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Light and Mass

**Relativity and Absoluteness as Intrinsically Connected Moments of the
Principle of Kinematic Relativity**

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Relativity and Absoluteness as Intrinsically Connected
Moments of the Principle of Kinematic Relativity¹**

Abstract

Our starting point is the question ‘What is mass’, what in particular enables mass to constitute *duration*: so that mass can be regarded as moving as well as at rest (the kinematic principle of relativity). In a *thought experiment*, this question is attacked here not from the perspective of mass itself, but from that of a *standing light wave*. In this model, *mass-analogous structures* can be re-constructed that can be in *relative motion to each other*. The (empirically known) *constancy* of the speed of light in every reference system is not assumed here. However, it can be shown that the speed of light is *identical* in all systems constituted by mass-analogous structures that move uniformly relative to each other and that the speed of light moreover is the *limiting speed* of motions. This independence of the motion of light from the system of reference, this character of *absoluteness*, and the *relativity* of mass motions thus prove to be contrary but inextricably linked moments of the kinematic principle of relativity: an interesting perspective concerning a *philosophy of relativity theory*. - The mathematical relationships are explained in more detail in the appendix.

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¹ Translated from the German by E. Kummert.

1. Introduction

A central consequence of the relativistic turn in 1905 was the *non-relativity* of the motion of light, a truly highly irritating aspect. This is in fact paradoxical: In contrast to the classical concept of the relativity of motion, in which motion is always relative motion, the theory of relativity maintains that the velocity of light is *absolute*. ‘Relative’ thereby means ‘dependent on the respective frame of reference’ and ‘absolute’ accordingly ‘independent of any frame of reference’. This strange fact of an absolute velocity of light is already shown in the Maxwell-equation for the propagation of electromagnetic waves in a vacuum, in which the velocity of light c appears as a *constant* and so as independent of the frame of reference, whereas velocity, according to the classical concept, is really the prototype of a *relative* quantity. This confusing situation was for Einstein – quite independent of the negative result of the Michelson-Morley experiment – one of the central motives for the development of the special theory of relativity, whereby Einstein’s great accomplishment, last not least, consisted in his proving the relativity of ‘normal’ motion and the absoluteness of the motion of light to be mathematically compatible. From the point of view of natural philosophy this absoluteness character of of light remains, of course, a fact to be clarified.

In the following I would first like to develop some general considerations on this point and then discuss a thought experiment in this connection.

2. The Motion of Mass and the Motion of Light

I would like to take up an argument here which I have developed with respect to Hegel’s discussion of the concept of motion²: The *logic* of the concept of motion requires that something must exist that motion *refers* to and that as such is determined as *at rest*. ‘At rest’ means, however, that something remains identical in motion, i.e., rest itself is connected to motion from the very beginning. This *unity* of rest and motion, which is supposed to be empirically realized in the *mass* of a body, results, according to Hegel, from the logic of the concept of motion: “The mass is the unity of the moments of rest and motion; both are contained in it, or it is indifferent to both, capable of both motion and rest” (9.65 addition)³. Hegel brings this in further connection with the

² Wandschneider, D. (1982) Raum, Zeit, Relativität. Grundbestimmungen der Physik in der Perspektive der Hegelschen Naturphilosophie. Frankfurt/M. 1982; Wandschneider, D. (1986) Relative und absolute Bewegung in der Relativitätstheorie und in der Deutung Hegels, in: Horstmann, R.-P. / Petry, M.J. (ed. 1986) Hegels Philosophie der Natur. Stuttgart 1986.

³ Quotations of this kind refer to: Hegel-Werkausgabe, ed. E. Moldenhauer and K. M. Michel. Frankfurt/M. 1969 ff, here vol. 9, p. 65 addition. In this connection Hegel’s image of a rotating circular surface is also suggestive. This would be a motion that simultaneously includes rest – namely in the centre of the circle, which is “the restored concept of duration, the motion which is extinct in itself”; “the *mass* is posited, which ... shows motion as its possibil-

characteristic of *inertia* (ditto); but in the present connection only the kinematic aspect is to be considered: A mass can be considered as ‘indifferent’ to rest and motion and thus as well as resting and moving, and that is exactly the content of the classical *principle of kinematic relativity*.

