Thinking in Perspective
The Critical Paradox

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Abstract

"Thinking about thinking", the topic of this section of our volume, leads us into an Escherian web of self-referential interconnections — which nevertheless make sense, at least if we do not shy away at once in the face of paradox, for fear of inconsistency. How do our critical faculties enter the picture? Adding a few conceptual dimensions to the merely linearly causal one may help us to shed light on the issue, and may help us also to understand some of the basic strategies "Western" thought has developed throughout its historical pathway in its conscious dealings with the 'real'. Of course, in a few pages it is impossible to do justice to this complicated subject. But I'll try to give the reader an as clear as possible description of the, in my view, crucial issues, pointing out the ways they resonate into the many fields of interest represented in the intended readership, and leave it to the reader to dig deeper through the given references.[1]

Introduction

1. The expression 'critical thinking', especially from a philosophical perspective, might sound somewhat like a pleonasm to the general reader. After all, we all like to take pride in our critical abilities, in our critical mindset or attitude as students, professionals or academicians. But when pressed to define or describe what the concept precisely means or implies, we find ourselves easily cought in a web of at first glance mutually inconsistent possibilities: from voicing an 'opinion', e.g., on a work of art, to the fact of rejecting or accepting a statement on a given subject matter not just considered as someone's personal opinion, but as concerning a matter of fact. The underlying idea then obviously is that the statement must be either true or false, and that a critical evaluation can decide in favour of either of these — like in the case of the hearing of witnesses before court.

2. We can see how these seemingly incongruent notions come together in the concept of 'judgment'. Exercising one's critical abilities seems to comes down to making a judgment of some kind. And a judgment implies an act of separation between the 'good' and the 'bad', the 'true' and the 'false', the "right" and the "wrong", the 'same' and 'other', the "beautiful" and the "ugly". This intuition is confirmed when we look at the etymology of the term. Indeed, the Greek adjective κριτικός (kritikos), "able to judge", stems from a verb κρινῶ, 'to divide, distinguish', and hence 'to arrange, to order, to select, to prefer' (Chantraine, 1999). So in what follows, we have to take a closer look at what it means, philosophically

[1] The paper started out as a set of notes for a workshop in the School of Thinking-program, in the context of which the present book appears.

speaking, to make a judgment, to use one's power of judgment or *Urteilskraft*, to put it in Kantian terms. Indeed, according to Kant, the faculty to judge and the faculty to think in the end come down to the same [*CPR,* A 80].[2]

3. So there are (at least) two fundamental aspects to reason, the cognitive ability to judge about and express (universal) truth, and the practical ability to judge and express (individual) choice (Martin, 2006). They each come with their concomitant basic human capacity, the capacity to mentally represent, and the capacity to bodily act, respectively. Kant calls them "pure" and "practical" reason, and dedicates the first two of his three famous *critiques* to them (Ward, 2006). It may be noted that it is far from self-evident how they interrelate (Rolf, 2020). The problem of the impossibility to rationally reconcile the 'ought' and the 'is' haunts "Western" philosophy since Antiquity — it could be argued on good grounds that the inevitable conclusion of its rational unsolvability is reached at the very moment we witness the implosion of the civilisation that produced it — but that would be a topic for another, be it highly relevant, debate.[3]

The Inner Eye

4. When we think about thinking, we are calling the very faculty we claim to investigate into action. Thus there is an inescapable recursive circularity build into this project, in the fundamental sense of defining the problem in terms of itself.[4] Another related, but again self-referential, question involves language: the question, "what is the relation between thought and language" obviously is an instance of both thought and language. So any attempt to frame the subject in a straightforwardly consistent way, by reasoning from supposed causes to verifiable effects, leads us into problems. We cannot anymore think about thinking in a detached, 'objective' way than it is possible to objectively observe our own observations, because we cannot separate ourselves from them. Even when we set up a scientific experiment to study a part of the 'outside world' this remains true, as we learned the hard way from quantum mechanics.[5] Stephen Hawking[6] puts it succinctly thus:

> *(...) we are not angels, who view the universe from the outside. Instead, we and our models are both part of the universe we are describing.*

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[2] Critical Theory, a school of thought within sociology and political theory, stresses the fact that individuals and groups have particular perspectives shaped by their cultural and economical needs in their spoecific historical contexts. Because of this, it tends to reject the notion of a unifying theory, and rubs shoulders with French postmodernism. It is based on the work of the well-known Frankfurterr Schule, but ultimatedly goes back to Kant's critical philosophical attitude. It is not our topic here, but I shall come back to it shortly towards the end of this paper. More on it in Bohman's SEP lemma "Critical Theory" (2019).

[3] A good starting point would be Bauman's book on "Liquid Modernity" (2000).

[4] The fact that this type of self-referentiality pops up everywhere from the moment we start to question our 'Self' and its implications is the subject of the engaging book by Hofstadter, *I am a Strange Loop* (2007).

[5] There is an enormous literature on this subject. Let me suffice by referring to the accessible standard handbook on QM as both a formal and physical theory by Hughes (1989).

[6] Cfr. ft. 76.

