

# Two Problems for Zylstra’s Truthmaker Semantics for Essence

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In his article ‘Making semantics for essence’ (*Inquiry*, 2019), Justin Zylstra proposed a truthmaker semantics for essence and used it to evaluate principles regarding the explanatory role of essence. The aim of this article is to show that Zylstra’s semantics has implausible implications and thus cannot adequately capture essence.

In recent years, truthmaker semantics has become an increasingly popular tool in various areas of metaphysics and the philosophy of language. In particular, the truthmaker framework is commonly used to devise semantics for ground.<sup>1</sup> The application to the closely related field of essence, however, has remained largely unexplored. In a recent article in this journal (‘Making semantics for essence’, 2019), Justin Zylstra offers the first proposal for a truthmaker semantics for essence. He then demonstrates how the account can help to illuminate the explanatory role of essence: By providing us with a joint semantic framework for essence and grounding, it allows us to investigate principles regarding the grounds and groundees of essentialist statements. In particular, Zylstra’s semantics would establish one main result: That the thesis of ESSENCE GROUNDS PREJACENTS—according to which an essentialist claim of the form ‘it is essential to  $\alpha$  that  $\phi$ ’ grounds its prejacent  $\phi$ —turns out to be false. ESSENCE GROUNDS PREJACENTS is a quite natural seeming claim that looms large in the debate and has recently become a matter of increasing controversy.<sup>2</sup> Apart from this application, Zylstra’s semantics would also promise to open up the possibility for further semantic research on the

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<sup>1</sup>See Correia (2010, 2016), Fine (2012a, 2012b, 2017b) and Krämer (2018).

<sup>2</sup>For the recent discussion, see in particular Kment (2014) and Rosen (2010) in favour of ESSENCE GROUNDS PREJACENTS, and Glazier (2017) against it.

connections between essence and various other phenomena of metaphysical interest, and especially on the connections between essence and metaphysical modality.

In this article, however, I will argue that the proposed semantics should not be adopted. The semantics implies results that are highly implausible, and in tension with all common views on essence. It thus does not afford us with a perspicuous way of formally representing essence.

I start out by introducing Zylstra's proposed semantics in Section 1. In the core part of the paper, Sections 2 and 3, I demonstrate the way in which the semantics allows us to derive problematic results. While the results obtained in Section 2 suggest that the conditions that Zylstra imposes on verifiers of essentialist statements are in a sense too weak, the results in Section 3 suggest that they are too strong in another. I end with some concluding remarks in Section 4.

## 1 Zylstra's semantics for essence

Zylstra's proposed semantics for essence builds on the truthmaking framework as developed by Kit Fine (2017a, 2017b). While the common possible worlds semantics associates sentences with the possible worlds at which they are true, truthmaker semantics associates sentences with the states that exactly verify them.

States can be complete (of the 'size' of a whole world) or incomplete ('smaller' than a world), and consistent or inconsistent. Some examples for states are the state of *snow's being white*, the state of  $1 + 1 = 5$ , the state of *Barcelona's being in Spain and Geneva's being in Switzerland*, and the actual world-state. States stand in relationships of parthood to another and fuse to larger states. Thus, the state of *Barcelona's being in Spain and Geneva's being in Switzerland* has the states of *Barcelona's being in Spain* and of *Geneva's being in Switzerland* as parts, and is the fusion of these two states.

The intuitive idea behind the notion of exact verification is that some state  $s$  exactly verifies  $\phi$  iff  $s$ 's obtaining would guarantee  $\phi$ 's truth, and moreover,  $s$  would be wholly relevant for  $\phi$ 's truth. (I shall drop the 'exact'-qualification in what follows.) Thus, *Barcelona's being in Spain and Geneva's being in Switzerland* would not count as a verifier of 'Barcelona is in Spain', since it does not meet the relevance-condition. Importantly, verification is not factive: False, and even inconsistent sentences can still have verifiers. For instance, the false sentence 'Barcelona is in Switzerland' would nevertheless have the state of *Barcelona's being in Switzerland* as a verifier.

To account for essence, Zylstra adds two elements to the standard truthmaking framework: First, a *set of items*, which represents the realm of potential bearers of essence and may include, e.g. Socrates, the singleton Socrates, and the number two. And, second, an *essence-making function*, which, intuitively, pairs sets of items with the propositions that express their collective constitutive essence.<sup>3</sup> Within this framework, Zylstra devises verification conditions for essentialist statements. Adopting Fine’s (2017b) account for grounding, he then shows ESSENCE GROUNDS PREJACENTS to fail.

