Accuracy, Logic and Degree of Belief

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Why care about being logical? Why criticize people for inconsistency? Must we simply take the normative significance of logic as brute, or can we explain it in terms of goals on which we have an independent grip: the merits of true (or knowledgeable) belief, for example? This paper explores Jim Joyce's argument for probabilism in the light of these questions---arguing that it provides a plausible route for *explaining* the value of consistency.

1. Logical norms, probability and accuracy

Is seems good to be consistent; and bad to be inconsistent. If you believe B, and also believe ~B, then *something has gone wrong*. Something has gone wrong, too, if B obviously follows from what you already believe, and you don't believe that B when the question is raised. These sorts of principles apparently report a *normative* role for logic. Logic is a source of principles about how we ought or ought not to conduct ourselves.

Some prefer to talk in terms of partial beliefs (whether as a replacement for, the explanatory basis of, or a supplementation to, talk of all-or-nothing belief). Here too we find similar theses what belief states should look like. Most familiar is *probabilism*: the doctrine that our partial beliefs *should* be representable as (or extendable to) a probability.

Probabilism, like logical norms, recommends or condemns certain *patterns of attitudes*. But the connection is even tighter: arguably, probabilism *embeds* logical norms. Probabilities can be "locally" characterized in terms of logical relations among partial beliefs---thus, each of the following are necessary for P to be a probability.

- If B follows from A, then P(A)≤P(B).
- If A and B partition C, then P(C)=P(A)+P(B). [2]
- If A is a logical truth, then P(A)=1.

If one's partial beliefs violate any one of the above, then they are not representable via a probability. So probabilism might be redefined "locally" as the view that partial beliefs ought to meet the particular constraints just given. My concern in this paper is with logical norms on partial belief, as expressed by this form of probabilism.

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[2] We say that A and B "partition" C, if we have a logical guarantee that C is true iff exactly one of A/B is true. For example, in a classical setting, when A, B are inconsistent, A and B will partition AvB, and hence the above constraint gives us a familiar additivity principle: P(A)+P(B)=P(AvB). I like this formulation rather than the additivity proper because it doesn't bake in assumptions about how particular connectives behave. For discussion, see Williams "Probability and non-classical logic", forthcoming in Hajek and Hitchcock *Oxford Companion to the Philosophy of Probability*.

(Some caveats: it's one thing to say that logic norms partial belief. It's another to say that probabilism describes these norms. Other formal treatments of belief functions might equally make play with logical notions, and can be taken as rival characterisations of the logical norms in play. Within the generally probabilist camp, one might wish to hedge and qualify matters in various ways---saying that we ought to avoid *obvious* violations of the probabilistic constraints, for example, rather than violations *simpliciter*. In order to have a clean model to work with, I'll set aside these complications for now.[3])

For the sake of argument, suppose we agree that probabilism sounds initially attractive. Then we face the question raised at the beginning: are we prepared to take these principles about how our beliefs ought or ought not to be as explanatorily basic, or do we seek an illuminating explanation?

What's the motivation to take the latter view? Niko Kolodny believes that *bare* logical norms would seem like a mere fetish for a certain patterning of mental states:

> "Simply put, it seems outlandish that the kind of psychic tidiness that … any… requirement of formal coherence, enjoins should be set alongside such final ends as pleasure, friendship, and knowledge"[4]

How much more attractive if we could reveal a commitment to psychic tidiness as *implicit in* or *following from* a respect for the kind of basic values Kolodny lists.

Kolodny's discussion is set within a wider discussion of the normativity of *rationality,* whose subject-matter includes not only logic but also coherence between beliefs, desires and intentions. We may assume for the sake of argument that we have identified a number of constraints (such as those axioms of probability theory) that must be met if a subject is to count as "perfectly rational". Call these *rationality requirements*. Crucially, we treat it as an open question whether someone *should* be rational---whether rational requirements are normative requirements. Perhaps "rational" seems too normatively loaded to allow one to hear the latter question as substantive. If so, let's simply take our candidate cluster of principles, and stipulate that they are to be treated as conditions of being "R"---with the substantive question being whether we *ought* to be R. If the answer is negative---if it is *not* the case we ought, even pro tanto, be rational---then we end up with a view whereby conforming to rational requirements is like membership of a kind of club. To be in that club, you have to do certain things or be a certain way (roll up your trousers in appropriate circumstances); but there's no obligation to join.[5]

The negative answer in the case of logic is perfectly possible. Harman and Maudlin think of logic as something like "the science of guaranteed truth-preservation"---consisting of facts about what follows from what, which are unrelated to questions

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[3] For alternative models of partial belief, see Halpern *Reasoning under uncertainty* MIT Press, 1995. For the formulation of the norms, see Field, "What is the normative role of logic?" *Proceedings of the Aristotelian Society* 2009, vol. 83(3), 251-268.

[4] Kolodny, "How does coherence matter"? *Proceedings of the Aristotelian Society*, 2007, p,241.

[5] See Kolodny, "Why be rational" *Mind* 2005; Broome "Does rationality give us reasons?" *Philosophical Issues* (2005).

about what patterns of attitudes one *ought* or *ought not* to have.[6] Clearly such answers avoid the challenge to explain the basis of logical norms, by denying their existence (though, to be fair, Maudlin at least allows a surrogate notion which *does* seem to have a normative role---and we can fairly raise the question about the source of those norms).

