

# On the Incompatibility of Dynamical Biological Mechanisms and Causal Graphs

Marcel Weber

## **Abstract**

I examine to what extent accounts of mechanisms based on formal interventionist theories of causality can adequately represent biological mechanisms with complex dynamics. Using a differential equation model for a circadian clock mechanism as an example, I first show that there exists an iterative solution that can be interpreted as a structural causal model. Thus, in principle it is possible to integrate causal difference-making information with dynamical information. However, the differential equation model itself lacks the right modularity properties for a full integration. A formal mechanistic model will therefore either have to leave out non-causal or causal explanatory relations.

## 1. Introduction

Recently, several authors have tried to show that devices such as causal graphs and Bayesian networks can be used for providing a formal account of mechanisms (Casini et al. 2011; Clarke, Leuridan, and Williamson 2014; Gebharder 2014; Gebharder and Kaiser 2014; Gebharder and Schurz 2016 in this volume; Casini 2016 in this volume). In contrast to traditional, qualitative thinking about mechanisms (e.g., Wimsatt 1976; Glennan 1996; Machamer, Darden, and Craver 2000), this new approach allows the incorporation of quantitative information in the representation of mechanisms. Indeed, quantitative information concerning in particular the dynamics of mechanistic processes is thought to be essential for explaining certain biological phenomena (Bechtel and Abrahamsen 2010). However, the mathematical tools used for representing this kind of information – typically differential equations – are different from those underlying formal accounts of causality, including the tools already mentioned as well as structural equations. This raises the question of how dynamical information (as well as spatial information, see Kaiser 2016 in this volume) can be assimilated into the formal causal framework, which basically only contains causal difference-making information.

Woodward (2013) has argued that dynamical as well as spatial information can always be integrated with causal difference-making information in order to represent complex mechanisms. In this paper, I want to examine to what extent such integration as imagined by Woodward is possible. My focus will be on mechanistic biological models with complex dynamics, while Kaiser (2016 in this volume) deals with mechanisms featuring complex spatial structures. Examining Goldbeter's (1995) differential equation model of an oscillating circadian clock mechanism with causal feedback, I will first show that there exists an iterative causal structural model that incorporates the relevant quantitative dynamical information. Thus, Woodward's claim about integration is partially confirmed. The victory is a partial one, however, because the iterative model is only an approximation and works with discrete time. Furthermore, in the iterative model all time derivatives of the concentration variables, which seem to play a crucially important explanatory role in the differential model, drop out of the picture.

So why not consider the differential equation model *itself* as a structural causal model? The problem with this, as I will show, is that it does not have the right modularity properties. Thus, the causal modeler is caught in a dilemma: Either she must consider only the approximate model as causal and the exact model as non-causal or she must admit that a formal causal model does not adequately represent the causal content of the exact differential model. Accepting either horn of the dilemma amounts to admitting an important limitation to the formal causal modeling approach to mechanisms as well as Woodward's integration strategy.

I shall proceed as follows. In Section 2, I shall briefly review the core notions used in the causal modeling literature, in particular the notion of a structural causal model. In Section 3, I analyze a dynamical model of a circadian clock mechanism that uses differential equations

and show that, while it has an approximate solution that can be represented as a structural causal model, the model itself cannot be so represented. In Section 4, I consider possible replies on behalf of the formal approach. Section 5 summarizes and integrates my conclusions with regard to the limitations of causal modeling of biological mechanisms.

## 2. Structural Causal Models

According to Pearl (2000), a structural causal model consists of an ordered triple  $\langle U, V, Q \rangle$  where  $U$  is a set of exogenous variables,  $V$  a set of endogenous variables, and  $Q$  a set of structural equations. These equations give the value of each endogenous variable as a function of the values of other variables in  $U$  and  $V$ . The variables may also be interpreted as nodes or vertices that are connected by directed edges in a causal graph. But in contrast to pure causal graphs, the structural equations also provide quantitative information as to how much some dependent variables change per unit change of the independent variable.

