An Interpretation of McCall's "Real Possible Worlds" and His Semantics for Counterfactuals

Alexandru Dragomir

Abstract: McCall (1984) offered a semantics of counterfactual conditionals based on "real possible worlds" that avoids using the vague notion of similarity between possible worlds. I will propose an interpretation of McCall's counterfactuals in a formal framework based on Baltag-Moss-Solecki events and protocols. Moreover, I will argue that using this interpretation one can avoid an objection raised by Otte (1987).

Keywords: counterfactuals, branching-time structures, dynamic epistemic logic events, possible worlds

I will begin with a presentation of the Stalnaker-Lewis semantics of counterfactual conditionals (Stalnaker 1968, Lewis 1973). Following McCall (1984), I will point out that they have some common underlying assumptions: that the truth of a counterfactual is decidable by inspecting the most similar possible worlds and that the notion of comparative similarity used is vague, therefore we will not be able to determine in all possible situations what worlds we are supposed to inspect. I will introduce the reader to McCall's (1984) "real possible worlds" and his semantics of counterfactuals, a type of semantics that does not use the notion of similarity, but searching for the closest real possible world that branched off the actual one. I will try to show how to generate such a branching-time structure using (1) a Dynamic Epistemic Logic with operators for ontic change (van Ditmarsch and Kooi 2008) and (2) protocols for such logics (Hoshi 2009, Hoshi and Yap 2009) and restate McCall's semantic definition using this formal apparatus. Further, I will consider one of Otte's (1987) objections towards McCall's theory of counterfactuals and argue that if we interpret this objection in a structure generated by the logical apparatus introduced, it will not hold.

The Stalnaker-Lewis Approach to the Meaning of Counterfactuals

This paper will be concerned with two types of semantics for counterfactuals, the Stalnaker-Lewis semantics and McCall's semantics (McCall 1984). A counterfactual will be a proposition of the following type:

(1) If *A* were true, then *B* would be true.

I will use Lewis' notation (Lewis 1973) for the counterfactual conditional operator, □→. As in the case of the material conditional, □→ is binary, connecting two propositions of a given formal language. Consequently, (1)'s logical form will be A □→ B.

According to Stalnaker (1968), A □→ B is non-vacuosly true in the actual world if and only if in the closest, meaning most similar to w, possible world in which A is true, B is also true. Following Stalnaker (1968), we can write this in a formal way, considering that: $Atoms(L)=\{p, q, r... \}$ is the set of atoms of a language L, M is a Kripke model composed of a set of possible worlds W, an accessibility relation $R \subseteq W \times W$, $V: Atoms(L) \rightarrow 2^W$ is a valuation function, $||A|| = \{w \in W \,/\, M, w \vDash A\}$ is the set of all the worlds of W that satisfy formula A (all A-satisfying worlds), and $f: L \times W \rightarrow W$ is a selection function that takes a formula A and a world w and picks out the most similar world to w that satisfies A:

$$M, w \vDash A \,\Box\!\!\rightarrow B \text{ iff } M, f(A, w) \vDash B$$

The definition above is read: A □→ B holds at world w of model M iff in w's closest possible world (meaning the most similar to w possible world) that satisfies A it is true that B. Alternatively, one can write the right side of the semantic definition as: $f(A, w) \in ||B||$ i.e. the world selected by f is a world belonging to the set of B-satisfying worlds.

To this definition Lewis (1973) has objected that it assumes that: (1) f will always pick at least one world, and (2) f will pick at most one world. Regarding (1), Lewis (1973, 19-21) offered the following example: imagine that in the actual world w there is a 1 inch line drawn on a blackboard. It is consistent with Lewis' theory concerning the nature of possible worlds[1] that there is a world u_1 such that in u_1 there is a 1.1 inches line drawn on the blackboard (everything else, except the length of the line, is identical to the state of affairs in w). However, there is also a world u_2 in which the line is 1.01 inches and a world u_3 in which the line is 1.001 inches long and so on, *ad infinitum*. Consequently, there is no one most similar possible world to the actual world. As for (2), it implies the validity of the conditional excluded-middle: $(A \,\Box\!\!\rightarrow B) \vee (A \,\Box\!\!\rightarrow \neg B)$. According to von Fintel (2012), a counter-example to the principle of the conditional excluded-middle can be found in Quine (1966, 15):

(a) If Bizet and Verdi had been compatriots, Bizet would have been Italian.
(b) If Bizet and Verdi had been compatriots, Verdi would have been French.

Quine argues that according to the principle of the conditional excluded-middle, either (a) or (b) should hold, yet none seems to be intuitively true.

