An Objection to Naturalism and Atheism from Logic

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Abstract: I proffer a success argument for classical logical consequence. I articulate in what sense that notion of consequence should be regarded as the privileged notion for metaphysical inquiry aimed at uncovering the fundamental nature of the world. Classical logic breeds necessitism. I use necessitism to produce problems for both ontological naturalism and atheism.

0 Introduction

An objection from logical considerations against atheism is one which attempts to show that some deliverance of logic is at odds with atheism or something strictly implied by atheism. The formulation of such an objection raises the question: which logic? Below I object to Rudolf Carnap's logical principle of tolerance (sect. 1), subsequently showing (in sect. 2), by appeal to the important work of Geoffrey Hellman, that there are good pragmatic reasons for adopting a classical logic. I then (in sect. 3) suggest that a deliverance of classical logic leads to what I call the new phenomenon of coordination (discussed in sect. 4). It is that phenomenon that causes problems for not only ontological naturalism, but also atheism.

1 Logical Monism or Something Near Enough

A classical first-order logic (CFOL) is a formal first-order language built on classical propositional logic (CPL), outfitted with the classical quantifier rules[1] and a specific formal semantics.[2] In addition, CFOL abides by the following three principles:

(Principle #1): Every well-formed formula's (*wff's*) truth-value on some interpretation $\mathcal{I}$ (where interpretations include non-empty universes of discourse) is completely fixed *via* the extension of the parts of that *wff* under $\mathcal{I}$.

[1] For a defense of the view that a system of logic should be specified (at least in part) by appeal to that system's rules of inference, see Rumfitt (2015, pp. 31-65).

[2] See Hodges (2001), and Hughes and Cresswell (1996, pp. 235-244), though they call it the "lower predicate calculus".

(Principle #2): There are only two truth-values, truth and falsehood.
(Principle #3): Every *wff* on any assumed interpretation $\mathcal{I}$ has exactly one truth-value.[3]

Some non-intuitionist, non-classical first-order logics affirm principles (2) and (3), but fail to obey (1). Most positive free first-order logics (P-FFOL) are like this. Well-formed formulas of those logics can have truth-values (on an interpretation) that do not depend on the extensions of the parts of those *wffs*, since some singular terms such as individual constants can fail to refer, and yet the expressions featuring those terms remain true.[4] Other free first order logics (FFOLs) are non-classical because they abandon the classical quantifier rules, or because they allow for truth-value gaps (*e.g.*, neutral free logic). Intuitionism yields a rejection of either (2), or (3), since from them it would appear to follow (in a classical metalanguage) that:

(Principle #4): Every *wff* on any one interpretation $\mathcal{I}$ is either true or false.[5]

Some logics reject (2) (*e.g.*, many-valued logics), and some others reject (3). Examples of the (3)-violating type include those non-classical FOLs which assert that some *wffs* under an interpretation are gappy in the sense that some *wffs* of those languages do not have a truth-value at all. Glut-laden non-classical first-order logics entail that there is a third truth-value (a truth-value glut) and so countenance *wffs,* which under an interpretation, possess neither truth (solely) nor falsehood (solely), but are, instead both true and false.[6]

Further distinguishing features of classical logic include its very elegant formal properties. For example, classical propositional logic is Post-complete, in that the system becomes inconsistent just as soon as one adds to it, as an axiom, a sentence that is a non-theorem of the system (Church (1996, pp. 109-110)). CFOL is not Post-complete, though with respect to that logic, Herbrand's theorem applies (Herbrand (1971)). Here's what I (roughly) have in mind. Suppose there's a valid *wff* of CFOL *p*, and a suitably correlative valid formula *q* of CPL. Herbrand's theorem entails that *p*'s validity is nothing over and above, *q*'s validity (*i.e.*, in some qualified sense, CFOL reduces to CPL). There are no applicable and general analogs of this theorem in non-classical logics such as intuitionism (Rumfitt (2015, p. 14)). Moreover, with regard to CPL, there are ways of justifying the inference rules of that logic from even a non-classical metalanguage.[7] Examples could be multiplied.

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[3] I have paraphrased these three principles from Grandy (2002, p. 531). Although Grandy uses the above principles to characterize what he calls, "standard logic", it is clear he has in mind classical logic, since he seems to use the relevant phrases interchangeably (see ibid.).

[4] If your choice P-FFOL is dual domain (one outer and one inner), then my comments in the main text should be taken to be about the inner domain of purely existing objects. The outer domain is thought to be "composed" of some non-existent objects. Dual domain approaches to free logic abound. Indeed, a completeness proof for free logic assumes it (see LeBlanc and Thomason (1968)).

[5] Priest (2008, p. 104) notes that intuitionism yields a failure to verify several "standard logical principles—most notoriously, some instances of the law of excluded middle…" My conception of classical logic departs from other treatments in the work of Field (2008) and Rumfitt (2015). What I've articulated above is closer to what Rumfitt calls "classical semantics" (ibid, pp. 10-11), although see (ibid., p. 17).

[6] See Priest's discussion of the relevant distinctions in Priest (2008, pp. 127-137).

[7] See Rumfitt (2015, pp. 153-219) who delivers a full justification of CPL given an intuitionistic metalanguage.

I am a necessitist. Necessitism is the view that necessarily, every entity/thing, is necessarily some entity/thing (*i.e.*, $\blacksquare\forall x\blacksquare\exists y(x = y)$).[8] I'm led to necessitism by an unflinching endorsement of classical quantified modal logic (CQML)—a quantified modal logic that respects (1)-(4) above—and a rejection of Rudolf Carnap's (1959) logical pluralist *principle of tolerance* (PT). PT says that there is no one true mathematics or logic. Rational cognizers may appropriate varying logics for multifarious pursuits, and logics need not be viewed as conflicting with one another since they are different languages constructed for different modes of inquiry.[9]

Logical pluralisms that affirm the PT are *strong logical pluralisms*. These pluralisms face an important problem. How does the classical logician stay classical with respect to the first-order calculus, and yet revert to a free logic in spheres of inquiry that have need of a *modal* first-order calculus? Quantified modal logic is assembled on a non-modal first-order logic. FFOLs have altogether different axiomatizations than CFOL. The difference in axiomatization produces differing lists of theorems. When the free and classical logicians disagree about axioms and theorems of first-order logic that disagreement is substantive. As Hartry Field stated, "[w]hen they disagree in their theorems (or at least, when one has theorems that the proponent of the other [logic] can be expected to disagree with), the dispute…seems a clearly factual one."[10] The theorems of FFOL and of CFOL cannot both be correct unless the strong pluralist is also a pluralist about logical truth. Pluralism about logical truth *in this context* amounts to the admission that in varying logical languages are varying logical truths. But again, that seems to suggest that—to take just one example—the conditional $\forall x(Fx) \rightarrow \exists x(Fx)$ is both a logical truth, and not a logical truth (it is in fact false in inclusive or universally free first-order logics[11]). That seems like a bad result. The theorems of CQML and non-classical (or more specifically universally free) quantified modal logics cannot both be correct. It would be problematic for one to embrace the theorems of CFOL and at the same time embrace the theorems of universally free quantified modal logics, since the legitimacy of the latter logics rests squarely upon the legitimacy of universally free first-order logics.

