DU CHÂTELET'S PHILOSOPHY OF MATHEMATICS

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ABSTRACT
Du Châtelet's main works articulate a stance on the nature of mathematical objects, our knowledge of mathematical truths, and how such truths apply to the physical world. I begin by outlining her ontology of mathematical objects, on which mathematical objects partly depend on acts of abstraction by finite minds. Next, I consider how this mind-dependence can be reconciled with her endorsement of necessary truths in mathematics. Lastly, I turn to her views on the relation between mathematics and physics, with a focus on how she can hold that geometry and nature correspond, even though in some cases mathematical representations are only approximately true.

At the beginning of her intellectual career, in 1734, Émilie Du Châtelet wrote to Maupertuis that she wished to become a "*géomètre*" (2018, I:133). Literally, this would mean becoming a mathematician or geometer, but the term could also have a broader sense, standing for mathematical physics. Du Châtelet in fact appears to have learned mathematics mainly in order to work on physics and its philosophical foundations. She explicitly frames mathematics as a "means" to natural philosophy and its philosophical underpinnings (1742, §xi, 12; all translations mine). Accordingly, her philosophical discussions of mathematics often focus on how mathematics plays a role in natural philosophy. Nevertheless, we'll see that she also advances distinctive and controversial claims about the metaphysics and epistemology of mathematics: on the nature of mathematical objects, our knowledge of mathematical truths, and how such truths apply to the physical world.[1]

This chapter focuses on three main themes. First, I aim to make sense of her view that mathematical objects are partly mind-dependent (section 1). Second, I consider the extent to which this makes mathematical objects and truths contingent, given that she asserts that many mathematical truths are necessary (section 2). Finally, I look at her views on the application of mathematics in natural philosophy, and especially her suggestion that at least some properties in the physical world—such as distances—correspond exactly to geometry (section 3).

1. Du Châtelet's Idealism about Mathematical Objects

To start with the general metaphysical picture, Du Châtelet holds that matter is not fundamental in the created order. There are more fundamental, immaterial simple substances. Yet we cannot observe simple substances directly, and physics deals merely with non-fundamental bodies.

Interpreters disagree on precisely how matter is grounded in simple substances, and in particular on whether matter is mind-dependent. On Marius Stan's reading, the existence of matter is grounded solely in simple substances that are not mind-like, so it "does not require any [non-divine] mental facts to ground it" (2018, 493). Caspar Jacobs, by contrast, argues that since extended matter is a mind-dependent construct, matter's existence is partly grounded in mental facts (2020, 69; 71). A similar reading is defended by Emily Carson (2004; forthcoming). Carson calls attention to Du Châtelet's claims that both geometrical extension and material "objects that fall under our senses" are confused representations of simple substances (1742, §134, 156). For Carson and Jacobs, an essential property of matter, namely extension, depends on finite minds. This means that finite minds are a necessary, though not sufficient, condition for the existence of matter. I focus on mathematical objects rather than matter, so I'll start off neutral on this debate. At the end, though, I'll suggest that my account raises some indirect reasons to view matter in Du Châtelet as mind-dependent.

While the status of matter is disputed, there is broad agreement that Du Châtelet's mathematical objects are mind-dependent in a distinctive way, independent of the status of material things. The reason for this is that geometrical extension and numbers are "formed by abstraction" from perceived material things (1740, 107).[2] By contrast, we do not distinctly perceive simple substances, and Du Châtelet does not suggest that material things are abstracted from simple substances. Abstraction, in turn, is an activity carried out by finite minds. Since she holds that extension and number are formed by abstraction, it follows that extension and number are partly dependent on activities carried out by finite minds.

This does not yet settle just what finite minds contribute in the process of abstraction. One way to understand abstraction is negative, so that abstracting merely disregards some properties of things in order to focus on their other properties. Mathematical objects would then be identical to magnitude properties that are already there in material things. In support of this reading, Du Châtelet does sometimes say that abstraction involves neglecting or leaving out properties (1742, §87, 112–13). The manuscript of the *Institutions*, goes so far as to directly characterize magnitudes as properties of material things, with no mention of further acts of abstraction. Specifically, magnitude or size is an "internal" property of a "thing," seemingly apart from its relations to other things or to finite minds (1737–40, ff. 33^r–33^v).

However, Du Châtelet's considered view must be that abstraction is not merely negative or subtractive. She repeatedly states that extension and number are not identical to the material things from which they are abstracted.

> Space is not the things themselves, it is a being that we have formed by abstraction…space is to real beings, as numbers are to things numbered. (1742, §87, 112–13)

> Number is not the things numbered…[time] is, like number, different than these things that follow one another in a continuous sequence. This comparison between time and number can help in forming for ourselves the true notion of time. (§103, 125).

Further passages show that she has in mind not just a numerical difference between mathematical objects and things, but different properties of these mathematical entities. Consider extension: its properties include being "similar, uniform, self-subsistent" and "immutable," and she does not attribute these properties to the material bodies from which extension is abstracted (1742, §85, 109). Extension has these properties in virtue of acts of abstraction. Therefore, abstraction does not merely subtract properties from bodies, but adds new properties.