Positive answers to the question "Why be logical?" come in various grades. Here are three. First, the grade one position: one says of each individual constraint, that there's a norm against violating it. This is a version that I read Hartry Field as subscribing to in the paper cited---though his main focus is on exactly what constraints it is that have this status. A grade two norm is to say *generically* that we ought not to violate logical constraints. Compare: generically you ought not to jump off cliffs---even though on a particular occasion (when escaping from a bear) jumping off a cliff may be the thing you ought to do. This is John Broome's favoured view. Finally, there is the lowest grade, debunking position, which claims that there are no logical norms *as such.* Rather, any violation of the constraints laid down will *guarantee* that some *other* norm is violated. Kolodny argues, for example, that believing an inconsistency guarantees that your beliefs are out of line with the evidence---hence failing to be logical is an invariable sign of failing to meet evidential norms on belief. The final position (by design) does not recognize a normative difference between the case of the perfectly rational agent who (perhaps because of odd priors) is out of line with the objective evidence, and one who has grossly inconsistent beliefs. That's why it debunks, rather than vindicates, logical norms.

As well as asking questions about the grades of normativity attaching to the logical norms, we can equally ask about the type of normativity in question---and in particular, the extent to which an agent's subjective take on what the relevant facts or values are, should factor into an evaluation. A hardliner might say it is *the facts* about whether attitudes or actions fit with what are *in fact* the proper values, that are relevant to normativity. If pleasure, friendship and knowledge are the aim, then actions not conducive to those ends are bad, independently of whether an agent has the mistaken view of the consequences of their actions (thinking that showing off will win them friends) or a mistaken take on what the proper ends are (thinking that power rather than pleasure is the goal, perhaps). Our subjective take needs to be accounted for somewhere, but we can argue about how to frame it. One view---advocated by Williamson in his account of the aim of belief[7]---is that the "fundamental" norm that one should believe p only if that would count as knowledge, gives rise to a "derivative" norm that one should believe p only if one believes it would count as knowledge. And I take it that part of the idea here is that we can explain our concern for derivative norms in terms of underlying concern for the fundamental norm (so at least in these cases they don't appear as a fetish for mental tidyness).

Kolodny recommends we describe this sort of case differently. Rather than admit there's anything "normative" to the rule that you should believe p only if you believe that this would count as knowledge, Kolodny highlights the fact that violating this rule means that *from the subjects' point of view* they will have violated the fundamental norm. One does not have to posit a "respect" for the derivative norm *as*

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[6] Harman, *Change in View*, Bradford, MIT Press, 1988. Maudlin, *Truth and Paradox* OUP, 2004.
[7] Williamson *Knowledge and its limits* OUP 2001, ch 11 for a discussion of fundamental and derivative norms of assertion; and p48 and passim for the claim that belief aims at knowledge.

such, to explain why violations of this rule are relevant---by calling the subjects' attention to violations, their respect for the fundamental norm kicks in.

There are really two issues here. First is whether we want to call the "rules" that arise from subjectivizing a fundamental norm themselves "normative". The second is exactly how to formulate the derivative rule. Notice, for example, that Williamson's formulation of a "derivative" norm talks about what subject believes they are in a position to know. So the rule is sensitive to subjects' limited information about what they know. But the rule is not sensitive to subjects' limited information about what the aim of belief really is. A subject who confidently believed (perhaps on good evidence) that the aim of belief was to cohere with the opinions of one's peers, would violate the derivative norm *even if they exactly satisfied, and knew they satisfied* what they take the fundamental norm to be. Kolodny's formulation is more thoroughgoingly subjective---it will seem to the subject just mentioned that they meet their obligations.
The contrast can be illustrated in the case of consequentialist evaluations of action. Take the fundamental norm on action to be that one should do what maximizes good consequences. A partial subjectivization is given by the rule to do what maximizes the expected goodness of the consequences. A full subjectivization is given by the rule to do what maximizes the expected *believed* goodness of the consequences.

Part of Kolodny's case that logic has only the third, debunking grade of normativity is that he doesn't think that the rules arising from subjectivizing fundamental norms *in general* have normative punch. His overall project is to display "rational requirements" as arising from factoring subjectivity into the fundamental norms---- roughly, even though there's strictly speaking no *reason* to be rational, it will always *appear to the subject* that there is such a reason.

The project to be pursued here cuts across this debate. What I intend to do is to look at prospects for explaining the apparent normativity of logic in terms of subjectivizations of fundamental norms on belief. The project is substantive and of interest whether or not we think that success would count as vindicating (derivative) normativity of logical constraints, or merely explaining away the appearances of normativity away, Kolodny-style. It is within this context I wish to examine Joyce's work on accuracy and probabilism.[8]

<u>2. Accuracy norms</u>

Joyce's starting point is a norm for belief that is not itself a norm of coherence – rather, it is a norm of *truth.* For all-or-nothing beliefs, the norm might simply have been that one ought to believe the true and disbelieve the false; Joyce generalizes this to partial belief, holding that a belief is *better* the *more accurate* it is – where the accuracy of a belief is a measure of how close it is to the truth value. An immediate issue is what this talk of "closeness to the truth value" amounts too; and this has been the focus of much of discussion. I want to concentrate on the downstream issues, so to fix ideas I will simply stipulate a particular accuracy measure: the *inaccuracy* of a degree of belief b(p) in the proposition p is given by $|t(p)-b(p)|^2$---and the inaccuracy of an overall belief state is simply the average inaccuracy (this is the "Brier score").

