Arguments for the Continuity of Matter in Kant and Du Châtelet

Aaron Wells
aaron.m.wells@gmail.com
(Draft—final version forthcoming in *Kant-Studien*.)

Abstract: In the *Metaphysical Foundations of Natural Science*, Kant attempts to argue a priori from the indefinite divisibility of space to the indefinite metaphysical divisibility of matter. This is one type of argument from the continuity of space—purportedly established by Euclidean geometry—to the continuity of matter. I compare Kant's argument to parallel reasoning in Du Châtelet, whose work he knew. Both philosophers appeal to idealism about matter in their reasoning, yet also face difficulties in explaining why continuity, though not some other properties from geometry, applies to matter. Both also risk inconsistency in adopting potentialist accounts of material parts, while also committing to realism about infinitesimals. An important difference between them is that Du Châtelet deploys at least three definitions of continuity; only one of these, amounting to indefinite divisibility, is shared with Kant.

Keywords: divisibility; potentialism; philosophy of geometry; Du Châtelet; Kant

Introduction: A Simple Argument for the Continuity of Matter

Kant's *Metaphysical Foundations of Natural Science* is committed to the continuity of matter. This raises the question of what matter's continuity amounts to. More systematically, one can ask whether a continuous matter theory is appropriate for the project of the *Foundations*. A further developmental puzzle is why Kant abandoned his earlier theory of discrete physical monads.

I aim to shed some light on these issues by critically considering a line of reasoning that moves from the continuity of space to the continuity of matter. Kant uses this reasoning, and it is also found earlier in Du Châtelet. Laying out this reasoning makes it easier to see common ground between these two philosophers, as well as more subtle differences between their views.

Here are four reasons why I consider Kant alongside Du Châtelet. First, they both argue on the assumption that continuous matter is only a phenomenon, and shares properties such as extension with mathematical objects—whereas the fundamental created substances are not extended. Both could therefore be said to treat the question of continuity within a framework of idealism about space and matter. Second, Du Châtelet is a plausible source for Kant's views. He responded to her in detail in his first publication on living force, where he cites her *Institutions de Physique* in Steinwehr's 1743 German translation.[1] Although I am not making the historical argument that Du Châtelet's work influenced Kant's *Foundations,* the plausibility of influence makes it particularly worth comparing their positions. Third, I defend an interpretation on which Du Châtelet rejects the rigid atoms and vacuum characteristic of Newtonian matter theory. This is also true of Kant's mature account of matter. At least on this issue, she is a possible non-Newtonian source for Kant's views.[2] Finally, putting Kant in dialogue with his French predecessor sharpens critical assessment of Kant's own work on continuity. I'll suggest that Du Châtelet considers various kinds of continuity, making her account in some ways richer than Kant's. Given his access to her work, Kant can be seen as missing an opportunity for further reflection on the concepts of continuity available in the eighteenth century. He might, for example, have considered possible relations of dependence between these continuity concepts, or argued explicitly that only one notion of continuity is in fact required.

The strategy of reasoning in question is a seemingly simple argument accepted by both Kant and Du Châtelet, but rejected by such prominent figures as Voltaire and Christian Wolff. This Simple Argument goes as follows:

(S1) Space is continuous;

(S2) If space is continuous, then matter is continuous;

∴ (S3) Matter is continuous.

Section 1 focuses on the first premise, which requires disambiguation because of the multiple senses of 'continuity.' On one understanding, space is continuous if any of its parts is divisible into further parts.[3] Call this the Divisibility Definition of 'continuity.' This is the conception Kant uses, and it is also important for Du Châtelet. Both take classical geometry to demonstrate that space is continuous in the sense of the Divisibility Definition. That is Kant's

---

[1] See GSK, AA 01:92. Ursula Goldenbaum ("How Kant was Never a Wolffian." In: *Leibniz and Kant,* ed. Brandon Look, Oxford 2021, 27–56) argues that Du Châtelet's *Institutions* is Kant's main target in his 1748 essay. Whether or not one agrees with this, he gives her work serious and detailed consideration.

[2] Du Châtelet was conversant with Newton's physics and framed her *Institutions* as in part expounding his system. Later, she translated his *Principia*. But as Marius Stan ("Newtonianism and the Physics of Du Châtelet's *Institutions de Physique*." In: Collected Wisdom of the Early Modern Scholar, ed. Anna Marie Roos and Gideon Manning, Cham 2023, 277–97.) has shown, much of the *Institutions*'s physics and metaphysics—including its matter theory—is not Newtonian, but part of a broadly Leibnizian tradition that includes Jacob Hermann and Christian Wolff. A further historical complication is that eighteenth-century Newtonian experimentalists often professed atomism without explicitly confronting metaphysical debates about infinite divisibility (Wilson, Catherine: "The Reception of Newton's Theory of Matter and his Atomism." In: *The Reception of Isaac Newton in Europe,* ed. Helmut Pulte and Scott Mandelbrote, London 2019, II, 437–38; 442–43). Still, these Newtonians are committed to metaphysical positions at variance with Du Châtelet's, since she rejects atomism.

[3] See Du Châtelet, Émilie: *Institutions physiques*. Paris 1742, 190; 193; Kant (KrV A524/B552). They deny that an actual infinity of parts is reached in division. Their statements that all parts can be divided should be read as saying that any part can be divided, rather than as quantifying over an actually infinite collection of parts.

main reason for accepting (S1). Indeed, he holds that divisibility *ad infinitum* is the criterion of any magnitude's continuity. Du Châtelet's position is more complicated. She also makes use of other conceptions of continuity, one based on the connectedness of parts, and another based on what she takes to be the definition of a continuous function. So she has a more expansive account of continuity than Kant does.

Section 2 turns to the second premise. Both Kant and Du Châtelet uphold this conditional in part because they are idealists about matter and space. But as I show, they each raise doubts about other conditional claims running from geometry to the physical world. These conditionals have a similar structure to premise (S2), suggesting that these doubts ought also to apply to (S2). For example, Du Châtelet denies that inferences from geometry to the physical world are always exactly correct. The relationship is instead one of approximation, opening room for doubt about (S2). Kant, meanwhile, is cautious about an reasoning from the unlimited and continuous character of geometrical space to a plenist conception of matter. In the *Metaphysical Foundations* he allows, unlike Du Châtelet, that empty spaces of a certain kind are possible. But if one inference from geometry to matter is suspect, why isn't the other?

