Inequivalent Vacuum States and Rindler Particles

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1. Introduction

The fundamental theories of contemporary physics are quantum field theories. The lowest energy state of a quantum field is the vacuum state. While the vacuum state does not necessarily have zero energy, it is the zero particle state: more energetic states of the field are states that "contain" particles. Recent work has appeared to indicate that which state of a field we account the vacuum state is dependent on our reference frame. While all inertial frames determine one vacuum state, called the Minkowski vacuum, a reference frame that uniformly accelerates (a Rindler frame) will, it is claimed, determine a different vacuum state, and the inertial vacuum will be a superposition of Rindler frame particle states i.e., for the Rindler observer, the inertial vacuum "contains" some indeterminate number of particles. This is quite a surprising prediction. One would certainly expect observers in different states of motion to disagree about the momenta and energies of particles, but for one observer to claim that the field "contains" particles where another says there are none is, at least, counter to our naive intuitions. In this paper we will argue that the naive intuitions are, in fact, correct. The claim that, for a uniformly accelerating observer, the inertial vacuum state "contains" particles is mistaken. This necessitates a reinterpretation of what happens when a particle detector uniformly accelerates through a field in the inertial vacuum state. While an accelerated detector will be excited by its interaction with the field, that excitation should not be interpreted as a particle detection event. However, another feature of the standard story about accelerating detectors does survive. If a detector is uniformly accelerated through the Minkowski vacuum it responds as if immersed in a bath of black body radiation with a temperature determined by the rate of acceleration. This phenomenon is interestingly analogous to the Hawking radiation of a black hole. Aside from its intrinsic interest, we feel the Rindler phenomenon is a good jumping off point for discussions of the nature of the vacuum, relating such issues as the existence of multiple inequivalent vacua, correlations in the vacuum, and spontaneous symmetry breaking.

2. A Brief Non-Technical Account of the Quantum Formalism

As is well known, quantum mechanics tells us that the outcome of many experiments cannot be reliably predicted. Instead, the theory offers us a specification of the possible outcomes and the probability of each. More generally, if we know a system is in a particular *quantum state*, we can compute the probability of any outcome of any type of experiment. For example, if we know the quantum state of a particle we can compute the probability of any possible outcome of a position measurement, and the probability of any outcome of a momentum measurement.

The quantum state of a system is represented mathematically as a vector. We can think of a vector as an arrow. The arrow has a direction, and a magnitude - the length of the arrow. We can picture the properties of vectors relevant to our discussion in the following way. Consider a three dimensional space with a Cartesian coordinate system consisting of three orthogonal axes X,Y, and Z. Each point p in the space has a unique set of coordinates (x,y,z). We can redescribe the space in terms of vectors as follows. With each point p we associate the vector $\underline{p}$ obtained by drawing an arrow from the origin of the coordinate system to p. We say that $\underline{p}$ is the sum of the vectors $\underline{x}$, $\underline{y}$ and $\underline{z}$ i.e., respectively, those vectors from the origin to (x, 0, 0), (0, y, 0), and (0, 0, z). Clearly, this decomposition of $\underline{p}$ relative to the directions picked out by these coordinate axes is unique. We term the set of unit vectors in the X, Y, and Z directions a *basis* for the space and the vectors $\underline{x}$, $\underline{y}$ and $\underline{z}$ are the *components* of $\underline{p}$ relative to that basis. Now, consider another set of coordinate axes (X', Y', Z') with the same origin as (X, Y, Z), obtained by rotating the original axes. Clearly, the point p will have different coordinates relative to this new set of axes. Correspondingly, the vector $\underline{p}$ is expressible as the sum of vectors $\underline{x}'$, $\underline{y}'$, and $\underline{z}'$.

