Kant, Infinite Space, and Decomposing Synthesis

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Abstract: Kant claims we intuit infinite space. There's a problem: Kant thinks full awareness of infinite space requires synthesis—the act of putting representations together and comprehending them as one. But our ability to synthesize is finite. Tobias Rosefeldt has argued in a recent paper that Kant's notion of *decomposing* synthesis offers a solution. This talk criticizes Rosefeldt's approach. First, Rosefeldt is committed to nonconceptual yet determinate awareness of (potentially) infinite space, but I argue that such awareness is (1) prima facie implausible, as well as contravened by textual evidence from Kant's (2a) definitions of 'infinity' and (2b) account of geometrical construction. Second, scrutiny of how Kant thinks awareness of potential infinity is afforded by geometrical construction (e.g., extending a line segment *ad indefinitum*) indicates no essential role for decomposing synthesis. Familiar synthesis from parts to wholes is sufficient.

Tobias Rosefeldt (2022) has recently laid out a highly original interpretation of Kant's doctrine that we have intuitions of infinite space. Among other things, Rosefeldt addresses a challenge to the coherence of Kant's doctrine. Kant asserts that being determinately aware of infinite space requires synthesis.[1] But our ability to synthesize is ineluctably finite. "It is not possible," Kant states, "to have experience…of an infinite space or infinitely flowing time" (AA 4:342).

As Rosefeldt points out, however, the way this problem is usually set up involves a crucial assumption. The assumption is that synthesis always runs from prior parts to a whole they compose. Rosefeldt gives compelling textual grounds to reject this assumption. In addition to the familiar case of synthesizing a whole out of parts, or 'composing' synthesis, Kant also describes "'decomposing' synthesis, i.e. an activity of the imagination by which we divide a previously given whole" (Rosefeldt 2022, 4). For example, we can become aware of two parts of a previously given circle by dividing it into two halves (6). In a minimal sense, all the parts of the circle are already given in the whole circle. Yet our conscious awareness of the circle's parts as determinate requires decomposing synthesis (see also Parsons 1983, 102; Sutherland 2022, 121–162).

So far, so good. The first question I want to address is whether, for Kant, awareness of infinite space can be afforded by decomposing synthesis as a "preconceptual" activity of the imagination (Rosefeldt 2022, 2). The second question is whether decomposing synthesis is required for awareness of space's infinity (regardless of whether such synthesis is conceptual). The answer to both questions, I will argue, is 'no.'

[1] Kant defines the "most general sense" of 'synthesis' as "the action of putting different representations together with each other and comprehending their manifoldness in one cognition" (A77/B103; see also B129–30). As Rosefeldt discusses, Kant also says we are aware of unlimited or potentially infinite time (A32/B48), but I leave this aside here.

I first summarize how, according to Rosefeldt, decomposing synthesis should get us a grasp of infinite space (section 1). Next, I introduce doubts about the in-principle plausibility of his account (section 2). Although preconceptual decomposing synthesis may give us a spatial manifold with no perceived boundary, this does not entail that the manifold is potentially infinite. In section 3, I lay out textual evidence that for Kant, if decomposing synthesis can discriminate between an infinite intuition and one that is merely very large, then such synthesis will no longer be preconceptual. I focus on Kant's definition of mathematical infinity and what he says about infinity in geometry. Section 4 argues that decomposing synthesis is not required for awareness of space's potential infinity.

1. How Decomposing Synthesis Works

Consider again the example of a circle: a finite bounded figure, assumed to be given in perception. Through a further act of decomposing synthesis, we divide the circle into two determinate halves. There may seem to be an important difference between the actual, given boundary of the circle and the parts of the circle. It's natural to assume that the latter are merely potential until they are constructed through acts of decomposing synthesis. The difference seems related to Kant's distinction between *Grenze* and *Schranke* (AA 4:352), and to an Aristotelian distinction between outer and inner limits of finite continua.

Rosefeldt makes the intriguing suggestion that this difference isn't as sharp as it seems. To be aware of the circle as a determinate figure, we must have already performed an act of decomposing synthesis that distinguishes the figure from its background (2022, 7). But in decomposing a figure such as a circle from its ground, one becomes determinately aware not only of the figure, but also of the ground. Crucially, this includes becoming aware of the ground as "unlimited" in extent—becoming aware, in other words, "that however large" a finite space we consider, it will always be within "a still larger and phenomenally unlimited space" (8; 10).[2] The 'that'-clause may suggest grasping a proposition, but Rosefeldt maintains that this synthesis is merely imaginative, "not…conceptual" (2022, 15). This means, first, that imaginative synthesis takes intuitions as inputs, and also produces intuitions as outputs (2). A second consequence is that, aside from its inputs and outputs, imaginative synthesis itself is not a "conceptual activity" (15). To keep things simple, I'll grant that Kant allows for synthesis without conceptualization (on this assumption see e.g. Allais 2009; Schulting 2015).

