Peirce's Arrow and Satzsystem: A Logical View for the Language-Game

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ABSTRACT— ***This article is an effort to understand how the Peirce's Arrow (Logical NOR), as a logical operation, can act within the concept of Ludwig Wittgenstein's language-game, considering that the language game is a satzsystem, i.e., a system of propositions. To accomplish this task, we will cover four steps: (1) understand the possible relationship of the thought of C. S. Peirce with the founding trio of analytic philosophy, namely Frege-Russell-Wittgenstein, looking for similarities between the logic of Peirce and his students (notably Christine Ladd and O.H. Mitchell) with a New Wittgenstein's approach, which sees Early Wittgenstein (Tractatus Logico-Philosophicus), Middle Wittgenstein and Last Wittgenstein (Philosophical Investigations) while a coherent way of thinking and not a theoretical break; (2) describe the operation of the Peirce's Arrow (Logical NOR) as a logical connective; (3) understand the notion of satzsystem (Middle Wittgenstein) and the possibility of applying the concept of language-game (Last Wittgenstein) on it; and (4) understand how the Logical NOR can operate within a satzsystem. The goal here is a search for the logic of the language-game and how the logical ideas of C. S. Peirce can help in this construction. And this construction might be interesting for a better understanding of the analytic philosophy of language.***

Keywords— C. S. Peirce, Ludwig Wittgenstein, Logic, Language-game

1. PEIRCE AND THE ANALYTIC PHILOSOPHY: UNIVERSALISM AND LOGICAL NOR

It is well known the position of Jaako Hintikka (1997, p. 145) that puts the logic of Peirce as a modal logician, i.e. model-theoretical tradition of logic, in opposition to the universalist logic's trio Frege-Russell-Wittgenstein. Hintikka sets, as a deciding factor of its position in the broad debate about Peirce in analytic philosophy of language, the question of metalogic. A metalogical approach would only be possible for the members of modal logic thanks to their beliefs in a non-linkage between language and world.

> One particular consequence of the universalist position is that our language and its logic can neither be self-applied, as a whole nor discussed in its entirety in a separate metalanguage (except for its purely formal features of course). This consequence, which was noted by van Heijenoort in his pioneering paper, offers some of the most useful tests of actual historical membership in the two traditions. If there were any lingering doubts about Peirce's allegiance to the model-theoretical tradition, they would hence be quickly dispelled by his willingness to discuss logic by means of logic (Hintikka, 1997, p. 148).

Hintikka uses, as an example for his theory, the existential graphs. Here there is a clear reference to the so-called gamma graphs. However, it is worth remembering that the alpha graphs have a mechanism almost identical to the Boolean logic and beta graphs are a first-order logic, such as that of Frege's.

The question we must ask here is if the metalogic of gamma graphs can assure a Peircean full turn to modal logic or whether it is, actually, a metalogical device used by an universalist logic, which is not uncommon. We can is an example of this in the concept of truth conditions (analogous to truth functions) presented in the *Tractatus*:

> We might say that the system of the *Tractatus* is reflexively self-destructive. In effect, Wittgenstein presents a metalanguage specifying the truth-conditions for a set of propositions that make up an object

> language. Matters are so arranged that the propositions in the metalanguage do not satisfy the conditions for propositions in the object language. In this way, standard paradoxes are avoided. If the complaint is now made that the object language is incomplete in not characterizing propositions of this kind that make up the metalanguage, Wittgenstein has a remarkable reply: although the propositions in the object language cannot say what the propositions in the metalanguage say, they make this things manifest simply by *embodying* the principles laid down in the metalanguage. So no loss occurs when these metapropositions are expelled from language, for the propositions of the object language are able to make known, without saying, what these metapropositions attempt to say. Metapropositions are a temporary expedient. Modern logic has not followed this course, but remains an idea of both depth and originality (Fogelin, 1995, p. 102).

