The Roles of One Thought Experiment in Interpreting Quantum Mechanics. Werner Heisenberg Meets Thomas Kuhn.

Maarten Van Dyck*
Centre for Logic and Philosophy of Science
University Ghent, Belgium
maarten.vandyck@UGent.be

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1 Introduction

As the present volume of articles testifies, recent years saw the rise of an interest in the roles and significance of thought experiments in different areas of human thinking. Heisenberg's gamma ray microscope is no doubt one of the most famous examples of a thought experiment in physics. However, its value has often been doubted, or even deemed to be totally misguided.

> ... a misleading attempt to "explain" the concept behind a purely quantum mechanical theorem ... [21, p.848]

> ... the consideration of optical analogies — such as ... Heisenberg's gamma ray microscope, are mistaken. Indeed, the reasoning in these cases is fallacious because it employs propositions belonging to optics, not to quantum mechanics — e.g. the formula for the resolving power of a lens ... [4, p.149]

Maybe this is one of the reasons this particular thought experiment has not received much detailed attention in the philosophical literature on thought experiments up to date.

As I want to argue in this paper, this neglect is to be regretted, as one of the philosophical accounts of the function of thought experiments can provide the clue to a most fruitful understanding of Heisenberg's thought experiment. In this way the philosophical discussion on thought experiments

*The author is Research Assistant of the Fund for Scientific Research — Flanders (Belgium).

could play a more exciting role than being a merely academic exercise on the relative merits of rationalism vs. empiricism. It can directly enhance our understanding of one of the most fascinating, but at the same time confusing, episodes in the history of science: the development of quantum mechanics and its interpretation. At the same time, the success of that philosophical account — which, as the title of the paper of course already gave away, is Thomas Kuhn's — in helping to interpret Heisenberg's thought experiment, should be counted as a point in its favor. As Kuhn's account is almost entirely based on the analysis of only one thought experiment in physics, i.e., Galilei's on the speed of objects, its present extension to another pivotal thought experiment should already be welcomed on that grounds alone. But since the extension at the same is a broadening, as will become clear, the role to be played by Heisenberg's thought experiment with respect to the philosophical discussion on thought experiments could be more inspiring.

Thus arises an interesting mutual influence between this particular thought experiment and the general philosophical discussion on thought experiments. Accordingly the aim of the present paper is twofold: to provide an interesting interpretation of the roles played by Heisenberg's gamma ray microscope in interpreting quantum mechanics, and to contribute to the ongoing discussions on the roles and significance of thought experiments in physics.

2 The gamma ray microscope

2.1 Heisenberg's uncertainty paper

Heisenberg's 1927 uncertainty paper "Über den anschaulichen Inhalt der quantentheoretische Kinematik und Mechanik" [10, 16] can be considered to be the interpretative counterpart of his epoch-making "Quantentheoretische Umdeutung kinematischer und mechanischer Beziehungen" [9], which was published one and a half year earlier. In the latter, Heisenberg had arrived at the basic equations of quantum mechanics by a symbolic translation of classical equations of motion, guided by a "sharpened application" of Bohr's correspondence principle.[1] The *symbolic character* of the translation was highlighted by the fact that the kinematic quantities lost their ordinary meaning and were interpreted in a *purely electromagnetic* manner, directly linked with the radiation emitted by excited atoms. The fourier components, classically characterizing a motion, were interpreted as giving a measure for the possible transitions between stationary states, and *not as components of a motion* (therefrom the "reinterpretation of kinematical and mechanical relations" in the paper's title). Moreover, the resulting mathematical

[1] [5] is an excellent analysis of the pivotal role played by the correspondence principle in the development of quantum theory.

scheme, which soon was remarked to be matrix algebra, was rather complicated, and contained at its center a peculiar anti-classical property. The multiplication between the matrices resulting from the translation of respectively position (q) and momentum (p) turned out to be non-commutative, and had to satisfy the following relation:

$$\mathbf{pq} - \mathbf{qp} = -i\hbar. \tag{1}$$

Since all classical quantities can be represented by real numbers, which of course satisfy a commutative rule of multiplication, this property looks very strange. Paul Dirac introduced the felicitous names of c–numbers and q–numbers, the former standing for classical numbers, the latter for quantum, or queer, numbers. But then, what does correspond in quantum mechanics to classical quantities like position? That is, *how are the q–numbers associated with physical quantities*, apart from their giving the right predictions about emitted spectra? The symbolic character of the new theory at first did not seem to allow an answer to these questions. This is why Schrödinger could refer to it as a "a formal theory of frightening, indeed repulsive, abstractness and lack of visualizability."[2] "Heisenberg's theory in its present form is not capable of any physical interpretation at all," was another claim made at the same time.[3] By 1927, a full blown transformation theory had been developed by Jordan and Dirac, encompassing Heisenberg's matrix scheme, as well as Schrödinger's wave mechanics. Drawing from this formulation of his initial theory, Heisenberg set out to counter these allegations in his uncertainty paper. In particular he gave an interpretation of equation (1), which was centered around his famous uncertainty relations and made critical use of the gamma ray microscope thought experiment (and other similar thought experiments).

I will follow Heisenberg's order of presentation in first introducing the thought experiment (although in a somewhat more elaborated form). Then I will explain in § 2.3 how Heisenberg thought this made possible a physical interpretation of the q–numbers, and in § 3, I will try to give a deeper analysis of the roles played by the thought experiment.

