Category Theory and the Ontology of Śūnyatā

Abstract
Notions such as śūnyatā, catuṣkoṭi, and Indra’s net, which figure prominently in Buddhist philosophy, are difficult to readily accommodate within our ordinary thinking about everyday objects. Famous Buddhist scholar Nāgārjuna considered two levels of reality: one called conventional reality, and the other ultimate reality. Within this framework, śūnyatā refers to the claim that at the ultimate level objects are devoid of essence or “intrinsic properties”, but are interdependent by virtue of their relations to other objects. Catuṣkoṭi refers to the claim that four truth values, including contradiction, are admissible in reasoning. Indra’s net refers to the claim that every part of a whole is reflective of the whole. Here we present category theoretic constructions that are reminiscent of these Buddhist concepts. The universal mapping property definition of mathematical objects, wherein objects of a universe of discourse are defined not in terms of their content, but in terms of their relations to all objects of the universe is reminiscent of śūnyatā. The objective logic of perception, with perception modeled as [a category of] two sequential processes (sensation followed by interpretation), and with its truth value object of four truth values, is reminiscent of the Buddhist logic of catuṣkoṭi. The category of categories, wherein every category has a subcategory of sets with zero structure within which every category can be modeled, is reminiscent of Indra’s net. Our thorough elaboration of the parallels between Buddhist philosophy and category theory can facilitate better understanding of Buddhist philosophy, and bring out the broader philosophical import of category theory beyond mathematics.

Keywords: Cantor; contradiction; emptiness; essence; figure; functor; Nāgārjuna; natural transformation; object; property; reality; relation; set; shape; structure; structure-respecting morphism; truth; value; Yoneda; zero.
Introduction

Buddhist philosophy, especially Nāgārjuna’s Middle Way (Garfield, 1995; Siderits and Katsura, 2013), is intellectually demanding (Priest, 2013). The sources of the difficulties are many. First it argues for two realities: conventional and ultimate (Priest, 2010). Next, ultimate reality is characterized by śūnyatā or emptiness, which is understood as the absence of a fundamental essence underlying reality (Priest, 2009). Equally importantly, contradictions are readily deployed, especially in catuṣkoṭi, as part of the characterization of reality (Deguchi, Garfield, and Priest, 2008; Priest, 2014). Lastly, reality is depicted as Indra’s net – a whole, whose parts are reflective of the whole (Priest, 2015). The ideas of relational existence, admission of contradictions, and parts reflecting the whole are seemingly incompatible with our everyday experiences and the attendant conceptual reasoning used to make sense of reality. However, notions analogous to these ancient Buddhist ideas are also encountered in the course of the modern mathematical conceptualization of reality. These parallels may be, in large part, due to “experience” and “reason” that are treated as the final authority in both mathematical sciences and Buddhist philosophy. Here, we highlight the similarities between Buddhist philosophy and mathematical philosophy, especially category theory (Lawvere and Schanuel, 2009). The resultant cross-cultural philosophy can facilitate a proper understanding of reality – a noble goal to which both Buddhist philosophy and mathematical practice are unequivocally committed.

Two realities

There are, according to Buddhist thought, two realities: the conventional reality of our everyday experiences and the ultimate reality (Priest, 2010; Priest and Garfield, 2003). In our conventional reality, things appear to have intrinsic essences. It is sensible, at the level of conventional reality, to speak of essences of objects, but at the level of ultimate reality there are no essences, and everything exists but only relationally. There is an analogous situation in mathematics. On one hand, mathematical objects can be characterized in terms of their relations to all objects, in which case the nature of an object is determined by the nature of its relationship to all objects. In a sense, there is nothing inside the object; an object is what it is by virtue
of its relations to all objects. The objects of mathematics are, as Resnik (1981, p. 530) notes, “positions in structures”, which is in accord with the Buddhist understanding of things as “loci in a field of relations” (Priest, 2009, p. 468). On the other hand, there is another level of mathematical reality, wherein we can speak of essences of objects (e.g. theories of objects; Lawvere and Rosebrugh, 2003, pp. 154–155). For example, one can characterize a set as a collection of elements or “sum” of basic-shaped figures (1-shaped figures, where $1 = \{\bullet\}$), with basic shapes understood as essences (Lawvere, 1972, p. 135; Lawvere and Schanuel, 2009, p. 245; Reyes, Reyes, and Zolfaghari, 2004, p. 30). Similarly, every graph is made up of figures of two basic shapes (arrow- and dot-shaped figures; Lawvere and Schanuel, 2009, pp. 150, 215).