Without getting into the investigation of possible dialogue between Peirce and Wittgenstein, we can see other situations that can make us see the Peircean logic outside the modal model defended by Hintikka. An example is given by Susan Haack (2002, p. 15) who says that Peirce wrote in his Logic Notebook, in 1909, that triadic logic is universally true. Thus, Peirce's triadic logic, developed a decade earlier, is a logical alternative like the many-valued logic of Lukasiewicz and Post.

Now, we can agree with Haack (2002, p. 269) who says that a system is considered a variant of another if it shares the vocabulary of the first, but has a different set of theorems/valid inferences. In this way, an "alternative logic" is a system that is a variant of classical logic. Thus, the many-valued logic of Lukasiewicz and Post, and the Peircean triadic logic, are both related with the bivalent semantic of the truth tables developed by Wittgenstein, which were made for an universalist logical way of thinking, in the tradition build by *Principia Mathematica* and *Begriffsschrift*, for his *Tractatus Logico-Philosophicus*.

Also, at the same time that the question of truth table was being laid by Peirce, his students were plotting considerations on truth conditions. Making an explicit criticism of W. S. Jevons, Christine Ladd (1883, p. 62) draws a table of truth condition for two arguments in a very similar way of the one made by the proposition 5.101 in the *Tractatus*. In this table, Ladd draws up 16 possible logical universes, using the word "universe" in Boole's tradition. Moreover, it does not observe these universes in a state of separation, as does Jevons (1874, p. 155-7), but in relationship. This "in relationship situation" makes this universe a path between 0 (absence of all) the universe 15 (the presence of all).

The result of this order results in a logic table by the manner of the classical logic, particularly the *Begriffsschrift* and the Aristotelian logic. Note that, even if it is possible to distinguish a metalogical case from Ladd's reflection, it would not be the same way as the one used by the modal logic of modalities, as there are no answers to the questions, for example, about the possibility of probability and necessity. The metalanguage here would be the same order as that found in the *Tractatus*' metapropositions.

Something similar to Ladd's tables are also found in O. H. Mitchell (1883, p. 75-6). The only difference between these two Peircean students is the lack of use of the symbol of the wedge by Mitchell, trampling more on the symmetry of the Aristotelian propositions studied by De Morgan.

However, it is interesting to note that Mitchell not only is influenced by the Peircean logic, but also by the logic designed by Ladd. He said that Ladd's wedge, with the addition of caret enables a cleaner use of Boolean logic without the use of the arbitrary symbol v (Mitchell, 1883, p. 97).

For Boole, the situations of v show all the bias of a given sentence letter. With the use of his wedge, Ladd eventually put that question in the sentential connective, i.e., the symbol of operation.

This Ladd's concern – confirmed as valid by Mitchell – in pursuit of sentence connectives which combine multiple logical operations on them seems to have been inherited from Peirce himself. In 1880, three years of the publication of Studies in Logic, Peirce wrote an unpublished text entitled "A Boolian Algebra with one constant" where their calls for a theoretical concern:

> Every logical notation hitherto proposed has an unnecessary number of signs. It is by means of this excess that the calculus is rendered easy to use and that a symmetrical development of the subject is rendered possible; at the same time, the number of primary formulæ is thus greatly multiplied, those signifying facts of logic being very few in comparison with those which merely define the notation. I have thought that it might be curious to see the notation in which the number of signs should be reduced to a minimum; and with this view I have constructed the following. The apparatus of the Boolian calculus consists of the signs, =, > (not used by Boole, but necessary to express particular propositions) +, -, *X*, 1, 0. In place of these seven signs, I propose to use a single one (CP 4.12).

Thus, Peirce develops the logical NOR, also called joint denial. This operation would be shown by the connective ↓, called Peirce's Arrow. As it was not published in his time, the logical NOR did not participate in the development of the field of logic in philosophy of language.

However, more than 30 years of discovery of the Peirce's Arrow, Henry M. Sheffer published in 1913, an article in which he reduces all of the sentence connectives used in the *Principia Mathematica* – and those used in other formal logics of his time – in a single one. He did this through the logical dual of Peirce's logical NOR: the logical NAND, also called alternative denial.