The state vector, which represents the system's quantum state, yields the possible outcomes of measurements in the following way. There is a basis corresponding to each type of measurement[1]. Each vector in the basis corresponds to one of the possible outcomes[2]. The state vector can be written as the sum of its components relative to that basis. The magnitude of the component corresponding to a particular outcome tells us the probability that such a measurement will have that outcome. Consider as an example a simple harmonic oscillator. We can think of this as a particle that is attracted to a point by a force that is proportional to its distance from that point. A weight suspended from a spiral spring is a classical system of this sort; if we pull the weight from its rest position the spring pulls the weight back with a force proportional to its extension. When we quantise such a system we find that the particle can occupy any of an infinite family of energy states. It may have energy 1/2 hν, 3/2 hν, 5/2hν, . . . and so on. Associated with each such energy value there is a vector, and this infinite set of energy state vectors $\underline{1/2\ h\nu}$, $\underline{3/2\ h\nu}$, $\underline{5/2h\nu}$,… constitutes a basis. Thus, whatever state the particle is in, that state is expressible as a sum over the energy basis vectors. Suppose the state vector $\underline{\Psi}$ is expressible as the sum, $\underline{\Psi} = (\sqrt{3}/2)\underline{1/2h\nu} + (1/2)\underline{5/2h\nu}$. The probability of measuring the particle as having energy 1/2 hν is then $(\sqrt{3}/2)^2 = 3/4$. The probability of measuring the particle as having energy 5/2hν is $(1/2)^2 = 1/4$. Two things are worth noting here. First, the rule for determining the probability of an outcome is to square the magnitude of that component of the state vector[3]. Secondly, the probability of the experiment having one or other outcome is 3/4 + 1/4 = 1 i.e. the state vector is always normalized to have unit magnitude. The vectorial representation of the quantum state is beautifully economical. All of the information about

[1]This is known from the Spectral Theorem but its justification does not concern us here.
[2]Since a measurement may have an infinite number of outcomes, the vector space, called a Hilbert space is typically infinite dimensional.
[3]In fact, Hilbert Space is a vector space over the field of complex numbers and the probability is the square of the modulus of the complex coefficient. But these details are irrelevant to our discussion.

measurement outcomes is coded up in the state vector for the system and can be extracted by simply specifying the basis for each type of measurement.

There is one further part of the quantum formalism we need to introduce. An operator on a vector space is a function that takes a vector as input and yields a vector as output. Each observable property of a system is associated with an operator on the state space. Thus, there is a momentum operator (P_{op}), a position operator (X_{op}), an energy operator (E_{op}), etc. For our purposes it is enough to note the following. These operators are defined so that, given the state vector of the system and the operator for a particular observable, we can compute the expectation value, or average, for measurements of that observable on a system in that state. For a system in a state Ψ, we write the expectation value of measurements of some observable A as $\langle A_{op} \rangle_{\Psi}$. Thus, returning to our simple harmonic oscillator with state vector $\underline{\Psi} = (\sqrt{3}/2)\underline{1/2h\nu} + (1/2)\underline{5/2h\nu}$, we have that $\langle E_{op} \rangle_{\Psi} = 3/4 \,.\, 1/2h\nu + 1/2 \,.\, 5/2h\nu = 13/8h\nu$.

3. Quantum Fields and the Vacuum State

In standard quantum mechanics, particles are ontological primitives. That is not to say that quantum mechanics entails the existence of particles that follow continuous trajectories as in classical mechanics. But the theory is, at least *prima facie*, about particles. The state vectors are associated with particles and the values of their momenta, positions, and spin components under measurement. In quantum field theory the ontological primitives are quantum fields, the quantum counterparts of classical entities such as the electromagnetic field. Each quantum field has its state vector. For a free field we find that one basis for the quantum field vector space is as follows: there is a lowest energy state, called the vacuum state; if the field is measured in such a state it will be found to have zero total momentum although it may not have zero energy. The other states in the basis also have definite momentum and energy values. But we have more to say about these states than that they merely have associated *total* momentum and energy values. Each state is characterized by a list of values of integer valued parameters n_k (the state vector is written $|n_1, n_2, \ldots, n_k, \ldots>$). For instance, there is a state with $n_1 = 2$ and $n_3 = 3$ and for all other n_k, $n_k = 0$. Such a state has two units of momentum of magnitude k_1 and 3 units of momentum of magnitude k_3 associated with it. The state also has 2 units of energy of the correct magnitude for a free particle of momentum value k_1 and 3 units of the energy for a particle of momentum k_3. In interactions, the field loses and gains energy and momentum by moving from one such state to another i.e. it gains and loses momentum and energy in the discrete lumps associated with changes in the values of the n_k parameters. Given these features it is natural to interpret n_k as the number of particles of momentum k associated with the field and to think of field interactions as being mediated by particles. The vacuum state is the zero particle state and the other basis states are states associated with aggregates of particles. This is not to say that the field just is an aggregate of particles; it is not. Nor need we commit ourselves to any claims about there being particles following continuous trajectories between measurements. But we can see that talk of detecting particles in field theory is on roughly the same footing as such talk in standard quantum mechanics.