This characterization of an object in terms of its contents, i.e. basic shapes or essences (Lawvere, 2003, pp. 217–219; Lawvere, 2004, pp. 11–13), can be contrasted with the relational characterization, wherein each and every object of a universe of discourse (a mathematical category; Lawvere and Schanuel, 2009, p. 17) is characterized in terms of its relationship to all objects of the universe or category (see Appendix A1). The relational nature of mathematical objects, as elaborated below, is reminiscent of the Buddhist notion of emptiness – an assertion that objects are what they are not by virtue of some intrinsic essences but by virtue of their mutual relationships.

**Emptiness**

According to Buddhist philosophy, everything is empty and the totality of empty things is empty. Here, emptiness is understood as the absence of essences. Things, in the ultimate analysis, are what they are and behave the way they do not because of [some] essences inherent in them, but by virtue of their mutual relationships (Priest, 2009). This idea of relational existence has parallels in mathematical practice. Mathematical objects of a given mathematical category (e.g. a category of sets) are what they are, not by virtue of their intrinsic essences but by virtue of their relations to all objects of the category. For example, a single-element set is a set to which there exists exactly one function from every set (Lawvere and Schanuel, 2009, pp. 213, 225). Note that the singleton set is characterized not in terms of what it contains (a single element), but in terms of how it relates to all sets of the category of sets. In a similar vein, the truth value set $\Omega = \{\text{false}, \text{true}\}$ is defined in terms of its relation to
all sets of the category of sets. The truth value set, instead of being defined as a set of two elements “false” and “true”, is defined as a set $\Omega$ such that functions from any set $X$ to the set $\Omega$ are in one-to-one correspondence with the parts of $X$ (ibid. pp. 339–344). To give one more example, a product of two sets is defined not by specifying the contents of the product set (pairs of elements), but by characterizing its relationship to all sets. More explicitly, the product of two sets $A$ and $B$ is a set $A \times B$ along with two functions (projections to the factors) $p_A: A \times B \to A$, $p_B: A \times B \to B$ such that for every set $Q$ and any pair of functions $q_A: Q \to A$, $q_B: Q \to B$, there is exactly one function $q: Q \to A \times B$ satisfying both the equations: $q_A = p_A \circ q$ and $q_B = p_B \circ q$, where “$\circ$” denotes composition of functions (ibid. pp. 339–344). The universal mapping property definition of mathematical constructions brought to sharp focus the relational nature of mathematical objects. It conclusively established that “the substance of mathematics resides not in substance (as it is made to seem when $\in$ [membership] is the irreducible predicate, with the accompanying necessity of defining all concepts in terms of a rigid elementhood relation) but in form (as is clear when the guiding notion is an isomorphism-invariant structure, as defined, for example, by universal mapping properties)” (Lawvere, 2005, p. 7). More broadly, Yoneda lemma (Lawvere and Rosebrugh, 2003, pp. 249–250; Appendix A1), according to which a mathematical object of a given universe of discourse (i.e. category) is completely characterized by the totality of its relations to all objects of the universe (category), is an unequivocal assertion of the relational nature of mathematical objects. Yoneda lemma, as pointed out by Barry Mazur, establishes that “an object $X$ of a category $C$ is determined by the network of relationships that the object $X$ has with all the other objects in $C$” (Mazur, 2008). Thus the Buddhist idea of emptiness or relational existence finds resonance in mathematical practice, especially in terms of universal mapping properties and the Yoneda lemma.