Pearl (2000, p. 160) gives the following "operational" definition of a structural equation:

An equation  $y = \beta x + \varepsilon$  is said to be structural if it is to be interpreted as follows: In an ideal experiment where we control  $X$  to  $x$  and any other set  $Z$  of variables (not containing  $X$  or  $Y$ ) to  $z$ , the value  $y$  of  $Y$  is given by  $\beta x + \varepsilon$ , where  $\varepsilon$  is not a function of the settings  $x$  and  $z$ .

According to this definition, it is obvious that structural equations *sensu* Pearl are linear equations in the sense of not containing derivatives of the variables. As we shall see, this feature constitutes a major limitation when it comes to modeling systems with complex dynamics.

Pearl's definition of a structural equation contains the idea of an "ideal experiment". This notion has been elaborated in great detail by Woodward (2003, 94-99), who defines it in terms of the notion of ideal intervention. On this account, an ideal intervention on some variable  $X$  with respect to some variable  $Y$  changes  $Y$  by changing  $X$  without changing any other variable that is a cause of  $Y$ , except such variables that lie on a path between  $X$  and  $Y$ .

In the following section, I show what problems are created for such structural causal models by dynamical mechanism of a kind that occurs frequently in biology, but also in physics and in other sciences. Kaiser (2016 in this volume) does the same for spatially complex mechanisms. Thus, while Kaiser's paper is about space, this one is about time.

### 3. It's About Time: Modeling Dynamic Processes

#### 3.1. Classic Examples of Dynamic Models in Biology

There is an important class of biological models that try to account for complex series of events in dynamical terms. A classic example is the Hodgkin-Huxley model of the action potential (Weber 2005, 2008). This model shows how changes in membrane conductance generate a temporary membrane depolarization that can spread along an axonal membrane and thus form the basis of information processing by neurons. A more recent example is Goldbeter's (1995) model of the circadian oscillations of the PER protein in *Drosophila*, which is the heart of a circadian clock mechanism that is found also in other organisms.<sup>1</sup> There are many more such models, but for the purposes of this paper I shall concentrate on the latter example.

#### 3.2. Bechtel and Abrahamsen on Dynamic Mechanistic Explanation

In a recent series of papers, Bill Bechtel and Adele Abrahamsen have provided a very illuminating account of models and mechanisms in circadian clock research, including Goldbeter's model and the PER circadian clock mechanism (Bechtel and Abrahamsen 2010; Bechtel 2013). Their account will prove to be useful for my analysis, which is why it will be briefly reviewed here. I take the gist of their account to be that circadian clock models provide what they call "dynamic mechanistic explanations". According to Bechtel and Abrahamsen, such explanations differ from other kinds of mechanistic explanations in providing quantitative information about the behavior of the systems in question. Dynamic mechanistic explanations often start with the identification of a number of parts and their sequential operations that could produce a phenomenon. For example, the PER circadian clock mechanism involves a nuclear gene called *per* for period first discovered in *Drosophila* mutants with altered circadian regulation. It is known that this gene makes a protein called PER, which is a transcriptional regulator that can, via some intermediate steps, inhibit its expression by binding to its own gene in the cell nucleus. Thus, there is a regulatory feedback loop, which may already be suspected to show oscillatory behavior.

An interesting feature of this sequential model according to Bechtel and Abrahamsen is the fact that it is possible to mentally rehearse the individual steps as well as their temporal arrangement. However, such a mental simulation fails to explain how the circadian system is capable of generating *stable* oscillations of a certain frequency. This is where the dynamical, quantitative model constructed by Goldbeter (1995) comes in. The model describes the change

---

<sup>1</sup> Here, I will consider only the model proposed by Goldbeter in 1995. Of course, research in chronobiology has advanced enormously since, revealing many more factors that are involved in circadian regulation. I trust that the foundational issues to be examined here are basically the same as for more complex models.

in cytoplasmic concentrations of PER mRNA ( $M$ ) as well as the different phosphorylation states of cytoplasmic ( $P_0, P_1, P_2$ ) as well as nuclear ( $P_N$ ) PER protein with the help of differential equations. The model uses standard Michaelis-Menten enzyme kinetics where the  $V_i$  are maximal reaction rates and the  $K_i$  the so-called Michaelis constants for the different biochemical reactions involved (the Michaelis constant gives the substrate concentration at which the reaction rate is half the maximal rate).

