Bohmian Collapse

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Abstract

I present and explain the Bohmian account of collapse in quantum mechanics.

1 Introduction

This paper is an explication of an idea due to Detlef Dürr and collaborators. The idea is striking: Bohmian mechanics has the formal and physical resources to fully account for the phenomenon of quantum collapse. This account is perhaps not as well understood, in philosophical circles at least, as it deserves to be. So my goal, in what follows, is to present a simple yet rigorous version of this account; hopefully, thereby, increasing its audience of appreciators.

But before doing so, it is worth making a brief remark about the passing of Detlef Dürr. There is joy, and also pain, in reading through this volume. The joy comes from seeing how many lives Detlef touched, and the community which formed around him. The pain comes from the reminder of his loss.

There is some comfort to be had, however, in Detlef's own research. For Bohmian mechanics suggests that when someone dies, their particles disperse according to a determined, coordinated dance, one in which—by virtue of nonlocality—we all participate. Just as clouds, gently floating across the sky, are carried towards the distant horizon by their particulate motions, so we are carried along by our composite particles, making tracks towards a horizon where everything familiar vanishes; a vanishing point through which Detlef has already passed, and through which all else passes too, and lucky for us that on the way there, our trajectories briefly crossed his.

2 Basics

In this section, I present the basics of Bohmian mechanics.[1] Roughly put, according to the version of Bohmian mechanics on which I focus here, the universe consists of some particles and a universal wave function. The resources used to describe all this, it turns out, can also be used to describe subsystems of the universe: in particular, those resources can be used to define the wave functions that subsystems have.

To start, let N be the number of particles in the universe. For each i from 1 to N, let $\mathbf{q}_i$ be a variable which ranges over the candidate positions of the ith particle. Let $q = (\mathbf{q}_1, \ldots, \mathbf{q}_N)$ be a variable which ranges over the candidate configurations of all particles in the universe. For each time t, let $\Psi_t(q)$ be the universal wave function at that time. In addition, for each i, let $\mathbf{Q}_i(t)$ be the actual position of particle i at time t. Let $Q(t) = \big(\mathbf{Q}_1(t), \ldots, \mathbf{Q}_N(t)\big)$ denote the actual configuration, at time t, of the particles in the universe.

There is an important difference between the symbols 'q' and '$Q(t)$'. The former is a generic variable which ranges over all candidate configurations of the universe's particles. The latter is, for any given time t, a constant which denotes a single configuration of the particles in the universe: the configuration which the particles, at t, actually have. So in Bohmian mechanics, q acts as a generic symbol which can be used to specify the domain of all possible particle configurations to which the universal wave function Ψ_t assigns a complex

[1]For an early formulation of Bohmian mechanics, see (Bohm, 1952a; 1952b).

number. $Q(t)$, however, specifies a specific particle configuration: the actual one.

Bohmian mechanics posits two equations: one describes the evolution of the universal wave function $\Psi(t,q) = \Psi_t(q)$, while the other describes the evolution of particle configurations. The evolution of the universal wave function is given by the Schrödinger equation.

$$i\hbar\frac{\partial \Psi}{\partial t} = H\Psi \tag{1}$$

The evolution of particle configurations is given by the guidance equation.

$$\frac{d\mathbf{Q}_i}{dt} = \frac{\hbar}{m_i} Im \frac{\Psi^* \nabla_i \Psi}{\Psi^* \Psi}(\mathbf{Q}_1, \ldots, \mathbf{Q}_n) \qquad\qquad i = 1, \ldots, N \tag{2}$$

In (2), each m_i represents the mass of particle i. Together, (1) and (2) describe how the entire universe evolves.

In addition to describing the behavior of the universe as a whole, Bohmian mechanics also provides the resources needed to describe subsystems. A 'subsystem' is simply a collection of particles. The 'environment' of a subsystem consists of all particles in the universe which are not in that subsystem.