5. So in a non-trivial way, observing the world is observing ourselves. Because you are embedded in the same world as is the system you study, you are related and can observe only a part of it, an aspect which you co-determine in advance by choosing a specific perspective on that part of the world by means of the set-up of your experiment.[7] Moreover, since quantum physics is a universal theory, the observer and his experimental apparatus obey the same quantum laws as all other physical phenomena. This implies that the observer is part of both his object theory (quantum mechanics) and its metatheory (interpretation), a feature characteristic of a very general category of systems that exhibit self-referential inconsistencies, and the formal expression of the fundamental properrty mentioned before.[8] In other words: what you think shapes what you see, just as much as what you see shapes what you think.[9] This has nothing to do with philosophical relativism, it is not a statement to the effect that "every proposition is equally true", or something similar. It does imply, however, that there are always more than one valid perspectives on the real, because, whatever the specific entity under consideration, the real can always be approached from more than one stand-point[10]:

> *The organisation of the living system defines "a point of view", a bias or posture from which the interactions take place; and determines the possible interactions."* (Maturana, 1972)[11]

[7] Karakostas and Zafiris come to a similar conclusion when they develop their categorical perspectivist approach on quantum mechanics, in their article on "Scientific Perspectivism" (2020).
[8] An excellent discussion of this complex self-referential state of affairs with respect to QM in P. Mittelstaedt (1998), especially the Introduction and Chapter 5.
[9] In quantum mechanics, this is expressed by Von Neumann's "collaps of the wave function", which is at the origin of the "Measurement Problem". see Hughes (1989), p. 272. Also Beltrametti, and Cassinelli (1981), p. 12. For Von Neumann's original formulation: Von Neumann (1971) , p. 211 sq.
[10] This is the backdrop against which the so-called "theory-ladenness" of scientific experimentation and the Duhem-Quine Thesis have to be seen. Cfr. Harding *(*1976).
[11] Quoted and discussed in Heylighen *et al.* (1990), p. 134.

6. In this sense, asking the question, "what is thinking" is a lot like looking at yourself: some parts of your body are clearly visible to you, but you can always see only a limited part of it, because 'you' and your embodied self are inseparable. Even more, some parts of you are inevitably inaccessible to your own direct observation, like your backside — you are 'situated' with respect to yourself, the perspective you 'choose' is already build-in. We can to some extend solve our limited visual self-image by broadening our observational scope, by creating an artifical optical separation by means of a mirror (or a photograph, if we invoke distance not only in space, but also in time). When we look at someone else, however, the situation is different. Not only is there more of her body — the whole front, her face — we can immediatedly see, but we can also choose to change our position with respect to her, and we shall be able, without the aid of a mirror, to see her backside as well — though not simultaneously with the front: it takes *time* to shift from one perspective to the other. Hence by taking *distance* and shifting standpoints, we create a more complete mental image of what surounds us; the real becomes a *world*, like our limbs and torso, which we experience individually in their animated actions related to a surrounding medium (an 'environment'), become, when looked upon from the outside, our body.[12] Even mental activity is associated directly to bodily fluids and breath. Our word "aesthetics", the subjective sensation of beauty, stems from a Homeric root αἴσθω, (aisthoo) 'I gasp, I breathe in' (Onians, p. 75) — a fact which, by the way, sheds another light on our notion of 'inspiration'. Again, exercising this power at distancing is problematic when it comes to observing ourselves. In the case of an external object this is (deceptively) easy, but in the case of our bodies it comes at a price: we have to somehow separate our mind from our body *inside our body* in order to see more of ourselves, but while doing so we literally *loose touch*. Consciousness becomes self-consciousness, a kind of internal mental metaphenomenon that literally imagines what it cannot actually see; it re-presents the absent as present before 'the mind's eye'[13]: "The Self as a subject that

Figure 1: Escher's impossible, self-referential stairs

[12] Interestingly, the Greek *soma*, body, originally means corpse. Apparently the first conscious dealings with a human body as a whole, as an object, and not as limbs in action (like, e.g., when running, or in a fight), are when it was dead (Snell 2000), p. 17. The living body, on the contrary, was perceived as a collection of fluids, like the *psyche*, in constant motion. See Onians (1994), p. 200 sq.

[13] An expression already used by Cicero, *De Oratore*, III, xli (163).

reflects upon itself as an object".[14] We have to be keenly aware of the fact that the subject and the object here only partially coincide, because the first 'Self' is purely mental, while the second one is the embodied self which can act in and upon the world. We nevertheless tend to reduce the latter to the former:

> *In part it is this ambivalent character of languages-as-logos that makes us think of words as a kind of activity (speech) or as a kind of structure deriving from visible marks, and tempts us to reduce the one to the other; but this ambivalence, reflected in the history of thought about thought, and made critical by the new technology of thought called writing, is actually a problem set for us by the ancient Indo-European two-fold division of knowing into kinetic knowing, as in "knowing how", (...) and mimetic knowing, as in "knowing that" (...)* (Tyler, 1984).

7. 'Taking distance' is really only an option when the sense of sight is involved and the body considered is not our own, for our mind cannot step out of its body — in that sense, all knowledge, even the most abstract, remains 'embodied'.[15] Moreover, this capacity of the eyesight to project and take in images carried over a distance is unique; none of the other senses allows for such a projective mental distance to be taken in any meaningful way. Such a reduction of knowledge to visualisation is not natural nor self-evident, it has to be learned.[16] From a very young age, an individual learns to 'see' things with his or her 'inner eye' by representing them in his inner mind-space[17], retracted from immediate experience. Technological innovations like literacy, although not not strictly visual in themselves,[18] tend to enhance this kind of reduction, and to enlarge the internal distance "within" the *psyche*.[19] On the other hand, there are communication technologies that depend directly on the capacity to 'leave the body behind' and partake mentally in visually re-presented events on a screen, like cinema. This possibility has been conceptualised, remarkably enough, already by Plato, in his Allegory of the Cave [*Republic,* 514a–520a]. The separation of the mind and the body, and since the body is part of the world, of the mind and the world, becomes the basic ontological conditon of the "Western subject" and the basis on which the metaphysical description of the world, the anchor point for classical philosophy for two millenia to come, will rest.

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[14] For an interesting discussion, see Rosseel, "Notes on Self and Self-Steering", in Heylighen *et al*., (1990), pp. 114-115. For a thorough historical overview of the ideas on the soul in the body seeing through the eyes, Biernoff (2002), especially chapter 4, "The Optical Body".