2.2 The thought experiment

Imagine we try to determine the physical properties of a microscopic particle using a light microscope. The accuracy with which the particle's position in a given x–direction can be measured is determined by the wavelength λ of the light (the smaller the wavelength, the more sensitive the microscope — if the wavelength is too large, the light no longer 'sees' the particle, just as sound waves can pass by small objects without any distortion), and in

[2] Quoted in a footnote in Heisenberg's paper [16, p.82].

[3] Norman Campbell in 1926, quoted in [1, p.30].

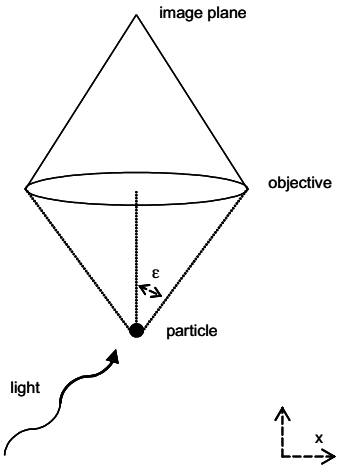

Figure 1: Schematized light microscope.

principle one could ascertain the position with the highest possible accuracy by using radiation of the *shortest possible wavelength* for the illumination. Thus, let us imagine we are in the possession of a gamma-ray microscope. (The shortest wavelengths actually used are near ultra-violet.) As a result, the only inaccuracy in the determination of the position would arise from the limited resolving power of any light microscope. The resolving power gives a measure for the 'blurriness' of the image of any object, that is, it tells how far two points on the object have to lie apart to be discerned as separate in the image. This is due to the fact that the light waves that give rise to the image of the object enter the objective of the microscope in an inverted cone as illustrated in Figure 1. Without going too much in detail, light is diffracted at the object and for a sharp image to be formed, high orders of diffraction must interfere at the image plane of the objective, but to capture higher orders of diffraction the cone must be wide enough. The resolving power is given by the Rayleigh criterion[4]:

$$\Delta x = \frac{\lambda}{2 \sin \epsilon} \tag{2}$$

Thus, the larger the numerical aperture ϵ, the closer two points can lie together and still be discerned as separate (remember that λ is already supposed to be as small as possible). Obviously this is a limit to the accuracy with which the position of a particle in a given direction can be determined, but again, in principle it might be made so small as to be negligible, by constructing a gamma ray microscope with a very *large numerical aperture* (ϵ close to $\pi/2$).

What about the particle's momentum? It is a well known fact that radiation shows particle-like behavior and that in the scattering of light from

[4]The medium between object and objective is supposed to be air.

a particle the Compton effect should be taken into account. (By 1927 this was an uncontestable experimental fact — in that year Compton received the Nobel prize for his experimental demonstration of the effect named after him.) This implies that there will be a recoil and a change in the momentum of the particle, this change being greater if the wavelength of the radiation is smaller, since the momentum of the incoming radiation is given by de Broglie's relation $p = h/\lambda$. This change in the particle's momentum is of course directly related to the change in the radiation's momentum, and could be calculated if the latter were known. But even if the momentum of the incident radiation were completely known, there still would be an upper limit on the accuracy with which this momentum change could ever be determined. Another look at Figure 1 immediately teaches that all one can ascertain about the scattered light is that it is situated *somewhere* in the light-cone arriving at the microscope's objective. Thus, all we can tell about the direction of the light's momentum after scattering is that lies in between $-\epsilon$ and ϵ. If we further assume that the change in wavelength due to recoil is negligible (that only the direction of momentum is changed), then the highest accuracy with which the change in the x–component of the light's momentum after scattering can be determined, becomes:

$$\Delta p_x = \frac{2h \sin \epsilon}{\lambda} \tag{3}$$

(The total momentum is always h/λ, and the extremal values for the x–component of the momentum are situated at the extreme ends of the cone, twice giving the value $(h/\lambda) \sin \epsilon$ for these x–components, once with positive and once with negative sign.) The only way to get an accurate determination of the particle's momentum along the x–axis is thus by using light of *long wavelength*, and a microscope with *small numerical aperture*.

As an immediate consequence of equations (2) and (3) we find a numerical equation expressing a mutual constraint on the lowest possible inaccuracies in position and momentum determination:

$$\Delta x \Delta p_x = h. \tag{4}$$

Obviously the demands of accuracy for the determination of position and momentum of a particle pull in opposite direction![5] Heisenberg comments in his paper:

[5]Heisenberg's original account was flawed in that he did not include the effect of the finite aperture of the microscope, and only considered the effect of the momentum (and thus wavelength) of the radiation. But a closer look at the determination of the momentum immediately shows that if it was not for the impossibility of determining the angle of scattering, the momentum change could be calculated, and thus there would be no resulting inaccuracy in the particle's momentum. In a note added in proof Heisenberg corrects this mistake, which was pointed out to him by Niels Bohr. As has often been remarked in the literature, and implicitly admitted by Heisenberg, his mistake was influenced by his desire to do away with all wave concepts in the discussion (for more on this, see [1,

> Thus, the more precisely the position is determined, the less precisely the momentum is known, and conversely. In this circumstance we see a direct physical interpretation ["direkte anschauliche Erlaüterung"] of the equation $\mathbf{pq} - \mathbf{qp} = -i\hbar$. [16, p.64]

2.3 The queerness interpreted

The direct physical interpretation Heisenberg alludes to consists in the fact that the thought experiment allows him to see that the q–numbers need not keep their symbolic character, but can be given a conceptual content that is closely linked with their original kinematic meaning. As the analysis of the hypothetical gamma ray microscope shows, there exists a certain latitude with which the classical concepts of position and momentum *can* be simultaneously *determined*. This fact obviously has implications for the sharpness with which these concepts *must* be *defined* at the theoretical level.