Three things are accordingly important: (1) The principle of the relativity of motion does not simply say that motion is always relative motion – in this form it would even be empirically false in respect to the motion of light. Rather, it contains the statement that a moving *mass* can be considered as at rest with the same right, and vice versa a resting one as moving. The principle of kinematic relativity formulates thereby, understood correctly, the relativity of the motion of *mass*. Thus: Motion can only exist relative to a resting instance of reference, but the *principle of kinematic relativity* maintains, in contrast, that the relation of rest and motion is *symmetrical*, insofar as we are dealing with the relation of the motion of *masses*. To put it differently, the *kinematic equivalence of all masses* is thus expressed.

(2) For this it is obviously decisive that a mass can always be considered as resting, too. The kinematic equivalence of all masses expressed in the principle of kinematic relativity is, therefore, based essentially on the *ability of the mass to be ‘at rest’* and, as such, to be a possible reference instance of motion. *How* a mass can be at rest seems to me to be one of the open questions in physics today⁴.

(3) When the principle of kinematic relativity says that the motion of mass is equivalent to relative motion, then this also implies that the motion of a *non-mass* – whatever that may be – must be a *non-relative* motion. This is a necessary, but not trivial consequence of the presented interpretation of the principle of relativity: The possibility of a *non-relative* motion is accordingly essentially *contained* in the statement of the principle of relativity! According to this view, the concept of relative motion is thus no longer in contradiction to the principle of relativity, but it is rather implied by the same.

But what is a non-mass? In his natural philosophy Hegel has given reasons for the fact that something of this kind must exist in nature and identified it as *light*. We will not go into this point at the moment⁵. In fact, it is empirically the case that light actually possesses energy but no ‘rest mass’. According to what has been said, it therefore cannot be at rest but *only in motion*. Certainly, motion is motion only relative to a reference instance realized by a mass, but in respect to light it is essential that the relation of motion in this

ity” (9.59 f add.); a detailed discussion on this by *Wandschneider, D.* (1993) *The Problem of Mass in Hegel*, in: *Petry, M. J.* (1993) *Hegel and Newtonianism*. London/Dordrecht/Boston 1993.

⁴ The so-called *Higgs-Mechanism*, with which the existence of rest masses of elementary particles is explained today, assumes the existence of a ‘Higgs-Boson’, which at present is still hypothetical; cf. *Bethge, K./Schröder, U. E.* (1986) *Elementarteilchen und ihre Wechselwirkungen*. Darmstadt 1986, 268 ff. However, the Higgs-particle itself is already supposed to possess a rest mass; insofar, the basic question of how a rest mass is constituted, does not seem to me to be explained by the Higgs theory in principle.

case – precisely because light possesses no rest mass – is no longer reversible as it is with the motion of masses. However, that now means that *mass* basically possesses no defined state of motion *relative to light*, and, in fact, independent of what state of motion a mass may have *relative to another mass*. In other words: The kinematic relation of mass and light is *identical* for all masses. As a consequence of this, the velocity of light cannot depend on the state of motion of a mass (i.e. relative to another mass), and this means that light must possess the *same* velocity relative to any mass, independent of its state of motion otherwise. It now becomes understandable that light as a ‘non-mass’ is a non-relative motion, i.e., a motion independent of any frame of reference.

If this were not the case, i.e., if the velocity of light would have different values, namely in respect to reference instances moving relative to each other, light could either not be a non-mass or its propagation would be bound to a medium (like, e.g., water waves) – a supposition that was, e.g., the basis of the Michelson-Morley experiment. Both possibilities have been empirically excluded. But then, according to the presented, improved interpretation of the principle of kinematic relativity – motion of mass is equivalent to relative motion – the consequence that the velocity of light has a non-relative, absolute character is inescapable.

Relative and absolute motion are therefore not only determined as compatible, but *at the same time* as the implication of the principle of kinematic relativity, as made clear in the presented manner. Nevertheless, basic questions remain open in this connection. One of the most important ones is certainly how the essential difference between mass and light presented is really founded: What enables a mass to be at rest in contrast to light? Or, to be linguistically neater: Why is *duration* constituted by mass, but not by light? In the following I would like to make some remarks on this with the use of a thought experiment.