Let's talk a little about the energy of the free field vacuum in Minkowksi spacetime. In fact, when we calculate it in a straightforward way we get an infinite value. This is because in quantum theory, energy is proportional to frequency, $E = h\nu$. We can, in a heuristic way, think of a state containing one particle of energy E as the vacuum being excited with a global vibration or wave with frequency ν. When n such particles are present, the excited wave has n times the amplitude. When particles of different energies are present the associated waves of differing frequencies have been excited. Now, although in this heuristic picture particle "waves" are absent from the vacuum, the quantum field cannot be completely turned off. For each frequency, there exists a ground state wave with energy $1/2\ h\nu$. Since there are an infinite number of possible excitation frequency values, ν, we get an infinite vacuum energy. We typically ignore this infinite energy, because all we can usually measure are energy differences—e.g. the difference between the energy of some n particle state and the vacuum—so that the infinities cancel out. However, there are circumstances where it cannot be ignored.

One is the Casimir effect, in which two parallel conducting plates are placed a distance L apart in the vacuum. Since the electric field must be zero at the conductor, the vacuum waves of the electric field must have zero amplitude there. So, the vacuum waves allowed to propagate when the plates are present will differ from when the plates are absent, and this will have an effect on the vacuum energy. For instance, in the region interior to the plates, some vacuum waves—those with a frequency that precludes them from having zero amplitude at both plates—that exist when the plates are not present, cannot exist and, intuitively, their absence should reduce the vacuum energy. The total vacuum energy for the system is the sum of contributions from the field between the plates, and from the field in the regions outside the plates. This is still infinite, but rigorous calculation shows how to extract the finite difference between the Minkowski vacuum energies with and without the plates present. This is the finite vacuum energy due to the presence of the conducting plates, and it cannot be ignored. Why? Because the energy is a function of the distance between the plates. Calculation shows that the closer the plates, the less the vacuum energy. So, there is an attractive force between the plates[4].

An analogous effect occurs if we consider a cylindrical two dimensional Minkowksi spacetime, where space is a one dimensional circle. Since space is circular the only allowed vacuum waves are periodic, each wave must join up with itself after going around the circle once. So, intuitively, there are less ground state waves in a cylindrical spacetime than in an infinite unrolled Minkowski spacetime. Again, as with the parallel

[4]Having computed the force between parallel plates as attractive, Casimir suggested that if the force on a spherical conductor was attractive, we might model a charged particle as a spherical conducting shell carrying a total surface charge equal to the charge of the particle. The Casimir force would balance the electrostatic repulsion produced by the sphere's charge. However, Boyer's 1968 calculation showed that, for a field in the presence of a spherical conductor, the force due to the Casimir effect pushes outward on the sphere, rather than pulling it inward.

plates, the energy difference is a function of length, in this case, of the circumference of the circle. For the scalar field, the energy difference decreases as the circumference decreases, causing space to contract, just as the plates are pulled together in the previous case. Interestingly, however, for the Dirac electron field the energy decreases as circumference increases, causing space to expand. The net effect on the universe would be given by the sum of vacuum energies of all the quantum fields.

Returning to our discussion of the quantum field formalism, for every observable there is a corresponding operator. There is a number operator that allows computation of the expectation value of particle numbers for a given field state. There is also a family of operators $\phi(x)$, each associated with the magnitude of the field at point x. In fact, all other observable operators are constructible from the $\phi(x)$ operators. Since we are concerned with relativistic quantum field theory, the x in $\phi(x)$ denotes a point in spacetime, or event. As might be expected, the expectation value of any $\phi(x)$ for the vacuum state is zero. But the vacuum expectation value of the product $\phi(x)\phi(y)$ for distinct points x and y is non-zero. So, $\langle\phi(x)\phi(y)\rangle_{vacuum} > \langle\phi(x)\rangle_{vacuum}\langle\phi(y)\rangle_{vacuum}$ i.e. the field value at point x is correlated with the value at point y[5]. These correlations are just an instance of a wealth of structure associated with the vacuum. For any local field observable (i.e. any function of the field that is zero outside of a bounded region of spacetime) we can prove the following results. Firstly, on a measurement of such an observable in the vacuum there is a non-zero probability of obtaining a non-zero value. Secondly, and even more remarkably, if we have an arbitrary local observable O_1 associated with the bounded region R_1, then for any

[5]Two physical variables X and Y are correlated if their values depend on each other in some way. For instance, if two particles with zero total momentum interact and then depart in opposite directions they have perfectly correlated momentum values; conservation of momentum requires that the momentum of one is the negative of the momentum of the other. So, if we measure the momentum of one we know the momentum of the other. The correlation expressed by the above relation between expectation values is not so exact; it's probabilistic. If we determine the value of the field amplitude $\phi(x)$ by measurement, we do not thereby determine the field amplitude $\phi(y)$, but our probability distribution for the various possible measurement outcomes for $\phi(y)$ is altered.

bounded region R_2, no matter how distant in space and time from R_1, there is a local observable O_2 associated with R_2 such that they are correlated[6].