Goldbeter wrote down the reaction rates for the different molecular species as follows (I use the dot notation for time derivatives):

$$\begin{aligned}
 (1a) \quad \dot{M} &= v_s \frac{K_I^n}{K_I^n + P_N^n} - v_m \frac{M}{K_m + M} \\
 (1b) \quad \dot{P}_0 &= k_s M - V_1 \frac{P_0}{K_1 + P_0} + V_1 \frac{P_1}{K_2 + P_1} \\
 (1c) \quad \dot{P}_1 &= V_1 \frac{P_0}{K_1 + P_0} - V_2 \frac{P_1}{K_2 + P_1} - V_3 \frac{P_1}{K_3 + P_1} + V_4 \frac{P_2}{K_4 + P_2} \\
 (1d) \quad \dot{P}_2 &= V_3 \frac{P_1}{K_3 + P_1} - V_4 \frac{P_2}{K_4 + P_2} - k_1 P_2 + k_2 P_N - v_d \frac{P_2}{K_d + P_2} \\
 (1e) \quad \dot{P}_N &= k_1 P_2(t_i) - k_2 P_N(t_i)
 \end{aligned}$$

Using numerical integration techniques, Goldbeter was able to show that for some parameter values there is indeed a limit cycle, in other words, a stable oscillation of the concentrations of mRNA and PER protein.

Bechtel and Abrahamsen stress that without this quantitative model, the sequential model provides no explanation for the *stability* of the circadian behavior. Without introducing quantitative parameters, the sequential model could produce all kinds of behavior, only some of which generate a limit cycle.

I will examine now how the sequential and dynamical models can be represented as structural causal models.

### 3.3 The Sequential Model as a Structural Causal Model

I shall first attempt to represent the sequential model within this causal framework. There is an apparent difficulty in that the sequential model is cyclical whereas causal graphs are acyclical. However, this problem is not new and solutions have been proposed by several authors (Kistler 2013; Gebharder and Kaiser 2014; Clarke, Leuridan, and Williamson 2014). Briefly, one way of doing this is by introducing a time index on some of the nodes of the causal graph structures. When a system comes to the end of a cycle, time has passed. This new state of the system should thus be represented by a different node, a variable that represents the state of the system at a later time. This way, the cyclical path is broken up and "rolled out" in time and presents no problems for the causal modeler.

However, it should be clear that such a causal graph fails to explain the explanandum phenomenon (stable circadian oscillations), because essential dynamical information is missing. The graph would merely represent what I referred to as the sequential model. In the next section, I shall examine how the dynamical model could be represented.

### 3.4 The Dynamical Model as a Structural Causal Model: An Iterative Solution

Could the same strategy that works for the sequential model also be used for representing Goldbeter's dynamical model by using causal graphs? Indeed, it could be suggested that the causal structure of the model is captured by the following time-indexed causal graph (Fig. 1):