> Thus only the uncertainty which is specified by equation [(4)] creates room for the validity of the relations which find their most pregnant expression in the quantum-mechanical commutation relations, $\mathbf{pq} - \mathbf{qp} = -i\hbar$. That uncertainty makes possible this equation without requiring that the physical meaning of the quantities p and q be changed. [16, p.68]

Heisenberg then proceeds to prove that the commutation relations indeed reflect a limitation on the sharpness with which concepts like position and momentum are simultaneously defined in quantum theory. He first derives the famous uncertainty relations from the Dirac–Jordan formulation of the quantum theory,[6] and then gives a more intuitive geometrical interpretation of that theory, in terms of the relation between experimental questions and the principal axes of matrices.

The Dirac–Jordan transformation theory assigns to any quantity a probability amplitude for finding the numerical value of that quantity in a given numerical interval, and it moreover gives rules for transforming these amplitudes, so that one could find the relation between e.g. the probabilities for finding a numerical value for q between q' and $q' + dq'$ and for finding p between p' and $p' + dp'$. If we have the probability amplitude $S(q')$ for the quantity q, then $|S(q')|^2 dq'$ gives the probability for finding a q–value between q' and $q' + dq'$. The average value of q then can be denoted as: $\overline{q} = \int q'|S(q')|^2 dq'$, and accordingly the uncertainty with which the value of

pp.70–74]). Apparently Heisenberg sometimes was a slow student, as he already nearly had failed his doctoral examination in 1923 by not being able to answer questions on the resolving power of light microscopes (see e.g. [19, Vol.2, pp.63–69]).

[6] I will present a slightly generalized and more straightforward derivation of the uncertainty relations, as can be found e.g. in [13].

q is known is given by:

$$\delta q = \left(2 \int (q' - \overline{q})^2 |S(q')|^2 dq' \right)^{1/2}. \tag{5}$$

The uncertainty of p can be defined analogously. These uncertainties reflect the fact that only a probability amplitude is given for the quantities, without further specification on the actual numerical values. Most important, the transformation rules relating the probability amplitudes for q and p imply that the uncertainties δq and δp are not independent. A straightforward calculation then shows that:

$$\delta q \delta p \geq \frac{h}{2\pi}. \tag{6}$$

This is the general form of Heisenberg's uncertainty relations for position and momentum.

Remark that only at this point one could unhesitatingly speak about position and momentum, physical quantities which had disappeared in Heisenberg's original 1925 paper. In the meantime these concepts had already reappeared in Born's statistical interpretation of the wave function, and its further elaboration by Pauli, Dirac and Jordan. Born's breakthrough consisted in the realization that quantum mechanics did contain information on the states of particles after scattering, albeit only statistical, as encoded by probability amplitudes. This insight was then quickly extended to include information on the position and momentum of particles. However, the precise conceptual content of the kinematical quantities was not entirely clear yet, as the non-commutative character still remained puzzling at the physical level. This is where Heisenberg's thought experiment comes in. His analysis of the hypothetical experimental situation shows that *even* when talking about the *classical quantities* of position and momentum, these quantities are *not unambiguously simultaneously determinable*. But this implies that, if one wants to apply these concepts to describe what happens in an experimental set-up, a certain leeway exists. As the analysis in § 2.2 shows, if one considers a situation in which the position of a particle can be ascertained with the highest possible accuracy, then one need not ascribe a definite momentum to the particle, as its possible value cannot show up in the same experiment; thus it becomes possible to interpret the uncertainty relations as stating that position and momentum are not simultaneously well-defined, that there exist a mutual and intrinsic uncertainty in their definition. One can take this freedom a step further and enunciate an operationalist attitude towards the definition of concepts: 'one need not' is then read as 'one can not', making the step from an epistemological to an ontological uncertainty compulsory. This is the strategy advocated by Heisenberg in his uncertainty paper, as the following quotation shows — but it is important to realize that the weaker reading already suffices for the reinterpretation to go through,

that is, one need not equate the means of measurement with meaning proper to argue that limitations in means of measurement imply a freedom in the meaning ascribed to terms (I will come back to the issue of operationalism in § 3.3).

> All concepts which can be used in classical theory for the description of a mechanical system can also be defined exactly for atomic processes in analogy to the classical concepts. The experiments which provide such a definition themselves suffer an indeterminacy introduced purely by the observational procedures we use when we ask of them the simultaneous determination of two canonically conjugate quantities. [16, p.68]

The exact form of the uncertainty relations, which is of course already important in its own right as a theorem of quantum mechanics, then serves to further justify the claims made by Heisenberg about the applicability of classical concepts, as it shows that the uncertainty that exists at the theoretical level (equation (6)) indeed is of the same order of magnitude as the leeway provided by the experimental procedures (equation (4)). This would become a basic theme in Niels Bohr's philosophy of quantum mechanics, as exemplified in his numerous remarks about the agreement between the possibilities of observation and those of definition.