3. A Thought Experiment: Standing Light Wave

The riddle of a non-relative motion is also based on the fact that the motions perceived in daily life are motions of a mass and are, therefore, relative motions. A non-relative motion in this perspective has something exotic or inexplicable about it. At the same time this factum brutum that duration is constituted by mass is not less a riddle.

In the following thought experiment I would like to attempt a *change of perspectives*: Instead of proceeding from the motion of mass I shall proceed from the motion of light and so possibly gain some understanding of what *duration* really is.

Let us assume two light waves running counter to each other, for the sake of simplicity in the form of plane waves of the same amplitude (which is be-

⁵ On this point cf. the works named in footnote 2.

sides put equal to 1). The superposition of two such waves running counter to each other has a mathematical formula⁶

$$(a) \quad \cos \omega_1 \left(t - \frac{x}{c} \right) + \cos \omega_2 \left(t + \frac{x}{c} \right),$$

with $\omega_i = 2\pi\nu_i$, the frequencies ν_i and the velocity of light c . x , t are the space and time coordinates. For equal frequencies $\omega_1 = \omega_2$ the superposition of both waves yields a so-called *standing wave*, as is well-known. This is characterized by the fact that there are *knots*, i.e., *stationary positions* with the field intensity zero, thus in the case of light in the electric field strength, and by the fact that between these knots the field strength *oscillates* periodically between its extreme values. One has thereby a series of equidistant positions in which the field intensity is constant to zero ('knots') and between them there are periodical oscillations of the field strength. A space-time reference system is thereby realized, so to speak: a regular sequence of *stationary positions* connected by *synchronic oscillations*⁷.

To avoid a misunderstanding: For the description of the wave motion a 'normal' reference system S has been taken as a basis first of all, and within this framework the observation can be made that under the given circumstances a standing wave exists and that its knots represent stationary positions, and the periodic oscillations between them represent synchronous clocks, so to speak. With these characteristics the frame realized by the standing wave can now take over the function of the frame S itself. In this manner we have, as it were, a model with which we can study what makes up a frame of reference and how especially *duration* is characterized.

In respect to this we can infer directly from this model that *duration* is something like identity in change. The supplement is important, namely, that we are dealing with an identity maintained in change. This is, in fact, a characteristic of a *standing wave*. A *single wave* also shows something unchangeable, namely, the quantity of the field intensity of a certain wave phase that remains unchanged in the motion of the wave. But that can only be determined, so to speak, '*from outside*' by recourse to an external reference system. Taken alone, the single wave simply contains a fixed succession of phases without any time reference and, therefore, without any affinity to rest and motion. Only the standing wave contains an *intrinsic time moment* and, in fact, in the form of periodic oscillations between the knots. In this sense we can study the character of duration on the described model.

Furthermore, the following could be considered: Every wave phase (of a single wave) is obviously the result of a disturbance of the state preceding it. For demonstration purposes, we can imagine dominos set up in a row: If one falls, the next one falls, too, and this one onto the next one, etc. This is a con-

⁶ The mathematical relations are presented more precisely in the appendix.

⁷ The oscillations are synchronic if the counter running waves have the same velocity, which is here the case by presupposition.

tinually newly self-producing disturbance of a state, from which we must assume that – be as it may – it is in equilibrium without this disturbance. That is basically the situation with a *single* wave (whereby the domino analogy must naturally be transferred to *every* wave phase): This is, so to speak, a ubiquitous ‘tipping process’ that continually creates itself anew.

From this point of view the *standing* wave appears in a new light: The continual tipping process connected with a wave is continually ‘tipped back’ by the wave running counter, as it were. If the single wave is the continual disturbance of an equilibrium, then the wave running counter is the continual restoration of the same. *Duration* is here, accordingly, the restored equilibrium in the form of a standing wave, negation of the asymmetry in connection with a single wave motion and the restoration of the original symmetry of a state that can *exist out of itself*. Every phase of a *single* wave is, in contrast, the opposite of duration. As a tipping process that continually produces itself anew, it can *only be in motion*.