Subsystems and their environments can be represented by variables and constants, in the following way. Take any subsystem of M particles, where $M < N$. Index all the particles in the universe so that particle 1, particle 2, ... , and particle M, are all and only the particles in this subsystem. For each i from 1 to M, let $\mathbf{x}_i = \mathbf{q}_i$: so each $\mathbf{x}_i$ is a variable which ranges over the candidate positions of particle i in the subsystem. Let $x = (\mathbf{x}_1, \ldots, \mathbf{x}_M)$ be a variable which ranges over the candidate configurations of the subsystem's particles. Similarly, for each i from $M + 1$ to N, let $\mathbf{y}_i = \mathbf{q}_i$. So each $\mathbf{y}_i$ is a variable which ranges over the candidate positions of particle i in the environment. Let $y = (\mathbf{y}_{M+1}, \ldots, \mathbf{y}_N)$ be a variable which ranges over the candidate configurations of the environment's particles. In addition, for each time t and each i from 1 to M, let $\mathbf{X}_i(t) = \mathbf{Q}_i(t)$: so for each time t, each $\mathbf{X}_i(t)$ is a constant which denotes the actual position of particle i in the subsystem. For each time t, let $X(t) = \big(\mathbf{X}_1(t), \ldots, \mathbf{X}_M(t)\big)$ be a constant which denotes the actual configuration, at

t, of the subsystem as a whole. Finally, for each time t and each i from $M+1$ to N, let $\mathbf{Y}_i(t) = \mathbf{Q}_i(t)$: so for each time t, each $\mathbf{Y}_i(t)$ is a constant which denotes the actual position of particle i in the environment. And for each time t, let $Y(t) = \big(\mathbf{Y}_{M+1}(t), \dots, \mathbf{Y}_N(t)\big)$ be a constant which denotes the actual configuration, at t, of the environment as a whole.

Some more notation will be helpful. For any subsystem of M particles as described above, the variable q—which ranges over candidate configurations of the universe—may be rewritten as $q = (x, y)$. This equation conveniently represents the split between (i) the candidate configurations of the subsystem, and (ii) the candidate configurations of the environment. Similarly, for each time t, the constant $Q(t)$—which represents the actual configuration of the universe—may be rewritten as $Q(t) = \big(X(t), Y(t)\big)$. This equation conveniently represents the split between (i) the actual configuration of the subsystem, and (ii) the actual configuration of the environment.

These resources can be used to define a particular sort of wave function—called the 'conditional wave function'—for any given subsystem (Dürr et al., 1992, p. 864).[2] To see how, take the subsystem of M particles described above. Let x, Y, t, and Ψ_t be as defined earlier. Then for any given time t, the conditional wave function of this subsystem is the function $\psi_t(x)$ defined as follows.

$$\psi_t(x) = \Psi_t\big(x, Y(t)\big) \tag{3}$$

In other words, the wave function[3] of a given subsystem at a fixed time is obtained by (i) taking the actual positions of the particles in the subsystem's environment, and (ii) plugging those positions into the universal wave function.[4]

This feature of Bohmian mechanics—that it contains the resources required to formu-

[2]For a more accessible account of wave function collapse, see (Goldstein, 2010). For more discussion of different ways to interpret wave functions like these, and different ways to understand the physical significance of universal wave functions too, see (Goldstein & Zanghì, 2013).

[3]Note that (3) is not normalized. This does not matter, however. All wave functions related by a constant non-vanishing multiple may be regarded as physically equivalent.

[4]For more on conditional wave functions, see (Dürr et al., 2004, pp. 966-968).

late equation (3)—is striking. In more orthodox interpretations of the quantum mechanical formalism, certain functions are simply stipulated to be the wave functions of subsystems. Subsystems' wave functions are not defined in terms of anything else. Similarly, in fact, for other interpretations of quantum mechanics, such as the Everett interpretation. Bohmian mechanics, in contrast, can be used to define the wave functions of subsystems in terms of a few basic posits: the existence of a universal wave function, and the actual positions of the physically real particles which comprise the environment. So altogether, whereas Bohmian mechanics has the formal and physical resources to account for how certain wave functions are associated with certain subsystems, many other interpretations of quantum mechanics do not. And that is a significant point in favor of Bohmian mechanics.

3 Collapse

In this section, I discuss the Bohmian account of how the wave functions of subsystems—that is, conditional wave functions—collapse.[5] Then I briefly present the conditions under which conditional wave functions conform to a version of Schrödinger's equation. Finally, I explain why this version of Schrödinger's equation does not always describe how conditional wave functions evolve.