Lewis (1973) argued for a semantics that cannot be countenanced by these objections. Say w is the actual world, the world in which we need to

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[1] Recall that for Lewis (1979), a possible world is a way things might have been. Things surely might have been such that the drawn line had a different length. So the possible world in which the line has a different length exists.

evaluate $A \Box\!\!\rightarrow B$. According to Lewis' formal apparatus, all possible worlds can be arranged in spheres centered in w: $S_1^w, S_2^w, \ldots$, each sphere containing possible worlds similar to w. In case a sphere S_i^w is included in sphere S_j^w, the worlds in S_i^w are more similar to w than the worlds in S_j^w. So, $A \Box\!\!\rightarrow B$ is true in w iff (1) A is false in all worlds, or (2) there is a world u in a sphere S_i^w such that A is true in u and the conditional $A \rightarrow B$ is true in all worlds of S_i^w.

Note that in the above definition, the proximity of a possible world to the actual world is given by its similarity to the actual world. But how should we discern the similarity between two possible worlds? Lewis admits to the indeterminacy of the comparative similarity relation, and, since counterfactuals seem to have an innate vagueness, this vagueness can be explained as being an inherited attribute from the intuitive similarity relation used to define them. Moreover, Lewis argues that the intuitive notion of similarity used in the semantics is already entrenched in our language and, so, fit to be a brick in the construction of a semantic definition of the counterfactual conditionals:

> … such an account must either be stated in vague terms – which does *not* mean ill-understood terms – or be made relative to some parameter that is fixed only within rough limits on any given occasion of language use. It is to be hoped that this imperfectly fixed parameter is a familiar one that we would be stuck with whether or not we used it in the analysis of counterfactuals; and so it will be. It will be a relation of comparative similarity. (Lewis 1973, 1)

One could wonder, though, whether it could not be the other way around: why is it not the case that counterfactual constructions are more intuitive and "familiar" and, so, fit for offering an explication of comparative similarity?

However, is this a good trade? McCall (1984) argues it is not:

> The most obvious difficulty about these semantics lies in determining the degree of similarity a set of possible worlds bears to the actual world. Can possible worlds be inspected and compared? (McCall 1984, 463)

To put McCall's point in different words, Lewis' semantics is inadequate because it does not and cannot tell us in what worlds we are supposed to check whether the two arguments of "$\Box\!\!\rightarrow$" hold or not, and the reason for this lies in the vagueness of the intuitive notion of similarity put to use. To countenance this type of objection, McCall proposes a semantics of counterfactual conditionals that does not rely on similarity between possible worlds.

McCall's Semantics of Counterfactuals

In order to put forth a semantics of counterfactual conditionals that does not need to use comparative similarity, McCall reconsidered the metaphysical framework used by Stalnaker and Lewis. Though McCall adheres to a realist concept of a possible world[2], the possibility set of a world *w* will be composed only of worlds physically possible relative to *w*. In other words, the accessibility relation *R* will link *w* to *u* if only if *u* is a physically possible temporal continuation of *w*. In order to design this branching-time model, McCall identifies possible worlds with histories of time-instants (McCall 1984, 464-465). If *w* is what McCall names "a real possible world", it can be described as a structure $(w(t_1), w(t_2), w(t_3)...)$, where each $w(t_i)$ is a time-instant in *w* (McCall 1984, 464). Suppose we are in $w(t_2)$. Then, things go on as $w(t_3)$, but from $w(t_2)$ things could have gone as $u(t_3)$ or $v(t_3)$ and so on. Of course, since $w=(w(t_1), w(t_2), w(t_3)...)$, the set of all $u(t_1)$, $u(t_2)$, ... is world *u* and $\{v(t_1), v(t_2), ...\}$ is world *v*. We will say that world *u* branches off world *w* when their histories coincide until instant t_i and diverge afterwards. In the case above, *w* and *u* coincide until t_2 and diverge afterwards.

Now, how do we interpret a counterfactual conditional $A \;\Box\!\!\rightarrow B$? McCall's proposed answer is the following: $A \;\Box\!\!\rightarrow B$ is true in the actual world *w* iff in *w*'s closest branching worlds that satisfy *A* it is true that *B*:

> ...we stipulate that the possible worlds in which the antecedent is true must branch off the actual world as close as possible to the time of the antecedent. (McCall 1984, 467)

And:

> In asking whether "If A had been the case, B would have been" is true or false, we simply identify the worlds closest to ours in which A holds, and inquire whether in them B holds, without imposing on them any further condition whatsoever. (McCall 1984, 468)

Now, I will try to sketch a formal model *M* in which to evaluate the truth of a counterfactual conditional. The formal model *M* in which we evaluate counterfactuals would be a model *(W, R, V)* constructed following McCall's concept of a real possible world:

1. *W* is a set of possible worlds, each one of them being a possible history of the actual world. One possible way to represent this is by letting each *w* of *W* be a structure $(w(t_1),w(t_2),w(t_3) ...)$, each $w(t_i)$ being a time-instant in world *w*.
2. *wRu* iff *u* is a physically possible temporal continuation of *w*, meaning that if *w* and *u* have the same history until an instant t_i (meaning that for $t_j \leq t_i$, we have that $w(t_j) = u(t_j)$), they will diverge afterwards: if $t_j > t_i$, then $w(t_j) \neq u(t_j)$.
3. *V* will have to assign atoms to instants $w(t_i)$ of worlds *w* in *W*.