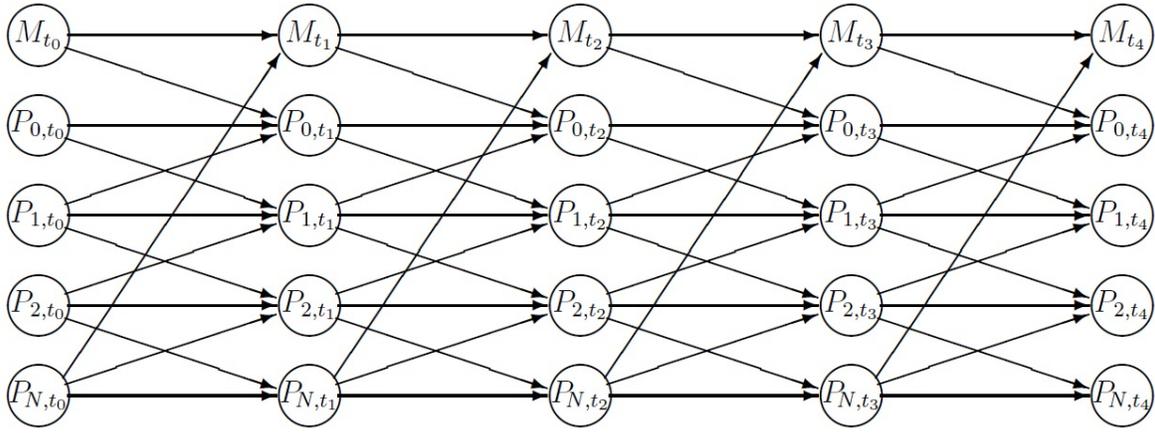


Figure 1. Proposed time-indexed DAG representing the causal dependencies in the Goldbeter model. The vertices of the graph represent the concentrations of the different molecular species at different discrete time points, time moving from left to right.

It could be argued that this DAG contains all the causal relations posited by the Goldbeter model. But how can we incorporate the necessary quantitative information into this causal model? I suggest that this could be done by writing down rules for updating the values of the salient variables from each discrete time point to the next:

$$(2a) \quad M(t_{i+1}) = M(t_i) + \left[ v_s \frac{K_I^n}{K_I^n + P_N^n(t_i)} - v_m \frac{M(t_i)}{K_m + M(t_i)} \right] \Delta t$$

$$(2b) \quad P_0(t_{i+1}) = P_0(t_i) + \left[ k_s M(t_i) - V_1 \frac{P_0(t_i)}{K_1 + P_0(t_i)} + V_1 \frac{P_1(t_i)}{K_2 + P_1(t_i)} \right] \Delta t$$

$$(2c) \quad P_1(t_{i+1}) = P_1(t_i) + \left[ V_1 \frac{P_0(t_i)}{K_1 + P_0(t_i)} - V_2 \frac{P_1(t_i)}{K_2 + P_1(t_i)} - V_3 \frac{P_1(t_i)}{K_3 + P_1(t_i)} + V_4 \frac{P_2(t_i)}{K_4 + P_2(t_i)} \right] \Delta t$$

$$(2d) \quad P_2(t_{i+1}) = P_2(t_i) + \left[ V_3 \frac{P_1(t_i)}{K_3 + P_1(t_i)} - V_4 \frac{P_2(t_i)}{K_4 + P_2(t_i)} - k_1 P_2(t_i) + k_2 P_N(t_i) - v_d \frac{P_2(t_i)}{K_d + P_2(t_i)} \right] \Delta t$$

$$(2e) \quad P_N(t_{i+1}) = P_N(t_i) + [k_1 P_2(t_i) - k_2 P_N(t_i)] \Delta t$$

This iterative model computes, for each molecular species, the concentration increment from time  $t_i$  to  $t_{i+1} = t_i + \Delta t$  by approximating the exact time integral for the concentration function  $\int_{t_i}^{t_{i+1}} \dot{M}(t) dt$  by  $M(t_i) + \Delta M \Delta t$  and for the other functions accordingly. In fact, this is also the standard approach to numerically integrating differential equations such as the ones that constitute the Goldbeter model, which have no analytic solution. In this approach, the  $\Delta t$ s can be made sufficiently small for the iterative model to be a good approximation for the continuous function.

This iterative model corresponds to the proposed causal graph (Figure 1) and appears to satisfy all the conditions for a structural causal model. Furthermore, the iterative model provides a way for numerically integrating the differential model devised by Goldbeter and thus for testing it. The system of differential equations as such is not analytically solvable and therefore cannot be used for making predictions. Thus, the differential Goldbeter model can only be tested with the help of the iterative model.

