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ORIGINAL RESEARCH

Ad hocness, accommodation and consilience: a Bayesian account

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Abstract

All of us, including scientists, make judgments about what is true or false, probable or improbable. And in the process, we frequently appeal to concepts such as evidential support or explanation. Bayesian philosophers of science have given illuminating formal accounts of these concepts. This paper aims to follow in their footsteps, providing a novel formal account of various additional concepts: the likelihood-prior trade-off, successful accommodation of evidence, ad hocness, and, finally, consilience—sometimes also called "unification". Using these accounts, I also provide a new Bayesian analysis of how someone such as Charles Darwin hypothetically could have reasoned in favor of evolution over special creationism. Lastly, I explore how these accounts relate to other topics and accounts in philosophy, and I chart out some areas for further research.

Keywords Auxiliary hypotheses · Ad hocness · Consilience · Unification · Likelihood-prior trade-off · Bayesianism · Darwinian evolution

1 Introduction

Every day, we make important epistemic judgments about what is true or false, or what is probable or improbable. This is true in mundane ordinary contexts: we make important judgments about whether a particular medication is probably safe to take, for instance. But it is also true in scientific contexts: for example, science accepts the truth of evolutionary theory, and this has had revolutionary implications for how we understand ourselves, our historical origins and our place in the world.

In making these judgments, we often appeal to various concepts, such as evidential support, explanation or causation. For example, we might believe evolutionary theory

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because it *explains* various facts, such as the genetic similarity between species, and each of these facts thereby provides *evidential support* for evolutionary theory.

Bayesian philosophers of science have attempted to explicate concepts like these in terms of probabilities—where "probabilities" are understood as the degrees of belief of some agent (actual or ideal), an understanding presupposed throughout this paper. Consider, for example, the concept of evidential support. According to one prominent account of this concept, some evidence e supports or "confirms" a hypothesis h if and only if e raises the probability of h, and it raises the probability as such when $P(h|e) > P(h)$ (where $P(h|e)$ is the probability of h conditional on e and $P(h)$ is the probability of h prior to the receipt of the evidence e).[1] This account represents an attempt to explicate one concept (evidential support) in terms of a quantitative account (probability-raising).

Note that these concepts are things which we might initially have only informal qualitative judgments about. Presumably scientists have judged for centuries that evidence can support one hypothesis over another, but they presumably did not always think of this "support" in precise quantitative terms—well, perhaps at least not before they became conceptually equipped to think in terms of probability.

For the most part, the program of Bayesian philosophy of science has been to explicate imprecise qualitative judgments about such concepts in more precise quantitative terms.

The task of this paper is to then extend this program to provide a unified account of three further concepts: successful accommodation, ad hocness and consilience. Here, "successful accommodation" refers specifically to when a hypothesis successfully accommodates some (new) evidence.[2] This typically looks like an attempt to reconcile the putative truth of the hypothesis with what would have otherwise been counter-evidence to that hypothesis. The second concept, ad hocness, then concerns unsuccessful attempts to save a hypothesis from such counter-evidence by appealing to "ad hoc" hypotheses. The third concept, consilience, concerns when a theory successfully explains multiple different kinds of evidence.[3] (This consilience is also sometimes called *unification*.)[4] The account of these concepts is unified in the following sense: typically, a hypothesis successfully accommodates some evidence just in case it does not appeal to ad hoc auxiliary hypotheses, and often a hypothesis successfully unifies or consiliates a body of evidence when alternative hypotheses would make ad hoc appeals to auxiliary hypotheses in trying to accommodate that same body of evidence.

However, from a Bayesian point of view, there is not necessarily a notion of "appeals to hypotheses", "accommodation" or "ad hocness" baked into the formalism. Instead, the task is to consider the relevant informal concepts of what people do in

[1] Sprenger and Hartmann, *Bayesian Philosophy of Science* (2019).

[2] Thus, this paper is not about "accommodation" as it pertains to the prediction vs. accommodation debate.

[3] Nola, "Darwin's Arguments in Favour of Natural Selection and Against Special Creationism" (2013); Laudan, "William Whewell on the Consilience of Inductions" (1971); Yeo, *Defining Science: William Whewell, Natural Knowledge, and Public Debate in Early Victorian Britain* (1993).

[4] Myrvold, "A Bayesian Account of the Virtue of Unification." (2003); Kitcher, "Explanatory Unification" (1981); Blanchard, "Bayesianism and Explanatory Unification: A Compatibilist Account" (2018).

non-quantitative terms and to then connect this to some illuminating formal account in quantitative Bayesian terms.

Insofar as scholars are interested in Bayesianism—the reigning orthodoxy in philosophy of science—so too may they be interested in whether, and how, Bayesianism can give an account of these and related concepts. But while I presuppose that the reader has an interest in how a Bayesian might account for these concepts, it is not a defense of Bayesianism, nor do I claim Bayesianism is the only valuable framework for understanding these concepts.

The structure of this paper is then as follows. In Sect. 2, I give a Bayesian account of successful accommodation and ad hocness. In Sect. 3, I give a Bayesian account of consilience, and show how it relates to the account of ad hocness. In Sect. 4, I explore how these accounts relate to the philosophical literature, including existing accounts of ad hocness and unification. There, I will argue that, unlike previous literature, the accounts in this paper provide some simple and interrelated quantitative formalisms for assessing ad hocness and unification. Finally, in Sect. 5, I explore some objections and unresolved questions for future research.

This paper also aims to be somewhat comprehensive and accessible to both specialists and non-specialists, including students in my introductory philosophy of science courses. Hence, the reader should not read this paper linearly but should expect to skip some content because it is aimed either at specialists (such as comparisons to other technical accounts) or at non-specialists (such as examples of what a hypothesis is, or details about the probability of a conjunct being no higher than that of the conjunction).

2 Successful accommodation and ad hocness

In science and in everyday life, we typically consider a range of hypotheses.[5] In science, some historically prominent examples of hypotheses are the hypotheses that Newtonian mechanics provides a true account of physics, that the species evolved through natural selection and that the earth is older than 12,000 years. In everyday life, we might also entertain more mundane hypotheses about various things: take, for instance, the hypothesis that some medication is safe for you to consume, or that it rained outside while you were in a movie theater.

Sometimes these hypotheses confront counter-evidence. In science, for example, Mercury's orbit is now regarded as counter-evidence to Newtonian mechanics. This was because Newtonian mechanics predicted that Mercury should orbit the sun in a particular way, but this was different to Mercury's actual orbit. Then there are more mundane everyday examples of counter-evidence: you might experience adverse symptoms after taking some medication, and you might think this is counter-evidence to your hypothesis that the medication is safe.

Hypotheses frequently encounter counter-evidence, and we often try to reconcile the hypotheses with the counter-evidence.

[5] Some scientists distinguish between theories and hypotheses and would regard these examples as theories, not hypotheses. However, for our purposes in this paper, it is not necessary to distinguish these terms here.

Typically, we do this by appealing to *some other hypothesis* aside from the one we are mainly interested in. For example, to reconcile the safety of the medication with the adverse symptoms, we might appeal to another hypothesis: the hypothesis that something else caused the symptoms, such as an allergy to your dinner. Hypotheses like these are often appealed to in order to "explain away" the counter-evidence—as some scholars say.

Philosophers of science call such hypotheses *auxiliary hypotheses*: they are hypotheses that are distinct from the central hypothesis that one is interested in, but they nevertheless have implications for how the evidence relates to that central hypothesis.

An auxiliary hypothesis is typically called "ad hoc" when it represents an unsatisfactory attempt to save a theory from some counter-evidence, although there are different accounts of precisely why these attempts are unsatisfactory.[6]

An example of this ad hocness concerns Newtonian mechanics and the orbit of Mercury. In 1859, Urbain Le Verrier recognized that Mercury orbited the sun in a way which conflicted with the predictions of Newton's law of physics. In an attempt to reconcile Mercury's orbit with Newton's laws, scientists advanced various hypotheses. Le Verrier himself also advanced one: that there was an undetected planet that had a gravitational influence on the orbit of Mercury, and this influence supposedly explained away the apparent conflict between Newton's laws and Mercury's orbit. However, such a planet was never found. A range of other hypotheses were proposed, each with their problems. Various commentators appraised these hypotheses as "ad hoc": they attempted to salvage theories in ways that were in some sense unsatisfying.[7]

However, ad hoc hypotheses are not confined only to discussions in science. The comedian Bill Hicks describes his conversation with a young earth creationist who believed the world was 12,000 years old.[8] Hicks asked him what he thought about dinosaurs and dinosaur fossils. The creationist reportedly replied, "God put those there to test our Faith". In this story, we have an example of an ad hoc hypothesis in an everyday context: someone tries to salvage some hypothesis (young earth creationism) by appealing to another one (God testing our faith). And we might think there is something bothering about this ad hoc hypothesis, as Hicks himself does:

> Does that bother anyone here? The idea that God might be f**king with our heads… that he's running around, burying skulls [saying] 'We'll see who believes in me now?'

Who knows whether this conversation occurred in the way that Hicks describes it, but the point is that we are all familiar with attempts to save beliefs with ad hoc hypotheses like these.

[6] That said, some other authors use the term "ad hoc" in a *neutral* way to denote any auxiliary hypothesis that is picked out solely for the purpose of rescuing some central hypothesis from disconfirmation. On their accounts, ad hocness is not necessarily a bad thing; a hypothesis can be both ad hoc and perfectly rational to propose and accept. In any case, I will call an auxiliary hypothesis ad hoc only if it unsuccessfully accommodates the evidence. For the neutral use of the term, see Strevens, "The Bayesian Treatment of Auxiliary Hypotheses." (2001); Howson and Urbach, *Scientific Reasoning: The Bayesian Approach* (2006).

[7] Brush, "Prediction and Theory Evaluation: The Case of Light Bending" (1989).

[8] Bill Hicks, "Dinosaurs," *Arizona Bay* (album, Austin, TX: Rykodisc) (1997).

However, attempts to save a hypothesis from counter-evidence are not always ad hoc. The story of Le Verrier himself offers an example of this. Well before discovering the conflict between Newtonian mechanics and Mercury's orbit, he had discovered another tension: Uranus also did not orbit the sun in the way that scientists expected if Newtonian mechanics was a true description of the world. As a result, some suspected that Newtonian mechanics was false. However, Le Verrier thought that a solution to this conflict laid elsewhere: he hypothesized that there was a specific undetected planet that affected Uranus' orbit, one which could resolve the tension within a Newtonian framework. He then deduced that this undetected planet could be observed at a particular time and location in the solar system. When astronomers tested this prediction, they did indeed find the planet that Le Verrier had postulated. This planet is now known as Neptune.

The postulation of Neptune is now regarded as one of science's best success stories—far from an ad hoc attempt to save a theory from some counter-evidence.[9]

Clearly, then, some appeals to auxiliary hypotheses successfully accommodate the evidence while others do not. And when these auxiliary hypotheses do not succeed, we often call them "ad hoc".

What then distinguishes successful attempts to save a theory from other attempts that are ad hoc? More precisely, under what conditions is an auxiliary hypothesis ad hoc?

There are various candidate answers to this question, some of which I explore in Sect. 4 and 5. However, I want to focus on one candidate: a Bayesian account.

2.1 The likelihood-prior trade-off

Thus, I will outline a Bayesian account of when attempts to accommodate auxiliary hypotheses are successful versus when they are not—and, in particular, when they are 'ad hoc'.

An attempt to accommodate counter evidence typically appeals to an auxiliary hypothesis to save some central hypothesis. By this, I mean that when someone confronts counter-evidence to some central hypothesis, they may start to entertain—in their mind and perhaps also in their speech—some other auxiliary hypothesis. When Newtonian mechanics was threatened by Mercury's orbit, Le Verrier appealed to the existence of Vulcan. When Newtonian mechanics was threatened by Uranus' orbit, Le Verrier appealed to the existence of Neptune. And when the creationist's worldview was threatened by the existence of dinosaur fossils, he appealed to God testing our faith. In each case, some other hypothesis—distinct from the central hypothesis—is entertained in one's mind or discussions to accommodate the evidence.

We can use probabilistic notation to formalize this. We can symbolize the central hypothesis with the letter h, the counter-evidence with e and the auxiliary hypothesis with a. For example, h is the hypothesis that the earth is 12,000 years old, e is the fact that dinosaur fossils exist, and a is the hypothesis that God put fossils here to test our faith. An appeal to an auxiliary hypothesis means that we are no longer entertaining

[9] Howson and Urbach, *Scientific Reasoning: The Bayesian Approach* (2006).

how h fares *by itself* with respect to e, but rather how the *conjunction of* h *and* a fares with respect to e.

Typically, the aim of the appeal is to show that the counter-evidence is more likely given the conjunction of h and a than it appears to be when we consider just h by itself. For example, even though Uranus' orbit did not at first seem probable if Newtonian mechanics was a true description of nature, it certainly seemed more probable if Newtonian mechanics was true *and* there was another planet which influenced Uranus' orbit in accordance with Newtonian mechanics. Here, h is the central hypothesis that Newtonian mechanics is true, a is the auxiliary hypothesis that some other planet (with specific properties) exists, and e is the orbit of Uranus. Let us use the notation $P(e|h)$ to denote the likelihood that Uranus would orbit the way it did given that Newtonian mechanics is true. And let $P(e|h\&a)$ denote the likelihood of Uranus' orbit given that Newtonian mechanics is true *and* there is another planet which influenced Uranus' orbit. In probabilistic terms, the appeal to the auxiliary hypothesis aimed to show this: even if $P(e|h)$ appeared low and Uranus' orbit did not at first seem likely if Newtonian mechanics was true, $P(e|h\&a)$ was much higher and the orbit was more likely given that Newtonian mechanics was true *and* there was another planet which influenced Uranus' orbit in a specific way.