The discovery of the logical NAND - symbolized by ↑, called Sheffer's stroke – was crucial in the modifications made by Russell in a new version of the *Principia Mathematica*. Both Russell and Wittgenstein were thinking about this job open by Sheffer: a reduction that would solve all the problems of logic. In his *Notes of Logic*, written in a self-imposed exile in Norway, Wittgenstein tried to find such a reduction with Sheffer's discovery, but failed. "Jean Nicod found it in 1916. Nicod demonstrated that only one axiom, together with the rule of uniform substitution and one other inference rule governing Sheffer's stroke, suffices to generate *Principia*'s sentential calculus" (Landini, 2011, p. 344-5).

Despite not having achieved this reduction immediately, Wittgenstein's *Tractatus* is highly influenced by Sheffer. The N-operator of the proposition 6 – "The general form of truth function is $[\bar{p}, \bar{\xi}, N(\bar{\xi})]$. This is the general form of proposition" (TLP 6) – is the analogue of Sheffer's stroke, which has just expanded its power by Wittgenstein to deny all the elements of a given set. Thus, the *Tractatus* tells us what is possible, using the NAND in the totality of logical propositions, derive any sentence's logic.

Something similar is Quine (1981) in the 1940s, but this time using the logical NOR. In fact, Quine (1981, p. 49) reports that Peirce not only discovered the logical NOR in 1880, but was already aware of the Sheffer's stroke in 1902, a decade before the publication of the article.

Here it is interesting to note that both logical operations - which, ultimately, may be considered fundamentally Peircean - the logical NOR and the logical NAND were extremely important for two constitutive theories that are considered universal by Hintikka (1997, p. 142): Wittgenstein's and Quine's. In addition, through the discovery of the Sheffer's stroke Russell decided to upgrade his *Principia Mathematica* (Landini, 2011, p. 345).

Then we have at least three findings that contradict the classification of Peirce by Hintikka as a model-theoretic logician: (1) the question of universality of the triadic logic, (2) the presence of truth tables and truth functions defined by Peirce as much by his students of logic, and (3) the essential use of Peircean sentential connectives in the so-called universal logic. In addition, we think the very semiotics itself as universal, especially on the issue of Universal Semiotics as represented by the term " pan-semiotic vision of the world" (Nöth, 1995, p. 61).

So we can imagine that there is a need to check other sort of logic that Peirce can bear its multiple characters. This check can provide a better understand on how much influence Peirce has on universalism. Maybe we have to resort to the question of psychologism.

When studying the question of psychologism in logic, Susan Haack (2002, p. 310) notes three positions: the antipsychological (Frege), strong psychologism (Boole) and the intermediate position of the weak psychologism. Each can be set to a position: (i) the logic is descriptive of mental processes (it describes how we think, or maybe we should think), (ii) the logic is prescriptive of mental processes (it prescribes how we should think), (iii) logic has nothing to do with mental processes; You can call these positions strong psychologism, weak psychologism and antipsychological, respectively. Examples: Kant argued something like (i), Peirce, a version of (ii), and Frege (iii).

Peirce calls that logic is normative in relation to reasoning. In the weak psychologism, we are not talking about ideas (as in strong psychologism) or propositions (as in antipsychological). In it, the logic is in the service of the sentences, based on worldly behavior that is equally objective and accessible.

In this way, it becomes easier to understand how Peirce and Wittgenstein can find a possible dialogue. Peirce, being between Boole and Frege, provides an important framework for understanding the Wittgensteinian concepts, especially in a new dimension of Wittgenstein that promotes a coherent junction between the *Tractatus* and *Philosophical Investigations*.

Now, if in the *Tractatus*, Peirce's connective – by Sheffer's article – has a central role, this form of operation of thought also becomes reinforced in the reading of the *Philosophical Investigations* where the field of weak psychologism opens in Wittgensteinian framework. Here, Wittgenstein deeply regrets a complete denial of psychologism in the *Tractatus* as it becomes clear in *Investigations'* paragraph 292: "Don't always think that you read off what you say from the facts; that you depict these in words according to rules! For you would still have to apply the rule in the particular case without guidance" (PI §292).