Therefore we will have to evaluate counterfactuals at instants of time: $M, w(t_i) \vDash A \;\Box\!\!\rightarrow B$ iff in every closest $u(t_i)$ *A*-satisfying worlds branching out of $w=(w(t_1),.., w(t_{i-1}))$ it is true that *B*. This, because the valuation function *V* is

[2] Following the (Stalnaker 1976) exposition of Lewis' realism: possible worlds exist, they are as irreducible to other kinds, they are not qualitatively different from *our* world, and actuality is an indexical notion (wherefrom *our* world is the world we live in).

constant only when defined on instants of time. Note that function V can assign truth to A at $w(t_1)$ but it can make A false in $w(t_2)$.

However, this is just a sketch of a formal model, inspired by McCall's (1) directions on how to evaluate a counterfactual conditional, and (2) intuitions regarding in what type of model to evaluate them. We will be able to generate a precise formal model (one resembling Epistemic Temporal structures) that satisfies McCall's intuitions using the apparatus presented below.

Events (BMS-actions) with Postconditions

I will begin this section with presenting the apparatus of Baltag-Moss-Solecki actions (Solecki, Baltag, Moss 1999).[3][4] Event models were introduced in Epistemic Logic by Solecki, Baltag, Moss (1999) as a means of describing the way an agent's knowledge set changes as a result of learning truths about the world or about other agents' knowledge. This change in an agent's knowledge set is realized by changing the agent's epistemic possibilities. As an example, if an agent considers that worlds w and u are equally plausible candidates for the status of the actual world, and p is true in w but not in u, then, after receiving information that p is true in the actual world, then world u should be eliminated from the set of the agent's epistemic possibilities. However, these structures were supplemented so as to allow for ontic change (van Ditmarsch and Kooi 2008), meaning that propositions about the world, and not only facts about what agents know about the world, can change their truth-values as a result of executing an event model in a Kripke model.

> **Definition** (van Ditmarsch and Kooi 2008, 91). An event model is a structure $!Act = (S, \sim, pre, post)$, such that:
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> 1. S is a set of event points,
> 2. $\sim$ is an equivalence relation on S
> 3. $pre : S \rightarrow L$ is a precondition of an event's execution in a world.
> 4. $post : S \rightarrow (Atoms(L) \rightarrow L)$ is a function that changes the valuation of atoms.

The following construction aims at representing the result of executing an event in a Kripke model:

> **Definition** (van Ditmarsch and Kooi 2008, 94). The Modal Product of a Kripke model $M = (W, R, V)$ and an event model $!Act = (S, \sim, pre, post)$ is a structure $M \otimes !Act =(W', R', V')$ such that:
>
> 1. $W'=\{(w, a) \mid w \in W, a \in S, \text{and } M, w \vDash pre(a)\}$
> 2. $<(w_1,a_1), (w_2, a_2)> \in R'$ iff $(w_1, w_2) \in R$, $(a_1, a_2) \in \sim$, for $(w_1,a_1) \in W'$, $(w_2, a_2) \in W'$
> 3. $M', (w, a) \vDash p$ iff $M, w \vDash post(a)(p)$

Some observations are due:

[3] Also see (van Ditmarsch et al. 2006) for an introduction.

[4] Called *event models* hereafter.

(a) The result of executing an event in a Kripke model is a Kripke model.

(b) The apparatus of restricted modal products allows for recording the history of a possible world in terms of a sequence whose first element denotes a member of *W*, a possible world, and all the others denote events that were executed in the possible world. This feature makes this device useful in representing McCall's real possible worlds.

(c) As we can see from condition (1), a world *(w, a)*, meaning world *w* after the execution of *a*, will be part of the domain *W'* iff *w* satisfies the precondition of event *a*. This condition seems intuitive: certain events cannot happen unless some prerequisites are met. For example, one cannot speed their car unless one does not drive a car.

Also, note that because of condition (3), if in $w \in W$ it is false that A ($w \notin V(A)$), then, in *(w,a)* it will be true that *A*, if $post(a)(A) = \top : (M \otimes !Act), (w,a) \vDash A$, although: $M, w \nvDash A$.