In any case, the iterative model shows that it is in principle possible to integrate dynamical information and causal difference-making information to some extent. As we have seen, the structural equations of the iterative model represent the dynamics as well as the causal dependencies of the PER mechanisms. However, the iterative model suffers from at least three different drawbacks. First, it is an approximation and uses discrete time intervals instead of continuous time. While the time intervals can be made arbitrarily short, thus minimizing the prediction error, it remains a discrete-time model and a coarse-grained description of reality. The causal structure shown in Figure 1 may only show up in this coarse-grained picture. Second, the time derivatives of the concentration variables, which seem to play a crucial explanatory role in the differential model, have completely dropped out in the iterative model. This constitutes a potential loss of an explanatorily salient property. Third, the iterative model is *derived* with respect to the differential model, the latter being in this sense more fundamental. Furthermore, it is the differential model that explanatorily connects the mechanism to general theories of biophysical chemistry, in particular enzyme kinetics. If the formal approach could not represent the differential model itself, it would be missing a lot. Therefore, I shall now examine if it could also be interpreted as a structural causal model.

### 3.5 The Differential Model Itself as a Structural Causal Model

The question is now whether the differential equations constituting the Goldbeter model could themselves be interpreted as a structural causal model. The DAG associated with such a model, presumably, might look as shown in Figure 2.

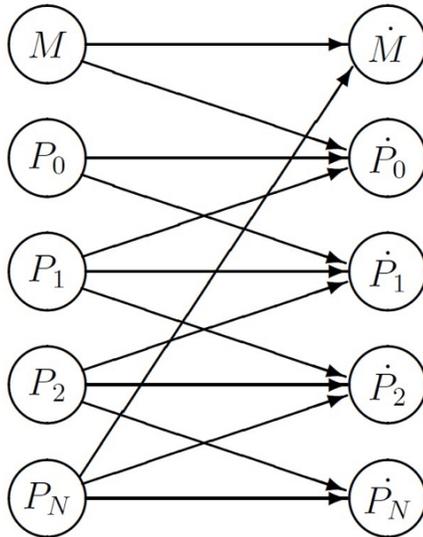


Figure 2. Proposed DAG for the differential model. The dots on the variables indicate time derivatives.

Interpreting Goldbeter's equations causally in the way indicated in Figure 2 requires that we can somehow write the equations in the form of structural equations, e.g., in the sense of Pearl (2000). That this cannot just be done straightforwardly should be clear from the fact that such an equation requires exogenous variables, that is, variables that are controlled from outside the mechanism. Mathematically, a variable in a set of equations may be considered as exogenous if its time derivative does not depend on any other variable in the system. Obviously, none of the concentration variables in the Goldbeter model satisfies this requirement. Thus, at least some of the equations need to be replaced. In fact, several techniques have been published in the machine learning literature for deriving causal graphs and structural equations from differential equations.

A classical approach is the causal ordering algorithm developed by Herbert Simon. Using this approach, Iwasaki and Simon (1994) propose two strategies for inferring causal relations from differential equations, namely "equilibrating" and "exogenizing". The first method is available for systems that contain equations that restore equilibrium very quickly after a disturbance compared to other equations. The second method may work when there are equations that change so slowly compared to the others that they may be replaced by a fixed value. There are several reasons why none of these approaches give a satisfactory solution for the Goldbeter model. First, there are no equations that change at a significantly different rate. In particular the dynamic equilibrium is characterized by all concentration variables oscillating at the same frequency. Second, these techniques introduce approximations and idealizations of

their own and will thus be subject to the same limitations as my iterative solution from the previous section.

Another technique consists in introducing brute-force interventions by setting time derivatives to zero and the corresponding variable to a fixed value (e.g., Mooij, Janzing and Schölkopf 2013). In this approach, at least one of the equations in a system of differential equations  $dX_i/dt = f_i(X_i)$  is replaced by an equation of the form  $X_i = \xi_i$  where  $\xi_i$  is a constant (thus  $dX_i/dt = 0$ ). This is the equivalent of an ideal intervention in ordinary structural equations. What this yields are structural equations describing static equilibrium states for the system as well as the effects that setting variables to fixed values has on these equilibrium states. Any dynamical behavior (i.e., behavior where the system still undergoes change) is thus lost, including in particular the dynamic equilibria.<sup>2</sup> Thus, these structural models do not represent the causal relations that operate while the system is undergoing change; they merely represent the effects that certain interventions have on a system that has been frozen in some state.

