

Nāgārjuna's *catuṣkoṭi*

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The *catuṣkoṭi* or tetralemma is an argumentative figure familiar to any reader of Buddhist philosophical literature. Roughly speaking it consists of the enumeration of four alternatives: that some propositions holds, that it fails to hold, that it both holds and fails to hold, that it neither holds nor fails to hold. The tetralemma also constitutes one of the more puzzling features of Buddhist philosophy as the use to which it is put in arguments is not immediately obvious and certainly not uniform: sometimes one of the four possibilities is selected as ‘the right one’, sometimes all four are rejected, sometimes all four are affirmed. It seems that this confusion is only exacerbated by the plethora of treatments we find in the modern commentarial literature, many of which try to analyze the tetralemma by recourse to notions of modern logic. There

is no agreement about whether the four alternatives are to be understood as quantified¹ or unquantified propositions,² whether any quantification is to be understood substitutionally or referentially,³ whether the Law of the Excluded Middle holds for them,⁴ or whether they should be formalized in classical,⁵ intuitionist⁶ or paraconsistent logic.⁷

Despite some important work done during the last decades⁸ a comprehensive study of the origin and development of the *catuṣkoṭi* from its use in the earliest Buddhist literature up to its later employment in the Buddhist philosophical works of Tibet, China, and Japan remains yet to be written. The present paper is obviously not intended to fill this gap, but has the specific objective of giving an interpretation of Nāgārjuna's employment of the tetralemma⁹ which makes both logical sense and is in accordance with his general philosophical position.¹⁰

¹Robinson (1967, 57–58).

²Schayer (1933, 93).

³Tillemans (1990, 75).

⁴Murti (1955, 146), Staal (1975, 46–47), Napper (1989, 672–673, note83).

⁵Robinson (1957).

⁶Chi (1969, 162-3).

⁷Priest and Garfield (2002).

⁸One of the most thorough treatments pertaining to its usage in the Mādhyamika context is given by Ruegg (1977).

⁹All examples of Nāgārjuna's use of the *catuṣkoṭi* will come from the *Mūlamadhyamakakārikā*.

¹⁰To this extent the present discussion differs importantly from treatments like Robin-

The following discussion will be divided into four main parts. Firstly I will discuss the Indian distinction between two kinds of negation which will be of central importance for understanding the interrelations of the nested negations found in the tetralemma. In the second section I consider what might be taken to be a simplified case of the *catuṣkoṭi*, namely Nāgārjuna's rejection of two alternatives, of a position and its negation. Once the resources for understanding the argumentative rôle of this argument-schema are in place we can move on to the third section, in which Nāgārjuna's use of the tetralemma proper as the negation of four alternatives is considered. In this section I concentrate on three main questions: whether the four alternatives are logically independent, what the status of third, 'contradictory' alternative is, and how instances of the *catuṣkoṭi* applied to properties are to be related to those applied to relations. In the fourth and final section I conclude my discussion with an account of the positive tetralemma, in which all four alternatives are affirmed.

son's (1975) who explicitly restricted his investigation to the formal aspects of Nāgārjuna's arguments (295).

1 Two kinds of negation

The Indian philosophical tradition distinguishes two kinds of negation which are referred to as *prasajya* and *paryudāsa*, respectively. The origin of this distinction is grammatical; in *prasajya*-negation the negative particle connects with a verb (as in *brāhmaṇa nāsti*, ‘This is not a brahmin’), in *paryudāsa*-negation it connects with a noun (as in *abrāhmaṇa asti*, ‘This is a non-brahmin’).¹¹

This grammatical distinction corresponds to an important semantic distinction. If we refer to somebody as a non-brahmin we negate the term ‘brahmin’ and simultaneously affirm that he is a member of one of the three other castes. If, however, we simply say ‘This is not a brahmin’ we negate a proposition (i.e. ‘This is a brahmin’), rather than a term (‘brahmin’) and do not imply that we speak about a person belonging to one of the three lower castes; in fact we do not have to speak about a person at all.¹² In the Indian philosophical discussion (and particularly in the Mādhyamika context) it is this semantic distinction between implicational term-negation and non-implicational propositional negation which the terms *paryudāsa* and *prasajya* are supposed to mark. It is therefore not necessarily the case that e.g. non-implicational propositional *prasajya*-negation is expressed as ver-

¹¹Oberhammer et al., II:163.

¹²Renou (1942, II: 11), Cardona (1967, 40), Kajiyama (1973, 167–174).

bally bound.¹³

In fact this distinction is very familiar to contemporary philosophers. Not only can the grammatical distinction from Sanskrit be easily replicated in English, but the semantic distinction between the two types of negation also features prominently in the current discussion, particularly concerning the notion of a category mistake. Given that numbers are abstract objects it is clear that claiming ‘The number seven is green’ is a category mistake. But what about ‘The number seven is not green’? This depends on how we take negation to operate in this case.

It has been argued by a variety of authors¹⁴ that we have to distinguish two kinds of negation, called *choice negation* and *exclusion negation*. A choice negation presupposes that an object falls under a property or its opposite. Presupposing that the apple on the table has some colour or other it must either be red or non-red. If we negate one alternative we affirm the other. Exclusion negation, on the other hand, ‘is supposed to reject merely what is denied, without making any presuppositions as to the fulfillment of sortal specifications’.¹⁵ Thus if we deny that the apple on the table is divisible by three we do not presuppose that it is the kind of thing which *could* be divided by three, but still (correctly) assert that it does not fall under the

¹³Ruegg (1977, 5), (2002, 20–21).

¹⁴Mannoury (1947), Pap (1960), Routley (1969), Sommers (1965).

¹⁵Thomason (1972, 242).

property ‘divisible by three’. It is then evident that ‘The number seven is not green’ is a category mistake only if the negation employed is taken to be choice negation, not if it is exclusion negation.

While the distinction between choice negation and exclusion negation gives us a good model for understanding the distinction between *paryudāsa* and *prasajya*, it should certainly not be identified with it, as there is no textual evidence that Indian thinkers connected the distinction between the two kinds of negation specifically with categorial considerations. Rather, the difference between choice and exclusion negation should be considered as *one example* of the difference the pair *paryudāsa* and *prasajya* indicates. This is the difference between negations carrying with them the presuppositions implied by the propositions they negate, and those which deny these presuppositions. Thus reading the ‘not’ in ‘The number seven is not green’ as choice negation carries with it a presupposition ‘The number seven is green’ makes, namely that seven is a thing which could be green. This assumption is denied if the ‘not’ is read as exclusion negation.

Examples of these different kinds of negation which do not rely on sortal considerations are not hard to come by. There are two ways of negating the assertion that the present King of France is bald, one making the negation true, the other false or meaningless¹⁶, similarly there are two ways of negating

¹⁶The first being ‘It is not the case that there is somebody who is both the King of France

the accusation of continuing to be an alcoholic, one asserting that one has stopped drinking now, the other also denying the implication that one ever was a heavy drinker.¹⁷

As I will argue below the best way of interpreting Nāgārjuna's arguments is based on understanding the concepts *paryudāsa* and *prasajya* in this particular way. That is, *paryudāsa*-negations will be regarded as negations which continue to endorse the presuppositions made by the proposition they negate, while the purpose of *prasajya*-negations is to be able to formulate negations which explicitly reject some of these presuppositions.

