7.2

Comments on 'Barriers to Implication'

Peter B. M. Vranas

1. Introduction

I was quite excited when I first read Restall and Russell's 'Barriers to Implication', for two reasons. First, because the chapter provides rigorous formulations and formal proofs of *implication barrier theses*, namely 'theses [which] deny that one can derive sentences of one type from sentences of another'. Second (and primarily), because the chapter proves a general theorem, the *Barrier Construction Theorem*, which unifies implication barrier theses concerning four topics: generality, necessity, time, and normativity. After thinking about the chapter, I am satisfied with its treatment of the first three topics, namely generality, necessity, and time. But I am not satisfied with its treatment of normativity, so my comments are exclusively on that topic.

My comments are divided into three parts. First, I go over Hume's Law – and by 'Hume's Law' throughout this chapter I refer to what Restall and Russell call the 'Normativity Formulation of Hume's Law'. Second, I raise two problems with Hume's Law: a technical problem and a substantive one. Third, I address two responses that Restall and Russell might make to the substantive problem.

2. Hume's Law

Hume's Law is the (is/ought) thesis that *no satisfiable (i.e., consistent) set of descriptive sentences entails a normative sentence.* This may look simple enough, but the terms 'descriptive' and 'normative' are given precise, technical definitions – which, except when I specify otherwise, I adopt throughout this chapter – by Restall and Russell, and it will take some work to explain these definitions. First, a sentence A is by definition *descriptive* exactly if it is *preserved* under normative translations; in other words, for every model M that satisfies A, every normative translation of M also satisfies A. (I will explain in a moment what a normative translation is.) Second, a sentence

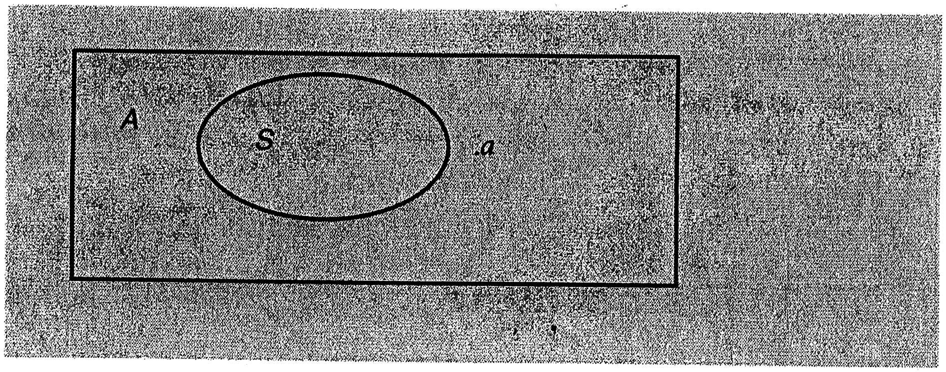

Figure 1 A model (and a sentence *A*).

A is by definition *normative* exactly if it is *fragile* under normative translations or extensions; in other words, for every model *M* that satisfies *A*, some normative translation or extension of *M* does not satisfy *A*. To explain these definitions, I will use a diagram.

In Fig. 1, every point in the shaded area represents a world of a model under consideration. The point labelled *a*, in particular, represents the actual world (of the model). The rectangle represents the set of worlds at which a particular sentence *A* is true, and thus represents the sentence *A*. The ellipse represents the set *S* of morally satisfactory worlds (of the model). Formally, a model is an ordered triple <*W*, *S*, *a*>, where *W* is a set of worlds, *S* is a subset of *W*, and *a* is a member of *W*.[1] A sentence *A* is by definition *obligatory* (in a model) exactly if it is true at *every* morally satisfactory world (of the model); in other words, exactly if the set *S* of morally satisfactory worlds is included in the set *A* of worlds at which the sentence is true (as in Fig. 1). Moreover, *A* is by definition *permissible* (in a model) exactly if it is true at *some* morally satisfactory world (of the model); in other words, exactly if the sets *S* and *A* have a non-empty intersection (as in Fig. 1). These definitions of what it is for a sentence to be obligatory or permissible correspond to definitions that are pretty standard in deontic logic.