Here there is a double attack: (i) to the antipsychological that somehow implies a logical solipsism; and (ii) to a strong psychologism that would lead to a language of behaviorism. Wittgenstein conclude that a antipsychological approach is not viable because the language is a domain shared by the rules of language-games and that a strong psychologism approach is not viable because there is not a metaphysical ideal to be followed (a guide), only the rules of interaction.

Now we can see how the Wittgensteinian language game is at the service of the sentences, whether its rules are statements as sentences of their activities. As *Investigations'* paragraph 202 indicates: "That's why 'following a rule' is a practice. And to *think* one is following a rule is not to follow a rule. And that's why it's not possible to follow a rule 'privately'; otherwise, thinking one was following a rule would be the same thing as following it" (PI §202).

Now we know that, as indicated Baker and Hacker (2005, p. 46), Ludwig Wittgenstein, in his return to philosophical activity in the 1930s, became interested in the controversy between Frege and the formalists called (Heine, Thomae and Weyl). These formalists believed that arithmetic was a game played with combinatorial rules and signs without content. There is here an analogy between numbers and the chess pieces, where the two are just signs of an external set of rules, signs of a function.

Thus, Wittgenstein's language-game comes from his answer to the Vienna Circle about this debate. Wittgenstein ends up taking the side of formality and extends the analogy between languages – it is appropriate to recall, in the Fregean tradition, the investigation of arithmetic is an investigation of the foundations of language - and chess. There is just a glimpse of what Wittgenstein set as a conceptual tripod for the language-game: rules, grammar and calculus. Baker and Hacker (2005, p. 47-8), reviewing the whole of Wittgenstein's work, reported the formation of tripod points on the analogy between language and chess, which strengthens the position of the first two feet and questions the character of calculus in this type of formulation.

Thus, we are steeped in language-games that are based on two situations: calculus as use (there is a well-defined way to calculate: play the language-game is, ultimately, the meaning is in use and not a substrate of reality) and the obedience of rules while the plurality of follow-up (there is not a right way to play the language-game).

The language-game is, first and foremost, a method for observing the inclusion of languages in life, and it consists of various protocols with their own grammars. We experience each of these protocols by using a certain set of language. Thus, we can observe how fruitful it can be a weak psychologism reading applied in the concept of the language-game. Also fruitful, is to observe how the ideas of Peirce's logic can help to reinforce this path. It must reinforce this path mainly as a bridge between the *Tractatus*, which seeks to see the limits of language as limits of the world, and the *Philosophical Investigations*, which notes that the constitution of the language-world is grounded in the use of language-games, much like the sentences' issues. So, rather than merge these two phases by the issue of therapeutic philosophy, like the "New Wittgenstein" does, this bridge is made by a joint issue of language and logic.

As we have seen over the last pages, Peirce and his students, indirectly, are already present in the universalist strand of analytic philosophy that Wittgenstein is present in its two phases. Thinking in this way, their logical considerations are ideals for build this Wittgensteinian bridge. The most notorious example is the logic built by Christine Ladd. It manages to combine the analysis of truth conditions of the *Tractatus* and the reduction of the sentence connectives without resorting to the Picture Theory, which would indicate an antipsychological instance.

So, maybe, this little treatise on logic, within the Peircean field, will make us able to see the possibilities of a new vision of language-game. But we must first understand the logical operation of the Peirce's Arrow and, also, understand the language-game, especially as a *satzsystem*, a concept of the Middle Wittgenstein.

2. THE LOGIC OF PEIRCE'S ARROW

Ordinarily, the logical operation led by logical NOR – which is known as Peirce's Arrow because of its logical connective (↓) and its creator, C. S. Peirce, who introduced the symbol, but the called it *ampheck*, a noun derived from the Greek word that means "two ways" – can be defined very simply: in a situation with two logical values of two propositions (of p and q, e.g.), it produces a value of True when both propositions are false. In the other three scenarios, it produces a value of False.