However, note that according to the Buddhist doctrine of emptiness, not only is everything empty, but the totality of empty things is also empty (Priest, 2009). In other words, even the notion of relational existence is empty, i.e. emptiness is not the essence of existence; emptiness is also empty. This idea of emptiness being empty is much more challenging to comprehend. When we say that objects are empty, we are saying that objects are mere locations in a network of relations. But when we say that the totality of empty things is empty, we are asserting that the existence of totality is
also relational just like that of the objects in the totality. What is not immediately clear is how we to think of relations especially when all we have is the totality, i.e. one object. Within mathematics, note that the totality of all objects (along with their mutual relations) forms a category. More importantly, categories are objects in the category of categories (Lawvere, 1966), and hence the totality of objects, i.e. category, is also empty or relational as much as the objects of a category. Thus the idea of śūnyatā (everything is empty) resonates with the relational nature of objects and of the totality of objects (within the mathematical framework of the category of categories).

Equally importantly, Nāgārjuna’s Middle Way, having gone to great lengths to distinguish two realities (conventional essences vs. ultimate emptiness) identifies the two: “There is no distinction between conventional reality and ultimate reality” (Deguchi, Garfield, and Priest, 2008, p. 399). Contradictions (such as these) within Buddhist philosophy, on a superficial reading, are diagnostic of irrational mysticism. However, as we point out in the following, contradictions also figure prominently in the foundations of mathematical modeling of reality. In light of these parallels, “contradiction” may be intrinsic to the nature of reality, which is the common subject of both Buddhist and mathematical investigations, and not a sign of faulty Buddhist reasoning.

**Contradiction**

Within the Buddhist philosophical discourse, one often encounters contradictions and these contradictions are treated as meaningful (Deguchi, Garfield, and Priest, 2008; Priest, 2014). There is an analogous situation in mathematics. Although not every contradiction is sensible, there are sensible contradictions such as the boundary of an object A formalized as “A and not A” (Lawvere, 1991, 1994a, p. 48; Lawvere and Rosebrugh, 2003, p. 201). More importantly, within mathematical practice, it is now recognized that contradictions do not necessarily lead to inconsistency (an inconsistent system, according to Tarski, is where everything can be proved; Lawvere, 2003, p. 214). Of course, admitting a contradiction invariably leads to inconsistency in classical Boolean logic. In logics more refined than Boolean logic contradiction does not necessarily lead to inconsistency. This recognition is very important,
especially since contradiction plays a foundational role in mathematical practice. Briefly, Cantor’s definition of SET is, as pointed out by F. William Lawvere, “a strong contradiction: its points are completely distinct and yet indistinguishable” (ibid. p. 215; Lawvere, 1994a, pp. 50–51). Zermelo, and most mathematicians following him, concluded that Cantor’s account of sets is “incorrigibly inconsistent” (Lawvere, 1994b, p. 6). Lawvere, using adjoint functors, showed that Cantor’s definition is “not a conceptual inconsistency but a productive dialectical contradiction” (Lawvere and Rosebrugh, 2003, pp. 245–246), which is summed up as the unity and identity of adjoint opposites (Lawvere, 1992, pp. 28–30; Lawvere, 1996).

A related notion is catuṣkoṭi, which is routinely employed in Buddhist reasoning (Priest, 2014; Westerhoff, 2006). To place it in perspective, in the familiar Boolean logic, any proposition is either true or false. Put differently, there are only two possible truth values, and they are mutually exclusive and jointly exhaustive. Unlike Boolean logic, in Buddhist reasoning more than two truth values are admissible. In the Buddhist logic of catuṣkoṭi, a proposition can possibly take, in addition to the familiar truth values of “true” or “false”, the truth values of “true and false”, or “not true and not false”. Given a proposition A, there are four possibilities: 1. A; 2. not A; 3. A and not A; 4. not A and not not A. Here contradiction is admissible, i.e. “A and not A” is a possible state of affairs, which is reminiscent of the boundary operation and the unity and identity of adjoint opposites in mathematics, alluded to earlier. Moreover, double negation is not same as identity operation as in the case of, to give one example, the non-Boolean logic of graphs (Lawvere and Schanuel, 2009, p. 355). Note that if not not A = A, then the fourth truth value of catuṣkoṭi is equal to the third.