2 Rejection of two alternatives

The distinction between the two kinds of negation helps to understand an important methodological tool which is used extensively throughout Nāgār-

and bald', the other 'The present Kind of France is not bald (i.e. has a full head of hair)'. The difference between the two is drawn in terms of the scope of the negation operator, that is, put formally, as the difference between $\neg(\exists!)(Kx \wedge Bx)$ and $(\exists!)(Kx \wedge \neg Bx)$.

¹⁷Shaw (1978, 63–64) notes the interesting idea of representing the proposition a sentence expresses as an ordered set, the last member of which is the sentence itself, the preceding one expressing the presuppositions that sentence makes, the one preceding this its presuppositions in turn and so on. A *paryudāsa*-negation can then be understood as negating the final member of the set only, whereas a *prasajya*-negation negates both it and some (possibly all) of its predecessors.

juna's writings. Consider verse 18.10 from the *Mūlamadhyamakakārikā* which begins by claiming that

whatever comes into being dependent on some object is not identical to that object, nor is it different from that object.¹⁸

If we ascribe to Nāgārjuna anything like the standard conception of identity (i.e. that identity is the relation everything bears to itself, and nothing bears to any other object) it seems hard to make sense of it, at least if we want to stay within the domain of classical logic. Nāgārjuna considers the property 'being identical to the object it depends on for coming about' (which we will abbreviate to 'being identical to a ') and denies that it applies to any object, and also denies that it fails to apply to any object. Expressed semi-formally this gives

1. *For all x which come into being depending on some particular object, not (identical-to- $a(x)$ or not identical-to- $a(x)$).*

But, applying the familiar laws of logic (in this case DeMorgan's law and double negation elimination), this can easily be seen to be equivalent to

2. *For all x which come into being depending on some particular object (not identical-to- $a(x)$ and identical-to- $a(x)$),*

¹⁸*pratītya yad yad bhavati na hi tāvat tad eva tat / na cānyad api [...].*

which is a contradiction.

How can this interpretation be avoided? The key lies in the distinction between the two kinds of negation. We have to assume that the two occurrences of ‘not’ in 1. do not in fact refer to the same concept of negation, but rather that the first is a *prasajya*-negation, a presupposition-cancelling negation,¹⁹ the second a *paryudāsa*-negation, understood as a presupposition-preserving negation.

Taking the first negation as *prasajya* is also suggested by Matilal,²⁰ who claims that on this interpretation ‘the apparent contradiction of the joint

¹⁹That the first kind of negation is supposed to be *prasajya* is stated both by Candrakīrti in the *Prasannapadā* (La Vallée Poussin, 1903–1913, 36–39) (see Ruegg (2002, 18–24) for a translation and commentary), as well as earlier by Bhāvaviveka in the *Prajñāpradīpa* (Walleiser, 1914, 10:8). Candrakīrti does not explicitly say what kind of negation the *second* negation is. It is evident, however, that for him it cannot be *prasajya*-negation too. Considering the first two alternatives of the tetralemma Candrakīrti argues against the claim that the negation of the first alternative (*A*) logically implies (*prāpnoti*) the second alternative (not *A*). If the ‘not’ in this ‘not *A*’ was indeed taken to be *prasajya* it would be obviously entailed by the *prasajya*-negation of *A*. Since it is not so entailed, however, it cannot be a *prasajya*-negation as well. I therefore think that it is plausible to regard the second kind of negation as *paryudāsa*, an assumption which, as we shall see, also makes a good deal of exegetical sense.

²⁰Matilal (1971, 164).

negation' disappears.²¹ Staal claims that such attempts to avoid inconsistency are unsuccessful, as 'it is not true that contradictions do not arise between *prasaṅgya*-negations'.²² However, this disregards the fact that Matilal's point was that only the outer negation of the two alternatives is to be regarded as *prasaṅgya*, while the negations employed *within* the statement of the alternatives are supposed to be *paryudāsa*-negations. It is therefore the entire set of two mutually exclusive alternatives which is negated, and on this interpretation there is indeed nothing inconsistent about it.

In order to see the motivation for this employment of two kinds of negation we have to understand that one of Nāgārjuna's main aims in the *Mūlamadhyaṃakakārikā* as well as elsewhere is to demonstrate the deficiency of some key concepts of our conceptual scheme (such as causation, motion, identity

²¹Matilal refers both to the negation of two and of four alternatives (the *catuṣkoṭi*). As we will see below the interpretation of the latter involves additional complications Matilal does not seem to be aware of.

²²See Staal (1975, 46). He also claims that the principle of contradiction *only* holds for *prasaṅgya*- and not for *paryudāsa*-negations, a claim which is backed up by reference to his discussion of the Mīmāṃsā concept of two kinds of *paryudāsa* (Staal, 1962, 60–61). But this can only serve to show that the Mādhyamika concept of *paryudāsa* is quite different as it is manifestly taken to be subject to the principle of contradiction (as is the notion of narrow negation employed above to explicate the Mādhyamika concept of *paryudāsa*-negation). Compare the characterization of *paryudāsa* by Avalokitavrata given in Kajiyama (1973, 169–172).

and so forth).²³ Their deficiency is taken to be due to a presupposition failure: in the same way that we spot a deficiency in calling the number seven yellow (because the presupposition that numbers are things which could possibly have a colour is not fulfilled), Nāgārjuna regards commonsense concepts like causation to be deficient because they presuppose the existence of *svabhāva*, the independent existence of objects,²⁴ which, Nāgārjuna argues, is a presupposition which is not fulfilled.²⁵ It then becomes easy to see that 1. should be interpreted along the lines of

3. *For all numbers x , not (yellow(x) or not yellow(x)).*

If the outer negation is taken to be exclusion negation and the second to be choice negation we cannot just read this as implying the contradictory statement that all numbers are both yellow and not yellow. Rather we will read it as denying (in a *prasaṅga*-manner) that the property yellowness and its (*paryudāsa*) opposite (which would imply that numbers were of some other colour) fail to be applicable to numbers.²⁶ Interpreted in this way, since

²³Ganeri (2001, 45–47).

²⁴What precisely the Mādhyamika concept of *svabhāva* entails is a complex issue which cannot be resolved in the context of the present paper. A good starting-point for distinguishing the different aspects of *svabhāva* is Ames (1982).

²⁵Ruegg (1977, 51).

²⁶See Galloway (1989, note 13, 29–30). ‘ x is yellow’ and ‘ x is not yellow’ are contraries when the referent of x is sortally incorrect (since they are both false). If the referent is

the outer negation is read as exclusion negation, 3. does also not presuppose that any *other* property is in fact applicable to numbers. In the context of 3. this neutrality is not particularly important, as we usually would want to claim that there are other (mathematical) properties which are applicable to numbers. It is, however, important for 1. since Nāgārjuna wants to extend his arguments to all other *svabhāva*-presupposing concepts (which, according to Nāgārjuna, are all the concepts we usually operate with).

If we therefore read the first ‘not’ in Nāgārjuna’s statement as *prasajya* and the second as *pariyudāsa*, the following interpretation emerges:

4. It is denied that either the concept ‘identical-to-a’ or its choice negation ‘different-from-a’ can be ascribed to any object x which comes into being depending on some particular object, without assuming that there is any pair of a concept and its choice negation one of which *can* be applied to such an object.

In order to demonstrate the deficiency of a concept Nāgārjuna has then to examine both the concept and its *pariyudāsa*-negation and show that both are not applicable to the objects under discussion, in the same way in which we argue that the concept ‘yellow’ is not applicable to numbers because numbers, not being material objects, cannot have a property like yellowness

sortally correct they are contradictories. Cf Raju (1954, 710–711).