There are also passages directly treating abstraction, which do not depict it as just mentally subtracting properties that are already in material objects. Instead, abstraction is a creative "power" of "forming…imaginary beings that contain just the determinations that we want to examine" (1742, §79, 105; §86, 111). If abstraction merely subtracted properties, it would be unclear why Du Châtelet characterizes it as actively forming beings.

For further textual support, observe first that Du Châtelet directly characterizes properties (not just objects) as abstract or concrete. Mathematical objects, as products of abstraction, thereby have "abstract" properties which are distinguished from properties of matter (1742, §86, 111). Token properties of bodies, by contrast, are "concrete" (§86, 111). The size of a particle, understood as a physical magnitude, is located somewhere and somewhen, but a number is not. Geometrical lines have no width, and do not exist in material nature, since all bodies do have width; a similar point applies for geometrical surfaces (§86, 111; §174, 196–97). To take another example, we can assume that a geometrical solid is homogeneous and has exact boundaries, whereas matter is actually an irreducibly heterogeneous mixture that lacks exact boundaries (191–92; 200–201). Again, material things do not possess these abstract properties, so merely subtracting other properties from material things by acts of abstraction will not yield abstract properties.

Second, Du Châtelet holds that mathematical and material entities have different identity conditions:

> When one divides a [geometrical] line in whatever way, and into however many parts, as one wishes, the same line always results from reassembling the parts, however one transposes these parts among themselves: it is the same for surfaces [i.e., planes] and for geometrical bodies [i.e., solids]. (1740, §78, 99; see also 1742, §169, 190)

So geometrical objects preserve their identity and properties across certain rearrangements, or decompositions and reaggregations, of their parts. In particular, these are operations that preserve the external limits of the geometrical object in question (as opposed to, for example, reassembling the parts of a given line segment into multiple shorter segments). But we cannot assume that the same body always results from reassembling its parts in a different order. The result may be a body of a different natural kind, with a different density and volume (1742, §198, 216–17).[3] A related contrast is that geometrical objects are taken to have their properties independent of their surroundings: for example, Euclidean geometry assumes that figures can be superimposed without deformation. The shape of a body, however, partly depends on causal interaction with its surroundings (§184, 207). Du Châtelet seems to hold that an object's identity conditions are essential to it.[4] Given that on her view mathematical and material entities have different identity conditions, it would follow that these two kinds of entity differ essentially. And in that case, merely subtracting properties from a material entity cannot yield a mathematical entity, since the mathematical entity will have some essential properties that the material thing does not.

Third, in describing how the natural numbers are formed, Du Châtelet mentions not just negatively abstracting *from* material objects, but also further acts of aggregation or concatenation. In a first step, one abstracts away from most of the determinate properties of a material thing and considers it as a mere "being" and a "unit," that is, the number 1 (1742, §103, 125).[5] This abstraction process requires reference to a category or "class" of thing, so it is not just arbitrary which portions of matter come to be considered as a being or unit (§103, 125). Since matter is indefinitely divisible, there are no material substances in the strict sense. The only true created substances are non-extended simple beings (§152, 166; Gireau-Geneaux 2001). Material objects can only be considered as essentially unified wholes, and therefore as countable, if they are brought under a "class" or concept. This indicates that the property of being a unit is not just there to be found in material objects. In a second step, which also depends on the activities of finite minds, units are aggregated into numbers. This involves grasping a "relation" among all of the units that make up the number (§103, 125). The number seven, on this account, is not directly abstracted from collections of material things, such as seven apples on a tree, because mere apples are not aggregated. While a definition of the natural numbers as an aggregate or multitude of units already appears in Euclid, Du Châtelet stresses that the units aggregated are themselves ideal or abstracted entities. This means that the aggregation of these ideal entities depends on the contingent acts of finite minds:

> To make a number one combines some units, the combination of which is not in the least necessary, but merely possible. (1742, §46, 70)

While I'll argue in the next section that there is still a sense in which Du Châtelet allows numbers to be necessary, the key point to stress here is that aggregating units is an *additive* process that concludes in grasping the number as a whole. This is different in kind from neglecting or removing properties from a material thing by abstraction.

Before continuing, it will be worth commenting on her occasional characterization of mathematical objects as not just abstract or ideal, but *imaginary*. This might suggest that mathematical beings are on a par with daydreams or fantasies. Furthermore, Du Châtelet seems to equate (at least some) "abstractions" with "fictions," and warns against "illusions of our imagination" (1742, §86, 111; §2, 17). However, at the time, fictions referred to products of human creation in a broad sense. She praises the usefulness of imaginary representations, which "help us infinitely" in scientific inquiry, and are "one of the greatest resources for solutions to the most difficult problems, which the understanding alone cannot achieve" (1742, §86, 111–12). So these fictions, which are applied to solve problems in natural philosophy, need not be what she calls a "heap of fables" (§71, 93).