Put simply, attempts to accommodate the counter-evidence typically amount to the claim that $P(e|h\&a)$ is much higher than $P(e|h)$ such that $P(e|h) \ll P(e|h\&a)$ for some central hypothesis h, counter-evidence e and auxiliary hypothesis a. Technically, the term for conditional probabilities like these are 'likelihoods'—that is, the probability of some evidence given some hypothesis (or hypotheses). Appeals to auxiliary hypotheses then have the potential advantage of raising the *likelihood* of the evidence in this particular sense.[10] (That said, this sense does not imply that the likelihood of the evidence given h—or $P(e|h)$—itself raises or changes; $P(e|h)$ stays the same before and after the appeal to the auxiliary. Instead, to say the likelihood of the evidence raises is to loosely refer to the fact that to help h accommodate the evidence, the *focus shifts* from a lower likelihood $P(e|h)$ to another higher likelihood $P(e|h\&a)$. In only in this sense can the likelihood of the evidence rise when appealing to an auxiliary hypothesis.)

However, such appeals are not without their costs. The reason for this is that the probability of two hypotheses being simultaneously true is almost always lower than the probability of just one of them being true. For example, no matter how probable Newtonian mechanics was at the time, one could not be as confident that Newtonian mechanics was true *and* another planet like Neptune existed. This is because the truth of this conjunction relies not only on the probability that Newtonian mechanics is true, but it further relies on the also less-than-certain probability that the other planet exists. And uncertainty coupled with even more uncertainty in this way can only increase one's uncertainty: the probability of the conjunction has to be lower than the probability of either one of its conjuncts, just as two coin-flips landing heads is

[10] Note that some philosophers, such as James Hawthorne, refer to $P(e|h)$ as the "likelihood of the evidence given the hypothesis", while others, such as Robert Nola, refer to $P(e|h)$ as the "likelihood of the hypothesis on the evidence". I have followed Hawthorne as I think this is a less confusing way of speaking about likelihoods. Hawthorne, "Confirmation Theory" (2011); Nola, "Darwin's Arguments in Favour of Natural Selection and Against Special Creationism." (2013).

necessarily less probable than a single coin-flip landing heads. However, an exception to this is when one is certain that both hypotheses are true, or they are certain that one hypothesis is true conditional on the other, or they are certain that at least one hypothesis is false. Otherwise, there is always a cost in the sense that the probability of the conjunction is lower than the probability of the conjunct.

So we have seen that an attempt to accommodate counter-evidence will typically raise the likelihood of the evidence by appealing to some auxiliary hypothesis a such that $P(e|h) < P(e|h\&a)$.

And this typically comes with a cost because the probability of $h\&a$ is almost always lower than the probability of h by itself. In other words, often $P(h) > P(h\&a)$ where $P(h)$ is the probability of the hypothesis and $P(h\&a)$ is the probability of the conjunction of the central hypothesis and the auxiliary hypothesis. This also is true when we interpret $P(h)$ and $P(h\&a)$ as *prior* probabilities—that is, as the probabilities of the hypotheses prior to receiving some particular evidence. For example, Le Verrier might have had a probability of 0.8 that Newtonian mechanics was true *prior* to learning about Uranus' orbit, but this probability might have increased to 0.9 *after* he updated on the evidence that Neptune exists and could be deduced from Newtonian mechanics and Uranus' orbit. Here, the prior probability is 0.8 while the so-called *posterior* probability is 0.9 (where the "posterior probability" of a hypothesis is the probability of the hypothesis *after* updating on the evidence). The claim, then, is that $P(h) > P(h\&a)$ often holds for prior probabilities too.

Consequently, in a sense, appeals to auxiliary hypotheses involve a trade-off: by appealing to some auxiliary hypothesis, one is now considering two propositions $h\&a$ which give the evidence a *greater* likelihood than the evidence would have had with just one hypothesis h (since $P(e|h) < P(e|h\&a)$), but they are also now considering two propositions $h\&a$ which have a lower *prior probability* than the prior probability of just one hypothesis h (since $P(h) > P(h\&a)$). Put simply, $h\&a$ make *e more likely* than just h, but $h\&a$ has a *lower prior probability* than just h. That is the likelihood-prior trade-off when appealing to an auxiliary hypothesis: appealing to an auxiliary raises the likelihood in one way, but it lowers the prior probability in another.[11] We can articulate this more formally with a principle:

Likelihood-prior trade-off principle

(1) If $P(e|h\&a) > P(e|h)$, it is nevertheless the case that $P(h\&a) < P(h)$ (given that $0 < P(h)$ and $P(a|h) < 1$)

(For completeness's sake, proof can be found in the appendix.)

[11] Again, the prior probabilities for h and for h & a do not change; instead, the relevant prior probability is lowered only in the loose sense that again the *focus shifts* from a higher probability $P(h)$ to a lower prior probability $P(h\&a)$. Furthermore, I claim only that the trade-off applies to *appeals* to auxiliary hypotheses which, as described above, are when one tries to accommodate some evidence e with some hypothesis h by appealing to some additional hypothesis a. However, it is not to say that any *proposition* comes with a trade-off in the sense that any proposition giving the evidence a high likelihood must also have a low prior probability. For example, it is clearly possible that a central hypothesis h may give some evidence e a high likelihood $P(e|h)$ while also having a high prior probability $P(h)$. I claim only that if one attempts to give the evidence an even *greater* likelihood by appealing to some auxiliary in the sense that $P(e|h\&a) > P(e|h)$, then the prior probability of the hypothesis and the auxiliary will be *lower* in the sense that $P(h\&a) < P(h)$.

This is a simple principle, the truth of which will be very obvious to many specialists in probability—and simply by virtue of the truth of the consequent.

However, I suspect it is not as obvious to the general public, including people who would otherwise fall prey to the conjunction fallacy—the fallacy whereby a conjunction is regarded as more probable than one of its conjuncts.[12] In my experience, sometimes people appeal to auxiliaries to accommodate counter-evidence, and they act as though this is permissible and completely cost-free so long as the auxiliary is merely possibly true. (Think of the creationist sort, for example.) However, according to the principle, attempts to accommodate counter-evidence always come at a cost for the prior probabilities—namely, the likelihood-prior trade-off. Highlighting this trade-off can direct attention to how much lower the prior probability of the conjunction is—and not merely whether the conjunction is possibly true or gives the evidence a greater likelihood.

Sometimes this trade-off is unsuccessful. Consider Hicks' creationist again. The creationist endorsed the central hypothesis h—that the world is 12,000 years old. He was then confronted with the counter-evidence e of dinosaur fossils. He then appealed to an auxiliary hypothesis a to raise the likelihood of this evidence: God put dinosaur fossils on earth to test our faith. And this did raise the likelihood of the evidence: after all, *if* we suppose God put dinosaur fossils there to test our faith, *then* we would not be so surprised to see dinosaur fossils, even if the world is 12,000 years old. The problem with the creationist's reasoning, however, is that it raised the likelihood of the evidence only by appealing to an auxiliary hypothesis which—to many of us at least—is obviously implausible. Consequently, the likelihood was raised, but only by lowering the relevant prior probability: we are now considering not just the already improbable proposition that the world is 12,000 years old, but we are now considering the much less probable conjunction of that proposition with the claim that God put dinosaur fossils on earth to test our faith. (Of course, for the creationist, the prior probability of God testing our faith may not be so low, but I shall discuss the topic of rational constraints on prior probabilities in Sect. 5 of this paper.)

So some trade-offs are not successful, but others are successful. To take an earlier example, suppose you develop some adverse health symptoms after taking some medication. You initially believed the hypothesis that the medication was safe, but if this was the case, then it seems to you that the symptoms would be unlikely. However, you might remember that you had an allergen in your dinner, and so you successfully accommodate this evidence by appealing to the auxiliary hypothesis that you are having an allergic reaction to your dinner instead. By your lights, this is a successful accommodation: it makes the symptoms more likely, and it does so by appealing to an auxiliary hypothesis that—we suppose—you have good evidence to be confident in. In this sense, the trade-off is successful: it raises the likelihood of the evidence, but without drastically reducing the prior probability of the propositions you are entertaining. This illustrates that a conjunction of some central and auxiliary hypothesis is not always concerningly improbable, since both or either hypothesis may be sufficiently supported by evidence.

[12] Tversky and Kahneman, "Judgments of and by Representativeness" (1982); Tversky and Kahneman, "Extensional versus Intuitive Reasoning: The Conjunction Fallacy in Probability Judgment." (2002).

What we want, then, is a formal account of when appeals to auxiliary hypotheses are successful versus when they are not. An answer to this might tell us when the likelihood-prior trade-off worth it in formal terms. That is the task of the next section where we will explore an account of successful accommodation and ad hocness,

2.2 Successful accommodation and ad hocness

I will offer a couple of accounts in this paper, each focused on specific contexts.

Such accounts, however, explicitly mention the *comparative* nature of inquiry: when we are considering the probability of some central hypothesis h_1, we are always considering it in *comparison* to some *alternative* hypothesis h_2. And we can see that this is true because often this alternative hypothesis is simply the hypothesis that the central hypothesis is false: if the central hypothesis is h_1, then the alternative hypothesis h_2 is such that $\neg h_1 \equiv h_2$. But sometimes the alternative is just one of a variety of ways in which the central hypothesis could be false: h_1 might be that Newtonian mechanics is true, and h_2 might be an alternative hypothesis that relativity theory is true, even if there are further alternatives such as the hypothesis h_3 that some as yet undiscovered quantum theory is true. In this case, we could suppose $\neg h_1 \equiv (h_2 \vee h_3)$.

Given the comparative nature of inquiry, we can articulate an account of successful accommodation as such. Let h_1 and h_2 be two mutually exclusive hypotheses and let e be some evidence such that e is putative counter-evidence to h_1. Then,

Account of successful accommodation

(2) An auxiliary hypothesis a and central hypothesis h_1 accommodate some evidence e as successfully as—or more successfully than—some alternative hypothesis h_2 just in case:

$$\frac{P(h_1)}{P(h_2)} \leq \frac{P(h_1 \& a|e)}{P(h_2|e)}$$

Put informally, this principle compares the relative probabilities of h_1 and h_2. On the left side, it compares their relative probabilities *without* conditioning on the evidence. On the right side, it compares their relative probabilities *given the evidence*, *but* with h_1 conjoined to a. The account then says that h_1 and a accommodate e as successfully as—or more successfully than—h_2 just in case the conjunction of h_1 and a fares at least as well in the light of the evidence (relative to h_2) as h_1 fared relative to h_2 prior to conditioning on the evidence.

Furthermore, let us say that h_1 and a successfully accommodate the evidence *in absolute (non-comparative) terms* just in the special case where $\neg h_1 \equiv h_2$ and (2) holds true. In other words, if $\frac{P(h_1)}{P(h_2)} \leq \frac{P(h_1 \& a|e)}{P(h_2|e)}$ and $\neg h_1 \equiv h_2$, then we can say that $h_1 \& a$ successfully accommodate the evidence simpliciter.

We can also define ad hocness: if some auxiliary hypothesis a is picked out to rescue some hypothesis h_1 from the counter-evidence, but the conjunction of $h_1 \& a$ fails to successfully accommodate the evidence (in absolute terms) as per the above account, then we may say that a is ad hoc (with the degree of ad hocness being proportional to the magnitude of the failure).

This account, then, attempts to define or explicate our informal and qualitative concepts of successful accommodation and unsatisfactory ad hocness in formal and quantitative terms.

Of course, one could think of other possible accounts of successful accommodation, and I will discuss one of these later, but this account is adequate for our current purposes.[13]

What, then, is the motivation for this account? It is the same motivation as for many other quantitative explications of qualitative concepts: it either coheres with our intuitions about applications of that concept or we can explain why those intuitions are unreliable when it does not conform.[14] Put differently, the motivation for the account is that it says successful accommodation—or at least this kind of it—applies in all and only those cases where we intuitively think successful accommodation applies, provided that those intuitions are not defeated by good reasons to think they are unreliable.

The above account seems to me to satisfy this criterion for a large class of intuitions about accommodation, but it is not possible to argue thoroughly for this here. That, I think, would be quite boring and lengthy, requiring an enumeration of intuitions as well as arguments about how the account relates to those intuitions. However, I will provide some support for this account by illustrating its application in specific cases where our intuitions cohere with the account (namely, Hicks' creationist and, in Sect. 4.1, the postulation of Neptune).

To do that, we can also derive a useful theorem from the probability calculus. This will make it easier to see how the account applies in many cases.

Theorem of successful accommodation

(3) $\frac{P(h_1)}{P(h_2)} \leq \frac{P(h_1\&a|e)}{P(h_2|e)}$ iff $P(e|h_2) \leq P(e|h_1\&a)P(a|h_1)$

Proof of the theorem is in the appendix. In words, what it implies is that h_1 and a accommodate e as successfully as—or more successfully than—h_2 just in case the likelihood of e given h_2 is less than the product of two terms: (1) the likelihood of the e given h_1 and the auxiliary hypothesis, and (2) the probability of the auxiliary hypothesis given h_1. This theorem can help us to determine whether the cost of the likelihood-prior trade-off is worth it.

In a sense, though, this theorem also has two other aims: *explaining* and *constraining*. More specifically, it explains what it is in virtue of that successful accommodation and ad hocness apply: namely, $h\&a$ successfully accommodate the evidence in virtue of the combination of (1) the evidence being sufficiently likely given $h\&a$ and (2) the auxiliary being sufficiently probable given h—otherwise, appealing to a is unsatisfactorily ad hoc.[15] Furthermore, it constrains in the sense that it specifies a constraint we

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[13] In particular, one might think that a better account is $\frac{P(h_2)}{P(h_1)} \leq \frac{P(h_2\&a|e)}{P(h_1\&a|e)}$. I will discuss this later on.