(which is exclusively had by material objects), nor can they have any other colour (the *paryudāsa*-negation of the concept ‘yellow’).²⁷

3 Rejection of four alternatives

As will be obvious to any reader of Nāgārjuna’s writings far more common than the case just discussed, where two alternatives (a concept and its *paryudāsa*-negation) are both rejected is the rejection of *four* alternatives: the rejection of the application of a concept, of the application of its negation, of the application of both the concept and its negation, and finally of the application of neither the concept nor its negation. For example we read in *Mūlamadhyamakakārikā* 22:11

‘Empty’ should not be asserted, ‘Non-empty’ should not be asserted, both or neither should not be asserted, as these are only said for the purpose of designation.²⁸

²⁷Raju (1954, 701–702) employs this argumentative procedure to show that neither the concept ‘positive’ nor its *paryudāsa*-negation ‘negative’ is applicable to the number zero (*śūnya* in Sanskrit) and claims that similarly for the Mādhyamika no concept is applicable to emptiness (*śūnyatā*). We have to note, however, that there is no evidence in the Mādhyamika literature of an explicit connection between the mathematical concept *śūnya* and the metaphysical concept *śūnyatā* having ever been made. See Ruegg (1977, 69, note 154), (1978), Galloway (1989, 27–28, note 7).

²⁸*śūnyam iti na vaktavyam aśūnyam iti vā bhavet / ubhayaṃ nobhayaṃ ceti prajñāpty*

The same argumentative pattern of the rejection of four alternatives is also applied to ‘permanence’ and ‘finitude’ concerning the Buddha (in 22:12), to the existence of Nirvāṇa (25), to the existence of persons in the past (27:13), to their permanence (27:15–18), and to the finitude of the world (27:25–28).

The employment of the tetralemma can be traced back to the earliest Buddhist scriptures. In the *Kandaraka Sutta* the four alternatives are employed as a classificatory tool for distinguishing four classes of ascetics, those which torment themselves, which torment others, which torment both and which torment neither.²⁹ In this case the fourth alternative is explicitly recommended by the Buddha as the ideal to be emulated.

A case of the rejection of the four alternatives concerning the question whether the Tathāgata exists after death³⁰ by the Buddha can be found in the *Aggivacchagotta Sutta*³¹ and the *Cūlamālunkya Sutta*.³² Although the relationship between the use of the tetralemma in early Buddhism³³ and its employment by later Mādhyamika authors is complex and will not be investigated here it is nevertheless important to note at least that two different

arthaṃ tu kathyate.

²⁹Trencker (1888, 1:341), Bikkhu Nalamoli and Bikkhu Bodi (2001, 445).

³⁰Nāgārjuna considers the same question in *Mūlamadhyamakakārikā* 22:12.

³¹Trencker (1888, 1:484–485), Bikkhu Nalamoli and Bikkhu Bodi (2001, 591).

³²Trencker (1888, 1:431), Bikkhu Nalamoli and Bikkhu Bodi (2001, 536).

³³For some material on this see Gunaratne (1980).

motivations can be discerned in Buddha’s rejection of the four alternatives. One motivation is pragmatic; deciding which of the four position holds regarding specific questions (such as whether the Tathāgata exists after death, whether the world is finite etc.) is seen to be irrelevant for the attainment of liberation. Buddha therefore wants to set these questions aside, as is illustrated in the well-known simile of the poisoned arrow.³⁴ The other motivation is systematic; Buddha argues that the predicates applied in the four alternatives under consideration are in fact not applicable to their respective subjects, in the same way as any specification of spatial co-ordinates is not applicable when being asked where the extinguished flame of a candle went.³⁵ All members of an exhaustive set of applications of such predicates (which the four alternatives are taken to be) have therefore to be rejected.

3.1 Distinctness of the four alternatives

There are a variety of *prima facie* difficulties in interpreting four statements in the tetralemma. The first difficulty concerns the distinctness of the four alternatives. It is fairly common in the Western commentarial literature to express the tetralemma in propositional form, so that in verse 22:11 just given (letting *A* stand for the proposition “Empty’ should be asserted’) Nāgārjuna

³⁴Trencker (1888, 429), Bikkhu Nalamoli and Bikkhu Bodi (2001, 534–535).

³⁵Trencker (1888, 486–487), Bikkhu Nalamoli and Bikkhu Bodi (2001, 593).

is taken to say that all of the following propositions are to be rejected:³⁶

1. A
2. $\neg A$
3. $A \wedge \neg A$
4. $\neg(A \vee \neg A)$

³⁶See e.g. Schayer (1933, 93), Galloway (1989, 16), Ng (1993, 93), Tillemans (1999, 134).

In some cases the equivalent form $(\neg A \wedge \neg\neg A)$ is given for the fourth alternative.

The reader might wonder why we expressed the fourth alternative as ‘not (A *or* not A)’ rather than ‘not (A *and* not A)’, i.e. as the negation of the third alternative, which would be equivalent to ‘A *or* not A’. If we look at the way the fourth alternative is formulated in the *Mūlamadhyamakakārikā* we realize that there is a considerable amount of variation which seems to allow both formalizations. We sometimes find it formulated as *na ubhayam* ‘not both [the first and second alternative]’ (22:11, 25:17, 25:23, 27:13) which supports the reading as ‘not (A *and* not A)’ and sometimes as *naiva ... naiva ...* ‘not even ..., not even ...’ (18:8, 25:15–16) or *na ... na ... ca* ‘not ... and not ...’ (25:22) which seems to support the reading as ‘not (A *or* not A). The reason for this variation is not that Nāgārjuna had problems distinguishing ‘and’ and ‘or’ but rather that the context makes it clear that ‘not (A *or* not A)’ is intended. If we read the fourth alternative as ‘not (A *and* not A)’ this leaves us with three possible ways in which it could be true: either A obtains and not A does not, A does not obtain and not A obtains, or A does not obtain and not A does not obtain either. Given that the first two possibilities would be inconsistent with the rejections of the first two alternatives earlier in the argument we are left with the third possibility, which just says the same as ‘not (A *or* not A)’.

It is, however, easy to see that on this understanding the final two alternatives come out as logically equivalent.³⁷ Given the prominent place which the tetralemma occupies in Mādhyamika literature we would have to charge both Nāgārjuna as well as later Mādhyamika authors with remarkable logical naïvety for not realizing that instead of considering four possibilities, they were in fact only dealing with three.

In order to see how to solve this difficulty it is important to realize that once the fourth alternative is rejected, we are dealing with a statement with three nested negations, namely $\neg\neg(A \vee \neg A)$.³⁸ If we read the negation-symbols just as straight truth-functional negation both this as well as the negation of the third alternative turn out to be equivalent to $A \vee \neg A$, and it is obvious that this is not the conclusion Nāgārjuna wants to draw.³⁹ I

³⁷Applying DeMorgan’s law to the fourth alternative, $\neg(A \vee \neg A)$, we get $(\neg A \wedge \neg\neg A)$, which, by Double Negation Elimination, is equivalent to $A \wedge \neg A$, i.e. the third alternative. Robinson (1967, 57) is one of the surprisingly few authors to have picked up on this very problematic issue.

³⁸After rearranging the relevant parts of verse 22:11 of the *Mūlamadhyamakakārikā* it is straightforward to see the three stacked occurrences of negation it contains (here highlighted in bold): **na** *vaktavyam* **na** *ubhayam śūnyam* **aśūnyam**. See also 27:13.

