Pragmatic encroachment theories have a problem with evidence. On the one hand, the arguments that knowledge is interest-relative look like they will generalise to show that evidence too is interest-relative. On the other hand, our best story of how interests affect knowledge presupposes an interest-invariant notion of evidence.

The aim of this paper is to sketch a theory of evidence that is interest-relative, but which allows that 'best story' to go through with minimal changes. The core idea is that the evidence someone has is just what evidence a radical interpreter says they have. And a radical interpreter is playing a kind of game with the person they are interpreting. The cases that pose problems for pragmatic encroachment theorists generate fascinating games between the interpreter and the interpretee. They are games with multiple equilibria. To resolve them we need to detour into the theory of equilibrium selection. I'll argue that the theory we need is the theory of risk-dominant equilibria. That theory will tell us how the interpreter will play the game, which in turn will tell us what evidence the person has. The evidence will be interest-relative, because what the equilibrium of the game is will be interest-relative. But it will not particularly undermine the story we tell about how interests usually affect knowledge.¹

1 Encroachment, Reduction and Explanation

I'll start with an argument for a fairly familiar disjunctive conclusion: either knowledge is interest-relative, or scepticism is true. The argument will resemble arguments to the same disjunctive conclusion in Hawthorne (2004) and Fantl and McGrath (2009). Indeed, it is inspired by those discussions. But I think we can get the argument to work with less controversial premises than they use.

The argument starts by considering a game, one I'll call the red-blue game. Here are the rules of the game.

- 1. Two sentences will be written on the board, one in red, one in blue.
- 2. The player will make two choices.
- 3. First, they will pick a colour, red or blue.
- 4. Second, they say whether the sentence in that colour is true or false.
- 5. If they are right, they win. If not, they lose.
- 6. If they win, they get \$50, and if they lose, they get nothing.

¹Thanks to audiences at the Arizona State University Pragmatic Encroachment Conference in February 2017, Arché in June 2017, and the Episteme Workshop in July 2017 for very helpful feedback. Thanks particularly to Shyam Nair for comments at the Episteme Workshop.

Our player is Parveen. She is an epistemologist who works on pragmatic encroachment, and (as will become important in a minute), she has frequently cited both *Knowledge and Lotteries* (Hawthorne, 2004), and *Knowledge and Practical Interests* (Stanley, 2005). She knows the rules of the game, and no other relevant facts about the game. When the game starts, the following two sentences are written on the board, the first in red, the second in blue.

- 1. Two plus two equals four.
- 2. Knowledge and Lotteries was published before Knowledge and Practical Interests.

Intuitively, there is a unique rational play in this game: Red-True. That is, Parveen announces that she will evaluate the truth value of the red sentence, and then announce that it's true. That's a sure \$50.

On the other hand, in normal circumstances, we would say that Parveen does know that *Knowledge and Lotteries* was published before *Knowledge and Practical Interests*. After all, she has looked up their publication dates many times in checking over her papers.

There is a puzzle in reconciling these intuitions. The pragmatic encroachment theorist has a solution to these puzzles. In normal circumstances, Parveen does know that *Knowledge and Lotteries* was published before *Knowledge and Practical Interests*. But these are not normal circumstances. Right now, it matters whether her reason to believe that *Knowledge and Lotteries* was published before *Knowledge and Practical Interests* is as strong as her reason to believe that two plus two equals four. And (unless something very weird is happening), that isn't true for Parveen. So she knows that red-true will win, she doesn't know any other play will win, so she should play Red-True.

If we reject pragmatic encroachment, and we are not sceptics, we should say that Parveen does know that *Knowledge and Lotteries* was published before *Knowledge and Practical Interests*. And then it is a mystery why playing Red-True is more rational than playing Blue-True. After all, Parveen knows the rules of the game, and she knows (by hypothesis) the blue sentence is true, so if she can do even basic logical reasoning in a knowledge preserving way, she knows she will get as good a result as possible by playing Blue-True. So it is a bit of a mystery why it would be anything other than maximally rational to play Blue-True.