Next, Rosefeldt suggests that awareness of unlimited extent through the imagination is all Kant meant in saying space is given to us as infinite (2022, 11–12). Thus in one passage, Kant explicates space's infinite magnitude as mere "boundlessness in the progress [*Fortgange*] of intuition" (A25; compare 29:980–91). There is no need to "immediately see" the completed infinity of space in a single intuition (Rosefeldt 2022, 11). I will also grant this for the sake of argument.

Once these assumptions are granted, it looks like decomposing synthesis elucidates how Kant thinks we synthesize our way to an intuition of infinite space, while still respecting the finitude of our synthesizing abilities. With a single act of decomposing synthesis, we can acquire a determinate intuition of infinite space (in the potentialist sense), while respecting the finiteness of Kantian synthesis.

[2] Something like this awareness is also discussed, but without reference to decomposing synthesis, by Parsons (1983, 103), Carson (1997, 499), and Shabel (2003, 53). This raises the suspicion, which I return to in section 4, that the relevant kind of awareness does not depend on decomposing synthesis after all.

2. Nonconceptual Awareness of the Unlimited

As compelling as this picture is, it rests on the problematic idea that we can have nonconceptual awareness of the unlimited or potentially infinite. We've seen that for Rosefeldt, nonconceptual synthesis produces intuitions as outputs. So in this case, nonconceptual decomposing synthesis produces the intuition of a background to a finite figure, such as a circle. Plausibly, through this synthesis, one becomes more explicitly aware of the background *as* falling outside the finite figure. One may also gain some awareness of the unity or topological connectedness of the background. Most importantly for our purposes, the background is not intuited as having any determinate boundary.

But that does not get us potential infinity. Not intuiting a determinate boundary for X, as a result of decomposing synthesis, does not entail that X is unlimited—that is, that X can be extended *ad infinitum*. Given the finitude of synthesis for beings like us, any intuition of X produced by decomposing synthesis will be finite. So it will always be possible for X to have a limit that lies outside this intuition. Decomposing synthesis yields only a disjunction: the background space may be unlimited, *or* it might instead have a limit beyond what is intuited.

To underscore the point, consider responding to someone endorsing a version of Aristotelian finitism (which may or may not be Aristotle's view). Aristotelians first put forth metaphysical arguments that bodies must have limits (cf. *De Caelo* I.5–I.7). Next, they argue on metaphysical grounds that all of our spatial claims, even in geometry, are ultimately about bodies (cf. *Metaphysics* M and N). They conclude that space has limits: space is not even potentially infinite.

These Aristotelians cannot be refuted just by pointing to our intuition of the background of a finite object such as a circle. For they can insist that space is big—perhaps bigger than we can ever intuit all at once, such that no finite intuition could determine its size. Instead, metaphysical grounds are needed to settle whether space is finite.

An illustrative standard objection to this Aristotelian view is that it conflicts with mathematics. For example, the finitude of space conflicts with Euclid's definition of 'parallel straight lines' (*Elements* I def. 23), which appeals to the production of lines without limit (*εἰς ἄπειρον*). This has bite because Aristotle himself takes conflict with mathematics to be a major problem for philosophical views he opposes (e.g., atomism in *De Caelo* III.1). But this objection appeals to mathematical concepts, such as <parallel straight line>. The general lesson is that the Aristotelians have advanced metaphysical theses, on the basis of metaphysical arguments. All of this is thoroughly conceptual. Properly engaging with these arguments requires going beyond what's delivered in mere intuition.

Now, I don't take this to be a knock-down philosophical objection to the idea of nonconceptual awareness of the unlimited.[3] Rather, I've introduced a *prima facie* philosophical problem, which sharpens the need to look in Kant's texts either for a response to the problem, or else for a way to avoid it. In the next section, I lay out textual evidence

[3] I take it that on Rosefeldt's reading, Kant could respond by invoking the so-called productive exercise of the imagination (B152; AA 7:167). This might afford awareness of the unlimited without the determinate application of concepts. Rosefeldt also suggests that the Husserl of *Experience and Judgment* convincingly addresses the problem by appealing to free variation in the imagination (2022, 11). These issues are beyond the scope of this talk. However, see the final paragraph of section 3 for an argument that a key passage from Kant—which Rosefeldt cites for the sufficiency of non-conceptual imagination in awareness of infinite space—actually need not support his reading.

that Kant avoids the problem: he is not committed to determinate nonconceptual awareness of the unlimited.