This framework allows for formulas that state that after the execution of an event, a formula becomes true (van Ditmarsch et al. 2006, 151):

(1) $M, w \vDash [!(Act, a)]A$ iff: if $M, w \vDash pre(a)$, then $(M \otimes !Act), (w,a) \vDash A$

(2) $M, w \vDash \langle !(Act, a) \rangle A$ iff $M, w \vDash pre(a)$ and $(M \otimes !Act), (w,a) \vDash A$

The semantic definition (1) is read: in model *M*, at world *w*, it is true that after the execution of event *!Act* it becomes true that *A* if and only if: if the precondition of the event *a* of *!Act* is satisfied at *w*, then in the model obtained after the execution of *!Act*, in world *(w, a)* it is true that *A*, and (2): in model *M*, at world *w*, it is true that the event model *!Act* can be executed and *A* is true if and only if world *w* satisfies the precondition of event *a* of *!Act* and in the product model, in world *(w, a)* it is true that *A*. Now, formulas like the below will express the fact that even though *A* is false, it becomes true after the execution of *!Act*:

$M, w \vDash \neg A \,\&\, [!(Act, a)]A$

How is this apparatus useful in representing McCall's real possible worlds? Recall that a real possible world is a history $(w(t_1), w(t_2), w(t_3) \ldots)$, so we can equate such a history with a sequence ($w, a_1, a_2, a_3 \ldots$) composed of a possible world *w* (of an initial singleton model) and a sequence of events that were executed in *w*. Now, why is it important to have a method of changing ontic truths (truths about the world)? Take two time-instants $w(t_i)$ and $w(t_{i+1})$. They could have the same valuations for their atoms, or, if things changed from $w(t_i)$ to $w(t_{i+1})$ they could differ in their valuations. Since postconditions can only change one atom, we will assume that two consecutive time-instants can only differ in at most one truth. Then, if we equate histories $(w(t_1), w(t_2), w(t_3) \ldots)$ with sequences ($w, a_1, a_2, a_3 \ldots$), the ontic difference between a time-instant $w(t_i)$ and $w(t_{i+1})$ can be represented in terms of executing an event a_{i+1} in ($w, a_1, a_2, \ldots, a_i$). In other words, we can represent the change that one event brought to a time-instant $w(t_i)$ as the execution of an event a_{i+1} in a sequence $(w,a_1,a_2,\ldots,a_i)$.

Now, since two time-instants can differ in at most one atom, for each atom A in the language, we will construct two events models, named $!A$ and $!\neg A$, the first making A true and the second making A false, defined as following:

Definition. Action $!A$ is a singleton event model $(\{a\}, aRa, pre, post)$, such that $pre(a) = \top$, $post(a)(A) = \top$.

Definition. Action $!\neg A$ is a singleton event model $(\{a\}, aRa, pre, post)$ such that $pre(a) = \top$, and $post(a)(A) = \bot$.

Because the precondition of any such action is $\top$, all such events will be executable in any possible world. Note that event $!A$ will change the truth value of A to *true* and event $!\neg A$ will change A's truth value to *false*. In addition, we will need an event that does not change the valuation of formulas:

Definition (van Ditmarsch et al. 2006, 150). Event $!^*$ is an event model $(\{a\}, aRa, pre, post)$ such that $pre(a) = \top$, and $post(a)(A) = id$, for *id* the identity function.

Up to this point, executing events in possible worlds cannot give rise to the kind of branching-time structure McCall considers necessary for his semantics of counterfactuals. If we start with a singleton Kripke model and execute singleton events, what we will obtain is a sequence of models: model M with w (the initial world), model $(M \otimes Act_1)$ with world (w,a_1), model $(M \otimes Act_1 \otimes Act_2)$ with world (w,a_1, a_2) and so on. In order to obtain a tree structure, we can make use of the notion of a protocol for Dynamic Epistemic Logics[5] (hereafter: DEL), as presented in Hoshi (2009). A protocol is a set of sequences of events, each sequence describing what events can be executed in a possible world of the model and in what order (Hoshi 2009, Hoshi & Yap 2009, 262). Let *Prot* be the class of all event models: $\{!(Act, a) \mid !Act$ is an event model and a is an event of the domain of $!Act\}$ and *Prot** the class of all the finite sequences constructed out of elements of *Prot*. Then, a protocol π is a subset of *Prot**, closed under finite prefix (meaning that if ab is in π, then also a is in π). Now, given a protocol and a Kripke model M, we can construct *the protocol model* (Hoshi 2009), a model that contains all possible evolutions of M as a result of executing the events in the sequences of protocol π (and in the order specified by π). All the possible evolutions of initial model M are also Kripke models (because executing an event in a Kripke model generates, by the restricted modal product, a Kripke model), so the end result is a Kripke forest, a structure composed of Kripke models. Let us see the construction of the protocol model, using (Hoshi 2009) and (Hoshi and Yap 2009, 262-263):

Given a Kripke model $M = (W, \sim, V)$ and protocol π, the protocol model $M^{\sigma,\pi} = (W^{\sigma,\pi}, \sim^{\sigma,\pi}, V^{\sigma,\pi})$, is constructed by induction on the length of σ, a sequence in π (by σ_n we will denote a sequence of n event models, and by $\sigma_{(n)}$ the n^th^ element of