Are these limitations just inherent in the methods used by causal modelers so far, or are there fundamental reasons for thinking that some differential equation models resist a full representation as structural causal models? I will argue now that the latter is the case.

The main problem is that the Goldbeter equations do not have the right modularity properties that are required by structural causal models. Modularity requires that it be possible to intervene on each one of the equations that describe a causal structure without changing any other equations (Woodward 2003, 48-49, 327-39). The question of whether modularity is a necessary feature of any causal model has been debated; several authors have presented counter-examples (e.g., Cartwright 2001; Mitchell 2009). Kuorikoski (2012) has shown that any causal model must at least be *modular in variables*. This means that an intervention on one of the cause variables must not change the parameter set nor the functional forms of the equations in the model. Failure of variable modularity prevents any prediction of the effects of the intervention, which is the *raison d'être* of a causal model according to interventionism. Without such predictions, the causal model cannot answer “what if things had been different”-questions. But the ability to answer such questions is what defines a model as causal according to the interventionist approach.

I claim that the differential Goldbeter model lacks variable modularity. To see this, consider, for example, equations (1b) and (1c). Suppose we wished to intervene on  $P_0$  in (1c) in order to manifest the causal dependence of  $dP_1/dt$  on  $P_0$ . We could try to set  $P_0$  to an exogenously determined value  $\xi_0$  and keep it fixed at this value. But to do this, we would have

---

<sup>2</sup> It may be possible to model the effects of interventions on certain parameters on the dynamic equilibria. For example, Goldbeter (1995) has shown that modifying the decay rate for PER protein has a strong effect on the period of the oscillation. This kind of intervention seems to be representable by a causal structural model. However, this kind of model only describes the causal role of certain parameters, not of the variables of the model.

to set  $dP_0/dt = 0$  and thus wipe out (1b), which constitutes a violation of (variable) modularity. Also, this would freeze the system in a static equilibrium and thus obliterate the explanandum phenomenon (circadian oscillation). Alternatively, we could try to push  $P_0$  to some value  $\xi_0$  at some time  $t_0$  (e.g., by adding protein) and then let it go and let (1b) determine the value of  $dP_0/dt$  for all  $t > t_0$ . Let's call this a "push and let go" (PLG) intervention. While a PLG intervention might preserve the functional forms of the other equations, it would not enable us to predict the effect of this intervention because this would require an analytic solution to the differential equation system, which demonstrably doesn't exist. Of course, we could use a numerical integration procedure to make the prediction but this would mean that we would use the iterative and not the differential model in order to determine the causal structure. PLG interventions on the differential model itself do not provide any *quantitative* causal information. But this is the kind of information we would need to fully integrate the differential model with causal difference-making information.

The failure of modularity is even more dramatic for some other variables. For instance, suppose we wished to intervene on  $P_N$  in eqn. (1a) such as to manipulate the value of  $dM/dt$ . This should manifest the causal feed-back of the PER protein in the transcription of its own gene, which is a crucial feature of the model. But because we cannot manipulate  $P_N$  independently of  $dP_N/dt$ , we would either have to replace eqn. (1e) by a different equation, for example  $dP_N/dt=0$ , thus freezing the system, or accept that  $dP_N/dt$  is changed along to some value consistent with (1e) by a PLG intervention. Thus, once again, it would in principle not be determinable quantitatively what effect this intervention has on the subsequent evolution of the concentration variables by using only the differential model. These differential equations just don't work like structural equations.

I suggest that this failure of variable modularity prohibits a full assimilation of the dynamical information provided in the differential model into the formal causal framework. Thus, formal causal models and dynamical equation models remain complementary, but not fully compatible perspectives that both contribute to our understanding of the biological phenomena.

In the following section, I will consider the possible replies to my argument open to proponents of the formal approach to mechanisms.