[8] Koldony himself discusses Joyce---albeit very breifly---in his 2007, p.256.

How much is built into the postulation of a norm of accuracy or truth? Must we, for example, buy into an ambitious teleological account of epistemic normativity, on which every epistemic norm on belief must ultimately be defended in terms of its producing the "final value" of truth? It's clear, I hope, that nothing so ambitious can be read off the mere postulation of a truth or accuracy norm on belief (even at a "rock bottom" level). For one thing, the existence of such a norm doesn't exclude the existence of other dimensions along which to assess belief; for another, we haven't made any commitment here to the general project of reducing justification (say) to the norm of truth. The assumption that we've made is comparable, in the ethical case, to claiming that the goodness of a state of affairs is measured by how much welfare it produces; that alone doesn't suffice to commit one to a teleological account of the rightness of action in terms of the production of welfare. Even if we did think that all epistemic norms are to be understood in relation to a norm of truth (compare Wedgwood on the aim of belief[9]) we're not thereby committed to explicating that relationship in aggregative or maximizing/satisficing terms.

However, there is something explicitly aggregational about the Brier score as a norm of accuracy (and also with all ways of measuring inaccuracy that meet Joyce's 1998 axioms). After all, the inaccuracy of a total belief state is a straight average of the inaccuracy of individual beliefs. Whether this form of aggregation is problematic should be measured by its fruits---but notice how limited a thesis it is. An aggregational theory of a *single person's* welfare *at a time*, doesn't commit one to aggregational theory of a single person's welfare *across time,* nor of aggregational theories of overall welfare across persons. Likewise, we shouldn't assume that an aggregational theory of the goodness of a belief state at a time commits one to aggregating the value of truth over time or over whole communities.[10]

With these observations in mind, let's stick with the Brier score as a measure of overall epistemic goodness of belief states. Joyce then proves a theorem:

- **Accuracy domination.** If a set of partial beliefs B is not a probability, then there's a probability, C, such that C is more accurate (so, better) than B no matter which world is actual.

We say in such a case that C "accuracy dominates B". One can also show:

- **Probabilistic anti-domination.** No probability function C is accuracy dominated by any belief state.

(It's easy to show that the every probability function "minimizes expected inaccuracy" by its own lights (the Brier score is "proper", in the jargon). It's also easy to see that when x accuracy-dominates y, the expected inaccuracy of x is strictly less than the expected inaccuracy of y. Anti-domination follows.).

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[9] Wedgwood, "The aim of belief", *Philosophy and Phenomenological Research*, 2002.
[10] Thanks to Selim Barker for discussion here. For discussion, see Barker's "Epistemic teleology and the separateness of propositions", *Philosophical Review* 122 (2013): 337–393.

Joyce uses accuracy-domination to argue for probabilism: and it's certainly suggestive---violating probabilistic norms means that in a certain sense you are being *needlessly inaccurate*. Joyce is content to take as a further premise that this accuracy-domination "gives us a strong, purely epistemic reason to prefer the [dominating credence] over the [dominated credence]".[11] Perhaps---but it'd be nice to have the case for this spelled out. What sort of epistemic reason is it, and how do such reasons arise out of the underlying concern for accuracy? Our question here is how the (alleged) vindication of probabilism ensues in detail.

[*Aside*: part of my interest in investigating Joyce's argument is that it generalizes very nicely. Joyce proves accuracy domination for *classical* probabilities assuming *classical* truth value assignments. But his argument generalizes to show accuracy domination for a specific sorts of *generalized* probabilities assuming specific sorts of *non-classical* truth value assignments. Moreover, one can use non-classical logic to spell out "localizations" of the generalized probabilities, just we familiarly use classical logic to localize classical probability.[12] So a route from accuracy domination to the corresponding probabilism promises a *very general* bundling together of truth values, logic and norms on belief.]

3. Accuracy and logical norms as such.

Can we turn accuracy domination into a defence of grade-one probabilistic norms? And if not, how far can we get along this route? In this section, I argue in the light of Joyce's result that:

(i) Probabilism is true for agents who are *a priori omniscient* (APO agents), so long as the strategy for "deriving" subsidiary norms from fundamental norms given earlier is ok.

(ii) There's no such direct case for probabilism for ordinary agents (you and me). But there's a specific sense in which according to the logical constraints is a *virtue* for such an agent---which can form the basis for a defence of a Broome-style norm of *generic* logicality.