As we said earlier, Peirce introduces the Boolean operator as a universal, capable of performing all logic operations required within a certain algebraic formulation. The Peirce's Arrow alone can, by successive operations, replace the other logical connectives, setting itself a formal logical system. Thus, as the logical NAND (Sheffer's stroke), the logical NOR is functionally complete as we can see in the following operations. These operations are the four real functions given by Jean Nicod (1917, p. 32), in his analysis of Sheffer's stroke and its application in *Principia Mathematica*, while elementary and necessary in classical logic:

$\sim p \equiv p \downarrow p$

$p \wedge q \equiv (p \downarrow p) \downarrow (q \downarrow q)$

$p \vee q \equiv (p \downarrow q) \downarrow (p \downarrow q)$

$p \supset q \equiv ((p \downarrow p) \downarrow q) \downarrow ((p \downarrow p) \downarrow q)$

With that, the logical NOR establishes a universality within a given system of propositions. And it would not be restricted to operations with two sentence letters. Peirce himself performs operations on three propositions (CP 4.265) and indicates that this is not the limit: it is only necessary the vinculum (i.e. parentheses) and the *ampheck* to perform such logical operations.

It is the universality that Wittgenstein seeks to apply into logic with the *Tractatus'* N-operator as mentioned earlier. The question is, if it can operate in any system of propositions, how the Peirce's arrow could operate within the language game if we consider it a logical system? To answer this question, we must better understand the *satzsystem* side of the language-game.

3. *SATZSYSTEM* AND LANGUAGE-GAME

To initiate the development of this section, we must accept a challenge presented in Ludwig Wittgenstein's *Zettel*: "Compare: to invent a game - inventing a language - to invent a machine" (Z § 327). This paragraph participates, in depth, in the broad debate in the philosophy of language between natural languages and formal languages.

This debate, that opposes human language with the machines, causes Wittgenstein to put the third question: invent the game. Interestingly, in *Zettel*, we find the beginning of the formulation of language-game and the presence of such statement anticipates the plurality of language-games described by the *Philosophical Investigations*. In PI, the closest definition that Wittgenstein gives us of a language game is in paragraph 7:

> In the practice of the use of language (from PI §2) one party calls out the worlds, the other acts on them. However, in instructions in the language the following process will occur: the learner *names* the objects; that is, he utters the word when the teacher points at the stone. – Indeed, there will be an even simpler exercise: the pupil repeats the words after the teacher – both of these being speech-like processes.
>
> We can also think of the whole process of using words in §2 as one of those games by means of which children learns their natives languages. I will call these games "*language-games*" and will sometimes speak of a primitive language as a language-game.
>
> And the process of naming the stones and of repeating words after someone might also be called language-games. Think of certain uses that are made of words in games like ring-a-ring-a-roses.
>
> I shall also call the whole, consisting of language and the activities into which it is woven, a "language-game" (PI §7).

If you look closely, the three items are here: natural language, formal language (primitive language, as symbolic as the logical languages started with Boole and *Principia Mathematica* and that led to Turing) and language-game (the language-game of PI §2). Such a distinction also appears in Paragraph 23 of *Investigations*, where there is an enumeration of the possible language-games. Thus, there is a series of language protocols that are beyond the traditional conception of natural languages and formal languages. See, for example, the question of "representing theater." Representing theater is an exercise of *mise en scène*, performance, beyond the use of natural language, and we are not talking here only the example of plays that use only body language. Every play has its own protocol to be recognized as a theatrical activity (and here is the root of the answer the question "What is Theatre?" Or at least "What is the Theatrical Language?").

Thus we see that the whole language is not formed only by what we might call the language, but also because these language protocols. And the language-game, nothing more nothing less, is the structural unit that makes such activities constitute what we call language.

Here is the question of Wittgensteinian *satzsysteme* (systems of propositions; when it is a system of propositions, is *satzsystem*). For Shanker, the *satzsysteme* is Wittgenstein's answer to the problem pointed out by Ramsey in the

Tractatus. In this book, Wittgenstein, according to his Cambridge colleague, in his try to prove the logical necessity by the problem of colour-exclusion, undermines the very necessity to draw a logical conclusion because it could be easily answered by metaphysical needs (time, space, matter, ether). Thus, the logical form of the *Tractatus* would be paradoxical, preventing its function of separation between logical truth and empirical truth.