Moreover, although the representations formed by abstraction are imaginary, "the determinations that we want to examine," that is, the content of these representations, need not be merely imaginary. She suggests that the understanding is also involved in mathematics, and she seems to specifically link the understanding to logical demonstration, which can help avoid errors of the imagination. While this is not the place to fully explicate the relationship between understanding and imagination in Du Châtelet, an initial point is that imaginary representations seem essentially spatial, while representations of the understanding are not.[6] So geometry typically involves the imagination, whereas algebra is based "only" in the "understanding" (1742, §ii, 3). Algebraic geometry combines these approaches: a function can be presented in a spatial, imagistic way through the imagination, but it can also be presented by the understanding alone, as an algebraic formula. In this way, the determinations of a figure can be expressed without the use of the imagination. Indeed, she holds that some geometrical figures can be conceived through the understanding, but not adequately imagined

(§135, 158–59). An example might be the solid that Torricelli showed to have infinite surface area but finite volume. We can partially represent this solid in the imagination, or with a diagram. But it cannot be given a complete finite spatial representation. So perhaps in this case, an imaginative representation aims at representing an object conceived through the understanding, but does not do so completely.

As it stands, Du Châtelet's conception of mathematical objects remains vulnerable to an objection. If mental activities like abstraction play such a crucial role in generating mathematical content, mathematics might seem to become subjective and arbitrary. Why assume that our powers of abstraction track the truth? Du Châtelet invokes the principle of contradiction as a constraint. But this cannot provide an adequate response. She acknowledges that mathematics does not get all of its content from the principle of contradiction: a rigorous Euclidean demonstration that takes the form of a reductio ad absurdum also requires, for example, showing how to construct figures (1742, §3, 18–19). Her account of mathematical necessity, as I detail in the next section, introduces further constraints on the content of mathematics, reducing the force of the objection. But this could bring Du Châtelet's account closer to realism about mathematical objects than it first appears to be.

2. Mathematics and Necessity

Du Châtelet holds that at least some mathematical truths are necessary. Our acts of abstraction appear to be contingent, so the necessity of mathematical truth would seem to need another source. She assigns the divine intellect an important role in fixing mathematical truths. But I take her to hold that at least some mathematical objects and mathematical truths are only actualized in virtue of contingent activities of finite intellects.

Since Leibniz's metaphysics is an important part of the background to Du Châtelet's *Institutions*, I will begin by briefly surveying related puzzles about mathematical necessity that arise for Leibniz. On the one hand, in many texts he is clear that at least some mathematical truths hold with "absolute" necessity (Leibniz 1875–90, IV:391). Leibniz often depicts these truths as actual, intentional objects of the divine understanding, though he also indicates a role for other divine attributes, such as immensity and eternity (II:49; II:305; V:210; VII:184; VII:275–78, 2001, 334; 2011, 123; 308–9; 337–38). Either way, mathematical truths depend on essential properties of God apart from volitions. They are independent of God's choice of which world to create, so these mathematical truths hold in every possible world. This need not imply that our mathematical knowledge requires direct access to the divine intellect. Leibniz sometimes offers an alternative preformationist account: since God is the creative cause of our mental faculties, our contingent acts of abstraction are designed to be able to arrive at mathematical truths (Leibniz 2011, 228; Reichenberger 2021, 342).

On the other hand, there are some texts where Leibniz suggests that mathematical objects and truths just depend on finite minds. Take numbers: Leibniz asserts in in 1676 that they

> are true entities only when they are thought about by us…for they can always be multiplied by perpetually reflecting on them, and so they are not real entities, or possibles, except when they are thought about. (Leibniz 2001, 83)

The passage continues by denying that there are as many things as there are numbers, since the "multiplicity of things is something determinate, that of numbers is not" (2001, 83). From the mind-dependence of numbers, then, this passage seems to conclude that they do not form a given infinite whole, but are determined by the mental activities of finite thinkers. The passage also implies that whatever can be "understood" *of* numbers—including truths about numbers—also depends on mental activity (83). While this discussion of numbers is from an early phase of Leibniz's development, Samuel Levey (1999) argues that Leibniz's later work regularly assumes that number and extension are finite and dependent on the activities of finite minds. It is in any case striking that Leibniz makes these claims about the dependence of numbers on finite thought while also taking necessary mathematical truths to be in some sense grounded in God.

This passage from 1676 also hints at a way of easing the tension between mind-dependence and mathematical necessity. Leibniz's exact claim is not that the existence of numbers depends on finite minds tout court, but rather that numbers are "true" or "real" entities only when thought about by finite minds. So they might have some minimal existence independent of this. Whether or not this reading is correct—one still needs to account for Leibniz's suggestion that even *possible* numbers depend on our thought—I think it can serve as a helpful point of comparison with Du Châtelet. For she also appears to say both that mathematical facts are necessary in virtue of depending on the divine intellect, *and* that some mathematical objects depend on finite minds. I will suggest that she can hold both views consistently, even if the resulting position may strike us as odd.

First, however, I deal with some texts that could suggest Du Châtelet accords strict logical necessity to many mathematical truths. If that were right, the dependence of mathematical objects and truths on finite minds would seem to be ruled out. A passage early in Du Châtelet's *Institutions* hints at the view that all geometrical truths are just logical truths:

> In geometry where all truths are necessary, one only makes use of the principle of contradiction. For in a triangle, for example, the sum of the angles is determinable only in a single manner, and the angles absolutely must equal the sum of two right angles. (Du Châtelet 1740, §8, 24–25).