One way we might try to resolve this mystery is by saying that although Parveen knows that Blue-True will win \$50, she super-knows that Red-True will win \$50. What do we mean here by *super knowledge*? Think of this as a placeholder for certainty, or knowledge that one knows, or anything other epistemic state that you think might be relevant to her practical decision making. Perhaps the fact that she

super-knows what two plus two is, but doesn't super-know when the epistemology books were published, could be the explanation for why Red-True is the unique rational play.²

But no such explanation can work, because Parveen doesn't super-know that playing Red-True will win \$50. She super-knows that two plus two is four. But we have not assumed that she super-knows the rules of the game. So she doesn't super-know that Red-True will win, she just knows it. And she also, by hypothesis, knows that Blue-True will win. So looking at any kind of super-knowledge can't break the intuitive asymmetry between Red-True and Blue-True.

Put another way, if Parveen knows that *Knowledge and Lotteries* was published before *Knowledge and Practical Interests*, then she knows that she is playing the following game.

- 1. Two sentences will be written on the board, one in red, one in blue.
- 2. The player chooses to play either Blue-True, Blue-False, Red-True, or Red-False.
- 3. If they play Blue-True, they win \$50.
- 4. If they play Blue-False, they win nothing.
- 5. If they play Red-True, they win \$50 if the red sentence is true, and nothing otherwise.
- 6. If they play Red-False, they win \$50 if the red sentence is false, and nothing otherwise.

And is is rational to play Blue-True in that game. (It might also be rational to pay Red-True if the red sentence is *Two plus two equals four*, but it is always rational to play Blue-True.) Yet it is not rational to play Blue-True in the original game. So Parveen does not know, when she plays the game, that *Knowledge and Lotteries* was published before *Knowledge and Practical Interests*.

So to avoid pragmatic encroachment here we must deny that Parveen ever knew that Knowledge and Lotteries was published before Knowledge and Practical Interests. On its own, that's not a sceptical conclusion: lots of people don't know that. But once we go down that path, it looks like not much knowledge will be left. After all, we can repeat the game with any number of different things in the place of the blue sentence. If we adopt the constraint that Parveen only knows p, right now, if it is rationally permissible for her to play Blue-True when p is the blue sentence, no matter what the red sentence is, then either we have to say very unintuitive things about rational plays of the game, or we have to say she knows very little.

²That we need some kind of super-knowledge for action, and not mere knowledge, is a natural conclusion to draw from the examples offered, e.g., Jessica Brown (2008) and Jennifer Lackey (2010).

So we've got the conclusion that either pragmatic encroachment is true, or scepticism is true. Since I'm not a sceptic, I'm happy to conclude that pragmatic encroachment is true. But note that we've done this without any reference to high stakes situations. The stakes in Parveen's game are just \$50. That's not nothing, but it's not 'high stakes' in the way that phrase is normally used.

The version of pragmatic encroachment we get is that what matters for knowledge are not the stakes involved in any bet on p, but the odds.³ Parveen loses knowledge because she is being asked, in effect, to make a super long odds bet on a fact about publication schedules. She is in no position to rationally make a bet at those odds. So she doesn't know the fact about publication schedules.

And that's the general principle: agents only know a proposition if they are in a position to rationally bet on that proposition at the odds currently being offered to them. In practice, high stakes situations tend to feature bets at long odds, so in practice much knowledge dissipates in high stakes cases. But the explanation of the dissipation is the odds the agent faces, not the stakes.

More precisely, I endorse these principles as constraints on knowledge:

- If the agent knows that *p*, then for any question they have an interest in, the answer to that question is identical to the answer to that question conditional on *p*.
- When an agent is considering the choice between two options, the question of which option has a higher expected utility given their evidence is a question they have an interest in.

Those principles are meant to not merely be extensionally adequate. They are meant to explain why agents lose knowledge when considering some sets of options, like in the Red-Blue game. In some sense, they are meant to be part of reductive explanations. These reductive explanations take as primitive inputs facts about the agent's evidence, and facts about evidential probability. I'm going to set aside worries about the metaphysics of evidential probability, and just focus on evidence. Because it turns out that there is a real problem in getting a plausible theory of evidence that can function as an input to that reductive explanation.