Recall that Joyce's underlying norm was one of accuracy: of getting one's degrees of belief as close as possible to the truth values. There's thus an immediate, debunking, but rather uninteresting sense in which we ought to be probabilists. For the *best* belief state to adopt, by the Joyce norm, is one that exactly matches the truth-value distribution. And such credence functions are (limiting cases) of probabilities---*extremal* cases where every probability is either 1 or 0. The most fundamental point here is that this rationale is so demanding as to be uninteresting. For *any* responsible (non-godlike) believer presumably will have intermediate credences in some things. And thus we *all* violate the norm of "matching the truth values". So there's no basis here for discriminating criticism---we're all condemned.

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[11] Joyce Joyce "Accuracy and coherence" in *Degrees of Belief* Huber and Schmidt-Petri, Kluwer, 2009, p.285

[12] See Williams "Probability and non-classical logic" (op cit) for an overview; "Generalized probabilism" *Journal of Philosophical Logic* 2012 for a discussion of the relevant generalized Brier-score domination results (originally due to de Finetti); and "Gradational accuracy" *Review of Symbolic Logic* 2012 for discussion of Joyce's own argument in the generalized setting.

Rather than appeal to the accuracy norm itself, we'll appeal to the rule that says that when you know that C is more accurate than B, you ought to have belief state C rather than B. This is a derivative norm in the Williamsonian sense given earlier---and while Kolodny may deny it is a *normative,* if we can explain the (apparent) normativity of probabilistic constraints in terms of the (apparent) normativity of this rule, that can feed into Kolodny's eliminative project.

It's pretty clear that APO agents will violate the derivative norm, if their belief state B is not a probability, by the following argument:

Core Argument:

1. B is not a probability. (Premise)
2. B is accuracy dominated. (by accuracy domination theorem and 1).
3. An APO agent will be able to calculate a specific x (call it C) that she can tell is more accurate than B. (from 2).
4. If one knows of C that it is more accurate than B, then one ought not to have belief state B. (Premise).
5. An a priori-omniscient agent ought not to have belief state B.

In sum: the APO agent will always know of a better belief state, than any candidate improbabilistic state she might consider adopting.[13]

For APO agents, violations of probabilistic constraints guarantee violation of the derivative accuracy norm. But furthermore, we can argue that the derivative norm is violated only when the probabilistic norms are. Probabilistic anti-domination doesn't give us this---for that tells us only that there's no state which is *necessarily* more accurate than one that meets probabilistic constriants, and for all that, the agent with belief state B could know of another that it is more accurate than C. However, because the accuracy-measure is "proper", we can show that the *expected inaccuracy* of B is less than that of any other belief state---by the lights of B. So if the agent was to know---therefore believe---of C that it is more accurate than B, this belief must be combined with C having *no higher* expected accuracy than B. The relation between flat-out belief and partial belief is vexed, but I think we're entitled to assume that this situation won't arise. So---for APO agents---we have a grade-one vindication of probabilistic norms so long as we suppose there is a *derivative* accuracy norm.

[13] Two notes on this. Though we can argue for the contrastive claim that one ought have C rather than B in these circumstances, and it follows that one ought not have belief state B, it doesn't follow that one ought to adopt belief state C. After all, there might be an even better belief state C*. Hajek notes, for example, that midway between the probability C and the probability B there will be many improbabilities that accuracy-dominate B and are accuracy dominated by C. Plausibly, we ought to adopt C* rather than B, but in fact we ought to have neither of these. Hajek notes that if probabilities were accuracy-dominated in turn (perhaps with an unending-chain of accuracy domination) we might never reach a belief state that it's permissible to adopt. Fortunately, for the Brier score (and proper scoring rules in general) we can show that no probability will be accuracy-dominated.
Even if our "target" probability accuracy-dominates and is not itself accuracy dominated, it still doesn't follow that we ought to adopt it. After all, for all we've said, many probabilities may accuracy-dominate B---with the APO agent aware of each. It seems bad to stick with B, but ceteris paribus permissible to shift to any one of the dominating probabilities. So the most we can hope for, I suspect, is the permissibility of shifting to C, alongside the impermissibility of sticking with B. (Thanks to Al Hajek, Samir Okasha and Richard Pettigrew for discussion of these issues).

The interest of such results is limited, given the restriction to APO agents. What we'd really like to know is whether logical norms constrain *us*, not some hypothetical idealization. The trouble is to use the above argument for ordinary agents, we'd need to weaken (3)---perhaps to say that it is "in principle possible" to calculate an accuracy-dominating credence; and correspondingly weaken the antecedent of (4). But then (4) will no longer be supported by our derivative norm, nor by the spirit of "subjectivizing" what we fundamentally care about. Suppose that the oracle tells me that some miners are in a specific mine shaft iff the answer to some horrendous mathematical puzzle is 42. I have to choose almost instantly whether to take action that will save all of them if they're in that shaft, or kill them all if they're not---or I can opt out of the decision, letting a small fraction die but saving the majority. If they are in fact in the shaft, I should "objectively" take the action, but "subjectively" I ought to opt out since it'd be grossly irresponsible for me to take the risk of them all dying. It's the actual information state, not the "in principle available" one that is relevant to the subjective norms.

4. Persistent partial beliefs

In this section, I'm going to add an extra element to the setup. Ultimately, this allows us to say something about the way accuracy arguments make logic constrain ordinary, non-ideal agents. But along the way we get other interesting results.