As an illustration of how the four truth values of catuṣkoṭi could be a reflection [of an aspect] of reality, we consider the category of percepts. Perception involves two sequential processes of sensation followed by interpretation (Albright, 2015; Croner and Albright, 1999). So, we define the category of percepts as a category of two sequential functions of decoding after coding. The truth value object of the category of percepts has four truth values (Appendix A2). Thus the objective logic of perception, with its truth value object of four truth values, is reminiscent of the Buddhist logic of catuṣkoṭi (see Linton, 2005).
Indra’s net and zero structure

Another important concept in Buddhist philosophy is the idea of Indra’s net, wherein reality is compared to a vast network of jewels such that every jewel is reflective of the entire net (Priest, 2015). In abstract terms, reality is characterized as a whole wherein every part is reflective of the whole. Admittedly, this Buddhist characterization of reality sounds mystifying, but there is an analogous situation, involving part-whole relations, in mathematics.

How can a part of a whole reflect the whole? First, note that mathematical objects of all sorts can be modeled (as structures) in the category of sets (Lawvere and Schanuel, 2009, pp. 133–151). Sets have zero structure (Lawvere, 1972, p. 1; Lawvere and Rosebrugh, 2003, p. 1, 57; Lawvere and Schanuel, 2009, p. 146). Negating the structure (cohesion, variation) inherent in mathematical objects, Cantor created sets: mathematical structures with zero structure (Lawvere, 2003, 2016; Lawvere and Rosebrugh, 2003, pp. 245–246). In comparing his abstraction of sets with zero structure to the invention of number zero, Cantor considered sets as his most profound contribution to mathematics (Lawvere, 2006). Sets, by virtue of having zero structure, serve as a blank page – an ideal background to model any category of mathematical objects (Lawvere, 1994b; Lawvere and Rosebrugh, 2003, pp. 154–155). However, structureless sets are a small part – the only part – of the mathematical universe that reflects all of mathematics. It seemed so until Lawvere axiomatized the category of categories (Lawvere, 1966; Lawvere and Schanuel, 2009, pp. 369–370). Along the lines of Cantor’s invention of structureless sets, Lawvere defined a subcategory of structureless (discrete and/or constant) objects within a category by negating its structure (cohesion and/or variation; Lawvere, 2004, p. 12; Lawvere and Schanuel, 2009, pp. 358–360, 372–377). Thus, within any category of mathematical objects, there is a part, a structureless subcategory, that is like the category of sets in having zero structure, and hence serves as a background to model all categories of mathematical objects (Lawvere, 2003; Lawvere and Menni, 2015; Picado, 2008, p. 21). Modeling a category of mathematical objects requires, in addition to the subcategory with zero structure, another subcategory objectifying the structural essence(s) of the objects of the category, i.e. the theory of the given category of mathematical objects. Finding the theory subcategory also depends on the structureless subcategory, by way of contrasting or negating the structureless.
subcategory (Lawvere, 2007). Once we have the subcategory with zero structure and the subcategory objectifying the essence (theory) of a given category, interpreting the theory subcategory into the structureless subcategory gives us models of objects of the given category. Thus, thanks to the recognition of significance of Cantor’s zero structure, every mathematical category can be modeled in any category of the category of categories.