4. Describing the Field in Different Frames

As is well known, we can describe physical situations in terms of different sets of coordinates. For example, if I am moving with velocity v relative to you, then an object that is stationary in your reference frame and, thus, has constant spatial coordinates, will have velocity -v relative to my coordinates. However, depending on the theory, we may privilege the description of one reference frame over that of another. Thus, on the standard interpretation of Newtonian mechanics, particles are taken to have an absolute velocity which is their velocity relative to absolute space. Even though one can describe the physics of a system using the coordinates for a frame in motion relative to absolute space, the velocity determined relative to that coordinate system is not the absolute velocity. Since absolute velocity is a fundamental physical property, and velocity relative to some arbitrary reference frame is not, we have a privileged reference frame that is "the right frame" in the sense that the values of the quantities expressed in terms of its coordinates are the values of the fundamental physical quantities.

On the standard Einsteinian interpretation of relativity, there is a democracy of reference frames. There is no absolute velocity, only velocity relative to a particular reference frame, and we do not privilege the description provided by any particular frame. Correspondingly, one speaks of the quantum field state relative to a particular reference frame (or set of frames that agree with regard to the field state). While it is not essential to our argument to adopt the Einsteinian interpretation, that interpretation is implicit in the standard discussions of the phenomena that concern us, and in this paper we have followed

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[6]For example, the electric charge density in region R_1 could be correlated with the electric charge density in the spacelike related region R_2.

suit. Thus, we speak of the quantum state of the field, relative to a particular reference frame, or, interchangeably, of the field state for a given observer.

Consider two observers in different inertial reference frames. Each one will define the quantum state of the field differently - just as observers in different inertial reference frames will impute different momenta to a classical particle, such quantities as the probability of detecting a quantum mechanical particle of a particular momentum will be reference frame dependent. If one inertial observer has a 100% chance of detecting a particle of momentum p (and 0% chance of detecting any other particle) and I am moving relative to that observer, then I will have a 0% chance of making such a detection but a 100% chance of detecting a particle with a momentum value, $\Lambda(p)$, given by the Lorentz transformation rules of special relativity. If the system is a quantum field then the probability of detecting a quantum field particle of a particular momentum will be similarly frame-dependent. We can represent this situation using our quantum state vector space. For you the field is in a state that contains one particle at momentum p. For me the field is in a state that contains one particle at the Lorentz-transformed momentum $\Lambda(p)$. So, we ascribe different state vectors to the system. There is, however, one state that we would expect to agree on: we should both agree on the no-particle vacuum state. The number of particles present is not something that we expect to be frame dependent and so we speak of the Minkowski vacuum common to all inertial observers in Minkowski space.

However, when we consider the uniformly accelerating Rindler frames something surprising happens. Following the standard quantisation procedure using the Rindler observer's coordinates we determine a vacuum state that differs from the inertial observer's vacuum state. Moreover, the inertial vacuum state is apparently expressible as an infinite sum of Rindler particle states. If this is correct we can only conclude that the state the inertial observer classifies as the vacuum is one which the uniformly accelerating observer classifies as "containing" particles. So, surprisingly, the presence or absence of particles is not an invariant feature of the underlying physics, but can only be correctly asserted when describing the world relative to a particular reference frame. Indeed, for a uniformly accelerating detector, there is apparently a finite probability of detecting particles of any energy and momentum[7].

But this standard interpretation of the relation between the Rindler and Minkowski quantum field theory is mistaken. Not all infinite sums of vectors are to be trusted. This fact has assumed considerable significance in understanding the physics of other quantum systems. We now turn to one such system, the Ising model in two dimensions.