The solution for this paradox – which is the beginning of the Wittgensteinian real turning point for Shanker – is quite refined:

> In order to correct this defect, Wittgenstein introduced two major innovations which he believed would enable him to resolve the colour-exclusion problem in such a way as to preserve the *Tractatus'* rigid demarcation between logical and empirical truth. His first step was to abandon the *Tractatus'* sweeping model of a single amorphous calculus underlying natural language, and to shift to what he described as a *Satzsysteme* conception, in which language was seen as comprising a complex network of interlocking calculi: autonomous "propositional systems" each of which constitutes a distinct "logical space" (Shanker, 1987, p. 6-7).

These logical spaces build by the *satzsysteme* are the language-games: "artificial microcosms of language whose sole purpose was to clarify various aspects of actual linguistic practice" (Shanker, 1987, p. 9). Thus, "For a given logic (*Beweissystem*), each *Satzsystem* can be formalized as a theory that defines the ontology of a narrow subject. The multiplicity of *Satzsysteme* implies that any word that is used in more than one system will have a different sense in each." (Sowa, 2010, p. 15).

Thus, we can understand the provocation of paragraph 327 of *Zettel*: natural languages and formal languages are *beweissysteme* protocols in which the language act. To understand these protocols – especially in the character of its definition in his "what", i.e., its ontology – it is necessary to draw logical spaces of the system of propositions [*satzsystem*], which is nothing less than the language-games.

Here there is the possibility of a logical analysis to observe the functioning of these *satzsysteme* that are language-games. As Wittgenstein himself says in his conversations with the Vienna Circle, the *satzsystem* is the main way of defining something in the face of reality. In a meeting with the Vienna Circle at Christmas 1929, recorded by Waismann, Wittgenstein compares the change between the *Tractatus'* calculation and the *satzsystem* idea:

> Once I wrote, "A proposition is laid against reality like a ruler. Only the end-points of the graduating lines actually *touch* the object that is to be measured". I now prefer to say that a *system of propositions* is laid against reality like a rule. What I mean by this is the following. If I lay a ruler against a spatial object, I lay *all the graduating lines* against it at the same time.
>
> It is not the individual graduating lines that are laid against it, but the entire scale. If I know that the object extends to graduating line 10, I also know immediately that is does not extend to graduating lines 11, 12, and so forth. The statements describing for me the length of an object form a system, a system of propositions. Now it is such an entire system of propositions that is compared with reality, not a single proposition (Wittgenstein *apud* Waismann, 1979, p. 63-4).

Thus, a *satzsystem*, i.e. language game, indicates the logical space of the definition of something. That is made through the operations made by the truth functions and their own logical necessity. That is the moment when we see this essay's point: that *Tractatus*' Wittgenstein is not totally different from *Investigations'* Wittgenstein. And, the curious thing is that what separates the two Wittgenstein – that is, the medium phase with the lectures and conversations with the Vienna Circle – is what binds them together in a single way of thinking. Like Wittgenstein says in *Philosophical Remarks* § 85: Language means the totality of propositions. We could say that a proposition is that we can apply the truth functions. - The truth functions are essential to language.

In this paragraph – the number 85 in the *Philosophical Remarks* – clearly demonstrates that union between all Wittgensteins (Early, Middle and Later) by Wittgenstein's own letter. We see it not only the question of the truth functions and the limits of language as limits of world (TLP 5.6), both central to the *Tractatus*, but also throughout the whole conception of language through language-games while *satzsysteme*, the great junction between Middle and Later Wittgenstein.

4. LOGICAL NOR IN THE LANGUAGE-GAME: A COMMUNICATIONAL EXAMPLE

After covering the relationship between Peirce and Analytic Philosophy, as well as the concept of the Logical NOR and *satzsystem*, we are able to analyze the operational logic of Peirce's Arrow within a language-game. For this, we use the language-game of communication described by the mathematical model of Claude Shannon and Warren Weaver.