3. Kant on Infinity and Geometry

First, a clarification. The key issue Rosefeldt wants to address is how we acquire determinate awareness of infinite space. His primary focus is not the way in which space may be always implicitly given to us, prior to synthesis, as a form of intuition. Merely as a form of intuition, space has structural features, including unlimited or infinite extent. "No concept," Kant writes, could play the role of this intuitive representation (A25; see also AA 4:285). This ensures that for Kant, substantive claims about space, including the claims of geometry, must be synthetic. However, Rosefeldt's focus is on how synthesis is required to make space a *determinate* object of our awareness (2022, 5–6). So the question on the table is not whether the original, intuitive representation of space is conceptual (it isn't). The question is what role concepts play, for Kant, in our determinate awareness of space and its properties—in particular, of the property of infinity. Rosefeldt maintains that this determinate awareness is preconceptual, whereas I deny this. So while in this section I'll be stressing the role of geometrical construction for determinate awareness of infinite space, I do *not* assume that geometrical construction is prior to the original intuitive representation of space (for critique of this assumption, see Carson 1997, 495–500).

I now want to raise two textual problems. The first stems from Kant's definitions of "the concept of the infinite," which conflict with Rosefeldt's non-conceptualist assumptions (AA 28:568). "The infinite," Kant holds, "is a magnitude for which no determinate [*bestimmtes*] measure can be specified [*angegeben*]" (28:568). Space and time, which have what he calls 'mathematical' infinity, fall under this definition. Substituting space into the definition, what's required to determinately grasp the infinity of space is to judge that space is a magnitude for which no determinate measure can be specified. This is a universal negative judgment, relating the concept of space to the concept <determinate measure>. So having a determinate grasp that the magnitude of space satisfies the definition of 'infinite' requires the application of concepts.

Kant's discussion even suggests that we can do merely conceptual, analytic reasoning with this concept of <mathematical infinity>, since this concept "must…be predicated of space" (AA 29:980).[4] It differs from the concept of a world that is actually infinite in spatial extent, which Kant suggests is self-contradictory (A793/B820–1; AA 18:402). What we cannot do by mere conceptual analysis is establish that <mathematical infinity> is instantiated, or has what Kant calls objective reality (see Carson 1997, 501ff.).

Kant's gloss on infinity as "boundlessness in the progress of intuition" may seem more promising for a non-conceptualist. For it refers only to intuitions, not concepts. But this leads to a second textual issue. As I'll now argue, conceptual activity must play a role in what Kant has in mind. To begin with, what Kant claims is boundless is progress *ad infinitum*, rather than any singular intuition that results from this progress (see further Shabel 2003, 52ff.). His notes connect this to our unbounded *capacity* for intuition (AA 17:638; 17:641). But the unboundedness of a capacity cannot be expressed by any single finite intuition.

For an example of progress in intuition, consider extending a line segment *ad indefinitum*. This example is essential to Rosefeldt's account of how decomposing synthesis actually gets us to an intuition of infinite space (2022, 9–11). He shows how extending two

[4] Also see A715/B743, AA 4:506–7, 29:837, and 29:864.

perpendicular segments can, with the help of decomposing synthesis, yield the intuition of a two-dimensional space, the area of which can likewise be extended *ad indefinitum*. Yet Rosefeldt does not explain in detail why extending segments does not involve conceptual activity.

A line segment is a determinate, finite one-dimensional space. And in the Transcendental Aesthetic, Kant remarks that determinate, finite spaces such as segments are "thought in" (not just intuited or imagined in) space as a form of intuition (A25/B39). More specifically, each iterated extension of the line is a geometrical construction. The construction procedure is summarized in Euclid's second postulate, namely "to produce a limited straight line in a straight line" (Mueller 1981, 318). This postulate arguably expresses the important assumption that two straight lines cannot have a common segment, an assumption that is exploited in the very first proof in the *Elements*.

Kant thinks geometrical construction is unlike ordinary conceptual reasoning in that its outputs can be singular intuitions (Friedman 2010, 589). In this case, the output will be "*einer äußern Anschauung*," namely a particular line segment (A33/B50). Geometrical construction can have singular results in virtue of what Kant calls schemata, which give sensible conditions for employing concepts. But schemata also have an irreducible "intellectual" aspect, attributed to the "understanding," in virtue of which they "attain the generality of the concept" (A138/B177; A141/B180–81). A schema in geometry is conceptual to the extent that it contains precise instructions for constructing an arbitrary figure via basic Euclidean construction operations (Jauernig 2013; De Boer 2020, 176). This conceptual character is clearest in complex cases, such as Euclid's construction of the icosahedron (*Elements* XIII.16) or the construction of a 96-gon (AA 8:212). But it is present even in the simple case of extending a straight line.

