The Best Question

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Question-answer contexts are prone to self-reference and ungroundedness, as when we ask:

(1) What is your answer to this question?

or when we respond:

(2) This is the answer to your question.

This sort of phenomenon is familiar from the literature on erotetic logic and provides good evidence for Belnap and Steel's celebrated *Hauptsatz* [1]:

(3) Ask a foolish question and you get a foolish answer.

The trouble is that sometimes our foolishness escapes us.

Here is a way of putting the problem, which Ned Markosian [3] calls the "paradox of the question". An angel shows up at a conference of the world's leading philosophers and gives them a unique opportunity to ask a question of their choice. The angel comes to them as a messenger from God and will answer their question truthfully—but she will only answer one question. What should the philosophers ask to make the most of their opportunity? Clearly they want to learn something about a truly important and intrinsically interesting subject. But which subject they should address is itself something they would like to know. After much debating, the proposal of a bright young logician is approved. So they ask:

(4) What's the ordered pair whose first member is the best question we could ask and whose second member is the answer to that question?

The angel listens carefully and responds:

(5) It is the ordered pair whose first member is the question you just asked me and whose second member is this answer I am giving you.

Thereupon the angel disappears, leaving the philosophers in mute despair. Could they be more foolish?

As Ted Sider [4] has pointed out, the angel's answer is actually a cheat. For if (5) were the right answer to (4) then (4) would have to be the best question, contradicting the fact that learning (5) is useless. Moreover, Sider has shown that (4) cannot be *the* best question. The bright young logician should have asked:

(6) What is an example of an ordered pair whose first member is one of the best questions we could ask and whose second member is one of its correct answers?

Unfortunately (6) itself generates a paradox similar to the familiar semantic paradoxes (either (6) is, or it is not, one of the best questions, and both suppositions lead to contradiction), so that question would not be any better. Eventually, Sider settles on the following:

(7) What is the true proposition (or one of the true propositions) that would be most beneficial for us to be told?

This is definitely a good question. But is this the best that we could do without infringing the *Hauptsatz*?

Luckily we can do better. For one thing, why should we ask for only one proposition? If there are several propositions that would be most beneficial for us to be told, i.e., more propositions such that there are no better ones for us to be told, let's ask for all of them. The angel is allowing for a single question but she has not put any constraint on the answer. So the following appears to be an improvement of (7):

(8) What are all the true propositions that would be most beneficial for us to be told?

There is no reason to stop here, either. For why should we be content with the most beneficial propositions? There are lots of other propositions such that it would be interesting enough to know that they are true, including propositions about the identity of Shakespeare or about the best practical way to change a car's oil. Of course we could not ask for *all* the true propositions, for then we would never get to the end of the answer. Worse, among the true propositions there are an infinite number of pretty silly and useless ones, and if the angel's answer begins with an infinite series of those then we will never get to any of the good stuff in anyone's lifetime. But this difficulty can be overcome. For we can set an upper bound on the length of the propositions we are interested in, and we can require that they all consist of words from a fixed, finite vocabulary. (The latter requirement is necessary because there is no upper bound to the number of numerals that can be constructed in English, or to the number of English names that can be constructed using quotation devices, for instance.) So let n be a fixed number and let V be a fixed finite set of English words. Then we could ask:

(9) What are all the true propositions statable in at most n words from vocabulary V?

The answer to this question is bound to be of finite length, and it is certainly more informative than (8) or its predecessors. The larger the vocabulary and the admissible number of words in a proposition are, the more informative the answer is. (And we can choose a *very* large number and a *very* large vocabulary, if we wish—e.g., the set of all English words tokens of which have occurred in print at least once). Of course (9) might not be the question that is most beneficial to *us*, for our lives might be over before we can even start exploiting some of the good things the angel is telling us. But we are supposed to be far-sighted, and we care for our children. The best question for us *to ask* is the one that is most beneficial to us, to our children, and to the generations to follow. So as long as we make sure that we put everything the angel says on record, and we instruct our descendants to keep on doing so, we can be assured that our question will serve its purpose.