The defender of strong pluralism or something like PT in the context of classical and free modal logics has a typical line of response. They may attempt to show that the free logician's understanding of the connectives is fundamentally different than the classical logician's understanding, and that therefore there is no real disagreement between the free and classical modal logicians. But this response fails. The two in fact have the same take on the meanings of the logical connectives. There is also no real disagreement about the meanings of the quantifiers. Free logicians modify their quantifier rules (*e.g.*, by rejecting $\vdash \forall x(Fx) \rightarrow \exists x(Fx)$, or by adding axioms (as in negative free logic)) so as to avoid licensing undesirable inferences.[12] But even if the free and classical logician disagreed about the meanings of the connectives, it still seems there's

[8] See the discussions in Linsky and Zalta (1994), (1996); and Williamson (1990), (2000), (2002), (2013). For criticism, see Hayaki (2006) and Sider (2009), (2016). See also the important discussion of the Barcan formula in Parsons (1994).

[9] Carnap (1959, p. xv). Later he would add, "*In logic there are no morals*. Everyone is at liberty to build up his own logic, i.e. his own form of language, as he wishes." (ibid., p. 52 emphasis in the original) Carnap used his view of logic to help enforce a ban on metaphysics, and to resolve debates about the foundations of mathematics (see Richardson (1994) for commentary and especially p. 69 on various uses of differing languages).

[10] Field (2009, p. 358).

[11] See Lambert (2001a, pp. 259-260) on the theme of universally free logic.

[12] In Nolt's (2014, sect. 1.2) explication of the differences between free and classical predicate logic he never cites any differences having to do with how to understand the connectives or the quantifiers. He notes, what I have, that the quantifier rules are restricted in free logic.

substantial disagreement. For as Field (2009, p. 345) has pointed out, it does not follow from a difference in meaning (in the weak sense I have in mind) that there is no substantial dispute between those who appropriate differing logics. In the history of physics, H.A. Lorentz disagreed with Albert Einstein about the meaning of the term 'simultaneity', but that does not mean they didn't have conflicting accounts of the phenomenon of simultaneity in special relativity (Field uses the example of disagreement over the meaning of the term 'electron' in the theories of J.J. Thomson and E. Rutherford).

I also reject a milder, though non-trivial, form of logical pluralism in the work of J.C. Beall and Greg Restall (2000), (2006).[13] According to them, a characterization of logical consequence in deductive logic amounts to a statement about validity.

> (**Logical Consequence Schema (LCS)**): A deductive argument is valid, just in case, there is no *situation* in which the premises are true and the conclusion false.

Varying deductive logics yield varying *admissible* accounts of logical consequence in so far as they provide numerous ways of understanding the notion of a *situation* in LCS.[14] Those alternative specifications of *situation* suggest differing accounts of validity. That is to say, an account of logical consequence provides a means whereby one can properly judge what propositions are logical consequences of some collection of propositions or perhaps the empty set of propositional parameters. The modes of evaluation waver in a manner that is dependent upon the assumed precisification of *situation*.[15]

If the situations are complete and consistent possible worlds or Tarskian models[16] (there is pluralism even within one language for Beall and Restall), then classical logic is appropriate (Beall and Restall (2000)). However, if the situations are, for example, constructions of a mathematical sort and are therefore incomplete though consistent, then the logic is constructive or intuitionist.[17] What precisification you appropriate depends upon your goals and aims. As Beall and Restall (2006, p. 99) put it when discussing the logic of choice for their study:

> The pluralist claim is that, given a body of informal reasoning…, you can use different consequence relations in order to analyse the reasoning. As to ***which*** relation we wish our own reasoning to be evaluated by, we are happy to say: any and all (admissible) ones!…It depends, of course, on whether the given kind of verification preservation is important to the task at hand. (ibid. emphasis in the original)

With respect to free logic, however, what are situations like? The positive free logician can understand 'situation' in LCS in terms of a set of worlds including those in which some singular term that is an individual constant '*t*' fails to denote though at those worlds it is (or can be) true

[13] Call this brand of pluralism *weak pluralism*. For criticisms of this pluralism see Bueno and Shalkowski (2009, pp. 296-306). And for another recent articulation of a logical pluralism I will not be engaged with, see Shapiro (2014).

[14] What makes a precisification of 'situation', and therefore the account of consequence employing that precisification *admissible* is the fact that it is (a) suitably formal (see Beall and Restall (2006, pp. 41-43)), and (b) normative, (c) necessary, and (d) such that it plays the consequence role (ibid., pp. 40-44).

[15] See Beall and Restall (2006, p. 29, p. 35, pp. 75-83).

[16] See Tarski (1983, p. 417)

[17] These examples show up in Bueno and Shalkowski (2009, p. 295) as well.

that F*t*. In other words, while *t* is empty, it could be true to predicate in the way suggested by F*t*.[18] This is because on every positive free logic *the principle of independence* (PI) holds, where PI states that entities may have properties even if those entities do not exist.[19]

On weak pluralism, one could side-step some metaphysical positions like necessitism by simply refusing to countenance whatever situation-types were appropriated in the definition of logical consequence in the metaphysical inquiry that leads to necessitism.[20] As I will show below, CQML gives you necessitism. If the weak pluralist agrees that classical logic is the logic of choice for metaphysical inquiry, but not in other contexts, I will not demur (hence the subtitle of this section). However, if they reject classical logic as the choice logic for that inquiry, we will need a reason why. In other words, if one is a weak pluralist, it will be important for one to provide justification for your choice precisification of 'situation' in LCS *for metaphysical inquiry*. In the absence of independent reasons, shifting logics to avoid necessitism seems like cheating.

Can the weak pluralist avoid the charge of inconsistency leveled against strong pluralism? Beall and Restall (2006, p. 100) answer yes. Their pluralism is a pluralism about logical consequence and (in a qualified sense) logical truth, not a pluralism about truth *simpliciter*. This is because for them, a logical truth *p* of free quantified modal logic (FQML) is just one that holds in all suitably precisified situations (call these situations$_{PF}$, for positive free logical consequence). Proposition or statement *p* does not conflict with a logical truth of CQML *q*, since *q* holds at every *classically precisified* situation (call these situations$_{C}$).

It is a substantive metaphysical question whether or not there are situations$_{PF}$. First, I might conceive of exemplification as a nexus tying together concrete particulars—understood as Aristotelian substances—and properties. That nexus or relation does not exist without *relata*. How can an object stand in an exemplification relation, or nexus if that object does not exist? There are, of course, Bradley-style regresses to worry about with regard to this conception of exemplification, but that regress may be benign, or else not a regress at all.[21] The metaphysical view just articulated seems inconsistent with the principle of independence.

Second, suppose I affirmed a (universal) truth-maker thesis, that every truth *p* has a truth-maker that *de re* necessitates its truth. We say that an entity *e de re* necessitates a proposition *p* if and only if *e's* existing entails (in some non-*de dicto* sense) *p*. Relative to situations$_{PF}$, what would make the claim <Santa Claus gives gifts.> true? The question is a difficult one. As we shall see, analogous questions can be raised in the context of an evaluation of intuitionism. It is a substantive metaphysical question, whether or not there are constructive and consistent situations (situations$_{I}$). I will return to this issue in sect. 2.