5. Multiple Vacuum States and the Inequivalence of the Minkowksi and Rindler Vacua

When we think of a physical system and the possible states it could occupy, we typically think that each state must be accessible from any other. If we pump in or take out enough energy or momentum then we can transform the system as desired. However, in physics we often consider infinite systems. An example of such a system is the Ising Model. Imagine an infinite two dimensional lattice, with a spin at each point on the grid. Each spin has two possible states, spin up and spin down. We specify a nearest-neighbor interaction between the spins: each spin may directly interact with its nearest neighbors but is insensitive to the orientation of any more distant spin. The interaction is such that the spins tend to line up i.e. if two adjacent spins have the same orientation then they are in a lower energy state than if they are oriented antiparallel. When all of the spins are aligned,

[7]A note on the use of the terms "observer" and "detector". When we speak, for example, of the value of a particle's velocity determined by an observer in some particular reference frame we are implying that any properly functioning velocity measurement apparatus, in the state of motion corresponding to the observer's reference frame, will yield that value for the particle's velocity. Similarly, when we speak of the particles detected by a uniformly accelerating detector, there is an implicit assumption that any properly functioning particle detector detects the particles "contained" in the field state defined by the coordinates of the corresponding uniformly accelerating observer. So, it would follow from the correctness of the Rindler expansion of the Minkowski vacuum that a properly functioning particle detector, uniformly accelerated, would have a probability of detecting particles with the energy and momentum values that appear in the Rindler expansion.

the system has the lowest energy possible. For such a system there are two distinct lowest energy states. One where all of the spins are pointed up, and another, where all of the spins are pointed down. Each has its family of states "close by", where the orientations of finite numbers of the spins differ from that of the infinite majority. The curious feature of such a system is that neither ground state is accessible from the other. There is no physical system that could start off in one ground state and evolve into the other. Nor could a system in any of the family of states accessible from each ground state evolve into any state accessible from the other state. There is a simple reason for this. Since the system in question is an infinite lattice, to go from a state where an infinite number of spins point down to a state where an infinite number point up would require an infinite amount of energy. Such transitions are not allowed in quantum mechanics[8]. So, while either of the ground states is possible, it is not possible for a system in one ground state to evolve into the other. The mathematical description of the Ising model reflects this situation. Instead of one vector space for the quantum states there are now two: each space contains one of the ground states and all of the states obtainable from that state by flipping a finite number of spins.

An analogous situation obtains for quantum fields. Consider states that contain an infinite number of particles, $|n_1 n_2 ... n_k ...>$, where an infinite number of the n_k are non-zero. We can see that such states are separated from the vacuum by an infinite energy gap and, as in the case of the Ising model, are inaccessible without the expenditure of an infinite amount of energy[9]. So, as with the Ising model, there can exist distinct state spaces for the

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[8]One might object that such an infinite energy gap does not preclude the evolution from one vacuum to another. A universe that contains an infinite lattice might also contain an infinite energy source—albeit distributed such that its energy density is finite—that could boost the field across the energy gap. However, this is not a possible evolution within quantum theory. There is no unitary evolution of the field state that will take the field across an infinite energy gap.

[9]Such states should not be confused with states formed by taking an infinite sum of finite-particle number states. When we construct a state space for a quantum field we find a vacuum state and then iteratively build particle states from that vacuum. For any finite particle number there is a corresponding state. The state space is infinite dimensional and any vector that we can form by summing over vectors in the infinite dimensional state space is also a vector in that space. Even a denumerably infinite sum of vectors is a member of the space, provided that it is normalised. However, such an "infinite-sum" state does not "contain" an

quantum field, inaccessible or inequivalent to each other in the following two ways. A state in one space cannot be expressed as a sum of states in any other space, and a state in one of them cannot evolve, according to the laws of quantum evolution, into a state in any other. The existence of such distinct spaces, and therefore of multiple vacuum states is of great importance in contemporary particle physics. It is the basis of spontaneous symmetry breaking which is crucial to the currently accepted standard model of the weak and electromagnetic interactions.

In each case of spontaneous symmetry breaking there exists a symmetry of the equations of the field (the dynamics) that is not a symmetry of the vacuum. For example, while rotation is a symmetry of the Ising model dynamical equations, it is not a symmetry of the vacuum. Rotating one of our vacuum spin distributions through 180^{o} does not leave the vacuum unchanged, but transforms it into the other vacuum. With regard to quantum fields, rigid space and time displacements are a symmetry of both the inertial vacuum and the dynamics of the quantum field. However, for certain quantum fields there are more abstract symmetries that leave the dynamics unchanged but not the vacuum. In such a case we can prove—it's called the Goldstone theorem—that certain fields have a vacuum expectation value, and associated with this, they must contain particles with zero rest mass. These massless particles are crucial to making the current electroweak theory work. It is conjectured that in the early universe, shortly after the big bang, there was no distinction between the electromagnetic and the weak forces - both were long range forces "mediated" by massless particles. But as the universe expanded and cooled, a kind of phase transition occurred, giving rise to spontaneous symmetry breaking. A field whose vacuum had been invariant under a dynamical symmetry transformation lost that symmetry. In the process, the field acquired a vacuum expectation value and multiple vacua. The vacuum