[14] For examples of scholars who support their accounts of probability using intuition, see Kyburg and Teng, *Uncertain Inference* (2001); Bacchus et al., "From Statistical Knowledge Bases to Degrees of Belief." (1996). Jaynes also uses intuition to argue against frequentist probabilistic methods in Jaynes and Kempthorne, "Confidence Intervals vs Bayesian Intervals." (1976). For a more detailed discussion of their arguments, see Wilcox, "From Relative Frequencies to Bayesian Probabilities." (2016).

[15] One might think that the theorem explains successful accommodation and ad hocness similar to how the law of likelihood explains confirmation. The law of likelihood explains that confirmation of h_1 over

should aim to satisfy when aiming to be rational: namely, when attempting to accommodate the evidence, we should aim to appeal to auxiliary hypotheses that not only make the evidence sufficiently likely, but also are themselves sufficiently probable given the central hypotheses that we care about.

Ultimately, then, whether an auxiliary hypothesis is ad hoc largely depends on how probable it is. If it has a sufficiently low prior probability, it is ad hoc, and if it has a sufficiently high prior probability, it may not be. This is intuitive, for unsatisfactory ad hocness presumably has some connection to the implausibility of the ad hoc hypothesis, as other philosophers have noted.[16]

But probabilities change, and so if ad hocness is a matter of probability, then it is only reasonable that ad hocness should change too. More specifically, I propose that if the probability of an ad hoc hypothesis later increases sufficiently, perhaps to the point of certainty, it is no longer ad hoc. Ad hocness, then, is relativized to a given time: a hypothesis may be ad hoc at one time, even if further evidence serves to vindicate it later. However, some philosophers, notably Christopher Hunt, may think this renders ad hocness meaningless.[17] But I do not see how this is necessarily so. Given some new evidence, a hypothesis may go from being ad hoc to non-ad hoc similarly to how it may go from being improbable to probable. Yet neither ad hocness nor improbability are thereby meaningless or useless concepts merely because their application changes and depends on the evidence.

Taking stock, then, the account and theorem of successful accommodation have several potential benefits: *defining* when an accommodation is successful and when an auxiliary hypothesis is ad hoc, *explaining* what it is in virtue of that successful accommodation and ad hocness apply, and *constraining* rationality by specifying a constraint we should aim for in accommodating evidence and proposing auxiliary hypotheses.

We can now further illustrate and support the account and theorem by applying it to our examples, starting with Hicks' creationist.

2.3 Illustrative application: Hicks' creationist

Let the central hypothesis h_1 be that the world is 12,000 years old. Let the alternative hypothesis h_2 be that the world is 4.5 billion years old, as well as the background theory that serves as the foundation for the current scientific understanding of the earth's age, origin and development. (And we can suppose this background theory is what allows us to assign a value to $P(e|h_2)$.) Let us make the simplifying assumption that h_1 and h_2 are the only competing hypotheses, so $\neg h_1 \equiv h_2$ (this enables us to phrase successful accommodation in absolute, non-comparative terms, as mentioned earlier).

Footnote 15 continued
h_2 applies in virtue h_1 making the evidence more likely than h_2. The theorem explains that successful accommodation applies in virtue of the aforementioned combination.

16 See, for instance, Howson and Urbach, *Scientific Reasoning: The Bayesian Approach* (2006, p. 124). However, as will be discussed later, their views contain more nuance than what is described here.

17 Hunt, "On Ad Hoc Hypotheses." (2012).

In Hick's case, the creationist was confronted with the counter-evidence e of dinosaur fossils.

This evidence is not so improbable if h_2 is true: after all, if the earth is very old, then we would not be surprised to see such dinosaur fossils. For illustration's sake, let us say then that $P(e|h_2) = 0.5$. The specific value is not so important here because, as we will see, all that matters is that the value is higher relative to other specific probabilities, including $P(e|h_1)$.

And on that note, it clearly seemed to Hicks that $P(e|h_1)$ is low: if the earth is only 12,000 years old, then it seems unlikely that we would observe dinosaur fossils that make it look much, much older. As Hicks describes it, the creationist then appealed to an auxiliary hypothesis a to save his preferred theory: God put those dinosaur fossils on earth to test our faith. Arguably, the probability of such fossils is then much higher given the conjunction of h_1 and this auxiliary hypothesis: if the earth was 12,000 years old *and* God really did put dinosaur fossils here to test our faith, then of course there would be such dinosaur fossils. $P(e|h_1\&a)$ is then very high—arguably a probability of 1, in fact!

So $P(e|h_1) < P(e|h_1\&a)$, and we have a classic case where one attempts to save a hypothesis by appealing to an auxiliary hypothesis to raise the likelihood of the evidence.

But as we have seen, this comes at a cost, and—in this case—a dire one. Even though $P(e|h_1\&a)$ is high, the probability of the auxiliary hypothesis is very low: it seems very implausible that God would put dinosaur fossils here just to test our faith. In this vein, Hicks expresses his incredulity at this auxiliary hypothesis: "Does that bother anyone here? The idea that God might be f**king with our heads… that he's running around, burying skulls [saying] 'We'll see who believes in me now?'".

I will assume that the reader shares an incredulity towards this hypothesis, in which case we can suppose $P(a)$ is very low: for illustrative purposes, say, $P(a) = 0.01$. And let us suppose that this is still true even if we suppose that the world is 12,000 years old. In other words, nothing about the world being 12,000 years old would by itself lead us to expect that God would want to test our faith *prior* to learning of the existence of dinosaur fossils. Then, $P(a) = P(a|h_1) = 0.01$.

In a sense, perhaps even a creationist would agree with this prior probability too. If a creationist never knew of the existence of dinosaur fossils, they might agree that God is unlikely to do something as extreme as test our faith with misleading evidence about the age of the earth. If you asked them about this in advance of them learning the fact, they might even be quite confident that a loving God would not mislead his creatures so severely. But of course, for various reasons, one might not let their prior probabilities affect their later attitudes in appropriate ways: one might reason backwards from their desired conclusions to reject any premise or counter-evidence they want, all without regard to any prior probabilities. (Of course, one might think the above probability assignments beg the case against the creationist who may have different priors; again, though, we will discuss the topic of rational constraints on prior probabilities in Sect. 5)

But in any case, many of us would agree that $P(a)$ is very low, and we can proceed to analyze how we might make sense of the creationist's ad hoc maneuver in a Bayesian framework.

Within our framework, we can then see whether the attempt to save the creationist hypothesis is successful. Recall that according to the account of successful accommodation (and assuming $\neg h_1 \equiv h_2$), an auxiliary hypothesis a successfully accommodates the evidence with h_1 just in case:

$$\frac{P(h_1)}{P(h_2)} \leq \frac{P(h_1 \& a|e)}{P(h_2|e)}$$

In other words, h_1 and a successfully accommodate the evidence just in case the conjunction of h_1 and a fares at least as well in the light of the evidence (relative to h_2) as h_1 fared relative to h_2 prior to conditioning on the evidence. According to the theorem of successful accommodation, then, this is true just in case $P(e|h_2) \leq P(e|h_1 \& a)P(a|h_1)$.

We can then plug in our values above to see whether the attempt does indeed successfully accommodate the evidence:

$$P(e|h_2) = 0.5 > P(e|h_1 \& a)P(a|h_1) = (1)(0.01) = 0.01$$

Here, we can see that the attempt is not successful: it does indeed raise the likelihood of the evidence, but only by appealing to an implausible auxiliary hypothesis. Since the auxiliary hypothesis is not successful as such, we can then say that it is ad hoc. And intuitively, this makes sense: some evidence should undermine a hypothesis when that hypothesis can raise the comparatively low likelihood of the evidence only with the help of sufficiently improbable auxiliary hypotheses. The Bayesian analysis of this appears intuitively plausible, and to that extent, this case provides some support for endorsing the account in this paper.

Note also two simplifying features of this account. First, in cases where the likelihood of the evidence given the auxiliary and accommodating hypotheses is 1 (such as $P(e|h_1 \& a)$ above), assessing ad hocness reduces to a simple comparison of two quantities: the likelihood of the evidence given the competing hypothesis (such as $P(e|h_2)$ above) and the probability of the auxiliary hypothesis given the accommodating hypothesis (such as $P(a|h_1)$ above). This simplifying feature also implies a second one: specific quantities need not be assigned in order to make the comparison, since all that is required is that one quantity is greater than the other. In order words, the comparison or *relative* difference matters, but not the absolute quantities. Some may find these simplifying features satisfying.[18]

3 A Bayesian account of consilience

So far, we have seen that attempts to accommodate evidence typically come at a trade-off, and we have specified a condition under which those attempts are successful—at least according to one account. As it turns out, however, this also has implications for another topic in the philosophy of science: the virtue of *consilience* or, as it's sometimes called, *unification*.

[18] I thank an anonymous reviewer for their insightful observation of these two features.

In its Latin roots, "consilience" literally means "jumping together", and it was introduced by William Whewell—the person who also coined the term "scientist".[19] Whewell used the term "consilience" to refer when multiple chains of reasoning lead us to the same conclusion. Whewell further claims that "[t]his Consilience is a test of the truth of the Theory in which it occurs."[20] Other philosophers of science have also endorsed the value of consilient or unifying explanations, including Carl Hempel, Clark Glymour, Branden Fitelson and Thomas Blanchard.[21] More recently, Robert Nola has construed the term "consilience" to refer to when a hypothesis explains multiple classes of fact.[22] We will adopt Nola's construal here. (Although note that others, notably Stathis Psillos, use the term *consilience* differently.)[23]

The task of this section is to give a Bayesian account of the virtue of consilience and to relate that account to the topic of ad hoc auxiliary hypotheses.

I will next describe the account in general terms and I will then illustrate it using the example of evolutionary theory.

3.1 Successful consilience

The account of consilience offered in this paper applies only in particular contexts that have two features. The first feature is that there are two competing and central explanations for some evidence. For example, as we shall see, evolutionary theory provided an explanation of multiple types of evidence, and the second central alternative to this was special creationism—the hypothesis that God created each species in a separate creative act. The second feature is that one of these competing hypotheses putatively offers a unified explanation of the evidence while the other has to appeal to additional controversial auxiliary hypotheses to accommodate that evidence. For example, evolution arguably could explain various kinds of evidence in a unified way, while special creationism would have to appeal to additional auxiliary hypotheses to accommodate that evidence (although I will add some caveats about this in response to a potential objection in Sect. 5.1). The account then applies only when some unifying explanation h_1 then competes with a set of alternative hypotheses, one of which is the central alternative h_2.

19 Whewell is also recognized as the first systematic writer on the nature and history of science, and he coined other terms like "physicist". Yeo, *Defining Science: William Whewell, Natural Knowledge, and Public Debate in Early Victorian Britain* (1993).

20 *The Philosophy of the Inductive Sciences founded upon their History* (2d ed., London: John W. Parker, 1847), p. 469. Quoted in Laudan, "William Whewell on the Consilience of Inductions." (1971).

21 Glymour, *Theory and Evidence* (1980); Hempel, *Philosophy of Natural Science* (1966); Fitelson, "A Bayesian Account of Independent Evidence with Applications" (2001); Blanchard, "Bayesianism and Explanatory Unification: A Compatibilist Account," (2003).

22 Nola, "Darwin's Arguments in Favour of Natural Selection and Against Special Creationism." (2013). Some might also use the term "unification" to refer to this same phenomenon, but this term is also used in other ways which are very different from that of consilience. See, for example, Jones, "Unification." (2014).

23 Psillos uses consilience to refer to when an explanation fits with background knowledge. Psillos, "Simply the Best: A Case for Abduction." (2003).

The below account explicitly connects the success of a unifying explanation to the probability of the alternative hypotheses. And it does this in a way which closely resembles the account of successful accommodation above.

More generally, then, let h_1 and h_2 be competing central hypotheses, let $e_1\&\ldots\&e_n$ be a conjunctive statement denoting n items of evidence, and let $a_1\&\ldots\&a_m$ be the conjunction of $m \leq n$ central or auxiliary hypotheses intended to accommodate the evidence when conjoined with h_2. Then, we can say that:

Account of successful consilience (in relative terms)

(4) h_1 successfully *consiliates* the evidence $e_1\&\ldots\&e_n$ relative to some central hypothesis h_2 and set of auxiliary hypotheses $\{a_1, ..., a_m\}$ just in case:

$$\frac{P(h_1)}{P(h_2)} < \frac{P(h_1|e_1\&\ldots\&e_n)}{P(h_2\&a_1\&\ldots\&a_m|e_1\&\ldots\&e_n)}$$

Note that the auxiliaries $a_1\&\ldots\&a_m$ are included in right side of the inequality but not on the left. The reason for this is because of the context for which the account applies: as mentioned, it applies only to cases in which the auxiliary hypotheses $a_1\&\ldots\&a_m$ are appealed to *in order to help h_2 accommodate the evidence,* but such appeals do not occur *prior* to the receipt of the evidence (we shall see an example of this with special creationism below). The formalism then concerns how the appeals to additional auxiliary hypotheses impact the posterior probability of the resulting conjunction. What the account says, then, is that h_1 successfully consiliates the evidence relative to some central alternative h_2 just in case $h_2\&a_1\&\ldots\&a_m$ cannot as successfully accommodate the evidence.

We could also say that h_1 successfully consiliates some diverse evidence *in absolute terms* just in case every piece of that evidence makes h_1 more probable. Put more precisely in formal terms,

Account of successful consilience (in absolute terms)

(5) h_1 successfully *consiliates* some diverse evidence $e_1\&\ldots\&e_n$ just in case, for every subset of the evidence $\{e_i, \ldots, e_r\} \subseteq \{e_1, \ldots, e_n\}$ and for every proper subset of that subset $\left\{e_j, \ldots, e_q\right\} \subset \{e_i, \ldots, e_r\}$, it is the case that $P(h_1|e_i\&\ldots\&e_r) > P(h_1\middle|e_j\&\ldots\&e_q)$.