[15] Tanaka, "The notion of embodied knowledge", In Stenner *et al.,* (2011), pp.149-157. Consciousness cannot be measured in a direct way. Firing neurons may cause consciousness, but not as some separate physical event; it is rather a state of my present brain as a whole. In this respect consciousness as a phenomenon resembles gravity, clearly a physical phenomenon, but not directly measurable in itself (what we measure are falling bodies, not gravity). Interestingly, already Newton himself saw and studied this parallel. Iliffe (2017), pp. 105-122. For coinsciousness, see Searle (1997), pp. 6-10.

[16] The acquisition process in children is described in Keil (1992).

[17] On the construction of the "inner mind-space", see Jaynes (1976), p. 54. sq.

[18] Not incidentially, "reading aloud" is what children and even many adults do naturally. For the interesting evolution of the practice of internalised, voiceless reading, cfr. Saenger, (2000).

[19] Ong, (2013), especially Ch. I, "Orality, Writing and Disjuncture", p. 17 sq.

> *Greeks like Aristotle and we to-day have apparently attained to greater 'detachment', power of thinking in cold blood without bodily movement, as we have to a sharper discrimination and definition of the aspects and phases of the mind's activity. It is with the consciousness, the knowing self, the spectator aware of what happens within and without (...) that a man would tend more particularly to identify himself.* (Onians, p. 18)

8. From that point onwards, people start to describe the world 'laid out before' (*ex-plicare*: un-folding) them in order to explain it. Our relation to the Self becomes part of our relation the "outside world", conceptualised as an enormous stage that unfolds before our inner eye, and simultaneously a mental mirror in which our own bodies remain invisible — the reader will remember that what we cannot see of ourselves when we look into the mirror tells us a lot about who we are. Knowledge becomes disembodied up to the point where we can imagine our consciousness to be infused into some mechanical brain, or where we mentally participate in an 'almost real' 3-D reconstruction of some virtual world set up around our sensorily stimulated bodies. But the body that I see on the screen is not *my* body.[20] The relation between 'I' and 'World' is thus profoundly re-shaped by means of the representational intervention appropriatedly called a "worldview" (Tyler, 1984, pp. 28-29). It explains why eyesight becomes the primary sense at the dawn of philosophy [Plato, *Timaeus*, 45b, Aristotle, *Metaphysics A*, 980a 22-28].[21]

> *Ainsi le film est comme le miroir. Mais en un point essentiel il diffère du miroir primordial: bien que, comme en celui-ci, tout puisse venir se projeter, il est une chose, une seule, qui ne s'y réflète jamais: le corps propre du spectateur* (Metz, 1977, p. 65).

Worldviews and (Meta)physics

9. This brings us, indeed, to philosophy and to science. The attempts to descirbe 'the Real' in an encompassing over-view of its totality — a "Theory of everything (TOE)" — is from the start the central question at the heart of the philosophical enterprise of "the West": the construction of metaphysical worldviews from Plato to Hegel, and scientific theories form Newton to Einstein. The mind and the body as related to the 'I' leave the stage for a auite a few centuries. After philosophical inquiry lost its hopes to achieve this goal after the critical onslaught by Kant and its reverberations into what would become Postmodern and critical philosophy, the task was passed on to natural science, where it continues to determine the agenda in the search for M-Theory, the latest installment of TOE (Zimmerman, *History*, 2019).

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[20] This crucial aspect is absent from the discussion of the topic of " 'I' represented elsewhere" in Hofstadter's already cited book, due to the fact that he conflates subject and object selves into the 'I' as a symbol. See the sections on "being hosted by others" and "telepresence", pp. 261-271. This displacement of 'I' does indeed open up a vista of new possibilities, but it is also highly disturbing, and can have pathological consequenses, as was already pointed out by Barglow (1994).

[21] Interestingly, the verb θεωρέω (cfr. our "theory"), 'to look at, to observe, to examine', comes from a noun θεωρός, 'someone sent to watch, consult the oracle, to attend the religious feast', hence the later meaning of 'spectator' in the theatre (Chantraine, 1999, pp. 433-434).

Paradoxes plagued the attempts at describing the the world right from the start. To the experience of the ancient mind, not only the body, but also the world moves constantly.[22] Thales famously said all nature was "as water" [DK 11A 27].[23] Heraclitus compared the ever-changing condition of our being-in-the-world to us stepping into an ever flowing river of being and non-being [DK 22B 49a]. This coincidence of opposites implies that the world is in constant flux, Plato points out [*Cratylus*, 402(a)], and it poses a serious problem to knowledge:

> *Now it seems that what has in fact come to light is that, if all things are in change, any answer that can be given to any question is equally right: you may say it is so and is is not so — or 'becomes'.* [*Theaetetus* 182(e)-183(a)]

10. The discovery that the natural world itself was not free from such paradoxes was demonstrated by Zeno, a student of the philosopher Parmenides of Elea.[24] He shows with acuity in his famous arguments that each time we try to describe physical processes involving plurality or motion, we end up in seemingly inescapable contradictions between what he calls "the large" and "the small".[25] Two millenia of attempts to refute Zeno only testify to the strength of his argument.[26]

> *Two great warring traditions regarding consistency originated in the days of the Presocratics at the very dawn of philosophy. The one, going back to Heraclitus, insists that the world is* not *a consistent system and that, accordingly, coherent knowledge of it cannot be attained by man. (...) The second tradition, going back to Parmenides, holds that the world* is *a consistent system and that knowledge of it must correspondingly be coherent as well, so that all contradictions must be eschewed.* (Rescher and Brandom, 1980, Introduction).

How to deal with this problematic situation? Plato's stroke of genius was to turn the tables: since it cannot be that the real world is riddled by inconsistency, these paradoxes must be a consequence of our faulty sensual perception, of our muddled experience of it. He expounds this novel idea brilliantly in his famous allegory of the cave [*Republic*, 514a–520a)]. What we need, says Plato, is an overall description of reality that avoids these "errors" and gives us back a picture of the world that is consistent, i.e., paradox-free. Plato's solution was to create a new kind of re-presentation based on what he calls "forms"

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[22] Many textual examples and a thorough discussion of their implications in Onians (1994), Ch. VIII, "The World: Beginnings of Greek 'Philosophy' ", pp. 247-253.