A final question, which I touch on in **Section 3**, is raised by the conclusion of the Simple Argument (S3): What does matter's continuity mean for the priority relation between material parts and wholes? Du Châtelet and Kant agree that insofar as matter is continuous, it lacks actual simple parts that would be prior to wholes they compose. Instead, both adopt potentialist accounts of material parts, grounding them in material wholes. This raises the question of how the priority of wholes can be squared with their realism about infinitesimal quantities in physics (which suggests that infinitesimals literally sum to finite quantities). **Section 4** concludes with some brief evaluative remarks on each philosopher's approach to the simple argument for continuity.

1. The Continuity of Geometrical Space

Du Châtelet lays out three distinct definitions of continuity. This threefold account of continuity makes the interpretation of her version of the Simple Argument especially complicated. It also raises the question whether she takes one definition of continuity to be primary. Kant, by contrast, stipulates that continuity is "a single quality [*eine einzige Qualität*]" of all magnitudes, and amounts to indefinite divisibility.[4]

[4] KrV A176/B218; see also A715/B743. On Kant's classification of continuity as a quality, see Büchel, Georg: *Geometrie und Philosophie*. Berlin 1987, 132–59.

I therefore begin with the Divisibility Definition of continuity. Both philosophers endorse it, and they also agree that the key evidence for continuity qua divisibility stems from Euclidean geometry. Du Châtelet asserts that “all geometrical extension is divisible to infinity.”[5] This is based, in her view, on geometrical proofs—credited notably to John Keill—that lines are divisible *ad infinitum*.[6] So she accepts that geometrical space is continuous in the sense that it is divisible *ad infinitum*.

Meanwhile, already in the 1760s Kant takes “infallible proofs of geometry” to provide a demonstration that no part of space is simple; he reaffirms in 1790 that simple parts of matter are demonstrably “absolutely impossible,” leaving any philosophical challenges as mere sophistry.[7] In the earlier work, Kant sketches a proof of infinite divisibility, which he attributes to Keill.[8] Consider the straight line *l*, and two segments perpendicular to it, *AB* and *CD*, each of which has an endpoint on *l*. For *reductio,* assume that geometrical space has indivisible elements. If so, *AB* would have a ‘first’ indivisible element *X*, closest to its endpoint *A*. Now since by Euclid’s first postulate a straight line can be drawn between any two points, a diagonal segment *CJ* can be constructed that intersects *AB* at *X* and continues on to intersect *l* at *J*. But since *l* is indefinitely extensible, it is always possible to construct another diagonal segment, also starting at *C*, that intersects *l* at some further point *K*. Given our assumptions, *CK* must *also* intersect *AB* at *X*, just as *CJ* did—but this violates the geometrical principle that “not more than one straight line can pass through two given points.”[9] Therefore, Kant concludes, geometrical space does not have indivisible elements.

Kant also gives an argument for space’s continuity that does not appeal explicitly to geometrical proof. He starts with the assumption that any one-dimensional part of space can only be given as enclosed between points. He has in mind the endpoints of a line segment. Points alone cannot compose a space, on his view. So every one-dimensional part of space is not a point, but a region. Since two-dimensional spaces can only be given as enclosed

[5] Du Châtelet, *Institutions*, 191.

[6] Du Châtelet, *Institutions*, 190.

[7] “*Untrüglicher Beweise der Geometrie*” (UD, AA 02:287); “*schlechterdings unmöglich*” (ÜE, AA 08:203). See further MAN, AA 04:506; KrV A439/B467.

[8] See Figure 1. Compare MonPh, AA 01:478 with John Keill (*Introduction to Natural Philosophy*, London 1733, 26–27) and J. C. Gottsched’s 1721 Königsberg thesis *Dubia circa monades leibnitianas*, which can be found in Pasini, Enrico: “La prima recezione della Monadologia,” *Studia settecenteschi*, XIV, 1994, 107–163. Leonhard Euler makes a similar argument, while more explicitly addressing the key assumption that a line can be indefinitely extended (*Lettres à un Princesse d’Allemagne*, St. Petersburg 1768, II, 203–4)

[9] “*Dass durch zwei gegebene Punkte nicht mehr als eine gerade Linie gehen könne*” (ÜE, AA 8:202). Keill calls this principle “an Axiom in the *Elements*” (*Introduction*, 27). Actually, it is only tacit in Euclid, but becomes an axiom in influential later editions, such as Clavius’s from 1574.

between one-dimensional spaces, the argument generalizes. He concludes that no part of space is simple, that is, that space is a continuous quantity.[10] To justify the premises of this argument, Kant would presumably appeal again to geometry.

To be clear, both philosophers think geometrical truths have further metaphysical foundations, which include the fundamental properties of space. Yet geometry is supposed to evince clear *knowledge* that space is infinitely divisible (what Kant calls a *ratio cognoscendi*), even if in a metaphysical sense, space partly grounds geometry (as the *ratio essendi* of most geometrical truths).[11] As Du Châtelet puts it, cases from geometry allow us to "see" clearly that continuity holds with "extreme exactness."[12]

Problems loom for (S1), however. Euclid neither defines continuity nor assumes that geometrical figures are continuous. A tradition dating to Aristotle and Proclus does nevertheless takes a key result of geometry to be that considering incommensurable geometrical magnitudes shows that geometrical figures are continuous.[13] Even granting this, however, the continuity of Euclidean figures does not entail the continuity of space. Euclidean construction operations by compass and straightedge do not suffice to construct a two-dimensional continuum. So Euclidean constructions cannot demonstrate that infinite divisibility holds for every part of space. Recent scholarship has established that the early moderns were partly aware of this problem. For example, critics held that Euclid fails to prove

---

[10] See KrV A169/B211; V-Met-L1/Pölitz, AA 28:201; Refl, AA 14:131–2. Kant's main reason for denying that points can compose a region is that in that case, the region would depend on an actual infinity of points, which he thinks is inadmissible (KrV A169–70/B211; A439/B467; MAN, AA 4:506; V-Met-L1/Pölitz, AA 28:208). Instead, points are mere limits, dependent on regions (Prol, AA 04:352; V-Met-L1/Pölitz, AA 28:204; V-Met-L2/Pölitz, AA 28:570; V-Met/Mron, AA 29:842). Therefore, his definition of spatial continuity states that between any two points, there is not a point but a region. This is typical of Aristotelian continuity (see *Physics* VI.1). By contrast, contemporary set-theoretic definitions of density in terms of open intervals state (roughly) that there always exist points between points.