This is a rough definition, and some other more precise definition may be better. But this has the advantage of implying that h_1 does not necessarily successfully consiliate in absolute terms just because there may be some trivial set of sufficiently improbable alternative hypotheses $\{h_2, a_1, \ldots, a_m\}$ such that the inequality in (4) holds. Even if h_1 successfully consiliates relative to some set of alternative hypotheses $\{h_2, a_1, \ldots, a_m\}$, the evidence might make h_1 less probable overall. This could happen if the inequality in (4) fails due to some other better set of hypotheses $\{h_3, a_{m+1}, \ldots, a_l\}$ which are confirmed by the evidence in ways that reduce h_1's overall probability.

In any case, it turns out that the inequality in (4) is satisfied whenever $h_2\&a_1\&\ldots\&a_m$ fails to successfully accommodate the evidence à la the theorem of successful accommodation. We can then extend this to articulate the following theorem:

Theorem of successful consilience (in relative terms)

$$(6)\ \frac{P(h_1)}{P(h_2)} < \frac{P(h_1|e_1\&\ldots\&e_n)}{P(h_2\&a_1\&\ldots\&a_m|e_1\&\ldots\&e_n)}$$

iff

$$P(e_1\&\ldots\&e_n\,|h_2\&a_1\&\ldots\&a_m)P(a_1\,|a_2\&\ldots\&a_m\&h_2)\ldots P(a_m\,|h_2) < P(e_1\&\ldots\&e_n\,|h_1)$$

The proof of this is again found in the appendix.

Like the theorem of successful accommodation, we again have a relatively easy way to see whether attempts to consiliate the evidence are successful.

But again, the theorem also has two other aims: to explain consilience in terms of the prior probabilities of competing auxiliaries and to constrain in the sense of recommending sufficiently probable auxiliary hypotheses when trying to offer alternatives to consiliating explanations.

And again, the motivation for this account of consilience is the same as the account of ad hocness: it either coheres with our intuitions about applications of that concept or we can explain why those intuitions are unreliable when it does not conform. Put differently, the motivation for the account is that it says successful consilience applies in all and only those cases where we intuitively think successful consilience applies—provided those intuitions are not defeated by good reasons to think they are unreliable.

So let us motivate the account by illustrating it more concretely, this time using the example of Darwin's observations in favor of evolution.

3.2 Illustrative application: Darwin's observations in favor of evolution

But before we get into the details, a disclaimer is in order: the purpose of this section is not to reconstruct how Darwin *actually did* reason, but rather to provide a Bayesian analysis of how one *could have* reasoned in order to reach Darwin's same conclusion in favor of evolution.

With that in mind, let us examine his argument in more detail.

Darwin was largely arguing against *special creationism*, the hypothesis that a God created each species in a separate creative act. According to special creationism, God created the chickens in one act, the lizards at another, and the bonobos at another. On this picture, it is not as though God created one species and that species then evolved into another.

Note, however, that Darwin did not present his argument as an argument against religion or the existence of God in general. Indeed, writing in the 6th edition of the *Origin of Species*, he states quite clearly:

> I see no good reason why the views given in this volume should shock the religious feelings of any one.... A celebrated author and divine has written to

> me that 'he has gradually learned to see that it is just as noble a conception of the Deity to believe that He created a few original forms capable of self-development into other and needful forms, as to believe that He required a fresh act of creation to supply the voids caused by the action of His laws.'[24]

It then seems that Darwin took himself to be arguing not against religion in general, but rather against a *specific religious view*—special creationism—that was held by some followers of religion, but not all of them.

His preferred alternative hypothesis was that the species evolved by natural selection. According to this hypothesis, species vary in their biological features, and some variations are more conducive to survival in their environmental circumstances. Darwin believed, for example, that birds with webbed feet had an advantage over those without webbed feet in aquatic environments. Consequently, species with advantageous features were more likely to survive and reproduce in their environments, and this forms the basis for the "natural selection" of those features in those environments. Such processes of variation continue, and consequently some species give rise to other species. For example, some species of non-aquatic birds that live on land evolved from aquatic birds with webbed feet, and this is why those non-aquatic birds have webbed feet even though they live on land. All species, then, ultimately evolved via this process of variation and natural selection and, for this reason, many species shared common ancestors (Darwin thought probably all organic life descended from one common ancestor, but he never committed to this, and he thought animals may have descended from "at most four or five progenitors").[25]

This view strongly contrasted with special creationism. On Darwinian evolutionary theory, humans and other primates evolved through variation and natural selection from a *common* ancestor. On special creationism, humans and other primates were made by God in *separate* creative acts—and not by descent from a common ancestor. The conclusion of Darwin's argument is that evolutionary theory is a better explanation of various phenomena than special creationism.

What, then, are the phenomena which evolutionary theory sought to explain?

Well, there are many. Some of these are concisely discussed by Nola, and I summarize examples below.[26]

3.2.1 The dissimilarity between blind insects in American and European caves

Darwin notes that blind insects in the caves of America and Europe have similar environments, and so we might have expected similarities in their biology if God created these insects for similar environments but separately from the insects outside of the caves. However, he notes that this is not the case. The blind insects in the American caves are quite dissimilar from those inside the European caves. Furthermore, the blind insects in the American caves more closely resemble the non-blind insects outside of those caves in America, and a similar point holds for the European insects. Darwin then

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24 Darwin, *The Origin of Species: By Means of Natural Selection, or the Preservation of Favoured Races in the Struggle for Life* (1876, pp. 421–422).

25 Darwin (1876, p. 424).

26 Nola, "Darwin's Arguments in Favour of Natural Selection and Against Special Creationism." (2013).

believes that the better explanation for this is that the blind insects inside the American caves evolutionarily descended from ancestors immediately outside of those American caves, and similarly for the European insects. This, Darwin thinks, favors the theory of natural selection over special creationism.

3.2.2 Web-footed, aquatic birds

Darwin notes that if each creature is “created as we now see it”, as special creationists believe, then it is surprising when animals have biological structures that do not match their habits. He mentions numerous examples of this. One of them was alluded to earlier: some geese live on dry land and seldom, if ever, go near the water, but despite that, these geese have webbed feet. There is then a mismatch between their structure and their habits—between the fact that they have webbed feet on the one hand and the fact that they rarely if ever go near water on the other.

He thinks this is quite probable given the theory of evolution by natural selection, for such geese may still have other evolutionary advantages in the non-aquatic habitats they dwell in. However, he thinks special creationism does not offer a satisfactory explanation:

> He who believes in separate and innumerable acts of creation will say, that in these cases it has pleased the Creator to cause a being of one type to take the place of one of another type; but this seems to me only restating the fact in dignified language.[27]

One might think, as Nola does, that Darwin here is making a stronger claim: that special creationism offers *no* explanation here at all.[28] Whatever the case is, Darwin clearly prefers the explanation offered by evolution by natural selection.

3.2.3 No leaps in structure

Darwin notes that the organs of species often appear to be related to earlier transitional organs. The evolutionary explanation of this is that such biological structures evolved from each other by “by short and sure, though slow steps”.[29] Hence, we would expect many existing species to be predated by earlier transitional forms. However, he asks why, if special creationism is true, “should not Nature take a sudden leap from structure to structure?”[30] Again, he instead prefers the explanation offered by evolutionary theory.

[27] Darwin, *The Origin of Species: By Means of Natural Selection, or the Preservation of Favoured Races in the Struggle for Life* (1876, p. 143).

[28] Nola, “Darwin’s Arguments in Favour of Natural Selection and Against Special Creationism.” (2013).

[29] Darwin, *The Origin of Species: By Means of Natural Selection, or the Preservation of Favoured Races in the Struggle for Life,* (1876, p. 156).

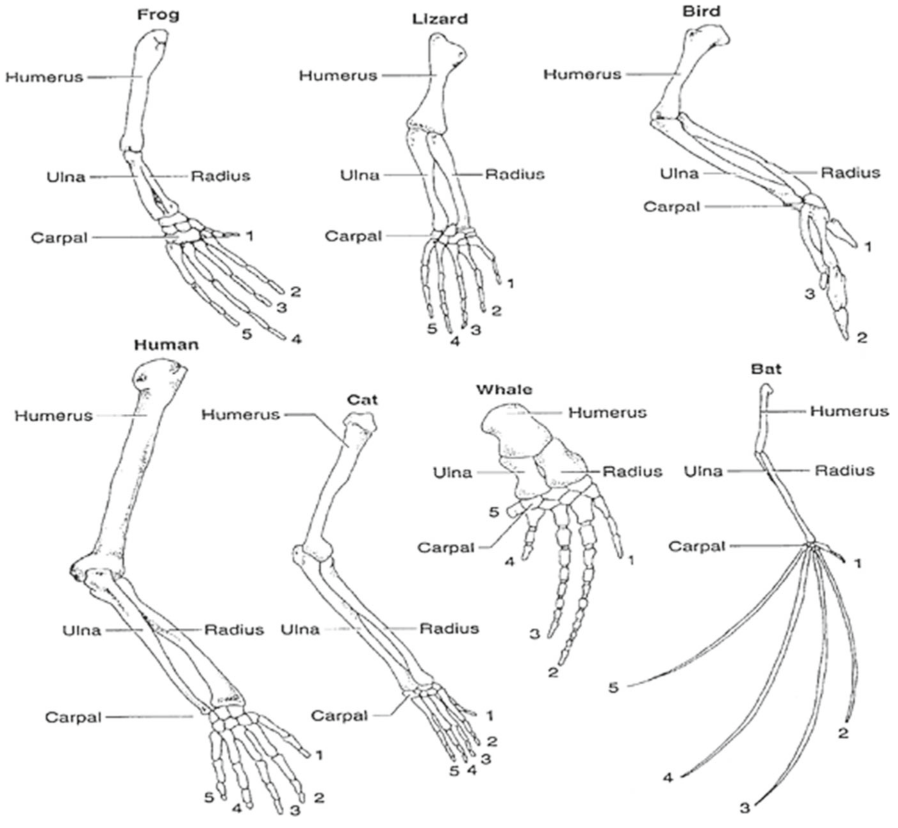
[30] Darwin (1876, p. 156).

3.2.4 Similarities in morphology

In biology, a "homology" is a technical term that refers to a similarity between a pair of structures due to the fact that the structures both descended from a common ancestor. Darwin discusses an example of a homology: the shared bone structure between species. He states:

> What can be more curious than that the hand of a man, formed for grasping, that of a mole for digging, the leg of the horse, the paddle of the porpoise, and the wing of the bat, should all be constructed on the same pattern, and should include the same bones, in the same relative positions?[31]

The below graph, reproduced from Nola, illustrates this phenomenon across various species.



We can then see numerous homologies between these species: the fact they all share one humerus bone preceding an ulna and a radius; the fact that (most of) these species share five digits or fingers, and so forth. Darwin claims that the explanation of this is

[31] Darwin, p. 382.

clear given evolution by natural selection: these similarities in structure exist because the species evolved by gradual steps from a common ancestor. Again, however, Darwin expresses dissatisfaction with the special creationist alternative:

On the ordinary view of the independent creation of each being, we can only say that so it is;—that it has pleased the Creator to construct all the animals and plants in each great class on a uniform plan; but this is not a scientific explanation.[32]

3.3 Darwin's inference in favor of evolution

Those are a few of the many phenomena that Darwin appeals to in support of evolution by natural selection.

How, more specifically, does he then infer that evolutionary theory is the preferable explanation?

Well, there are a couple of interpretations available here.

One is the claim that evolutionary theory offers an explanation of the observations whereas special creationism offers no explanation at all. As Darwin says in one case, special creationism may appeal to the possibility that it just "so pleased the Creator to construct each animal and plant", but he complains in another case that this "seems to me only restating the fact in dignified language".[33] Some might interpret these statements as saying that evolution offers an explanation whereas special creationism does not.

Another interpretation is that regardless of whether special creationism offers an explanation, evolutionary theory nevertheless offers a consiliating explanation: it explains diverse classes of fact in a way that is so satisfactory that it cannot be false. This interpretation appears to be clearly affirmed by Darwin himself:

> It can hardly be supposed that a false theory would explain, in so satisfactory a manner as does the theory of natural selection, the several large classes of facts above specified. It has recently been objected that this is an unsafe method of arguing; but it is a method used in judging of the common events of life, and has often been used by the greatest natural philosophers. The undulatory [wave] theory of light has thus been arrived at; and the belief of the revolution of the Earth on its own axis was until lately supported by hardly any direct evidence.[34]

Here, he expresses incredulity that a "false theory" could explain the above phenomena and others in "so satisfactory a manner". He further thinks that a similar "method of arguing" is used in both the "common events of life" as well as in natural philosophy—where "natural philosophy" refers to what we now call "science". And he mentions two success stories in science as support of this: the wave theory of light and the universal law of gravitation, both of which are discussed by Whewell. (In fact,

[32] Darwin, p. 383.

[33] Darwin, *On the Origin of Species by Means of Natural Selection, or the Preservation of Favoured Races in the Struggle for Life,* 1st edition (1859, pp. 435 and 186).

[34] Darwin, *The Origin of Species: By Means of Natural Selection, or the Preservation of Favoured Races in the Struggle for Life* (1876, p. 421).

Nola argues that Whewell actually inspired Darwin's conception of science and the scientific method here.)

So Darwin gives us some insight into his reasoning process.[35]

However, we are less interested here in explaining how Darwin actually did reason and more how one *could have* reasoned in a Bayesian framework. This, then, is the topic of the next subsection.