#### **4. Possible Replies On Behalf of the Formal Approach to Mechanisms**

An obvious reply open to the structural causal modeler consists in denying that the differential equations are even contenders for representing causal dependencies. For starters, the differential equations express synchronic and not diachronic dependencies. Causation as traditionally understood is diachronic. In a similar vein, it can be argued that equations need to be integrated in order to predict or explain physical events. Surely, when we want to discuss the causal content of models such as Goldbeter's we have to consider suitably integrated forms

of the equations. As the model cannot be analytically integrated, we have to consider an approximation such as the iterative model that I have discussed in Section 3.4. Indeed, as we have seen, this model qualifies perfectly as a structural causal model. This model thus reveals all the causal relations that the dynamical mechanism contains, so all is well.

My answer to this reply is the following: If this is correct, then what is the *differential* Goldbeter model for? Is it just a set of equations that can be used to generate a causal model? If it has no causal content itself, as this reply contends, then what *is* its content? Perhaps it could be suggested that, indeed, the differential model itself makes no causal claims, it just states some lawful dependencies between the variables. While this would be an interesting claim (which I won't try to defend here), it doesn't seem to work in favor of the formal approach to mechanisms. For this seems to imply that there is something in the mechanism, something that is explanatorily salient, that the formal causal representations leave out. So the representation of the mechanism would be at best incomplete. This is the first horn of the dilemma that I outlined in the introduction.

The second horn arises if we try to consider the differential equations as directly targeting a set of causal dependencies after all. The problem with this, as I have shown, is the failure of variable modularity, which prevents a full representation of the differential equations as structural equations. Thus, the formal causal approach can at best consider the variables of the model as interrelated but not as *causally* dependent. But if this is true then the formal approach misses out on some causal dependencies that make up the mechanism and is therefore also incomplete.

Thus, any way we cut the cake: The formal framework permits only a partial representation of the dynamical biological mechanism. It leaves out either non-causal explanatory relations<sup>3</sup> or causal relations.

## 6. Conclusions

I have shown that information about the dynamics of biological mechanisms cannot always be fully assimilated into an interventionist formal causal framework. Even though approximate, discrete-time models succeed in integrating dynamical and causal difference-making information, the structural causal modeling approach leads to a dilemma when trying to account for certain differential equation models. Because these equations do not have the right modularity properties, they must either be considered as non-causal to begin with, or as exhibiting a kind of causality not representable in the formal framework. In either case, the formal causal framework provides only a partial perspective on the mechanism.

---

<sup>3</sup> Proposed non-causal explanatory relations include constitutive relevance (Craver 2007), association laws (Kistler 2013) or metaphysical grounding (Correia and Schnieder 2012).

To be fair, it must be acknowledged that some followers of the formal approach to causality and mechanisms, in particular Gebharter and Kaiser (2014), admit from the outset that the formal models can provide only a partial perspective. Furthermore, some causal theorists such as Woodward (2003) do not understand their framework as universal accounts of causality. Here, I hope to have provided additional justification for their caution in showing where exactly the limitations of the formal interventionist approach lie.

## Acknowledgements

I wish to thank in particular Marie Kaiser, Lorenzo Casini, Alexander Gebharter, Naftali Weinberger, the audience at the PSA 2014 symposium "How Adequate Are Causal Graphs and Bayesian Networks for Modeling Biological Mechanisms?", and the anonymous reviewers for many helpful suggestions and criticism. Versions of this paper were also presented at the Department of Philosophy, University of Neuchâtel, the Institute of Philosophy, Leibniz-University Hannover, the Biology Club, University of Fribourg, and at the EPSA 2015 meeting in Düsseldorf.