[23] Not "is water", as many readings have it. It is an analogy, not a naturalist hypothesis. The *phusikoi* did not trrade in primitive physicalist worldviews. They couldn't have, since the separation between 'I' and 'world' did not yet exist: *Il n'y a pas de cosmogonie malgré l'apparence*, *parce qu'il n'y a pas de représentation (...).* cfr. Bollack and Wismann, 1972, p. 49. With respect to Thales see Kirk *et al*., 1983, pp. 90-91.

[24] Plato dedicated one of his most profound dialogues to him. A commented translation in Chrysakopoulou and Hedley (2010). Also Fonterotta (1998).

[25] For a detailed textual analysis of Zeno's arguments, see Verelst (2006).

[26] The myth that Zeno has been "refuted" is tenacious, but you should have none of it. "*No one has ever touched Zeno without refuting him, and every century thinks it worthwhile to refute him*" (Whitehead, 1948, p. 87).

or "ideas", which he finds by a method of conceptual division or *diairesis* not incidentially closely related to Zeno's divisional procedure, but now applied to concepts, not to extended objects (Dixsaut, 2001, pp. 156-158). To achieve this, he cuts the 'world' in two layers with a different status in reality, where 'identity' and 'change' can be relocated separatedly in a clear hierarchy. Since then, every rational "world view", even our present-day scientific theories, consists of a two layered world-model, in which the static 'Eleatic' layer of stable identity precedes the dynamic 'Heraclitean' layer of instability and change.

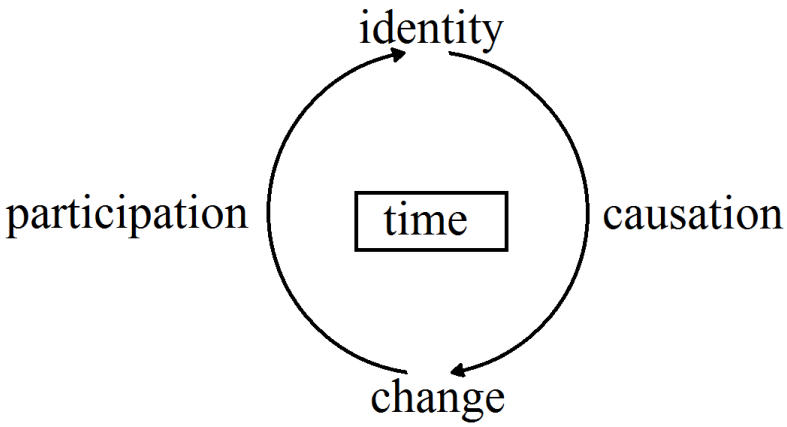

Figure 2: A *metaphysical theory* always involves a procedure which divides the "world" into a layer of stability (being) and one of change (non-being), connected by a relation of causal participation (Verelst, 2008).

11. So Plato and Aristotle develop their metaphysical systems on both the ontological and the epistemological level The fundamental principle governing the correct use of language is the Principle of (non-)Contradiction (Priest et al., 2004). It reflects on the linguistic level the 'fact' that in realty paradoxes are not supposed to occur — which is why Aristotle lays it down as an axiom to his metaphysics [*Met.* IV, 1005b(8-34)] and states expressly its unprovability. This is the origin of our present-day obsession with logic as the universal criterion for soundness of argument, and also of the age-old question on the precise nature of the "correspondence" between language and the world (David, 2016). The endless attempt to reconcile a consistent world-picture with our paradox-riddled experience of it continues even in the most advanced scientific theories. We are trained to see inconsistencies as immediatedly invalidating arguments completely. In the sciences this is moderated to some extend by the fact we use mathematics rather than logic as a reasoning tool: it is clear that mathematics formalises a richer implicit ontology than logic, whose procedures are finite, discrete and rigorously consistent. The seventeenth century debate on the foundations of mathematics offers a case in point.

12. Hence we can take a short cut and jump from Antiquity directly into the sixteenth century, where the basis of modern natural science was laid. Scholastic philosophers had been discussing for centuries which fundamental building blocks to take as starting point for the best possible metaphysical description of the world, whether those blocks were 'real' or 'ideal' in nature, and which causal relations governed the processes that constituted

them into a world. They had been broadening, deepening and honing philosophy's logical arsenal in the process. But they ended up in internal inconsistencies or incompatibilities between their descriptions and nature time and time again — booktitles like "yes and no", and "The Guide for the Perplexed" indeed do speak volumes[27]. In the mean time, the broadening scope of maritime travel, the observations done at sea and in the skies, as well as developing technologies like ballistics, made the need for an adequate understanding of physical phenomena related to motion ever so urgent (Dijksterhuis, 1961). Early modern natural philosophers like Galilei, Kepler (Bucciantini, 2008) and Descartes reject the basis of Aristotelian metaphysics when they realise that its concepts fail to adequatedly describe processes of motion and change in the light of these new mathematical and experimental discoveries. But metaphysics (in the sense of the structure we described above) has never been abandoned, only adapted: now experimental observation combined with mathematics will decide what the basic elements of each new theory should be (Clericuzio, 2000). Aristotle's principle of contradiction continues to reign supreme. A metaphysical theory with such a rationalised perceptive, observational procedure is called a scientific worldview. This is why debates on matter and space take up center stage during this transition known as "the Scientific Revolution".[28]