3.4 A Bayesian analysis of consilience and evolutionary theory

Let us frame the debate about evolution in probabilistic terms.

More specifically, let h_{NS} be the Darwinian hypothesis that the species evolved by natural selection from a common ancestor, and let h_{SC} be the hypothesis that every species was created separately and as is by God. Then, let the following symbols denote the different pieces of evidence above:

e_{BI} = the blind insects in the American caves are quite dissimilar from those inside the European caves, and the blind insects in the two continents more closely resemble the non-cave dwelling insects outside their respective caves
e_{WB} = some birds have webbed feet but are non-aquatic
e_{LS} = there are no leaps in structure between the species, and instead many biological structures appear to be predated by earlier transitional forms
e_H = the species share many similarities (now called "homologies"), including the common bone structure in the arms of humans, horses, moles, bats and other animals

These pieces of evidence together constitute some of the phenomena that Darwin appeals to in his argument for evolution. Let us denote these pieces of evidence with the conjunctive statement $e_{BI}\&e_{WB}\&e_{LS}\&e_H\&\ldots$.

We can then use this notation to reason about how the evidence affects the probability of the central hypotheses h_{NS} and h_{SC}. Darwin appears to think that the probability of the evidence is relatively high on evolution by natural selection. After all, if the species evolved by natural selection, then we would not be surprised to see that blind insects in the American caves are quite dissimilar from those inside the European caves, that some birds have webbed feet but are non-aquatic, and so on and so forth. For illustrative purposes, then, we might say that $P(e_{BI}\&e_{WB}\&e_{LS}\&e_H\&\ldots|h_{NS}) = 0.2$ (although one might think the probability should be much higher or much lower). But again, in this context, what matters is not so much the specific value but more so that it is higher than other quantities, including the likelihood given the alternative hypotheses.

So, then, how likely is the evidence given special creationism?

Well, it seems not very likely. After all, the mere hypothesis that God created every species separately does not by itself lead us to expect that blind insects in the American caves are quite dissimilar from those inside the European caves, that some non-aquatic birds have webbed feet and so on.

35 This example also played an important role in Kitcher's early account of scientific explanation as unification. Kitcher, "Explanatory Unification." (1981).

Darwin then anticipates some auxiliary hypotheses which the special creationist might offer. In discussing the webbed feet of non-aquatic birds, he says: "He who believes in separate and innumerable acts of creation may say, that in these cases it has pleased the Creator to cause a being of one type to take the place of one belonging to another type".[36] This paves the way to a generic response to the evidence which a creationist might give: for any fact about the species, the special creationist could say that it simply "pleased" God to create the species in that way. In our notation, then, let the auxiliary hypothesis a_{NL} denote the similar proposition that God simply desired to make it so that there are no leaps in structure between the species, let the auxiliary hypothesis a_H denote the proposition that God simply desired to make it so that the species share many similarities, and so on for the other pieces of evidence.

Then, the special creationist might claim that even though $P(e_{BI}\&e_{WB}\&e_{LS}\&e_H\&\ldots|h_{SC})$ appears low, $P(e_{BI}\&e_{WB}\&e_{LS}\&e_H\&\ldots|h_{SC}\&a_{BI}\&a_{WB}\&a_L$ is much higher—a probability of 1, in fact.

Again, we see an attempt to accommodate the evidence by appealing to auxiliary hypotheses to raise the likelihood of the evidence. And in this case, clearly something seems suspicious about this attempt.

What is it then?

Darwin offers a critique, but it is not very detailed. For example, he discusses the creationist explanation that it simply "pleased the Creator" to make aquatic birds "take the place" of the non-aquatic birds. What is his complaint about this? He says nothing more than this: "this seems to me only restating the fact in dignified language". It seems, then, that his criticism is that it does not really explain the fact in question, but it seems to be a mere restatement of it.

Of course, we might not be satisfied with Darwin's complaint. To state that some mainland birds have webbed feet seems quite distinct from stating that *God desired there to be mainland birds with webbed feet*. One statement seems distinct from the other, and so it is not exactly clear how one is merely a restatement of the other. Perhaps, however, there is some more compelling or charitable interpretation of Darwin here: one might think that he is somehow getting at the idea that the auxiliary hypothesis is not supported by independent evidence, or something to that effect.

In any case, as mentioned above, the point is not to describe and evaluate what Darwin thought.

Instead, it is to provide a formal Bayesian account of how someone, like Darwin, *could have hypothetically* reasoned about this case. So, then, is there a different criticism of these auxiliary hypotheses?

Here is my suggestion: evolution by natural selection successfully consiliates the evidence, and the auxiliary hypotheses are ad hoc!

To see this, let us apply the above account of successful consilience to our example.

According to the account, evolution by natural selection h_{NS} successfully *consiliates* the evidence $e_{BI}\&e_{WB}\&\ldots$ relative to a set of alternative hypotheses

[36] Darwin, *On The Origin of Species: By Means of Natural Selection, or the Preservation of Favoured Races in the Struggle for Life,* 1st edition (1859, pp. 185–186).

$\{h_{SC}, a_{BI}\&a_{WB}\&\ldots\}$ just in case:

$$\frac{P(h_{NS})}{P(h_{SC})} < \frac{P(h_{NS}|e_{BI}\&e_{WB}\&\ldots)}{P(h_{SC}\&a_{BI}\&a_{WB}\&\ldots|e_{BI}\&e_{WB}\&\ldots)}$$

What this says, then, is that h_{NS} successfully consiliates the evidence relative to special creationism h_{SC} just in case special creationism cannot successfully accommodate the evidence with its auxiliaries.

And according to the theorem of successful consilience, special creationism will be successful as such just in case:

$$P(e_{BI}\&e_{WB}\&\ldots|h_{SC}\&a_{BI}\&a_{WB}\&\ldots)P(a_{BI}|a_{WB}\&a_{LS}\ldots\&h_{SC})\ldots P(a_i|h_{SC}) < P(e_{BI}\&e_{WB}\&\ldots|h_{NS})$$

(where a_i is the final auxiliary hypothesis that helps special creationism accommodate the evidence)

Crucially, then, to determine whether special creationism successfully accommodates the evidence, we must consider the probability of each of the auxiliary hypotheses. And on the Bayesian analysis, this is the locus of the problem in special creationist reasoning: each auxiliary hypothesis is less than certain, and consequently the appeals to such hypotheses progressively compromise the ability of special creationism to accommodate the evidence.

For illustrative purposes, let us assign values to the probability of each auxiliary hypothesis. Suppose that prior to learning of the evidence, we had no strong reason to expect God to want the evidence that way. For example, prior to learning about there being no leaps in structures, suppose we have no reason to expect God to make it so that there were no leaps in structures, and similarly for the other items of evidence. For that reason, then, we can suppose $P(a_{LS}) = 0.5$, and similarly for the other evidence about the dissimilarity between insects in different caves, the non-aquatic birds with webbed feet and so forth. (Again, the value of 0.5 is debatable, and some might think it should be much lower, but it is used merely for illustrative purposes.) And furthermore, let us suppose for illustrative purposes that all of these auxiliary hypotheses are probabilistically independent: for instance, learning that God wants non-aquatic birds to have webbed feet would not affect the extent to which we would expect insects in the caves of Europe and America to be different. Put formally, then, let $P(a_i) = P(a_i|a_1\&\ldots\&a_k)$ for any auxiliary hypotheses a_i and a_j such that $i \neq j$ for any $1 \leq j \leq k$.

Using the above values, we can then show formally both that evolutionary theory successfully consiliates the evidence while special creationism and its auxiliaries fail to successfully accommodate it.

$$P(e_{BI}\&e_{WB}\&\ldots|h_{SC}\&a_{BI}\&a_{WB}\&\ldots)P(a_{BI}|a_{WB}\&a_{LS}\ldots\&h_{SC})\ldots P(a_i|h_{SC}) = (1)(0.5)(0.5)(0.5)(0.5)\ldots < P(e_{BI}\&e_{WB}\&e_{LS}\&e_H\&\ldots|h_{NS}) = 0.2$$

As we can see, the product of the left of the inequality is less than 0.0625, while the value on the right hand of the inequality is 0.2. By implication, then, Darwinian evolutionary theory does a good job of consiliating the evidence while special creationism and its auxiliaries offer only an ad hoc attempt to accommodate the evidence, an attempt that makes too many costly likelihood-prior trade-offs. To my mind, this framework represents a satisfactory analysis of how one *could* have reasoned about evolution and creationism in Bayesian terms, while bearing in mind that the exact values are debatable and used merely for illustrative purposes.

4 Implications and relevance to the literature

In the preceding sections, I have outlined a Bayesian account of when hypotheses successfully accommodate counter-evidence, when they are ad hoc and when they successfully consiliate disparate evidence. The task of this section is to explain how these accounts relate to the philosophical literature, often by outlining how they resemble or differ from extant accounts.

4.1 Other accounts of auxiliary hypotheses and ad hocness

Philosophers have given various accounts of ad hocness and similar concepts. There are too many to discuss here, but Samuel Schindler provides a helpful overview of some of the main accounts elsewhere.[37]

These other accounts often differ in that they do not provide *quantitative* analyses of ad hocness. Instead, ad hocness is described in terms of qualitative criteria. According to Karl Popper, for example, an ad hoc hypothesis is one which is not independently testable: that is, it does not lead us to expect other consequences that can be independently tested.[38] This account is purely qualitative: the ad hocness of a hypothesis is assessed simply by whether it meets this criterion of independent testability. However, the account in this paper is quantitative in a particular sense: it incorporates some specific quantities—probabilities—that determine whether an auxiliary hypothesis is ad hoc.

Of course, there are some Bayesian philosophers of science who discuss auxiliary hypotheses, but none of them thoroughly articulate a quantitative account of ad hocness. For example, Jon Dorling provides an excellent discussion of how evidence bears on the probabilities of both central and auxiliary hypotheses, yet he does not use this to determine which auxiliary hypotheses are ad hoc.[39] Colin Howson and Peter Urbach discuss ad hocness in their pioneering work *Scientific Reasoning: the Bayesian Approach.* However, all they do is claim that there is nothing wrong with ad hoc hypotheses, at least when they are understood as—to use Carl Hempel's

[37] A review of various accounts of ad hocness can be found in Schindler, *Theoretical Virtues in Science* (2018).

[38] Popper, *Objective Knowledge; an Evolutionary Approach* (1972); Bamford, "Popper's Explications of Ad Hocness: Circularity, Empirical Content, and Scientific Practice." (1993).

[39] Dorling, "Bayesian Personalism, the Methodology of Scientific Research Programmes, and Duhem's Problem." (1979).

words—hypotheses that are introduced “for the sole purpose of saving a hypothesis seriously threatened by adverse evidence”.[40] They do claim, however, that the Bayesian approach can explain why some react with “incredulity” when “certain ad hoc hypotheses are advanced”—namely, because they are “struck by the utter implausibility” of those hypotheses.[41] I do not disagree with the truth of this claim; rather, the point of this paper is to affirm its truth and integrate it into a formal account of unsatisfactory ad hocness. Despite this, however, Howson and Urbach do not themselves offer a formal litmus test for unsatisfactory ad hocness.

The closest one finds to a formal discussion of ad hocness in the literature is Michael Strevens' discussion of auxiliary hypotheses. Hence, we will examine it in detail.

Strevens aims to address the Quine-Duhem problem, which he understands as the problem of finding “a method for ‘apportioning the blame’ between h and a” when a prediction of $h\&a$ is falsified.[42]

He also applies his account to a discussion of “ad hoc auxiliary hypotheses”. He claims that when some evidence e falsifies the conjunction $h\&a$, h is “rescued from falsification” just in case there is some alternative to a, denoted with a', such that $h\&a'$ is consistent with e.[43] He says that “[w]hen a' is picked out specifically for its ability to rescue h from e, it is called ad hoc”.[44]

Importantly, then, for Strevens, an ad hoc hypothesis is not necessarily a bad thing. For example, he alludes to the postulation of Neptune as an ad hoc hypothesis which was a “glorious rescue” of Newtonian mechanics.[45] Instead, then, the key evaluative distinction for Strevens is not about whether a hypothesis is ad hoc or not, but rather about whether an ad hoc hypothesis provides a “desperate rescue” which “dent[s] the credibility of h” or instead provides a “glorious rescue” which “greatly increase[s] the probability of a'”.[46]

When is a rescue glorious or desperate as such?

Strevens suggests Howson and Urbach provide an answer which is “at best incomplete”.[47] Their answer, he claims, is that a rescue is glorious when $P(a^{'}|e)$ is high (so that $P(h\&a^{'}|e)$ will tend to be high), and a rescue is desperate when $P(a^{'}|e)$ is low (so that $P(h\&a^{'}|e)$ will tend to be low). Yet he claims this answer does not explain why the probability of the auxiliary hypothesis a' increases in a glorious rescue while the probability of the central hypothesis h decreases in a desperate rescue.

To fill in this explanatory gap, Strevens then offers an explanation in terms of prior probabilities:

> In summary, a glorious rescue occurs roughly when $P(h)$ is considerably higher than $P(a)$, while a desperate rescue occurs roughly when $P(h)$ is considerably

[40] Hempel, *Philosophy of Natural Science* (1966).

[41] Howson and Urbach, *Scientific Reasoning: The Bayesian Approach* (2006, p. 124).

[42] Strevens, p. 518.

[43] Strevens, p. 533.

[44] Strevens, p. 533.

[45] Strevens, p. 534.

[46] Strevens, p. 534.

[47] Strevens, p. 534.