Nor is a question along the lines of (9) the best we can do. For the order in which the angel lists the true propositions might be crucial to our survival, hence to the survival of our descendants. For instance, there might be a proposition such that, if we don't learn of its truth within the next year, we accidentally blow up the planet. Hopefully this proposition is statable in at most n words from vocabulary V, but there is no guarantee that it is one of the propositions that we will learn from the angel's answer to (9) in the next year. Let us, then, ask the angel to do some ranking for us. Let R be the *more-beneficial-than* ordering defined on the set of all relevant propositions, so that p precedes q under R just in case it is more beneficial for us to know that p before knowing that q. This is certainly a strict ordering, though some pairs of true propositions may not be comparable under R. Let's extend it to a total ordering (or chain) $<_R$ in some fixed way. For example, let $<_R$ be defined so that $p <_R q$ just in case either (i) p precedes q under

R, or (ii) p and q are not comparable under R but p is shorter than q, or (iii) p and q are not comparable under R and have equal length, but p precedes q in the alphabetic order. This yields a chain, the least element of which is the first shortest R-minimal true proposition in the alphabetic order. (Other conventions would be equally good.) Then we could ask the angel:

(10) What is the sequence, under the ordering $<_R$, of all the true propositions statable in at most n words from vocabulary V?

This would not only prevent us from accidentally blowing up the planet. It would also have a lot of other advantages, for all the good things will come first. Suppose, for example, that long life would result from a diet of pasta puttanesca and kumquats. Then the angel would put that early on the list, we would follow the diet, live longer, and get to explore more of the list. Moreover, reference to R will have the advantage of excluding from the initial portion of the answer all tautologies and other sorts of useless logical truths. They will show up in the list. But their ranking under R, and therefore under $<_R$, will be pretty low and so they will show up only towards the end of the angel's answer.

There is still a possibility that the angel will squander our time with long series of redundant propositions. If we are told that snow is white, we don't need to be told, in addition, that either snow is white or grass is both green and not green. We can effectively figure out all tautological equivalences by ourselves, and lots of other logical equivalences as well. So let S be our favorite logical theory for the fragment of English with vocabulary V among those whose relation of logical equivalence $\equiv_S$ is known to be decidable. Then we could improve (10) along the following lines:

(11) What is the sequence, under the ordering $<_R$ and including only the first member from each equivalence class under $\equiv_S$, of the true propositions statable in at most n words from vocabulary V?

Of course, at this point we could consider further entailment relations (besides logical equivalence) among the members of the list. As when we define a theory, we might want the sequence of true propositions supplied by the angel to form a nice, elegant set of axioms from which every other relevant true proposition follows. But I am not sure such improvements are all that important. Getting the angel's answer to a question like (11) would surely be an extraordinary achievement regardless.

One significant improvement is still possible, however. For the answer to (11) is just going to be a long series of propositions. The most important ones will come first, and that is the good thing about (11); but we might still have to do a lot of work before we can come up with a good search engine that can find the mildly beneficial true propositions that will meet our needs as these arise. We might need to know whether it is better to check the oil when the car is hot or when it is cold, but the truth about this rather unimportant issue will be buried amidst the sequence. Wouldn't it be better if the angel's answer already came in a good format, matching each true proposition with the question or questions that it would answer? Besides, just as the true propositions can be ranked according to $<_R$, we can come up with a similar ranking for the set of all the questions that we could ask. Certainly those questions are strictly ordered by some *more-beneficial-than* relation R', and we know how to extend this to a total order $<_{R'}$ (the conventions used for $<_R$ would do). We may also suppose that our logical theory S is good enough to be applicable to interrogative clauses, at least insofar as we are interested in the relation of logical equivalence. Asking whether snow is white is equivalent to asking whether it is the case that either snow is white or grass is both green and not green, and our favorite logical theory will say so. Let us, then, make one more revision and redraft (11), or any of its logical improvements, as follows:

(12) What is the sequence, under the lexicographic order induced by $<_{R'}$ and $<_R$, of all the ordered pairs x, y such that x is the $<_{R'}$-first member of some $\equiv_S$-equivalence class of questions that we could ask using at most n words from vocabulary V and y is the $<_R$-first member of some $\equiv_S$-equivalence class of correct answers to x statable in at most n words from vocabulary V?

(The lexicographic order is defined so that x_1, y_1 precedes x_2, y_2 if and only if either $x_1 <_{R'} x_2$, or $x_1 = x_2$ and $y_1 <_R y_2$. Perhaps we could simplify things by requiring the relevant sequence to be a function, the value of each argument x being defined as the $<_R$-first member of the class of *all* relevant answers to x. In that case the lexicographic order would be determined exclusively by $<_{R'}$, but we would forfeit the possibility that some questions have multiple non-equivalent answers, as when we ask for a good restaurant in Rome. So let's not push it.)