A *normative translation* of a model by definition 'changes' the set *S* of morally satisfactory worlds (but keeps the set of worlds unchanged). For example, *S* may 'move' from inside to outside the rectangle, as in Fig. 2. Note that in such a case *A* 'remains' true, because the actual world remains inside the rectangle. So *A* is *preserved* under normative translations and is thus descriptive. On the other hand, the sentence '*A* is permissible' may 'become' false, since if the new set *S* is outside the rectangle its intersection with *A* is empty (as in Fig. 2). So '*A* is permissible' is *fragile* under normative translations and is thus by definition a normative sentence.

I hope it is now clear what a normative translation is. A normative *extension*, by contrast, adds new worlds to the model, including possibly some new morally satisfactory worlds. So if one starts with the same model as

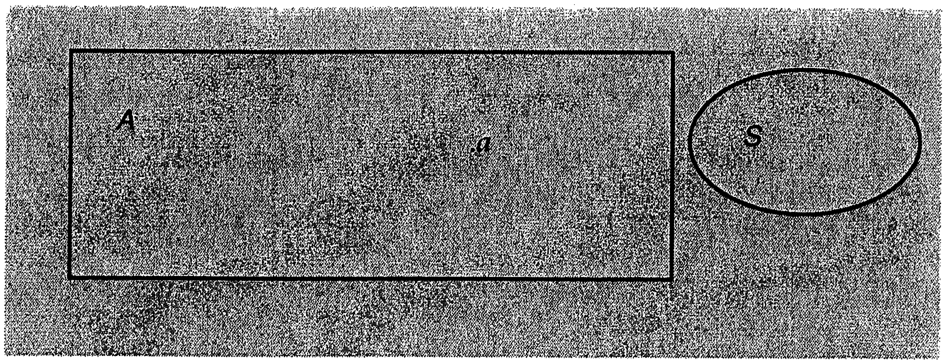

Figure 2 A normative translation of the model in Figure 1.

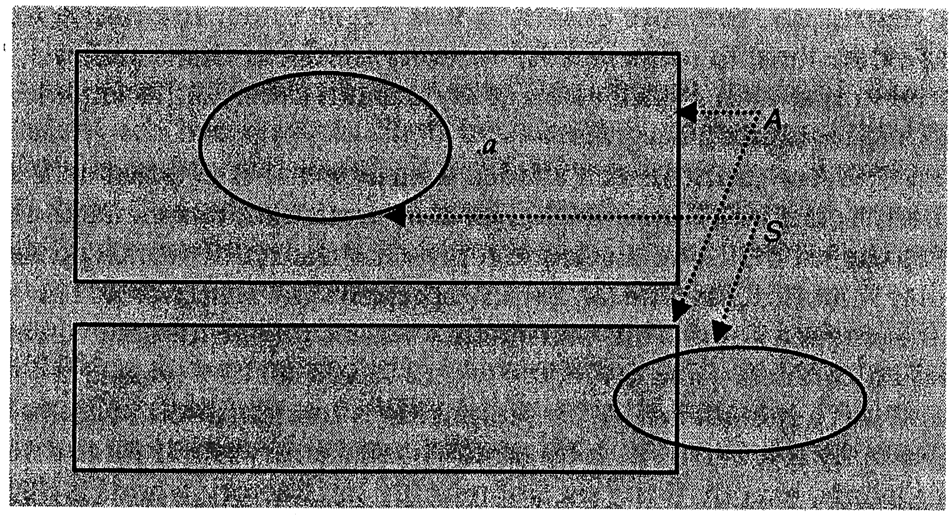

Figure 3 A normative extension of the model in Figure 1.

before (Fig. 1), Fig. 3 gives a normative extension of the model. The new shaded area represents the additional worlds, and the new ellipse represents those of the additional worlds that are morally satisfactory. The new rectangle represents those of the additional worlds at which the sentence *A* is true. Note that, if the sentence '*A* is permissible' is true in a model, it remains true in every normative extension of the model: if *S* and *A* have a nonempty intersection, as in Fig. 1, the intersection remains nonempty no matter what new worlds one adds. So the sentence '*A* is permissible' is preserved under normative extensions. Nevertheless, as we saw, the sentence '*A* is permissible' is fragile under normative *translations*, and this is enough to make it a normative sentence, because recall that by definition a sentence is normative if it is fragile under normative translations *or* extensions. Now if the sentence '*A* is obligatory' is true in a model, it may become false in a normative extension of the model, as in Fig. 3: some of the new morally satisfactory worlds are not inside *A*. So '*A* is obligatory' is fragile under normative extensions and is thus also a normative sentence.