> lower than $P(a)$. In words, a glorious rescue occurs when the auxiliary hypothesis receives most of the blame for a false prediction, and is rightly discarded by researchers in favor of some other auxiliary hypothesis that makes the correct prediction. (The degree of glory, I remark in passing, is perhaps inversely proportional to the prior probability of the ad hoc hypothesis.) A desperate rescue occurs when the central hypothesis receives most of the blame for a false prediction, but where researchers cling to the central hypothesis and discard the evidently superior auxiliary.[48]

Put simply, then, Strevens sees the situation as follows. Suppose $h\&a$ make a prediction that is refuted by some evidence e. An ad hoc auxiliary hypothesis a' is one which, by Strevens lights, is picked out specifically for its ability to save h from e. Then, appealing to auxiliary a' to rescue h constitutes a glorious rescue when h has a considerably higher prior probability than a and consequently a deserves most of the blame for the false prediction, and it constitutes a desperate rescue when h has a considerably lower prior probability than a and consequently h deserves most of the blame instead of a.

That, then, is Strevens' account.

One similarity between my account and Strevens' is that each employs the same Bayesian mathematics. Strevens affirms this for his paper when he says, "I cannot emphasize strongly enough that the proposals made in this paper do not revise the mechanics of Bayesian confirmation theory one bit".[49] This ensures some degree of consistency in our approaches, at least insofar as the mathematical results are concerned.

However, there are three salient differences in our approaches.

The first concerns the terminological characterization of ad hoc hypotheses and where we draw the key evaluative distinctions. As mentioned, for Strevens, an ad hoc auxiliary hypothesis is simply any hypothesis that is "picked out specifically for its ability to rescue" some h from some falsifying evidence.[50] Consequently, ad hocness is not necessarily a bad thing, and he thinks the postulation of Neptune is an example of this. Instead, he thinks what matters is whether an ad hoc hypothesis provides a desperate rescue or a glorious one.

The approach in this paper is different. As mentioned above, ad hocness is understood as *unsatisfactory ad hocness*. Thus, the challenge is to specify precisely when and why it is unsatisfactory, and this is where many authors differ. However, I do not claim that Strevens is wrong in how he uses the term "ad hoc". Instead, all I claim is that there are different uses that are permissible, and my alternative use of the term is defensible because it accords with how many prominent treatments of ad hocness use the term. Consider, for example, Karl Popper himself, the philosopher who arguably first pushed the topic of ad hocness centerstage in philosophical debates. He clearly disavowed ad hocness in theories that have been modified with unsatisfactory auxiliary hypotheses. For example, he states: "One can show that the methodology of science (and the history of science also) becomes understandable in its details if we assume that the aim of science is to get explanatory theories which are as little ad hoc

48 Strevens, pp. 535–536.

49 Strevens, p. 525.

50 Strevens, p. 533.

as possible: a 'good' theory is not ad hoc, while a 'bad' theory is."[51] Furthermore, he claims that "it is well known that ad hoc hypotheses are disliked by scientists: they are, at best, stop-gaps, not real aims."[52] That said, Popper has different views about what makes a hypothesis ad hoc and about the relationship between ad hocness and probability. Various other philosophers likewise use the term ad hocness to denote unsatisfactory auxiliary hypotheses.[53] Consequently, I do not think it is objectionable for this paper to also use the term in this way.

The second difference between my paper and Strevens' concerns the conditions under which the accounts hold. As mentioned, Strevens aims to address the Quine-Duhem problem, which he understands as the problem of finding "a method for 'apportioning the blame' between h and a" when a prediction of $h\&a$ is falsified.[54] He thinks this requires focusing specifically on the impact of e on h solely in virtue of falsifying $h\&a$. Consequently, as he explains, he restricts his attention to a class of cases for which $P(e|\neg(h\&a)) \approx P(e|h\&\neg a)$ in the sense that $\frac{P(e|\neg(h\&a))}{P(e|h\&\neg a)} = 1 \pm \epsilon$ for some small $\epsilon > 0$.[55] However, the account and theorem of successful accommodation in this paper are not restricted to these cases. Consequently, it provides a more general characterization of ad hocness and successful accommodation.

For example, consider the case of Hicks' creationist, this time using notation that more closely resembles Strevens'. Suppose h is the hypothesis of young earth creationism—that the earth is 12,000 years old, a is the auxiliary that God did not put dinosaur fossils on earth to test our faith and $a^{'}$ is the ad hoc auxiliary hypothesis that God put dinosaur fossils on earth to test our faith (so $\neg a \equiv a'$). We can suppose that e is the evidence that there are actually dinosaur fossils. Now suppose e falsifies $h\&a$, the conjunction of young earth creationism and the auxiliary hypothesis that God did not put dinosaur fossils on earth to test our faith. To rescue young earth creationism and accommodate the evidence, the young earth creationist then appeals to the auxiliary hypothesis $a^{'}$—the hypothesis that God *did* put dinosaur fossils on earth to test our faith.

Strevens' account does not apply here. The reason for this is that, as he himself states, he restricts his attention to cases where $P(e|\neg(h\&a)) \approx P(e|h\&\neg a)$. However, the young earth creationist case is one where this condition is not satisfied. This is because $P(e|\neg(h\&a)) = P(e|\neg h\&a)P(\neg h\&a) + P(e|h\&\neg a)P(h\&\neg a) + P(e|\neg h\&\neg a)P(\neg h\&\neg a)$, but $P(e|\neg h\&a)$ will be sufficiently different to $P(e|h\&\neg a)$ to make it so $P(e|\neg(h\&a))$ is not approximately equal to $P(e|h\&\neg a)$. And this is because if young earth creationism is false and God did not put dinosaur fossils on earth to test our faith (i.e. if $\neg h\&a$ is true), then it's not obvious that dinosaur fossils would exist, so, at the very least, $P(e|\neg h\&a) \ll 1$; but if young earth creationism is true and God did put dinosaur fossils on earth to test our faith (i.e., if $h\&\neg a$ is true since $a^{'} \equiv \neg a$), then dinosaur fossils would certainly exist, so $P(e|h\&\neg a) = 1$. So, given that $P(e|\neg h\&a) \ll 1 = P(e|h\&\neg a)$ and that $P(e|\neg(h\&a)) =$

[51] Popper, *Conjectures and Refutations* (1962, p. 61).

[52] Popper, p. 287.

[53] See the review in Schindler, *Theoretical Virtues in Science* (2018).

[54] Strevens, "The Bayesian Treatment of Auxiliary Hypotheses" (2001, p. 518).

[55] Strevens, p. 532.

$P(e|\neg h\&a)P(\neg h\&a) + P(e|h\&\neg a)P(h\&\neg a) + P(e|\neg h\&\neg a)P(\neg h\&\neg a)$, I think it is reasonable that $\frac{P(e|\neg(h\&a))}{P(e|h\&\neg a)} \neq 1 \pm \epsilon$ and the conditions for the application of Strevens' account do not apply.

However, the account in this paper does apply to this case. In brief, it says that h and a' successfully accommodate the evidence just in case $P(e|\neg h) \leq P\left(e|h\&a'\right)P\left(a'|h\right)$ (where, as mentioned earlier, we made the simplifying supposition that $\neg h$ is equivalent to the current scientific theory and background theory that dates the earth at 4.5 billion years old). And, as we saw, h and a' do not successfully accommodate the evidence, and a' is unsatisfactorily ad hoc, because even if young earth creationism was true, there is a low prior probability that God would test our faith with dinosaur fossils, so $P\left(a'|h\right) = 0.01$ and $P(e|\neg h) = 0.5 > P\left(e|h\&a'\right)P\left(a'|h\right) = (1)(0.01) = 0.01$ (see Sect. 2.3 for a more detailed explanation of these values).

Of course, this is not to say that Strevens' account is wrong, but only that the accounts in this paper apply to a broader range of cases than Strevens'.

However, there is also a third and deeper difference between our accounts: he appraises some cases favorably while I do not (at least initially). To illustrate this, I will use the only example of an ad hoc hypothesis that he appraises favorably (or even appraises at all): the postulation of Neptune circa 1845. Let us imagine we are in that time and suppose that h is the central hypothesis of Newtonian mechanics (that is, $h \equiv h_N$), a is the hypothesis that there are no extra planets in the solar system, and a' is the rescuing hypothesis that an extra planet like Neptune exists (that is, $a' \equiv a_N$). On his account, a_N is a glorious rescue of Newtonian mechanics, since he claims a has a considerably lower prior probability than h_N.

On my account, however, a_N would qualify as unsatisfactorily ad hoc, but only *at first*. To see how this is so, let us use a probability distribution to model the situation, albeit perhaps not in a way that Strevens would see it.[56] Suppose that $P_p(.)$ models the scientist's degrees of belief prior to receiving the evidence e_U of Uranus' anomalous orbit, $P_e(.)$ then models the scientist's degrees of belief after receiving that evidence e_U and $P_a(.)$ models the scientist's degrees of belief after learning that a_N is true. Then, for illustration's sake, we can specify a probability distribution as follows:

[56] Strevens explicitly discusses a, the proposition that implies that it is not the case that "there are more planets". Strevens. $\neg a$ then is equivalent to a long disjunction of propositions specifying the existence of many extra planets where "a vanishingly small proportion of the possible configurations of any extra planets would give rise to the observed perturbations". Strevens, p. 535. Strevens uses this to justify his claims that $P(e|\neg(h\&a)) = P(e|h\&\neg a) \approx 0$ and so $\frac{P(e|\neg(h\&a))}{P(e|h\&\neg a)} \neq 1 \pm \epsilon$. Strevens consequently depicts the scenario as though there are an extremely large number of alternatives to a and, of course, this would be difficult to model in tabular form as I do here.

Possible outcome	Conjunction of the atomic propositions (or their negations) that are true in that outcome	Prior probability measure $P_p(.)$	Posterior probability measure $P_e(.)$	Posterior probability measure $P_a(.)$
s_1	$h_N \& e_U \& a_N$	.07	.7368	.9986
s_2	$h_N \& e_U \& \neg a_N$	.02	.2105	0
s_3	$h_N \& \neg e_U \& a_N$	0	0	0
s_4	$h_N \& \neg e_U \& \neg a_N$	.9	0	0
s_5	$\neg h_N \& e_U \& a_N$	.0001	.0011	.0014
s_6	$\neg h_N \& e_U \& \neg a_N$	.0049	.0516	0
s_7	$\neg h_N \& \neg e_U \& a_N$	.0025	0	0
s_8	$\neg h_N \& \neg e_U \& \neg a_N$	.0025	0	0

According to this probability distribution, h_N was highly probably at first, since $P_p(h_N) = .99$, while a_N was quite improbable at first, since $P_p(a_N) = .0726$. Once the scientist then learns of the anomalous orbit of Uranus, and they then appeal to the existence of a planet like Neptune, they then become less confident in Newtonian mechanics, since $P_p(h_N) = .99 > P_e(h_N) = .947$. The reason that the probability of Newtonian mechanics is reduced is partly because $P_p(e_U|h_N \& \neg a_N) \approx .02 < 1 = P_p(e_U|h_N \& a_N)$ and so Newtonian mechanics needs to appeal to an antecedently improbable auxiliary hypothesis a_N to accommodate the evidence. As mentioned, this appeal comes at a cost.

So, according to my account, the postulation of Neptune is initially ad hoc if the existence of Neptune has a sufficiently low prior probability.

Some might worry that this is an unintuitive outcome, but I think this worry can be allayed by two thoughts.

First, whether the postulation of Neptune is ad hoc depends largely on how antecedently improbable it was. If the auxiliary hypothesis was antecedently probable, however, it would not incur such a cost. For example, let us suppose that $P_p(a_N) \approx 0.95$. The below probability distribution is an example of this:

Possible outcome	Conjunction of the atomic propositions (or their negations) that are true in that outcome	Prior probability measure $P_p(.)$	Posterior probability measure $P_e(.)$	Posterior probability measure $P_a(.)$
s_1	$h_N \& e_U \& a_N$	.95	.95	.9999
s_2	$h_N \& e_U \& \neg a_N$	.02	.02	0
s_3	$h_N \& \neg e_U \& a_N$	0	0	0
s_4	$h_N \& \neg e_U \& \neg a_N$	.02	0	0
s_5	$\neg h_N \& e_U \& a_N$	.0001	.0001	.0001
s_6	$\neg h_N \& e_U \& \neg a_N$	.0049	.0049	0
s_7	$\neg h_N \& \neg e_U \& a_N$	.0025	0	0
s_8	$\neg h_N \& \neg e_U \& \neg a_N$	.0025	0	0

In this case, a_N now has a high prior probability, and far from disconfirming h_N, e_U now confirms h_N, since $P_P(h_N) = 0.99 < P_e(h_N) = 0.995$. This is intuitive, for it seems plausible that the ad hocness of an auxiliary hypothesis should be partly a function of how improbable it was and how much it consequently affects the probability of the central hypothesis when it is conjoined to that central hypothesis.

Second, as alluded to earlier, to say that a hypothesis *was* ad hoc does not mean that it must remain so once further evidence strongly supports the truth of that hypothesis.

Putting these two thoughts together, it is consistent to say that the postulation of Neptune *was* ad hoc *if* it had a sufficiently low prior probability which dented the credibility of h when h was conjoined to it, but it was no longer ad hoc once further evidence vindicated the existence of Neptune. On my account, then, unsatisfactory ad hocness depends largely on the probability of the accommodating auxiliary hypothesis, and if this probability is high—initially or at least eventually—it will not be regarded as ad hoc in the long-term. In this way, this paper's account of ad hocness can explain why the postulation of Neptune is now regarded one of science's greatest success stories—*now,* at a time when its probability is virtually as high as it could be.

But the account in this paper also has an additional advantage: if the postulation of Neptune was never vindicated—if it was never found, and its probability remained low, if not lower, while permanently denting the credibility of Newtonian mechanics—then presumably we would now regard it as an ad hoc hypothesis, just as we do for the Vulcan hypothesis. The account in this paper can explain why this is the case, but it is not obvious that Strevens' account does, since Strevens' account relies solely on the prior probabilities, regardless of what the evidence may say later on.