13. In debating these issues, the two giants of seventeenth century natural philosophy, Newton and Leibniz, fueled the debate in its new guise (Verelst, 2014). The heroic battle between the two founders of infinitesimal calculus was paralleled by an at least as heroic, be it somewhat less known, battle on natural philosophy.[29] Newton defended the idea that space is a *vacuum*, and that gravity works as action at a distance (Henry, 2011). on the bodies contained in it. Leibniz (and Huygens) on the other hand stick to the idea of the mechanical *plenum*, "vortices" which exert contact forces only (Lanczos, 1970). consequently, Newtonian gravity is proportional to mass, Leibnizian gravity to volume. This led to different conceptions on the constitution of matter — with Newton hard atoms, with Leibniz elementary particles of a more elastic nature —, and to different conservation laws: momentum vs. force. Famously, Newton succeeded in proving the universality of his gravitational inverse square law, unifying his description of the tides and that of planetary motion by means of it. The qua descriptive power aequivalent, but conceptually different mathematical formalisms (vectorial vs. analytical mechanics) that go with these positions reflect their different underlying philosophies accuratedly. Interestingly, this is also true for their respective theological points of view: Newton's open-ended, actually infinite universe needs to be replenished regularly by its all-powerful Creator in order to prevent it from coming to a grinding thermodynamical halt (Iliffe, 2017, pp. 95-105). Leibniz's God, on

[27] *Sic et Non* is a commented collection of opposing philosophical and theological opinions of the Church Fathers by the twelfth century logician and philosopher Peter Abelard. *The Guide for the Perplexed* is the appropratedly named book written by twelfth century Jewish philosopher and theologian Maimonides, to help the educated to navigate their way between the truths of the Scriptures and those of ancient Greek philosophy, especially Aristotle.
[28] See, e.g., Lindberg, 2008. The notion "Scientific Revolution" is revisted critically in Osler, 2000.
[29] For a history, Bertoloni Meli, 1993. Also Koyré and Cohen, 1962. One of the best papers on the methodological debate is Shapiro, 2004.

the other hand, puts his neatly balanced machinery in motion right from the start, and then contents himself looking at the perfection and beauty of his work — which looks back at him.

14. In order to sum up, it turns out that even in physics equivalent descriptions ('models') exist, each with its own fundamental theoretical presuppositions, and based on mathematical formalisms adapted to its needs. Hence they offer compatible, but not simply mutually reducible, perspectives on space and time — the ongoing debate on the nature of light offers a nice example, where we see the opposing standpoints of Newton and Huygens (corpuscular vs. wave) reappear at the beginning of the twentieth centurry. Experiment should be able to decide which one is ultimatedly right, but things turned out to be a bit more complicated, with the discoveries of quantum mechanics and relativity theory respectively. The well-known incompatibility between these two is in itself just a new manifestation of the old problems concerning the nature of matter and space (Heisenberg, 1958).

The Come-back of the Mind

15. The next great attempt at reconciliation of the 'absolute' vs. the 'relative' point of view came with Immanuel Kant. That there would be not one, consistent, all encompassing theory of the world[30], and therefore no answer to

> *"this question, of how the faculty of understanding achieves this conformity with the things themselves"*[31]

was an unacceptable scandal to him, exactly as with Plato. In his so-called "pre-Critical" works, Kant sets out to reconcile the two rivalling schools in natural philosophy which he labeled "the mathematical", and "the metaphysical", basically again Newton and Leibniz.[32] Each had, according to Kant, its strengths and its weaknesses.[33] Newtonian theory had its ideas about absolute space and time right, and also its stress on empiricism[34], but it had the wrong conservation laws and failed to describe the universe as a whole, as a complete and closed system without the need for a Newtonian all-powerful God to constantly entertain it (Cooper, 2002, p. 7, p. 12). The Leibnizian school allowed for a full mechanical description thanks to the notion of dynamical conservation of force, but its theory of matter

[30] A discussion of the philosophical relevance for Kant of the "world as a totality" in (Kerzberg, 1997).
[31] In a *letter to his friend and former student, Marcus Herz,* quoted in Rohlf, 2020.
[32] A detailed analysis of Kant's pre-Critical endeavours can be found in Shimony, 2013. For the opposition mentioned, see pp. 11-13.
[33] *This project was the expression of Kant's desire to create a systematic philosophy of nature. He wanted to harmonize scientific and metaphysical perspectives such that the paradigm of explaining nature, Newtonian physics, could be reconciled with the deepest metaphysical convictions.* Schönfeld, 2000, Preface.
[34] Cfr. Kant's comments on David Hume in the foreword to his *Prolegomena to any future Metaphysics*. Cfr. Kant (2002).

and its conception of space suck (Cooper, 2002, p. 13). During his pre-Critical career, Kant tries to find a way out by systematically developing all the alternative cosmologies, i.e., theories of the universe including its history, that would follow out of the opposing viewpoints he encountered in his predecessors, but he continues to run, like his predecessors, in paradoxes and inconsistencies (Shimony, 2013, pp. 18-23). Desillusioned by this defeat (Schönfeld, 2000, p. 229 sq.), he choses a radically different approach: the critical investigation of the conditions under which a full metaphysical description of the Universe is possible, if at all.[35] In his first "critical" work, the *Critique of Pure Reason*, het starts from the ground up, by listing the insurmountable problems natural philosophers encounter again an again when they try to formulate a consistent "theory of everything" (a metaphysics) in a concise summary: the Four Antinomies of Reason (Al-Azm, 1972).

> *It was not the investigation of the existence of God, immortality, and so on, but rather the antinomy of pure reason... that... first aroused me from my dogmatic slumber and drove me to the critique of reason itself, in order to resolve the scandal of ostensible contradiction of reason with itself (letter to Christian Garve, October 1798, 12:257-58)* (Shimony, 2013, p. 8).

The first two of these four antinomies are basically just versions of Zeno's familiar plurality and motion paradoxes: they concern the size of the world and the divisibility of space and time. The last two concern causality and the existence of a "necessary being", i.e., God.[36] They are not just errors or figments of the imagination, no, they are "not arbitrarily invented but founded in human reasoning as such", so that "all the metaphysical art of the most subtle distinction cannot prevent this opposition." (Al-Azm , 1972, p. 7) Plato's project, to defeat the paradoxes by providing a two-layered world-model in which being and non-being are neatly separable, is declared dead once and for all.