If we compare the category of categories to Indra’s net, then categories within the category of categories would correspond to jewels in Indra’s net. Just as in the case of Indra’s net, wherein every jewel in the network of jewels is reflective of the entire network, in the category of categories every category (part) of the category of categories (whole) reflects the whole. For example, the category of dynamical systems is a part of the category of categories. Within the category of dynamical systems, we have the constant subcategory (obtained by negating the variation) of dynamical systems (wherein every state is a fixed point), which is like the category of sets, and within which any category can be modeled. Similarly, the category of graphs is another part of the category of categories. Within the category of graphs there is the discrete subcategory (obtained by negating the cohesion) of graphs (with one loop on each dot), which is also like the category of sets, and hence can model every category. Thus, we find that within the category of categories, every part is reflective of the whole, which is reminiscent of the Buddhist depiction of reality as Indra’s net: a whole with parts reflective of the whole.

Conclusion

There are similarities between Buddhist philosophy and mathematical practice, especially with regard to essence vs. emptiness, contradictions, and part-whole relations. These similarities might be a natural consequence of identical objectives – understanding reality and commitment to truth – and identical means – experience and reason – employed toward those ends. It is in this respect that the practices of the two – mathematicians and Buddhists – can be compared. Our exercise, on that score, can help better appreciate the rationality of Buddhist reasoning. Oftentimes, admission of contradiction (as in catuṣkoṭi) tends to be equated with irrational mysticism. However, as we have seen, contradictions are also an integral and
indispensable part of the mathematical understanding of reality. Equally importantly, in drawing parallels between Buddhist thought and mathematical practice, we hope to have brought out the broad philosophical import of category theory beyond mathematics.
Appendices

A1. Yoneda lemma

We begin with an intuitive introduction to the mathematical content of Yoneda lemma (Lawvere and Rosebrugh, 2003, pp. 175–176, 249). Informally speaking, Yoneda lemma can be understood as a mathematical conceptualization of commonsense, as in the African ‘Ubuntu’ meaning: I am because we are! In mathematical terminology: Every object (of a category) is completely determined by its relations to all objects of the category. Let us now look at simple illustration of the Yoneda lemma. For ease of exposition, let us consider a single-morphism category (one object along with its identity morphism). Consider a one-morphism category C, with \(1_{\bullet} : \bullet \to \bullet\) as its morphism. Next, consider a set-valued functor \(F : C \to S\), with \(F(\bullet) = A\), a set, and \(F(1_{\bullet}) = 1_A\). Yoneda lemma says: \(A \approx N\), the set of natural transformations \(\eta: C(\bullet, -) \to F\), where \(C(\bullet, -): C \to S\) is a functor assigning to each object in the category C, the set of \(\bullet\)-shaped figures in the object (see Lawvere and Rosebrugh, 2003, p. 241 for the definition of natural transformation). Since there is only one object \(\bullet\), along with its identity morphism \(1_{\bullet} : \bullet \to \bullet\) in C, we calculate the natural transformations \(\eta_{\bullet}: C(\bullet, \bullet) \to F(\bullet)\).

\[
\begin{array}{c}
C \\
\downarrow \hspace{2cm} \downarrow \\
S \\
\end{array} 
\quad \quad 
\begin{array}{c}
C(\bullet, \bullet) \\
\downarrow \eta_{\bullet} \downarrow \\
S \\
\end{array} 
\quad \quad 
\begin{array}{c}
F(\bullet) \\
\end{array}
\]

Since \(C(\bullet, \bullet) = 1\) and with \(F(\bullet) = A\), a set, we obtain natural transformations \(\eta_1\).

\[
\begin{array}{c}
C(\bullet, \bullet) = 1 \\
\downarrow 1_1 \\
C(\bullet, \bullet) = 1 \\
\end{array} 
\quad \quad 
\begin{array}{c}
\cdots \eta_1 \cdots \rightarrow \\
1_1 \\
\cdots \eta_1 \cdots \rightarrow \\
\end{array} 
\quad \quad 
\begin{array}{c}
F(\bullet) = A \\
\downarrow 1_A \\
F(\bullet) = A \\
\end{array}
\]
With the set $A = \{a, b\}$, we find the two natural transformations corresponding to the two elements of the set $A = \{a, b\}$.