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infinite number of particles. It is a state that could yield any of an infinite number of finite particle numbers under measurement i.e. it "contains" some indeterminate but finite number of particles.

expectation value coupled to the weak force giving it a mass so that it is no longer long range. The electromagnetic force remains long range, and the two are no longer equivalent.

We find another example of inequivalent vacua when we look at Rindler quantisation. The putative expansion of the inertial vacuum in terms of Rindler states includes states that "contain" an infinite number of particles[10]. So, the expansion cannot be taken seriously. The Rindler and Minkowski vacua are mutually inaccessible and belong in distinct state spaces with every vector in the inertial observer's state space orthogonal to every vector in the Rindler observer's state space[11]. Therefore, since the inertial vacuum state is not expressible as a sum of Rindler particle states, uniformly accelerating a detector though a field in the inertial vacuum state cannot result in the detection of Rindler particles.

In fact, the Rindler particle states—as we now see, a misnomer but we'll stick with the name—are the states for a quite different physical system[12]. There remains the question of what does happen when we accelerate a detector through the Minkowski vacuum. Some of the standard interpretation does survive, and it is interestingly related to other phenomena such as Hawking radiation.

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[10]This is not immediately obvious from the form in which the expansion is usually written (see, for instance, Unruh and Wald (1984)). We can express a many-particle state as a product of states that correspond to the occupation number for particles with some particular momentum value e.g. $|n_1, n_2>$ can be rewritten as the product $|n_1>|n_2>$. The purported expansion of the inertial vacuum in terms of Rindler states is typically written as a product of sums of such states. The product is taken over the infinite set of occupation numbers and thus contains terms of the form $|n_1>|n_2> \ldots . |n_k> \ldots .$The sum is over all of the possible values of the occupation numbers (n_i =1, 2, 3, . . .). So, the expansion of the inertial vacuum state includes states that have an infinite number of non-zero occupation numbers.

[11]The Rindler and Inertial state spaces are inequivalent representations of the field algebra. See Gerlach 1989 for a demonstration.

[12]Described in Gerlach 1989.

6. Accelerating a Detector through the Minkowski Vacuum

As discussed above, the existence of particles is standardly associated with the field being in a particle number state. There are good reasons for this: incrementing the particle number of the field state increases the field energy and momentum in discrete units; further, the energy and momentum values associated with the field particles satisfy the special relativistic energy-momentum relation for particles. If the existence of field particles requires that there be a corresponding particle number state, and there is no obvious alternative criterion, then there are no Rindler particles. Consequently, we cannot describe the response of any particle detector to the field in terms of detection of such particles.

But what does happen when we uniformly accelerate a detector through the field? According to the standard interpretation, the Minkowksi vacuum state contains a thermal distribution of Rindler particles i.e. the expectation value of the number of particles as a function of particle energy satisfies the Planck distribution for a temperature T, proportional to the acceleration of the frame. While we must reject this interpretation the response of an accelerated model detector to the field has been computed in a way that does not depend on the existence of Rindler particle states in the inertial state space[13]. The predicted excitations of the detector also conform to the Planck distribution as if the detector is indeed moving through a thermal bath of particles. However, the detector response can be explained in the absence of Rindler particles.

First we must recognize that the detector is an active player in the system. For it to respond to the field it must be coupled to the field, and so, when we accelerate the detector