³⁹It is interesting to note that the Tibetan commentarial tradition tried to avoid this difficulty by plugging in various modifiers, such as ‘ultimately’ (*don dam par*) or ‘conventionally’ (*tha snyad du*). Abbreviating these by U and C respectively the tetralemma is taken to assert that all of the following should be rejected:

have now argued above that the two instances of negation in such statements as verse 18:10 of the *Mūlamadhyamakakārikā* should be regarded as different kinds of negation, namely that the outer had to be taken as *prasajya*-negation and the inner as *paryudāsa*-negation. Since it is evident that the negation involved in the rejection of the four alternatives is meant to be *prasajya*-negation⁴⁰ the rejection of the fourth alternative would then have to be read as

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1. UA
 2. $C\neg A$
 3. $UA \wedge C\neg A$
 4. $(\neg UA \vee \neg C\neg A)$

It is thereby denied that A obtains ultimately, that it conventionally fails to obtain, that it both ultimately obtains and conventionally fails to obtain, and finally that it neither ultimately obtains nor conventionally fails to obtain. Tillemans (1999, 134–137) gives an example (slightly more intricate than the above) of such an interpolation procedure from Se ra rje btsun chos kyi rgyal mthsan’s *sKabs dang po’i spyi*. It is evident that on this account the third and fourth alternative are not in turn equivalent to the Law of the Excluded Middle, without requiring us to assume that negation behaves non-classically. While the dGe lugs interpolation procedure here (as well as in other contexts) provides a very interesting interpretation of the Mādhyamika arguments there seems to be no textual evidence that Nāgārjuna expected qualifications of the above kind to be supplied when interpreting the tetralemma.

⁴⁰As stressed in the Prasannapadā (La Vallée Poussin, 1903–1913, 36–39).

prasajya- \neg *prasajya*- \neg ($A \vee$ *paryudāsa*- $\neg A$).

If we now assumed that *prasajya*-negation obeyed Double Negation Elimination, i.e. that an even number of such negations cancelled each other out this would mean the rejection of the fourth alternative entailed the assertion of either A or its *paryudāsa*-negation, which is clearly not what Nāgārjuna wants to say. We therefore want to claim that this assumption is indeed not justified, i.e. $\neg\neg A \equiv A$ does not hold when the negation is taken to be *prasajya*-negation.

It is sometimes remarked in contemporary commentarial literature that the notion of negation at work in Mādhyamika arguments should be understood along the lines of intuitionist negation, which famously does not accept the equivalence $\neg\neg A \equiv A$.⁴¹ It has to be kept in mind, however, that the intuitionist rejection of $\neg\neg A \equiv A$, which went hand in hand with a negation of the Law of the Excluded Middle was motivated by very specific mathematical reasons. Since the negation symbol was interpreted as expressing our ability to give a *reductio ad absurdum* of the mathematical proposition to be negated, while the assertion of an unnegated proposition was taken to imply our ability to provide a proof of that proposition, $\neg\neg A$ could not entail A , as a demonstration that we cannot disprove a proposition does not

⁴¹For an exposition of intuitionist logic see Heyting (1971). The intuitionist reading was considered by Chi (1969, 162-3) and Staal (1975, 47).

amount to a proof of that proposition.⁴² Moreover, given the existence of undecided mathematical sentences the intuitionist is unwilling to accept that we are able to provide either a proof or a refutation of each mathematical proposition, which is what $A \vee \neg A$ means for him.⁴³ It is obvious that these problems in the ontology of mathematics were not problems Nāgārjuna was concerned with.⁴⁴ What speaks furthermore against the intuitionist interpretation of Mādhyamika negation is the fact that while it is sensible to argue that *prasajya*-negation does not obey $\neg\neg A \equiv A$ in order to make sense of the tetralemma, I do not think Nāgārjuna also rejected the Law of the Excluded Middle for it.⁴⁵ For even if some property (or indeed all properties) should turn out to be inapplicable to an object, this means that the *prasajya*-negation of the ascription of the property to the object should be affirmed. And given Nāgārjuna does not express any doubts about our ability to check whether properties are in fact applicable to objects in general, it appears to be unproblematic to affirm that $A \vee \neg A$ holds for *prasajya*-negation, that

⁴²See Heyting (1971, 17–18) for an example.

⁴³Heyting (1971, 99–100).

⁴⁴In (1974, 297) Richard Chi agrees with this point, calling his earlier intuitionist analysis of the tetralemma a ‘mistake’: ‘Despite the superficial resemblance, it is incredible that Nāgārjuna and Brouwer could possibly think in the same way. Dialectics and pure mathematics are, after all, two different disciplines. The agreement of the two systems is a sheer coincidence; they reach the same result for different reasons’.

⁴⁵Nor did Tsongkhapa. See Napper (1989, 61).

is to assume for any property and any object, that either this property is applicable to the object or it is not.⁴⁶

There does not seem to be any direct textual evidence in Indian Mādhyamika literature stating that *prasajya*-negation does not obey $\neg\neg A \equiv A$.⁴⁷ If we consider one example of a presupposition-cancelling *prasajya*-negation discussed above, namely the case of choice negation it seems plausible that $\neg\neg A \equiv A$ does not hold for it. Remember that when we use exclusion negation to negate a statement like ‘The apple is red’ we are merely saying of the apple that it has some other colour. To use set-theoretic terminology we assert (within the domain of coloured things) that the apple belongs to the complement of the set of red things. Now if we use choice negation twice, saying ‘The apple is not not red’ we are just saying that the apple belongs

⁴⁶That Nāgārjuna accepts the Law of the Excluded Middle is also argued by Ruegg (1977, 48-49). His argument there is, however, based on the erroneous presupposition (also made by Staal (1975, 47)) that the intuitionist has to assume the existence of a third truth-value (see Dummett (1998, 178), (2000, 11)).

⁴⁷An interesting case of a Tibetan rejection of this principle is provided by the Sa skya pa scholar Go rams ba bsod nams seng ge. As Tillemans (1999, 137) has argued, on the mainstream dGe lugs approach the Tibetan analogues of *prasajya*- and *paryudāsa*-negation, *med dgag* and *ma yin dgag*, were assumed to obey $\neg\neg A \equiv A$. (See e.g. Tsong kha pa bLo bzang grags pa (1973, 43–44).) Go rams pa bSod nams seng ge (1988, 51–52), however, rejects this principle, precisely to make sense of the tetralemma without the dGe lugs-style interpolations.

to the complement of the complement of the set of red things, which is of course the set of red things itself. In brief, we just say that the apple is red.

Exclusion negation, on the other hand, would be used to negate a statement like ‘The number seven is yellow’, thereby claiming that yellowness not just fails to be true of the number seven, but is indeed not applicable to it. If we then iterate this exclusion negation we say that it is not applicable to assert of the number seven that the property of yellowness is not applicable to it — and whatever this means, it seems quite distinct from saying that the number seven is yellow.

Be this as it may, I think there is a more elegant way to dissolve the above difficulty of iterated negations. This involves the notion of *illocutionary negation*.⁴⁸ The underlying idea is that propositions expressing a content can be prefixed by illocutionary operators forming assertions, commands, requests, promises and so on. Thus an ascription of the property of being open to the window produces the assertion ‘The window is open’ when prefixed by the assertion operator, the command ‘Open the window!’ when prefixed by the command operator and so on. It is now important for our purposes to note that when one of these results is negated it makes a difference whether or not

⁴⁸Searle (1969, 31–33); the distinction of illocutionary force from content goes back to Frege. The relevance of illocutionary negation to this problem was suggested by Jayatilleke (1963, 346, 475), (1967, 81), Chakravarti (1980) and Matilal (1986, 66–67, 88–90).

the negation operator is within the scope of the illocutionary force operator, i.e. whether we say ‘I promise not to open the window’ or ‘I do not promise to open the window’. Similarly there is a distinction between ‘I assert that the window is not open’ and ‘I do not assert that the window is open’ — the first involves familiar propositional negation, the second illocutionary negation.