At this point, one could complain that the foregoing does not really make clear how all this provides "a direct physical interpretation" of the non-commutative character of the quantities, which is after all an algebraic property. The real interpretative work rather seems to be done by a *consequence* of this property, viz. the uncertainty relations (6). However, Heisenberg proceeds by giving a physical interpretation of certain algebraic properties of the quantum mechanical scheme in the Dirac–Jordan formulation. The basic insight that can be gained from the thought experiment is the realization that every experimental situation "divides physical quantities into "known" and "unknown" (or more or less accurately known quantities) in a way characteristic of the experiment in question" [16, p.70]. In quantum mechanics, on the other hand, the value of a quantity is given by the diagonal terms of the corresponding matrix, and the fact that two matrices cannot be always simultaneously diagonalized — a direct consequence of the non-commutative character — then reflects the fact that the corresponding quantities belong to mutual exclusive experimental situations. As a result any experimental situation can be associated with a direction in a multidimensional matrix space (in which directions correspond with possible principle axes of the matrices). The non-commuting character of the q–numbers corresponding with q and p can now be understood as reflecting the fact that both quantities cannot be simultaneously determined with unlimited accuracy.

3 A function for thought experiments

3.1 Conceptual transformations

One of the earliest philosophical discussions of thought experiments is Thomas Kuhn's 1964 "A Function for Thought Experiments" [18]. In his paper Kuhn discusses what afterwards has been called Kuhn's paradox of thought experiments [17, p.1].

> Granting that every successful thought experiment embodies in its design some prior information about the world, that information is not itself at issue in the experiment. On the contrary, if we have to do with a real thought experiment, the empirical data upon which it rests must have been both well-known and generally accepted before the experiment was even conceived. How, then, relying exclusively upon familiar data, can a thought experiment lead to new knowledge or to new understanding of nature? [18, p.241]

The first suggestion Kuhn considers is that a thought experiment is primarily aimed at uncovering a confusion in the scientist's conceptual apparatus. Its function would be to show that some of the concepts used by the scientist are inconsistent. The paradox disappears because the knowledge gained is solely about the conceptual apparatus, not about nature itself. And obviously no new empirical information is needed, since the inconsistency was there already, slumbering as it were, only waiting to be revealed. Moreover, a superficial glance at some famous thought experiments immediately shows that they are aimed indeed at unveiling some kind of contradiction in the scientist's mode of thought.

Nevertheless, Kuhn disagrees with this view since it rests on too naive a view on the use and definition of concepts. However, having brought into focus the issue of conceptual transformation, he can proceed to a more sophisticated view, which still allows him to disentangle the apparent paradox. The problem with the suggested solution is that it rests on some kind of analytic/synthetic distinction that cannot be maintained, as Kuhn already famously argued in his "Structure of Scientific Revolutions". Concepts never come free from physical implications, and as such their use always provides information about what the world is like. This of course implies that they can err in a non-logical sense, i.e., that the world is not exactly as presupposed by the concept, and exposing these kind of errors is the function Kuhn ascribes to thought experiments. In this they are not very different from ordinary, natural, laboratory, real — or whatever one likes to call them — experiments, the major difference being that thought experiments entirely rely on empirical information that was already at hand, but was not completely assimilated yet. By making this information explicit in an imagined situation, it can be shown that the conceptual apparatus used by the scientist does not fit situations to which it should apply. In this way thought

experiments can teach conceptual reformation, which in its turn can teach something about the world, and not only about our conceptual apparatus.

> The concepts "corrected" in the aftermath of thought-experiments displayed no *intrinsic* confusion. If their use raised problems for the scientist, those problems were like the ones to which the use of any experimentally based law or theory would expose them. They arose, that is, not from his mental equipment alone but from difficulties discovered in the attempt to fit that equipment to previously unassimilated experience. Nature rather than logic alone was responsible for the apparent confusion. [18, p.261]

Kuhn tries to defend his claim, that the contradiction exposed in a thought experiment is not of a purely logical origin, by an analysis of Galilei's thought experiment on the speed of bodies. In this thought experiment Galilei tries to establish that the Aristotelian concept of speed can give rise to incompatible assessments of the same situation. The thought experiment could only be effective if the Aristotelians were prepared to accept that their concepts should apply to the situation presented, *and* that our world is like the situation presented. On the other hand, if the thought experiment was aimed at a logical inconsistency, the aim would be to establish the contradiction in all *possible worlds*. But then why the emphasis on the fact that our world is indeed like it is supposed to be in the hypothetical situation? Moreover, there is not a hint of an argument that all possible worlds should be alike in the relevant aspects that cause the contradictory assessments; and quite understandable, since it is rather easy to think of possible worlds in which the contradiction would never arise, thus making the Aristotelian concepts perfectly well suited. The empirical presuppositions underlying the applicability of the Aristotelian concepts of speed could have been satisfied, but as it turns out, this is not the case in our world.