16. Something drastic needed to be done, and Kant's solution in his first *Critique* is radical: he basically reverses the traditional relation "real - ideal" in our connection to the world (Shimony, 2013, p. 13). The Self is back in the game, be it in the at first glance hardly recognisable form of the "Transcendental Subject". It is an illusion to think that "infinite space" or "infinitely divisible matter" are given to us as objects of the senses. We can think them because they are structures of our thought, but whether they are real properties of the world, we cannot know: they are literally "too large" for our (synthetic) understanding and "too small" for our (analytic) reason (Shimony, 2013, pp. 91-100). The world as such becomes appearance before a "transcendental subject", not because of our fallible senses, but because our mind has an universal perceptive structure build into it, an "a priori intuition" which is imposed on all our interactions with the 'outside'. You can never see the world "as it is", but only take a specific perspective on it, a basic perspective that we, as

[35] It is important to stress that Kant's "critical turn" was a direct consequence of his failed precritical work, and intended as a preliminary to future, more succesful attempts. A commented selection that presents the relevent material in its internal unity is Schmidt, 1993. This viewpoint is also taken by Shimony, 2013.

[36] Cfr. Al-Azm, 1972, in the section *Kant's arguments*, under the respective headings.

humans, nevertheless universally share. To Kant, this imposed structure is infinite three-dimensional Euclidean space and infinitedly linear time, not incidentially the basis of Newton's gravitational theory (Parsons, 2006). It should nevertheless be clear that Kant was not simply a Newtonian. He succeeds in saving both the Newtonian cabbage and the Leibnizian goat, by declaring a final decision on the horns of the dilemma posed in his four antinomies forever out of human reach, as being literally "too large" and "too small" (in a far from idle reminiscence of Zeno) for human reason and understanding. Whatever the picture of the world you'd like to build, it will be marked by the higher mentioned "transcendental categories", which will give a certain shape to your knowledge, independently of your desires.

17. There is some irony in the fact that Kant has been claimed to be the most ardent of Newtonians while simultaneously being reproached for staying steeped deeply into Leibnizian 'metaphysics', and both for good reasons.[37] But, altough extremely influential in a broader sense, Kant's idea of a "transcendental observer" was not taken up by the scientists of his day, supposedly because it concerned "metaphysics", not a subject a serious scientist would still embroil himself with. We see here to what extend "science" and "philosophy" had already parted ways, merely a century after Newton. But, even if we would not take his "transcendental categories" on face value, Kant's fundamental idea, the inexorable situatedness of the observer *of* the world *in* the world, clearly opens up possibilities with respect to the problems discussed before.[38]

18. The situation basically stayed stuck there up to today. The succesful development of "Newtonian" science throughout the nineteenth century led even the brightest minds to believe precisely in what Kant had diagnosed already as impossible: a consistent "theory of everything" based on mechanical principles and experimental observations alone, a desire expressed with astute clearity by Lord Kelvin in his famous 1900 "Clouds"-speech (Zimmerman, 2019). In theoretical physics, we strive still today for "Grand Unified Theories" that unify the cosmological large and the microscopically small while causally bridging the gap between the static and the dynamic level of physical reality by means of different kinds of forces (Barbour, 1982). It is well know, and in the end not surprising, that such attempts have hitherto failed, and that theories which claim succes are riddled with inconsistencies, neatly tucked away behind "singularities" of all stripes.[39] The property of "semantical completeness"[40] which Mittelstaedt defines for quantum mechanics

[37] For Kant's Newtonianism, see Cooper, 2018. For Kant's anti-Newtonianism, Watkin, 2013.
[38] Kant's influence on science should not be underestimated. For a discussion of Kant's impact on the founders of quantum theory, see Jammer, 1966. For Heisenberg specificially, see Camilleri, 2005. Also Mittelstaedt, *1998,* pp. 92-93.
[39] See the lecture of Stephen Hawking cited above. For a more technical discussion, Zee, 2003, p. 72 sq. On the topic of epically failing grand theories, I cannot recommend enough the exhilarating (but sound) book by Smolin, *The Trouble with Physics* (2008).
[40] "*The measuring process plays a twofold rôle: it serves as a means to interpret the quantum object theory, and it is also a real physical process that belongs to the domain of phenomena described by the object theory*"; "*The property whereby a theory incorporates the theory of its own measuring instruments*". Cfr. §5 above. Mittelstaedt (1998), p. 4; pp. 103-105.

clearly must hold for all universal theories. And that necessarily implies some form of self-referential inconsistency, because as observers we always belong to both the object- and the metatheory simultaneously, since we are inexorably part of the world we describe. This was precisely Kant's original discovery. The reader will have no difficulty to recognise the conceptual shape of Kant's antinomies in the background of issues like the QM-paradoxes, the Measurement Problem, or — especially — the incompatibility between the quantum mechanically Small and General Relativity-like Large, which is only very partially solved in QFT.

19. Stephen Hawking's comments on this state of affairs, in a remarkable lecture at Dirac's centennial at Cambridge, are merciless:

> *In the standard positivist approach to the philosophy of science, physical theories live rent free in a Platonic heaven of ideal mathematical models. That is, a model can be arbitrarily detailed and can contain an arbitrary amount of informaton without affecting the universes they describe. But we are not angels, who view the universe from the outside. Instead, we and our models are both part of the universe we are describing. Thus a physical theory is self referencing, like in Gödel's theorem. One might therefore expect it to be either inconsistent or incomplete. The theories we have so far are both inconsistent and incomplete* (Hawking, (2002).