You are free to balk at the underlying metaphysical theses used to give the relevant (in the sense of contextual salience) logicians pause. The point remains: metaphysics can drive a rejection of the existence of the referents of the suitably precisified situations. I believe this is because logic is

[18] What Beall and Restall (2000, pp. 480-481 note 8); (2006, pp. 75-77) actually suggest is that the free logician might be able to understand 'situation' in terms of Phillip Bricker's (2001) world classes. A world class is nothing above and beyond a single possible world with causally isolated and detached spatio-temporal parts themselves understood as Lewisian concrete possible worlds. If LCS is modified such that it affirms that a deductive argument is valid, just in case, there is no world class in which the premises are true and the conclusion false, and there is an empty world class, then one is dealing with a free logic (Beall and Restall (2000, p. 481. n. 8)). Since Bricker's approach seems to require some of the equipment of Lewis's (1986) possibilism, I refer the reader to footnote 39 below.

[19] Paśniczek (2001). Lambert (2001b, p. 246) puts the principle this way, "an object can have properties without having being".

[20] Unless the arguments provided could go through on even the relevant competing logics.

[21] See Moreland's (2001, p. 116) very nice response to the regress problem for exemplification.

a type of metaphysics (*à la* Russell (1920, p. 169); Sider (2011, pp. 97-98); Williamson (2013)), or as Rumfitt (2015, p. 219) put it, "[m]etaphysical considerations cannot be extruded from rational decisions between rival logical systems." I will assume that the best metaphysical account of properties and concrete substances ties them together via a nexus of exemplification. I will therefore be entitled to reject the principle of independence and so the existence of $situations_{PF}$. This is in the spirit of searching for that notion of logical consequence that cuts at the deep joints of reality.[22] This is in the spirit of trying to answer the question: which, among the many admissible precisifications of 'situation' in the LCS, is the *fundamental*, or *joint carving* precisification? The answer "none", betrays the fact that logic is a type of metaphysics, and the promulgater of that answer leaves themselves open to the following objection:

The Success Argument for Classical Logical Consequence:

(1) The correct admissible precisification of 'situation' in LCS in physics (including mathematical physics) is $situation_C$.
(2) It would be a "miracle" if the (i) justification, (ii) content, and (iii) deliverances of our most empirically successful theories in physics were verisimilitudinous, though the admissible precisification of logical consequence at work in (i)-(iii) with respect to the empirically successful sectors of physics was not fundamental (or joint carving).
(3) If (1) and (2), then interlocutors in the ontology room (the place in which the standards of precision are set appropriately for metaphysical inquiry into the hierarchy of being and what exists) should appropriate $situation_C$ as their choice admissible precisification of 'situation' in LCS.
(4) Therefore, interlocutors in the ontology room should appropriate $situation_C$ as their choice admissible precisification of 'situation' in LCS.[23]

I will leave justification of (1) to my discussion in sect. 2. Premise (2) is reminiscent of a premise in a formulation of the (no) miracles or success argument for scientific realism in the philosophy of science.[24] It would be a very strange fact indeed, if the justification of, and mathematical formalism peculiar to empirically successful physical theories required distinctively classical reasoning and inference, though that type of inference or logical consequence was somehow disconnected from the fundamental nature of the world. If anything informs us about the deep structure of the world it is empirically successful physics and physical theorizing (including the underlying mathematics). That theorizing requires $situations_C$ (again see sect. 2). That fact seems to justify the use of $situation_C$ in LCS in spheres of inquiry concerned with the fundamental nature of reality (hence premise (3)).[25]

[22] Sider (2011). I am assuming a realism about the deep joints of nature. Defending that realist assumption in this context would take me too far afield. Thanks to Andy Arana for motivating me to make this assumption more explicit.

[23] I consider this argument to be one very much in the spirit of Putnam's (1979, pp. 72-73) famous work. Like him, I am seeking to provide a new analog of the success argument for scientific realism.

[24] For discussions of the (no) miracles/success argument for scientific realism, see Devitt (2005); Laudan (1981); Putnam (1978), (1979); and van Fraassen (1980, pp. 23-25; 34-40).

[25] One might wonder what logic I'm assuming in my defense of (4). The answer is any logic able to countenance modus ponens, and conjunction.

2 The Pragmatic Case for Classical Logical Consequence

The strong pluralism of Carnap faces an inconsistency charge. The weak pluralism of Beall and Restall can be appropriated by the opponent of necessitism, if that weak pluralist agrees that the choice admissible precisification of 'situation' with respect to LCS in the context of metaphysical inquiry is the classical one. This section will argue against the weak pluralist who would attempt to appropriate a different notion of logical consequence in that self-same inquiry.

My reasons for preferring classical logic are pragmatic. One cannot properly underwrite mathematical physics without classical logic.[26] Therefore, one cannot properly do (pragmatic) physics without classical logic. Classical logic was conceived for the purpose of making explicit and rigorous the sense of validity employed by mathematicians in mathematical reasoning. As Burgess stated,

> Classical logic was developed by Frege, Peano, Russell, Hilbert, Skolem, Gödel, Tarski, and other founders as an extension of traditional logic mainly, if not solely, about proof procedures in mathematics.[27]

Classically based mathematics (a mathematics that requires classical logic) is that which pure ZF-set theory is approximating (Burgess (1992, p. 18)).

By far, the most far reaching and substantive attempt to recapture certain spheres of applied mathematics for non-classical logics has come from intuitionist-based constructive mathematics (constructive mathematics just is math done with an intuitionistic logic (Bridges (1999, p. 440); and see Bridges and Palmgren (2013)). The problem is that their efforts come up short. Douglas S. Bridges has remarked, "[i]t is clear that a constructive examination of the mathematical foundations of quantum physics does reveal substantial problems."[28] Let us look at some of the details.

First, consider both bounded and unbounded linear Hermitian or symmetrical essentially self-adjoint operators in non-relativistic and relativistic quantum mechanics (QM).[29] These are operators that provide the means whereby one transmutes a vector into yet another vector in the complex linear vector space called a Hilbert space $\mathcal{H}$. They are put into service as devices that help mathematically represent real physical quantities such as momentum, position, and energy.[30] The non-Hermitian annihilation (A) and creation (A*) operators, as well as the Hamiltonian operator (which is Hermitian) can be understood as functions of momentum and position operators. These *quantum operators* are therefore very important for understanding the dynamics of relativistic and non-relativistic QM.[31]

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[26] Brian P. McLaughlin (1997, p. 219) has said, "…no one knows how to do calculus without classical logic, and no one knows how to do physics without calculus." This may be a bit of an overstatement. Constructive mathematicians have not only developed ways of "doing the calculus", but they have also gone beyond calculus to functional analysis (see Beeson (1985); Bishop (1967); and Bridges (1979)).

[27] Burgess (1992, p. 9).

[28] Bridges (1981, p. 272).

[29] They are linear because they abide by a set of conditions articulated in Shankar (1994, p. 18); bounded because the Hilbert space involved is finitely dimensional; symmetrical because the operator T is such that $T^* \supseteq T$, self-adjoint because $T^* = T$, and essentially self-adjoint because $T \subset T^{**} = T^*$, given certain conditions on T^*, for which see Jauch (1968, p. 41).

[30] Weinberg (2013, p. 61).

[31] Hellman (1993b, p. 240, p. 247. n. 5); Weinberg (2013, p. 78).