The overarching idea is that in adopting doxastic attitudes to a proposition, we incur commitment to *persist* in those attitudes if no new evidence is forthcoming (where persistence is understood as *not changing one's mind*---i.e. not adopting a different attitude to the same proposition. I discuss cases of simply ignoring the proposition below). In the limiting case, consider a situation where one simply moves from one moment to the next, with no new input or reflection. It would be bizarre to change ones (non-indexical) beliefs in such circumstances. Insofar as action, over time, is based on one's beliefs, it would mean that a course of action started at one time might be abandoned (since it no longer maximizes expected utility) without any prompting from reflection or experience.

Persistence might be construed as a (widescope) diachronic *norm* on belief. Alternatively, a disposition to retain an attitude to the proposition over time might be *constitutive* of belief. If what makes something count as a belief is its functional role, then the reflections on extended action above motivate this kind of claim.

In what range of circumstances must one's beliefs be persistent? It's clear that when new empirical evidence comes in, persistence is off the table---when the evidence changes, your mind changes. Less clear is a case where one learns something new, by "pure reflection". On the one hand, it is clear that when we figure out how to prove p, it is perfectly proper to shift from agnosticism over whether p, to endorsing p. That might seem to be a violation of persistence. I'll argue in a little while that this appearance is misleading, but for now let's set this aside, and consider an intermediate case.

Between the clear case where there's no change at all, and the questionable cases where the only relevant changes are as a result of pure reflection, there are other cases in which (I contend) we are committed to persist in attitudes. In particular, there are cases where the only changes are shifts in attention: where one simply stops taking an attitude to a proposition, because it is no longer relevant to one's purposes (one may of course still be disposed to adopt that same attitude, should the question rearise). Suppose an agent believes that she'll win the lottery to degree 0.1; that she'll have sweet potato for lunch to degree 0.9; and that she'll be given a job offer to degree 0.5. Now, without new evidence coming in, if she starts ignoring the proposition about the lottery, it'd be completely bizarre if she thereupon lowered her credence in sweet potato for lunch, or raised her credence in the obtaining of a job offer. As before, it's questionable whether an agent who is disposed to shift their attitudes to p in these circumstances really had a degree of *belief* in the first place.

I'll assume persistence in this still very restricted sense. Then we get the following principle: if an agent has belief state B, over propositions P, then if P* is a subset of P, they are committed to adopting the belief state B*, where B* is simply the restriction of B to P*, in the event that their attention shifts so that P* is now the only propositions they have attitudes towards. The sense of "commitment" I'll need is at least this: if it's bad to have belief state B* over P*, it's also bad to be committed to belief state B* over P*. This turns out to be a powerful assumption in this context. Let's start to put it to work.

To begin with, there's an assumption in Joyce's formal arguments that may seem worrying. His argument for accuracy-domination works *on the assumption that the belief states in question are defined over a finite algebra of propositions*.[14] But one might think this is a serious restriction. On many conceptions of belief, I presently believe, not only that the number of my hands is two, but also that for each integer n other than two, the number of my hands is not n. The trouble is that we have no argument for accuracy domination for infinite belief states of this kind.

In comes persistence. If in having a specific partial belief in each of infinitely many propositions, I am committed to the finite subsets of those beliefs, then we can meaningfully appeal to accuracy domination. Suppose that some finite subset of my infinite beliefs violate one of the probabilistic constraints. One cannot apply Joyce's theorem to my actual belief state as a whole. But we can consider the restricted belief state to which I am committed, concerning only the finite subset in question. And this belief state is accuracy-dominated, hence bad. By the assumption that it's bad to be committed to a bad belief-state, any locally improbabilistic infinite belief state is a bad thing to adopt.

A second application is the following preface-paradox-like situation. Modest agents should perhaps concede that they violate probabilistic constraints. So suppose we now detect some specific local inconsistency, and see how to tweak our beliefs to regain local consistency. Does the above give us motivation for doing so? One would like to argue that the original beliefs have the vice of being accuracy-dominated; and we can remove this flaw by the tweak. But if we're pretty confident that the local inconsistency in question isn't the *only* such inconsistency in our overall belief state,

[14] For cousins of Joyce's arguments in the infinite setting---though not of the dominance form I have been discussing---see Easwaren "Expected Accuracy supports Conditionalization---and Conglomerability and Reflection" *Philosophy of Science* 2013: 119-142.

we'll believe the resulting belief state is accuracy-dominated, even after tweaking. So what exactly is gained?

Again, persistence can help. If we see that our attitudes to P, Q, R violate probabilistic constraints, then tweaking them so they no longer do so at least does this: it means we can be confident that the *restricted* belief state involving only P, Q, R is not accuracy-domianted, whereas the original belief state we knew was. So even if we haven't improved matters for the belief state as a whole, we've at least removed one identifiable respect in which our commitments are open to criticism.