[11] Kant uses the *ratio essendi*/*cognoscendi* distinction at e.g. KpV, AA 05:4n. and V-Met/Mron, AA 29:748. Though Du Châtelet also takes geometry to reveal the metaphysics of space, unlike Kant she does not consider space and time primitive forms of intuition. She instead emphasizes the grounding role of God, the "eternal geometer," who providentially ensures harmony between material nature and geometry (Du Châtelet, *Institutions*, 34; Wells, Aaron: "'In Nature as in Geometry': Du Châtelet and the Post-Newtonian Debate on the Physical Significance of Mathematical Objects." In: *Between Leibniz, Newton, and Kant*, ed. Wolfgang Lefèvre, Dordrecht 2023, 69–98).

[12] Du Châtelet, *Institutions*, 33.

[13] Vincenzo De Risi argues that Euclid did not use the concept of continuity to characterize intersections between plane figures ("Gapless Lines and Gapless Proofs." In: *Apeiron*, 54(2), 2021, 233–259). For a different view, compare Sattler, Barbara: *The Concept of Motion in Ancient Greek Thought*. Cambridge 2020, 292–95. On the Aristotle-Proclus tradition, see Heath, Thomas: *The Thirteen Books of Euclid's Elements*. Cambridge 1908, I, 268.

that there exist points of intersection between figures, because he tacitly relies on intuition rather than deduction from axioms and definitions.[14]

Du Châtelet's second definition of continuity focuses on *connectedness*, rather than divisibility, so I'll call it the Connectedness Definition. Connectedness is required for a "being" to be called continuous, where the paradigm case is a body.[15] Specifically, she defines what it is for two parts of an extended being to be continuous with one another. Here continuity is a two-place relation, whereas divisibility is a one-place property. To be continuous, two parts must (1) touch each other and (2) be "linked together" in virtue of some "internal reason" that prevents them from being separated.[16] If condition (1) is met but not (2), then the two parts are merely *contiguous*, not continuous. She goes on to state that "we must represent space to ourselves as continuous," and also that "the continuity of bodies is actual," in that bodies satisfy the Connectedness Definition.[17]

These two-place definitions of continuity and contiguity are indebted to Aristotle.[18] He deploys similar definitions to make sense of the motion of bodies or physical substances. Two continuous bodies, for Aristotle, will move together. Du Châtelet also applies the continuity–contiguity distinction to bodies. She gives the example of two metal hemispheres that are in contact, but are only contiguous. If the hemispheres are not just touching but are fused together under high heat, then they will be continuous in the connectedness sense—although the sphere remains physically divisible.[19] A difficulty is that her distinction between contiguity and continuity is also supposed to apply to parts of space and geometrical figures, but it's unclear what that distinction amounts to outside a physical context.[20] We could in

[14] See De Risi, Vincenzo: "Leibniz on the Continuity of Space." In: *Leibniz and the Structure of Sciences*, ed. Vincenzo de Risi, Cham 2019, 111–170; "Has Euclid Proven Elements I, 1?". In: *Reading Mathematics in Early Modern Europe*, ed. Philip Beeley et al., London 2021, 12–32. Despite this, Kant famously embraces the justificatory role of intuition in geometry.

[15] Du Châtelet, *Institutions*, 106.

[16] Du Châtelet, *Institutions*, 106.

[17] Du Châtelet, *Institutions*, 107. She's correcting the first edition, where we read that "this contiguity" is actual (Du Châtelet, Émilie: *Institutions de Physique*. Paris 1740, 103).

[18] See *Physics*, V.3 (227a10–12); Metaphysics B.5 (1002a28–b10). Aristotle's two-place definition was a crucial starting point for Leibniz (Arthur, Richard: *Monads, Composition, and Force*. Oxford 2018, 40; De Risi, "Continuity"). Notoriously, though, Aristotle *also* defines continuity as infinite divisibility (e.g. *De Caelo*, I.1, 268a). On Aristotle's two definitions see Sattler, *Motion*, 295–334; Castelli, Laura: *Aristotle: Metaphysics Book Iota*. Oxford 2018, 26–31; and Pfeiffer, Christian: *Aristotle's Theory of Bodies*. Oxford 2018, 53–65, 89–121, 147–193.

[19] Du Châtelet, *Institutions*, 106–107.

[20] On these issues in Aristotle, see Pfeiffer, *Bodies*, 159–182) and Sattler, *Motion*, 294); for their uptake in Leibniz see Arthur, *Monads*, 52–3, 179–96 and De Risi, "Continuity;" on related issues in Wolff and Baumgarten, see Moretto, Antonio: "La rilevanza matematica della discussione sui concetti di continuo e di funzione nella filosofia tedesca dell'età dell'illuminismo." In: Fenomenologia e società, 18/2-3, 1995, 117–20.

principle give a causal account of how the hemispheres are bonded together, whereas no such account seems available for two hemispherical solids in geometry. Moreover, Du Châtelet states the Connectedness Definition essentially without argument, and it is hard to say how it fits into her relationalist theory of space and location.[21]

Leaving these difficulties aside, Du Châtelet importantly assumes that evidence from geometry is independently sufficient for accepting the infinite divisibility of space and matter, apart from the Connectedness Definition.[22] Kant, meanwhile, does not offer a definition of continuity in terms of the connectedness of parts. This is understandable in the case of connections between material parts or the states of these parts. These connections require causal action or interaction, not just compositional structure. For pure space, however, Kant is committed to a non-causal connection between parts, which ensures that spatial parts cannot move relative to each other. He could avail himself of a connectedness definition of continuity for pure space, but does not do so.[23]

A third definition of continuity appears early in the *Institutions*. This definition pertains to the continuity of processes, and relies on properties of mathematical functions. This definition has an important kind of priority. From it, Du Châtelet suggests, one can derive continuity as divisibility. On this definition, a function is continuous if its graph has a tangent at every point.[24] Call this the Continuous Function definition of continuity. A function with a smooth graph, she assumes, satisfies this condition, even at inflection points.[25] Graphs with angular bends fail this condition, since a tangent is not well-defined at the bend. She is gesturing at the idea of what we would now call an everywhere differentiable function. In

---

[21] Du Châtelet, *Institutions*, 116.