Shannon thought, in 1948, in a communicational model focused on transmission issues, worrying about something that might be called mathematical mechanics of communication. Weaver, a year later, popularized the ideas of Shannon, seeing "philosophical" implications to the field of communication.

Thus, Shannon and Weaver became known with a communication model that consists of six elements – information source, transmitter, channel, receiver, destination and noise source – working with two elements: the message and signal. As Shannon himself (1948, p. 381) writes, these parts of the communication model must be represented "as mathematical entities, and idealized by their physical counterparts." These mathematical entities possessed a very definite scheme and closed explanation:

> 1. An *information source* which produces a message or sequence of messages to be communicated to the receiving terminal. The message may be of various types: (a) A sequence of letters as in a telegraph of teletype system; (b) A single function of time $f(t)$ as in radio or telephony; (c) A function of time and other variables as in black and white television — here the message may be thought of as a function $f(x;y;t)$ of two space coordinates and time, the light intensity at point $(x;y)$ and time t on a pickup tube plate; (d) Two or more functions of time, say $f(t)$, $g(t)$, $h(t)$—this is the case in "threedimensional" sound transmission or if the system is intended to service several individual channels in multiplex; (e) Several functions of several variables—in color television the message consists of three functions $f(x;y;t)$, $g(x;y;t)$, $h(x;y;t)$ defined in a three-dimensional continuum—we may also think of these three functions as components of a vector field defined in the region — similarly, several black and white television sources would produce "messages" consisting of a number of functions of three variables; (f) Various combinations also occur, for example in television with an associated audio channel.
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> 2. A *transmitter* which operates on the message in some way to produce a signal suitable for transmission over the channel. In telephony this operation consists merely of changing sound pressure into a proportional electrical current. In telegraphy we have an encoding operation which produces a sequence of dots, dashes and spaces on the channel corresponding to the message. In a multiplex PCM system the different speech functions must be sampled, compressed, quantized and encoded, and finally interleaved properly to construct the signal. Vocoder systems, television and frequency modulation are other examples of complex operations applied to the message to obtain the signal.
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> 3. The *channel* is merely the medium used to transmit the signal from transmitter to receiver. It may be a pair of wires, a coaxial cable, a band of radio frequencies, a beam of light, etc.
>
> 4. The *receiver* ordinarily performs the inverse operation of that done by the transmitter, reconstructing the message from the signal.
>
> 5. The *destination* is the person (or thing) for whom the message is intended (Shannon, 1948, p. 380-1).

The sixth, the noise source, as explained by Weaver (1949), is simply those things that are unintentionally added to the source of information. Moreover, Weaver believes that Shannon's communicational model is a metaphor of the human as a machine: "When I talk to you, my brain is the information source and yours is the destination, my voice system is the transmitter and your ear and eighth nerve is associated with the receiver".

The problem – one of the problems, in fact – to believe that this is the only analytical way to see the communication in question is the point of view by Shannon and Weaver used to examine the communication. Mauro Wolf (1992, p. 102) call it, precisely, as the "point of view of the postman" because they saw it as syntactic system. This means that this system is merely an organizer, without contemplating the problem of message and its communicative dimension. As Wolf points out, the information – key feature to Shannon – must not to be confused with meaning.

However, within the dimension of the philosophy of language that the present work is affiliated, this information system is not appropriate and, ultimately, may be considered erroneous. In its flux – as is well represented by the arrows of his scheme – we do not see the relationship of interactional language, especially language and things in the world. So

what Umberto Eco (1972) proposes is an update of this model, making sure that the code transforms itself from "a significant system to a system of meanings."

Eco's model (2007, p. 48 and 90-1) is simple and can be divided into four major groups: transmitter's functions, receiver's functions, communicational interaction between transmitter and receiver, and the external communicational common. From a mere telegraphic way, as seen by Shannon, Eco draws functions that are similar to each other and set up a multi-faceted way of communicational flux.