A subset (call it σ) of the collection of *quantum operators* on the Hilbert space are closed, linear, and unbounded when $\mathcal{H}$ is infinitely dimensional[32], and when the operator is only defined over a restricted domain of that space that is dense. In fact, it is a theorem of mathematical physics that any closed operator that is linear and that is defined over all of the space must be bounded (see Riesz and Sz.-Nagy (1990, pp. 296-299, pp. 306-307)). We have already reached a shortcoming of constructive mathematical physics, the proofs for this theorem are distinctively classical and non-constructive (Hellman (1993b, p. 223)), and I am unaware of constructive surrogate proofs. Ignore this for now. Let our choice collection σ, also include quantum operators that fall under the theorem of Marian Pour-El and Ian Richards (1983), or rather, an extension of that theorem as supplied by Hellman (1993b, p. 228). Our members of σ will therefore also be computable (in the sense Pour-El and Richards have in mind) and defined over a Banach space. This will mean that every member of σ satisfies constructive extensions of the Pour-El and Richards axioms for a computable operator as supplied by Hellman (1993b). These revised axioms replace the occurrence of 'recursive function(s)' in those axioms with something constructively acceptable, *viz.*, 'natural number constructive function'. Hellman (1993b) notes that given such replacement, the proof provided by Pour-El and Richards no longer works, and as a result the operators that fall under that theorem are non-constructive items or objects. The proof of Pour-El and Richards is distinctively non-constructive and classical. This fact has not been resisted by constructive mathematicians. For example, Bridges' (1995, p. 559) reply to Hellman ends with the assertion that the proof's inclusion of the convergence of a certain series is non-constructive and distinctively classical in that the reasoning used to "establish" it is classical. Elsewhere, particularly in Bridges' (1999) general treatment of constructively based mathematical physics, he confesses:

> …while *there may be* some significant, fully constructive analogue of the First Main Theorem, a careful analysis…reveals that the Pour-El and Richards proof of that theorem has little or no significant constructive content. (Bridges (1999, p. 445) emphasis mine)

Ye's (2000) study attempts to bring many theorems of physical interest back under the banner of constructivist mathematics. The Pour-El and Richards theorem is conspicuously missing from that study, as is a discussion of those closed linear unbounded operators that fall under that theorem. Hellman's result seems well in hand.

Constructivists attempt to resist, not the result, but the classicists' interpretation of it. Hellman (1997) has observed that if constructivist mathematics is unable to recognized members of σ, two things follow. First, dynamical evolutions from a quantum state ψ of a quantum physical system represented by the relevant operators cannot be modeled by constructivist QM. Second, states such as ψ should be such that they can take expectation values of a variety relevant to measurable quantities that are themselves peculiar to such states. But that fact is precluded if the math cannot handle operators in σ that have ψ as their prejacents.[33]

Constructivist mathematics has a problem establishing certain of the singularity theorems in general relativity. More specifically, it cannot be used to prove the broader theorems of Stephen

[32] Prugovečki (1971, p. 180) tells us that "most of the operators of interest in quantum physics are unbounded".

[33] The general point is made by Heathcote (1990). Its application to constructivist QM was noted by Hellman (1997, p. 123).

Hawking and Roger Penrose.[34] This is an important methodological constraint, one that appears to count against going constructivist in one's mathematical physics. Defenders of constructivist mathematics have once again refrained from resisting the result. As Billinge's reply to Hellman concluded:

> I concede that the Hawking-Penrose singularity theorems of General Relativity are likely to be non-constructive since they tell us merely that singularities exist, and do not provide any further information about the nature of such singularities. Billinge (2000, p. 316)

There are, therefore, strong pragmatic considerations from physics in favor of endorsing classical (over constructive) logic in physical inquiry.

There are other problems too. Consider the mathematics of the calculus of variations. Therein lives the extreme value theorem (EVT) which states that "a continuous function on a compact domain assumes its maximum (minimum) value at some point."[35] There is no constructive proof of this important theorem of calculus. Constructive mathematicians do not resist this judgment either. We can infer from the conclusions of Troelstra and van Dalen (1988, pp. 292-295) that if there were a constructive proof of EVT, there would exist a constructive proof of something that is known to be non-constructive.[36] That is impossible.

We could pile on with more examples, but I leave it to the reader as homework to study the debate (see particularly Hellman (1993a,b), (1998)). Specific cases aside, we should not forget that after the development of intuitionism in the early 20th century, the proposed revision of classically based mathematics given by constructive mathematics was "rejected by the vast majority of mathematicians…".[37] When one adds that in the relevant history, the underlying motivating philosophy was likewise rejected as deeply problematic, the prospects of justifying the appropriation of intuitionistically precisified situations_I in the mathematical-physical sphere look dim. As justification for the latter claim about the underlying philosophy, consider the fact that Brouwer and Dummett (two of the early pioneers of intuitionism) were both verificationists about truth and meaning. Each affirmed that every truth is verifiable, and that such verifiability is even a necessary condition for the meanings of substantive declarative sentences. That underlying verificationism implies that all truths are known, given the famous Church-Fitch knowability result (for which see Church (2009); Fitch (1963); Hart and McGinn (1976, pp. 205-206); Kvanvig (2006, pp. 8-14); and Salerno (2009a), (2009b)), and while ordinary derivations of that result are classical, Priest (2009, pp. 98-99) has found a way to intuitionistically derive a correlative result that yields a conclusion that intuitionists should not accept (*viz.*, $(\sim Kp \rightarrow \sim p)$).

Set aside problems with the underlying motivation. As was suggested earlier, logic is a type of metaphysics. There can be metaphysical reasons in favor of precluding the intuitionist precisification of 'situation' in LCS. Not only that, there can exist other philosophical reasons, reasons from the philosophy of language, for abandoning the existence of certain admissible precisifications of 'situation' in LCS. Let me illustrate these two points about metaphysics and philosophy of language constraining logic by way of an objection to intuitionism from those two sub-disciplines of philosophy.

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[34] See Hawking and Penrose (1970). As Hellman (1998, pp. 436-437) has noted.

[35] Hellman (1998, p. 431).

[36] Thanks to Andy Arana for help here.

[37] Burgess (2009, p. 121).

Assume a metaphysics of propositions according to which, propositions are mind-independent entities, and assume that propositions are fundamental truth-value bearers such that, a a non-propositional entity only gets to be true or false because it expresses or better (following King (2002)) designates a true or false proposition (call such views, realist-F views), or else it stands in some appropriate relation to propositions (*e.g.*, beliefs might be true or false because they have as their contents true or false propositions). On intuitionism, however, there is something like a verification, proof, or constructive condition on truth (van Dalen (2001, p. 224)) such that a sentence P that is true in an intuitionistically precisified situation$_I$, is one for which there exists a proof, or construction, or verification that P. But do sentences only begin to designate true propositions just as soon as there begins to exist a verification of them, or a verification that there's no verification of them (as in the case of intuitionistically interpreted ~P)? There is no theory of proposition-designation that would suggest as much. We typically regard as proposition-designating linguistic items—like that-specifying clauses within declarative sentences laden with the notion 'proposition' (*e.g.*, 'Russell believed the proposition that God does not exist.'), or linguistic items in sincere utterances (or sentences expressing denials, assertions, objects of belief)—those entities that stand in logical relations etc., because we believe propositions just are the objects of denials, assertions, beliefs, etc. (following Bolzano, Frege, Russell and many others).

Let us suppose that the conditions of true proposition-designation are intimately connected to the existence of a proof. It would follow that I could never truthfully assert, believe, or deny without it being the case that there exists a proof of what I'm asserting etc., or a proof that there is no proof of what I'm denying etc. This seems wrong. We were in the business of truthfully asserting, denying, agreeing etc. before the existence of verification and proof procedures. The conditions of true proposition-designation have nothing to do with the existence of certain types of mental entities or activities that are proofs or verifications.