[22] As Marij van Strien reads Du Châtelet, continuity is understood *only* in terms of the Connectedness Definition, and this is compatible with denying the infinite divisibility of matter ("Continuity in Nature and in Mathematics." In: *EPSA15 Selected Papers*, ed. Michela Massimi et al., Dordrecht 2017, 75). I take van Strien's idea to be that two indivisible material atoms might touch each other and be linked together, satisfying the Connectedness Definition. However, there is a long Aristotelian tradition arguing that the Connectedness Definition *does* entail infinite divisibility: see Aristotle's anti-atomist arguments at *Physics* VI.1, *De Caelo* III.4, and *De Generatione et Corruptione* I.2. Although Du Châtelet does not present such arguments, she does not reject them either—and she in any case asserts the infinite divisibility of matter elsewhere. So van Strien's conclusion is premature.

[23] On causal connections between material parts and their states, see KrV B219 and B233, the telegraphic footnote at B201–202, and V-Met/Mron, AA 29:824. Kant states that the parts of pure space are unmovable at KrV A41/B58. It's possible that Kant avoids using the Connectedness Definition because it would apply to pure space, but not to matter. Compare the case of the continuity of change, where he explicitly makes the formal structure of pure time a condition for the continuity of material and psychological changes (A199/B244). I thank Marius Stan for helpful suggestions here.

[24] Du Châtelet, *Institutions*, 24; 32.

[25] Both of these moves may be indebted to Leibniz, but space does not permit entering into details. For a different reading of Du Châtelet on continuous functions, see Pelayo, Areins: "Certitude et loi de continuité dans les Institutions de physique d'Émilie du Châtelet." In: *Les études philosophiques*, 146(3), 2023, 13–14.

modern parlance, a function with angular bends can indeed be everywhere continuous, even though it is not everywhere differentiable. While she does not disentangle these issues, she recognizes the relative strength of the Continuous Function definition, because an angular graph need not display any gap that would violate the Divisibility or Connectedness definitions of continuity.[26]

Du Châtelet uses this conception of continuity to argue that space, time and matter are continuous in the Divisibility sense. In geometrical space, the motion of a point in accordance with a function "engenders a continuous succession by its fluxion," namely a line.[27]. This is motion in an abstract sense, which she treats as prior to static geometrical continua such as lines or circles. Next, she generalizes the idea to the motion of concrete physical things. A moving body or beam of light will pass through all *conceivable* locations and times in its path.[28] Du Châtelet takes it to be uncontroversial that the indefinite divisibility of space, time, and matter is at least conceivable. Given that motion satisfies the Continuous Function definition—bodies move through every conceivable location and time—she concludes that space, time, and matter really are divisible *ad infinitum*. This indicates that she takes the Continuous Function definition to be more basic in the order of explanation than the Divisibility Definition.

This definition differs from Kant's views in at least two key respects. First, the property she labels 'continuity' is roughly differentiability everywhere, and to my knowledge there is no such conception of continuity in Kant.[29] Second, she takes the continuity of processes and functions to be prior to the infinite divisibility of both space and time. Kant, by contrast, derives the continuity of change from the continuity of time.[30]

---

[26] She links the principle of sufficient reason to her assumption that any function that can be expressed by a single function or "law" has a well-defined tangent at every point (Du Châtelet, *Institutions*, 34; see further van Strien, "Continuity," 73–75; Wilson, Mark: *Imitation of Rigor*. Oxford 2022, 135–141. Conversely, she thinks any graph with angular bends is a "bastard figure" resulting from multiple functions (34). Both of these assumptions are wrong, so her route from the principle of sufficient reason to what we'd call differentiability everywhere is obscure. For other connections between continuity and sufficient reason in Du Châtelet, see Wells, "Post-Newtonian Debate," 89–94.

[27] Du Châtelet, *Institutions*, 127; compare 33.

[28] Du Châtelet, *Institutions*, 32.

[29] Kant does argue in the 1770s that if a point travels around a triangle, its motion must be discontinuous at the vertices (MSI, AA 02:400; V-Met-L1/Pölitz, AA 28:201–4). When the point reaches a vertex, its trajectory must switch from one direction to another, and given time's infinite divisibility, "there is a [finite] time" between these two states of motion, during which the point is "at rest," violating the continuity of motion (MSI, AA 02:400). Although the triangle example could be considered in terms of differentiability, in fact this flawed argument uses the Divisibility Definition of continuity.

[30] His derivation of the continuity of change parallels the Simple Argument for the continuity of space, though perhaps with additional premises. He usually suggests that at least as a "*formale Bedingung*," continuity of change can be established "*völlig a priori*" from metaphysical or mathematical grounds (KrV A209–210/B254–56; cf. MSI, AA 02:402; ÜE, AA 08:206). But one passage in the first *Critique* (A171/B212–13) implies that

2. From Continuity in Geometry to the Continuity of Matter

Recall (S2): If geometrical space is continuous, then matter is continuous. This premise relies on the assumption that if a property holds for geometrical space, then it also holds for matter. There are at least two prima facie questions about this line of reasoning, however. First, both philosophers assume that classical geometry has shown geometrical space as a whole to be continuous. The results of classical geometry, however, concern particular plane figures and solids rather than geometrical space as a whole. So their reasoning may commit a fallacy of composition.[31] A second question, which will be my main focus here, is whether Du Châtelet and Kant really hold that *all* properties established in geometry can be assumed to hold for matter. It seems that they do not: both reject other inferences from geometrical properties to properties of matter. Absent criteria for distinguishing acceptable from unacceptable inferences from geometry to matter, their doubts threaten to undermine (S2).

First consider Du Châtelet's views on these issues. While she explicitly denies that all substances are continuous (since some substances are simple), matter and its properties are partly mind-dependent phenomena. Given this partial mind-dependence, she concludes that "the same thing happens in nature as in geometry."[32] Geometrical space is continuous and therefore matter is continuous.[33] She argues for this as follows:

(G1) Assume for reductio that matter has indivisible parts.

(G2) To compose matter, these parts would have to take up space.

(G3) But then these parts would themselves have parts, and would not be indivisible, contradicting G1.