In Zylstra’s semantics, *essence-making models* are construed as quintuples  $\langle S, \mathcal{I}, \sqsubseteq, \mathcal{M}, |\cdot| \rangle$ , whereby:

- $S$  is the *set of states*.
- $\mathcal{I}$  is the *set of items*.
- $\sqsubseteq$  is the *parthood relation* on  $S$ , a partial order.

The *fusion* of a set  $T \subseteq S$  is defined as the least upper bound of  $T$  with regard to parthood. That is,  $T$ ’s fusion has all members of  $T$  as parts and is part of every state that has all the members of  $T$  as parts. The fusion of the members of a set  $T$  is denoted by ‘ $\bigsqcup T$ ’, and the fusion of some states  $s_1, s_2, \dots$  by ‘ $s_1 \sqcup s_2 \sqcup \dots$ ’. It is required of all models that every subset of  $S$  has a fusion, that is, each  $T \subseteq S$  has a least upper bound.

Zylstra adopts a so-called *regular unilateral conception of propositions*, i.e. he identifies propositions with sets of states  $P \subseteq S$  that fulfil the following two conditions:

- CLOSURE UNDER FUSION:  $P$  is closed under fusion, i.e. for every nonempty  $T \subseteq P$ ,  $\bigsqcup T \in P$ .
- CONVEXITY:  $P$  is convex, i.e. for all  $s_1, s_2, s_3 \in S$ , if  $s_1 \sqsubset s_2 \sqsubset s_3$  and  $s_1, s_3 \in P$ , then  $s_2 \in P$ .

As Zylstra notes, however, it has been argued in the recent literature on grounding that one should drop the assumption of CONVEXITY and merely demand of propositions that they be closed under fusion.<sup>4</sup> Moreover, Zylstra’s semantics is independent of CONVEXITY—the semantics could be set up in an entirely parallel way and would exhibit the same relevant features if CONVEXITY was not assumed. To show that my arguments do not hinge on CONVEXITY and would equally apply to a version of Zylstra’s semantics that dropped it, I will remain neutral with regard to CONVEXITY here. That is, I shall work with both candidate conceptions of propositions whenever the difference will matter, adopting the relevant definitions in the non-convex

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<sup>3</sup>See Fine (1994a) on collective essence, and Fine (1994b) on constitutive essence.

<sup>4</sup>See Krämer and Roski (2015) and Correia (2016). Fine (2017b) remains neutral, but largely adopts CONVEXITY for technical reasons.

case from Fine (2017a, 2017b). I will use the following notation: For some set of states  $T \subseteq S$ , the symbol ‘ $T_f$ ’ stands for  $T$ ’s closure under fusion (i.e. the smallest set that contains  $T$  and is closed under fusion), and the symbol ‘ $T_f^c$ ’ for  $T$ ’s regular closure (i.e. the smallest set that contains  $T$  and is both closed under fusion and convex). The symbol ‘ $T*$ ’ serves as a placeholder for  $T_f^c$  under the assumption of CONVEXITY and for  $T_f$  otherwise. The symbol ‘ $\mathcal{S}$ ’ stands for the set of propositions on the relevant conception.

- $\mathcal{M}$  is the *essence-making function* from subsets of  $\mathcal{I}$  to subsets of  $\mathcal{S}$ , i.e. from sets of items to sets of propositions. Two conditions are imposed on  $\mathcal{M}$ :
  - UPWARD CLOSURE: If  $P, Q \in \mathcal{M}(I)$  for some  $I \subseteq \mathcal{I}$ , then:  $P \wedge Q \in \mathcal{M}(I)$  and  $P \vee Q \in \mathcal{M}(I)$ .
  - DOWNWARD CLOSURE: If  $P \wedge Q \in \mathcal{M}(I)$  for some  $I \subseteq \mathcal{I}$ , then:  $P, Q \in \mathcal{M}(I)$ .<sup>5</sup>

Where:  $P \wedge Q := \{s \mid s = s_1 \sqcup s_2 \text{ for some } s_1 \in P, s_2 \in Q\}*$ ,  $P \vee Q := \{s \mid s \in P \text{ or } s \in Q\}*$ .

- $|\cdot|$  is the *valuation function*.  $|\cdot|$  maps every (singular or plural) name in the language to a subset of  $\mathcal{I}$ , i.e. to a set of items. And  $|\cdot|$  maps every sentential constant to some proposition  $P \in \mathcal{S}$ , its set of *verifiers*.