This strikes me as a pretty good question to ask, clumsy as it might sound. It is a single question, but its response will contain every possible question-answer pair that we could ever possibly be interested in, ordered in terms of beneficialness, and without logical redundancies. To be sure, the response might still contain some foolish question-answer pairs like (1), (2) , but these will show up towards the end and would be harmless. What is certain is that nowhere will the answer include the pair consisting of (12) together with the angel's answer; for the latter will consist of more than *n* words. And nowhere will the angel's answer include the pair (4),(5) . For (5) is a correct answer to (4) only if (4) is the best question—and that is definitely not the case.

Alas, there is still a chance that things go wrong. For (12) is based on the pragmatic presupposition that the angel will deliver her answer in a sensible way, and that is not so obvious. For one thing, as Sider has pointed out, one can always answer a question of the form 'What is the . . . ?' by saying 'It is the . . .'. This is not as bad as answering (1) with (2) but the result is equally uninformative. And, as it stands, our question is subject to this possibility. Sider's suggestion is that this sort of problem could be overcome by requesting explicitly that the answer be given in some canonical form, and perhaps that suggestion could be applied to our case. For instance, we could require the angel to produce the actual list under some Gödelian coding. On the other hand, perhaps it is also fair, in this regard, to rely simply on the assumption that the angel will produce her response in compliance with certain basic conversational norms. Answering in the indicated way would certainly violate Grice's undisputed *Cooperative Principle* [2]:

(13) Make your conversational contribution such as is required, at the stage at which it occurs, by the accepted purpose or direction of the talk in which you are engaged.

And we may assume that our divine angel—an angel, not a evil spirit—is not going to violate (13).

Unfortunately there are other ways in which the angel's truthful response might still fail to be beneficial for us, even if the angel complied with all the standard conversational maxims. For what do we know about angelic practices? We are asking for a sequence of finite length, and each ordered pair in the sequence is going to be reasonably short, but the angel's actual answer might turn out to be unreasonably long nevertheless. For instance, the angel might take an infinite amount of *time* to tell us the first member of the first ordered pair in the sequence, so that neither us nor our descendants will ever hear the answer to the question that the angel ranks as most beneficial to us. (The angel might do so by uttering the relevant expression at an increasingly lower speed: *n* minutes to do the first half of the job, 2*n* minutes for the next quarter, 4*n* minutes for the next eighth, etc.) Or the angel might take an infinite amount of *space* to tell us the first member of the first pair in the sequence. (She might do so by writing the relevant expression in increasingly bigger and bigger handwriting on an infinitely large piece of paper.) Perhaps only an insane angel would do such things. But the general worry is real: just as we have tried to be careful and rule out the possibility that the angel will squander our time with a long series of redundant propositions, we want to be sure that the angel will not squander our time in these other ways. In fact, there is also a risk that the angel will deliver the answer in a way that will have (perhaps unexpectedly) devastating effects on us. For example, she might begin her response in such a loud voice that all existing humans will die immediately from the enormous force of the sound waves.

It's hard to come up with a general solution to this sort of problem. To the extent that we are worried that our angel might be unkind, or up to mischief, we are bound to accept some risk. But perhaps there is still room for some improvement. So let's make one last, prudential move and let's modify (12) along the following lines:

(14) If I were to read off the sequence, under the lexicographic order induced by $<_{R'}$ and $<_R$, of all the ordered pairs x, y such that x is the $<_{R'}$-first member of some $\equiv_S$-equivalence class of questions that we could ask using at most n words from vocabulary V and y is the $<_R$-first member of some $\equiv_S$-equivalence class of correct answers to x statable in at most n words from vocabulary V, what would it sound like?

This should work fine. For surely, in the nearest possible world in which I read off the relevant sequence, I do so in a way that avoids the problems mentioned above. And we are assuming that the angel is a Gricean creature. She should not, therefore, pretend to miss our point. At least, she should not fulfil her promise with the hideous, sarcastic reply:

(15) It would sound like a pedantic philosopher talking about a lot of important stuff.

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References

[1] Buel D. Belnap, Jr., and Thomas B. Steel, Jr., *The Logic of Questions and Answers*, New Haven and London, Yale University Press, 1976.

[2] Paul H. Grice, ‘Logic and Conversation’, in *Syntax and Semantics*, Volume 3 (Peter Cole and Jerry L. Morgan, eds.), New York, Academic Press, 1975, pp. 51–58.

[3] Ned Markosian, ‘The Paradox of the Question’, *Analysis* 57 (1997), 95–97.

[4] Ted Sider, ‘On the Paradox of the Question’, *Analysis* 57 (1997), 97–101.