³Jessica Brown (2008, 176) shows that pragmatic encroachment theories that rely just on the stakes involved are subject to serious counterexample. Katherine Rubin (2015) argues that if we have a 'global' version of pragmatic encroachment, where all our epistemic notions are interest-relative, then it is implausible that it is the stakes the subject faces that matter for knowledge. Since I'm defending such a global version of pragmatic encroachment, Rubin's arguments show that it is important that I'm relying on odds, not stakes. Baron Reed (2014) argues that if it is stakes alone that matter to pragmatic encroachment, then agents who the pragmatic encroachment theorist takes to be perfectly rational would be subject to a Dutch Book.

2 The Problems with Evidence

Go back to the red-blue game. Consider a version of the game where:

- The red sentence is that two plus two equals four.
- The blue sentence is something that, if known, would be part of the agent's evidence.

I'm going to argue that there are cases where the only rational play is Red-True, but the blue sentence is something we want to say that, ordinarily, the subject knows. And I'll argue that this is a problem for the kind of reductive explanation I just sketched. If pragmatic effects matter to what the evidence is, we can't take the evidence as a fixed input into an explanation of how and when pragmatic effects matter.

Let's have Parveen play the game again. She's going to be playing the game in a restaurant, one in Ann Arbor where she lives. Just before the game starts, she notices an old friend, Rahul, across the room. Rahul is someone she knows well, and can ordinarily recognise, but she had no idea he was in town. She thought Rahul was living in Italy. Still, we would ordinarily say that she now knows Rahul is in the restaurant; indeed that he is in the restaurant. It would be perfectly acceptable for her to say to someone else, "I saw Rahul here", for example. Now the game starts.

- The red sentence is *Two plus two equals four*.
- The blue sentence is *Rahul is in this restaurant*.

Now we have a problem. On the one hand, there is only one rational play here: Red-True. If you haven't seen someone for a long time, then you can't be completely certain it's them when you spot them across a restaurant. It would be foolish to be as confident that it's Rahul as that two and two make four. It looks like this is a case where pragmatic effects defeat knowledge.

On the other hand, our story for why Parveen loses knowledge here has run into problems. I wanted to tell a story roughly like the following. She can't play Blue-True when the probability of the blue sentence, given her evidence, is less than the probability of the red sentence, given her evidence. That explanation can only go through if the blue sentence is itself not part of her evidence, since the probability of anything given itself is one. So we need a story about how it is that it is not part of Parveen's evidence that Rahul is not in the restaurant.

That story can't be the one that presupposes facts about what is in Parveen's evidence. So it can't use facts about the probability of some proposition given her evidence; at least not in any simple way. If we can independently identify Parveen's

evidence, then we can go back to using evidential probability. But until we've done that, we're stuck.

There are two options here that seem possible for the pragmatic encroachment theorist, but not particularly attractive.

One is to say that propositions like *Rahul is in this restaurant* are never part of Parveen's evidence. Perhaps her evidence just consists of things like *I am being appeared to Rahul-like*. Such an approach is problematic for two reasons. The first is that it is subject to all the usual objections to psychological theories of evidence (Williamson, 2007). The second is that we can re-run the argument with the blue sentence being some claim about Parveen's psychological state, and still get the result that the only rational play is Red-True. A retreat to a psychological conception of evidence will only help with this problem if agents are infallible judges of their own psychological states, and that is not in general true (Schwitzgebel, 2008).

Another option is to deny that a reductive explanation is needed here. Perhaps pragmatic effects, like the particular sentences that are chosen for this instance of the Red-Blue game, mean that Parveen's evidence no longer includes facts about Rahul, but this isn't something we can give a reductive account of. We shouldn't assume that everything will have a simple reductive explanation, so this isn't so bad in theory. The problem in practice is that without a reductive explanation, we don't have a predictive theory of when pragmatic effects matter. And that seems to be a bad thing. For instance, the following theory is completely consistent with Parveen's case as described.

- 1. E=K; i.e., one's evidence is all and only what one knows.
- 2. Someone does not know *p* if the evidential probability of *p* is not close enough to one for current purposes.
- 3. Since it is part of Parveen's evidence that Rahul is in the restaurant, the probability that he is there is one, so it is close enough to one for current purposes.
- 4. So this is not a case where pragmatic effects change what she knows.