There are various reasons why someone may employ illocutionary negation. One example is obviously when the proposition to be negated carries an unwelcome presupposition which propositional negation would preserve. Thus we will be happy to say ‘I do not assert that the number seven is yellow’ (presumably together with ‘I do not assert that the number seven is not yellow’), but not ‘I assert that the number seven is not yellow’. In other words, one motivation for using illocutionary negation is the desire to employ a *prasajya-* rather than *paryudāsa-*negation because we want to reject a particular presupposition made by the sentence to be negated. Note, however, that this is not the only reason why we might use illocutionary negation. Another obvious candidate is lack of evidence. We might say ‘I do not assert that the continuum hypothesis is true’ in order to indicate that we have no good evidence either way; in this case the presupposition-cancelling consideration involved when discussing the colour of the number seven does not come into play. We do not want to say that the continuum hypothesis is not the kind of thing which could be true or false. A third case in which we might want to

apply illocutionary negation to a proposition A is when A is not part of our language and we have no way of translating it. In this case we would want to assert A because we do not know which situation would make it true, and which would make it false. It is therefore evident that illocutionary negation is a more general notion than presupposition-cancelling *prasajya*-negation: it incorporates these, but it subsumes other considerations as well.

It is now tempting to interpret the tetralemma as asserting that illocutionary negation⁴⁹ should be applied to the following positions:

1. A
2. $\neg A$

⁴⁹It is interesting to note that in the Pali sources we sometimes find the four alternatives denied by the phrase *na h' idam*, 'it is not so' (for example in the *Anguttara Nikāya*, (Morris, 1888, 2:163)), and sometimes by the phrase *mā h' evam*, 'do not say so' (*Saṃyutta Nikāya*, (Feer, 1888, 2:19–20)). Some have argued that there is a semantic distinction between the two uses and that 'it is not so' is employed when the predicate in question is applicable to the situation discussed, but giving an affirmative answer to any one alternative would be misleading, while 'do not say so' is used where the predicate is not applicable to the situation (Jayatilleke (1963, 346), Gunaratne (1980, 231–231), Bharadwaja (1984, 312–313)). This second use corresponds to the illocutionary negation just introduced; interestingly enough this is employed in the passage from the *Saṃyutta Nikāya* just cited to reject the four alternatives claiming that suffering is produced by oneself, others, both or neither.

3. A and $\neg A$

4. I do not assert (A or $\neg A$)

Here the negation-operator \neg is to be read again as *pariyudāsa*-negation. The focus of our attention is of course the negation of the fourth alternative, which now features two illocutionary negations in a row, i.e.

I do not assert that I do not assert that (A or $\neg A$).

The advantage of replacing the *prasajya*-negations in this way by illocutionary negations is that it allows us to see straightaway that the two negations do not reduce to an unnegated proposition, that is that $\neg\neg A \equiv A$ does not hold. Declining to assert a proposition which in turn asserts that we decline to assert a third proposition does not amount to an assertion of this third proposition.

Tillemans has argued that the illocutionary reading of *prasajya*-negation in the context of the tetralemma has the ‘serious philosophical drawback’ that it gives the impression of the Mādhyamika’s refusing to adopt either a positive or negative position on some subject-matter. This impression would be misleading, however, since it seems apparent that Nāgārjuna and his commentators wanted to assert ‘*some form* of a negated proposition’⁵⁰ when setting out the arguments for rejecting the different parts of the tetralemma.

⁵⁰Tillemans (1990, 74).

Fortunately this problem can be easily dissolved. While the application of illocutionary negation to some proposition entails that we want to be ‘uncommitted to the truth or falsity of it’⁵¹ it also means that we want to assert a negative proposition when speaking *about* the proposition concerned. For example we might want to deny that there is enough evidence available for deciding it, or that we can translate it into our language, or that it carries with it a presupposition we do not want to assert. It is of course this last justification for using illocutionary negation the Mādhyamika wants to adopt, as he wants to deny the existence of *svabhāva* presupposed by the four positions in the tetralemma. It is therefore unproblematic to assert that the Mādhyamika declines to assert any of the four positions, while still ‘asserting *some form* of negated proposition’.

A further objection one might make at this point is that the interpretation in terms of illocutionary negation is not able to account for one important feature we would want to ascribe to the tetralemma, namely that the four alternatives are logically disjoint. It is evident that if I refuse to assert some proposition *A* (that is, negate it illocutionarily) this will entail that I also refuse to assert its conjunction with some other proposition. It could not be the case that I refused to assert the continuum hypothesis but would be happy to assert both the continuum hypothesis and Riemann’s hypothesis. But in

⁵¹Tillemans (1990, 74).

this case the illocutionary negation of the first alternative will imply that of the third, so that any distinct argument for rejecting the third possibility would be superfluous.

This argument of course depends on the assumption that the ‘and’ in the formulation of the third alternative behaves like the truth-functional operator of conjunction, so that the third alternative entails the first. We will argue shortly that this is not generally the case. To do this, however, we must first have a closer look at the status of the third alternative itself.

3.2 The status of the third alternative

A further problem in interpreting the tetralemma is connected with the rejection of the third alternative, which asserts the applicability of a property and its *paryudāsa*-negation. Why, we might well ask, does Nāgārjuna think we have to consider this contradictory option as well, as if this constituted a real possibility?⁵²

Robinson suggests that a way of dealing with this problem is to interpret the four alternatives not in a propositional, but in a quantificational way.⁵³

⁵²There are clear cases of *paryudāsa*-negation in the *Mūlamadhyamakakārikā* where Nāgārjuna assumes the Law of Non-contradiction (e.g. 7:30 and 8:7). The third contradictory alternative should therefore not constitute a genuine possibility. See also Robinson (1967, 50–52), Ruegg (1977, 48–49), Galloway (1989, 19–22).

⁵³Robinson (1967, 57–58).

If F is the property under consideration the four alternatives to be rejected become:

1. Everything is F .
2. Everything is not F .
3. Something is F and something is not F .
4. **Not**: Something is F or something is not F .

Here all negations are *pariyudāsa*, apart from the one in the fourth alternative set in boldface, which is a *prasaṅgya*-negation.

It is evident that when formulated in this way the third alternative is ambiguous, depending on whether we take the two occurrences of ‘something’ to refer to the same object. If we take them to refer to different objects, the third alternative is not any more problematic than saying that chesspieces are both white and not white, where this is to mean that some are white, and some are not white. This interpretation, however, does not fit well with the employment of the tetralemma by Nāgārjuna. His aim is to investigate the applicability of various concepts (such as emptiness, permanence, finitude etc.) to objects. If the third alternative was taken to mean ‘the concept under discussion is applicable to some objects, and not to others’ this would not be an argumentationally interesting option for Nāgārjuna, as the application

of the concept to some objects, and its non-application to the others would then have to be investigated individually in any case. On this interpretation the third alternative would merely present a complex statement of two argumentative options which Nāgārjuna will want to investigate separately. Richard Robinson remarks:⁵⁴

It is a striking feature of the *Stanzas* that all predicates seem to be asserted totally of the whole subject. Existential quantifications are denied because the discussion is concerned, not with the denial or affirmation of commonsense assertions such as ‘Some fuel is burning, and some is not’, but with the concepts of own-being and essence. What pertains to part of an essence must of course pertain to the whole essence.