I want to begin by showing that, in context, her claim that "only" the principle of contradiction is used in geometry is not as strong as it might seem.

To be sure, Du Châtelet takes logical demonstration to be crucial in geometry and science more generally (1740, §2, 17). It is a necessary condition for mathematical truth. If it did not hold,

> There would no longer be any truth, even in numbers, and each thing could be or not be, according to the fantasy of each, so that two and two could make four just as much as six, or even both at once (§2, 18–19; see also 1737–40, 29^{v}–30^{r})

Like Leibniz (1875–90, IV:363), she rejects a Cartesian account on which intuition of eternal truths and a "lively, internal sentiment of clarity and evidence" are the basis for mathematical reasoning (Du Châtelet 1742, §2, 17).[7] While Du Châtelet grants that we appear to have clear and distinct ideas of mathematical objects such as equilateral triangles, she holds that these appearances may mislead us. We should rely instead on rigorous deductive proofs (17).

Nevertheless, she does not construe geometrical proof as resting on logic alone. While her views here are not unusual in an eighteenth-century context, it will nevertheless be worth clarifying them. First, her discussions of geometrical examples make clear that the logical necessity she has in mind is hypothetical. Geometrical proof relies on logical consequence relations, but it must begin with irreducible, non-logical facts. One of her examples is the problem of finding the unknown length *L* of one side of a trapezoid. If we are given the values for the three other side lengths and the two angles opposite to *L*, then *L*'s length will be "determined by these givens," as it "follows from" them with hypothetical necessity (1742, §41, 66).[8] But the givens themselves are contingent, rather than logically necessary:

> These givens do not in the least have intrinsic determinations, which determine them to be together, and their magnitude [*grandeur*] can vary and be such as he who gives the problem decides. (1742, §41, 66)

In other words, it is possible for there to be trapezoids with the same angles and different side lengths, or vice versa. At least some of the givens in a geometrical problem, this passage suggests, are not logically necessary.

Second, an essential part of a Euclidean geometrical demonstration is to show "how things must be done in order to construct" the relevant geometrical objects (Du Châtelet 1742,

§3, 18–19). Some or even all demonstrations are only complete in virtue of this sort of constructive element, which can be realized by diagrams.

Third, Du Châtelet does not take basic operations on numbers to be purely logical. On her view, the number 1 is the unit or "a non-composite number" on which all other natural numbers depend (1740, §120, 133). However, this is not logical dependency. Instead, the number 1, as a unit, gives the "sufficient reason" for higher numbers, by way of acts of aggregating units, as discussed above (§120, 133). This is significant because the principle of sufficient reason is, in Du Châtelet's view, non-logical and irreducible to the principle of contradiction (§8, 22–26). Therefore, the basic laws of arithmetic are not logical.

Even with these clarifications in hand, the passage we began with seems in tension with Du Châtelet's idealism about mathematical objects. The passage, though not committing her to a purely logical account of geometry, does state that "all" geometrical truths are necessary. In context, though, this passage is making a fairly narrow point about the two principles she considers fundamental for knowledge and reasoning, namely the principle of contradiction and the principle of sufficient reason. The principle of contradiction is the only one needed for necessary truths, but contingent truths also require the principle of sufficient reason (Du Châtelet 1742, §7, 22–23). In saying that in geometrical proofs, "only the principle of contradiction is used," she is pointing out that geometry only makes use of the principle of necessary truths, that is, the principle of contradiction.

Given her idealism about mathematical objects, however, the objects of geometry are partly mind-dependent. They depend on acts of abstraction, carried out by minds like ours. But it does not seem necessary that such minds exist, let alone that they actually carry out the relevant acts of abstraction. For example, humans might contingently have failed to use the power for abstraction to develop geometry. If mathematical truths themselves depend only on contingent acts of abstraction, then it is hard to see how these truths are logically necessary.

One way to avoid this tension would be to read Du Châtelet as holding that all mathematical truths concern merely possible objects. Mathematical claims would then be made true by God, as the ground of what is possible. She does state that possibilia are "in" the divine understanding, "the eternal region of truths" that "contains everything…possible" (Du Châtelet 1740, §49, 68–69; see also 1742, §50, 74). Truths about the essences of possible things are necessary, and would hold even if nothing was ever created (1742, §46, 70–71). On this reading, she might argue as follows: all mathematical truths concern possible objects; truths about possible objects are *de dicto* necessary; therefore all mathematical truths are *de dicto* necessary.

Unfortunately for this reading, the necessary truths in question are not first-order mathematical truths. They are second-order metaphysical truths *about* possibility and impossibility. For Du Châtelet, all such truths about possibility are necessary. To use her own example:

(a) *Necessarily, it is possible that Alexander the Great did not invade Persia* (1742, §23, 45).

Even in possible worlds where Alexander does not exist, it is true that he might have existed and not invaded Persia. This fact is grounded in the divine understanding, so it is independent of God's voluntary choice to create the actual world (§23, 45–48). It is necessary because God is a necessary being, and facts about the divine understanding are grounded in the divine essence. Now, Du Châtelet sees that this doctrine also applies to mathematical truths, noting for example that

(b) *Necessarily, it is possible that triangles have three sides* (60).