If you interpret the constructive condition on truth in intuitionism as a condition on the truth of propositions, then the object language that is intuitionistic logic should be outfitted with propositions instead of sentences. This would be problematic on many realist-F views of propositions. For example, Merricks (2015) believes that propositions do not have logical forms, and do not have logical connectives as constituents. For Stalnaker (1976, pp. 79-80); and Lewis (1986, p. 53), propositions are the very sets of worlds at which the sentences expressing those propositions are true. Given that situations$_I$ are possible worlds, it is unclear how those entities are fine-grained enough to allow for a difference between intuitionistic equivalences, or the various theorems and/or axioms of intuitionistic logic, all of which one might think hold at all admissible intuitionistically precisified situations$_I$, where again these are now being understood as worlds.[38] Moreover, it looks as if p would be identical to ~~p on such a view of propositions. That would seem to make it difficult to affirm that p and ~~p are not equivalent. On intuitionism, both (p v ~p) the law of excluded middle, and (p → ~~p) are rejected as theorems or axioms. This is because the addition of either statement as an axiom yields the theoremhood of the other statement in the intuitionist system (see Burgess (2009, 127-130) for the proofs). Double negation elimination is therefore intuitionistically unacceptable.

Perhaps one shouldn't think of situations$_I$ as possible worlds. Beall and Restall (2000, p. 477) only recommend a possible worlds precisification of 'situation' in LCS for a distinctive *classical* understanding of logical consequence, not for intuitionist logical consequence. But even on the

[38] There is nothing about intuitionism that should cause one to deny that situations$_I$ are possible worlds. Priest (2008, pp. 105-107) has articulated a possible worlds semantics for intuitionism.

Fineness of grain problems have already been articulated in numerous places, but see Soames (2009).

supposition that possible worlds are only suitable for classical and admissible precisifications of 'situation' in LCS, it remains true that if we appropriate a realist-F view firmly within the Stalnaker-Lewis tradition, p = ~~p. The lesson here is that one's metaphysics of propositions, and one's views about proposition-designation can constrain one's views about logic and logical consequence.

Without situations_I, we don't have intuitionism as a choice among the plurality of logics to choose from for mathematical-physical spheres of inquiry. But nothing comes even close to recovering the results of mathematics and physics outside of constructive math and classical math. Thus, we have good reasons then for affirming that the correct and admissible precisification of 'situation' in LCS in physics (including mathematical physics) is situation_C. I have already justified the remaining two premises of the success argument for classical logical consequence. We are left then with the conclusion that interlocutors in the ontology room should appropriate situation_C as their choice precisification of 'situation' in LCS. Let us now visit the ontology room with situations_C in hand.

3 From Classical Logic to Necessitism in the Ontology Room

From classical assumptions, I will prove that[39]:

$$\text{(NNE): } \blacksquare\forall x\blacksquare\exists y(x = y)$$

That is to say, necessarily for any *x*, necessarily there is at least a *y*, such that *x* is identical to *y*. In Williamson's (2013, p. 2) slogan, NNE says that "necessarily everything is necessarily something". That NNE follows from classical logic may strike one as a truly confounding claim. How can it be that classical considerations yield such a shocking truth? There are several routes to NNE from classical reasoning. Let's start with a route that brings in a more heavy-duty assumption and then slim down.

Many philosophers believe that S5 is the system of modal logic that correctly captures our intuitions or correct ideas about the nature of metaphysical necessity and possibility.[40] Interestingly, S5 classical constant domain QML entails necessitism as a theorem. Here's the proof (the complete and sound tableaux system in this case is from Priest (2008, pp. 6-11; 45-46; 266-277; 308-315; 350-352). It would correspond to an S5(NI) proof system):

$$\sim\blacksquare\forall x\blacksquare\exists y(x = y), 0$$
$$\blacklozenge\sim\forall x\blacksquare\exists y(x = y), 0$$
$$\sim\forall x\blacksquare\exists y(x = y), 1$$
$$\exists x\sim\blacksquare\exists y(x = y), 1$$
$$\sim\blacksquare\exists y(a = y), 1$$
$$\blacklozenge\sim\exists y(a = y), 1$$
$$\sim\exists y(a = y), 2$$
$$\forall y\sim(a = y), 2$$

[39] I will ignore Lewis's (1986) possibilism in this paper. For objections to the counterpart theory that is at the heart of Lewis's possibilism, see Fara and Williamson (2005). I also ignore the approach to QML that restricts it to haecceities (as in Plantinga (1974, pp. 70-87)). For objections to that view, see Williamson (2013, pp. 267-296), and see especially (ibid., pp. 288-289) which takes you from haecceities to necessitism.

[40] For arguments along these lines see Hale (2013, pp. 127-131).

$$\sim(a = a), 2$$
$$(a = a), 2$$
$$X$$

Therefore, $\vdash_{S5\ CQML} \blacksquare \forall x \blacksquare \exists y(x = y)$.[41]

In fact, NNE is a theorem on a much weaker system of modal logic, *viz.*, system K. Consider:

(1) $\blacksquare(\forall x)(\exists y)(x = y)$ [Theorem]

Here is a tableaux proof showing that $\vdash_{CK} \blacksquare(\forall x)(\exists y)(x = y)$:[42]

$$\sim\blacksquare(\forall x)(\exists y)(x = y), 0$$
$$\blacklozenge\sim(\forall x)(\exists y)(x = y), 0$$
$$0r1$$
$$\sim(\forall x)(\exists y)(x = y), 1$$
$$(\exists x)\sim(\exists y)(x = y), 1$$
$$\sim(\exists y)(a = y), 1$$
$$(\forall y)\sim(a = y), 1$$
$$\sim(a = a), 1$$
$$(a = a), 1$$
$$X$$

(2) $(\forall x)(\exists y)(x = y)$ [Nec. Elim. (1)]
(3) $(\exists y)(v = y)$ [UI (2)]

But what follows from what is necessary, must itself be necessary:

(4) $\blacksquare(\exists y)(v = y)$
(5) $(\forall x)\blacksquare\exists y(x = y)$ [UG (4)]

And again, what follows from what is necessary, is itself necessary. So,

(6) $\blacksquare(\forall x)\blacksquare(\exists y)(x = y)$[43]

Of course, (6) is NNE. Thus, NNE follows from classical constant domain QML given just K (the weakest normal modal logic).[44]

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[41] "On the fixed domain interpretation, the sentence $\forall x \Box \exists y(x = y)$ (which reads 'everything exists necessarily') is valid". Garson (1991, p. 112)

[42] The 'CK' stands for the classical constant domain K system of quantified modal logic. I adopt the classical K tableaux system of Priest (2008).

[43] There must be a proof like this in the literature somewhere. The nearest and most similar reasoning I could recently find is in Hale's (2013, pp. 207-208) work. See also Linsky and Zalta (1994, p. 452, n. 11).

[44] This is hardly surprising. "The constant domain assumption implies that whatever exists in the actual world exists necessarily, that is in all possible worlds." Schurz (2002, p. 468)

Let contingentism be the thesis that NNE is false (following Williamson (2013, p. 2)). The contingentist will point out that the above proofs assume constant domain modal logics. According to such logics, the census of individuals does not change from world to world since the domain does not vary among accessible worlds. It is therefore no surprise that NNE holds on such logics. What we should ask is whether or not there are *classical varying* domain QMLs that provide an escape for the classical contingentist? There are not (at least there are no unproblematic logics fitting that description).[45] Every classical normal QML validates the converse Barcan formula ($\blacksquare \forall x \phi \rightarrow \forall x \blacksquare \phi$) (CBF), which can be used to prove NNE (see Hale (2013, p. 207)).[46] Let me explain.