To be shure, after what has been said the question arises why a *superposition* of waves is at all necessary to exemplify duration, or put differently: Why does the *restored* equilibrium only possess the character of duration and not the original undisturbed equilibrium before it assumed in our considerations – whatever we mean by that? The answer here is probably that a state that is to function as an instance of reference must somehow be different from its neighboring states in order to make such a reference possible, and that is only possible in a form of energy because we are acting here on an energy level. A completely homogeneous vacuum cannot be a reference system, because there are no differences in it. With the energy contained in the *single* wave differences have already been realized, to be sure, but as indicated, in the form of continually ‘tipping’ states. Only the *standing* wave can stop this tipping process and lead to equilibrium again that can exist out of itself and thereby constitute *duration* – but not in the manner of the homogeneous uniformity of a vacuum, rather in energy differences in space, which as such can only be instances of reference. Accordingly, the standing wave constitutes duration by the realization of states of equilibrium which are simultaneously realized as energy differences in space. This *energy* aspect is clearly the basis for the *moment of time* too that has just been asserted. The fact that the standing wave performs internal oscillations is essentially an expression of the energy contained in it.

Admittedly, these are reflexions that are not empirically founded, suggesting to us, however, to be attempts at an interpretation of the observed standing wave model. At the same time, the change of perspectives has thereby been completed: The starting point is no longer the mass, which as such always contains a moment of duration, but the motion of light waves on which we can study, as it were, how something like duration, hence a characteristic of mass, can be constituted by the superposition of waves. In this sense I would like to

speak of *mass-analogue structures*⁸. In this perspective the motion of waves appears as primary and duration, as connected with the character of *mass analogue* wave knots, appears as an *epiphenomenon* of the wave motions. The question thereby arises whether the *analogue of a moving mass* can also be reconstructed in this model.

4. Mass-Analogue Motion

As shown, a standing wave with stationary knots, that rest in the reference system S and, therefore, can themselves be considered as constituents of this frame, results for equal frequencies $\omega_1 = \omega_2$ of the counter running waves. For different frequencies $\omega_1 \neq \omega_2$ on the other hand, *moving* knots result, which move with a constant velocity $u = \frac{\omega_1 - \omega_2}{\omega_1 + \omega_2} c$ in the direction of *the* wave with

the higher frequency (see appendix). From this a remarkable fact directly results, namely, that u is always smaller than the light velocity c here, i.e., in the observed model the velocity of light plays the role of a *limiting velocity* which cannot be exceeded by the mass-analogue motion of moving knots in principle – certainly not an unimportant fact for the attractiveness of the model⁹.

The knots moving in the frame S with the constant velocity $u < c$ constitute a new frame S' in which they themselves are at rest. S is presupposed as an inertial frame of reference; S' is then a reference system moving uniformly with respect to S and thereby also an inertial system¹⁰. In this sense two pairs of counter running waves are presupposed: The one realizing the frame S , the other the frame S' moving uniformly relative to the first one. All waves concerned have the same velocity c in S according to the presupposition. We may

⁸ Cf. (see footnote 3) Hegel's image of a rotating circular plane that includes motion as well as rest – namely, the centre of the circle, which is “the restored concept of duration, which is motion extinct in itself”; “the *mass* is posited, the continuation, which shows motion as its possibility” (9.59 f add.); cf. Wandschneider (1993).

⁹ It is interesting that moving knots occur as well, which move at the velocity $u^* = \frac{\omega_1 + \omega_2}{\omega_1 - \omega_2} c$, i.e., *only with super light velocity*; in the borderline case of equal frequen-

cies $\omega_1 = \omega_2$ these non-stationary zero transits are moving with an infinite velocity, or stated more precisely: In this case they are identical with the whole x-axis and so form the synchronous zero transit of the oscillations mentioned already between the knots of the standing wave.