This quote is remarkable for two reasons: firstly, the reference to Gödel's theorem, on which we shall come back, and secondly, the admission of the defective nature of our most fundamental physical theories. It appears that somehow the attempt to escape paradoxes by creating observation-based, logically consistent, all-encompassing worldviews backfired by going full circle. So maybe this self-referential inconsistency, rather than a problem, should be part of the solution. As far as mathematics is concerned, Gödel's theorems show the way: whether one choses the incompleteness or the inconsistency wing out of the dilemma,[41] the deep connection between Gödel's result and logical paradoxes like Richard's paradox, Berry's paradox (Hofstadter, 2007, p. 139) or the Liar is a given, because of the structure of the undecidability proof itself, as Gödel himself points out very clearly in his introduction:

> *The analogy between this result and Richard's antinomy leaps to the eye; there is also a close relationship to the "Liar" antinomy, since the undecidable proposition [R(q);q] is not provable. We are therefore confronted with a proposition which asserts its own unprovability* (Gödel, 1992, pp. 40-41).

Only to add in a footnote:

> *Every epistemological antinomy can likewise be used for a similar undecidability*

[41] "*Thus mathematics is either inconsistent, or imcomplete. The smart money, is on imcomplete*", so Hawking in the same lecture. But this choice remains a *choice*; there is no intrininsic reason in the proof of the theorem that obliges one to make it.

proof (*id.*, ft. 14)

It has been shown since that this not only holds good for systems much weaker than Peano arithmetic, but also that reformulation in terms of 'truth' instead of 'provability' is perfectly possible (all be it technically somewhat intricate).[42]

20. The deep interconnection between these paradoxes and Theories of Everything (TOEs) becomes even more clear because of something Bertrand Russel discovered already in 1905: for Liar-like inconsistencies to pop up you need self-referentiality *and* a reference to the totality of the set implicit in it:

> *In each contradiction something is said about all cases of some kind, and from what is said a new case seems to be generated, which both is and is not of the same kind as the cases of which all were concerned in what was said* (Russell, 1989, p. 61).

So let us bring this insight back to physics: since quantum mechanics is a universal theory and definitedly belongs to the Gödelian category of "related systems" to number theory (as do all physical theories; cfr. Feferman, 2006), we strongly suspect that it has to manifest formally this type of self-referential inconsistency somewhere as well — and indeed it does, in two different, but doubtlessly related ways. The first is the The Kochen-Specker theorem[43], which formalises explicitly the idea that it is inconsistent within the confines of QM that a locally embedded observer can have a consistent global perspective on all other observers simultaneously.[44] The other telling formal connection is that cognitive processes oscillating between truth and falsehood in which the cogniser interacts with the coginitive entity, can be described accuratedly by Hilbert space quantum formalism — in other words: the Liar paradox is structurally embedded in QM-formalism (Aerts *et al.*, 1999).

21. It should be clear that to state this fact does not come down to stating that "everything is equally true" — Barwise and Etchemendy, in their brilliant book about the Liar paradox, make precisely this case.[45] It is about time that we leave that age-old superstition build into classical logic behind. The truth is, as as a metaphysical theory, classical logic is simply wrong. Its usefulness is real, but within very specific limits and under conditions of

[42] Smullyan, 1992. Especially the Introduction and chapter 2.
[43] Specker's inspiration for his theorem went beyond QM: *Very similar questions had already been raised by scholastic theology. Ernst Specker was influenced by these scholastic thoughts in his approach to the question of hidden variables. Indeed, as has aleady been mentioned (...) in 1960, he was considering the question of whether it might be possible to consistently define elements of physical reality "globally" which can merely be measured "locally".* Svozil, 1998, pp. 79-85 for a non-technical description, p. 85 sq. for a formal one. Quote on p. 79.
[44] Karakostas and Zafiris discuss the connection between the KS-theorem and perspectivism at length, be it form a somewhat different... perspective. See "Perspectivism", section 2.
[45] They make its relevance clear: " *Logicians, it is said, abhor ambiguity but love paradox. Perhaps that is why they are so inclined to give for- mal prescriptions for avoiding the famous Liar paradox, but so loathe to diagnose the underlying problem that gives rise to it. Despite its antiquity, and its genuine importance, no adequate analysis of it has ever been given, or so we feel."* Barwise and Etchemendy, 1987, p. 3.

possibility (to use a Kantian concept) which, for many practical purposes, are fulfilled in everyday life, but which do not strecth out to all cornes of the universe. Useful as it may be, classical logic becomes a burden when applied where it should not. As we saw, one of these places where it should not is self-consnciouness.

The Liar tells the truth...

22. So it appears that after all, the Cretan told the truth. Even if Kant's critical contribution, which caused a tectonic shift away from metaphysics and into what would become "critical" philosophy, was largely neglected in the scientific community, his fundamental discovery, that it is impossible to construct a objective, unique, consistent and true picture of the totality of the world of which you are part, remains valid for all of us. If there be any "transcendental category" build into our being-in-the-world, it would be this one. So let us take it as a starting point, and accept by principle the — literal — relativity of all individual perspectives. This is not "philosophical relativism", because it nowhere implies that "anything goes", nor that "everything is equally true". It matters what we say, even tough it is always situated in a context. Contexts, however, are not prisons, because we all share the same world, like different waves that share the same sea.[46] Kant also shows that this fundamental insight is deeply connected to the ancient paradoxes of plurality and motion. Kant thus forces us to go full circle and reconnect the results of the most high-end contemporary theories based on particle accelerators at CERN with the humble beginnings of Heraclitus's flowing river.

23. We can, of course, choose to simply accept accept Kant's pre-Critical conclusion, take for granted the failure of two millenia of philosophical attempts, and retract ourselves on the cynical, icy hights of some Subject that lost its transcendency, where theories are mere figments of the mind, and true knowledge forever out of reach. Or we can follow the path of what became later known as critical philosophy, accepting Kant's conclusion as an invitation to philosophise in an entirely different manner; an invitation taken up by different strands of structuralism and constructivism, which are by the way far from meaningless gibberish if one understands where they come from, but they are nevertheless not entirely satisfying when it comes to our deeply rooted desire for knowledge and truth. Alternatively, we can continue our scientific endeavours like if Kant never happened, as is mostly the case in the scientific community. There is no problem with that as long as we stay away from foundations and totalities, for in most scientific daily practice, the classical limit is good enough an approach. But the fact remains that with respect to such all-embracing theories, this comes down to continuing to do metaphysics in an awkward way, under the guise of mathematics and with, as far as cosmology is concerned, vey distant support of scientific experience.[47] In the end, Kant did happen, and I am afraid that we shall have to deal with his insights and conclusions at some point, also as scientists.