That theory seems to me to be badly mistaken, since it goes on to predict that it is rationally permissible to play Blue-True. But we need a pragmatic account that says that it is mistaken, and says something about which alternative situations would not threaten Parveen's knowledge. We don't yet, as far as I can see, have such an account. The aim of the rest of this paper is to provide one.⁴

⁴You can read this paper as a reply to the challenge posed by Ichikawa et al. (2012). They note that there are challenges facing the pragmatic encroachment theorist whether they make evidence interest-relative, or interest-invariant. I'm going to show how to have an interest-relative theory of evidence, and keep what was desirable about pragmatic encroachment theories.

3 A Simple, but Unsatisfying, Solution

Let's take a step back and look at the puzzle more abstractly. We have an agent S, who has some option O, and it really matters whether or not the value of O, i.e., V(O) is at least x. It is uncontroversial that the agent's evidence includes some background K, and controversial whether it includes some contested proposition p. It is also uncontroversial that $V(O|p) \ge x$, and we're assuming that for any proposition q that is in the agent's evidence, V(O|q) = V(O). We'll make one other large assumption. Say there is a prior value function (like the mystical prior probability function) V^- . Then for any choice C, $V(C) = V^-(C|E)$, where E is the evidence the agent has.

Now we're in a position to state the solution. Let p be the proposition that the agent might or might not know, and the question of whether $V(O) \ge x$ be the only salient one that p is relevant to. Then the agent knows p only if the following is true:

$$\frac{V^{-}(O|K)+V^{-}(O|K \wedge p)}{2} \geq x$$

That is, we work out the value of O with and without the evidence p, and if the average is greater than x, good enough!

That solves the problem of Parveen and Rahul. Parveen's evidence may or may not include that Rahul is in the restaurant. If it does, the Blue-True has a value of \$50. If it does not, then Blue-True's value is somewhat lower. Even if the evidence includes that someone who looks a lot like Rahul is in the restaurant, the value of Blue-True might only be \$45. Averaging them out, the value is less than \$50. But you'd only play Blue-True if it was worthwhile it play it instead of Red-True, which is worth \$50. So you shouldn't play Blue-True.

Great! Well, great except for two monumental problems. The first problem is that what we've said here really only helps with very simple cases, where there is a single decision problem that a single contested proposition is relevant to. We need some way to generalise the case to less constrained situations. The second (and in my opinion bigger) problem is that the solution looks completely ad hoc. Why should we use the arithmetic mean of these two things rather than any other formula that would have implied the intuitively correct result in the Parveen-Rahul case? Pragmatic encroachment starts with a very elegant, very intuitive, principle: you only know the things you can reasonable take to be settled for the purposes of current deliberation. It does not look like any such elegant, intuitive, principle will lead to some principle about averaging out the value of an option with and without new evidence.

Happily, the two problems can be given a common solution. But the solution requires a detour into some technical work. It's time for some game theory.

4 Gamifying the Problem

We can usefully think of some philosophical problems as games, and hence subjects for study using game theoretic techniques. This is especially when the problems involve interactions of rational agents. Here, for example, is the game table for Newcomb's problem, with the human who is usually the focus of the problem as Row, and the demon as Column.⁵

	Predict 1 Box	Predict 2 Boxes
Choose 1 Box	1000, 1	0,0
Choose 2 Boxes	1001, 0	1, 1

This game has a unique Nash equilbrium; the bottom right corner.⁶ And that's one way of motivating the view that (a) the game is possible, and (b) the rational move for the human is to choose two boxes.

Let's look at a more complicated game. I'll call it The Interpretation Game. The game has two players. Just like in Newcomb's problem, one of them is a human, the other is a philosophical invention. But in this case the invention is not a demon, but The Radical Interpreter. To know the payouts for the players, we need to know their value function. More colloquially, we need to know their goals.

- The Radical Interpreter assigns mental states to Human in such a way as to predict Human's actions given Human rationality. We'll assume here that evidence is a mental state, so saying what evidence Human has is among Radical Interpreter's tasks. (Indeed, in the game play to come, it will be their primary task.)
- Human acts so as to maximise the expected utility of their action, conditional on the evidence that they have. Human doesn't always know what evidence they have; it depends on what The Radical Interpreter says.