To start, here is the account of how conditional wave functions evolve in accord with the collapse postulate of quantum mechanics. Let t_1 be a time shortly before a measurement occurs. Suppose that at time t_1, the subsystem's conditional wave function $\psi_{t_1}(x)$ is in a superposition of the eigenstates $\psi_{t_1,\alpha_1}(x)$, $\psi_{t_1,\alpha_2}(x)$, $\ldots$, $\psi_{t_1,\alpha_n}(x)$ of the observable being measured. So for some constants c_{α_1}, c_{α_2}, $\ldots$, c_{α_n}, the following holds.

$$\psi_{t_1}(x) = \sum_{\alpha=\alpha_0}^{\alpha_n} c_\alpha \psi_{t_1,\alpha}(x) \tag{4}$$

[5] This discussion is based on the theory developed in (Dürr et al., 1992; Dürr & Teufel, 2009; Goldstein, 2010).

In addition, suppose that before measurement of the observable, the subsystem and the environment do not interact with one another. Moreover, let us assume that at time t_1, there is a function $\phi_{t_1}(y)$ such that the universal wave function is $\psi_{t_1}(x)\phi_{t_1}(y)$.[6] So (4) implies that at time t_1, the universal wave function is

$$\Psi_{t_1}(x,y) = \psi_{t_1}(x)\phi_{t_1}(y) = \sum_{\alpha=\alpha_0}^{\alpha_n} c_\alpha \psi_{t_1,\alpha}(x)\phi_{t_1}(y) \tag{5}$$

In other words, before measurement, the universal wave function is in a superposition of the wave functions $\psi_{t_1,\alpha}(x)\phi_{t_1}(y)$, where each $\psi_{t_1,\alpha}(x)\phi_{t_1}(y)$ represents a universal wave function in which the subsystem's state is $\psi_{t_1,\alpha}(x)$ and the environment's state is $\phi_{t_1}(y)$.

Now to describe some post-measurement wave functions which will be relevant in what follows. Let t_2 be a time right after the measurement occurs. Take any α from $\alpha_0, \ldots, \alpha_n$. Then from t_1 to t_2, the wave function $\psi_{t_1,\alpha}(x)\phi_{t_1}(y)$ evolves to a new wave function $\psi_{t_2,\alpha}(x)\phi_{t_2,\alpha}(y)$, where $\psi_{t_2,\alpha}(x)$ and $\phi_{t_2,\alpha}(y)$ have several important properties. First, $\psi_{t_1,\alpha}(x) = \psi_{t_2,\alpha}(x)$: this corresponds to the fact that measurements of a given observables' eigenstates do not alter those eigenstates. Second, $\phi_{t_2,\alpha}(y)$ is the wave function associated with the environment recording the fact that the subsystem is in state $\psi_{t_2,\alpha}(x)$: that is just part of what it is for the event in question to count as a measurement of the observable in question.[7] Third, for all α' such that $\alpha \neq \alpha'$, the support of $\phi_{t_2,\alpha}(y)$ is macroscopically disjoint from the support of $\phi_{t_2,\alpha'}(y)$:[8] basically, this too is just part of what it is for the

[6]This assumption is unrealistic: the universal wave function generally does not factorize into a product state of functions $\psi_{t_1}(x)$ and $\phi_{t_1}(y)$. But as it turns out, this assumption is not really necessary, even approximately. It is sufficient that the universal wave function satisfies $\Psi_t(x,y) = \psi_{t_1}(x)\phi_{t_1}(y) + \Psi_{t_1}^{\perp}(x,y)$, where $\phi_{t_1}(y)$ and $\Psi_{t_1}^{\perp}(x,y)$ have macroscopically disjoint y-supports (Dürr et al., pp. 861-864).

[7]For more discussion of why the state $\psi_{t_2,\alpha}$ must record the state of the subsystem—which is based on considerations of what it is to conduct a measurement—see (Albert, 1992, pp. 74-79).

[8]In other words, if $\phi_{t_2,\alpha}(y)$ is non-zero for some configuration y, then $\phi_{t_2,\alpha'}(y)$ is zero for all configurations y from which that former configuration is macroscopically indistinguishable. And if $\phi_{t_2,\alpha'}(y)$ is non-zero for some configuration y, then $\phi_{t_2,\alpha}(y)$ is zero for all configurations y from which that former configuration is macroscopically indistinguishable. Put in intuitive terms, this all amounts to the following: the configurations of the environment, which record the outcome of the measurement, are macroscopically distinct from one another. In other words, the measurement device never enters a state in which it is somehow recording two distinct experimental outcomes.

event in question to count as a measurement.[9] So each term $\psi_{t_1,\alpha}(x)\phi_{t_1}(y)$, in (5), evolves to a wave function $\psi_{t_2,\alpha}(x)\phi_{t_2,\alpha}(y)$ such that (i) $\psi_{t_1,\alpha}(x)$ is $\psi_{t_2,\alpha}(x)$, (ii) $\phi_{t_2,\alpha}(y)$ says that the system is in state $\psi_{t_2,\alpha}$, and (iii) the $\phi_{t_2,\alpha}(y)$ have macroscopically disjoint supports.