∴ (G4) Matter does not have indivisible parts.[34]

---

continuity of alteration is an empirical issue, perhaps because Kant here understands alteration not as a sheer succession of states but as causal and exemplified by motion (see B291–92). In his 1758 "New Doctrine," Kant goes so far as to say that some causal interactions are temporally continuous (as in the case of gravity) but others are not (as in the case of impact), and that therefore, the general assumption that physical change is continuous can be "*widerlegen*" on empirical grounds (NLBR, AA 02:21). On these complexities see Jankowiak, Timothy: "Kant on the Continuity of Alterations." In: *Canadian Journal of Philosophy*, 50(1), 2020, 49–66; McNulty, Michael Bennett: "Continuity of change in Kant's dynamics." In: *Synthese*, 196, 2019, 1595–1622.

[31] Thanks to Marius Stan for suggesting this.

[32] Du Châtelet, *Institutions*, 34; see 176 on the mind-dependence of matter.

[33] Du Châtelet, *Institutions*, 107; 435–36.

[34] This is a reconstruction of an argument at Institutions, 139; also see 107; 199–203. She stresses that this argument is relatively uncontroversial for large bodies, and there is no reason why it does not also hold at smaller scales (139–40). Other passages may appear to deny that matter is infinitely divisible, leading some to recommend atomist readings (van Strien, "Continuity"; Brading, Katherine and Marius Stan: *Philosophical Mechanics in the Age of Reason*. Oxford 2024, 136). But what Du Châtelet actually wants to rule out is that matter is composed of an infinity of "determinate and actual" parts that are prior to the whole (Du Châtelet, *Institutions*, 190; see further Coissard, Guillaume: "Du Châtelet entre monadisme et atomisme." In: *Revue*

A key assumption in the argument is that if something takes up space, it is divisible. This assumption follows from the divisibility of the parts of geometrical space, plus the principle that matter and geometrical space have “the same” compositional structure.

But she also cautions that inferences from geometry to “physical effects…always” involve “a considerable loss of exactness and precision.”[35] This suggests that mathematical representations are only approximately true of the material world, at least insofar as they seek to represent causal goings-on. For the broader scientific project of her *Institutions*, this need not be a problem. Approximations can be accurate enough that no appreciable error results, and more positively, simplification reasoning in a more “intelligible” way.[36] However, the infinite divisibility of matter is all-or-nothing. If the relationship between mathematics and matter is only approximate, then it’s unclear whether she is licensed to infer matter’s infinite divisibility from that of geometrical space.

Admittedly, Du Châtelet’s reference to physical effects in this discussion leaves room for construing the infinite divisibility of matter as just independent from physical causes and effects. In that case, the claim about approximation, as only holding for physical causes and effects, would not pertain to infinite divisibility. But Du Châtelet herself does not pursue this strategy, or attempt to sharply separate questions about the divisibility of matter from causal questions.

Now consider Kant. As seen in section 1, he thinks we can know a priori that the space of classical geometry is continuous.[37] In turn, the continuity of geometrical space entails the continuity of matter. “Nature,” he writes in 1770, “is completely subject to the prescriptions of geometry, in respect of all the properties of space which are demonstrated in geometry”; in 1790, he reaffirms that “concrete time and space” are “subject to that which mathematics demonstrates of its abstract space,” so that anything occupying a space “can be divided into just as many things…as…the space…which it occupies.”[38] As a supporting argument, Kant

d’histoire des sciences, 74(2), 2021, 297–329; Wells, “Post-Newtonian Debate”). This is compatible with matter’s continuity.

[35] Du Châtelet, *Institutions*, 417–18.

[36] Du Châtelet, *Institutions*, 395; 250.

[37] As I remarked there, the epistemological priority of geometry for our knowledge of continuity does not entail that it is metaphysically prior to space as a form of intuition (Carson, Emily: “Kant on Intuition in Geometry.” In: *Canadian Journal of Philosophy*, 27(4), 1997, 489–512; Sutherland, Daniel: *Kant’s Mathematical World*. Cambridge 2022, 125–26).

[38] MSI, AA 02:404; ÜE, AA 08:202 (“*Nun kann man hier nicht die Ausflucht suchen, die konkrete Zeit und der konkrete Raum sei demjenigen nicht unterworfen, was die Mathematik von ihrem abstrakten Raume…als einem Wesen der Einbildung beweiset…So lässt sich ebenso apodiktisch beweisen, dass ein jedes Ding im Raume…sobald sie einen Theil des Raumes…einnehmen, grade in so viel Dinge…geteilt werden, als in die der Raum…welche sie einnahmen, geteilt werden*”). See also KrV A165/B206; A439/B467; V-Met/Mron, AA

urges that if geometrical space and matter did not have the same structure, then mathematical physics would make erroneous claims about the world. Like Du Châtelet, Kant takes the compositional structures of space and matter to be in harmony.

This derivation from geometry is further worked out in the first *Critique*. A key premise is that any material object or body is a mere appearance in space. Appearances, in Kant's technical terminology, are representations through intuition.[39] Therefore, appearances partly depend on space and time as forms of intuition. In particular, the fact that bodies are essentially extended wholes depends on space. Since bodies fill space by being extended wholes, Kant concludes that the compositional structure of any body is the same as the compositional structure of the space it fills.[40] This line of reasoning is a priori. It establishes what Kant calls the metaphysical divisibility of matter. So whereas Du Châtelet sees a relation of approximation between the compositional structure of matter and geometrical space, Kant holds that these strictly correspond.

The *Metaphysical Foundations* gives a further argument that matter is not just metaphysically but also physically divisible *ad infinitum*. Physical divisibility requires a mechanism by which every part of matter can be moved away from its adjacent parts, but this does not follow from mere metaphysical divisibility.[41] This is why the *Foundations* argues at length against absolutely impenetrable parts of matter (atoms), and against discrete point-particles that exert repulsive and attractive force (physical monads), instead of relying on (S2). Still, Kant treats matter's metaphysical divisibility as a necessary prerequisite for its physical divisibility.[42] To establish metaphysical divisibility, Kant repeats a version of the Simple Argument from the *mathematical* continuity of space to the continuity of matter. A key supporting premise is that matter is merely an appearance.

29:930. In the 1750s, he assumes without argument that Euclidean constructions "can be done not only in the geometrical sense but also in the physical sense" (MonPh, AA 1:478). But the mature Kant does not quite assert the identity of concrete physical space and abstract geometrical space. Following Anja Jauernig, I think his argument requires only that these spaces have the same topological and compositional structure ("The Labyrinth of the Continuum." In: *The Sensible and Intelligible Worlds*, ed. Karl Schafer and Nick Stang, Oxford 2022, 207).