## References

- Bechtel, William (2013), "From Molecules to Networks: Adoption of Systems Approaches in Circadian Rhythm Research", in Hanne Andersen, Dennis Dieks, Wenceslao J. Gonzalez, Thomas Uebel and Gregory Wheeler (eds.), *New Challenges to Philosophy of Science*, Berlin: Springer, 241-261.
- and Adele Abrahamsen (2010), "Dynamic Mechanistic Explanation: Computational Modeling of Circadian Rhythms as an Exemplar for Cognitive Science", *Studies in History and Philosophy of Science Part A* 41:321-333.
- Cartwright, Nancy (2001), "Modularity: It Can—and Generally Does—Fail", in Maria Carla Galavotti, Patrick Suppes and Domenico Costantini (eds.), *Stochastic Causality*, Chicago: The University of Chicago Press, 65-85.
- Casini, Lorenzo (2016), "How to Model Mechanistic Hierarchies", *Philosophy of Science*, in this volume.
- , Phyllis McKay Illary, Federica Russo, and Jon Williamson (2011), "Models for Prediction, Explanation and Control: Recursive Bayesian Networks", *Theoria. An International Journal for Theory, History and Foundations of Science* 70 (1):5-33.
- Clarke, Brendan, Bert Leuridan, and Jon Williamson (2014), "Modelling Mechanisms With Causal Cycles", *Synthese* 191:1651-1681.
- Correia, Fabrice, and Benjamin Schnieder (2012), "Grounding: An Opinionated Introduction", in Fabrice Correia and Benjamin Schnieder (eds.), *Metaphysical Grounding. Understanding the Structure of Reality*, Cambridge: Cambridge University Press, 1-36.

- Craver, Carl (2007), "Constitutive Explanatory Relevance", *Journal of Philosophical Research* 32.
- Gebharter, Alexander (2014), "A Formal Framework for Representing Mechanisms?", *Philosophy of Science* 81:138-153.
- and Marie I. Kaiser (2014), "Causal Graphs and Biological Mechanisms", in Marie I. Kaiser, O. Scholl, D. Plenge and A. Hüttemann (eds.), *Explanation in the Special Sciences. The Case of Biology and History*, Berlin: Springer.
- and Gerhard Schurz (2016), "A Modeling Approach for Mechanisms Featuring Causal Cycles", *Philosophy of Science*, in this volume.
- Glennan, Stuart S. (1996), "Mechanisms and The Nature of Causation", *Erkenntnis* 44:49-71.
- Goldbeter, Albert (1995), "A Model for Circadian Oscillations in the *Drosophila* Period Protein (PER)", *Proceedings of the Royal Society of London. B: Biological Sciences* 261 (1362):319-324.
- Iwasaki, Yumi, and Herbert A. Simon (1994), "Causality and Model Abstraction", *Artificial Intelligence* 67:143-194.
- Kaiser, Marie I. (2016), "Modelling Spatially and Structurally Complex Mechanisms", *Philosophy of Science*, in this volume.
- Kistler, Max (2013), "The Interventionist Account of Causation and Non-Causal Association Laws", *Erkenntnis* 78:1-20.
- Kuorikoski, Jaakko (2012), "Mechanisms, Modularity and Constitutive Explanation", *Erkenntnis* 77:361-380.
- Machamer, Peter, Lindley Darden, and Carl Craver (2000), "Thinking About Mechanisms", *Philosophy of Science* 67 (1):1-25.
- Mitchell, Sandra D. (2009), *Unsimple Truths. Science, Complexity, and Policy*. Chicago: The University of Chicago Press.
- Pearl, Judea (2000), *Causality. Models, Reasoning, and Inference*. Cambridge: Cambridge University Press.
- Weber, Marcel (2005), *Philosophy of Experimental Biology*. Cambridge: Cambridge University Press.
- (2008), "Causes without Mechanisms: Experimental Regularities, Physical Laws, and Neuroscientific Explanation", *Philosophy of Science* 75:995-1007.
- Wimsatt, William C. (1976), "Reductive Explanation: A Functional Account", in R.S. Cohen, C.A. Hooker, A.C. Michalos and J.W. van Evra (eds.), *PSA 1974 (Proceedings of the 1974 Biennial Meeting, Philosophy of Science Association)*, Dordrecht/Boston: Reidel, 671-710.
- Woodward, James (2003), *Making Things Happen: A Theory of Causal Explanation*. Oxford: Oxford University Press.
- (2013), "Mechanistic Explanation: Its Scope and Limits", *Proceedings of the Aristotelian Society Supplementary Volume* 87 (1):39-65.