibid*., 19: «Ubi in Physicis moralis Evidentiae auxilio bene cognita habemus Phaenomena, id est, ubi constat nos horum Phaenomenon habere ideas, quae cum rebus ipsis conveniunt, ratiocinia circa has ideas Mathematice certa erunt, conclusionesque ad res ipsas poterunt applicari».

as easily as in mathematics.[75] In theology, this inexactness was present as well. Even worse, there was according to ’s Gravesande no way to give mathematical evidence that God «would have declared his divine announcement to mankind». Yet, when we had established via moral evidence that there was in fact such a divine announcement, ’s Gravesande asserted that the «the stability of the reasonings [that we base upon this divine announcement] will be mathematical» again.[76]

What we see is that ’s Gravesande discussed methodologies of different disciplines, and did so by comparing all of them to mathematics. Mathematics was the science where we could apply our inexorable logic most effectively, «but the method that mathematicians use, [could] be applied to all sciences», he claimed.[77] In theology, it was only the deductive logic that we could copy from mathematics, and only once we had established some stable starting points via moral evidence. In physics, observation and analogy were to provide these starting points for deductive reasoning as well, but we also had the benefit of dealing with quantities, the ideas with which we could avoid error easily. In metaphysics, on the other hand, we could not reason with quantities, but instead had the gain of being able to establish our first principles via mathematical evidence, that is, simply by demonstration from self-evident ideas.

75 *Ibid.*, 14-16.

76 *Ibid.*, 17-18: «an suprema & infinita Intelligentia voluntatem suam Hominibus peculiaribus declaraverit Oraculis […] hoc ex simplici idearum collatione nunquam determinari poterit: ubi autem de Oraculis constat, conclusiones, ex iis deducendae […] ad ideas spectabunt; & ratiociniorum stabilitas Mathematica erit». ’s Gravesande of course implied that such an announcement could be found in Scripture.

77 *Ibid.*, 10: «Methodus autem, qua utuntur Mathematici, omnibus scientiis potest applicari».

5. Conclusion

As has become clear in this article, 's Gravesande regarded the methodology of mathematics as a prototype of stable and certain reasoning. Mathematics and mathematical reasoning provided the way to achieve true knowledge in philosophy in general and in physics in particular. The 'art of reasoning' was best learnt through mathematics because it was in that science that we found its clearest application. Mathematics had three characteristics that led us to avoid errors: building trains of thoughts via rigorous deduction; handling only abstract ideas for which we did not need knowledge of the things themselves; and pertaining to quantities, the simplest ideas. All other sciences lacked at least one of these characteristics, but according to 's Gravesande would best follow the mathematical method with regards to the others as much as possible.

In its application to physics, mathematics played a methodological role, too, and it is here that we find the sort of unitary answer that Gori has asked for. First, 's Gravesande carried out quantitative measurements with great regard for detail. Because mathematics was according to 's Gravesande the study that related to quantities, these measurements were inevitably mathematical. Yet, 's Gravesande also applied higher level mathematics to his physics in order to compare different concepts, or ideas in his own vocabulary, to each other quantitatively. Contrary to what has been claimed in recent literature, he had solid reasons for calling his physics 'mathematical' as mathematics helped him to derive more abstract generalizations from his measurements. Clearly, we would, according to 's Gravesande's philosophy, have moral evidence for such generalizations. If we had been careful enough, these generalizations were to be considered as certain, and could be used as axioms or principles on which we could build further reasoning. For 's Gravesande, the role of mathematics was clearly to provide a rigorous and certain methodology to physics.

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Acknowledgements

This article has benefitted from comments by Steffen Ducheyne, Pieter Present, Yannick Van den Abbeel and the anonymous referees. I am grateful to Steffen and Pieter for their help with translations from Latin and to all participants of the workshop in Paris for useful discussion. My research has been made possible by funding of the Research Foundation - Flanders (project G002913N).

Quâ Sphæræ Segmenta, & Coni, ex datis Diametris, conferuntur, diviso Hæmisphærio in mille partes, & hujus Diametro in Centum. 867.

Diam.	Segment. Profund.	Segm.	Coni.	Diam.	Segment. Profund.	Segm.	Coni.
35.			23.	68.	13.	97.	172.
36.			25.	69.	14.	104.	179.
37.			27.	70.	14.	111.	187.
38.			30.	71.	15.	118.	195.
39.			32.	72.	15.	126.	203.
40.			35.	73.	16.	134.	212.
41.			38.	74.	16.	143.	221.
42.			40.	75.	17.	152.	230.
43.			43.	76.	17.	162.	239.
44.			46.	77.	18.	173.	249.
45.			49.	78.	19.	184.	259.
46.			52.	79.	19.	196.	269.
47.			56.	80.	20.	208.	279.
48.			60.	81.	21.	221.	290.
49.			64.	82.	21.	235.	301.
50.	7.	26.	68.	83.	22.	250.	312.
51.	7.	28.	72.	84.	23.	266.	323.
52.	7.	30.	77.	85.	24.	283.	335.
53.	8.	33.	81.	86.	24.	301.	347.
54.	8.	36.	86.	87.	25.	320.	359.
55.	8.	39.	91.	88.	26.	341.	372.
56.	9.	42.	96.	89.	27.	363.	385.
57.	9.	45.	101.	90.	28.	387.	398.
58.	9.	48.	106.	91.	29.	414.	411.
59.	10.	52.	112.	92.	30.	442.	425.
60.	10.	56.	118.	93.	32.	473.	439.
61.	10.	60.	124.	94.	33.	508.	453.
62.	11.	64.	130.	95.	34.	547.	468.
63.	11.	69.	136.	96.			483.
64.	12.	74.	143.	97.			498.
65.	12.	80.	150.	98.			514.
66.	12.	85.	157.	99.			530.
67.	13.	91.	164.	100.	50.	1000.	546.

TABLE 1: 'S GRAVESANDE 1742, I, 247. 's Gravesande referred his readers to this table in his famous *vis viva* experiment. The first column gives the diameter of the cavity, the second its computed height, and the third its volume in proportion to the volume of the ball itself. The fourth column concerns another experiment.

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