Put briefly, given that Nāgārjuna wants to inquire into the applicability of particular concepts to objects *tout court* we should also consider the four alternatives as giving alternative ways of the application of particular concepts to objects *tout court*, rather than as implying their application to some objects, but not to others.

We therefore have to interpret the two occurrences of ‘something’ as pertaining to the same object, i.e. the third alternative claims that ‘something is

⁵⁴Robinson (1967, 54). See also Gunaratne (1986, 225–226).

F and the same something is not F '. Whether this is contradictory depends on how we understand the application of the properties F and not F . For example, it is straightforward to assert that a chess board is black and not black, if we mean by this that some parts of it are black, and others not black. On this reading the contradiction is avoided by relativising the two properties involved to different mereological parts. The same result can be achieved by relativizing to different respects or perspectives under which the object is considered, for example if we assign different utilities to an alternative in a decision problem under different descriptions.⁵⁵

To see that these kinds of relativizing interpretations are present in Nāgārjuna it is instructive to look at the reasons by which the third alternative is generally rejected. Here we can distinguish two varieties. In the first case Nāgārjuna rejects it because its claim is as contradictory as asserting of a single object that it is wholly black and not black. For example we read in *Mūlamadhyamakakārikā* 25:14:⁵⁶

How could *nirvāṇa* exist and not exist? Like light and darkness
these two cannot be at the same place.⁵⁷

⁵⁵The *Dīgha Nikāya* (1:31) asserts that the world is both not existent and not non-existent; the former because it ceases, the latter because it arises. Cf. Jayatilleke (1967, 79), Robinson (1969, 75), Gunaratne (1980, 221).

⁵⁶Further examples can be found in 7:30, 8:7 and 27:28.

⁵⁷*bhaved abhāvo bhāvaś ca nirvāṇa ubhayam khatham / na tayor ekatrāstitvam*

In the second case Nāgārjuna rejects the third alternative since it would combine the difficulties facing the first and second alternative (which have already been rejected earlier in the argument). This point is clearly made by Candrakīrti:⁵⁸

Things do not originate from both themselves and from other things. This is because the problems stated for both positions [i.e. the first and second alternative] will arise together.

It is clear from this way of rejecting the third alternative that it is here not understood to be contradictory, but that Candrakīrti takes it to be perfectly possible that something could be caused partly by itself and partly by other things. (One straightforward account of this consists in conceiving of an effect as a potential in a cause which is only actualized given the right background conditions).⁵⁹ This is rejected because the presence of these two ways of causing would imply the difficulties of both causation from itself, and of causation from other things, both of which Nāgārjuna has rejected as *ālokatamasor yathā*.

⁵⁸*dvābhyāmapī nopajñāyante bhāvāḥ ubhayapakṣābhihitadoṣaprasaṅgāt pratyekamutpādāsāmarthyāc ca* (La Vallée Poussin, 1903–1913, 38), (Ruegg, 2002, 73). Candrakīrti makes the same point when commenting on *Mūlamadhyamakakārikā* 12:9. See Schayer (1931, 20).

⁵⁹Garfield (1995, 106–107).

unsatisfactory earlier.⁶⁰

We therefore have to conclude that Nāgārjuna applies the argumentative figure of the tetralemma both to cases where he takes a concept and its *pariyudāsa*-negation (i.e. the conjuncts of the third alternative) to be contradictory, as in the first case just mentioned, as well as where he considers it to be possible that both can be applied to an object, as in the second case.

Obviously only in the second case recourse to the tetralemma would have been strictly necessary, as in the first case a consideration of two alternatives (of the concept and its *pariyudāsa*-negation) would have been sufficient,

⁶⁰These two ways of rejecting the third alternative are also distinguished in Ghose (1987, 296–297). He also mentions a third way where the third alternative is rejected because ‘it attributes to the conjunction some properties which are common to both the conjuncts’. As an example Ghose discusses verse 25:12 from the *Mūlamadhyamakakārikā* where Nāgārjuna claims that ‘if *nirvāṇa* was both existent and non-existent, it would not be non-dependent, as it would depend on both’. Nāgārjuna here does not refute this alternative by saying that it is contradictory for something to be both existent and non-existent, but by arguing that since existence and non-existence both presuppose dependence, *nirvāṇa* would be dependent, which it is not. However, it is evident that this is just an example of the second way or rejecting the third alternative too. Nāgārjuna has already rejected (in verses 6 and 8) that *nirvāṇa* is either existent or non-existent, because it would be dependent in each case. The third alternative is thus rejected because it implies the difficulties of both the first and second alternative, which happen to be the same difficulty in this particular case.

given that both of them together are regarded as contradictory in any case. We might perhaps explain the fact that Nāgārjuna uses the four alternatives nevertheless on rhetorical, rather than logical grounds. If it was assumed that all four alternatives of the tetralemma applied to a particular notion were positions actually propounded by some school of thought⁶¹ it would be heuristically useful, if not logically necessary, to go through all of them individually, even if this included an alternative which the Mādhyamika regarded as logically contradictory.

But if we thus regard the second case as the domain of the tetralemma proper (and the first only as a rhetorical expansion of the rejection of two alternatives) it is clear that in the tetralemma proper the third alternative does not entail the first. Consider the case of the tetralemma applied to causation. Here the first alternative claims that things are exclusively caused by themselves. The third alternative constitutes a compromise between the first and second alternatives: it says that things are partly self-caused, and partly caused by other objects. But this obviously does not imply the first alternative, no more than saying that a chess-board is partly black and partly white implies that it is black all over. For this reason the illocutionary negation of the first alternative also does not imply that of the third, since the third is not a truth-functional conjunction of the first alternative and

⁶¹See note 73.

something else.

We should also note that according to the quantificational reading given above the third and fourth alternative are logically distinct, as 3. says that some objects instantiate both the property F and its complement, whereas 4. says that neither is in fact instantiated. Finally, as we conceive of the initial two negations in the negated fourth alternative as illocutionary negations, so that they do not cancel each other out the rejection of the fourth alternative is not equivalent to ‘Something is F or something is not F ’.

It therefore becomes evident that what Nāgārjuna wants to say in *Mūla-madhyamakakārikā* 22:11 is that the following four alternatives should all be rejected:

1. ‘Empty’ should be asserted of all objects.
2. ‘Empty’ should be denied (in a *pariyudāsa* fashion) of all objects.
3. ‘Empty’ should be asserted of some objects, and should be *pariyudāsa*-denied of the same objects.
4. **Not:** ‘Empty’ should be asserted of some objects, or ‘Empty’ should be *pariyudāsa*-denied the same objects.

Nāgārjuna’s usual argumentative procedure (as we will see below) is to argue that each of the four alternatives leads to an absurd consequence, so

that the whole set is to be rejected. In this case, however, he does not discuss the four alternatives individually but dispatches them with a single argument, namely by saying that all assertions listed in the four alternatives ‘are only names’.⁶² Nāgārjuna is therefore making a semantic point: while it is of course true for the Mādhyamika that every right-minded person should assert emptiness of all objects, this should not be done by assuming that there are some objectively existent objects out there, referred to by a similarly objective reference relation, and that these objects have the property of emptiness.⁶³ Statements of emptiness should not be understood according to the standard semantic theory.⁶⁴ With such a theory in mind it is neither

⁶²Since Nāgārjuna does not give reasons for the rejection of the four alternatives individually we cannot say whether he would have wanted to reject the third alternative because he considers it to be contradictory (‘nothing can be empty and not empty at the same time’) or because it would combine the difficulties inherent in the first two alternatives. It is certainly conceivable that someone might adopt the third alternative by arguing that phenomena are empty in some respects but not in others, e.g. one might claim that they are empty insofar as they are causally produced, but not empty insofar as they exist independently of us.