One standard way of connecting normal *varying* domain QMLs with the classical quantifier rules involves adding in the increasing domains principle (also called the nested domains constraint). Let 'R' be the accessibility relation between worlds w and w* with respective domains D_w and D_{w*}. The increasing domains principle says that necessarily, $D_w \subseteq D_{w*}$, given that R_{ww*}, or necessarily if *w** is accessible from *w*, then the domain of world *w* is a subset of the domain of world *w**. The principle is necessary because varying domain classical and normal QMLs cannot validate the CBF without it. But again, dispensing with CBF would sacrifice either the classicality or (inclusive) the normality of the QML in question, because every normal and classical QML validates the CBF.[47]

But even varying domain classical normal QMLs that affirm the increasing domains principle should not be accepted for reasons having to do with an argument from Garson (1991, p. 113, p. 115) and Schurz (2002, pp. 468-469). Their objection may be paraphrased as follows: Suppose that 'h' names Han Solo, that all proper names are rigid designators, and that '@' picks out the actual world. Given contingentism, Han Solo will not be a member of the domain of @ (note that $h \notin D_@$, $V(h) \notin D_@$, and $V(h) \notin V_@(F)$, where F is a monadic predicate).[48] Assume, however, that every entity that is a member of @'s domain has F. Thus, $(\forall x)(Fx)$ holds at @, though Fh does not hold. Since our logic is classical, ~Fh holds at @. But ~Fh conflicts with $(\forall x)(Fx)$, since from it one can derive $(\exists x)\sim(Fx)$. Thus, a varying domain QML seems to be incompatible with the classical quantifier rules.

Furthermore, normal classical QMLs with the nested domains constraint implies a *constant domain* QML, so long as the following principle holds:

(7) $p \rightarrow \blacksquare \blacklozenge p$ [Axiom B]

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[45] What about Kripke's (1971) system that did away with singular and virtually all other referring terms? Did he not show how one could keep the classical quantifier rules and yet work inside a varying domain QML? Kripke's 1971 system gave up on the unrestricted rule of necessitation, not just constants and/or singular terms. This makes Kripke's resulting system non-normal (*q.v.*, n. 47). If he had kept that rule in his system one could derive in it $\blacksquare(\exists y)(x = y)$ from the empty set of propositional parameters (Garson (1991, p. 114)) read (as Kripke reads it) in terms of its universal closure.

[46] "The converse Barcan formula…is a Q1K-theorem." Schurz (2002, p. 464, *cf.* p. 468); *cf.* Cresswell (2001, pp. 150-151); Garson (1991, pp. 114-115).

[47] A normal modal logic is one that is closed under *modus ponens*, and that holds on to (a) the Kripke rule $\blacksquare(p \rightarrow q) \rightarrow (\blacksquare p \rightarrow \blacksquare q)$, (b) the classical inference rules, and (c) the *unrestricted* rule of necessitation.

Several authors have attempted to preserve the classical quantifier rules while embracing a normal varying domain QML *via* an appeal to the nested domain constraint (see Bowen (1979, pp. 8-15, but particularly p. 8); Gabbay (1976, pp. 44-60, but particularly p. 44); *cf.* the discussion in Schurz (2002, p. 468)).

[48] Where 'V' is the valuation function.

i.e., so long as the accessibility relation of the logic is symmetric. This is a well-known result in the literature on quantified modal logic.[49]

It's clear then. Classical (normal) quantified modal logic implies necessitism. Noted philosophical logicians have already realized this and have on that basis pushed for the adoption of a free modal logic. Garson writes,

> …the stipulations required in order to preserve the classical principles do not always sit well with our intuitions. Our conclusion, then, is that there is little reason to attempt to preserve the classical rules in formulating systems with the objectual interpretation and world-relative domains. The principles of free logic are much better suited to the task.[50]

I have already argued that positive free logic requires the principle of independence, and that that commitment yields its implausibility for metaphysical reasons. But there are also negative and neutral free logics. Negative free logics imply that sentences featuring singular terms that fail to denote are false, while neutral free logics say of such sentences that they take truth-value gaps. Gappy logics are non-classical on account of a denial of (Principle #3). They, like non-free but non-classical logics, cannot properly underwrite mathematical physics since they give up on the law of excluded middle.[51] We should therefore forgo on adopting neutral free logics.

Negative free modal logics have severe problems, for some such systems are crafted in such a way that there is only one domain (a domain of existing objects/entities) and yet sentences involving modal predication to non-existent objects (objects not in the domain of the actual world) must be understood in such a way that they express falsehoods. So consider,

(8) ⧫Bh

where 'Bx' means '*x* is brave', and where '*h*' is once again Han Solo. Single domain negative free QMLs deliver the verdict that (8) is false since '*h*' fails to refer. However, the falsehood of (8) entails that it is impossible that Han Solo is brave, and that seems counter-intuitive. Consider now proposition (9):

(9) ⧫Eh

where 'Ex' means '*x* exists'. Again, the negative free modal logician must say of (9) that it is false. But that entails that *h* could not possibly exist. In fact, the following is appropriated as an axiom of single domain negative free quantified modal logic[52]:

(10) (∀x)(~Ex → ■~Ex)

Proposition (10) is clearly incredible if necessitism is false.

[49] See Schurz (2002, p. 468). If your choice QML is as strong as S5, the accessibility relation will be symmetric, and the same entailment will hold. This is not the case for S4, since the accessibility relation in that system is merely reflexive and transitive (Sider (2010, p. 140)).

[50] Garson (1984, p. 261). See also the comments in Garson (1991, p. 111). See also Hale (2013, p. 209) who opts for a negative free logic.

[51] Again see Hellman (1998, p. 441) and the discussion there of the Intermediate Value Theorem, though he has in mind constructive math and classically based math.

[52] See Schwarz (2013, p. 35).

Tim Crane (2013, p. 55) has recently voiced some powerful objections to negative free logics. Crane asks us to consider a case in which someone (Brandon) thinks about Han Solo. Unsurprisingly, in such a scenario, the sentence 'Brandon is thinking about Han Solo.' seems to come out obviously true. However, the sentence expresses a statement with a polyadic relational predicate. One of the singular terms next to that predicate is empty (*viz.*, 'h' for Han Solo), and so negative free logic will demand that it be dismissed as false, *reductio ad absurdum*.

What of two domain negative free quantified modal logics?[53] I'm afraid such systems are underdeveloped. In fact, I cannot find a complete presentation of any such logic. Embracing free logic seems therefore to be an implausible way of avoiding necessitism.

4 From Necessitism to the Falsity of Naturalism and then to Theism

Here is a way of making the atheist uncomfortable *via* necessitism. Assume that it is implausible to regard all actual concrete entities as necessarily concrete. Given necessitism, the sense in which I could have failed to exist is best characterized in terms of my failing to be concrete. Thus, I and the entire host of the cosmos, including the cosmos itself, are all contingently concrete. Let us build that rather plausible view into necessitism or at least suggest that it is a truth one should affirm alongside necessitism.[54] Now consider the following argument:

(1) If necessitism is true, then ontological naturalism is false.
(2) Necessitism is true.
(3) Therefore, ontological naturalism is false.