¹⁰ The transition from the mass-analogue knots resting in S to moving ones by a change in frequency may be understood as an *acceleration* in S , too, produced either by an enhancement of frequency and thereby an influx of energy from outside to one of the two counter running waves (comparable to an acceleration, say, by an impact) or by reducing the frequency of one of the two waves, which corresponds to an emission of energy into the outside (comparable to an acceleration, say, by the energy consumption of a motor drive).

now ask what velocity they have in relation to S' (i.e., for the moment we ignore the empirical fact that the velocity of light is independent of the frame of reference). To answer this question the *transformation* from S to S' must be observed more closely. General demands on this are¹¹: (1.) It must be homogeneous and linear; (2.) the principle of relativity requires the reciprocity of the transformations from S to S' and vice versa. Both requirements are basically fulfillable by a rotation of the coordinate systems relative to each other. This leads to a general ansatz for the transformation in the form

$$(b) \quad \begin{pmatrix} x \\ t \end{pmatrix} = \begin{pmatrix} \gamma & \alpha \\ \beta & \gamma \end{pmatrix} \begin{pmatrix} x' \\ t' \end{pmatrix},$$

with the constants α, β, γ still to be determined. The classical Galilei transformation is in contrast much more special, since it leaves the time coordinate unchanged, $t = t'$, while here $t = \beta x' + \gamma t'$, t thus containing a factor γ and in addition to this a spacial element $\beta x'$. If then the transformation expressions (b) for x and t are inserted into the pair of waves $\cos \omega_1 \left(t - \frac{x}{c} \right), \cos \omega_2 \left(t + \frac{x}{c} \right)$ then relations between the coefficients α, β, γ as well as the unknown quantities ω_1', ω_2', c' result. At this point, however, only c' is certainly of interest. Now the knots moving in S are supposed to be *at rest* in S' , which, according to what has been said before, corresponds to the condition $\omega_1' = \omega_2'$. If this condition is taken into consideration, then the result is, that the velocity of light in the frame S' is the same as in the original frame S moving relative to S' ,

$$(c) \quad c' = c.$$

In other words, the velocity of light proves itself to be an *absolute* quantity, i.e., *independent of the reference system* in the model considered, too – a result, which here has been deduced from very general presuppositions.

Now the requirement of the equality of the frequencies, $\omega_1' = \omega_2'$, that leads to $c' = c$ simply corresponds to the precondition that the mass-analogue knots moving in the frame S are at rest in the transformed frame S' and so simultaneously constitute S' . Expressed differently: The frames S and S' constituted by the mass-analogue knots, moving uniformly relative to each other, are both equally determined as resting or as moving. But that is nothing other than the statement of the principle of kinematic relativity in the form made precise at the beginning: ‘Motion of mass is equivalent to relative motion’. If the frame S' is now considered as resting, then the same velocity of light results from this as in the frame S considered as resting. In other words, the fact formulated in connection with Hegel’s concept of motion that the absoluteness of the velocity of light is implied by the principle of relativity is confirmed in the observed model of standing light waves, too.

¹¹ Cf. *Hund, F. (1969) Grundbegriffe der Physik. Mannheim 1969, p. 87 ff.*

Accordingly, the velocity of light is an identical quantity for all frames of reference and thereby a comprehensive characteristic common to all frames. This *comprehensive independence of any reference system* is not at all surprising, for the principle of relativity itself already makes a comprehensive statement independent of the very frame and in this sense an *absolute* statement: ‘All reference instances can be moving as well as resting’. The variety of possible motions also refers, in the end, to a *comprehensive unity* of nature.

The question remains, why this absolute noticed in nature is a *motion* and not a spacial, temporal or any other quantity. The answer is: There exists a principle of relativity for *motion*, thus for the connection of space and time but not for space and time, each for itself. This is obviously based on the fact that the being of nature is characterized essentially by processes, which as such contain not only a spacial but also a temporal moment. An object in nature is always spacial *and* temporal and in this sense basically determined as *moving*.

5. Conclusion

The discussed *thought experiment* on the question what mass is and, especially, how it is capable of constituting duration, has brought the following results: The question is attacked here in a reversal of the perspectives, i.e., not from the perspective of the mass itself but from that of a *standing light wave*. In this model *mass-analogue structures* can be reconstructed which can be in *relative motion*. Here it is not presupposed that the velocity of light is independent of the frame of reference (as it is known as an empirical fact). Nonetheless it can be shown, that the velocity of light is the *limiting velocity* of such relative motions, further that this is the case in all frames of reference constituted by mass-analogue structures moving uniformly relative to each other, and that the velocity of light is identical in all these frames. Thus the absoluteness of the velocity of light and the relativity of mass motions indeed prove themselves to be different but intrinsically connected moments of the principle of kinematic relativity.