For ease of presentation, Zylstra restricts himself to toy language fragments that include conjunction and disjunction as truth-functional connectives, but not negation. As Zylstra notes, however, his account could be extended to languages with negations in the common way.<sup>6</sup>

To account for conjunction and disjunction, the range of the valuation function  $|\cdot|$  gets extended from the sentential constants to truth-functionally complex sentences in the following way:

$$|\phi \wedge \psi| = \{s \mid s = s_1 \sqcup s_2, \text{ for some } s_1 \in |\phi|, s_2 \in |\psi|\}*$$

$$|\phi \vee \psi| = \{s \mid s \in |\phi| \text{ or } s \in |\psi|\}*$$

The core of Zylstra’s semantics is given by the proposed verification conditions for statements of

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<sup>5</sup>These two conditions ensure that essentialist claims exhibit the inferential behaviour that one would pre-theoretically expect them to have. For instance, DOWNWARD CLOSURE has the effect that its being essential to  $\alpha$  that  $\phi \wedge \psi$  entails that it is essential to  $\alpha$  that  $\phi$ , in the sense of ‘entailment’ relevant within truthmaker semantics (see p. 5 of this article).

<sup>6</sup>For this, a bilateral rather than unilateral account of propositions would have to be adopted, i.e. an account on which each sentence would be associated with both a set of verifiers and a set of falsifiers, rather than merely with a set of verifiers (see Fine 2017a). This modification, however, would have no bearing on the relevant features of the account and make the presentation substantially more lengthy. Since the problems I present here already arise for the negation-free case and would obviously transfer to the more complex case, there is no need for us to go into these additional complications.

constitutive essence. These statements are taken to be of the form  $\Box_\alpha\phi$ , where  $\alpha$  is a (singular or plural) name,  $\phi$  a sentence, and  $\Box_\cdot$  the essence-operator.<sup>7</sup> The conditions are:

If  $|\phi| \notin \mathcal{M}(|\alpha|)$ ,  $|\Box_\alpha\phi| = \emptyset$ .

If  $|\phi| \in \mathcal{M}(|\alpha|)$ ,  $|\Box_\alpha\phi| = \{\sqcup|\phi|\}$ .

That is, if  $|\phi| \notin \mathcal{M}(|\alpha|)$ ,  $\Box_\alpha\phi$  has no verifiers in the model. If, by contrast,  $|\phi| \in \mathcal{M}(|\alpha|)$ , then  $\Box_\alpha\phi$  has exactly one verifier:  $\sqcup|\phi|$ , the fusion of all verifiers of  $\phi$ , called the *subject-matter* of  $\phi$ .

Zylstra adopts Fine’s (2017b) truth-conditions for statements of ground. Say that a proposition  $P$  entails another proposition  $Q$  iff  $P \subseteq Q$ . Correspondingly, say that a sentence  $\phi$  entails another sentence  $\psi$  iff  $|\phi|$  entails  $|\psi|$ , that is, iff all verifiers of  $\phi$  are also verifiers of  $\psi$ . Say that a proposition  $P$  or sentence  $\phi$  is *verifiable* iff  $P$  is non-empty or iff  $|\phi|$  is non-empty, respectively. Using ‘<’ as a symbol for (worldly, full, strict, non-factive) grounding, we have:<sup>8</sup>

$M \models \phi_1, \phi_2, \dots < \psi$  iff, in  $M$ :

(i) VERIFIABILITY:  $\phi_1, \phi_2, \dots, \psi$  are verifiable.

(ii) ENTAILMENT:  $\phi_1 \wedge \phi_2, \dots$  entails  $\psi$ .

(iii) CONTAINMENT:  $\sqcup|\phi_1| \sqsubset \sqcup|\psi|$ ,  $\sqcup|\phi_2| \sqsubset \sqcup|\psi|, \dots$ <sup>9</sup>

It is straightforward to see that, on the combined semantics, ESSENCE GROUNDS PREJACENTS fails, as Zylstra wishes to show. That is, there are no models of the semantics in which  $(\Box_\alpha\phi) < \phi$  turns

<sup>7</sup>While Zylstra allows for the embedding of any kind of sentence under the essence-operator, statements of ground should arguably be excluded from the range of embeddable sentences: Since, on the assumed account of grounding, statements of ground are not associated with verifiers,  $|\chi_1, \chi_2, \dots < \mu|$  and consequently also  $\sqcup|\chi_1, \chi_2, \dots < \mu|$  would be left undefined.

<sup>8</sup>See Fine (2012a, 2017b) on the relevant notion of grounding.

<sup>9</sup>Assuming CONVEXITY, Fine (2017b) originally provides the following condition (iii)\* in place of (iii), but then proves the equivalence of (ii) & (iii) and (ii) & (iii)\*:

(iii)\* For all  $\phi_i$ : There are no propositions  $Q_1, Q_2, \dots$  such that:  $|\psi| \wedge Q_1 \wedge Q_2 \wedge \dots$  entails  $|\phi_i|$ .