[5] See (van Ditmarsch et al. 2006) for an introduction.

sequence σ), following the rules (see (Hoshi and Yap 2009, 262-263), for rules (1)-(3) and (van Ditmarsch and Kooi 2008, 94), for rule (4)):

1) $W^{\sigma 0,\pi} = W$, $\sim^{\sigma 0,\pi} = \sim$, $V^{\sigma 0,\pi} = V$
2) $w\sigma_{n+1} \in W^{\sigma n+1,\pi}$ *iff*
   (a) $w \in W$,
   (b) $M^{\sigma n,\pi}, w\sigma_n \vDash pre(\sigma_{(n+1)})$,
   (c) $\sigma_{n+1} \in \pi$
3) $\forall(w\sigma_{n+1}, u\sigma_{n+1}) \in W^{\sigma n+1,\pi} : (w\sigma_{n+1}, u\sigma_{n+1}) \in \sim^{\sigma n+1,\pi}$ *iff* $(w, u) \in \sim$,
4) $\forall p \in Atoms(L)$: $V^{\sigma n+1,\pi}(p)=\{ w\sigma_{n+1} \in W^{\sigma n+1,\pi} \mid M^{\sigma n,\pi}, w\sigma_n \vDash post(\sigma_{(n+1)})(p)\}$

This construction will generate all the possible ways in which an initial model will evolve as a result of executing the events in the protocol. As a consequence of applying rules 1-4, $M^{\sigma,\pi}$ will be composed of other Kripke models, each one of them representing a possible state the initial *M* might evolve into as a result of executing the events in the protocol. For example, if *π={!A!B, !A!C}* and the initial model *M* is a singleton composed of world *w*, then the protocol model will contain: *M* – with domain *{w}*, *M ⊗ !A* – with domain *{w!A}*, *M ⊗ !A ⊗ !B* – with domain *{w!A!B}* and *M ⊗ !A ⊗ !C* – with domain *{w!A!C}.* Rule (2) assures us that any new possible world (history) added in the domain of a newly added model meets the prerequisite imposed by the precondition of the event that was executed a step before in the construction. Rule (4) allows for changing the truth value of an atom as a result of executing an action whose postcondition is not the identity function.[6]

In order to evaluate formulas that state what truths change in the model as a result of executing some events, Hoshi (2009, 62) and Hoshi and Yap (2009, 263) chose to use ETL models (Epistemic Temporal Logic models) generated by the protocol model. But in an ETL model the valuation of atoms remains unchanged, so we will call the structure generated by the protocol model defined above a *pseudo-ETL-model*.

A pseudo-ETL-model $\mathbb{H} = (H, \sim', V')$ generated by the protocol model $M^{\sigma,\pi} = (W^{\sigma,\pi}, \sim^{\sigma,\pi}, V^{\sigma,\pi})$ is constructed as below (Hoshi 2009, 62, Hoshi and Yap 2009, 263):

1) $H = \{h \mid h = w\sigma \in W^{\pi,\sigma}$ *with* $w \in W, \sigma \in \pi\}$
2) $(h, h') \in \sim'$ *iff* $(h, h') \in \sim^{\sigma,\pi}$, *for* $\sigma \in \pi$ *and every* $h, h' \in H$ *such that* $h=w\sigma$ *and* $h'=u\sigma$
3) $h \in V'(p)$ *iff* $h \in V^{\sigma,\pi}$, *for* $p \in Atoms(L), \sigma \in \pi, h=w\sigma$.

Recall that the protocol model presented above constructs a series of models out of an initial Kripke model. The above rules grant that a pseudo-ETL-model will include the worlds of all models of a protocol model. As a consequence, *H* will be a set of worlds (represented as sequences of events executed in the world of the initial singleton model) and not a set of Kripke

[6] A postcondition that is an identity function leaves the assignment unchanged (van Ditmarsch & Kooi 2008, 91-92).

models. The epistemic accessibility relation ~ will play no role in our construction, since we will only use singleton models and singleton events. But for each sequence, we will introduce relations R by the following rule:

4) $(h, h') \in R_{!\sigma(n+1)}$, for each $h = w\sigma_n$ and $h' = w\sigma_{n+1} = w\sigma_n!\sigma_{(n+1)}$

This relation holds between two worlds h and h' (two time-instants of the same real possible world) iff h' is a possible future state of h, or, in other words, if h' can be obtained by executing an event in h. Now we have a formal representation of the set of physically possible temporal continuants of a world h as being the set $\{h' \mid$ there is an action $!a$ such that $hR_{!a}h'\}$.