[39] Kant equates appearances with "*bloße Vorstellungen*" (KrV A491/B519), though he also says appearances are indeterminate objects *of* empirical intuitions, i.e. objects of representations (A20/B34). For current purposes, the key point is that essential formal properties of appearances are grounded in space and time as forms of intuition.

[40] For dependence claims, see KrV B202 and A525/B553. For sameness-of-structure claims, see A525/B553, A170/B212, V-Met-L1/Pölitz, AA 28:202–203, and Refl, AA 18:410.

[41] For the claims discussed in this paragraph see MAN, AA 04:503–506. Arthur, *Monads*, 39–43 considers similar definitions of physical divisibility in Leibniz and various Cartesians. Physical divisibility in this sense is also treated in Du Châtelet: see footnote 51 below.

[42] See further Pollok, Konstantin: *Kants 'Metaphysische Anfangsgründe der Naturwissenschaft': Ein Kritischer Kommentar*. Hamburg 2001, 257–64.

However, Kant does not assume the soundness of just any inference from general structural features of geometrical space to features of matter. A striking example is his discussion of empty space in the *Foundations*. Geometry, Kant thinks, presupposes an unbounded space, lacking either outer limits or internal gaps. But Kant does not conclude that matter is actually an unbounded plenum:

> The well-known question as to the admissibility of empty spaces in the world may serve as our conclusion. The *possibility* of such spaces cannot be disputed. For space is required for all forces of matter, and since it also contains the conditions of the laws of diffusion of these forces, it is necessarily presupposed prior to all matter. Thus attractive force is attributed to matter insofar as it *occupies* a space around itself, through attraction, without at the same time *filling* the space. Thus the space can be thought as empty.[43]

Neither geometry nor the metaphysics of attractive forces settle the question whether there exist dynamically empty spaces—that is, spaces in which no repulsive forces act. This is a point of contrast with Du Châtelet, whose principle of sufficient reason rules out empty spaces.[44] Kant instead indicates that *if* the question of dynamically empty spaces were to be settled, then empirical evidence for or against empty spaces would need to be available to us.[45] By contrast, Kant does not think establishing the infinite metaphysical divisibility of matter requires empirical evidence.[46] It is unclear why Kant accepts one inference from geometry to matter, but not the other.

This suggests that he should have taken more seriously how empirical considerations might bear on the compositional structure of matter.[47] While I cannot go into the topic in

[43] "*Den Beschluss kann die bekannte Frage wegen der Zulässigkeit leerer Räume in der Welt machen. Die Möglichkeit derselben lässt sich nicht streiten. Denn zu allen Kräften der Materie wird Raum erfordert und, da dieser auch die Bedingungen der Gesetze der Verbreitung jener enthält, notwendig vor aller Materie vorausgesetzt. So wird der Materie Attractionskraft beigelegt, sofern sie einen Raum um sich durch Anziehung einnimmt, ohne ihn gleichwohl zu erfüllen, der also selbst...als leer gedacht werden kann.*" (MAN, AA 04:534–35).

[44] Du Châtelet, *Institutions*, 96; 207ff.

[45] Kant denies the consequent: "*alle Erfahrung giebt uns nur comparativ leere Räume zu erkennen*" (MAN, AA 04:535). By modus tollens, we can't be sure if empty spaces exist. The conclusion of the *Foundations* returns to this topic, however, and seems more hopeful about empirically ruling out dynamically empty spaces, in favour of an aether (MAN, AA 04:563–64). On this apparent conflict see Pollok, *Kommentar*, 341–46; 360–70).

[46] More precisely, the following conditional proposition needs no empirical evidence: If matter exists, then it is infinitely metaphysically divisible.

[47] Michael Friedman argues that Kant did attend to empirical considerations—from Euler's fluid dynamics—in switching from physical monads to continuous matter ("Synthetic History Reconsidered." In: *Discourse on a New Method*, ed. Mary Domski and Michael Dickson, Chicago 2010, 604–10). But Friedman's supporting text is buried in a remark on cohesion (MAN, AA 04:526ff.) that does not support the main argumentative structure of

detail, Kant's embrace of the continuity of matter in the 1760s may be linked to his abandonment, around the same time, of a relationalist account of space. He was partly inspired by Newton, but as his contemporaries appreciated, the dynamics of Newton's *Principia* has serious limitations. Marius Stan has argued, in turn, that Euler's extension of Newton's dynamics is the best fit, among the historically available options, for the *Metaphysical Foundations*. But Euler's rotational dynamics crucially involves a torque law, and for Kant to be able to derive this law from premises he'd accept, he would probably have to abandon his continuum account of matter.[48] Given these empirical factors, Kant might have been better served by sticking with a version of his early physical monadology, on which matter is composed of mass-points exerting distance forces.

3. Continuity, Parts, and Wholes

So far, I've focused on two main senses in which matter may be said to be continuous: infinite divisibility and connectedness of parts. Kant and Du Châtelet accept infinite divisibility, while stressing that matter is not composed of an infinite number of actual, prior parts. They hold that the parts of matter, like the parts of space and time, are merely possible or potential. Both of them draw the further conclusion that potential parts are not prior to composites, but instead are dependent on wholes.[49] Yet there is a tension here that's still relatively neglected, because these philosophers both adopt a realist stance on infinitesimals.

For Du Châtelet, the number of parts in a geometrical object is absolutely indeterminate, as the objects of geometry only have potential parts.[50] We've seen that she argues against the possibility of indivisible parts of matter by reasoning that, because such

the *Foundations*. Kant's official justification of matter's infinite divisibility stems from mathematics and metaphysics, not fluid dynamics.

[48] Euler's torque law says that $\tau = \frac{dL}{dt}$, where τ is net torque and L is angular momentum. For Kant to ground this law, he arguably would need to assume the principle that forces between particles are not only equal and opposite, but central: acting on a straight line between the particles (Stan, Marius: "Kant and the object of determinate experience." In: *Philosophers' Imprint*, 15(33), 2015, 12). This assumption needs a model of matter where discrete particles interact at a distance.