Kuhn's analysis thus suggest that one important function of thought experiments is the role they play in teaching conceptual transformation. Before revisiting the gamma ray microscope, it is worth quoting Werner Heisenberg on conceptual transformation, to bring out the close affinities that exist between his and Kuhn's views on this issue. I think this can help to underscore the fact that Heisenberg was not just the brilliant but philosophically naive physicist, whose conceptual analysis of quantum mechanics, as exemplified by his thought experiment, only served self-justifying purposes. His closeness to Kuhn's views on conceptual transformation, but thirty years predating them, makes it very plausible to assume that his use of the thought experiment was indeed intended to play the role Kuhn ascribes to thought experiments in general.

> [T]he validity of classical physics is limited by the lack of precision of the concepts contained in its axioms. ... [T]here is no criterion

> allowing an *a priori* assessment, as to whether the application of a term is objectionable or not. ... [T]herefore the only possible progress for science seemed to lie in the unhesitating use, in the first place, of existing terms for the description of experience, and the revision of these terms from time to time as demanded by new experiences. [12, p.43]

As Kuhn reminds us, these "new experiences" can also come to us in the form of a thought experiment, incorporating well known, but not completely assimilated, empirical knowledge. This is exactly what happens in the gamma ray microscope thought experiment.

3.2 The microscope revisited

Heisenberg's thought experiment has frequently been criticized as a misguided attempt to give a classical explanation of a quantum mechanical phenomenon (e.g. the quotations given in § 1, but more examples could easily be found). The underlying idea is that a consistent treatment of measurements made with a gamma ray microscope should invoke only quantum mechanical considerations (bracketing, of course, the perennial problems surrounding the notion of measurement in quantum mechanics). Heisenberg's analysis, on the other hand, has to be bluntly inconsistent since he uses the idea of a particle having both position and momentum to argue that it cannot have these properties...

Following Kuhn's analysis of the function of thought experiments, and Heisenberg's own understanding of conceptual transformations, we can now see how misguided these critiques are (but see § 3.3 for a qualification of this claim). The thought experiment presents us with a situation to which the classical concepts are naively thought to apply, and then shows some unexpected consequences. A fully quantum mechanical treatment would teach us nothing on the conceptual level, clearly contrary to Heisenberg's intentions. However, whereas in Kuhn's original treatment, these unexpected consequences were supposed to be contradictions, in Heisenberg's case one is confronted with an ambiguity. Probably this is the main reason why the logic of the argument is not always clearly understood. But nothing in Kuhn's analysis seems to exclude such an extension, since the function assigned to thought experiments is teaching conceptual transformation. Clearly this can be achieved via different routes, i.e., by exposing an ambiguity as well as an inconsistency, since both can be thought to be signs of a confusion in the conceptual apparatus as applied to our world. As in Kuhn's and Heisenberg's views on conceptual transformation, this ambiguity could not be assessed *a priori*, but only by confronting the conceptual apparatus with some well known empirical facts, i.e., the formulas expressing the resolving power of a light microscope and the Compton effect. By

devising an imaginary experimental situation in which this empirical knowledge has to be taken into account, Heisenberg can force his contemporary "Aristotelians" — the defenders of a classical worldview — to admit that our world is such that position and momentum need not be simultaneously ascribed to microscopic objects; and thus, that the q–numbers can still be given a kinematical interpretation.

Maybe I have been overstating the point by calling this an ambiguity in the conceptual apparatus of classical physics as applied to our world? After all, there seems to be nothing ambiguous in applying concepts where they need not be (that position and momentum are not simultaneously unambiguously determinable — this follows uncontestable from the thought experiment — does not imply that they are only ambiguously applicable). However, I believe that the assessment of this claim must be altered in view of the existing physical knowledge at the time of Heisenberg's thought experiment. If one takes serious the insight that classical physics cannot tell the complete story about the world, and that much was agreed upon already at the first Solvay conference in 1911, where the attendants seemed to reach a consensus on the fact that the by then known quantum phenomena could not be dealt with in the classical scheme — a conclusion which of course had been extensively confirmed by the subsequent developments of physics — then the fact that there exists a leeway in the applicability of classical concepts should be taken serious as well. If one accepts that classical physics has its limits in the microscopic domain, then the discovery that position and momentum need not be simultaneously ascribed in this domain should not be met with a shoulder-shrugging "don't care about what needn't be done". Something must be done, that much was for sure, and here was the place were things could be done. Against that background, I think it is not too strong to call the conceptual leeway an ambiguity. After all, what else is an ambiguity than an underdetermination by the facts that potentially causes problems? The introduction of Heisenberg's uncertainty paper explicitly discusses the suggestion that the simultaneous ascription of position and momentum in a discontinuous world might cause insurmountable problems, thus adding to the plausibility of the idea that it is indeed here that lies a source of many of the interpretative problems.