A final application of persistence involves the more *general* project of defending accuracy scores. We've been assuming that the Brier score measures inaccuracy. But Joyce (2009) proves a more general theorem, which one might wish to have as a backstop if one is worried about the specific Brier-score proposal. Joyce shows one can prove accuracy-domination for any accuracy score which at least makes belief states meeting probabilistic norms *admissible* (i.e. which gives us probabilistic anti-domination, as described earlier) and which satisfies a certain "truth-directedness" principle (in that, if two belief states agree on every propositions but P, and B is linearly closer to the truth value on P than is C, then B is overall more accurate than B). And those conditions are really quite weak constraints to impose on an accuracy score. However, there's a crucial qualification: Joyce proves this theorem only for belief states defined over propositions that form a *partition* (are pairwise inconsistent and mutually exhaustive---he shows that accuracy-domination follows if the sum of one's credences across the partition is other than 1). But there's an obvious worry: what is the relevance of the result to belief-states like ours, where we take attitudes not only to *grass being green* and *grass not being green*, but also to conjunctions and disjunctions thereof?

It turns out that any improbabilistic belief state will be such that, on some partition, credences in propositions in that partition do not sum to 1. We can focus on the restricted belief state where we adopt attitudes only to propositions in that partition. Given persistence, we are committed to a belief state over that partition alone, which sums to other than 1. Joyce's (2009) theorem can then kick in---*this restricted state* is accuracy-dominated. And since it is bad in this sense, the overall belief state believing which committed you to it is derivatively bad.

Persistence in the extreme limiting case is extremely plausible. The slight extension to cases of ignoring seems intuitively well-motivated; and adding it to our account increases the explanatory power of the setup considerably. This itself, I think, gives us reason to think it is on the right track.

But what of the more contentious case mentioned earlier: persistence under pure reflection? As noted above, this seems a different case at first glance. Whatever one might say about ideal agents, for real agents like you and me, pure reflection can produce what seems like new information. If we're faced with sealed boxes, and told that there is a bar of gold inside the box that is labelled with a root of polynomial P; then the pure reflection required to solve the polynomial will change me from a state where (intuitively) I should divide credences evenly as to which box contains the gold; to a state where I'm relatively certain which one contains it. There seems little wrong with that movement in thought; it certainly doesn't seem constitutive that we avoid it.

There is another way to look at it, however. To be sure, the change is a positive one, given the information made available by reflection; in the situation in question, once we've reflected, we should change our belief state to incorporate the now-manifest information. But that isn't inconsistent with the persistence principle applied to this case. For that principle should be understood in a wide-scope way: one shouldn't both have credence d in p at t; and have credence other than d in p at t*, given that only pure reflection occurs in the meantime. It is quite consistent with the truth of this normative claim that one *should* have credence other than d in p at t*: one obvious way to make this the case arises when one was wrong to have credence d in p at the earlier time. So, consistently with the pure reflection persistence principle, we can see the change in view from 50-50 credence in the proposition that n is a root of the polynomial, to credence 1 in that proposition, as something to be recommended but only necessary because of a flawed initial state.

Underlying this is the recognition that, in cases of pure reflection like the solving of a polynomial, the information in question was *at the earlier time* within one's epistemic reach. One has to *wait* on empirical information---until the sense data impact, the empirical information is in a strong sense *unavailable*. But information reached by pure reflection was *already* available, at least in principle. (Notice that the case of the polynomial is special in that there's an algorithm one can use to get the answer in a short period of time. Cases where there's no decision procedure like this might well be treated differently).

I think the *obvious* criticism of a norm of persistence under pure reflection is misguided, therefore. But is there anything to the idea? The cases where one culpably violates this norm will be those where one adopts a set of attitudes, being fully aware that pure reflection would lead one to change them---that is, one knowingly adopts unstable beliefs (an example is the gold bar/polynomial case above). Such a course is often excusable: there are competing demands on our energy and resources, and practical agents can't spend all day in a priori reflection. But there's a tradition in philosophy of considering impractical agents---Descartes' pure enquirer, for example. As Bernard Williams describes this thinker, she is one whose sole project is to gain beliefs; whose sole goal is truth; and who has created a space (practically) free of distractions or limitations of time and resources.[15] The pure enquirer commits herself to not appealing to the sort of excuses just mentioned. Such an agent, I think, should have stable belief states under pure reflection, and their beliefs should persist under such reflection.

The pure enquirer is not a priori omniscient---she may very well (as Descartes recommends) be initially agnostic about a whole range of questions, including those settleable by pure reflection. Nevertheless, once she adopts an attitude, she is committed to persisting with that state through any pure reflection (if she gives it up at any point, that will simply reflect badly on her initial adoption of the attitude). Despite not being a priori omniscient, we can use the accuracy-domination results to argue that she should be a probabilist. Suppose she had improbabilistic belief state B.

[15] Williams, B *Descartes: the project of pure enquiry* Penguin 1978.

Pure reflection alone allows the agent to know, of C, that it accuracy-dominates B. And the result is improbabilistic belief state B is unstable under pure reflection. Pure enquirers, therefore, shouldn't adopt B.

We are not pure enquirers. But the relation between us and pure enquirers is interestingly different between us and a priori omniscient agents. A priori omniscient agents differ in capacity from us---so we can't sensibly hold ourselves to their standards. On the other hand, pure enquirers have the same capacities as us---we could in principle become the pure enquirer---and so insofar as we share their goals, we can sensibly hold them up as a model of what our beliefs should look like, if only untainted by other interests. Dispositions possessed by the pure enquirer will be *epistemic* virtues in us. And since improbabilistic credences are bad for the pure enquirer, the proper conclusion is that probabilism is an epistemic virtue.