⁵In these games, Row chooses a row, and Column chooses a column, and that determines the cell that is the outcome of the game. The cells include two numbers. The first is Row's payout, and the second is Column's. The games are non-competitive; the players are simply trying to maximise their own returns, not maximise the difference between their return and the other player's return.

⁶A Nash equilibrium is an outcome of the game where every player does as well as they can given the moves of the other players. Equivalently, it is an outcome where no player can improve their payout by unilaterally defecting from the equilibrium.

⁷The Radical Interpreter feels like they should be a humanesque character in *Alice in Wonderland* or *The Phantom Tollbooth*, but for now they are resolutely abstract.

The result is that the game is a coordination game. The Radical Interpreter wants to assign evidence in a way that predicts rational Human action, and Human wants to do what's rational given that assignment of evidence. Coordination games typically have multiple equilibria, and this one is no exception.

Let's make all that (marginally) more concrete. Human is offered a bet on p. If the bet wins, it wins 1 util; if the bet loses, it loses 100 utils. Human's only choice is to Take or Decline the bet. The proposition p, the subject of the bet, is like the claim that Rahul is in the restaurant. It is something that is arguably part of Human's evidence. Unfortunately, it is also arguable that it is not part of Human's evidence. We will let K be the rest of Human's evidence (apart from p, and things entailed by $K \cup \{p\}$), and stipulate that Pr(p|K) = 0.9. Each party now faces a choice.

- The Radical Interpreter has to choose whether *p* is part of Human's evidence or not.
- Human has to decide whether to Take or Decline the bet.

The Radical Interpreter achieves their goal if human takes the bet iff p is part of their evidence. If p is part of the evidence, then The Radical Interpreter thinks that the bet has positive expected utility, so Human will take it. And if p is not part of the evidence, then The Radical Interpreter thinks that the bet has negative expected utility, so Human will decline it. Either way, The Radical Interpreter wants Human's action to coordinate with theirs. And Human, of course, wants to maximise expected utility. So we get the following table for the game.

$$\begin{array}{c|cccc} & p \in E & p \notin E \\ \hline \text{Take the bet} & 1, 1 & -9.1, 0 \\ \text{Decline the bet} & 0, 0 & 0, 1 \\ \end{array}$$

We have, in effect, already covered The Radical Interpreter's payouts. They win in the top-left and lower-right quadrants, and lose otherwise. Human's payouts are only a little trickier. In the bottom row, they are guaranteed 0, since the bet is declined. In the top-left, the bet is a sure winner; their evidence entails it wins. So they get a payout of 1. In the top-right, the bet wins with probability 0.9, so the expected return of taking it is $1 \times 0.9 - 100 \times 0.1 = -9.1$.

There are two Nash equilibria for the game - I've bolded them below.

Take the bet
$$p \in E$$
 $p \notin E$ Decline the bet $0, 0$ $0, 1$

The mathematical result that there are two equilibria to this game should not come as a surprise. In discussing games like thie earlier, we said that general principles connecting evidence, knowledge and action are not predictive; they are consistent both with p being part of the evidence, and with it not being part of the evidence. The general principles we had stated rule out, in effect, non-equilibrium solutions to games like this one. But they are not predictive in cases where there are multiple equilibria.

To make more progress, we need to turn to more contested areas of game theory. In particular, we need to look at some work on equilibrium choice. We'll introduce this material via a game that is inspired by an example of Rousseau's.

Equilibrium Selection Principles

At an almost maximal level of abstraction, a two player, two option each game looks like this.

$$\begin{array}{c|cccc} & a & b \\ \hline A & r_{11}, c_{11} & r_{12}, c_{12} \\ B & r_{21}, c_{21} & r_{22}, c_{22} \end{array}$$

We're going to focus on games that have the following eight properties:

- $r_{11} > r_{21}$
- $r_{22} > r_{12}$
- $c_{11} > c_{12}$
- $c_{22} > c_{21}$
- $r_{11} > r_{22}$
- $c_{11} \geq c_{22}$
- $\begin{array}{ccc}
 & \frac{r_{21}+r_{22}}{2} > \frac{r_{11}+r_{12}}{2} \\
 & \frac{c_{12}+c_{22}}{2} > \frac{c_{11}+c_{21}}{2}
 \end{array}$

The first four clauses say that the game has two (strict) Nash equilibria: Aa and Bb. The fifth and sixth clauses say that the Aa equilibria is Pareto-optimal: no one prefers the other equilibria to it. In fact it says something a bit stronger: one of the players strictly prefers the Aa equilibria, and the other player does not prefer *Bb*. The seventh and eighth clauses say that the *Bb* equilibria is **risk-optimal**. Riskoptimality is a somewhat complicated notion in general; see Harsanyi and Selten (1988) for more details. But for our purposes, we can focus on a simple characterisation of it. Neither player would prefer playing A/a to playing B/b if they thought it was a coin flip which equilibrium the other player was aiming for.