Whether or not there actually are any triangles, (b) is true in virtue of God's understanding. There is no doubt that Du Châtelet endorsed (b), but the interesting question is whether Du Châtelet thought all mathematical truths are necessary *simpliciter*, as in

(c) *Necessarily, triangles have three sides*.

Her general doctrine that, if it is possible that *p*, then necessarily it is possible that *p* does not, however, entail the necessity of mathematical truths like (c).

A second reading would take the necessary truth of propositions such as (c) to be solely based in facts about the divine intellect. Then, mathematical truths are true in all possible worlds, insofar as they are grounded in the divine intellect. By contrast, the laws of nature, even if they hold necessarily given the creation of the actual world, might not hold if God had chosen to create a different possible world. Such a reading would do justice to her claim that in geometry, all truths are necessary. Since it grounds all consistent mathematics in God's intellect, it also fits nicely with her view that anything logically possible is, in a minimal sense, a being or object that is grounded in the divine intellect (1742, §35, 61; cf. Descartes 1964–76, VII:116).

Yet Du Châtelet maintains a distinction between mathematical objects that are actual and those that are merely possible. There are merely "possible numbers," independent of actual created things (Du Châtelet 1742, §87, 113). That is, even if there were no actual created things to count, there would still be possible numbers. The ground for the possibility of numbers, like that of other possibilities, is God. Since the divine understanding is infinite, there could be infinitely many possible numbers. This affords a partial response to a different objection to abstractionist approaches to mathematics often raised by Platonists, which is that there are many more mathematical objects than can be produced by finite minds and their activities of abstraction. Moreover, it seems that God must know necessary conditional claims about worlds that would instantiate possible numbers, if those worlds were made actual. For example, before creating any actual world, God has yet to choose whether the actual world will contain more than seven things, or whether it will contain finite beings that can grasp numbers. But God can already grasp that *if* such a world is created, *then* necessarily, some further truths about the number seven will hold. However, she suggests that the divine intellect alone does not suffice to bring about actual mathematical objects, such as numbers. God must create agents with minds like us, and in turn our activities of abstraction are partly responsible for actual numbers.

The question is then why she is committed to actual numbers, rather than merely to possible numbers. The first reason is that actual, "real and existent" numbers must be grounded in concrete, countable things (1742, §87, 113). For example, one requirement for the number seven to be real and existent is for there to be seven things in the actual world. Here it might seem most parsimonious to identify "real and existent" numbers with actual, countable things. Yet as I argued in Section 1, Du Châtelet does not do this, instead stressing that numbers differ qualitatively from the things numbered. We can now see that she distinguishes three sorts of entity: possible numbers, things numbered, and actual numbers themselves. The latter are dependent on mental acts of abstraction by finite minds. This provides a second reason why actual numbers are not identical to possible numbers: the latter exist even if actual finite minds do not.

Although Du Châtelet explicates this distinction in a consistent way, it has surprising consequences. Mathematical objects, namely actual numbers, turn out to be contingent, since they depend both on how many things are created and on contingent abstraction by finite minds. It is "hardly necessary" that minds like ours, which form these mathematical objects by abstraction, actually exist (1742, §46, 70). Therefore, since actual mathematical objects depend on there being powers of abstraction, it is not necessary that some mathematical objects exist. It could even be that some mathematical objects are not actual at all times, with acts of abstraction producing larger and larger actual numbers as human history progresses. But even if we don't ascribe this picture to Du Châtelet, the thought that actual mathematical objects are contingent seems to conflict with her dictum that all truths in geometry are necessary.

The conflict can be eased when we note that her claim about necessity in geometry is conditional. The crucial claim is that there are essences of geometrical objects, from which consequences necessarily follow. For example, if something is a triangle, then necessarily, its angles must be equal to the sum of two right angles (1742, §46, 70; Jacobs 2020, 68n13). She makes a similar point for natural numbers (§46, 70–71). The necessity in question concerns the essences of triangles and numbers: triangles and numbers themselves need not exist necessarily. So her claims are consistent with propositions about triangles or natural numbers being false or vacuous if triangles or natural numbers do not actually exist. This reading fits well with her view that the principle of contradiction plays a role not just in mathematical proofs, but also for the identity conditions of essences.

Even if the internal tensions in Du Châtelet's position can be eased, questions remain about how to understand it. One such question is whether contingency just affects mathematical objects, or true mathematical propositions as well. From the contingency of actual numbers, it does not automatically follow that propositions about actual numbers are contingent as well.

Yet there are reasons to think that if actual mathematical objects are partly dependent on finite minds, then actual propositions and demonstrations referring to that object will also depend in part on finite minds. Du Châtelet accepts the antecedent of this conditional, because she thinks (actual) mathematical objects partly depend on finite minds. And she seems to accept the whole conditional claim as well. She cautions, for example, against accepting putative "demonstrations" that move from the part–whole structure of geometrical space to the structure of actual, material things (1742, §87, 113). I take her to be drawing attention to the fact that which actual parts we take geometrical space to have is, to some extent, up to us: they are distinctively mind-dependent. If care is not taken in our inferences from claims about geometrical space, we may get entangled in "ingenious sophism[s]," such as an alleged proof that "with a single grain of sand we could fill the entire universe" (§171, 194). By contrast, which actual parts bodies have is not up to us. The existence of these parts must be "demonstrated by experience" (§171, 194). So Du Châtelet seems to accept that at least some mathematical propositions and demonstrations partly depend on finite minds. Since finite minds are contingent, these propositions and demonstrations are contingent too.