After giving this rather extensive background, I turn to the second part of my comments, in which I raise two problems with Hume's Law.

3. Two problems with Hume's Law

The first problem I wish to raise is that Hume's Law is false because the (set consisting of the) descriptive sentence '*A* is necessary' (which for some *A* is consistent) entails the normative sentence '*A* is obligatory'. First, the sentence '*A* is necessary' is descriptive because it is preserved under normative translations: its truth-value cannot change just by changing the set of morally satisfactory worlds. Second, as we just saw, the sentence '*A* is obligatory' is normative because it is fragile under normative extensions. Finally, here is why the sentence '*A* is necessary' entails the sentence '*A* is obligatory': for every model which satisfies '*A* is necessary' (i.e. every model at whose actual world '*A* is necessary' is true), *A* is true at every possible world of the model, and is thus true at every morally satisfactory world of the model, so the model also satisfies '*A* is obligatory'.[2]

But now one may ask: how can Hume's Law be false if it is a special case of the general Barrier Construction Theorem that Restall and Russell prove? Is the general theorem also false? No. It turns out that Hume's Law is the *only* formal implication barrier thesis in Restall and Russell's chapter that does not follow from the general theorem (plus definitions) *alone*: to prove Hume's Law Restall and Russell also use a lemma, which they call 'Lemma 26', and the problem is with that lemma. More specifically, Lemma 26 amounts to the claim that every descriptive sentence is preserved under normative extensions, and this claim is false because the sentence '*A* is necessary' is descriptive (as we just saw) but is not preserved under normative extensions (because *A* may be false at some possible world that the extension adds to the original model).

Restall and Russell might respond that my criticism is not quite fair, because in the part of their chapter that deals with Hume's Law they consider a formal language that does not contain a necessity operator, so there is no such sentence as '*A* is necessary' in that language. But then, I reply, they need to find a way to avoid my counterexample in a richer language that does contain a necessity operator. Maybe they can do this; for example, Gillian Russell (in conversation) suggested that she might contest an (implicit) assumption that I have made so far and that I have used in arguing that '*A* is necessary' entails '*A* is obligatory', namely the assumption that every morally satisfactory world is a possible world. I find this way out unattractive: I find it implausible to say that some morally satisfactory world is not a possible world. Moreover, although by rejecting the above assumption Restall and Russell would avoid my counterexample, they would still need a proof of Hume's Law in a language that contains a necessity operator. But let me not dwell on this issue, because to my mind this first, technical problem with Hume's Law is minor in comparison with the second, substantive problem that I will raise shortly.

Before I raise the substantive problem, I need to go over an objection to Hume's Law, an objection patterned after a classic objection (to implication barrier theses) due to Arthur Prior (1960, p. 202). Take a descriptive sentence D and a normative sentence N. Consider their disjunction, $D \vee N$. Is the disjunction normative or descriptive? If it is normative, then Hume's Law is false because the descriptive sentence D entails the normative sentence $D \vee N$. On the other hand, if the disjunction is descriptive, then Hume's Law is false because the descriptive sentence $D \vee N$, in conjunction with the negation of D (this negation is descriptive;[3] assume that it is compatible with $D \vee N$), entails the normative sentence N. Therefore, in either case, Hume's Law is false – or so the objection goes. Restall and Russell reply in effect that the objection relies on a false dichotomy: the disjunction may be *neither* descriptive *nor* normative. In fact, let me now explain why such a disjunction may be neither descriptive nor normative.[4]

Consider the disjunction of a descriptive sentence A with the normative sentence 'A is obligatory'. Take first a model in which A is false but 'A is obligatory' is true, so that the disjunction is true. Consider a normative translation of that model in which one moves the set of morally satisfactory worlds from inside to outside A. Then 'A is obligatory' becomes false and A itself remains false, so the disjunction becomes false. So the disjunction is not preserved under normative translations and is thus by definition not descriptive. Take next a model in which A is true, so that the disjunction is true. Then A, and thus also the disjunction, is true in every normative translation or extension of that model.[5] So the disjunction is not fragile under normative translations or extensions and is thus by definition not normative.