This construction allows for different formulas to be evaluated in possible worlds of $\mathbb{H}$ (Hoshi 2009):

1) $\mathbb{H}, h \vDash A$ iff $h \in V'(A)$
2) $\mathbb{H}, h \vDash \neg A$ iff $h \notin V'(A)$
3) $\mathbb{H}, h \vDash A \,\&\, B$ iff $\mathbb{H}, h \vDash A$ and $\mathbb{H}, h \vDash B$
4) $\mathbb{H}, h \vDash \langle !Act \rangle A$ iff $h!Act \in H$ and $\mathbb{H}, h!Act \vDash A$
5) $\mathbb{H}, h \vDash [!Act]A$ iff: if $h!Act \in H$, then $\mathbb{H}, h!Act \vDash A$

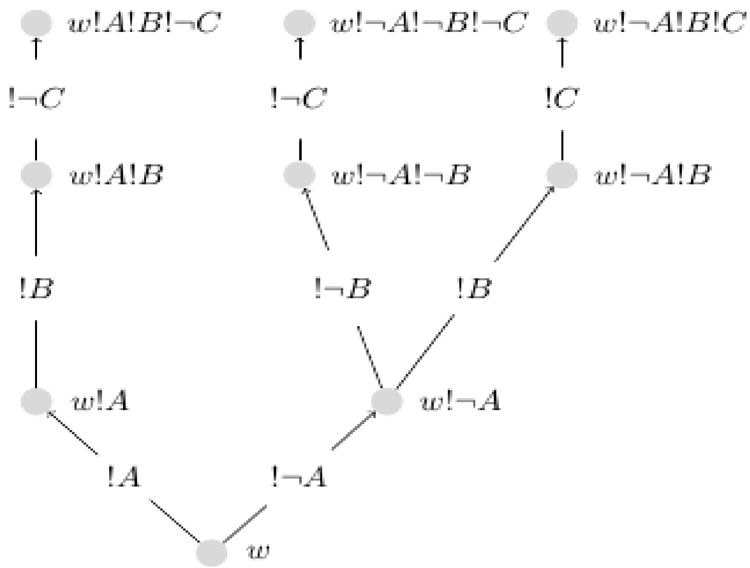
Definition (5) will be read: in model $\mathbb{H}$, at world h, it is true that after the execution of event *!Act* it is true that A iff: if *h!Act* (meaning world h after the execution of event *!Act*) is a part of H (meaning that *h!Act* is a possible state h might evolve into, according to the protocol), then in *h!Act* it is true that A.

6) $\mathbb{H}, h = w\sigma_{n+1} \vDash BEFORE(A)$ iff $\mathbb{H}, h' = w\sigma_n \vDash A$

This semantic definition is read: in model $\mathbb{H}$, at world h, meaning a sequence composed of w and $n+1$ events, it is true that before the last event was executed it was true that A iff in the same model, in the immediate past of h (meaning w followed by the first n events) it is true that A. This definition makes sense only in case the model is a structure in which each node has only one parent. But such a structure is the one McCall argues for using in interpreting counterfactual conditionals.

Now, we can use the apparatus presented above to illustrate the process of obtaining McCall's real possible worlds. Note that if the updated model is a singleton model, then the structure of a pseudo-ETL model will be a tree-like structure, similar to the one recommended by McCall to establish the truth value of a counterfactual. Also, as already proposed, McCall's real possible worlds will be sequences $w!a_1!a_2...$, with w in W, each $!a_i$ an event, and $!a_1!a_2,...$ a sequence of events (a historical description of that world).

Let us see how we can obtain the model used by McCall (1984, 470) to prove that his semantics invalidates the transitivity principle. The protocol for this model is $\pi = \{!A!B!\neg C, !\neg A!\neg B!\neg C, !\neg A!B!C\}$ and the initial model is a singleton domain Kripke model, $M=(W=\{w\}, wRw, V)$. By applying the rules for generating the protocol model, we will obtain a Kripke forest composed of the following singleton Kripke models: (1) M, the initial model, (2) $M \otimes !A$, (3) $M \otimes !A \otimes !B$, (4) $M \otimes !A \otimes !B \otimes !\neg C$, (5) $M \otimes !\neg A$, (6) $M \otimes !\neg A \otimes !\neg B$, (7) $M \otimes !\neg A \otimes !\neg B \otimes !\neg C$, (8) $M \otimes !\neg A \otimes !B$, (9) $M \otimes !\neg A \otimes !B \otimes !C$. Now, the pseudo-ETL structure generated will only contain the worlds of each model in the protocol model: (1) w, (2) $w!A$, (3) $w!A!B$, (4) $w!A!B!\neg C$, (5) $w!\neg A$, (6) $w!\neg A!\neg B$, (7) $w!\neg A!\neg B!\neg C$, (8) $w!\neg !A!B$, (9) $w!\neg A!B!C$:



Let us see McCall's argument that his view on how counterfactuals should be understood makes "□→" non-transitive. The argument focuses on trying to find a situation in which though A □→ B and B □→ C, it is not the case that A □→ C.