Unfortunately, Du Châtelet does not go into very much detail about which mathematical propositions are contingent, or how this contingency ought to be understood. One example might be propositions in calculus. In some of her formulations, calculus appeals to fictional entities that are not even possible objects, in her view, such as "infinitely small straight lines" (1742, §290, 265). Other examples are propositions in plane geometry, such as the Euclidean definition of *parallel line*, that essentially take "extension" as "unlimited" or even "infinite" (§84, 109). But she also seems committed to contingency in a more puzzling case, already mentioned above. Truths about "actual" numbers depend on the existence not only of multiple "things," but also of minds like ours that form units by abstraction and then unify them into numbers (§87, 112–13).

This brings us to a second question: do her contingency claims pertain to all truths about actual numbers, or just to some truths about them? There is textual evidence that she only saw some of the truths about actual numbers as contingent. Each actual number, she writes, has a place in a temporally successive series (1742, §97, 121). This is one way of giving an *ordinal* account of natural numbers, where numbers correspond to positions in a linearly ordered sequence. This property of numbers seems to derive from properties of time, on her view. But she also thinks time is dependent on finite minds, and need not exist in all possible worlds. Properties of natural numbers that depend on time then look to be contingent. However, natural numbers can also be given what we now call a *cardinal* characterization, on which cardinal numbers answer "how many" questions. This cardinal characterization seems implicit in Du Châtelet's account of number as an aggregate of units that is used in reference to things numbered. These cardinal properties of natural numbers might hold in all possible worlds, for example if they turn out to be logical truths.

Despite these caveats, one might worry that her account of actual mathematical objects and truths runs together the conditions under which we acquire mathematical beliefs with the grounds of mathematical truth itself. That is, it might seem as if eternal truths about possible mathematical objects already settle which mathematical truths hold, and there is no work left to be done by actual mathematical truths. What might underlie this worry is a further suspicion about the very idea of distinguishing actual and possible in the case of mathematical objects. Standard marks of actuality might include being situated at a particular time and place, or being causally active. Mathematical objects are often thought to lack these features, dissolving any distinction between possibility and actuality for mathematical objects.

I take it that Du Châtelet would reject some assumptions behind these objections. She distinguishes possible and actual mathematical objects, but does not give up on classifying mathematical objects as abstract. So she may not hold that the necessary conditions for actuality include causal activity or being at a determinate time and place.

But this leads to a third question: how *does* she understand the relationship between actual mathematical objects and individual finite minds? On a more extreme reading, Du Châtelet would hold that actual mathematical objects do not stand entirely apart from causes, since these objects must be brought into being by the acts of finite minds. As I noted earlier, actual mathematical objects could even be seen as coming into existence at a determinate time, since they are generated by acts of abstraction. In that case, there would be a robust distinction between possible and actual mathematical objects. A cost of this reading would be that some mathematical objects (and truths) contingently depend on the mental activities of individual thinkers. This seems counterintuitive.

A more moderate reconstruction of her view, which I favor, would begin by distinguishing different types of possibility. Recall that she thinks that grounds in the divine intellect just give all the logical possibilities. Some philosophers of mathematics, such as Mark Balaguer (1998), would say that this settles the crucial question of logical consistency, which is all that mathematics needs. But that is not a foregone conclusion: more demanding senses of possibility could also be relevant for mathematics. These might include nomological possibility (what is compatible with general physical or metaphysical laws) or some notion of geometrical possibility (what can in principle be constructed in Euclidean geometry). A mathematical proposition could be logically possible—that is, consistent—while also being impossible in certain worlds. While it's now common to assume that in the case of mathematics only logical possibility needs to be considered, Du Châtelet need not share this assumption. Her chief concern is with applied mathematics. Truths about mere logical possibilities do not entail substantive truths about the actual world, or about those possible worlds containing matter and minds similar to ours. This may lead her to focus on more demanding senses of possibility. Her talk of establishing truths about actual and not just possible numbers might then be glossed in terms of showing that mathematical truths are possible in more demanding senses that are not entailed by logical possibility. In that case, truths about actual numbers would not need to be grounded exclusively in the mental activities of individual thinkers. Instead, while not absolutely necessary, they could be grounded in general facts about possible worlds. These worlds might be constituted in such a way that, if they are actualized, some possible mathematical truths become actual—even if the means of their actualization involves individual thinkers.