Now I come to my main point. Let me for a moment switch to using the terms 'descriptive' and 'normative' in their everyday senses (rather than in Restall and Russell's technical senses). Is it plausible to say that the disjunction of a descriptive with a normative sentence is neither descriptive nor normative? In some cases it is indeed plausible. For example, consider the sentence: 'The sky is blue or you should not lie'. It seems indeed that this is a 'mixed' sentence, neither descriptive nor normative. But in other cases it is not plausible. For example, consider the sentence: 'If Jane is a citizen, she ought to vote' (in obvious notation: $Cj \rightarrow OVj$). This is a paradigmatically moral sentence, and yet, because the material conditional is equivalent to a disjunction, the sentence is neither descriptive nor normative on Restall and Russell's definitions of 'descriptive' and 'normative' (to which I switch back from now on). Similarly, consider the sentence: 'Every citizen ought to vote'. This is a universally quantified sentence (in obvious notation: $\forall x(Cx \rightarrow OVx)$), but (again, because the material conditional is equivalent to a disjunction) one can adapt my reasoning in the previous paragraph to show that the sentence is neither descriptive nor normative. Similarly for this sentence: 'No student may cheat'. These sentences, and many others like them, are paradigmatically moral, and yet they are not normative according to Restall and Russell's definition of the term 'normative'. Nor are they descriptive, so they are excluded from the scope of Hume's Law.

Why is this a major problem? Because we *want* a law which says that such paradigmatically moral sentences are *not* entailed by descriptive sentences. So Hume's Law as formulated by Restall and Russell is not a sufficiently general or sufficiently powerful implication barrier thesis. Here is another way to put the point. In the beginning of their chapter, after they explain Prior's objection to implication barrier theses, Restall and Russell say that a standard reaction to such objections is to retreat to a *weakened* implication barrier thesis; an example would be a thesis to the effect that moral sentences like '*A* is obligatory' (rather than moral sentences in general) are not entailed by descriptive sentences. Restall and Russell propose to do better. In a sense they in fact do better, because it turns out that Hume's Law is not only about sentences like '*A* is obligatory', but is also about sentences like '"*A* is obligatory" is permissible'. But such sentences are of limited usefulness, so Restall and Russell do only marginally better. The fact that Hume's Law is silent about sentences like 'Every citizen ought to vote' shows that Hume's Law is too far away from being a sufficiently strong implication barrier thesis. So, contrary to their announced intentions, Restall and Russell have in effect also retreated to a weakened implication barrier thesis. This is what I take to be the major problem with their chapter.

I turn now to the third and final part of my comments, in which I address two responses that Restall and Russell might make to the problem I just raised.

4. Two responses to the second problem

First, Restall and Russell might defend their definition of the term 'normative' by means of the intuitive motivation that they provide for 'the idea that normative sentences are fragile over changes in situation through which descriptive sentences are preserved'. They start with a situation in which it is obligatory that Alice refrains from hitting Bob. Then they 'extend' (in an informal sense) the situation to one in which Alice and Bob are training for a boxing tournament, so that it is *not* obligatory that Alice refrains from hitting Bob. Finally, they extend further the situation to one in which 'a suicidal anti-boxing protester... has informed Alice that she will kill all three of them if Alice hits Bob', so that it *is* again obligatory that Alice refrains from hitting Bob.

I have two replies. First, even if this example succeeds in motivating Restall and Russell's definition of the term 'normative' for *obligation* sentences, it is irrelevant to *permission* sentences. This is because the example is only about *extensions* (informally), but as we saw, permission sentences are supposed to be *preserved* under normative extensions; they are supposed to be fragile under normative *translations*. Similarly, the example does not succeed in motivating the understanding of *mixed* sentences as non-normative, which was the source of the substantive problem that I raised for Hume's Law. Second, in the example we have a sentence, namely 'Alice refrains from hitting Bob', which is *not* obligatory in a model but becomes obligatory in an informal extension of the model. But according to the formal definition of a normative extension this cannot happen: if '*A* is obligatory' is false in a model then some morally satisfactory worlds are outside *A*, and these remain outside no matter what *other* worlds we add to the model, so '*A* is obligatory' is false in *every* normative extension of the model. I conclude that Restall and Russell's example *cannot* motivate their definition of the term 'normative' because it fails to capture the formal properties of their concept of a normative extension of a model.