I have already argued for premise (2). However, why believe that premise (1) holds? Let ontological naturalism be the idea that everything that exists is material, or else strongly supervenes upon the material. A material entity is any entity that is itself non-mental, non-mathematical (in the sense that it is not an abstract mathematical object), and yet it is in some way grounded, built up, determined, or composed of ordinary objects commonly investigated by successful physical inquiry (*i.e.*, particles, fields, and various other forms/structures of matter and massive and massless bodies).[55] Assuming the material is non-modal, following Hale ((2013, p. 86) with some adjustments), I will express the idea that the modal strongly supervenes upon the material via the following:

[Fx: x has a non-modal material property; Mx: x has a modal property; when F is a part of a quantifier it ranges over non-modal properties, and when M is a part of a quantifier it ranges over modal properties]

(4) $\blacksquare\forall x\forall y(\forall F(Fx \equiv Fy) \longrightarrow \blacksquare\forall M(Mx \equiv My))$ [Strong Supervenience]

English: Necessarily for any entity *x* and for any entity *y*, [(if for any material property *F*, (*x* has *F*, just in case, *y* has *F*), then necessarily, for any modal property *M*, (*x* has *M*, just in case, *y* has *M*)].

[53] See Bencivenga (2002, pp. 298-299) on the theme of two domains. See also LeBlanc and Thomason (1968).

[54] See Linsky and Zalta (1996); Williamson (1998).

[55] This doctrine may be close to a type of physicalist thesis, but it is one standard way of understanding ontological naturalism (see the discussion in Papineau (2016, sect. 1.1)).

But (4) clearly and trivially entails:

(5) $\blacksquare \forall x \forall y \big(\forall F(Fx \equiv Fy) \longrightarrow \forall M(Mx \equiv My)\big)$ [Weak Supervenience]

> English: Necessarily for any entity *x* and for any entity *y*, [(if for any material property *F*, (*x* has *F*, just in case, *y* has *F*), then for any modal property *M*, (*x* has *M*, just in case, *y* has *M*)].

If one can show that some modal properties do not even weakly supervene upon the non-modal, then it will follow that modal properties do not strongly supervene either. Barring the clearly mistaken view that modal properties are themselves material beings, ontological naturalism will come out false.

Suppose that necessitism holds. If an individual F_1 is an abstract individual that is possibly a fermion, and B_1 is an abstract individual that is possibly a boson, and each object exists (as abstract individuals) at the actual world @, both F_1 and B_1 would be similar or indiscernible with respect to their non-modal profiles at @ (they will have the same material properties). However, the property ***is possibly a fermion*** would not weakly supervene on the material, for B_1 does not have that property despite being non-modally and materially similar to or indiscernible from F_1. Because strong supervenience entails weak supervenience, the above result is bad news for ontological naturalism (the argument could be extended to many other modal properties).[56]

What about atheism (the thesis that theism is false, or that there does not exist a supernatural, omnipotent, omniscient, omnibenevolent creator of the universe)? A strong case can be made for the claim that modal notions and modal properties play indispensable roles in our best physical theories (*contra* Sider (2011)). Here are but a few examples, only one of which I elaborate on here.

There exist configuration spaces in the formalism of Hamiltonian mechanics, classical statistical mechanics, and non-relativistic quantum mechanics. Points in these spaces represent *possible* states of physical systems. In the case of non-relativistic QM, one's physical theory must be outfitted with both the notions of a wave function ψ and a quantum state in order to have a proper interpretation.[57] One defines the former notion over configuration space: "[t]he wave function of a system is…a complex-valued function *on the configuration space*, i.e. a function which assigns a complex number to each possible configuration" (quoting Maudlin (2003, p. 462 emphasis mine)), and its dynamics is given by the fundamental (to QM) dynamical Schrödinger equation: $i\hbar\frac{\partial\psi}{\partial t} = H\psi$. Configuration spaces are therefore indispensable entities laden with modal structure in wave QM.[58]

As a second example, consider the modal notions that show up indispensably in the general theory of relativity (GTR). GTR represents *our* spacetime via a triple (M, $g_{\mu\nu}$, $T_{\mu\nu}$), where $T_{\mu\nu}$ indirectly represents the matter fields of spacetime, where $g_{\mu\nu}$ is the Lorentz (1, 3) signature-metric which itself represents the metric and/or inertio-gravitational field, and where M represents

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[56] And here I'm indebted to Williamson (2013, pp. 385-389). Williamson would go on to suggest that his argument could be resisted given a radical type of anti-essentialism. I think the anti-essentialism he sketches is too radical. Space constraints do not permit criticism here.

[57] Maudlin (2003, p. 463) "[a]ll 'interpretations' of quantum theory…employ a quantum state or wave-function."

[58] For a similar point, see Malament (1982).

the curved four-dimensional differentiable, smooth spacetime manifold featuring spacetime points or relativistic "events". The geometric structure of *M* includes smooth curves represented as maps in the formalism (*e.g.*, the map $\gamma: I \rightarrow M$) that can be causal, timelike, or null-like. The timelike curves—when they are the straightest they can be in the curved spacetime—represent the *possible* paths of gravitating free massive bodies (*i.e.*, bodies that aren't under the influences of any external forces). The null-like curves of extremal length represent the *possible* paths or trajectories of free photons or massless bodies. As Malament (2012, p. 121) notes in a related, but only slightly different context (he's concerned with images of curves), the modality involved is essential. Not every geodesic actually is a path an appropriate body follows. Geodesics are themselves features of the geometric structure of *M* induced by the metric $g_{\mu\nu}$ (see Wald (1984, pp. 41-47)).[59] That free massive and massless bodies traverse some actual geodesics of the spacetime metric is a dynamical law of GTR, one that follows from Einstein's fundamental (to GTR) interactive-dynamical field equations: $R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} + \Lambda g_{\mu\nu} = 8\pi G T_{\mu\nu}$.

Modality enters our best physical theorizing by way of the fundamental dynamical laws of those theories. But given that modal properties float free from the natural order of things in that they do not even supervene upon the non-modal, why should we expect that fact on atheism? That the laws of nature should be coordinated with modal properties that hang free of the physical world seems a bizarre coincidence, one that pleads for an explanation. That coordination does not seem to be logically or metaphysically necessary, given that the laws of physics are themselves contingent. These considerations lead to what I call the ***new coordination problem*** in the philosophy of physics.[60] It is a problem that I believe not only involves modality and natural nomicity or lawfulness, but also metaphysical laws governing relations required by our best science (*e.g.*, realization relations in Boltzmannian statistical mechanics *inter alia*). The problem runs so deep, that I believe it exists even if we grant ontological naturalism (I develop this idea in an unpublished manuscript "The Argument from Metaphysical Teleology"). But let us clearly state the problem and apply pressure:

> (**The New Coordination Problem**): Why is it that modal properties and notions enter the verisimilitudinous fundamental dynamical laws of our best and most empirically successful physical theories given that modal properties do not weakly supervene upon the physical or material? (or) How is it that the material world came to be ordered in such a way that it evolves in a manner that is best captured by modally laden physical theorizing or dynamical laws given that modal properties do not even weakly supervene upon the material and non-modal?