So we have, for f_A a function that selects the closest branching A-satisfying possible world (this selection is possible, given the construction of the model):

1) $M, w!\neg A!\neg B!\ \neg C \vDash A$ □→ B iff $f_A(w!\neg A!\neg B!\neg C) = w!A!B!\neg C$ and $M, w!A!B!\neg C \vDash B$.
2) $M, w!\neg A!\neg B!\neg C \vDash B$ □→ C iff $f_B(w!\neg A!\neg B!\neg C) = w!\neg A!B!C$ and $M, w!\neg A!B!C \vDash C$

Note that the $f_B(w!\neg A!\neg B!\neg C)$ is not $w!A!B!\neg C$, since this world branches off from the actual world earlier than $w!\neg A!B!C$.

3) $M, w!\neg A!\neg B!\neg C \vDash \neg(A$ □→ $C)$ because $f_A(w!\neg A!\neg B!\neg C) = w!A!B!\neg C$ and $w!A!B!\neg C \nvDash C$

Now, that we have a formal model M that corresponds to McCall's intuitions, we can state his semantic definition as following:

> *M, wσ* ⊨ *A* □→ *B* iff in all worlds wτ such that: (1) *τ* and *σ* are of equal length and (2) *wτ* are the closest *A*-satisfying branching off of *wσ* worlds, it is true that *B*.

Although there are sound and completely axiomatized logical systems of Dynamic Epistemic Logic with ontic change (van Ditmarsch & Kooi 2008, 96) and of Epistemic Logic with Protocols (see, for example, Hoshi's Temporal Dynamic Epistemic Logic and Temporal Arbitrary Dynamic Epistemic Logic in (Hoshi 2009) and (Hoshi and Yap 2009)), a sound and complete logic that incorporates both ontic change operators and protocols is still due.

An Objection to McCall's Semantics of Counterfactuals

In this section I will offer an interpretation in the logical framework presented above of one of the objections raised to McCall's semantics of counterfactuals by Otte (1987). I will argue that the objection, as interpreted, will not hold.

Otte (1987, 422) imagines the following situation. Suppose Franz is a very bad skier who fraudulently secured himself a place at the World Ski competition. Most of the track is extremely tough, so he finishes the last. Now, consider Otte's counterfactual:

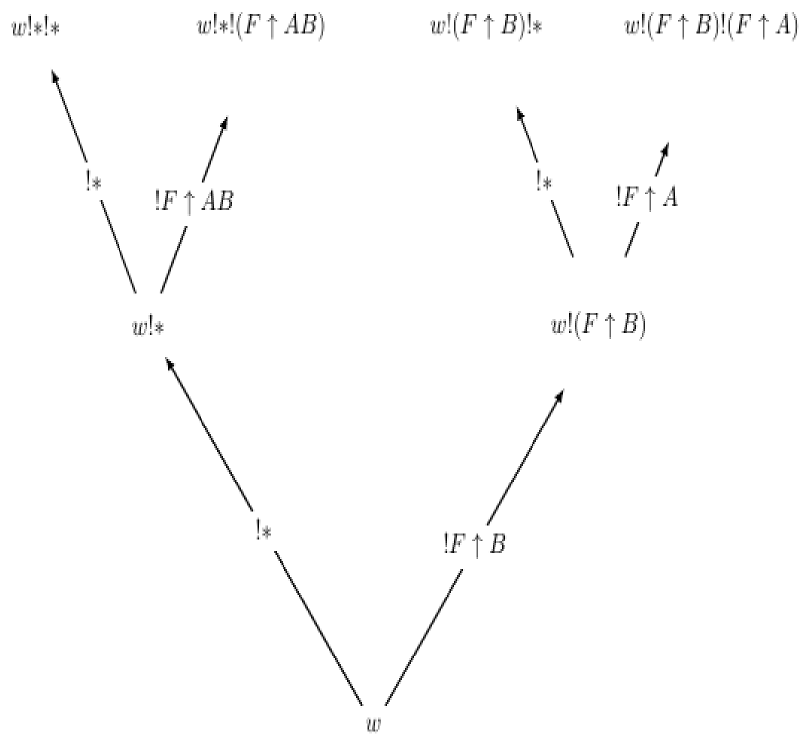
> If Franz had won the race, all of the other skiers would have been ahead of him throughout the race until just short of the finish line (Otte 1987, 422).

Otte argues that this counterfactual comes out as true by McCall's semantics, because the closest world that branches off the actual one in which Franz wins is a world in which every other racer is ahead of him until short of the finish line. However, intuitively, this counterfactual should not hold.