If this is all accepted, then we can use it and accuracy-domination to argue for a norm binding on actual agents: one should meet logical constraints. For any belief state that violates such constraints will be accuracy-dominated, and so pure reflection would put us in the position of the APO agent discussed earlier. The improbabilistic APO agent directly violates a derivative accuracy norm, and so believes badly. The improbabilistic pure enquirer is committed to violating this norm, and so believes badly. The improbabilistic ordinary-joe fails to implement the rules that govern the pure enquirer, and to that extent, betrays an *epistemic vice*.

5. Probabilistic evidence

Kolodny (2007, p.256) discusses Joyce's results---though his main focus is on logical norms for all-or-nothing, rather than partial belief. The dialectic is somewhat involved by that stage of the paper, but he seems to at least be open to the idea they can play a role in arguing that "the set of degrees of belief that epistemic reason requires are probabilistic". The focus here is not fundamental norms of accuracy, but norms of evidence---of proportioning one's credence properly to the evidence.

This section will explore how Joyce-style reasoning can assist in arguing for something like probabilism in a revised setting where matching evidence, rather than truth-value, is taken as one's fundamental aim. So let us assume in a given context each proposition has a specific *degree of evidential support*.[16] In the partial belief case, we'll assume an analogous form of evidentialism: that evidential support norms partial belief, in that one's degree of belief in p *should match* the corresponding degree of evidential support for p.

If we could assume that degrees of evidential support are in fact probabilistically structured, then we could immediately conclude that degrees of belief should be probabilistically structured. This would fit the Kolodny-esque debunking pattern. For the entire normative weight of this claim would hang on *matching one's evidence.* Someone who met all logical constraints, but had probabilities that are out of whack

[16] The context will be relativized to the agent----partly because different agents will bring different background evidence to bear; and partly because there may be non-evidential facts about an agent (the "stance" they adopt) that factor into the ideal belief to have given the evidence available (which is how I'm thinking about "evidential support"). One's initial priors, attitudes to epistemic risk, and the like, are candidates for non-epistemic determinants of support.

with the evidence, would not have anything more going for them on than someone with improbabilistic beliefs. If there's no more to be said, then there would be no logical norms as such---though nevertheless anybody whose beliefs violates logical constraints would ipso fact not be believing as they should.

This is very similar to a picture we considered and dismissed as uninteresting in our initial discussion of Joyce's accuracy norm---based on the observation that truth value distributions were probabilities, and that matching truth values minimizes inaccuracy. But in that context, we could say the same about violations of *extremality*, and since all reasonable agents will violate the latter, all reasonable agents will equally violate the norm---depriving it of any discriminating purpose.

But the evidential norm is much better placed. For a start, it's not a non-starter to think that some of us do properly align our credences to the evidence. Given this, one cannot straight away show it to be undiscriminating. At the very least, we don't have the "bad company" of the putative constraint "be extremal"---for there's no reason to think that violations of extremality violate the evidence norm. (Presumably, whether or not it's a practical option for agents to follow the evidence norm depends on how "externalistically" we construe evidence. A Williamson-like knowledge-based conception of evidential probability might make most if not all ordinary agents fail to meet the norm; on more internalistic conceptions perhaps it is easier to respect).

However, this whole case hangs on the assumption that *degrees of evidential* support are themselves structured probabilistically (as does the central argument of Kolodny 2007 p.234; cf. footnote 14). But that looks like it assumes the point to explained, in our context. Certainly one can imagine cases where there's *prima facie* strong evidential support for P, and strong evidential support for ~P. Why couldn't it turn out that all-things-considered evidential support ranks assigns high degrees to each of P and ~P---so the sums of their degrees is greater than 1? Such evidential support couldn't be represented probabilistically. The task is to explain why such cases don't arise.

This is where Joyce's accuracy domination theorem can kick in again. Here's an assumption that simply seems *right* to me:

> **Plausible Premise:** if an assignment of degrees Q is demonstrably *closer to the truth* than R (in any world), then Q beats R as a candidate to systematize "evidential support".

But Joyce's results show (relative to his way of accuracy or "closeness to the truth") that *only* probabilities are immune from such "trumping" (and, on a proper scoring function like the Brier score, all probabilities are so immune). Given the Plausible Premise, we can conclude that evidential support will therefore be probabilistically structured. And given an evidential norm on belief, we can get the Kolodny-esque debunking explanation going.[17]

[17] I give an alternative route to the conclusion that degrees of evidence are structured probabilistically in "A non-pragmatic dominance argument for conditionalization" (ms.). However, the argument there *assumes* that the Joyce-style dominance situation is sufficient for a belief state being rationally flawed.

Moreover, the availability of this debunking explanation needn't *exclude* the points made earlier. For just as we may generalize a truth norm to a gradational accuracy norm, we may generalize our evidential norm into a graded form. We would say that B is better than C to the extent that B is *closer to the degrees of evidence* than C. The Brier score mentioned earlier is straightforwardly generalizable---closeness being a matter of minimizing the average square difference between the degree of evidence and credence (with the minimum achieved when credence and evidential support match). An evidential analogue to the accuracy domination theorem can be proved.[18] And then our earlier discussion can be rerun. Overall, then, in this setting we may get the following three justifications of forms of probabilism:

(i) We ought to have credences that meet logical constraints on partial belief, in virtue of the fact that we should match our degrees of belief to the evidence.