This, I take it, is a satisfactory outcome for this paper's account of ad hocness. After all, a proponent of even a highly probable hypothesis h can make an appeal to an unsatisfactorily ad hoc hypothesis a', even if that hypothesis h was considerably higher than some other auxiliary hypothesis a that was entertained at the time. On my account, what matters is not simply the prior probability of h relative to a, but rather whether appealing to an auxiliary a' thereby dents the posterior probability of h when it is conjoined to a' in light of the evidence. Unsatisfactory ad hoc hypotheses are those that come at a cost to the credibility of the hypotheses they rescue, at least initially. Some hypotheses are credible enough to afford that cost without tarnishing their credibility too much, and some will redeem such costs once further evidence later vindicates the no longer ad hoc hypotheses. However, in principle, any hypothesis that falls short of absolute certainty can become probabilistically bankrupt by too many costly appeals to ad hoc hypotheses. The formalism merely specifies whether there is a cost and indicates how severe it is. This, I take it, is an adequate role for an account of ad hocness.

4.2 Other analyses of evolution

This paper's analysis of ad hocness and evolution also differs from some analyses offered by others. Various philosophers have discussed why special creationism is defective, and there are several candidate answers.[57]

Elliott Sober, for example, claims that the problem with the special creationist hypothesis is that it tries to accommodate the data by appealing to auxiliary hypotheses that have no independent justification (a criticism that is similar to what Popper would say).[58] For example, while evolution could explain morphological similarities between species by appealing to evolution from a common ancestor, special creationism appeals to an otherwise unsupported hypothesis to explain the same evidence: namely, that it simply pleased God to create the species with such similarities. Sober then objects that the special creationist fails to offer independent justification for their hypotheses about the goals and abilities of God in this case.

Maarten Boudry and Bert Leuridan criticize Sober's view, claiming that sometimes appeals to auxiliary hypotheses are legitimate even if they lack independent justification.[59] Instead, they join Philip Kitcher and (seemingly) Darwin in claiming that the problem with intelligent design is that it lacks the virtue of *consilience* (although Kitcher, Boudry and Leuridan instead use the term *explanatory unification* to refer to what is essentially this same virtue).[60]

The theorem of successful consilience may to some extent jibe with the perspectives of all of these philosophers. According to the above analysis, evolutionary theory is a successful theory because it successfully consiliates the evidence. We can agree with

[57] Kitcher, "Explanatory Unification" (1981); Sober, *Evidence and Evolution: The Logic behind the Science* (2008); Boudry and Leuridan, "Where the Design Argument Goes Wrong: Auxiliary Assumptions and Unification" (2011); Nola, "Darwin's Arguments in Favour of Natural Selection and Against Special Creationism." (2013).

[58] Sober, *Evidence and Evolution: The Logic behind the Science* (2008).

[59] Boudry and Leuridan, "Where the Design Argument Goes Wrong: Auxiliary Assumptions and Unification." (2011).

[60] Kitcher, "Explanatory Unification" (1981); Boudry and Leuridan, "Where the Design Argument Goes Wrong: Auxiliary Assumptions and Unification" (2011); Nola, "Darwin's Arguments in Favour of Natural Selection and Against Special Creationism." (2013).

Darwin, Kitcher, Boudry and Leuridan on this much. And in contrast, special creationism falters as it requires numerous ad hoc auxiliary hypotheses—and consequently numerous costly trade-offs—to accommodate the same evidence. In this case, each auxiliary hypothesis has a relatively low probability prior to receiving the evidence: for example, what reason would we have for expecting God to create various similarities between species *prior to us knowing the fact of their similarity?* Of course, if we had independent justifications to believe this, then the prior probability would be higher. For that reason, we can agree with Sober to the extent that the auxiliary hypotheses would have been more appropriate if they had such justifications. But lacking such justifications, these auxiliaries come at a cost which compromises the ease with which special creationism can accommodate the evidence. Ultimately, though, the locus of the problem is the probability of the auxiliary hypotheses, and justifications matter only in virtue of this.

4.3 Ad hocness in other philosophical debates

Aside from evolution, the accounts in this paper may also have relevance to other areas. Consider, for example, Robin Collins' discussion of fine-tuning in the philosophy of religion.[61] Here, the main datum is that the physical laws and constants of the universe seem finely-tuned so as to permit life. Supposedly this datum supports theism—that is, in this context, the hypothesis that God exists. Fine-tuning arguments typically assert that such fine-tuning is more likely given theism (which we can denote with h_T) than given a single atheistic universe (which we can denote with h_A). For that reason, fine-tuning supports theism—or so Collins' argument goes. However, Collins is also aware that the evidence fine-tuning is not so unlikely given a reformulated version of the atheistic hypothesis. This reformulation is basically h_A conjoined to the proposition a that our universe is life-permitting. And one might be sympathetic to this reformulation: after all, the universe would need to have values tuned so as to permit life if it was indeed life permitting, so fine-tuning would not be surprising given this hypothesis. But Collins claims there is something fishy about this reformulation—-namely, that it suffers from what he calls *probabilistic tension*. Using our notation (not his), Collins would say that probabilistic tension occurs when a conjunct in the conjunction $h_A \& a$ is "very unlikely" when conditioned on the other conjunct.[62] Now he claims that the reformulated hypothesis suffers from such tension because it is very unlikely that the universe would be life permitting given a single atheistic universe. He then provides examples of probabilistic tension in other cases, and he uses these to intuitively motivate the idea that probabilistic tension provides a reason to "reject" a hypothesis.[63]

Now my point here is not to endorse or reject Collins' argument, especially when I have not discussed other objections or alternative explanations (like the so-called "multiverse" alternative explanation of fine-tuning which I think is much better than $h_A \& a$). Rather, my aim is to point out that the spirit of his account of probabilistic

[61] Collins, "The Teleological Argument: An Exploration of the Fine-Tuning of the Universe." (2009).

[62] Collins.

[63] Ibid.

tension is captured by the theorem of successful accommodation. On this account, an auxiliary hypothesis successfully accommodates the evidence only if $P(e|h_T) \leq P(e|h_A\&a)P(a|h_A)$. But if the auxiliary hypothesis is very improbable given the central hypothesis—if $P(a|h_A)$ is very low—then in Bayesian terms, *this* is the reason why appealing to the auxiliary hypothesis is blameworthy. In this sense, rightly or wrongly, Collins might say that the reformulated atheistic hypothesis is ad hoc because the probability of the auxiliary hypothesis is very low given the original atheistic hypothesis—that is, the probability of the universe permitting life is very low if we live in a single atheistic universe.

So the theorem captures the spirit of Collins' proposal, but it does so for a different reason—a stronger one, in my opinion. Collins motivates his principle with examples and intuition. The account of ad hocness is motivated with some intuition too, but in contrast, the theorem follows from axioms that are widely endorsed by Bayesians and by probability aficionados more generally. Of course, the theorem of successful accommodation also is broader than Collins constraint. Unlike Collins' account, it exhorts us to be sensitive to relevant parameters: for example, even if the prior probability of the auxiliary hypothesis is low, it may still successfully accommodate the evidence if the likelihood of the evidence given the other competing hypothesis is that much lower.

In any case, the point is not to criticize Collins' constraint, but just to provide a tangible example of how the theorem may be useful for debates in fields aside from the philosophy of science.

4.4 Other accounts of consilience

In comparison to ad hocness, consilience has received more explicit attention in the Bayesian literature. Perhaps the most recent treatment of the topic comes from Thomas Blanchard.[64] Blanchard aims to show that the Bayesian framework naturally gives weight to the virtue of consilience (a virtue which he calls "explanatory unification").

In a paper currently on my academic website, I compare my account to Blanchard's in some detail, detail which I omit here for sake of space. There, I surmise that our accounts are consistent, and even complementary, especially since they rest on the same Bayesian foundations, but I do not currently have any proof as such. That said, there are some notable differences: unlike Blanchard's, my account of consilience in relative terms applies only when it makes sense to designate one of the alternative hypotheses as the *central* alternative hypothesis, and Blanchard makes the useful distinction between hypotheses and the explanations offered by those hypotheses. Again, though, the details of this comparison can be found online.

5 Unresolved questions

So we have outlined some ideas which aim to elucidate the nature of successful accommodation, ad hocness and consilience. In my experience, particular readers may have

[64] Blanchard, "Bayesianism and Explanatory Unification: A Compatibilist Account," (2018).

some remaining worries: some may worry that every likelihood tacitly involves auxiliary hypotheses, and it is not clear whether all auxiliaries should be modelled explicitly or not; some may worry that the input or prior probabilities for these accounts are arbitrary, in which case the formalisms would be useless; and some may worry that there are alternative accounts of accommodation that are just as good. I will share some thoughts about these worries, although readers who are not so worried can safely skip this section.

5.1 The multiplicity and ubiquity of hypotheses, auxiliaries and otherwise

This paper has worked with a picture of reasoning that might look over-simplified. In particular, our examples often describe scenarios where one hypothesis involves an appeal to an auxiliary hypothesis while the other does not. This might give rise to two misconceptions which I wish to dispel.

The first misconception is that any central hypothesis involves only one hypothesis rather than many.

This misconception is false since many, if not all, hypotheses are comprised of multiple "sub-hypotheses", so to speak. For example, the modern hypothesis of natural selection h_{NS} is actually a collection of sub-hypotheses: these include the hypotheses that all species descended from one or a few ancestors h_{NS_1}, that these species diversified through random mutations that occur in the process of reproduction h_{NS_2}, that the environment will favor those mutations that conduce to survival and reproduction h_{NS_3} and perhaps others. Really, then, natural selection is a single *complex* hypothesis that is comprised of many other more simple sub-hypotheses. Many—if not all—hypotheses are the same, consisting of a set of sub-hypotheses rather than just one.

The second misconception is that one central hypothesis relies on auxiliary hypotheses whereas the other does not. In reality, however, one might argue *all* non-trivial hypotheses are intimately linked to auxiliary hypotheses.

For example, consider natural selection h_{NS}. The natural selection hypothesis might explain why butterflies with camouflaged wings are more prevalent in a particular forest than butterflies with wings that are more conspicuous to predators who might eat the butterflies. However, it would only do so by appealing to the auxiliary hypothesis that camouflaged wings confer an evolutionary advantage over conspicuous wings. Since natural selection could depend on an auxiliary like this in order to make predictions, it might look misleading to consider only $P(e|h_{NS})$ instead of $P(e|h_{NS}\&a)$ where e is the evidence of the prevalence of camouflaged wings in the particular forest, h_{NS} is natural selection and a is the auxiliary hypothesis that camouflaged wings have an evolutionary advantage.

So there are two ways in which there are more hypotheses than one might think: first, a central hypothesis can be composed of many sub-hypotheses and, second, central hypotheses often or always rely on auxiliary hypotheses as well.

However, it is important to note that, if done according to orthodox Bayesian norms, any calculations would automatically be sensitive to these two facts even if these other hypotheses are not explicitly represented in the calculations. This is because Bayesianism says all probabilities should conform to axioms of probability, and these give rise to theorems which imply that the relevant hypotheses should be appropriately integrated in Bayesian calculations.

Let us see how this is the case, starting first with sub-hypotheses and then looking at auxiliaries.

According to Bayesian norms, it is a theorem that the same probabilities should attach to equivalent propositions. As a result, any probability involving a complex hypothesis, like the likelihood of the evidence given natural selection $P(e|h_{NS})$, would be the same as the corresponding probability involving the equivalent conjunction of sub-hypotheses, such as the probability of the evidence given the simpler component hypotheses comprising the complex hypothesis of natural selection $P(e|h_{NS_1}\&h_{NS_2}\&h_{NS_3}\&\ldots)$. Consequently, even though the equivalence of a hypothesis and its sub-hypotheses may not be explicitly represented in the formalism, it should automatically be taken into account in the calculations.

As for auxiliaries: the values of any likelihood should also be coherent with the influence of any relevant auxiliary hypotheses. This is because of the theorem of total probability, which says that for any propositions q and r, $P(q) = P(q|r)P(r) + P(q|\neg r)P(\neg r)$. Applied to our context, the theorem in its conditional form implies that the likelihood of some evidence given any hypothesis should be sensitive to *any* auxiliaries: for any propositions e, h and a, $P(e|h) = P(e|h\&a)P(a) + P(e|h\&\neg a)P(\neg a)$. If the auxiliary is certainly true and $P(a) = 1$, then $P(e|h) = P(e|h\&a)$. If the auxiliary is not certain and $P(a) < 1$, then the theorem implies that $P(e|h)$ will need to be sensitive to the probability of the auxiliary $P(a)$ and the likelihood of the evidence given the central hypothesis and the auxiliary $P(e|h\&a)$. Ditto for the negation of the auxiliary too. In either case, Bayesianism and the probability calculus dictates that $P(e|h)$ should appropriately incorporate the influence of any relevant auxiliaries regardless. Consequently, the Bayesian account in this paper recommends that the influence of any relevant auxiliaries is appropriately incorporated into any calculations as per the orthodox Bayesian norms.

So Bayesian calculations should automatically take into account both the complexity of hypotheses and the ubiquity of auxiliary hypotheses.

However, a question arises as to *when* it is useful to explicitly model such complexities and auxiliaries in our probabilistic representations of the situation. I suspect that this will depend on the specific situation and that the relevant contexts and complexities are too numerous to enumerate here. That said, it seems clear that it is useful to model sub-hypotheses and auxiliaries when they illustrate key weaknesses in reasoning, controversial issues or other potential insights.

Nevertheless, this is perhaps not the final answer, and so this is a question for future research.