Here is a proof that the equivalence also holds if CONVEXITY is not assumed. First, note that the following holds:

(L) For all  $P, Q \in \mathcal{S}$ : If  $\sqcup P \not\sqsubseteq \sqcup Q$ , then there are no  $P_1, P_2, \dots \in \mathcal{S}$  such that  $P \wedge P_1 \wedge P_2 \wedge \dots$  entails  $Q$ .

Here is why: Let  $P_1, P_2, \dots \in \mathcal{S}$  and  $s_1 \in P_1, s_2 \in P_2, \dots$  be arbitrary. Now, we have that  $\sqcup P \sqsubseteq \sqcup P \sqcup s_1 \sqcup s_2 \sqcup \dots$ . So, if  $\sqcup P \not\sqsubseteq \sqcup Q$ , also  $\sqcup P \sqcup s_1 \sqcup s_2 \sqcup \dots \not\sqsubseteq \sqcup Q$ , and thus  $\sqcup P \sqcup s_1 \sqcup s_2 \sqcup \dots \notin Q$ . At the same time, we have that  $\sqcup P \sqcup s_1 \sqcup s_2 \sqcup \dots \in P \wedge P_1 \wedge P_2 \wedge \dots$ . Hence,  $P \wedge P_1 \wedge P_2 \wedge \dots$  does not entail  $Q$ .

(iii) & (ii)  $\rightarrow$  (iii)\*: Suppose that  $\sqcup|\phi_i| \sqsubset \sqcup|\psi|$ . This implies that  $\sqcup|\psi| \not\sqsubseteq \sqcup|\phi_i|$ . Then, by (L), there are no  $P_1, P_2, \dots$  such that  $|\psi| \wedge P_1 \wedge P_2 \wedge \dots$  entails  $|\phi_i|$ .

(iii)\* & (ii)  $\rightarrow$  (iii): By contraposition: Suppose that  $\sqcup|\phi_i| \not\sqsubseteq \sqcup|\psi|$ . This leaves us with two options: (a),  $\sqcup|\phi_i| \not\sqsubseteq \sqcup|\psi|$ , or, (b),  $\sqcup|\phi_i| = \sqcup|\psi|$ . If (a), by (L), there are no  $P_1, P_2, \dots$  such that  $|\phi_i| \wedge P_1 \wedge P_2 \wedge \dots$  entails  $|\psi|$ —pace (ii). If (b), by contrast, we get that  $|\psi| \wedge \{\sqcup|\phi_i|\}$  entails  $|\phi_i|$ —pace (iii)\*. To see the entailment, consider some arbitrary  $s \in |\psi| \wedge \{\sqcup|\phi_i|\}$ . Then,  $s = s' \sqcup \sqcup|\phi_i|$  for some  $s' \in |\psi|$ . Since  $s' \in |\psi|$ ,  $s' \sqsubseteq \sqcup|\psi| = \sqcup|\phi_i|$ . And therefore,  $s = s' \sqcup \sqcup|\phi_i| = \sqcup|\phi_i| \in |\phi_i|$ .  $\square$

out to be true. If  $|\phi| \notin \mathcal{M}(|\alpha|)$ , VERIFIABILITY fails, since, in this case,  $\Box_\alpha\phi$  has no verifiers. If  $|\phi| \in \mathcal{M}(|\alpha|)$ , by contrast, CONTAINMENT fails: We then have that  $\sqcup|\Box_\alpha\phi| = \sqcup\{\sqcup|\phi|\} = \sqcup|\phi|$ . Hence, the subject matter of  $\Box_\alpha\phi$  is identical to the subject matter of  $\phi$ , rather than being a proper part of it—as would be required for grounding.

In the remainder of the paper, however, I will show that proponents of ESSENCE GROUNDS PREJACENTS need not worry about this result: We have independent reasons to reject the proposed semantics for essence. The semantics allows us to derive results that are highly implausible, and incompatible with any common views on essence. These results suggest that the conditions that Zylstra provides for verifiers of essentialist statements are inadequate and that the semantics cannot capture essence in a convincing way.

## 2 The first objection

Let us start out by considering a set of plausibly satisfiable conditions on essence-making models. I will then show that, in any model  $M$  that meets these conditions, Zylstra’s semantics allows us to derive implausible grounding-claims:

(C1)  $M$  contains two states  $s_1, s_2 \in S$  which are not parts of one another, and some items  $a \subseteq \mathcal{I}$ .  $s_1$  is the only verifier of  $\phi_1$  and  $s_2$  the only verifier of  $\phi_2$ .  $\alpha$  designates  $a$ . Moreover,  $|\phi_1 \wedge \phi_2| \in \mathcal{M}(|\alpha|)$ .