Here is a second response that Restall and Russell might make to the substantive problem that I raised with Hume's Law. Sentences like 'Every citizen ought to vote', although admittedly moral, *must* be excluded from *any* is/ought thesis because they follow trivially from paradigmatically non-moral sentences. To adapt an example due again to Arthur Prior (1960, p. 202), the paradigmatically non-moral sentence 'No one is a citizen' entails 'Every citizen ought to vote'. Similarly, 'No one is a student' entails 'No student may cheat'. And so on. Therefore, Restall and Russell might conclude, although Hume's Law is admittedly not as general as one might wish, it is the best one can do: Prior's example shows that it is impossible to find an is/ought thesis that covers all sentences we want such a thesis to cover.

I reply that one *can* do better. To start with, notice that in Prior's example the entailment is in a sense 'vacuous'. To see this, notice that the sentence 'No one is a citizen' entails not only 'Every citizen ought to vote', but also entails 'No citizen ought to vote', also entails 'Every citizen ought to refrain from voting', also entails 'Every citizen ought to murder', and so on. So the idea is to try to formalize a distinction between vacuous and non-vacuous entailment, and to formulate an is/ought thesis as follows: no non-moral sentence *non-vacuously* entails a moral sentence. There has been some relatively recent formal work in this direction, and I particularly recommend the book *The Is-Ought Problem*, by Gerhard Schurz (1997a).[6] So I would like to close on a positive note: there is indeed hope for formulating and proving a sufficiently powerful is/ought thesis.

In conclusion let me make clear that, despite the problems that I raised with Restall and Russell's chapter, I do think it is a great contribution, and I was glad to have the opportunity to think about it.

Notes

This chapter is based on a presentation that I made at the Second Formal Epistemology Workshop on 27 May 2005; for the most part, the writing preserves the informal style of the presentation. I am grateful to Alan Hájek, Aviv Hoffmann, Greg Restall, and Gillian Russell for comments. Thanks also to Jason Alexander, Stephen Darwall, and James Pryor for help, and to my mother for typing the bulk of the chapter.

1. Strictly speaking, Restall and Russell take *S* to be a binary *relation* on *W*, not a *subset* of *W*; what I call 'morally satisfactory worlds' are the worlds that are morally satisfactory *with respect to the actual world*. Nothing of substance hangs on my simplification, which I adopt to make the exposition easier.
2. One might contest the claim that every necessary sentence is obligatory (think, e.g., of '2 + 2 = 4'), and in fact there is a relevant debate in deontic logic (see, e.g. Åqvist, 2002, pp. 156–7; Bailhache, 1991, pp. 17–19, 23–4; von Wright, 1951, pp. 10–11). This debate, however, does not matter for present purposes, since my point is that '*A* is necessary' entails '*A* is obligatory' *given* the definition of '*A* is obligatory' in §2.
3. Assuming that a model does not satisfy a sentence exactly if it satisfies the negation of the sentence, it can be shown that a sentence is descriptive exactly if its negation is descriptive.
4. The explanation that follows in the text is similar to (and was inspired by) Restall and Russell's explanation of why a disjunction of a 'semantically particular' sentence with a 'semantically universal' one may be neither semantically particular nor semantically universal.
5. Strictly speaking, in order to be true in every *extension* of the model, *A* must be unlike those descriptive sentences which (as we saw in the case of '*A* is necessary') are not preserved under normative extensions.
6. See also Schurz (1991a, 1994, and chs 7.1 and 7.3 in this volume). On similar proposals see Jackson (1974), Kurtzman (1970, pp. 497–8), Morscher (1984, pp. 430–1), Pigden (1989, pp. 133–7, and chs 5.2 and 7.2 in this volume), Prior (1960, reprinted in this volume as ch. 1.1), and Shorter (1961, reprinted in this volume as ch. 1.2); cf. Black (1964, p. 168) and Kurtzman (1970, pp. 493–4).

Hume on *Is* and *Ought*

Edited by

Charles R. Pigden
University of Otago, New Zealand

palgrave
macmillan