The transmitter's functions are: Source and Transmitter. They are the activities that start the communicative action. It ties the issue of shaping the world (i.e. source) and the linguistic engine that makes that cut (i.e. transmitter).

The receiver's functions are: Receiver, Source and Formal System, Semantic Receiver and Meaning. They are all functions in the consolidation of the shaping in the world, having an analogous to transmitter (i.e. receiver), a redefinition of the shape (i.e. source) on the plurality of possibilities (i.e. formal system) where only one is chosen (i.e. meaning) by the linguistic engine the receiver (i.e. receiver semantic).

The interactive features of communication between transmitter and receiver are: Transmitter Code, Medium and Receiver Code. It is the intersection of pure communication, where the two forms of consolidated statements (i.e. transmitter code and receiver code) are consolidated in one medium.

The functions of external communicational common are Rhetorical Apparatus, Ideological Apparatus, Physical Noise, Circumstance and Semantic Noise. These are all situations in a given space and time that are external to linguistic communicational process, but part of it. It is the piece of the *beweissystem* in the *satzsystem* which is within the language-game. It is the only part that is not seen by Shannon and Weaver's model and demonstrates the great progress made by Eco's semiotic model.

Thus, it is easy to see that the four functions of propositions are propositions that make up a particular language game. We must be careful in saying that there is *satzsysteme en abyme* placing a set of propositions. What is formed here is not a set, but a system. The warning here is required by the Wittgensteinian criticism over the Set Theory.

Now, seeing this, it is easy to associate these four functions to the logical space delimited by the sentential letters in a Venn diagram. Indicating that p is the transmitter and q is the receiver, we find the following configuration: (1) transmitter's functions are $p\sim q$, (2) receiver's functions are $\sim pq$, (3) interactive features of communication between transmitter and receiver are pq; and (4) the external communicational common is $\sim p\sim q$.

With this analysis, we can think about a logical operation within a language-game. The logical NOR is the reciprocal negation of disjunction. As we know by its truth table, obtaining the disjunction produces a logical value of true, for $p\sim q$, $\sim pq$ and pq, making $\sim p\sim q$ false. Uniting it with the algebraic reading of Umberto Eco's model, we can say that the disjunction is an affirmation of Shannon and Weaver's model, putting the external communicational common in the full discredit.

Thus, it indicates that logical NOR is the opposite of Mathematical Model of Communication by Claude Shannon and Warren Weaver. It is the appreciation of the universalist sign approach of C. S. Peirce, i.e. the broad scope of semiotics. We can say then that the Eco's external communicational common is the idea of C. S. Peirce's "pan-semiotic vision of the universe". In Peircean interpretation, signs are not a class of phenomena just like the other non-semiotic objects. Rather, the whole world is permeated with signs, if it is not composed exclusively of signs (CP, 5.448 *apud* Nöth, 1995, p. 62).

So, thinking about the articulation of the universal logic, we give due importance to the external communicational common outside the transmitter-receiver's relationship. Without the "outside", nothing would exist and this must be present as a trace in the heterogeneous linguistic mixture that manifests itself in a language-game. The universe of signs promotes a significance that goes to infinity. Here's *satzsystem en abyme*.

5. A FINAL REMARKS: THE LOGIC OF THE LANGUAGE-GAME

The articulation of Peircean ideas shows us a new articulation of Wittgenstein's thought: a consistent way by the logic and the calculus of language, mediated by the notion of *satzsystem*, which results in the language-game. This gives us that within language protocols, such as communicative language game, there is a satzsystemic grid that indicates a plurality of possibilities for a particular practice. These are the possibilities of a logical language-game.

All this is made possible, in large part, by an understanding of Peirce's logic as universal. This not only facilitates the insertion of his thinking in a larger field of debate in analytical philosophy of language, but also represents a gain in the mentioned field: the universe of signs as key to understanding the interaction between language and world.

Thus, the path, that we constructed here in the logical NOR's analysis, is an incentive to show the semiotic force in the construction of our reality. Reality that is only possible through complex web of signs.

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