6. Appendix:

Brief Mathematical Presentation of the Standing Wave Model Discussed

Our starting point are two counter running, plane light waves in a vacuum in a reference system S . For the sake of simplicity we omit the field strength vector and assume that both waves have the same amplitude 1; retained are the pure wave features: $\cos \omega_1 \left(t - \frac{x}{c} \right)$, $\cos \omega_2 \left(t + \frac{x}{c} \right)$; c is the velocity of light;

$\omega_1 = 2\pi\nu_1$, $\omega_2 = 2\pi\nu_2$; ν_1, ν_2 are the frequencies of the light waves. Superposing both waves yields

$$(1) \quad \cos \omega_1 \left(t - \frac{x}{c} \right) + \cos \omega_2 \left(t + \frac{x}{c} \right)$$

or rearranged

$$(1') \quad 2 \cos \frac{1}{2} \left((\omega_1 - \omega_2)t - (\omega_1 + \omega_2) \frac{x}{c} \right) \cdot \cos \frac{1}{2} \left((\omega_1 + \omega_2)t - (\omega_1 - \omega_2) \frac{x}{c} \right).$$

For $\omega_1 = \omega_2 = \omega$ the superposition (1') yields a *standing wave*, as is known. (1') then simplifies to

$$(2) \quad 2 \cos \frac{\omega}{c} x \cdot \cos \omega t.$$

On the basis of the left cos-function it is clear that there are zero positions of the whole functional expression (2) independent of time and thereby *stationary positions* of zero field strength: the '*knots*' of the standing wave. On the basis of the right cos-function it is clear that there are temporal, *periodic oscillations* between the stationary knots, whereby the whole expression periodically becomes zero and, indeed, independent of x , so that *these* zero positions are identical with the complete x -axis at a moment.

If $\omega_1 \neq \omega_2$, then there can be no standing wave in general, because there are no stationary knots. This is seen from the general expression (1'): Putting it equal to zero means that each of the two cos-functions can separately be zero. Putting the left cos-function equal to zero provides for the argument the condition

$$(3a) \quad \frac{1}{2} \left((\omega_1 - \omega_2)t - (\omega_1 + \omega_2) \frac{x}{c} \right) = k_1 \cdot \pi, \quad k_1 = 1, 2, \dots ;$$

putting the right cos-function equal to zero provides for the argument the condition

$$(3b) \quad \frac{1}{2} \left((\omega_1 + \omega_2)t - (\omega_1 - \omega_2) \frac{x}{c} \right) = k_2 \cdot \pi, \quad k_2 = 1, 2, \dots .$$

The zero positions of (1') are then given by

$$(4a) \quad x_a = \frac{\omega_1 - \omega_2}{\omega_1 + \omega_2} ct - \frac{k_1 \cdot 2\pi}{\omega_1 + \omega_2} c$$

and respectively

$$(4b) \quad x_b = \frac{\omega_1 + \omega_2}{\omega_1 - \omega_2} ct - \frac{k_2 \cdot 2\pi}{\omega_1 - \omega_2} c.$$

From (4a) it follows then: Because $\omega_1 \neq \omega_2$, there are no stationary knots but knots that *move* in the direction of the wave with the higher frequency at the velocity

$$(5a) \quad u_a = \frac{dx_a}{dt} = \frac{\omega_1 - \omega_2}{\omega_1 + \omega_2} c < c.$$

The velocity of light c thereby proves itself to be the *limiting velocity* for u_a . In contrast to that from (4b) it follows that

$$(5b) \quad u_b = \frac{dx_b}{dt} = \frac{\omega_1 + \omega_2}{\omega_1 - \omega_2} c > c,$$

i.e., *these knots can only move with super light velocity* (and insofar remind us of the concept of ‘tachyons’ discussed in physics). In the case of equal frequencies (see above) we are dealing here with synchronous zero transits of the periodic oscillations between the stationary knots, identical with the complete x-axis at a moment, as already noticed.

