Conditionals, Modals, and Hypothetical Syllogism

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Moti Mizrahi (2013) presents some novel counterexamples to Hypothetical Syllogism (HS) for indicative conditionals. I show that they are not compelling as they neglect the complicated ways in which conditionals and modals interact. I then briefly outline why HS should nevertheless be rejected.

Do natural language conditionals validate Hypothetical Syllogism (HS)? Let us assume that indicative and counterfactual conditionals can be regimented by means of the binary propositional connectives, $\rightarrow$ and > respectively, so that 'if A, B' is regimented as A $\rightarrow$ B, and 'if A had been the case, C would have been the case' is regimented as A > C.[1] Our question then is, are the following schemas valid

Indicative HS: $((A \rightarrow B) \wedge (B \rightarrow C)) \supset (A \rightarrow C)$

Counterfactual HS: $((A > B) \wedge (B > C)) \supset (A > C)$?

Moti Mizrahi (2013) has recently presented a new type of putative counterexample to Indicative HS. In §1 I outline Mizrahi's case against Indicative HS. I then show (§2) that considerations similar to the ones Mizrahi adduces can also be used to present a case against Modus Ponens. This is uncomfortable for Mizrahi as he takes Modus Ponens to be valid. In any case, one might think that giving up Modus Ponens is a large cost to pay in order to reject Indicative HS. In §3 I argue that such logical revisionism is unwarranted

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[1] Angelika Kratzer rejects this assumption: "The History of the conditional is the story of a syntactic mistake. There is no two place *if … then* connective in the logical forms of natural languages. If-clauses are devices for restricting the domains of operators" (Kratzer 2013: 106). The assumption is harmless in the present context, however, since Kratzer can replicate what I say in §3 within her own framework.

because the counterexamples to Modus Ponens and Indicative HS considered below fail to take account of the way in which conditionals and modals interact. Finally (§4), I briefly outline why Hypothetical Syllogism is nevertheless invalid for both indicatives and counterfactuals.

1. Mizrahi's Counterexample to HS

Mizrahi (2013) presents some new-style counterexamples to Indicative HS, such as the following:

1. If I am in Boston at time t, then I am in a city whose name starts with the letter 'B' at time t.

2. If I am in a city whose name starts with the letter 'B' at time t, then I might be in Baltimore at time t.

Therefore,

3. If I am in Boston at time t, then I might be in Baltimore at time t.

Mizrahi claims that (1) and (2) are true, but that (3) is false, and then concludes that (1)-(3) thereby invalidate Indicative HS.

After discussing HS in general, Mizrahi (2013: 41) says that for the purposes of his paper he'll focus on Indicative HS. This suggests that he thinks there are similar counterexamples to Counterfactual HS. Perhaps, for example, Mizrahi would be happy to offer the following as a counterexample to counterfactual HS:

4. If, at time t, I had been in Boston, then I would have been, at time t, in a city whose

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name starts with the letter 'B'.

5. If, at time t, I had been in a city whose name starts with the letter 'B', then I might have been, at time t, in Baltimore.

Therefore,

6. If, at time t, I had been in Boston, then I might have been, at time t, in Baltimore.

It seems as if there is a reading on which (4) and (5) are true and yet (6) is false, so perhaps Mizrahi's strategy extends to counterfactuals. But like Mizrahi, I'll focus on indicative conditionals, although what I say regarding (1)-(3) applies, mutatis mutandis, to (4)-(6) as well

As Mizrahi himself notes, advocates of HS have rejected previous putative counterexamples to HS on the grounds they involve an illicit change in context (cf. Wright, 1983). It is strange, then, that Mizrahi does not pause to consider whether there is a plausible contextualist response to his alleged counterexample. We can ignore such issues here, however, since I shall argue on independent grounds that Mizrahi's case against Indicative HS is unconvincing.