3. Applied Mathematics and Approximation

Several of Du Châtelet's influential contemporaries, including Voltaire and Christian Wolff, were sceptical about the use of mathematics to represent and reason about the physical world (Wells 2023). By contrast, she endorses the slogan that "the same thing happens in nature as in geometry": for example, both geometry and nature are governed by principles of continuity (Du Châtelet 1742, §13, 34). This is why "geometry is the key to all doors" in natural philosophy; the same goes for calculus, which she thinks Newton has made just as certain as classical geometry (2018, I.500; 1759/1990, 9). Science acquired "solid foundations" in the seventeenth-century scientific "revolution" because it made use of not just experiment but mathematics (1740, §xi, 12; §v, 5). For example, Newton's use of mathematics played a central role in making his inverse-square law of gravitation a "demonstrated truth," as opposed to Hooke's hypothesis of universal gravity, which she regards as a lucky guess (1759/1990, 6–8; Smith 2022, 269–70). Newton, unlike Hooke, was properly "guided by geometry," making his theory a fecund source of further discoveries (1759/1990, 6). Du Châtelet also links the methodologies of natural philosophy and mathematics. She defends what might now be called quasi-empirical methods in mathematics: even elementary arithmetical operations such as division employ hypothetical rather than strictly deductive reasoning (1740, §59, 81).[9]

Moreover, Du Châtelet has been read by George Smith as critical of theories that employ approximation for the sake of mathematical simplicity, such as Newton's assumption that the moon's orbit is circular. In Smith's terms, Du Châtelet prefers "mathematically exact solutions" (Smith 2022, 296). While the texts Smith cites are perhaps not decisive, a standard of mathematical exactness would fit nicely with the proposal that the same thing happens in nature and geometry.[10]

However, we also find passages where Du Châtelet suggests that mathematical representations are only approximately true of the material world.

> To reduce physical effects to mathematical calculations, we are always obliged to make a number of assumptions, and when we then wish to come back from mathematical calculations to physical effects, we find that there is a considerable loss [*bien du déchet*] of exactness and precision. (1740, §514, 394).

Her claim that we are always obliged to make simplifying assumptions is not, I think, merely rhetorical. Because of the limitations of our faculties, "we can only see objects by parts, and to consider the composite, it is always necessary for us to simplify it" (§496, 384). That is, the actual world is too complex, given our limitations, to be exactly represented. In this sense, "nature does not allow any precision," though nevertheless, "to make our reasoning more intelligible," we assume that mathematics really does represent the material world precisely (1740, §267, 239; also see Rey 2022, 361). This assumption then looks to be useful but false.

So we can ask how it is that the same thing happens in nature and geometry, if mathematics can only approximate material nature. I will focus on the relationship between geometrical extension and extended matter.

To begin with, note that the passage about loss of precision quoted above refers twice to mathematical accounts of physical effects. Similarly, when Du Châtelet states that there is no precision in nature, she is discussing collisions between bodies, stressing that however useful it may be to assume that bodies are perfectly elastic or perfectly hard, the bodies we have actually encountered always fall somewhere in between (1740, §267, 239). So while in the case of collision laws she seems to permit mathematical representations that are not exact, there is in principle room to read her claims about approximation as focused on "physical effects." This opens the possibility of exact fit between geometrical properties and properties of physical nature, just in case the latter properties can be disassociated from physical effects.

Distances are a plausible candidate for this sort of property. Du Châtelet's discussion of distances is part of her complex relationalist account of space, which I cannot treat in detail here.[11] The crucial point is that she allows geometrical and material objects to stand in relations of exactly equal distance. Her account of space begins with "the extension…of a geometrical body" (1742, §78, 103–104). The properties of geometrical extension include distance, and a geometrical line can measure a physical distance:

> When we wish to measure a distance, we can represent it to ourselves as a line without breadth or width, and without any internal determinations. (§86, 111)

> Since we determine a being's manner of existence by its distance to its coexistents, and since these distances are measured by straight lines, the limits [*extrémités*] of lines are points, so place should be considered as a point. (§89, 115–16)

As we saw above, she holds that no material object can have all of the properties of a geometrical line. A line has no breadth, width, or qualitative internal determinations, on her view, but every material object has breadth, width, and internal determinations. However, this does not entail that material objects cannot have *any* of the properties of a geometrical line. These passages suggest that, in fact, geometrical lines and material bodies can have precisely equal distances. For one further example, consider the definition of *situation* that concludes her chapter on space. Here a situation is roughly the collection of spatial relations that a number of objects bear to one privileged reference body. The key points for our purposes are, first, that sameness of situation is partly defined in terms of distance relations, and second, that she describes both physical objects (such as houses in a city) and geometrical objects (such as points) as having situations (§93, 117). This implies that physical and geometrical objects can stand in the very same distance relations. For distances, then, natural and geometrical objects can have the same relational properties: there can be an exact correspondence between nature and geometry.