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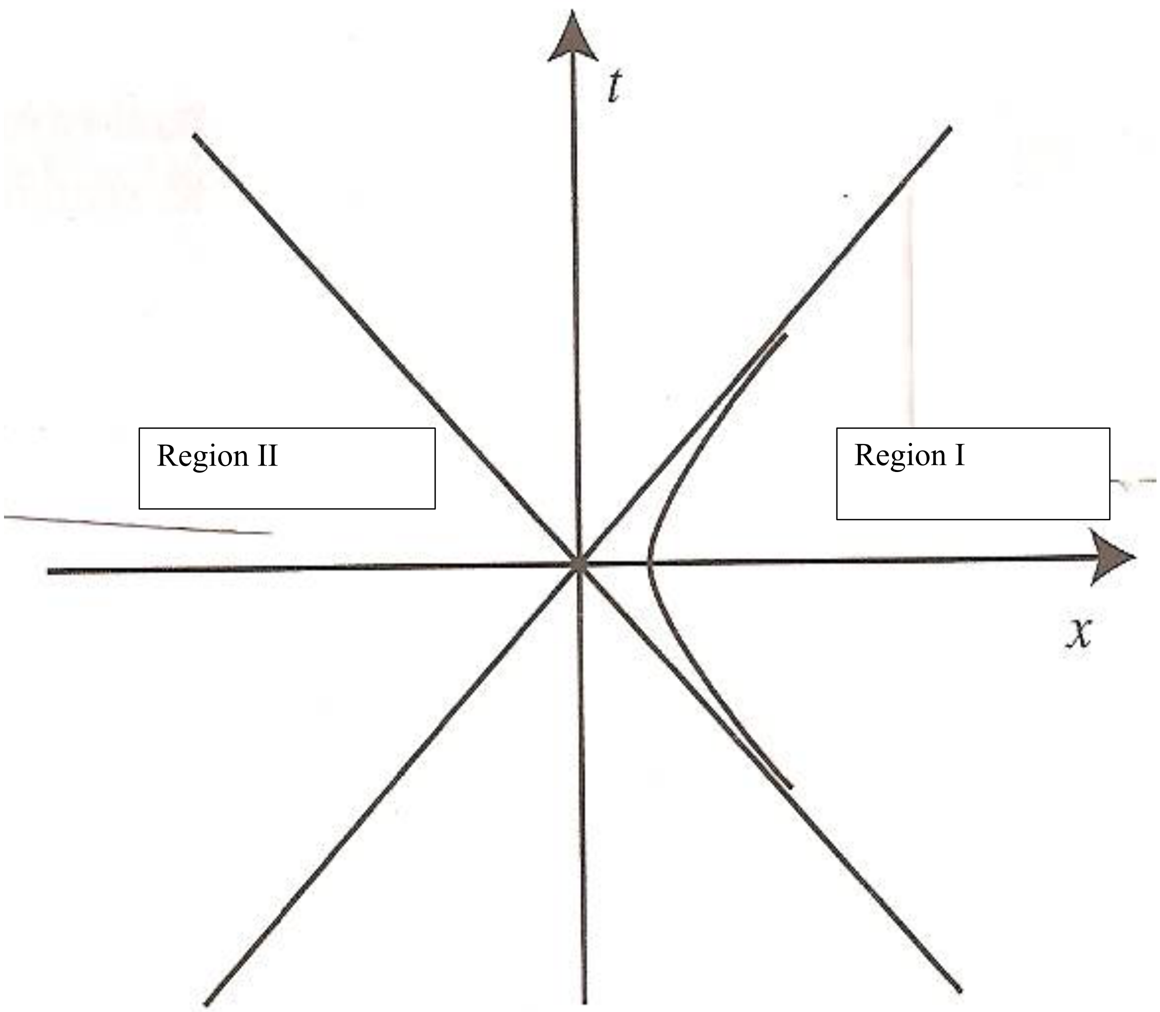
[13]While the Rindler particle number states form an inequivalent representation of the field algebra, the operators associated with the expansion of the field in Rindler coordinates are defined on both the inertial and Rindler state spaces. Thus, calculations of, for instance, the expectation value of the Rindler particle number operator in the Minkowski vacuum state are entirely legitimate. However, we can no longer interpret the number operator as a Rindler particle number operator. Just what we should interpret it as is not clear. However, the use of Rindler creation and annihilation operators in the interaction Hamiltonian for the detector and the field seems legitimate, relying as it does only on the assumption that the detector responds to the field as "perceived" in the accelerating frame.

it disturbs the field state. We can understand this in terms of the heuristic model employed in our discussion of the Casimir effect. We think of the vacuum state as containing waves and the presence of obstructions in the field constrains the sorts of waves that can propagate. A detector that uniformly accelerates through the field is a moving obstruction that Doppler shifts the waves associated with the field. The effect of the detector on the field is to shift it from the vacuum state into a superposition of multiple Minkowski particle number states i.e. an inertial observer would see the detector as emitting Minkowksi particles into the vacuum. The field, no longer in the vacuum state, reacts back on the detector, triggering the detector response. So, the energy to trigger the detector ultimately comes from the disturbance in the field produced by the detector itself, not from the vacuum, and the triggering of the detector is compatible with the non-existence of Rindler particles. However, given the absence of Rindler particles, there is no robust phenomenon that the detectors are detecting, and their response may crucially depend on the nature of their coupling to the field.