2. Modus Ponens

Mizrahi's suggested counterexamples to Indicative HS all essentially employ conditionals with modals in their consequents. It has long been known, however, that conditionals interact with modals in interesting ways, and that naïve treatments of conditionals containing modals are problematic.[2] For example, the following seems true:

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[2] See, for instance, Gillies (2010)

7. If Wanchen does kill her father, she should do it gently.

Now as it happens Wanchen does kill her father, so if Modus Ponens is valid, as Mizrahi (2013: 41) supposes, then it seems to follow from (7) that Wanchen should kill her father gently. But given that Wanchen's father is in good health, wants to live, and is a moral saint, it is not true that Wanchen should kill her father gently. From such simple thoughts we find ourselves with a contradiction![3]

Moreover, the phenomenon is not limited to conditionals containing deontic modals. Suppose that I know that Imran is either at work or in the park, but I don't know which, so that Imran might be at work and he might be in the park. It seems to follow from this that

8. If Imran is not at work, (then) he must be in the park.

As it happens Imran is not at work and so, by Modus Ponens from (8), Imran must be in the park. But this contradicts our earlier assumption that he might be at work![4]

These two examples show that conditionals such as (7) and (8) are inconsistent with the following three assumptions (given our background claims about what might be the case and what is permissible):

A. (7) and (8) have the logical form their grammar suggests, i.e. $(A \rightarrow B)$

B. Modus Ponens is valid for such conditionals

C. B means the same when it occurs in the consequent of (7)/(8) as it does when it has been detached from (7)/(8).

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[3] See Forrester (1984).
[4] See Gillies' (2010:13-14) discussion of Fact 1. I'm assuming here that the epistemic 'might' and 'must' are duals of one another, or at least that must P $\supset$ ~might ~P.

So independently of HS, we have reason to think that something strange is going on with conditionals containing modals. Until we have resolved this problem, it is reasonable to conclude that Mizrahi's example cannot bear the argumentative weight he places on it.

Nevertheless, that (7) and (8) are inconsistent with (A)-(C) does not directly tell against Mizrahi's rejection of Indicative HS. Moreover, one might respond to the examples above by giving up Modus Ponens. The bare case against Indicative HS that (1)-(3) present does not depend on the validity of Modus Ponens (although see Mizrahi's own reason for rejecting (3) below), so anyone convinced by Mizrahi's counterexample to Hypothetical Syllogism can reject Modus Ponens as well in response to the above puzzle.[5]

But many will think that giving up Modus Ponens in response to (7) and (8) is a large cost, even if they want to endorse Mizrahi's case against Indicative HS. Logical revisionism is not required to accommodate (7) and (8), however. Rather, we have reason to give up (C) instead. And once we realize this, we shall see why (1)-(3) can have the truth values that Mizrahi takes them to have, but that they nevertheless do not represent a counterexample to Indicative HS.

3. Embedded Modals

One simple-minded response to the inconsistency of (7) and (8) with (A)-(C) is to deny (A), and say that at the level of logical form, the modals in (7) and (8) take wide scope over the whole conditional. This certainly seems plausible because (7) and (8) seem true on such an interpretation. For example, (8) seems true because (given certain background facts) it is inconsistent with knowing that Imran finished work, that he be anywhere other than the park. But if the modals in (7) and (8) have wide scope, then we do not have a conditional which can be used in Modus Ponens, and so, for all we've said, (7) and (8) do not threaten Modus

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[5] Indeed, one may already reject Modus Ponens on independent grounds (see, for instance, Lycan, 2001, and McGee, 1985).

Ponens.

Similarly, if the modals in the consequents of (2) and (3) really take wide scope over the whole conditional, then (1)-(3) is not an instance of Indicative HS. On such an interpretation, (2) and (3) are not conditionals, but rather conditionals embedded within modals. So if the modals in (2) and (3) take wide scope, (1)-(3) do not represent a counterexample to Indicative HS, even granting that (1) and (2) are true, and that (3) is false. This is an especially acute problem for Mizrahi since (3) seems false because there are no epistemically possible worlds where I am in both Boston and Baltimore. On the other hand, (2) seems plausible because there are epistemically possible worlds where I am in a city beginning with 'B' and I am in Baltimore.[6] So unless we can reject that these modals take wide scope, Mizrahi's challenge to Indicative HS fails.