With all that as background, here is the Bohmian account of collapse. Since each $\psi_{t_1,\alpha}(x)\phi_{t_1}(y)$ evolves to $\psi_{t_2,\alpha}(x)\phi_{t_2,\alpha}(y)$, the linearity of the Schrödinger equation implies that the universal wave function in (5) evolves to the universal wave function below.

$$\Psi_{t_2}(x,y) = \sum_{\alpha=\alpha_0}^{\alpha_n} c_\alpha \psi_{t_2,\alpha}(x)\phi_{t_2,\alpha}(y) \tag{6}$$

By the definition of conditional wave functions from (3), the conditional wave function of the subsystem at time t_2 is obtained by substituting $Y(t_2)$—the actual configuration of the environment particles at t_2—for y in (6). The following results.

$$\begin{aligned}\psi_{t_2}(x) &= \Psi_{t_2}\big(x, Y(t_2)\big)\\ &= \sum_{\alpha=\alpha_0}^{\alpha_n} c_\alpha \psi_{t_2,\alpha}(x)\phi_{t_2,\alpha}\big(Y(t_2)\big)\end{aligned}$$

Since the functions $\phi_{t_2,\alpha}(y)$ all have macroscopically disjoint supports, at most one of the quantities $\phi_{t_2,\alpha}\big(Y(t_2)\big)$ is non-zero. Since each of the wave functions $\psi_{t_1,\alpha}(x)$ are eigenstates of the original conditional wave function $\psi_{t_1}(x)$, at least one of the quantities $\phi_{t_2,\alpha}\big(Y(t_2)\big)$ is non-zero. Therefore, for some α_j, the above sum reduces to $\psi_{t_2,\alpha_j}(x)\phi_{t_2,\alpha_j}\big(Y(t_2)\big)$ where $\phi_{t_2,\alpha_j}\big(Y(t_2)\big) \neq 0$. Dropping the unnecessary constant $\phi_{t_2,\alpha_j}\big(Y(t_2)\big)$[10]—and using the fact, mentioned earlier, that $\psi_{t_1,\alpha_j} = \psi_{t_2,\alpha_j}$—it follows that at time t_2, the conditional wave function of the subsystem is as follows.

$$\psi_{t_2}(x) = \psi_{t_1,\alpha_j}(x) \tag{7}$$

In other words, the conditional wave function of the subsystem after measurement is one of

[9]For more details, see (Dürr et al., 1992, pp. 863-866).

[10]Recall that as mentioned in footnote 3, wave functions related by a constant non-vanishing multiple are physically equivalent.

the eigenstates of the conditional wave function of the subsystem before measurement. The subsystem's conditional wave function has collapsed.

Basically, according to the Bohmian account, collapse results from two different features of subsystems, environments, and the universe as a whole. First, the wave function associated with any given subsystem is determined by (i) the wave function of the universe, and (ii) the actual positions of the environment particles. In other words, the wave function of any given subsystem is the conditional wave function given by (3). Second, after measurement, the universal wave function has the following property: when the post-measurement positions of the environment particles are plugged into the universal wave function, the resulting function is an eigenstate of the conditional wave function of the subsystem just before measurement. In slogan form: what it is to be a subsystem's wave function is to, among other things, exhibit collapse-like behavior.

Note that according to this account, collapse is real: the wave functions of subsystems really do, that is, undergo collapse. For when measurement occurs, a subsystem's wave function really does become an eigenstate of the wave function which the subsystem had before the measurement event. The subsystem starts out with one conditional wave function before measurement; after measurement, the subsystem's conditional wave function is an eigenstate of the conditional wave function from earlier. So collapse is a real, actual part of the physical world, according to the Bohmian account.

It is worth briefly explaining why conditional wave functions sometimes conform to a version of Schrödinger's equation. For conditional wave functions do not always collapse: they often exhibit Schrödinger evolution. Basically, that happens whenever the subsystem—corresponding to the conditional wave function in question—is suitably isolated from its environment.