⁶³It should therefore be noted that the last three alternatives are in a way more deficient than the first one. For a Mādhyamika the first assertion would be true if interpreted according to the right semantics, whereas the final three would still have to be rejected, because even with the right semantics they would be false.

⁶⁴Garfield (1995, 280).

correct to say that all things are empty, or that they all lack emptiness, or that some are both empty and not empty, or that the predicate ‘empty’ is not applicable to objects at all, in the same way as the predicate ‘yellow’ is not applicable to numbers.

Before leaving the subject of the quantificational interpretation of the tetralemma it might be useful to have a brief look at the analysis presented by Tillemans in an appendix to (1990). There the four alternatives are formalized as

1. $\neg(\exists x)(Fx)$
2. $\neg(\exists x)(\neg Fx)$
3. $\neg(\exists x)(Fx \wedge \neg Fx)$
4. $\neg(\exists x)(\neg Fx \wedge \neg\neg Fx)$.

Tillemans argues that it is straightforward to make sense of the simultaneous rejection of all four positions if we assume that there is no x , i.e. if the domain of quantification is empty.⁶⁵ While this reading makes the distinction between different kinds of negation in the tetralemma superfluous it also has a number of problems. On the one hand there is the familiar difficulty that the third and fourth possibility come out as logically equivalent. On

⁶⁵Tillemans (1990, 75).

the other hand (as noted by Tillemans himself) this interpretation implies that the Mādhyamika would also have to *accept* all the four positions of the tetralemma, since the corresponding universal statements are also true in the empty domain. But there is no textual evidence in Mādhyamika literature that the four positions of the tetralemma are simultaneously to be rejected and accepted.⁶⁶

Tillemans continues to argue that quantified statements accepted by the Mādhyamika are generally to be interpreted substitutionally, rather than referentially. (Interpreted referentially the statement ‘All x are F’ means that there is some set of objects such that every single one of them is F. Interpreted substitutionally it means that for every name substituted for ‘ x ’ in ‘ Fx ’ we get a true statement.) The Mādhyamika can therefore ‘use the world’s language to communicate about whichever day-to-day affairs the world concerns itself with: his *śūnyavāda*, however, dictates that he never accepts a referential interpretation of such language’.⁶⁷

This does not strike me as a good way of understanding what the Mādhyamika means. The difference between referential and substitutional quantifi-

⁶⁶As we will see in the final section of this essay there are cases in which all four positions are affirmed (the so-called positive tetralemma). This, however, serves a very different purpose from the negative tetralemma.

⁶⁷Tillemans (1990, 75).

cation is that between quantifying over objects in the world and quantifying over pieces of language. The Mādhyamika distinction between the two truths, however, which Tillemans wants to spell out in this way, is concerned with two different ways of interpreting the ontological status of objects, or, to put it differently, with two different accounts of what it means for a statement to be true. At the conventional level a statement is true if what it says is indeed the case, i.e. if there are objects with *svabhāva* related in the necessary ways. On the absolute level, however, the notion of *svabhāva* is to be found to be deficient and *svabhāva* is seen to be non-existent. But both the referential and the substitutional interpretation of a statement can be read either way: the objects quantified over can be seen as either existing with *svabhāva* or being empty, similarly the truth of the sentences featuring in the substitutional interpretation can be regarded as being made true by situations regarded at the level of conventional truth, or by situations regarded at the level of absolute truth, which are then seen as empty. It appears that what is important from the Mādhyamika perspective is not so much whether a quantified statement is read referentially or substitutionally, but the way in which the notions of ‘object’ and ‘true statement’ contained in these readings are spelt out.

3.3 Rejection of four alternatives: the case of relations

All of the examples of the rejection of four alternatives discussed concerned the rejection of one-place properties, such as emptiness,⁶⁸ permanence, or finitude. Nevertheless, some of Nāgārjuna's most famous arguments in fact involve the rejection of four alternatives concerning relations.

A very clear example of the employment of the tetralemma in this way can be found in the twelfth chapter of the *Mūlamadhyamakakārikā*. Nāgārjuna starts out by listing the four possibilities available when applying the concept of causation to suffering:

Some say that suffering is caused by itself, or by something else,
or from both, or that it arises in an uncaused way.⁶⁹

Now we could interpret this along the lines of the tetralemma concerning properties by just regarding it as about the *property* of self-causation, rather than about the *relation* of causation. The above verse would then amount to a rejection of the following four alternatives:

1. Everything is self-caused.

⁶⁸Garfield (1996, 6) is of course correct in pointing out that 'empty of' denotes a relation. But what Nāgārjuna has in mind is clearly emptiness of inherent existence, which is a one-place property.

⁶⁹*svayaṃ kṛtaṃ parakṛtaṃ dvābhyāṃ kṛtaṃ ahetukaṃ / duḥkhaṃ ity eka icchanti [...].*

2. Everything is not self-caused (i.e. caused by others).
3. Something is self-caused and (the same) something is not self-caused.
4. **Not:** Something is self-caused or (the same) something is not self-caused.

While this move allows us to treat the forms of the tetralemma dealing with properties and relations as exactly parallel, I think a more natural way of reading the above argument would run as follows.

The essential difference between a property and a relation is that a property (such as yellowness) will divide the set of objects it is applicable to (material objects) into two subsets, those which have the property (like lemons, bananas, curry powder and so on) and those which lack it (like strawberries, apples, chili powder and so on). A relation,⁷⁰ however, divides the set of objects it is applicable to into pairs of objects from the set which are related by the relation. There are various ways in which this set of pairs can be made up; it can consist

1. exclusively of pairs containing the same object twice, or
2. exclusively of pairs containing two different objects, or
3. of both pairs of identical and distinct objects, or finally

⁷⁰For the sake of simplicity we will confine ourselves here to two-place relations.

4. it can consist of nothing at all, i.e. it can be completely empty.

Which of these possibilities obtains determines the way in which the objects in the set are related by the relation. If, for example, we consider the ‘loves’ relation and a set of human beings then in the case of 1. we are dealing with a set of egoists, where people only ever love themselves, in 2. we deal with a set of altruists, where people only ever love other people, in 3. we have the (normal) situation of some people loving both themselves as well as others, and in 4. have an emotional vacuum: nobody loves anybody, neither themselves nor others.⁷¹

If we thus wanted to argue for the deficiency of the concept of a particular relation along the lines of the above argument, we would consider the four possibilities of that relation relating an object to itself, relating an object to something which is not itself (where the notion of negation involved is again of the *paryudāsa*-kind) relating an object both to itself and to other objects, and relating it neither to itself nor to other objects, that is, relating it to nothing at all. If we succeed in showing all four possibilities to be unsatisfactory we can then deny all four alternatives by a *prasaṅgya*-negation and thus

⁷¹It is important not to confuse this fourth case with the inapplicability of a relation to a set: in a set of people nobody may stand in the ‘loves’ relation, and nobody will stand in the ‘is the square root of’ relation. But it is at least possible that people could stand in the former relation, whereas it is impossible that they stand in the latter.

apply illocutionary negation to them. In this way we demonstrate the inapplicability of the concept of the relation to the objects under consideration.