Here is an example of a real-life case that would arguably exhibit this structure—that is, a case which, when implemented into the semantics, would yield an essence-making model that fulfils (C1): Let  $e$  be some electron. Like all electrons,  $e$  has a (rest) mass of  $m := 9.10938356(11) \cdot 10^{-32}$  kg, and a charge of  $c := -1.6021766208(98) \cdot 10^{-19}$ C. Let’s assume that it is essential to  $e$  that it has mass  $m$  and charge  $c$ . Let  $\phi_1$  stand for ‘ $e$  has mass  $m$ ’ and  $\phi_2$  for ‘ $e$  has charge  $c$ ’. Now, it seems plausible that the only states that should be taken to verify  $\phi_1$  and  $\phi_2$ , are the states of  $e$ ’s *having mass*  $m$ , and of  $e$ ’s *having charge*  $c$ , respectively.<sup>10</sup> Moreover, these two states plausibly are not parts of one

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<sup>10</sup>Note that if we were to assume that  $\phi_1$  and  $\phi_2$  express fundamental truths, we would automatically get the intended result that they have one single verifier each. This is since an ungrounded proposition can never have more than one verifier. To see this, consider an arbitrary proposition  $P$  with more than one verifier. Then,  $P$  will have  $\sqcup P$  as a verifier, plus at least one other verifier  $s \sqsubset \sqcup P$ .  $\{s\}$  entails  $P$  and  $\{s\}$ ’s subject-matter is a proper part of  $P$ ’s, so  $\{s\}$  grounds  $P$ . The assumption that  $\phi_1$  and  $\phi_2$  express fundamental truths, however, is not needed to set up a case of form (C1), because propositions with a single verifier can still fail to be fundamental. As an example, take, e.g. a proposition  $Q = \{s_1 \sqcup s_2\}$  with  $s_1$  and  $s_2$  non-overlapping. Clearly,  $Q$  has only one verifier and is grounded in the two propositions  $\{s_1\}$  and  $\{s_2\}$  taken together.

another. So, arguably, the case would be one which would be represented by essence-making models that fulfil the (C1)-conditions.

Now, let us return to the general abstract case of (C1). In any model  $M$  that fulfils (C1), Zylstra's semantics would yield the following result:

$$(R1) \ M \models \phi_1, \phi_2 < \Box_\alpha(\phi_1 \wedge \phi_2).$$

That is, Zylstra's semantics would imply that, in (C1) cases, the relevant essentialist statement with a conjunctive prejacent is fully grounded in the two conjuncts taken together.

*Proof.* For (R1), we need to show VERIFIABILITY, ENTAILMENT and CONTAINMENT.

VERIFIABILITY: Obvious for  $\phi_1$  and  $\phi_2$ . That  $\Box_\alpha(\phi_1 \wedge \phi_2)$  has a verifier follows from the fact that  $|\phi_1 \wedge \phi_2| = \{s_1 \sqcup s_2\} \in \mathcal{M}(|\alpha|)$ .

ENTAILMENT:  $|\phi_1 \wedge \phi_2| = \{s_1 \sqcup s_2\}$  and  $|\Box_\alpha(\phi_1 \wedge \phi_2)| = \{\llcorner |\phi_1 \wedge \phi_2|\} = \{\llcorner \{s_1 \sqcup s_2\}\} = \{s_1 \sqcup s_2\}$ . So, every verifier of  $\phi_1 \wedge \phi_2$  is also a verifier of  $\Box_\alpha(\phi_1 \wedge \phi_2)$ .

CONTAINMENT:  $\llcorner |\phi_1| = s_1 \sqsubset (s_1 \sqcup s_2) = \llcorner \{s_1 \sqcup s_2\} = \llcorner |\Box_\alpha(\phi_1 \wedge \phi_2)|$ . For the proper parthood between  $s_1$  and  $s_1 \sqcup s_2$ , recall that it was stipulated that  $s_1$  and  $s_2$  are not parts of one another. It follows that, since  $s_1 \sqcup s_2$  has  $s_2$  as a part while  $s_1$  does not,  $s_1 \sqcup s_2$  and  $s_1$  cannot be identical. The case of  $\llcorner |\phi_2| \sqsubset \llcorner |\Box_\alpha(\phi_1 \wedge \phi_2)|$  is entirely analogous.  $\square$