I'm going to offer an argument from Hans Carlsson and Eric van Damme (1993) for the idea that in these games, rational players will end up at Bb. The game that Human and The Radical Interpreter are playing fits these eight conditions, and The Radical Interpreter is perfectly rational, so this will imply that in that game, The Radical Interpreter will say that $p \notin E$, which is what we aimed to show.

Games satisfying these eight inequalities are sometimes called *Stag Hunt* games. There is some flexibility, and some vagueness, in which of the eight inequalities need to be strict, but that level of detail isn't important here. The name comes from a thought experiment in Rousseau's *Discourse on Inequality*.

[T]hey were perfect strangers to foresight, and were so far from troubling themselves about the distant future, that they hardly thought of the morrow. If a deer was to be taken, every one saw that, in order to succeed, he must abide faithfully by his post: but if a hare happened to come within the reach of any one of them, it is not to be doubted that he pursued it without scruple, and, having seized his prey, cared very little, if by so doing he caused his companions to miss theirs. (Rousseau, 1913, 209–10)

It is rather interesting to think through which real-life situations are best modelled as Stag Hunts, especially in situations where people have thought that the right model was a version of Prisoners' Dilemma. This kind of thought is one way in to appreciating the virtues of Rousseau's political outlook, and especially the idea that social coordination might not require anything like the heavy regulatory presence that, say, Hobbes thought was needed. But that's a story for another day. What we're going to be interested in is why Rousseau was right to think that a 'stranger to foresight', who is just focussing on this game, should take the rabbit.

To make matters a little easier, we'll focus on a very particular instance of Stag Hunt, as shown here. (From here I'm following Carlsson and van Damme very closely; this is their example, with just the labelling slightly altered.)

$$\begin{array}{c|cccc} & a & b \\ \hline A & 4, 4 & 0, 3 \\ B & 3, 0 & 3, 3 \end{array}$$

At first glance it might seem like Aa is the right choice; it produces the best outcome. This isn't like Prisoners Dilemma, where the best collective outcome is dominated. In fact Aa is the best outcome for each individual. But it is risky, and

Carlsson and van Damme show how to turn that risk into an argument for choosing *Bb*.

Embed this game in what they call a *global game*. We'll start the game with each player knowing just that they will play a game with the following payout table, with x to be selected at random from a flat distribution over [-1, 5].

$$\begin{array}{c|cccc}
 & a & b \\
\hline
A & 4, 4 & 0, x \\
B & x, 0 & x, x
\end{array}$$

Before they play the game, each player will get a noisy signal about the value of x. There will be signals s_R and s_C chosen (independently) from a flat distribution over [x-0.25, x+0.25], and shown to Row and Column respectively. So each player will know the value of x to within $\frac{1}{4}$, and know that the other player knows it to within $\frac{1}{4}$ as well. But this is a margin of error model, and in those models there is very little that is common knowledge. That, they argue, makes a huge difference.

In particular, they prove that iterated deletion of strictly dominated strategies (almost) removes all but one strategy pair.⁸ Each player will play A/a if the signal is greater than 2, and B/b otherwise.⁹ Surprisingly, this shows that players should play the risk-optimal strategy even when they know the other strategy is Pareto-optimal. When a player gets a signal in (2, 3.75), then they know that x < 4, so Bb is the Pareto-optimal equilibrium. But the logic of the global game suggests the risk-dominant equilibrium is what to play.

Carlsson and van Damme go on to show that many of the details of this case don't matter. As long as (a) there is a margin of error in each side's estimation of the payoffs, and (b) every choice is a dominant option in some version of the global game, then iterated deletion of strongly dominant strategies will lead to each player making the risk-dominant choice.