[49] The proposal that continua possess parts potentially, and lack an actual infinity of parts, goes back at least to Aristotle (*Metaphysics* Δ.13; Sattler 2020, 305–11). Aristotle seems to conclude that substances, as wholes, are ontologically prior to their potential parts (e.g. *De Anima* III.6). In the early modern period, Leibniz agrees that parts of continua are potential, and takes this violation of the usual priority of part over whole to be a major problem raised by the composition of matter (Jauernig, "Labyrinth," 189–97). But in the 1740s German *Monadenstreit* and most of Kant's discussions, the focal issue is not priority but *divisibility*, especially metaphysical divisibility. Kant nevertheless faces puzzles about part–whole priority. For instance, he thinks determinate parts of space, time, and matter are extensive magnitudes, which by definition involve some kind of priority of part over whole, despite being continuous (210; Sutherland, *Mathematical World*, 98–101). I here raise a related puzzle for Kant's physics.

[50] Du Châtelet, *Institutions*, 190.

parts occupy space, they must in principle be divisible. For metaphysical or in-principle divisibility, then, the number of parts in material things must also be indeterminate.[51] She takes this to solve a Zeno paradox that results from assuming that physical extension is composed of an infinite number of extended parts that exist prior to division. The implication is that potential parts depend on the wholes they make up, and also depend on acts of division. This is in tension, however, with her treatment of infinitesimal moments of force:

> An infinitely small living force…can only become a finite living force when it is repeated an infinite number of times, and accumulated by an infinity of successive pressures [*pressions*] in the body that receives the motion…this infinitely small force…is the element of the living force.[52]

This is no mere mathematical idealization. The infinitesimal moments of living force have physical reality, insofar as they are the effects of pressure or dead force. So she is committed to infinitely many infinitesimal moments. These are not mere potential parts or limits, but real non-Archimedean magnitudes, existing prior to the finite forces to which they 'sum.'[53]

Parallel issues arise in Kant's *Metaphysical Foundations*. On the one hand, "the parts" of matter, "as belonging to the existence of an appearance, exist only in thought, namely, in the division itself."[54] In other words: matter does not consist of infinitely many prior parts, but parts of matter are merely potential, coming into being when wholes are divided. Nevertheless, Kant sometimes commits himself to realism about infinitesimals. Some forces, such as gravity, are infinitesimally small at an instant, but 'sum' over time to a finite result. The infinitesimal "moment [*Moment*]" of gravity is a real "cause [*Ursache*]" rather than a mathematical idealization (KrV A168/B210). In these passages, then, Kant commits to

---

[51] This may seem hard to square with her statement that each physical object has a "fixed number" of determinate parts (Du Châtelet, *Institutions*, 191). As Coissard, "Monadisme" has shown, however, Du Châtelet distinguishes metaphysical divisibility from physical division. The latter requires a causal mechanism by which parts of matter can be "really separated" or moved away from their adjacent parts (1742, 222; 200). So she likely means that physical objects are actually *physically* divided into a fixed, finite number of parts, yet potentially *metaphysically* divisible into an indefinite number of parts.

[52] Du Châtelet, *Institutions*, 438.

[53] Fluxions and infinitesimals are sometimes used interchangeably by both Du Châtelet (*Institutions*, 127; 265; Cajori, Florian: "Madame Du Châtelet on fluxions." *The Mathematical Gazette*, 13, 1926, 252) and Kant (Refl, AA 18:167). Like many after Leibniz, she understands integrals as literal 'sums' of infinitesimals, akin to the indivisibles of Cavalieri and Torricelli (compare Leibniz 1849–63, V, 226–33). I put 'sum' in scare quotes because Du Châtelet's mathematical foundations, like Kant's, only define addition for Archimedean magnitudes, not for infinitesimals (*Institutions*, 132; Kant, V-Met-L2/Pölitz, AA 28:561). We know that she owned and consulted a calculus textbook written by l'Hôpital and Johann I Bernoulli. She may have assumed that l'Hôpital's approach to infinitesimals, based on ratios with non-finite terms, was sufficiently rigorous.

[54] "*Die Theile, als zur Existenz einer Erscheinung gehörig, existiren nur in Gedanken, nämlich in der Theilung selbst*" (MAN, AA 04:507). See further Pollok *Kommentar*, 264–65.

infinitely many infinitesimals that would be prior to finite quantities, rather than merely potential and dependent on them.[55]

Both philosophers are realists about infinitesimal moments of force, but potentialists about the parts of matter. This could suggest a philosophically relevant difference between, say, active force and passive matter. However, they each take matter to be essentially endowed with active force. Indeed, both seem to ground matter's extension—and thus its part–whole structure—in more basic forces.[56] So no clean distinction between active force and passive matter is available to them.

4. Concluding Assessment

By way of conclusion, let me sketch some successful and unsuccessful aspects of each philosopher's approach to continuity, which may suggest avenues for future research. Du Châtelet's discussion is richer than Kant's, in that it deals with not one but three notions of continuity. She thereby touches on ideas that were especially fruitful for later mathematics, including analysis and topology.[57] Moreover, she presents continuity as differentiability everywhere (and perhaps also continuity as Connectedness) as more basic than continuity as divisibility. The Simple Argument may be just one possible route to matter's infinite divisibility. Unfortunately, this is hard to assess: it is not always clear how Du Châtelet justifies her various accounts of continuity, nor what entailment relations might hold between them.

As for Kant, he holds that continuity is a unique, qualitative property and amounts to the divisibility of every part of a magnitude. There is only one sense in which matter is continuous. But to his credit, the logical structure of Kant's position is clearer than Du Châtelet's, as he takes care to give justifications and explications of each premise in the Simple Argument. Still, his narrow focus on divisibility has at least two negative consequences. One is a failure to engage in detail with relational definitions of continuity, or with the notion of continuity as differentiability everywhere. Another consequence is that his

---

[55] See also KrV A168/B210; NG, AA 02:168; and especially MAN, AA 04:551–52. Further similar texts are discussed by Sutherland, Daniel: "Continuity and Intuition in Eighteenth-Century Analysis and in Kant." In: *The History of Continua*, ed. Stewart Shapiro and Geoffrey Helman, Oxford 2021, 178–79. Sutherland concludes that given Kant's potentialist language elsewhere, he should be read on balance as handling infinitesimals in terms of limits. While this may be the most charitable reading, textual tensions remain.

[56] See Du Châtelet (1742, 165); Kant (MAN, AA 04:536, V-Met/Mron, AA 29:841).

[57] De Risi, "Continuity" explores the historical importance of what I've called the Connectedness Definition of continuity.

assumption that matter is continuous stands or falls with Simple Argument, and with its dubious assumption that classical geometry has proven space to be continuous.