Transition into another frame of reference: The knots in the reference system S moving with the constant velocity u_a are now to be considered as constituents of a new reference system S' in which they themselves rest. According to Hund (1969, see above), a transformation of S into S' must firstly fulfill the condition that the transformation equations are homogeneous and linear (because of the homogeneousness of space and time) and, secondly, are reciprocal in relation to the transitions from S to S' and vice versa (due to the principle of relativity). Both requirements are basically fulfillable by a rotation of the coordinate systems relative to each other. This leads to a general transformation ansatz of the form

$$(6) \quad \begin{pmatrix} x \\ t \end{pmatrix} = \begin{pmatrix} \gamma & \alpha \\ \beta & \gamma \end{pmatrix} \begin{pmatrix} x' \\ t' \end{pmatrix},$$

with the coefficients α, β, γ still to be determined. This transformation is much more general than the Galilei transformation, which, e.g., leaves the time coordinate unchanged, $t = t'$, while here $t = \beta x' + \gamma t'$, t hence containing the factor γ and in addition to this the spacial element $\beta x'$. Inserting the transformation equations (according to (6)) for x and t into the superposition (1) of the waves yields

$$(7) \quad \cos \omega_1 \left(\gamma - \frac{\alpha}{c} \right) \begin{pmatrix} t' - x' \frac{c}{\gamma - \alpha} \\ \gamma - \frac{\alpha}{c} \end{pmatrix} + \cos \omega_2 \left(\gamma + \frac{\alpha}{c} \right) \begin{pmatrix} t' + x' \frac{c}{\gamma + \alpha} \\ \gamma + \frac{\alpha}{c} \end{pmatrix}.$$

Now the superposition of both waves in the reference system S' has the general form

$$(8) \quad \cos \omega_1' \left(t' - \frac{x'}{c'} \right) + \cos \omega_2' \left(t' - \frac{x'}{c'} \right).$$

A comparison of the two expressions (7) and (8) yields

$$(9) \quad \omega_1' = \omega_1 \left(\gamma - \frac{\alpha}{c} \right), \quad \omega_2' = \omega_2 \left(\gamma + \frac{\alpha}{c} \right),$$

$$(10) \quad \frac{1}{c'} = \frac{\frac{\gamma}{c} - \beta}{\gamma - \frac{\alpha}{c}} \quad \text{and} \quad \frac{1}{c'} = \frac{\frac{\gamma}{c} + \beta}{\gamma + \frac{\alpha}{c}}.$$

Both expressions for $\frac{1}{c'}$ result in $\frac{2\alpha\gamma}{c^2} = 2\beta\gamma$ or

$$(11) \quad \beta = \frac{\alpha}{c^2}.$$

Besides, if use is made of the condition that the knots moving in S are *at rest* in S' , then it must be that $\omega_1' = \omega_2'$ and therefore, according to (9),

$$(12) \quad \omega_1 \left(\gamma - \frac{\alpha}{c} \right) = \omega_2 \left(\gamma + \frac{\alpha}{c} \right)$$

or

$$(13) \quad \gamma = \frac{\alpha}{c} \cdot \frac{\omega_1 + \omega_2}{\omega_1 - \omega_2}.$$

Our interest here is directed toward the velocity of light c' in the reference system S' moving relative to S : Inserting β and γ from (11) and (13) into (9) finally yields the *velocity of light as independent of the reference system*:

$$(14) \quad c' = \frac{\gamma - \frac{\alpha}{c}}{\frac{\gamma}{c} - \beta} = \frac{\frac{\alpha}{c} \cdot \frac{\omega_1 + \omega_2}{\omega_1 - \omega_2} - \frac{\alpha}{c}}{\frac{\alpha}{c^2} \cdot \frac{\omega_1 + \omega_2}{\omega_1 - \omega_2} - \frac{\alpha}{c^2}} = c$$

– a result, which was *not presupposed* by choosing *light-waves* for the model discussed. The explicit value of α is not at all necessary to get this result and therewith neither that of γ .

The relation $c' = c$ shows, moreover, that the transformation (6) is in fact the known ‘Lorentz transformation’ with γ as the ‘Lorentz factor’

$1/\sqrt{1 - \left(\frac{u}{c}\right)^2}$ and α as the relative velocity u' of the systems S and S' (in S'):

$u = \frac{dx}{dt} = \frac{dx}{dt'} \cdot \frac{dt'}{dt} = \frac{\alpha}{\gamma}$ (according to (6)), thus $\alpha = \gamma u$, which, according to the

Lorentz transformation, is equal to u' .