(R1), however, is obviously a result that we do not want our semantics to yield. Returning to our example of the electron, the mere two facts that electron  $e$  has mass  $m$  and that  $e$  has charge  $c$  taken together should not already provide us with a full ground of  $e$ 's having essentially mass  $m$  and charge  $c$ . While these two facts taken together do provide us with a metaphysical explanation for the fact that  $e$  has mass  $m$  and  $e$  has charge  $c$ , they do not explain that  $e$  has this mass and charge *essentially*. The essentialist truth is something that goes beyond the mere material truth. And thus, it asks for a different pattern of explanation. Plausibly, it requires either that different or at least further material be present in the explanans, or, alternatively, that the essentialist claim be considered a fundamental truth that demands for no metaphysical explanation in the first place. By ruling out both of these options in the case at hand, Zylstra's semantics turns out to be incompatible with all common accounts that have been proposed in the literature of essence thus far, be they primitive or reductive. For no account in the literature proposes that essentialist statements could be simply fully grounded in the grounds for their prejacent, and only few accounts would take them to be even partial grounds. Rather, commonly, reductive accounts take the grounds of essentialist statements

to include (the grounds for) its *necessitated* prejacent, plus further conditions such as naturalness or intrinsicity.<sup>11</sup>

(R1) thus suggests that Zylstra’s semantics cannot correctly capture the considered case—and, more generally, all cases that fulfil (C1)—and hence cannot provide a convincing account of essence.

### 3 The second objection

One way of seeing the objection that I have presented in the last section is this: Zylstra’s semantics provides conditions on verifiers of essentialist statements that are, in a sense, too easily satisfiable. States verify essentialist statements thus too easily, allowing us to infer unacceptable claims about the grounds of certain essentialist statements. The objection that I wish to present in this section might be seen as showing that the verification-conditions proposed by Zylstra are in another respect too strong. This leads to implausible entailments from essentialist statements to other statements and ultimately allows us to derive further unacceptable results.

More precisely, the problem is this: Zylstra’s semantics has it that whatever verifies an essentialist statement with an embedded disjunction also verifies the corresponding conjunction. That is, in every model  $M$ :

$$(R2) \quad \Box_\alpha(\phi \vee \psi) \text{ entails } \phi \wedge \psi.$$

*Proof.* We show that  $\bigsqcup|\phi \vee \psi| = \bigsqcup|\phi| \sqcup \bigsqcup|\psi|$ . (R2) then directly follows: On Zylstra’s semantics, the only potential verifier of  $\Box_\alpha(\phi \vee \psi)$  is  $\bigsqcup|\phi \vee \psi|$ . Moreover, due to CLOSURE UNDER FUSION,  $\bigsqcup|\phi|$  verifies  $\phi$ , and  $\bigsqcup|\psi|$  verifies  $\psi$ . Hence,  $\bigsqcup|\phi| \sqcup \bigsqcup|\psi| = \bigsqcup|\phi \vee \psi|$  verifies  $\phi \wedge \psi$ .

Note first that, for every  $T \subseteq S$ ,  $T_f^c = \{s \mid s' \sqsubseteq s \sqsubseteq \bigsqcup T \text{ for some } s' \in T\} =: U$ . It is evident that  $U$  contains all elements of  $T$ , is convex and closed under fusion. To see that it is also minimal in this respect, let  $\hat{T}$  be some arbitrary set that (i) contains all elements of  $T$ , is (ii) convex and (iii) closed under fusion. Due to (i) and (iii),  $\hat{T}$  contains  $\bigsqcup T$ . And due to this, (i) and (ii),  $\hat{T}$  must contain any state that lies between some element of  $T$  and  $\bigsqcup T$ , i.e., all elements of  $U$ .

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<sup>11</sup>For reductive accounts, see, e.g. Cowling (2013), Denby (2014) and Wildman (2013). Depending on one’s views on reductions, one may want to distinguish between accounts that are reductive in the strict sense—that is, provide us with a reductive analysis/real definition of essentialist statements—and accounts that merely state the grounds for essentialist statements. Arguably, however, reductive analysis/real definition requires (at least conceptual) grounding (cf. Rosen 2010, 2015; Fine 2015; Correia 2017).



We thus have that, for every  $T \subseteq S$  and  $s \in T_f^c$ ,  $s \sqsubseteq \sqcup T$ . And since  $\sqcup T_f^c$  is part of every state that has all elements of  $T_f^c$  as parts, we get that  $\sqcup T_f^c \sqsubseteq \sqcup T$ . Moreover, clearly,  $\sqcup T \sqsubseteq \sqcup T_f \sqsubseteq \sqcup T_f^c$ . Thus  $\sqcup T_f^c = \sqcup T_f = \sqcup T$ . Hence,  $\sqcup |\phi \vee \psi| = \sqcup \{s \mid s \in |\phi| \text{ or } s \in |\psi|\}^* = \sqcup \{s \mid s \in |\phi| \text{ or } s \in |\psi|\}$ .

What remains to be seen is that  $\sqcup |\phi| \sqcup \sqcup |\psi|$  is the fusion, i.e., the least upper bound of  $\{s \mid s \in |\phi| \text{ or } s \in |\psi|\} =: V$ . That is, (a), that every element of  $V$  is part of  $\sqcup |\phi| \sqcup \sqcup |\psi|$ , and, (b), that  $\sqcup |\phi| \sqcup \sqcup |\psi|$  is part of every state that has all elements of  $V$  as parts.