[59] You can also take Malament's case of the images of the smooth curves understood as worldlines. He wrote about such a case,

> "…the modal character of the assertions (i.e., the reference to possibility) is essential. It is simply not true…that all images of smooth, timelike curves *are*, in fact, the worldlines of massive particles. The claim is that, as least so far as the laws of relativity theory are concerned, they *could* be." Malament (2012, p. 121) emphasis in the original

[60] "New" because it is distinct from the old problem of coordination involving the attempt to reconcile the special science laws that appear to be asymmetric, causal, and ceteris paribus, with the exceptionless, time-reversal invariant, and perhaps non-causal laws of micro-physics.

The coordination cannot receive a natural explanation for such explanations feature in their *explanans* the very laws themselves. We are interested in explaining why the laws are coordinated with modal properties. We cannot explain that fact by appeal to laws themselves else our explanation will look circular or uninformative. But if the coordination fact stands in need of an explanation, and it is not necessary, nor explicable natural-scientifically, what other explanation could there be? One might argue that whatever explains the laws of nature explains why they happen to be coordinated with modal properties and modal structure in the way they are. Some defenders of the Mill-Ramsey-Lewis best-systems account of laws (I have in mind the Humeans) maintain that those laws receive an explanation by way of truth-making. The laws are made true by the Humean mosaic, a particular structure consisting of point-like objects (or some suitable physical surrogate entities such as strings), their qualitative, and categorical properties together with the spatio-temporal relations in which such entities stand (see Lewis (1986); Loewer (2012)). But that type of explanation removes puzzlement about the wrong *explanandum*. While, the mosaic may explain why the propositions expressing the laws are true, it does not explain why those laws feature the distinctive modal content they do given that the modal hangs free of the non-modal. Indeed, it is a doctrine of Humeanism that the modal supervenes upon the non-modal. We should therefore not expect such a view to be in the business of providing the requisite explanation.

If we were to survey other available types of explanation, we would see that nothing in the atheological wardrobe is fit for the job.[61] This is because atheism does not provide the necessary explanatory equipment to do the job. I leave it as homework for the atheist to try to solve the new coordination problem. I've cut off a few preliminary paths.

Let me briefly summarize my argumentation:

(6) Necessitism is true and modal properties are indispensable to our best physical theories.
(7) If (6), then there is a new phenomenon of coordination (NPC).
(8) Necessarily, (if there is an NPC, it has an explanation).
(9) Necessarily, [if possibly both (atheism is true and there is an NPC), then it is not possible that the NPC has an explanation].
(10) Therefore, atheism is false. [see the proof of validity in the appendix]

I have argued for (6)-(8) in preceding discussion. But why think (9) holds? I've argued that atheism does not have enough explanatory power to account for the new coordination fact or facts, but why is it that the mere possible truth of the conjunction <atheism holds and there is an NPC> entails that there *can't* be an explanation of the NPC? Assume S5, and grant the antecedent of (9). There is a possible world *w** at which atheism holds and there is an NPC. That there is a theistic explanation of the NPC at *w**, that the laws of *w** are coordinated with a non-supervening modal fabric of reality *because* God weaved the relevant portions of the contingent reality of *w** that way, seems perfectly possible. So now there is a world *w*, accessible from *w** at which God explains the NPC (*i.e.*, the very NPC that exists at *w**). However, at *w** the following holds: if there could be an explanation of the NPC, then there is an explanation of the NPC. This follows from a very plausible restriction of the principle of sufficient reason:

(C-PSR): Necessarily, for any contingent truth about the coordination of the physical with the non-physical *p*, if *p* could be explained, then *p* is explained.

[61] If I had space, I would argue that injecting the modal into the material will not work either.

I call this the coordination-PSR. It seems highly intuitive, and it enjoys some inductive support, though I'll leave the task of providing a full case for (C-PSR) for later. What we can infer, given C-PSR, is that the NPC at *w** has an explanation. The problem is that atheism is true there, and as I've argued, atheism seems to lack the resources to explain NPC. Thus, we face a contradiction unless we reject the thesis that there could be an explanation of NPC at *w**. Because the same reasoning we've applied here was applied to arbitrary worlds *w** and *w* with the relevant contents, and because the C-PSR is a necessary truth, it looks like we will be able to secure the same result at any world at which atheism holds and there is an NPC. Thus, from the perspective of any of those worlds, it must be impossible that the NPC has an explanation. That is premise (9).

One might counter that atheism could hold at *w** and yet some immensely (but finitely) powerful being, with an immense (though finite) amount of knowledge exercised control over the natural world, ensuring that it unfolded in a way that allowed for *w**'s natural world to be truthfully described and explained by modally laden laws. But Graham Oppy (2014) has made a very strong case for the view that there is but one unique concept (not conception) of god, and that according to that concept:

> …to be a god is to be a superhuman being or entity who has and exercises power over the natural world…in circumstances in which one is not, in turn, under the power of any higher ranking or more powerful category of beings (ibid., p. 1).

Very plausibly then, the imagined immensely powerful entity at *w**, just is a god, even if it does not fit the conception of God appearing in western theistic traditions.[62]

We therefore have a new type of logical consideration in favor of theism, one that is not a god-of-the-epistemic gaps argument. The suggestion here is that possibly there's something in need of an explanation, and that the explanatory gap *cannot* be bridged by means of some natural-scientific explanation. The gap is therefore ontological and not epistemic. Resolution of the issue will not come by waiting for a more informed science. We are interested in the very nature of science and modality themselves. More empirical scientific success will only strengthen the case for coordination and make the problem for naturalism and atheism even more potent.

5 Conclusion

Our path was long and winding, but we arrived finally at a new logical argument for theism. The argument constitutes a truly logical objection to atheism in so far as it hinges upon the deliverance of classical logic that is necessitism. There are many rejoinders to be sure, but I hope I have at least sparked some debate on the question of whether atheism is truly scientifically respectable since it looks to be infected with the inability to make sense of the deliverances of a truly scientifically (and logically) informed analytic natural philosophy.

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[62] One might counter that w* may be a world according to which a finitely immensely powerful and knowledgeable being was under the influence, or control, or command of a being belonging to a "more powerful category of beings". But then, that entity would fit the unique concept of god.

Appendix: Validity Proof of Argument (6)-(10)

Let N be the statement that necessitism is true.
Let 'I' stand for the statement modal properties are indispensable to our best physical theories.
Let C be the statement that there is an NPC.
Let E be the statement that the NPC has an explanation.
Let A be the statement that atheism is true.

(1) N & I	Premise	∴ ~A
(2) (N & I) → C	Premise	
(3) ■(C → E)	Premise	
(4) ■[⧫(A & C) → ~⧫E]	Premise	
(5) A	Assumption	
(6) C	MP (1), (2)	
(7) C → E	Nec. Elim. (3)	
(8) E	MP (6), (7)	
(9) ⧫(A & C) → ~⧫E	Nec. Elim (4)	
(10) ⧫E	Rule T1 (8)[63]	
(11) ~~⧫E	DN (10)	
(12) ~⧫(A & C)	MT (9), (11)	
(13) ■~(A & C)	Duality of Modal Operators (12)	
(14) ~(A & C)	Nec. Elim (13)	
(15) ~A v ~C	DeM (14)	
(16) ~~C	DN (6)	
(17) ~A	DS (15), (16)	
(18) A & ~A	Conj. (5), (17)	
(19) A → (A & ~A)	CP (5)-(18)	
(20) ~A	*Reductio* (19)	

Notice that it also follows from this argument that given atheism, atheism is incompatible with a certain fact, *viz.*, that there is an NPC.

[63] See Hughes and Cresswell (1996, p. 42).

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