Now whilst the wide scope treatment of conditionals such as (2)-(3), (7), and (8) has some plausibility, Dorr and Hawthorne (Forthcoming) have argued that this account is not well motivated. For example, what should the wide scope theorist say about

9. If this is an animal, it might be zebra and it's probably hungry?

Wide-scoping 'might', 'probably', or both fails to give the correct result.[7] For this and many other reasons (see n10), Dorr and Hawthorne argue that the modals in the consequents of conditionals are naturally read as *constrained modals*. That is, the epistemic modals above do not concern what is epistemically possible tout court, but rather what is epistemically possible given the antecedent. (Similarly, the deontic modal in (7) concerns not what is required tout court, but only what is required given the antecedent.) So for Dorr and

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[6] See Gillies' (2010) discussion of his Fact 3, namely that 'if A, might B' and might (A and B) seem to be true in the same circumstances.
[7] See also Gillies' (2010) discussion of his Facts 2 and 3 which together, he argues, make trouble for the wide-scoping approach.

Hawthorne there are readings on which:

10. If P, must Q is equivalent to If P, then in all epistemically possible worlds in which P, Q

11. If P, might Q is equivalent to If P, then in some epistemically possible world in which P, Q.

So what (8) says is that if Imran is not at work, (then) in all epistemically possible worlds where he is not at work, he is in the park. Given the truth of the antecedent and Modus Ponens, this only allows us to conclude the unobjectionable must (Imran not at work $\supset$ he is in the park). So Dorr and Hawthorne deny (C), since on its own 'Imran must be in the park' is naturally read as an unconstrained epistemic modal, true iff Imran is in the park in all epistemically possible worlds. And we can tell a similar story about (7), namely that all we can conclude by Modus Ponens is that the best worlds in which Wanchen kills her father are those where she kills him gently. And unlike the claim that she ought to kill her father gently, this is not objectionable.[8]

Returning to Mizrahi's alleged counterexample to Indicative HS, Dorr and Hawthorne's theory predicts that on the constrained reading of the modals (2) is true and that (3) is false just as Mizrahi claims they are. According to Dorr and Hawthorne, the truth conditions of (2) and (3) are given respectively by

12. If I am in a city whose name starts with the letter 'B' at time t, then there is an epistemically possible world where I am in a city whose name starts with the letter 'B'

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[8] Of course, these observations about embedded modals don't show that Modus Ponens is valid, and don't speak to the counterexamples to Modus Ponens provided by McGee (1985) or Lycan (2001). Indeed, the considerations adduced here do not, on their own, explain away Kolodny and MacFarlane's (2010) counterexample employing conditionals with deontic modals in their consequents. But I am not here defending the validity of Modus Ponens in general, and nor does my rejection of Mizrahi's argument against Indicative HS depend on it. Nevertheless, I do think that Modus Ponens is valid for both indicatives and counterfactuals. I have argued elsewhere (Walters 2009, Walters and Williams 2013) that the truth of P and Q is sufficient for the truth of P > Q, and this thesis entails Modus Ponens for counterfactuals.

at time *t* and I am in Baltimore at time *t*,

and

13. If I am in Boston at time *t*, then there is an epistemically possible world where I am in Boston at time *t* and I am in Baltimore at time *t*.

We can see that (2) is true because (12) is true – if we're in a city beginning with 'B', then there is some epistemically possible world in which we're in a city beginning with 'B' and we're in Baltimore.[9] Similarly, (3) comes out as false, because (13) is false: there is no epistemically possible world in which we're in Boston and Baltimore at the same time, and being in Boston won't change this.