The two natural transformations correspond to the two points (functions from the terminal set $\mathbf{1}$) of the set $A$.

Thus, with $F(\bullet) = A$ and $F(1\bullet) = 1_A$ we found that $A \approx N$, the set of natural transformations $\eta: C(\bullet, -) \rightarrow F$, where $A = \{a, b\}$ and $N = \{\eta^a, \eta^b\}$, in accord with the Yoneda lemma.
A2. Four truth values of the logic of perception

Conscious perception involves two sequential processes of sensation followed by interpretation:

Physical stimuli → Brain → Conscious Percepts

(Albright, 2015; Croner and Albright, 1999), which can be thought of as

X – coding → Y – decoding → Z

and objectified as two sequential processes:

A –\( f \) → B –\( g \) → C

Without discounting that the processes of sensation and interpretation are much more structured than mere functions, and with the objective of simplifying the calculation of truth value object, we model percept as an object made up of three [component] sets C, B, and A, which are sets of physical stimuli, their neural codes, and interpretations, respectively, and two [structural] functions \( f \) and \( g \) specifying for each interpretation in A the neural code in B (of which it is an interpretation) and for each neural code in B the physical stimulus in C (of which it is a measurement), respectively (see Lawvere and Rosebrugh, 2003, pp. 114–117). The logic of [the category of] perception, whose objects are two sequential functions is determined by its truth value object (Lawvere and Rosebrugh, 2003, pp. 193–212; Lawvere and Schanuel, 2009, pp. 335–357; Reyes, Reyes, and Zolfaghari, 2004, pp. 93–107). The truth value object of a category is an object \( \Omega \) of the category such that parts of any object \( X \) are in 1–1 correspondence with maps from the object \( X \) to the truth value object \( \Omega \). Since parts of an object are monomorphisms with the object \( X \) as codomain, for each monomorphism with \( X \) as codomain there is a corresponding \( X \)-shaped figure in \( \Omega \). In order to calculate the truth value object, first we need to define maps between objects of the category of percepts. A map from an object
A → B → C
to an object
A' → B' → C'
is a triple of functions
p: A → A', q: B → B', and r: C → C'
satisfying two equations
q ◦ f = f' ◦ p and r ◦ g = g' ◦ q
which make the two squares in the diagram commute, i.e. ensure that maps between objects preserve the structural essence of the category (Lawvere and Schanuel, 2009, pp. 149–150). Now that we have maps of the category of percepts defined, we can calculate its truth value object. The truth value object of a category is calculated based on the parts of the basic shapes (essence) constituting the objects of the category. In the category of sets, one-element set 1 (= {•}) is the basic shape in the sense that any set is made up of elements (see Posina, Ghista, and Roy, 2017 for the details of the calculation of basic shapes, i.e. theory subcategories of various categories). Since the set 1 is also the terminal object (i.e. an object to which there is exactly one map from every object; Lawvere and Schanuel,
of the category of sets, and since every set is completely determined by its points (terminal object-shaped figures), we can determine the truth value object of the category of sets by determining its points, i.e. maps from \(1\) to the (yet to be determined) truth value object. According to the definition of truth value object, \(1\)-shaped figures in the truth value object are in 1–1 correspondence with parts of \(1\). Since the terminal set \(1\) has two parts: \(0\) (\(=\{\}\)) and \(1\), the truth value set has two points (elements). Thus, the truth value object of the category of sets is \(2\) (\(=\{\text{false, true}\}\)). Along similar lines, let us calculate the terminal object of the category of percepts. Since there is only one map from any object (two sequential functions) to the object \(T\) (two sequential functions from one-element set to one-element set):

\[
1 \to 1 \to 1
\]

the terminal object of the category of percepts is \(T\). Since parts of the terminal object \(T\) correspond to the points of the truth value object, let’s look at the parts of the terminal object. The terminal object \(T\) has four parts:

Part 1 (\(\theta_0: 0 \to T\))

\[
\begin{array}{c}
0 & 1 \\
\downarrow & \\
0 & 1 \\
\downarrow & \\
0 & 1
\end{array}
\]

Part 2 (\(\theta_1: 0_1 \to T\))

\[
\begin{array}{c}
0 & 1 \\
\downarrow & \\
0 & 1 \\
\downarrow & \\
1 & 1
\end{array}
\]
Part 3 ($\theta_2: 0_2 \to T$)

\[
\begin{array}{c}
0 & 1 \\
\downarrow \\
1 \to 1 \\
\downarrow \\
1 \to 1
\end{array}
\]

Part 4 ($I: T \to T$)

\[
\begin{array}{c}
1 \to 1 \\
\downarrow \\
1 \to 1 \\
\downarrow \\
1 \to 1
\end{array}
\]

These four parts correspond to the four points (global truth values) of the truth value object, which means that the component set (of the truth value object) corresponding to the stage of interpretations is a four-element set $4 = \{0, 0_1, 0_2, I\}$. Since objects in the category of perception (two sequential functions) are not completely determined by points, we look for all other basic shapes that are needed to completely characterize any object of the category of two sequential functions. The other basic shapes, besides the terminal object $T$, are: domains of the parts $\theta_2$ and $\theta_1$ of the terminal object $T$, i.e. shape $0_2$

\[
\begin{array}{c}
0 & 1 \to 1
\end{array}
\]

and shape $0_1$

\[
\begin{array}{c}
0 & 0 & 1
\end{array}
\]

Since the basic shape object $0_2$ has three parts ($\theta$, $\theta_1$, and $I$), there are three $0_2$-shaped figures in the truth value object, and since the object $0_1$ has two parts ($\theta$ and $I$), there
are two 0₁-shaped figures in the truth value object, which means that the component set (of the truth value object) corresponding to the stage of neural coding is a three-element set $3 = \{0, 0₁, 1\}$, while the component set (of the truth value object) corresponding to the stage of physical stimuli is a two-element set $2 = \{0, 1\}$. Putting it all together we find that the truth value object of the category of percepts is:

$$4 \rightarrow j \rightarrow 3 \rightarrow k \rightarrow 2$$

We still have to determine the functions $j$ and $k$, which can be done by examining the structural maps between the basic shapes

$$0₁ \rightarrow c \rightarrow 0₂ \rightarrow d \rightarrow T$$

which as a subcategory constitutes the theory (abstract essence) of the category of two sequential functions. More explicitly, the incidence relations between the three basic-shaped figures in the truth value object are calculated from the inverse images of the parts of the basic shapes ($0₁, 0₂$, and $T$) along the structural maps ($d$ and $c$). The inverse images of each one of the four points ($0, 0₁, 0₂$, and $1$ corresponding to the four parts of the terminal object $T$) along the structural maps decoding $d$ and coding $c$ give for each one of the four global truth values $4 = \{0, 0₁, 0₂, 1\}$ its value in the truth value sets $3 = \{0, 0₁, 1\}$ and $2 = \{0, 1\}$ of the previous stages of neural codes and physical stimuli.

For example, the global truth value $0₂$ corresponds to the part $0₂$ of the basic shape $T$, and its inverse image along the structural map $d$: $0₂ \rightarrow T$ is the entire basic shape $0₂$, which corresponds to the truth value $1$ (of stage $3$); and the inverse image of the entire object $0₂$ along the structural map $c$: $0₁ \rightarrow 0₂$ is the entire basic shape $0₁$, which corresponds to the truth value $1$ (of stage $2$). Along these lines, we find that

$$j (0) = 0, j (0₁) = 0₁, j (0₂) = 1, j (1) = 1$$

$$k (0) = 0, k (0₁) = 1, k (1) = 1$$

which completely characterizes the truth value object
\[ 4 - j \to 3 - k \to 2 \]

of the category of percepts.
References