(a) Let  $s \in V$ . If  $s \in |\phi|$ , then  $s \sqsubseteq \sqcup |\phi| \sqsubseteq \sqcup |\phi| \sqcup \sqcup |\psi|$ . If  $s \in |\psi|$ , then  $s \sqsubseteq \sqcup |\psi| \sqsubseteq \sqcup |\phi| \sqcup \sqcup |\psi|$ .

(b)  $\sqcup |\phi| \in |\phi|$ , and hence  $\sqcup |\phi| \in V$ . Analogously,  $\sqcup |\psi| \in V$ .  $\sqcup |\phi|$  and  $\sqcup |\psi|$  are thus parts of every state that has all elements of  $V$  as parts. And since  $\sqcup |\phi| \sqcup \sqcup |\psi|$  is part of every state that has  $\sqcup |\phi|$  and  $\sqcup |\psi|$  as parts, (b) follows via the transitivity of parthood.  $\square$

But (R2)—that  $\Box_\alpha(\phi \vee \psi)$  entails  $\phi \wedge \psi$  in every model—is clearly a problematic result. Intuitively, given that the embedded sentence is *disjunctive*, a state that renders  $\Box_\alpha(\phi \vee \psi)$  true should not thereby automatically render the *conjunction*  $\phi \wedge \psi$  true. It should be possible for a state to verify a sentence of the former kind without verifying a sentence of the latter kind. To see this more clearly, let us again consider concrete cases. Zylstra himself provides an example of an essentialist statement with a disjunctive prejacent: It might be essential to the event of the 90th Academy Awards that Frances McDormand either won the best actress for Three Billboards or lost. Or, to borrow a different example from Glazier (2017), it might be essential to some binary Boolean variable in a computer program that it takes either value 0 or takes value 1. Clearly, both examples provide cases of disjunctive essence in which only one of the two disjuncts is actually the case. In fact, McDormand only won and did not also lose the prize. And, plausibly, our Boolean variable can have only either value 0 or value 1 at any given time, but not both of them. Schematically, we are thus confronted with cases in which we have that:

$$(C2) \quad \Box_\alpha(\phi \vee \psi) \text{ is true and } \phi \wedge \psi \text{ is false.}$$

And thus, Zylstra's semantics is intuitively in tension with the two example cases: A state that verifies the *true* essentialist claims in these cases should not automatically also verify the *false* corresponding conjunctive claims. But (R2) would demand precisely that.

To show that Zylstra's semantics is strictly incompatible with the two cases, however, a little bit more has to be said. For what would be needed for this would not be our previous result (R2), but, rather, the slightly different result that in every model  $M$ :

(R2\*) If  $\Box_\alpha(\phi \vee \psi)$  is true, so is  $\phi \wedge \psi$ .

The combination of Zylstra’s and Fine’s semantics provides us exclusively with truth-conditions for statements of ground. For the relevant sentences  $\Box_\alpha(\phi \vee \psi)$  and  $\phi \wedge \psi$ , by contrast, we are merely provided with verification-conditions. Thus, as yet, (R2\*) does not even enter the picture. It is straightforward to see, however, that we would get (R2\*) as soon as we were to devise truth-conditions for the relevant sentences in the standard way, based on the assumed verification-conditions for these sentences. For this aim, we would have to include a distinguished set of *obtaining* states in every model, and let a sentence be true in a model iff the model contains an obtaining verifier of the sentence.<sup>12</sup> And, clearly, since (R2) dictates that every verifier of  $\Box_\alpha(\phi \vee \psi)$  is also a verifier of  $\phi \wedge \psi$ , a model could not include an obtaining verifier of the former claim without also including one of the latter claim. That is, (R2\*) would follow from (R2). Hence, on the natural way of construing truth-conditions out of verification-conditions, Zylstra’s semantics turns out to be strictly incompatible with any cases with the same structure as those of the 90th Academy Awards and the Boolean variable—cases that seem perfectly coherent, and that are acknowledged by Zylstra himself.<sup>13</sup>

## 4 Conclusion

Taking stock, Zylstra’s semantics allows us to derive untenable results: As we have seen in Section 2, the semantics implies that, in certain classes of models, conjunctive essentialist statements are fully

<sup>12</sup>See Correia (2010) for an account to this effect. Note that we clearly could not adopt the simpler truth-conditions that a sentence is true in a model iff the model contains a verifier of the sentence. Our models have to include non-obtaining and even inconsistent states in order for them to yield the correct verdicts for grounding-statements. And, clearly, the existence of an inconsistent state that verifies a statement should not guarantee the truth of the statement.