4. Conclusion

One could still ask how this harmony between physical and geometrical objects is possible. Fully answering this question would require detailed discussion of Du Châtelet's metaphysics of space and matter, which I cannot undertake here. There is, however, a final point worth making about the interpretive debate considered at the start, which concerns the mind-dependence of matter in Du Châtelet. Sections 1 and 2 argued that Du Châtelet regards actual mathematical objects as mind-dependent. They depend on acts of abstraction, and have distinctive properties over and above the properties of material objects. In Section 3, I contended that she takes at least some properties of mathematical objects to be *the same* as properties of physical objects. If Du Châtelet did think that matter is mind-dependent, she would be well placed to explain how physical and geometrical objects harmonize: geometrical objects have a common partial ground, namely the activities of finite minds. This provides an indirect reason to prefer a reading on which Du Châtelet's matter is mind-dependent: such a reading is a good fit for her claims about the harmony of nature and geometry.[12]

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Notes

[1] I will focus on Du Châtelet's *Institutions de physique.* So I leave aside the large mathematical portion of her posthumously published *Principia* commentary, which offers calculus solutions to problems from Newton's work (Smith 2022). I also leave aside her unpublished treatise on optics (Du Châtelet 2017), which was apparently intended for a planned second volume of the *Institutions* (Toulmonde 2021). Further unpublished manuscripts, currently in private hands, apparently deal with mathematical topics such as the principles of arithmetic, conic sections, and Book I of Euclid's *Elements*.

[2] See further 1742, §72, 94–95; §79, 104; §83, 108–109; §87, 112; §§98–99, 122–23.

[3] This contrast does not seem to be as sharp as Du Châtelet presents it. For example, a body's mass is a magnitude, and is invariant across rearrangement and decomposition. So both bodies and geometrical objects can preserve a magnitude across certain rearrangements and decompositions. Still, she may be justified in holding that material bodies are "determined" by their parts in ways that geometrical objects are not (112; see further Reichenberger 2021, 350).

[4] Du Châtelet takes the principle of the identity of indiscernibles to hold necessarily of concrete bodies and simple substances (1742, §12, 30–31). What it is to be a body or simple substance is, among other things, to have different properties (beyond numerical distinctness) from all other bodies or simple substances. By contrast, extension is homogeneous, in the sense that its nonidentical parts *are* indiscernible (§78, 103). What it is to be a part of extension is, among other things, to be only numerically distinct from other parts of extension. So it is plausible that, for Du Châtelet and in these cases, identity criteria are essential to the kinds whose identities they determine.

[5] This account does not exclude unified, countable beings that are not material—our soul is presented to us by introspection on her view, and from the existence of our soul we can infer the existence of God a priori. However, Du Châtelet does not mention immaterial beings in her discussion of number. The implication is that typically, the beings from which we abstract units are material things.

[6] On imagination and error in Du Châtelet, see further Lascano (2021) and Rey (2022); on fictions in her account of mathematics, see further Wells (2023) and Sidzińska (2024).

[7] Antoine Arnauld's influential *Nouveaux éléments* exemplifies the pitfalls of this Cartesian approach. For example, he states that Euclid's parallel postulate has "enough clarity" to be assumed as an unproven axiom (1683/2009, 361). Indeed, 'straight line' need not be defined because this "idea is very clear in itself and…all men conceive the same thing by this term" (357–58).

[8] It is worth noting that this phrase is deleted from the revised 1742 edition. Du Châtelet also deletes all references to the *essence* of a particular trapezoid, and repeatedly replaces *following from* ("*les attributs découlent*") with *dependence* ("*les attributs dependent*") (1740, §52, 72; 1742, §52, 76). A hypothesis about why she made these changes begins with her view that the attributes or *propria* of a genuine or per se substance follow logically from its essence alone. A token trapezoid is not a per se substance, however, and is partly mind-dependent. She may think additional assumptions—for example about space as grounded in relations among substances—are needed for geometrical proofs about the trapezoid. She now speaks of the given properties of a figure as necessary conditions for solving a geometrical problem— "without [these properties] it would be impossible to solve the problem"—but leaves open whether these conditions are sufficient (1742, §42, 67).

[9] On quasi-empirical methods in mathematics, see Putnam (1975, 60–78). Historical examples Putnam discusses include infinitesimals as postulates or fictions (in Leibniz's calculus), and the postulation of real numbers (in the analytic geometry of Fermat and Descartes). In taking arithmetic and geometry to use hypotheses, Du Châtelet concurs with Christian Wolff (1726, §127; 1724, §112). The same proposal was later picked up by Kästner (1758, 17) and Kant (1980, 29:51–54).

[10] Smith's reading is partly based on an account of Newton's work on the three-body problem and assessments of it by Clairaut and others, and I cannot consider these issues here. Smith's key evidence, though, is a single passage from Du Châtelet (1759/1990, 98) that could instead be read as primarily descriptive of Newton's

methods, rather than explicitly critical. In support of this, a few pages later she accepts cases of approximation where readers can "see" Newton's assumptions; she focuses her criticisms on what she takes to be his failures to make explicit his reasoning and evidence (104). That is, her criticism might only be that Newton does not clearly flag and explain his approximations.

[11] Helpful recent discussions include Jacobs (2020), Brading (2023), Brading and Lin (2023), and Carson (forthcoming).

[12] Special thanks to Fatema Amijee, Katherine Dunlop, Jeff McDonough, Tuomas Pernu, Anat Schechtman, and two anonymous referees for written comments on earlier drafts, which led to many improvements. For very helpful correspondence, I am grateful as well to Emily Carson and the late George Smith. I presented some related material at the 2023 Berlin-Hamburg Workshop in Early Modern Philosophy. I am grateful to the participants and especially the organizers—Sebastian Bender, Dominik Perler, and Stephan Schmid—for an excellent discussion.