[46] Linguistically speaking, they behave like the "I" in Jacobson and Benveniste rather than the "ego" in Freud. An introduction to their respective linguistic theories in Kursell, 2010, pp. 217-236.
[47] Cfr. Arp, 1971. For more recent criticisms, see Smolin's already cited book.

24. Where to start? I believe that, as far as methodological attitude is concerned, Poincaré's ideas point in a viable direction. The problem he started from is that also scientific 'facts' cannot be simply taken for granted. Already before quantum mechanics enters the scene, Poincaré pointed out that we can never exhaust the real by means of experimental observation alone, because knowledge thus obtained will always remain partial. Not only does the theory determine what we can 'see', as Einstein famously held[48], but there will always be theory needed to complete the partial picture. Even if one takes all the available experimental data into account, there still remains an gap to be filled up conceptually — and there always will be a degree of liberty involved in this choice of theoretical perspective. This is the root of Poincaré's often misunderstood 'conventionalism' with respect to geometrical theories, a stance informed by the need to deal with the untenability of Kant's position on Euclidean geometry as a transcendental *a priori* (Ivanova, 2015). The debate on special Relativity aggravated the issue, because from Einstein's perspective, geometry now became a physical theory in its own right. Poincaré stresses precisely this point in his 1909 ULB-address:

> *Facts are subject to a number of interpretations, because they are never more than only partially known. Among this multitude, there are interpretations which are more likely than others, but sadly enough likelyhood is an extremely fugitive, subjective and delicate thing, on which even the finest minds will not always agree* (Poincaré, 1955, p. 98).

25. We are now better placed to understand and appreciate the subtlety of the adagium formulated by Poincaré on the same occasion, in which, according to many, the rational scientific attitude is epitomised, and which is claimed today by the two Brussels Free Universities as their ideological foundation.[49] It does not simply mean, "get the facts right, and once you get them, a unique and incontestable conclusion will follow" — far from it. It is to some point rather the contrary: "since we are part of this world, it will never be possible to exhaust reality completely by means of observation alone, you will always have to complete your picture of the world by means of theoretical concepts which do more than just translate observations into terms of the object theory — and, as long as they do not contradict the data, you can choose these theoretical elements freely." A lot less sexy, and a lot less easy to throw at your opponent in an ideological debate, but a lot more up to the demands of the issue at hand: the relation between science and true knowledge about our world. It nevertheless seems that, in order to to justice to the topic, we need to take it at least one step further, because it is not just incompleteness, but also inconsistency that vanquishes our attempts at total intellectual control. Mittelstaedt demonstrates in his book that the universality of quantum mechanics, which leads to its "semantical completeness" — the inclusion of the observer in the theory — entails its inconsistency as well: the

[48] Without theory, it would not even be possible to build the experimental set-up to start with. Quoted in Heisenberg, 1986, Ch. 5: "Die Quantenmechanik und ein gespräch mit Einstein", p. 92.
[49] *Thinking must never submit itself, neither to a dogma, nor to a party, nor to a passion, nor to an interest, nor to a preconceived idea, nor to anything whatsoever, except to the facts themselves, because for it to submit to anything else would be the end of its existence.* Poincaré, 1955, p. 96.

observer is part and not-part of the object theory. He represents this self-referential inconsistency by means of the figure of a *tribar*, an impossible triangle reminiscent of Escher's art (Mittelstaedt, 1998, p. 104). This combination of incompleteness and inconsistency brings to mind Gödel's theorems, where again the impossibility to attain a metalevel from where completeness and consistency can be achieved is at the heart of the argument. A similar case is the *Liar* paradox, that other paragon of ancient paradoxicality, in which a Cretan runs into inconsistencies when he tries to make a not too flattering statement about the totality of the population to which he belongs.[50] Gödel himself insists on pointing out the tlink between this paradox and his own procedure in the introduction to his notorious 1931 article (Feferman, 2006). But just as little as mathematics ceased to exist and being practiced after Gödel discovered his theorems, or quantum mechanics ceased to be the most succesful and at the same time challenging physical theory ever proposed after Kochen and Specker formulated theirs, so philosophy will not disappear once we accept the irreducible pluriformity of the one world we all inhabit. Claims at truth remain possible under the condition of being valid, but partial perspectives on the world, about which they all teach us someting essential, without any warranty that they are mutually commensurable, exactly as in quantum mechanics. There are always more than one valid perspectives on the real. We do not merely have to accept the world's fundamental paradoxicality, we have to learn to embrace it:

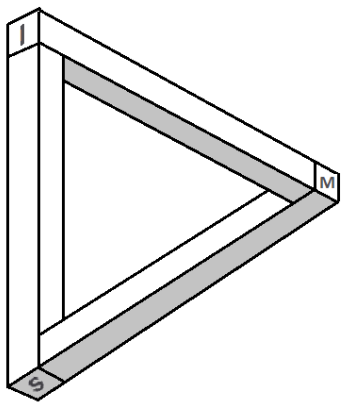

Figure 3: The observational tribar of the self-referential and inconsistent relations an observer entertains to himself, the 'outside' world and his interpretation of it.

"Paradoxically, the world is all that is the case."

[50] Cfr. Barwise and Etchemendy, 1987. Bertrand Russell stressed the crucial importance of the combination of self-reflexivity and totality in the *Liar*, in Russell, 1994, p. 61.

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