Despite the truth of (1) and (2), and the falsity of (3), Dorr and Hawthorne's theory doesn't license Mizrahi's conclusion that Indicative HS is invalid. This is because the proposition expressed by the modal in the consequent of (2) is not the same as the proposition expressed by the modal in the consequent of (3). In order to generate a counterexample to Indicative HS, what Mizrahi needs is not the falsity of (3), but rather the falsity of

14. If I am in Boston at time *t*, then there is an epistemically possible world where I am in a city whose name starts with the letter 'B' at time *t* and I am in Baltimore at time *t*.

But (14) seems true, at least given the presupposition required for the truth of (2), namely, that we don't know that we're in Boston. So if we adopt something like Dorr and Hawthorne's treatment of embedded modals, Mizrahi's argument against Indicative HS fails. And as Mizrahi has not considered such accounts, let alone argued against them, his putative

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[9] I take it that if we are in Boston or in Boise and we know we are, (2) has no plausibility.

counterexample to Indicative HS is far from decisive.[10]

Moreover, it seems that Mizrahi is not well-placed to reject a constrained reading of the modals in (2) and (3), as his case against Indicative HS seems to rely on it. Mizrahi's reason for rejecting (3) is as follows:

> If I am in Boston at time *t*, then it is not the case that I might be in Baltimore *at the same time*, since I am already in Boston at that time, and thus could not be in Baltimore *at the same time*. Once one is already in spatial location *L1* at time *t*, the window of *physical possibilities* is closed, as it were, for being in another spatial location *L2* at time *t*, since a human person cannot be in two different spatial locations simultaneously. If this is correct, then the antecedent of (3) is true [we might suppose] but the consequent is false, which means that (3) is false by [Modus Ponens] (Mizrahi, 2013: 42, my numbering).

But why think that the falsity of (3) follows from the alleged impossibility of a human person being in two places simultaneously? That is, why think being in Boston rules out it being the case that I *might* be in Baltimore? If we read (3) as a narrow-scoped unconstrained modal, as seems required to cause trouble for Indicative HS, then all that seems to follow from $\sim\Diamond(P \wedge Q)$ (and Modus Ponens) is $(P \rightarrow \sim Q)$, and not $(P \rightarrow \sim\Diamond Q)$ as Mizrahi claims. This is

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[10] Although I have chosen to illustrate the invalidity of Mizrahi's argument within Dorr and Hawthorne's framework, the same result can be replicated using Kratzer's (2013: Chapter 4) restricted quantifier approach, or with Gillies' (2009, 2010) context-sensitive strict conditional account. I chose Dorr and Hawthorne's framework because I am not convinced that indicative conditionals are epistemic or context-sensitive in the way that either Kratzer or Gillies supposes. Moreover, modals embedded within disjunctions and conjunctions, as well as within conditionals, can be treated by Dorr and Hawthorne's account of constrained modals. For example, as well as supporting (8), my minimal knowledge of the disjunction that Imran is either at work or in the park seems to entail (i) that either Imran is at work, or he must be in the park. Dorr and Hawthorne account for this by positing that the modal is constrained in such a way that 'he must be in the park' is true iff he is in the park in all epistemic worlds that are accurate with respect to whether he is at work. (i) is then equivalent to either Imran is at work or in every epistemically possible world where is not at work, he is in the park. And this is true. Kratzer's and Gillies' theories of conditionals, however, are silent on modals embedded within disjunctions and conjunctions. See Dorr and Hawthorne (Forthcoming) on this and other motivations for their account.

especially clear if we read the 'might' in (3) as expressing epistemic possibility, which seems overwhelmingly plausible. Of course, what Mizrahi seems to be claiming is that if I am in Boston at t then it is impossible for me to be in Baltimore at t *as well as* being in Boston at t. Granting Mizrahi his claim about multiple location, this is true, assuming that what is impossible doesn't become possible if I am in Boston. But why think that this is something that (3) is committed to on an unconstrained narrow scope reading of the possibility modal? On this reading, given its antecedent and Modus Ponens, (3) yields only

15. I am in Boston and I might be in Baltimore.

Now (15) may be Moore-paradoxical or otherwise infelicitous, but it does not prima facie entail that I can be located in two places simultaneously or any other putative impossibility. So, for all Mizrahi has said (3) is true, and so he has not made the case that (1)-(3) constitute a genuine counterexample to Indicative HS after all.