For example, take the subsystem of M particles once more. Suppose that the universal wave function factorizes such that for all times t, there is a function Φ_t such that $\Psi_t(x, y) = \psi_t(x)\Phi_t(y)$; or at least, suppose that the universal wave function approximately obeys an equation of this form.[11] In addition, suppose that there is negligible interaction between the subsystem and the environment; so the universal Hamitonian H may be written as $H = H^x + H^y$.[12] Finally, let ψ be defined by $\psi(t, x) = \psi_t(x)$. Then it can be shown that the following holds.[13]

$$i\hbar \frac{\partial \psi}{\partial t} = H^x \psi \tag{8}$$

The conditional wave functions of subsystems which are approximately in product states, in other words, conform to a version of Schrödinger's equation.

Before concluding, it is worth discussing two reasons why the Bohmian account of collapse is preferable to the account of collapse that orthodox quantum mechanics endorses. First, that other account—call it the 'orthodox account'—simply stipulates that collapse occurs. The phenomenon of collapse, in other words, is a primitive posit of the orthodox account. The Bohmian account, however, does not merely posit a collapse principle. Instead, the Bohmian account shows how collapse derives from other, more basic posits: namely, posits about actual configurations and universal wave functions.

Second, and relatedly, the orthodox account does not offer a clear method for associating wave functions with subsystems in the first place.[14] To illustrate, consider the following question: for any given subsystem of the universe, at any given time t, what wave function should be associated with that subsystem? The answer to this question, that Bohmian mechanics supports, is clear: given that (i) the wave function of the universe at t is Ψ_t, (ii) the subsystem in question is defined as the collection of particles with actual positions $\mathbf{X}_1(t), \ldots, \mathbf{X}_M(t)$, so that the environment particles have actual positions $\mathbf{Y}_{M+1}(t), \ldots, \mathbf{Y}_N(t)$, and (iii) a con-

[11]For the reasons mentioned in footnote 6, this assumption is unrealistic, but not necessary. It is made here merely in order to simplify the discussion.

[12]H^x is the contribution to H arising from terms involving only degrees of freedom from particles in the subsystem, while H^y is the contribution to H arising from terms involving only degrees of freedom from particles in the environment.

[13]See (Dürr et al., 1992, pp. 861-862).

[14]Arguably, this is true for other accounts too, like accounts of collapse suggested by some versions of the Everett interpretation.

dition about macroscopically disjoint supports of the sort mentioned earlier, it follows that (iv) the wave function which should be associated with this subsystem is the conditional wave function $\psi_t(x) = \Psi_t\big(x, Y(t)\big) = \Psi_t\big(\mathbf{x}_1, \ldots, \mathbf{x}_M, \mathbf{Y}_{M+1}(t), \ldots, \mathbf{Y}_N(t)\big)$. Orthodox quantum mechanics does not support an analogously clear answer to this question. For orthodox quantum mechanics does not provide clear, precise principles which, for any given subsystem, define the wave function associated with that subsystem in terms of anything as well-defined as actual configurations and universal wave functions. And so whereas Bohmian mechanics can be used to provide a satisfying answer to this question, orthodox quantum mechanics cannot.

This is, in my view, one of the most attractive features of Bohmian mechanics. It provides precisely the resources needed to clearly define the wave functions which should be associated with subsystems: those resources consist of a few simple posits about particles and a universal wave function. And in so doing, it supports an account of how collapse occurs.

In short, by helping itself to physically real particles, Bohmian mechanics clarifies decades of confusion surrounding quantum collapse. The orthodox account exacerbates that confusion, since it resists positing an actual configuration for any given subsystem's environment: so given the orthodox account, there is nothing to plug into a universal wave function, to obtain the wave functions associated with subsystems—that is, according to the Bohmian account, the conditional wave functions—which undergo collapse. Bohmian mechanics does posit an actual configuration for each subsystem's environment, however. And as a result, Bohmian mechanics supports an illuminating account of how, and why, collapse occurs.

4 Conclusion

Bohmian mechanics can be used to provide an attractive, elegant, and simple account of collapse. The account says, basically, that collapse is a consequence of how conditional wave functions evolve over time. Their evolution generates the phenomenon of collapse because of how the environment particles, and the universal wave function, evolve.

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