Acknowledgments
I owe special thanks to Stephen Howard, Jeff McDonough, Rudolf Meer, Lorenzo Spagnesi, Marius Stan, and an editor at *Kant-Studien* for written comments on earlier versions; to Emily Carson for correspondence that inspired several ideas; and to Silvia De Bianchi for giving me the opportunity to present some of this material at the University of Milan's PROTEUS seminar. For helpful exchanges, I am also indebted to Joe Anderson, Laura Follesa, Rima Hussein, Laura Marongiu, William Marsolek, Andrea Reichenberger, Anat Schechtman, and Jan-Willem van Holten.

Works cited

Arthur, R. T. W. (2018). *Monads, Composition, and Force*. Oxford: Oxford University Press.

Brading, K. and M. Stan (2024). *Philosophical Mechanics in the Age of Reason*. Oxford: Oxford University Press.

Büchel, G. (1987). *Geometrie und Philosophie*. Berlin and New York: De Gruyter.

Cajori, F. (1926). Madame Du Châtelet on fluxions. *The Mathematical Gazette*, *13*, 252.

Carson, E. (1997). Kant on Intuition in Geometry. *Canadian Journal of Philosophy*, *27*(4): 489–512.

Castelli, L. (2018). *Aristotle:* Metaphysics *Book Iota*. Oxford: Clarendon Press.

Coissard, G. (2021). Du Châtelet entre monadisme et atomisme: La matière dans les *Institutions de physique*. *Revue d'histoire des sciences*, *74*(2), 297–329.

De Risi, V. (2019). Leibniz on the Continuity of Space. In V. De Risi (Ed.), *Leibniz and the Structure of Sciences* (pp. 111–170). Cham: Springer.

De Risi, V. (2021a). Gapless Lines and Gapless Proofs: Intersections and Continuity in Euclid's *Elements*. *Apeiron*, *54*(2), 233–259.

De Risi, V. (2021b). Has Euclid Proven *Elements* I, 1? The Early Modern Debate on Intersections and Continuity. In P. Beeley, Y. Nasifoglu, and B. Wardhaugh, (Eds.), *Reading Mathematics in Early Modern Europe* (pp. 12–32). London: Routledge.

Du Châtelet, E. (1740). *Institutions de Physique*. First edition. Paris.

Du Châtelet, E. (1742). *Institutions Physiques*. Second edition. Paris.

Euler, L. (1768). *Lettres à un Princesse d'Allemagne sur divers sujets de physique & de philosophie*. Three volumes. St. Petersburg.

Friedman, M. (2010). Synthetic History Reconsidered. In M. Domski and M. Dickson (Eds.), *Discourse on a New Method* (pp. 571–813). Chicago: Open Court Press.

Heath, T. L. (1908). *The Thirteen Books of Euclid's Elements*. Cambridge: Cambridge University Press.

Goldenbaum, U. (2021). How Kant was Never a Wolffian, or, Estimating Forces to Enforce *Influxus Physicus*. In B. C. Look (Ed.), *Leibniz and Kant* (pp. 27–56). Oxford: Oxford University Press.

Jankowiak, T. (2020). Kant on the Continuity of Alterations. *Canadian Journal of Philosophy*, *50*(1), 49–66.

Jauernig, A. (2022). The Labyrinth of the Continuum: Leibniz, the Wolffians, and Kant on Matter and Monads. In K. Schafer and N. F. Stang (Eds.), *The Sensible and Intelligible Worlds* (pp. 185–216). Oxford: Oxford University Press.

Keill, J. (1733). *Introduction to Natural Philosophy*. Fourth edition. London.

Leibniz, G. W. (1849–63). *Leibnizens mathematische Schriften*. (C. I. Gerhardt, Ed.). Halle: Schmidt.

McNulty, M. B. (2019). Continuity of change in Kant's dynamics. *Synthese*, *196*, 1595–1622.

Moretto, A. (1995). La rilevanza matematica della discussione sui concetti di continuo e di funzione nella filosofia tedesca dell'età dell'illuminismo. *Fenomenologia e società*, 18/2–3, 109–153.

Pasini, E. (1994). La prima recezione della *Monadologia*. *Studi settecenteschi*, *XIV*, 107–163.

Pelayo, A. (2023). Certitude et loi de continuité dans les *Institutions de physique* d'Émilie du Châtelet. *Les études philosophiques*, *146*(3), 7–22.

Pfeiffer, C. (2018). *Aristotle's Theory of Bodies*. Oxford: Oxford University Press.

Pollok, K. (2001). *Kants 'Metaphysische Anfangsgründe der Naturwissenschaft': Ein Kritischer Kommentar*. Hamburg: Meiner.

Sattler, B. (2020). *The Concept of Motion in Ancient Greek Thought*. Cambridge: Cambridge University Press.

Stan, M. (2015). Kant and the object of determinate experience. *Philosophers' Imprint*, *15*(33), 1–19.

Stan, M. (2023). Newtonianism and the Physics of Du Châtelet's *Institutions de Physique*. In A.M. Roos and G. Manning (Eds.), *Collected Wisdom of the Early Modern Scholar* (pp. 277–97). Cham: Springer.

Sutherland, D. (2021). Continuity and Intuition in Eighteenth-Century Analysis and in Kant. In S. Shapiro and G. Hellman (Eds.), *The History of Continua* (pp. 123–58). Oxford: Oxford University Press.

Sutherland, D. (2022). *Kant's Mathematical World*. Cambridge: Cambridge University Press.

van Strien, M. (2017). Continuity in Nature and in Mathematics: Du Châtelet and Boscovich. In M. Massimi et al. (Eds.), *EPSA15 Selected Papers* (pp. 71–81). Dordrecht: Springer.

Wells, A. (2023). ‘In Nature as in Geometry’: Du Châtelet and the Post-Newtonian Debate on the Physical Significance of Mathematical Objects. In W. Lefèvre (Ed.), *Between Leibniz, Newton, and Kant*, 2nd ed (pp. 69–98). Dordrecht: Springer.

Wilson, C. (2019). The Reception of Newton’s Theory of Matter and his Atomism. In H. Pulte and S. Mandelbrote (Eds.), *The Reception of Isaac Newton in Europe* (vol. II, pp. 433–450). London: Bloomsbury.

Wilson, M. (2022). *Imitation of Rigor*. Oxford: Oxford University Press.