But let us grant, temporarily, that (15) is inconsistent as Mizrahi requires, so that, given Modus Ponens and the truth of the antecedent, (3) is false. In that case, Mizrahi still has no compelling counterexample to Indicative HS. This is because similar reasoning shows that (2) is false in those circumstances. That is, assuming I am in Boston, a city that begins with 'B', it follows from (2) by Modus Ponens that I might be in Baltimore, and this conjoined with the obvious fact about my location gives the allegedly inconsistent (15). So, if (15) is inconsistent, then (2) is false in the same circumstances in which we are granting (3) is false. So whether or not being in Boston is incompatible with it being the case that I might be in Baltimore, Mizrahi's objection to Indicative HS fails.

What Mizrahi needs to generate the falsity of (3) is not (15), but rather

16. I might be in Boston and in Baltimore.

If (3) entails (16), then (3) is committed to the epistemic possibility of my being in Boston and being in Baltimore at the same time. And we can concede that there is no such epistemic possibility. But (16) only follows from (3) if we accept something like Dorr and Hawthorne's constrained reading of the modal in (3). Now as we are treating (2) as true, the modal in (2) is not plausibly read as (16), and so the modal claims in (2) and (3) are distinct, as Dorr and Hawthorne predict. As a result, (1)-(3) do not represent an instance of Indicative HS, yet alone a counterexample to it. So the very reading that Mizrahi requires in order to account for the falsity of (3) is the same reading that prevents (1)-(3) invalidating Indicative HS. Mizrahi, then, has not provided a compelling new type of counterexample to HS.

4. Rejecting HS

Of course, the failure of Mizrahi's counterexample does not vindicate Hypothetical Syllogism. Indeed, Hypothetical Syllogism is invalid. Let me briefly illustrate this with respect to counterfactuals. One compelling thought about counterfactuals is that when the consequent is already true, and the antecedent is unrelated to the obtaining of the consequent, then (at least in deterministic contexts) the counterfactual is also true. As Pollock (1976: 26) observed, the conjunction 'it would not rain if he did not do a rain dance, but it would not rain if he did either' expresses the lack of a connection between his dancing and the weather. It is a short step in such cases to affirm something of the form $(A \lor \sim A) > C$, either because we think 'it would not rain if he did not do a rain dance or he did do a rain dance' expresses the requisite lack of connection, or because we are happy to infer this conditional from the previous two via the following standard theorem of counterfactual logic: $((A > C) \land (\sim A > C)) \supset (A \lor \sim A) > C$.

A second compelling thought about counterfactuals is that any counterfactual (with a possible antecedent) which has a necessary consequent is thereby true. So we also have B >

(A v ~A). Assuming the validity of Counterfactual HS, these two counterfactuals license B > C. So, from assuming that there is some true C and that there is some A of which it is independent, we have arrived at B > C for an arbitrary B. But in general the truth of a counterfactual's consequent is not sufficient for the truth of the counterfactual, and so Counterfactual HS must be rejected. And something similar can be said, I think, in the case of indicatives, since the two thoughts above are equally plausible for at least some indicatives. For example, assuming that witch doctors do not have the powers they claim and that it will rain, then it seems that it will not rain whether he does a rain dance or not, and that if B, he will do a rain dance or not. [11] This, then, and not Mizrahi's putative counterexample, is why Hypothetical Syllogism is invalid.[12]

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[11] See my Forthcoming for more details.
[12] Thanks to Matthew Bird, two referees, and an editor of *Thought* for helpful comments.

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