Nonetheless, Heisenberg seems to have been troubled by the worry that an ambiguity was not enough — an ambiguity of course can turn out to be harmless, and he firmly believed that *this* ambiguity was not harmless — as he added an extra premiss to his argument, i.e., an operationalist view on the definition of physical concepts, which makes the simultaneous ascription of position and momentum simply inconsistent, thus bringing the form of the argument even closer to Kuhn's analysis. However, at the same time this takes away a lot of the strength of the thought experiment, as this extra premiss is rather controversial. In opting for this strategy, Heisenberg might have been influenced by a kind of revolutionary zeal in which he not only

wanted to argue for the possibility of a conceptual reformation, but also for its necessity (see Mara Beller [1] for a rather unsympathetic account of this aspect of Heisenberg). On numerous occasions Heisenberg clearly stressed the *necessity* of leaving behind old ideas about causality, visualizability etc.; and he thought to have established such in his uncertainty paper, of which the last sentence reads:

> Because all experiments are subject to the laws of quantum mechanics, and therefore to equation (4), it follows that quantum mechanics establishes the final failure of causality. [16, p.83]

The failure of causality comes from the impossibility of knowing the present exactly, i.e., values for both position *and* momentum, thus excluding the possibility of completely predicting the future. This failure can only be taken to be final if the uncertainties are necessary, but remember that *this* only follows from the thought experiment on the extra assumption of operationalism; but why should *that* be necessary... So, should we follow Beller in her assessment of Heisenberg, and conclude that he simply wanted too much to follow from his example? That is, did he put the thought experiment to more work than could be done by it? As I will briefly argue in the following section, this question can not be given a straightforward answer: the claims about finality need not find its origin in the thought experiment, since they arise already from Heisenberg's more general views on the nature of scientific knowledge and methodology; however, the thought experiment did suggest a physical explanation for this finality, which made the conclusion more palatable and was frequently used by Heisenberg, but later on turned out to be susceptible to damaging criticism.

Focussing on the claims about the failure of causality — or what comes down to the same, about the completeness of quantum mechanics — will have the benefit of highlighting another role played by the thought experiment. Most importantly, I think that having exposed the precise function of the thought experiment in conceptual transformation, will help in being more careful in assessing some controversial issues surrounding the notion of disturbance in the interpretation of quantum mechanics.

3.3 Operationalism, finality, and disturbance

Let us begin by taking another look at the last sentence of the uncertainty paper, in which Heisenberg announces "the final failure of causality". The reason he cites for this conclusion is the fact that "all experiments are subject to the laws of quantum mechanics". This is of course highly significant. As explained in § 2.1, the main aim of the uncertainty paper was to provide a conceptual interpretation for the q–numbers, and one can take Heisenberg to have been successful on this point. Moreover, on the interpretation he proposes, quantum mechanics indeed implies the breakdown of (one idea

of) causality on the grounds mentioned by Heisenberg. (Of course one can have quarrels with Heisenberg's characterization of the idea of causality.) This could only be argued after the reinterpretation, i.e., classical kinematic concepts remain applicable, but with intrinsic uncertainties due to the fact that they are linked with different experimental situations. But once this interpretation is accepted, the thought experiment has no further role to play in the breakdown of causality, which becomes a consequence of the validity of quantum mechanics.

The foregoing paragraph can best be summarized by the statement that the uncertainty relations are of a purely theoretical nature, but that their nature as uncertainties follows from the interpretation partly based on the thought experiment — on another interpretation, the uncertainty relations might have another significance, as witnessed, e.g., by David Bohm's interpretation.

What about the finality of this failure? *On the supposition* that quantum mechanics in Heisenberg's interpretation is completely valid, this validity indeed implies this finality. All situations in nature then behave as prescribed by the theory, implying the universality of the uncertainties, and thus the finality of the failure. No situations can be found for which the principle of causality holds. A further, maybe surprising consequence of this fact is that operationalism now is shown to be true on *physical grounds*! The validity of quantum mechanics in Heisenberg's interpretation — and remember that operationalism need not be presupposed for his reinterpretation of classical concepts to go through — implies that the applicability of classical concepts is tied to their measurement in experimental situations (a fact highlighted by the talk about "observables" in standard quantum mechanics). Maybe it is even not too farfetched to suggest that Heisenberg first reached this conclusion, and only then inserted his operationalist remarks in the presentation of his thought experiment.[7] However, this does not take away from the fact that the significance of a physically based operationalism is entirely different from a that of a philosophical operationalism, and that Heisenberg clearly alludes to the latter in his paper. The major difference is that the former form of operationalism stands and falls with the physical theories on which it is grounded, whereas the latter is supposed to be independent of any changes in physical knowledge. It is clear that a philosophical operationalism is thus much stronger, and accordingly should be looked at with

[7]Although I want to question the view of the philosophically naive Heisenberg, it is clear that in his scientific papers he often used philosophical doctrines to give a *post hoc* justification for some of his more far-reaching results. A clear example of this can be found in the positivist discourse in his 1925 *Umdeutung* paper, in which he claimed to start from the principle that only observable quantities should enter a significant physical theory — Olivier Darrigol [5, pp.273–276] convincingly shows that this was not how Heisenberg reached his results (and luckily, since the scheme had to contain some non-observable phase factors to be successful).

much more suspicion.