<sup>13</sup>One might wonder whether it would be a theoretical option for Zylstra to bite the bullet and maintain that these standard truth-conditions simply could not be added to his framework. In particular, one might wonder about the following alternative condition in the case of essentialist statements:  $M \models \Box_\alpha \phi$  iff  $|\phi| \in \mathcal{M}(|\alpha|)$ . However, separating the truth-conditions of essentialist statements in this way from their verification-conditions is in tension with the guiding idea of truthmaker semantics. Moreover, it is also possible to derive implausible results from (R2) without relying on the addition of further truth-conditions (although in a slightly less direct way), and rejecting this addition would thus not suffice to eliminate the problem. We have the following as a consequence of (R2) in every model  $M$ :

(R2\*\*) For any verifiable  $\phi, \psi, \mu$  with  $\sqcup|\mu| \not\sqsubseteq \sqcup|\phi \vee \psi|$ : If  $|\phi \vee \psi| \in \mathcal{M}(|\alpha|)$ , then  $M \models (\Box_\alpha(\phi \vee \psi)) < ((\phi \wedge \psi) \vee \mu)$ .

*Proof.* VERIFIABILITY is clear, and ENTAILMENT a direct consequence of (R2). CONTAINMENT: ENTAILMENT gives us that  $\sqcup|\Box_\alpha(\phi \vee \psi)| \sqsubseteq \sqcup|(\phi \wedge \psi) \vee \mu|$ . Moreover, we have that  $\sqcup|(\phi \wedge \psi) \vee \mu| = \sqcup|\phi \wedge \psi| \sqcup \sqcup|\mu|$  (cf. the proof for (R2)), and hence that  $\sqcup|\mu| \sqsubseteq \sqcup|(\phi \wedge \psi) \vee \mu|$ . Since  $\sqcup|\mu| \not\sqsubseteq \sqcup|\phi \vee \psi| = \sqcup|\Box_\alpha(\phi \vee \psi)|$ , it follows that  $\sqcup|\Box_\alpha(\phi \vee \psi)| \not\sqsubseteq \sqcup|(\phi \wedge \psi) \vee \mu|$ .  $\square$

(R2\*\*) would e.g. yield the result that it’s being essential to the Boolean variable that it has value 0 or has value 1 fully grounds the following: The variable has value 0 and the variable has value 1, or snow is purple. And this is obviously an untenable result. The essentialist truth is not enough to guarantee the truth of the conjunction, and it is entirely irrelevant for the added disjunct.

grounded in the two conjuncts taken together. Thus, we might, for example, get the result that the two facts that some electron has mass  $m$  and that it has charge  $c$  taken together provide us with a full ground of its having essentially mass  $m$  and charge  $c$ . And as we have seen in Section 3, essentialist statements with a disjunctive prejacent entail the conjunction of the disjuncts. Under the standard way of construing truth-conditions out of verification-conditions, this renders it impossible that some disjunctive essentialist statement could ever be true and yet one of the disjuncts be false, thereby ruling out cases such as that of the 90th Academy Awards and the Boolean variable.

The obvious question to ask at this point is this: Are there any ways of modifying the specific way in which things are set up in Zylstra’s semantics, while still preserving its core underlying idea—viz., the combination of the essence-making function with independent verification-conditions for the essentialist claims? What might naturally come to mind would be to identify the verifiers of  $\Box_\alpha\phi$  simply with those of  $\phi$  if  $|\phi| \in \mathcal{M}(|\alpha|)$ . However, clearly, such a modification would not help us with the first objection as discussed in Section 2 and, in fact, make matters even worse: On the proposed modification, we would get that, for any model in which  $|\phi| \in \mathcal{M}(|\alpha|)$ , whatever grounds  $\phi$  also grounds  $\Box_\alpha\phi$ . So this modification looks like a clear non-starter. And it is hard to see how else one could modify the condition so as to circumvent the problems raised. For instance, taking the verifiers of  $\Box_\alpha\phi$  to be all states that are parts of the subject-matter of  $\phi$  would obviously not allow us to solve the first problem and, rather, give rise to results that are even worse than those of the accounts previously discussed. For, then, even more states would count as verifiers of  $\Box_\alpha\phi$ , and so it would be even easier to find counterexamples.

Thus, neither the semantics originally proposed by Zylstra, nor the modifications that would naturally come to mind can provide us with a convincing account of essence. While I cannot prove that there are no other options at Zylstra’s disposal, I think that we have strong reasons to be sceptical. This suggests that the source of the difficulties goes deeper than the specific details of Zylstra’s account: The very strategy of pairing an essence-making function with independent verification-conditions for essentialist statements looks unpromising, and it seems that we have to look elsewhere for a truthmaker semantics for essence.

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