I conclude from that that risk-dominant choices are rational in these games. There is a limit assumption involved here; what's true for games with arbitrarily small margins of error is true for games with no margin of error. (We'll come back to that assumption below.) And since The Radical Interpreter is rational, they will play the strategy that is not eliminated by deleting dominant strategies. That is, they will play the risk-dominant strategy.

In the case of Human, that means they will say that $p \notin E$. And in the case of Parveen and Rahul, that means they will say that it is not part of Parveen's evidence that Rahul is in the restaurant. And this is an interest-relative theory of evidence;

⁸A sketch of the proof is in Appendix One.

⁹Strictly speaking, we can't rule out various mixed strategies when the signal is precisely 2, but this makes little difference, since that occurs with probability 0.

had Parveen been playing a different game, The Radical Interpreter would have said that it is part of Parveen's evidence that Rahul was in the restaurant.

And from this point we can say all the things we wanted to say about the case. If it is part of Parveen's evidence that Rahul is in the restaurant, then she knows this. Conversely, if she knows it, then The Radical Interpreter would have said it is part of her evidence, so it is part of her evidence. Parveen will perform the action that maximises expected utility given her evidence. And she will lose knowledge when that disposition makes her do things that would be known to be sub-optimal if she didn't lose knowledge.

In short, this model gives us a way to keep what was good about the pragmatic encroachment theory, while also allowing that evidence can be interest-relative. It does require a slightly more complex theory of rationality than we had previously used. Rather than just say that agents maximise evidential expected utility, we have to say that they play risk-dominant strategies in coordination games. But it turns out that this is little more than saying that they maximise evidential expected utility, and they expect others (at least perfectly rational abstract others) to do the same, and they expect those others to expect they will maximise expected utility, and so on.

6 Objections and Replies

We'll end the body of the paper with some objections that might be raised to this model. And then the appendix will contain proofs of a couple of the formal claims.

Objection: The formal result of the previous section only goes through if we assume that the agents do not know precisely what the payoffs are in the game. We shouldn't assume that what holds for arbitrarily small margins of error will hold in the limit, i.e., when they do know the payoffs.

Reply: I think it's basically fine to use limit assumptions like this to resolve hard cases like Stag Hunt. But even if you don't think that's true in general, note that we don't need to make such an assumption here. What we really need is that Parveen doesn't know precisely the probability of Rahul being in the restaurant given the rest of her evidence. Given that evidence is not luminous, as Williamson (2000) shows, this is a reasonable assumption. So the margin of error assumption that Carlsson and van Damme make is not, in our case, an assumption that merely makes the math easier; it is built into the case.

Objection: Even if Parveen doesn't know the payoffs precisely, The Radical Interpreter does. They are an idealisation, so they can be taken to be ideal.

Reply: It turns out that Carlsson and van Damme's result doesn't require that both parties are ignorant of the precise values of the payoffs. As long as one party

doesn't know the exact value of the payoff, the argument goes through. I prove this in Appendix Two.

Objection: The formal argument requires that in the 'global game' there are values for x that make A the dominant choice. These cases serve as a base step for an inductive argument that follows. But in Parveen's case, there is no such setting for x, so the inductive argument can't get going.

Reply: What matters is that there are values of x such that A is the strictly dominant choice, and Human (or Parveen) doesn't know that they know that they know, etc., that those values are not actual. And that's true in our case. For all Human (or Parveen) knows that they know that they know that they know..., the proposition in question is not part of their evidence under a maximally expansive verdict on The Radical Interpreter's part. So the relevant cases are there in the model, even if for some high value of n they are known not to obtain.

Objection: This model is much more complex than the simple motivation for pragmatic encroachment.

Reply: Sadly, this is true. I would like to have a simpler model, but I don't know how to create one. The argument I gave earlier that our simple principles underdetermine what to say in cases like Parveen and Rahul's seems fairly compelling. So more complexity will be needed, one way or another. I think paying this price in complexity is worth it overall, but I can see how some people might think otherwise.

Objection: Change the case involving Human so that the bet loses 15 utils if p is false, rather than 100. Now the risk-dominant equilibrium is that Human takes the bet, and The Radical Interpreter says that p is part of Human's evidence. But note that if it was clearly true that p was not part of Human's evidence, then this would still be too risky a situation for them to know p. So whether it is possible that p is part of Human's evidence matters.