The role of schemata means that constructing a triangle or extending a line *ad indefinitum* is an irreducibly conceptual activity, though not a purely conceptual activity. Becoming determinately aware of "boundlessness in the progress of intuition," then, requires conceptual activity. So it is conceptual in one of Rosefeldt's senses (compare the second paragraph of section 1 above), even if its inputs and outputs are intuitions.

Kant's account of schemata is contentious, and space does not permit defending this reading in detail. But my textual case does not rest with a reading of schemata. Kant also states directly that concepts are involved in geometrical construction. "By means of" geometrical construction, he writes,

> I put together in a pure intuition…the manifold that belongs to the schema of a triangle in general and thus to its concept, through which general synthetic propositions must be constructed. (A718/B746; see also A734/B762)

So the synthesis involved in geometrical construction deals with what belongs to a geometrical concept, resulting in general geometrical propositions. This fits the example of Euclid's second postulate. Without grasping the concept <straight line>, I cannot construct its extension *ad indefinitum*.

Similarly, Kant writes in the *Prolegomena* that "the understanding…has itself constructed the [circle] in accordance with its concepts (namely, the equality of the radii)" (AA 4:320). He continues: "that which determines space into the figure of a circle, a cone, or a sphere is the understanding, insofar as it contains the basis for the unity of the construction of these figures" (4:321). And in the third *Critique*, he states that

> The figure of a circle is an intuition that can be determined [*bestimmt*] by the understanding in accordance with a principle; the unity of this principle, which I assume arbitrarily and, as a concept, make into a ground [*als Begriff zum Grunde*

> *lege*]…makes comprehensible the unity of many rules resulting from the construction of that concept [*sc.*, the concept of a circle]. (5:364)[5]

What comes across in these passages is that specific mathematical concepts are involved in geometrical construction, in addition to the understanding's ability to unify through synthesis. The role of geometrical concepts in construction helps make sense of Kant's claim that geometrical inferences proceed on the basis of the principle of contradiction (the principle of analytic judgments) (B14).

With these passages in hand, let's finally turn to a text from the B-Deduction that is important for Rosefeldt's defense of his nonconceptualist reading.

> Motion, as description of a space, is a pure act of the successive synthesis of the manifold in outer intuition in general through productive imagination, and belongs not only to geometry but even to transcendental philosophy. (B155n)

Motion so understood is not the physical motion of an outer object. Rather, it is a subjective, purely a priori "synthesis of the manifold in space" (B155; cf. A41/B58). Rosefeldt argues that extending a line *ad infinitum* is just a case of motion in this sense, such that extending a line requires the "productive imagination," but need not be conceptual (2022, 8–10). I agree that this passage highlights the role of the imagination in Kant's geometry, a role that probably could not be filled by mere conceptual analysis (for discussion compare Carson 1997, 510; Friedman 2010, 591; Jauernig 2013, 94–95). The texts I've reviewed suggest, however, that even if motion as an act of productive imagination is a necessary condition for geometrical constructions such as the extended line, it is not a sufficient condition. Schemata and concepts are required as well, and the passage from the B-Deduction does not imply otherwise.

4. Superfluous Synthesis

As we've seen, Rosefeldt holds that Kant's account of decomposing synthesis allows for an awareness of the infinity of space that doesn't depend on conceptual activity. Against this, I've argued that determining the results of decomposing synthesis to be infinite requires concepts, as indicated by Kant's definition of infinity and account of geometrical construction. Here it might be tempting to drop Rosefeldt's non-conceptualist assumptions, while keeping his basic appeal to decomposing synthesis.

But we're now in a position to appreciate another challenge for Rosefeldt's approach. His reading essentially relies, we've seen, not just on decomposing but also on composing synthesis, whereby line segments are extended indefinitely. The challenge is that, given Rosefeldt's potentialist reading of Kantian infinity, decomposing synthesis seems superfluous. The potentially infinite character of space arises from the capacity to extend line segments part by part: decomposing synthesis is used, in his account, *only* to move from two perpendicular segments to a two-dimensional space (Rosefeldt 2022, 10). Now consider an alternative construction. Euclid's third postulate states that a circle of arbitrary size can be constructed. With this postulate, arbitrarily large two-dimensional spaces can be directly constructed by composing synthesis alone. In avoiding decomposing synthesis, such a construction better respects Kant's dictum that geometry is "grounded" on *composing* synthesis (A163/B204). Decomposing synthesis thus plays no essential role in awareness of potentially infinite space.

[5] For still more textual evidence, see AA 5:241, 6:208, 8:212, and 20:411.

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