The claim that no situations can be found for which the principle of causality holds, or what comes down to the same on Heisenberg's interpretation, for which the uncertainty relations are violated, has profound implications. Therefore, it should be made plausible on independent grounds, and not only by referring to the successes of quantum mechanics in dealing with other kind of situations. This is another important task served by the thought experiment: it shows that, even if one does not take the validity of quantum mechanics for granted, generally accepted empirical laws imply that the uncertainty relations will be valid. Apparently nature is such as prescribed by the validity of quantum mechanics: it is at least very well possible that quantum mechanics would be valid and complete...[8] Apart from the critical role of showing classical concepts to be ambiguous, the thought experiment thus also serves a constructive "semi-empirical" role, as exemplified by the agreement between equations (4) and (6), which helps to argue for the general validity of the uncertainty relations.[9]

The real weight of the proclaimed finality of the failure of causality is carried by the complete validity of quantum mechanics. This validity can be made plausible by the number of empirical successes (which, however, was not very high at the time of Heisenberg's uncertainty paper) and semi-empirical successes, i.e., the thought experiments (as explained in the last paragraph); but in the end, believing in completeness comes down to an act of faith. As testified by his other writings, Heisenberg believed that there were some further good reasons for proselytization. I will not go into what one could call "internal reasons", having to do with the elegance, comprehensiveness, etc. of the mathematical scheme, taken together with Heisenberg's experience of the dreadful state-of-the-art before the advent of quantum mechanics. More important, in the present context, is the *methodological reason* that brought Heisenberg to believe in the *fruitfulness* of such a postulation. This has to do with

> [...] the really fundamental characteristic of a physical discovery. It is not the result of, but the precondition for a clear delineation of the range of applicability of the discovered concepts. [12, p.52]

[8] Maybe one could have quarrels with the claim that quantum mechanics is not taken for granted in the thought experiment, since it crucially uses the fact that light has both a wave and particle character, thus potentially making the consistency-check self-evident? This complaint neglects the fact that the wave-particle duality was an empirical fact, well established before the advent of quantum mechanics, as recounted in [22].

[9] Some quotations from Heisenberg's 1929 Chicago lectures [13] show that Heisenberg indeed intended them to play this role: "but this does not circumvent the uncertainty relation" (p.22) "The change in momentum which is necessarily produced by the last observation is subject to such an indeterminateness that the uncertainty relation is again fulfilled" (p.25) "The problem is therefore to determine the velocity in the y–direction, and it is to be shown that the knowledge of the y–coordinate is destroyed by this measurement to the extent demanded by the uncertainty relation." (p.26)

When a fundamentally new theory is put forward, this is bound to have profound implications for the way physical concepts are used. The prime example before quantum mechanics of course was Einstein's theory of special relativity. If one takes the stance that the new theory is complete, then, due to the possibility of a new delineation of the range of applicability of concepts, this provides one with new knowledge about what is observable in nature. After all, as Heisenberg recounts being taught by Einstein [15, pp.269–270], it is theory which determines what is observable. Even more significant in the case of the uncertainty relations, theory can also tell what is not observable! One of Heisenberg's favorite ways of expressing this insight was to enunciate the uncertainty relations to the status of principle, thus mimicking Einstein's principle of the finiteness of the speed of light.

> The restrictions of classical concepts as enunciated in the uncertainty relations acquire their creative value only by making them questions of principle. [12, p.47]

By making them questions of principle, it is posited that the uncertainty relations reflect a fundamental limitation, and thus that they indeed determine what is unobservable in nature. In this way, one clearly sees Heisenberg consciously postulating the theory's completeness for methodological reasons.[10] I will not discuss the merits of this fascinating strategy, as I only wanted to indicate why the "necessity" of the failure of causality does have other, more subtle, origins than the thought experiment — this is often overlooked by commentators. As to the charge of dogmatism, most effectively levelled by Beller, I would like to stress that Heisenberg clearly wanted this completeness to be understood in a non-absolute way; that is, to be complete the theory has to be correct *within the range of applicability of its concepts*, but new limitations on this applicability can be unveiled by new theories, and probably will be [12, p.51]. Heisenberg considered classical mechanics to be a complete theory, despite its limitations in domain of validity, and accordingly expected a similar limitation on the completeness of quantum mechanics.[11]

The necessity of the failure of causality thus can be traced to other, more subtle, origins than the thought experiment, and still Heisenberg often sug-

[10] See also [13, pp.3–4]: "The starting-point of the critique of the relativity theory was the postulate that there is no signal velocity greater than that of light. In a similar manner, th[e] lower limit to the accuracy with which certain variables can be known simultaneously may be postulated as a law of nature (in the form of the so-called uncertainty relations) and made the starting-point of the critique which forms the subject matter of the following pages. These uncertainty relations give us that measure of freedom from the limitations of classical concepts which is necessary for a consistent description of atomic processes."

[11] This brings Heisenberg in some respects surprisingly close to Einstein on the issue of the completeness of quantum mechanics — in particular if we follow the interpretation of Einstein's position as presented in Arthur Fine's [8].

gested it also finds its origins there. One might say that the suggestion flowing from the thought experiment simply turned out to be irresistible: the uncertainties are due to an unavoidable disturbance present in any measurement (remember the recoil suffered by the particle in the gamma ray microscope).

> The possibility of statistical inter-connections [as opposed to deterministic] is created only by regarding the effect of the measuring apparatus on the system to be measured as a partial disturbance uncontrollable in principle. [12, p.49]

Something very weird is going on here; Heisenberg is trying to do the impossible — to give a causal explanation why causal explanations are impossible, as it was aptly put by Karl Popper [20, p.248]...

The disturbance finds its origin in the classical description of the hypothetical measurement with the gamma ray microscope. It is a particle having both classical position and momentum that is being disturbed in the measurement process, thus rendering a simultaneous knowledge of both impossible. But how can Heisenberg claim at the same time that the uncertainty relations — implying the statistical character of the theory — find their origin in quantum mechanics ("all experiments are subject to the laws of quantum mechanics"), *and* in a classical description of the measurement process? Clearly he cannot, especially as his interpretation of the laws of quantum mechanics is explicitly anti-classical (a particle does not have simultaneous position and momentum).