In any case, the illustrations throughout this paper (such as the natural selection example) should not be biased merely because, say, natural selection is represented as one hypothesis rather than many or because any relevant auxiliaries are not represented explicitly. Even if such complexity and auxiliaries were represented more explicitly, the probability calculus entails the quantities should be the same regardless.

However, one might think that when such representations are more explicit, this warrants assigning lower (prior) probabilities than the ones I used in the illustrations.

This a fair concern, especially since I in no way claim that the probability assignments used throughout this paper are sacrosanct; they are intended merely as rough illustrations of the relevant formalisms, but other (prior) probability assignments may very well be better.

This, then, leads to a second problem which I discuss in the next section.

5.2 The problem of the priors

We have considered a novel Bayesian approach to ad hocness and consilience. Perhaps a part of the reason such an account has not already been proposed is that it appeals to prior probabilities, at which point we run head-first into the problem of the priors: how should these prior probabilities get their values?

This is also relevant to earlier questions about whether there are rational constraints on the assignment of priors in the case of the creationist: are we really to criticize the creationist when they may simply have different priors which can vindicate their views—even with the formalism?

The formalisms in this paper do not specify what is ad hoc or successfully consilient in absolute terms. In that sense, the illustrative examples I discuss are not intended to be universally persuasive arguments that any particular hypothesis is ad hoc or consilient (though I suspect many will be sufficiently sympathetic to the illustrative assignments to see the motivation for formalisms). Indeed, this would require detailed arguments for specific choices of prior probabilities and likelihoods—something I have not provided.

Instead, my purpose is merely to present the formalisms as a framework for specifying what is ad hoc or successfully consilient *conditional on particular likelihoods and prior probabilities*.

Yet, if the priors have no rational constraints, then one might think the formalisms seem useless as rational constraints.

A discussion of the Bayesian approach to ad hocness would then ideally feature a discussion of the problem of the priors. Of course, though, the problem of the priors deserves a book length treatment by itself and not merely the passing remarks in this paper.

However, I will offer two brief thoughts on the problem.

The first is that I think the priors *do* have rational constraints. If the reader is concerned about the problem of priors, or thinks prior probabilities constitute a problem for the formalisms in this paper, then I strongly encourage them to read another paper I have written on the topic.[65] There, I argue that in many contexts, we have empirical evidence to inform the relevant priors, and we can criticize priors as being more or less trustworthy depending on whether we have reason to think they are produced by cognitive processes that have a track record of "calibration" and that are "maximally inclusive" of the relevant evidence (albeit in a complex sense that I will not elaborate

[65] Wilcox, "Credences and Trustworthiness: A Calibrationist Account." (2022).

on here).[66] Consequently, I believe there is a good solution to the problem of the priors, but there is obviously no space to argue for this here when I do so in another paper.

The second response is that even if one thinks the priors have no rational constraints (which I do not), this might not be too objectionable for this paper given that many Bayesian accounts can live life comfortably while depending on priors—and without such constraints. For example, dominant accounts of confirmation depend on priors, as does Strevens' account of auxiliary hypotheses, as does Jon Dorling's, and so too does Myrvold and Blanchard's accounts of unification.[67] For that reason, I would think the problem of the priors does not constitute an especially strong objection to my paper—at least not for those interested in Bayesianism—since the problem similarly arises for much of the Bayesian program in philosophy of science.

Regardless, despite the problem, I hope that the merit of the accounts is apparent for reasons I described earlier: namely, that they can define, explain and constrain the relevant phenomena—namely, successful accommodation, ad hocness and successful consilience.

5.3 Alternative accounts of accommodation

Earlier, I mentioned that there are alternative accounts of successful accommodation. We might then ask which, if any, is the best. I think this is a topic for future research, but I will share a few thoughts here, at the very least because one might object to my discussion unless I do so. In particular, one might think that a better account of accommodation is the following:

Alternative account of successful accommodation

(7) An auxiliary hypothesis a and central hypothesis h_1 accommodate some evidence e as successfully as—or more successfully than—some alternative hypothesis h_2 just in case:

$$\frac{P(h_2)}{P(h_1)} \leq \frac{P(h_2\&a|e)}{P(h_1\&a|e)}$$

And it turns out this will be true just in case the following condition holds.

Alternative theorem of successful accommodation

(8) $\frac{P(h_2)}{P(h_1)} \leq \frac{P(h_2\&a|e)}{P(h_1\&a|e)}$

iff

$$P(e|h_1\&a)P(a|h_1) \leq P(e|h_2\&a)P(a|h_2)$$

66 Wilcox.

67 Sprenger and Hartmann, *Bayesian Philosophy of Science*, (2019); Strevens, "The Bayesian Treatment of Auxiliary Hypotheses," September (2001); Dorling, "Bayesian Personalism, the Methodology of Scientific Research Programmes, and Duhem's Problem" (1979); Myrvold, "A Bayesian Account of the Virtue of Unification," (2003); Blanchard, "Bayesianism and Explanatory Unification: A Compatibilist Account," (2018).

One might think this account is better for this reason: if an auxiliary hypothesis a is to successfully accommodate the evidence with h_2, then the evidence should not still raise the probability of h_1 relative to h_2 when h_1 is also conjoined to that same auxiliary hypothesis.

This account may very well have some merit in some particular cases.

However, I am not convinced it is better in *all* cases. For example, suppose we are considering natural selection h_{NS} and special creationism h_{SC}, but we learn just one piece of evidence—this time about the non-aquatic birds with webbed feet e_{WB}. Now one might try help h_{SC} accommodate the evidence by appealing to the auxiliary hypothesis a_{WB} that God simply desired there to be non-aquatic birds with webbed feet. We could then plug the relevant probabilities into the alternative theorem of successful accommodation. But this is clearly misguided because there is simply no point in comparing special creationism to evolution by natural selection, but with evolutionary theory *conjoined to the auxiliary hypothesis that God wanted there to be non-aquatic birds with webbed feet*: evolution simply does not need, and should not have, itself conjoined to the auxiliary hypothesis about God's desires. For that reason, the alternative account cannot be the correct account in all cases, even if it is correct for some of them.

Such considerations lead me to think that there might be multiple viable accounts of accommodation, but which is one is the best is *context-sensitive*: it depends on various features of the agent's context. What those features are, then, is a matter for future research. In any case, the accounts in this paper have their merits, and I suspect that the earlier "Account of Successful Accommodation" may suffice for many examples encountered in science and in everyday life.

6 Conclusion

In this paper, I have attempted to provide a formal Bayesian analysis of various concepts, including the likelihood-prior trade-off, successful accommodation, ad hocness and consilience. Consequently, insofar as one is interested in Bayesianism, so too may they be interested in whether, and how, Bayesianism can give an analysis of these concepts. I have also applied the Bayesian analysis to examine the reasoning which could have underpinned inferences in favor of evolution over special creationism—according to which evolution consiliates the evidence while special creationism makes too many costly likelihood-prior trade-offs. To my mind, the analysis represents a satisfactory and illuminating attempt to understand these concepts in ways which complement—but do not necessarily compete with—particular existing accounts on similar or related concepts. Some might also think the existence of such a Bayesian analysis speaks in Bayesianism's favor as a program in the philosophy of science.

Despite this, however, further work remains in understanding various questions. These include questions about when auxiliary hypotheses are useful to model in our formal representations and when alternative accounts of accommodation are warranted.

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Declarations

Conflict of interest The author has no conflicts of interest to declare.

Appendix: Proofs of main results

Likelihood-prior trade-off principle

(1) If $P(e|h\&a) > P(e|h)$, it is nevertheless the case that $P(h\&a) < P(h)$ (given that $0 < P(h)$ and $P(a|h) < 1$)

Proof Suppose $P(h) > 0$ and$P(a|h) < 1$. Then, $P(h) = n$ for some $n \in (0, 1]$ and $P(a|h) = m$ for some $m \in [0, 1)$. Now by the probability calculus,$P(h\&a) = P(h)P(a|h) = nm$. Since *n* is a non-zero value, and *m* is less than 1, *nm* will necessarily be less than *n*, and so$P(h\&a) < P(h)$. This is true regardless of the values of $P(e|h)$ and$P(e|h\&a)$, so it is true in the special case where$P(e|h\&a) > P(e|h)$. Consequently, if$P(e|h\&a) > P(e|h)$, it is nevertheless the case that $P(h\&a) < P(h)$ (given that $0 < P(h)$ and$P(a|h) < 1$).

Theorem of successful accommodation

(3) $\frac{P(h_2)}{P(h_1)} \leq \frac{P(h_2\&a|e)}{P(h_1|e)}$ iff $P(e|h_1) \leq P(e|h_2\&a)P(a|h_2)$

Proof By Bayes's theorem:

$$\frac{P(h_2)}{P(h_1)} \leq \frac{P(h_2\&a|e)}{P(h_1|e)} \text{ iff } \frac{P(h_2)}{P(h_1)} \leq \frac{(P(e|h_2\&a)P(h_2\&a))/P(e)}{(P(e|h_1)P(h_1))/P(e)}$$

We can then multiply both sides by $\frac{P(e)}{P(e)}$ (and since we are dealing with probabilities and none of the signs are negative, the inequality does not change):

$$\text{iff } \left(\frac{P(h_2)}{P(h_1)}\right)\left(\frac{P(e)}{P(e)}\right) \leq \left(\frac{(P(e|h_2\&a)P(h_2\&a))/P(e)}{(P(e|h_1)P(h_1))/P(e)}\right)\left(\frac{P(e)}{P(e)}\right)$$

We then have the following:

$$\text{iff } \left(\frac{P(h_2)}{P(h_1)}\right)(1) \leq \frac{P(e|h_2\&a)P(h_2\&a)}{P(e|h_1)P(h_1)}$$

By the probability calculus, we can express $P(h_2\&a)$ as $P(a|h_2)P(h_2)$:

$$\text{iff } \frac{P(h_2)}{P(h_1)} \leq \frac{P(e|h_2\&a)P(a|h_2)P(h_2)}{P(e|h_1)P(h_1)}$$

We can then derive the following:

$$\text{iff } \frac{P(h_2)}{P(h_1)} \leq \left(\frac{P(e|h_2\&a)P(a|h_2)}{P(e|h_1)}\right)\left(\frac{P(h_2)}{P(h_1)}\right)$$

Dividing both sides by $\frac{P(h_2)}{P(h_1)}$, we have the following:

$$\text{iff } \frac{P(h_2)/P(h_1)}{P(h_2)/P(h_1)} \leq \frac{((P(e|h_2\&a)P(a|h_2))/P(e|h_1))(P(h_2)/P(h_1))}{P(h_2)/P(h_1)}$$

We then have the following:

iff $1 \leq \frac{P(e|h_2\&a)P(a|h_2)}{P(e|h_1)}$
And if we multiply both sides by $P(e|h_1)$, we then have our desired conclusion:

iff $P(e|h_1) \leq P(e|h_2\&a)P(a|h_2)$
Therefore, $\frac{P(h_2)}{P(h_1)} \leq \frac{P(h_2\&a|e)}{P(h_1|e)}$ iff $P(e|h_1) \leq P(e|h_2\&a)P(a|h_2)$

Theorem of successful consilience (in relative terms)

$$(6)\ \frac{P(h_1)}{P(h_2)} < \frac{P(h_1|e_1\&\ldots\&e_n)}{P(h_2\&a_1\&\ldots\&a_m|e_1\&\ldots\&e_n)}$$

iff

$$P(e_1\&\ldots\&e_n|h_2\&a_1\&\ldots\&a_m)P(a_1|a_2\&\ldots\&a_m\&h_2)...P(a_m|h_2) < P(e_1\&\ldots\&e_n|h_1)$$

Proof By Bayes' theorem:

$$\frac{P(h_1)}{P(h_2)} < \frac{P(h_1|e_1\&\ldots\&e_n)}{P(h_2\&a_1\&\ldots\&a_m|e_1\&\ldots\&e_n)}$$

$$\text{iff}\quad \frac{P(h_1)}{P(h_2)} < \frac{\frac{P(e_1\&...\&e_n|h_1)P(h_1)}{P(e_1\&...\&e_n)}}{\frac{P(e_1\&...\&e_n|h_2\&a_1\&...\&a_m)P(h_2\&a_1\&...\&a_m)}{P(e_1\&...\&e_n)}}$$

By reasoning similar to the above proof:
iff $\frac{P(h_1)}{P(h_2)} < \frac{P(e_1\&...\&e_n|h_1)P(h_1)}{P(e_1\&...\&e_n|h_2\&a_1\&...\&a_m)P(h_2\&a_1\&...\&a_m)}$

Then, decomposing $P(h_2\&a_1\&\ldots\&a_m)$ into $P(a_1|a_2\&\ldots\&a_m\&h_2)...P(a_m|h_2)P(h_2)$:

iff $\frac{P(h_1)}{P(h_2)} < \frac{P(e_1\&...\&e_n|h_1)P(h_1)}{P(e_1\&...\&e_n|h_2\&a_1\&...\&a_m)P(a_1|a_2\&...\&a_m\&h_2)...P(a_m|h_2)P(h_2)}$

Which, by reasoning similar to the above, holds just in case:

$$P(e_1\&\ldots\&e_n|h_2\&a_1\&\ldots\&a_m)P(a_1|a_2\&\ldots\&a_m\&h_2)...P(a_m|h_2) < P(e_1\&\ldots\&e_n|h_1)$$

Therefore,

$$(6)\ \frac{P(h_1)}{P(h_2)} < \frac{P(h_1|e_1\&\ldots\&e_n)}{P(h_2\&a_1\&\ldots\&a_m|e_1\&\ldots\&e_n)}$$

iff

$$P(e_1\&\ldots\&e_n|h_2\&a_1\&\ldots\&a_m)P(a_1|a_2\&\ldots\&a_m\&h_2)\ldots P(a_m|h_2) < P(e_1\&\ldots\&e_n|h_1)$$

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