Reply: This is all true, and it shows that the view I'm putting forward is incompatible with some programs in epistemology. I don't think this is a problem because I don't buy into those programs, but it is a downside of the view. The worry is that how long the odds have to be for the practical situation to destroy knowledge differ depending on whether the proposition is (a) known, but for the practical situation, or (b) part of the agent's evidence, but for the practical situation. And that's incompatible, I think, with a strong version of the knowledge first program that reduces evidence to knowledge. I think it's perfectly plausible that propositions that are plausibly evidential are treated differently than those which are not, but if you don't think that, you shouldn't like the view defended here.

Objection: Carlsson and van Damme discuss one kind of global game. But there are other global games that have different equilibria. For instance, changing the method by which the noisy signal is selected would change the equilibrium of the global game. So this kind of argument can't show that the risk-dominant equilibrium is the one true solution.

Reply: This is somewhat true. There are other ways of embedding the game involving Human and The Radical Interpreter in global games that lead to different outcomes. They are usually somewhat artificial; e.g., by having the signal be systematically biased in one way. But what really matters is the game where the error in Human's knowledge of the payoffs is determined by their actual epistemic limitations. I think that will lead to something like the model we have here. But it is possible that the final result will differ a bit from what I have here, or (more likely) have some indeterminacy about just how interests interact with evidence and knowledge. The precise details are ultimately less important to me than whether we can provide a motivated story of how interests affect knowledge and evidence that does not presuppose we know what the agent's evidence is. And the method I've outlined here shows that we can do that, even if we end up tinkering a bit with the details.

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7 Appendix One: Carlsson and van Damme's Game

I haven't finished writing out the proofs yet, but here's a sketch of how they will go. I will **not** be presenting the proofs at the workshop in any case.

- The game is symmetric, so we'll just focus on showing that rationality requires that $T_R = 2$; it will follow that $T_C = 2$
- The only rule we will use is iterated deletion of strictly dominated strategies.
- The return to a strategy pair is uncertain; we'll say it is the expected value of the global game.
- T_R = 4.25 strictly dominates any strategy where T_R = y > 4.25, since it has a higher guaranteed return when $s_r \in [4.25, y]$, and the same return otherwise, and there is some chance that $s_r \in [4.25, y]$. So we can delete all strategies T_R = y > 4.25, and similarly all strategies for Column that have a tipping point above 4.25.
- If $s_r \in [-0.75, 4.75]$, then it is equally likely that x is above s_r as it is below it. Indeed, the posterior distribution of x is flat over $[s_r 0.25, s_r + 0.25]$.
- From this it follows that the expected return of A is s_r .
- For any y > 2, assume we know that $T_C \le y$. Then the expected return of playing B after seeing $s_r = y$ will be at most 2. That's because the probability of Column playing b will be at most 0.5. So given that $T_C \le y$, there will be some value ε such that $T_R = y \varepsilon$ dominates $T_R = y$. (And the same goes for Column.)
- Repeating this reasoning shows that iterative deletion of strictly dominated strategies rules out any tipping points other than $T_C = T_R = 2$.

 And Carlsson and van Damme show that we can generalise this result to other games; in general the risk-dominant strategy is the only one that survives deletion of strictly dominated strategies when it is embedded in a global game.

8 Appendix Two: The Modified Game

In this appendix I'll do the proof that Carllson and van Damme's result goes through even when one of the players gets perfect information about the payoff structure. The proof is similar to the previous proof, except we need to use different steps to do the dominance reasoning.

- For all y > 2, if we know that $T_R < y$, then we can find a z < y such that $T_C = z$ dominates any strategy with a higher payoff. (And the size of y z is great enough that repeated iteration won't lead to some asymptote north of 2.)
- For all y > 2, if we know that $T_C = y$, then $T_R = y$ dominates any strategy with $T_R > y$.
- So the iterated deletion has a zig-zag shape. We use the first bullet point to pull T_C down towards 2, then use the second bullet-point to have T_R 'catch up', then once T_R is caught up, we can lower T_C again, and so on until we get $T_C = T_R = 2$.