That the detector responds to the vacuum is, perhaps, not surprising once we realize that it is transferring energy to the vacuum waves. But that it responds as if immersed in a bath of thermal radiation is still puzzling. The paradigm of thermal radiation is a cavity filled with radiation that is in equilibrium with the walls of the cavity - the rate of radiation absorption by the walls equals the rate of radiation emitted. The equilibrium state is a state of maximum entropy or, intuitively, disorder. But the vacuum state is not a state of maximal disorder; there are correlations between the vacuum waves, even at spacelike separated points. Why does the Rindler detector's response not reflect this?

As we see from figure 1, a Rindler observer has access only to a certain patch of spacetime.



You need two Rindler coordinate patches to cover all of Minkowksi spacetime. Crucially, an observer in one patch cannot receive information from any point in the other Rindler coordinate patch. In particular, a uniformly accelerating detector in region I can receive no information from region II. Therefore, the description of the vacuum state constructible from the readings of such a detector is incomplete. All information about vacuum correlations between regions I and II is unavailable to the detector. So, the detector responds as if the vacuum were a disordered state.

Heuristically, we can understand this by analogy with another system. Consider a box that contains many gas molecules. We can, in principle at least, describe the state of the gas in two different ways. We can specify the position and momentum of each molecule in the box. We will term a state described in such a way a microstate of the gas. Alternatively, we can describe a state of the gas in less exact terms, for instance, by specifying the values of some macroscopic parameters. We could specify the temperature of the gas, its density, and the pressure it exerts on the walls of the containing box. Such a state is a macrostate of the gas. Clearly, there will be many microstates of the gas that instantiate such a macrostate. The more microstates associated with a macrostate, the higher the entropy of the macrostate.

How does the entropy relate to the gas being ordered or disordered? Intuitively, the more detailed and restrictive your specification of the macrostate of the gas, the more orderly the state described. And, the more restrictive your specification, the less microstates that will meet it. To take an example, if I specify a macrostate by requiring that the density of the gas be uniform throughout the box there will be many associated microstates. However, if I specify the macrostate by requiring uniform density and, in addition, that zero pressure be exerted on any but the top and bottom walls of the box, only a very small subset of the uniform density microstates will realize such a macrostate, in fact, only those microstates for which the molecules move on exactly vertical trajectories bouncing back and forth between the top and bottom walls in perpetuity. So, ordered states are low entropy states. And, if I specify that certain correlations must obtain between, say, the momenta of particles in different regions of the box, I am specifying a state with a certain amount of order and, consequently, a less than maximal entropy.

Now, imagine I have a box of gas with a partition in the middle, and, such that the momenta of the gas molecules in one half of the box are correlated with the momenta of the molecules in the other half but the momenta of the gas molecules in any one half of the box are not significantly correlated with each other. Then, if I restrict my attention to one half of the box I will consider the enclosed gas to be in a high entropy, disordered state. This is analogous to the situation of a detector uniformly accelerating through a field in the inertial vacuum state. There are correlations between regions I and II but the detector has no access to them. However, if I consider both halves together, with their extensive mutual correlations, then, I will adjudge the entire system to have a low entropy since the correlated macrostate of the entire system is compatible with a relatively small number of microstates. This is analogous to the view of an inertial detector.

There is some analogy here between the two Rindler coordinate patches and the interior and exterior of a black hole formed from the collapse of a massive body. An observer outside the black hole receives no information from the interior of the hole, just as an observer in region I receives no information from region II. (Albeit, disanalogously, someone inside the black hole can receive information from the region outside the black hole.) An observer free falling towards the black hole is analogous to an inertial observer in flat space-time. Far from the hole, where spacetime is almost flat, an inertial detector will respond as if immersed in a thermal bath. This is, again, a reflection of the fact that the detector lacks information about correlations between vacuum waves, in this case inside and the outside of the black hole. But the source of the radiation differs from the Rindler case. It is not the detector, which is inertial, that provides the energy for the detector excitation. Rather, it is due to the collapse of the black hole itself. As the black hole shrinks, the geometry of space-time changes and this disrupts the propagation of the vacuum waves. So, unlike the Rindler case, the energy is not provided by the detector.

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