I will not try to give a diagnosis of Heisenberg's apparent schizophrenia on this issue, and only remark that in this episode Mara Beller's idea of a "dialogical approach" to the history of science seems to be very fruitful. What I would like to do here, is to point out how my analysis of the thought experiment can help to disentangle some of the thorny issues surrounding the disturbance idea. It should be clear by now that the disturbance present in the thought experiment, that is, in the classical description of the measurement process, has an important role to play — it creates room for the validity of quantum mechanics in an apparently classical world. However, as such it plays a semantical role: it teaches us something about the possible meanings of classical concepts (in our world), it does not tell us what really happens in the world when we try to measure the properties of a particle (remember that the world is supposed to be such that quantum mechanics is true of it, and not classical mechanics). Seen in this way, one could say that the disturbance "causes" equation (4), but not (6), where only the latter expresses a genuine quantum mechanical law.[12] Heisenberg acknowledged

[12]Incidentally, if interpreted in this way, I think a large part of Heisenberg's schizophrenia can be seen to disappear; it turns out that when explicitly using disturbance language, he is often talking about the origins of equation (4), which is *not* the uncertainty principle, that is, not a law of quantum mechanics!

that the idea of disturbance played several distinct roles:

> This ... disturbance ... assumes importance in many different ways. To start with, it is the reason for the appearance of statistical laws of nature in quantum mechanics. Further it imposes a limit on the application of the classical concepts ... [11, p.15]

By keeping apart these two roles, we can thus have the disturbance playing its valid role in the conceptual transformation, without having to suppose that it does so by offering an inconsistent physical explanation where none is to be had.

The idea of disturbance causing the uncertainties is not only incoherent, in 1935 it was also shown to be false, on the supposition that all disturbances propagate locally, in the famous EPR thought experiment [7]. Niels Bohr's answer to this allegation is quite revealing: he insisted that the disturbance has to be understood as an "influence on the very conditions which *define* the possible types of prediction regarding the future behavior" of a system, as there is "no question of a mechanical disturbance of the system under investigation" [3, p.148] (my emphasis). The conditions which define the possible types of prediction, of course are the limits of application of the classical concepts. In their [2], Beller and Fine convincingly show the difficulties one runs into if one tries to salvage the idea of disturbance causing the validity of the uncertainty relations. But, as my discussion was intended to show, this does not imply that a mechanical disturbance cannot play another role — a role that is pivotal in introducing classical concepts in interpreting quantum mechanical laws, and that need not stand or fall with one's opinion on the values of operationalism.

4 Conclusion

Let me briefly try to recapitulate the main strands running through this paper. Heisenberg's gamma ray microscope can be seen to fulfill two important roles in interpreting quantum mechanics: it shows the way for a conceptual transformation of classical concepts that is suited for quantum mechanical laws, and it helps to argue for the validity of these laws (the semi-empirical successes referred to in § 3.3). It does so by presenting us with a hypothetical situation in which well known empirical laws convey some important insights; insights which could only be made explicit in this kind of situation. The implicit simultaneous ascription of both momentum and position to the particle in the thought experiment is not simply an inconsistency on behalf of Heisenberg, but is an essential part of his argument about the limited applicability of these concepts. Although this was often blurred by Heisenberg himself, the thought experiment need not be taken to be about tracing the origins of the uncertainty relations, since these come

from the theory. By focussing on Kuhn's analysis of the function of thought experiments, we could retract what is genuinely valuable about the thought experiment — the way it helps to argue for conceptual transformation — without having to accept all the conclusions drawn from the disturbance in the measurements made with the hypothetical gamma ray microscope, thus qualifying claims like the following in an important way:

> The concept of disturbance, inaugurated in Heisenberg's uncertainty paper, is an ill-fated and inconsistent one... [1, p.156]

Kuhn's analysis not only teaches something about Heisenberg's thought experiment, there is also a reciprocal relation: we can see that not only straight inconsistencies showing up in a thought experiment pave the way for conceptual transformation, but that ambiguities can play the same role in the right kind of context. Moreover, the thought experiment played more roles than the one function ascribed to it by Kuhn, as it also served as semi-empirical evidence for the correctness of the interpreted quantum mechanical laws.

There are several things I did not do in this paper. I did not enter upon discussions concerning visualizability in quantum mechanics.[13] This certainly is an important shortcoming, as these discussions are highly relevant with respect to the related ones on the roles of thought experiments — for instance, one could argue that the classical picture used in the thought experiment serves as a kind of "intuition enhancer" to prepare us to the quantum world where there is no simultaneous reality ascribed to quantities like position and momentum. I also did not comment on what should be the right interpretation of the uncertainty relations (are they linked with state preparation, with measurement, ..., do they apply only to ensembles, or also to individual particles, etc.). Heisenberg used the freedom in applying classical concepts which the thought experiment showed him to exist, but of course other strategies are also possible (and maybe preferable on other grounds). After all, the thought experiment is only a preliminary step towards an interpretation of quantum mechanics, showing "something" about classical concepts and our world, but interpreting this "something" requires genuine creativity.

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