Let us restate Otte's objection in the logical framework presented above. We will create a model for the situation in which Franz's opponents are *A* and *B*. First, we will have to create a vocabulary of atoms and other useful formulas and events:

- Atoms *F_is_in_front_of_A* will be true in all worlds in which *F* has passed *A*, and *F_is_in_front_of_B* will be true in all worlds in which *F* has passed *B*. Atom *F_wins* will be true in all worlds in which Franz wins.
- *F↑A* is a singleton event that makes atom *F_is_in_front_of_A* true.
- *F↑B* is a singleton event that makes atom *F_is_in_front_of_B* true.
- *(F↑AB)* is a singleton event that makes atom *F_wins* true.
- *!** is a singleton event that does not change anything in the model (its postcondition function is the identity function).

Now, let us state what worlds will be used in order to offer a model for Otte's objection. The protocol that will generate the model will be *π={!*!*, !*!(F↑AB), !(F↑B), !(F↑B)!(F↑A)}*, each of the worlds in its domain being:



- *w* - is the initial possible world,
- *w!** and *w!*!** - have the same valuations with *w*,
- *w!*!(F↑AB)* - is world *w!** after Franz passes both *A* and *B* and, so, wins,
- *w!(F↑B)* - is world *w* after which Franz passes *B*,
- *w!(F↑B)!** - is world *w!(F↑B)* after which no other atoms change their truth values,
- *w!(F↑B)!(F↑A)* - is world *w!(F↑B)* after which Franz passes *A* also (and therefore, wins).

Regarding the valuation function, *F_wins* is true in the following worlds: *w!*!(F↑AB)* and *w!(F↑B)!(F↑A)*.

In this model, world w!*!* is the actual world, the world in which Franz lost the contest as the last of all the competitors. Otte's objection can be interpreted as saying that the closest world in which Franz wins that branches off the actual one is *w!*!(F↑AB)*. We will argue that his objection does not hold because world *w!*!(F↑AB)* is inconsistent in the model created with the intention to reflect Otte's context in which a counterfactual comes out as true though it should intuitively not. In order to establish our argument, first, let us observe that the following formulas should be considered true in every possible world of the model:

(1) *F_wins → (F_is_in_front_of A & F_is_in_front_of_B)*

Its meaning is intuitive: if Franz wins, then he must be in front of every competitor.

(2) *F_is_in_front_of_A → BEFORE(<!F↑A>⊤) ∨ BEFORE(BEFORE(<!F↑A>⊤))*

(3) *F_is_in_front_of_B → BEFORE(<!F↑B>⊤) ∨ BEFORE(BEFORE(<!F↑B>⊤))*

The meanings of (2) and (3) are also intuitive: in order to be in front of each one of the competitors, the event of Franz's passing *A* and the event of Franz's passing *B* must have been executed in the model.[7]

Now, since at *w!*!(F↑AB)* Franz wins the race (this is the assumption of the model), *F_wins* must be true:

M, w!!(F↑AB)* ⊨ *F_wins*

However:

M, w!!(F↑AB)* ⊭ *BEFORE(<!F↑A> ⊤) ∨ BEFORE(BEFORE(<!F↑A> ⊤))*,

Moreover:

M, w!!(F↑AB)* ⊭ *BEFORE(<!F↑B> ⊤) ∨ BEFORE(BEFORE(<!F↑B> ⊤))*

This, because in each case both disjuncts are false in *w!*!(F↑AB).* Therefore *F_wins* should be false, reaching a contradiction with the assumption on which the model was constructed. As a consequence, *w!*!(F↑AB)* is not a consistent possible world. But (1) according to McCall, the accessibility relation only links physically possible worlds, and (2) logically impossible worlds are not physically possible worlds, therefore: world *w!*!(F↑AB)* is not accessible from the actual world, *w!*!**.

Conclusion

In this paper we have presented the Stalnaker-Lewis semantics of counterfactual conditionals. First, we introduced the reader to Stalnaker's (1968) semantics and presented some of the objections raised by Lewis (1973). We have presented Lewis' solution to the objections raised, in terms of a different semantic theory of counterfactuals. Different indeed, but following the same underlying intuition: that counterfactuals can be defined in terms of a comparative similarity relation between possible worlds. McCall (1984) addressed this issue – using comparative similarity, a vague concept, as a brick in the foundation of a theory of truth for counterfactual conditionals – and proposed a different semantics, devoid of the vagueness implicit in the first two. Using the apparatus of Event Models (Solecki, Baltag, Moss 1999, van Ditmarsch et al. 2006) with ontic change (van Ditmarsch and Kooi 2008), and that of protocols for Dynamic Epistemic

[7] Formula *<!F↑A>⊤* means: event *!F↑A* has been successfully executed (*⊤* is a tautology, therefore a formula true in all possible worlds).

Logics (Hoshi 2009, Hoshi and Yap 2009) we have presented a method to generate the branching-time structure that McCall uses to evaluate counterfactuals. Moreover, we have presented an interpretation of one of Otte's (1987) objections to McCall's theory in the formal apparatus introduced. We have argued that the interpretation of that objection can be countered using the logical apparatus introduced.

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