We can therefore read the first verse from the twelfth chapter of the *Mūlamadhyamakakārikā* as arguing that if it makes sense to use the concept of causation when talking about suffering at all, it would either have to be the case that causation related suffering to itself (i.e. that it was self-caused) or to another thing, or both, or that suffering was not causally related to anything. As is hardly surprising, Nāgārjuna sets out to argue that the concept of causation is not applicable in this context, and ends the verse by stating the conclusion to be established:

To consider [suffering] as produced is not appropriate.⁷²

In the remainder of the chapter Nāgārjuna then sets out to refute each of these possibilities. Verse 2 attempts to refute suffering's self-production, verses 3 to 8 production from another, and verse 9 the final two possibilities.⁷³

As we are here primarily interested in the argumentational mechanics of the

⁷²[...] *tac ca kāryaṃ na yujyate*.

⁷³It is sometimes argued (e.g. in ?, 11–12) that the four possibilities concerning causation mentioned in *Mūlamadhyamakakārikā* 12:1, and, more generally, in 1:1 represent the views of four different Indian schools of philosophy. Self-causation is ascribed to the Sāṃkhyas (Murti, 1955, 168–169), causation by others to the theory divine causation expounded in the Vedas and Brāhmaṇas (see Kalupahana (1975, 5) for some other examples of what he calls 'external causation'), causation by self and others to the Nyāyas and

tetralemma we can disregard the precise contents of these arguments. It is, however, important to note the generalization stated in the final verse of the chapter:

Not only does suffering not exist in any of the four possible ways described, but no other external entity exists in these ways either.⁷⁴

Thus, apart from being a specific argument about the suitability of using the concept of causation to talk about suffering, Nāgārjuna takes the contents of this chapter also to be an argument-schema, that is as a framework which can be employed to demonstrate the deficiency of other concepts when referring to external entities.⁷⁵

Vaiśeṣikas (Dasgupta, 1942, I:320), (King, 1999, 208) and finally absence of causation to the Lokāyatas (Kalupahana, 1975, 25). This last identification is denied by Schayer (1931, note 16, 20–21) who argues that the view of the Cārvākas only denies causality in the context of karma, but not all causal determinations, as they assert that things are determined by their intrinsic nature (*svabhāva*).

⁷⁴*na kevalaṃ hi duḥkasya cāturvidyaṃ na vidyate / bāhyānām api bhāvānām cāturvidyaṃ na vidyate.*

⁷⁵As Robinson (1967, 50) points out, Nāgārjuna frequently indicates that his arguments function as patterns into which other terms can be substituted. For examples from the *Mūlamadhyamakakārikā* see 3:8, 16:7, 19:4, and 10:15.

4 Affirming four alternatives: the positive tetralemma

As we saw above the tetralemma is usually employed in Mādhyamika argumentation to provide an enumeration of four exclusive and exhaustive logical alternatives, all of which are then shown to be deficient and thus rejected. There is, however, one notorious exception in Nāgārjuna’s writings, in verse 18:8 of the *Mūlamadhyamakakārikā*. There Nāgārjuna seems to *affirm* all four alternatives by claiming that

All is so, all is not so, both so and not so, neither so nor not so.

This is the Buddha’s teaching.⁷⁶

In the commentarial tradition following Candrakīrti this verse is generally understood as indicating the graded nature of Buddha’s teaching (*anuśāsana*).⁷⁷ The idea is that ‘all is so’ is taught to ordinary disciples, in order to convince them of Buddha’s insight into the nature of phenomena. ‘All not so’ is taught subsequently to inform them about the impermanence and

⁷⁶*sarvaṃ tathyaṃ na vā tathyaṃ tathyaṃ cātathyaṃ eva ca / naivātathyaṃ naiva tathyaṃ etad buddhānuśāsanam.*

⁷⁷See Ruegg (1977, 5–7). For another discussion of graded teaching by Nāgārjuna see verses 394–396 of his *Ratnāvalī* (Hahn, 1982), (Hopkins, 1998, 90–91, 147), as well as verse 30 of the *Yuktiśaṣṭikā* (Tola and Dragonetti, 1995, 38).

momentariness of all phenomena. ‘All is both so and not so’ is taught to show that what appears to be genuine and substantial from an ordinary perspective might not do so from the perspective of a Buddha’s disciple. Finally, ‘All is neither so nor not so’ is taught to show that neither of these terms is applicable to reality in ultimate terms, in the same way, Candrakīrti observes, as the adjectives ‘pale’ or ‘dark-skinned’ are not applicable to the son of a barren woman.⁷⁸

Neither of the four alternatives is therefore to be rejected in this context. They rather form an ascending series of views of increasing conceptual sophistication, each suitable for the purposes of a specific audience.⁷⁹

Garfield offers a different interpretation based on the dGe lugs interpolation procedure already discussed above.⁸⁰ Here the conflict between the four alternatives is dissolved not by relativizing them to different perspectives, as Candrakīrti does, but by adding the modifiers ‘ultimately’ and ‘convention-

⁷⁸La Vallée Poussin (1903–1913, 370–372).

⁷⁹Robinson (1967, 56–57), Ng (1993, 94–99). Ruegg (1977, 6–7, 63–64, note 71) argues that since each alternative improves on the preceding one and even the fourth alternative is only intended for the ‘scarcely obscured’ all four alternatives should nevertheless be rejected. (This interpretation is criticized by Wood (1994, 140–146)). Even if we accept Ruegg’s position it is clear that the four alternatives as given in 18:8 are quite distinct from all the other uses in Nāgārjuna’s writings, as in all other instances all four alternatives are negated and are not even assigned a heuristic value.

⁸⁰(1995, 250–251).

ally'. The passage is thus interpreted as saying that

1. Everything is *conventionally* real.
2. Nothing is *ultimately* real.
3. Everything is both *conventionally* real and *ultimately* unreal.
4. Nothing is either *conventionally* unreal nor *ultimately* real.

While Garfield does not deny that the conception of graded teaching is something 'with which Nāgārjuna would agree', he argues that such a discussion seems 'out of place' in the argumentative context of chapter 18. The reason for this is not quite clear. After all the sixth verse asserts that Buddha taught the teachings of self, non-self, and neither self nor non-self, and Garfield himself asserts that these three were meant to counteract specific wrong conceptions of the self in the mind of the listeners.⁸¹ On the whole the reading of the positive tetralemma in terms of graded teaching seems to be more satisfactory, as it does not commit us to making any additions to the text itself.⁸²

I hope the above remarks have made it plausible that to understand the *catuṣkoṭi* it is essential to keep apart the different kinds of nested negations involved. In this way it is possible to see that the four alternatives of

⁸¹(1995, 249).

⁸²See Tillemans (1990, 73).

the tetralemma are logically independent, as well as to understand how the rejection of the four alternatives (as illocutionary negations based on a presupposition failure) fits in with Nāgārjuna's general philosophical attempt to demonstrate the non-existence of *svabhāva*.

Compared to some accounts in the contemporary commentarial literature the interpretation presented above is logically very conservative. It does not involve anything beyond the resources found in classical logic and in particular gets by without rejecting the Law of the Excluded Middle or adopting a paraconsistent logic. While I think there are some aspects of Nāgārjuna's works (for example the notoriously complex issue of the emptiness of emptiness) which are best explained by reference to some variety of dialetheism, such as the one put forward by Priest and Garfield⁸³ this does not apply to the methodological foundations of Nāgārjuna's arguments in the *catuṣkoṭi*. These can be explained entirely within the framework of classical logic. *

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⁸³(2002).

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