(ii) Ideal agents are subject to logical norms as such (irrespective of whether they manage to match their credences to the evidence).

(iii) Ordinary agents' credences are epistemically virtuous if they meet the requirements; and epistemically vicious if they don't. Ordinary agents should be disposed to meet logical norms (irrespective of whether they manage to match their credences to the evidence).

5. The accuracy score.

I've now finished discussing what we can extract from Joyce's accuracy domination result *if we spot him an appropriate accuracy score* (we have worked with the Brier score). But many have thought the discussion of the accuracy score itself the weak point of Joyce's whole discussion. I'm much more sanguine than others appear to be; and I close this paper by explaining why.

One thing that we need to be clear on is the dialectical role we want the accuracy-domination considerations to play. One conception of their role is as a bludgeon to use against theorists who endorse some rival to probabilistic norms---so conceived, Joyce would need to offer *suasive* considerations for each element of his setup, and would have to watch out for "begging the question" against his rivals. In particular, we certainly couldn't appeal to the probabilistic formulation of logical norms in support of a choice of accuracy score.

Here is a different project. Start by *assuming* certain that logic norms partial belief---at least for APO agents or pure enquirers. Advocates of *different* formulations of requirements logic places on belief will thus already have got off the boat. Our task is not to *convince the unconvinced* that those norms are in force, but to *explain where they get their normative punch from*. It's then legitimate here to consider various candidate explanatory hypotheses. Accuracy domination suggests that the Brier score (for example) is in a position to explain the probabilistic norms in the relevant setting-

[18] The generalized forms of domination argument given in Williams (2012) and Williams (2012a.) can be interpreted to this effect---the focus there is on nonclassical truth value assignments, but the results are easily reinterpreted. The strategy of replacing truth with another "aim" in evaluating belief states is also explored in Pettigrew "Accuracy, Chance and the Principal Principle", where the role here played by evidential support is there played by Chance.

--as well as dispositional version of that norm for ordinary agents. Of course, since we were initially unsure about the right formulation of the probabilistic norms and what strength to target, there would be some back and forth as the data is refined in the light of theoretical considerations. But the essential epistemic structure is that the accuracy score is not *independently* justified at all, nor does it need to be.

This may nevertheless have some teeth against rival conceptions of logical norms on credence. For we can legitimately ask fans of alternative models to provide an account of the source of the norms *they* think obtain, that achieves comparable success to our model here. Exploring whether (for example) advocates of Dempster Shafer theory can do this is a worthwhile project.

Patrick Maher objects that Joyce hasn't provided a sufficient justification for favouring scoring functions that will support his theorem, over the simple "linear score" that will not.[19] Advocates of Joyce need to fight Maher in the trenches if do intend to convince the unconvinced to be probabilists. But challenge from the linear score, at least has an easy answer within the explanatory project. The reason that we favour the Brier over the linear score is that *only the former is even a candidate explainer of the norms we're trying to explain*.

While Maher objected to Joyce on grounds of *bad company* --- prima facie reasonable measures that don't deliver the needed results; Aaron Bronfman and others have objected on grounds of too much *good company*.[20] Bronfman's objection trades on an *overabundance* of candidate measures of accuracy---all of which, we may assume, allow us to derive accuracy domination (and probabilistic anti-domination). The trouble that Bronfman points to is that *these may disagree*. If one's credences B violate probabilistic constraints, then candidate accuracy measure 1 might tell you that C accuracy-dominates B; candidate accuracy measure 2 might tell you that D does so. It's an open possibility that the first accuracy measure says that D is in some worlds less good then B; and the second accuracy measure says the same about C. If you're getting *conflicting advice* from the various candidates, then, intuitively, your best bet might be to stick with B, rather than taking a risk on actually lowering your overall accuracy.

Exactly how we respond to the objection depends on how it is supposed to work. One reading (suggested by Joyce's characterization) is that the worry is that it turns out *indeterminate* which accuracy measure is the right one. The point then is that for all the candidate-by-candidate accuracy domination results tell us, there need be no probability P which is *determinately* more accurate than B. If that is the worry, then the position could be stabilized by *denying* that this kind of indeterminacy exists: this is in effect the proposal that Joyce (2009) and Huber (2007) advocate. (Joyce 1998 lays out a number of axioms for the accuracy score. Several different accuracy measures satisfy these axioms: the various quadratic loss scores, for example. If one thought of the axioms as *exhausting* the conventions governing the use of the term then there might be a prima facie case that there was "no fact of the matter" which satisfier deserves the name "accuracy". The alternative conception of the enterprise has it as purely epistemic: if Joyce's arguments work, we know that the One True accuracy measure satisfies the axioms he lays down, but that is all.)

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[19] Maher "Joyce's argument for probabilism" *Philosophy of Science* (2002).

[20] Cf. Huber 2007, Joyce 2009

I'll come back to the indeterminacy version of Bronfman's objection shortly. But I focus first on a variant that threatens even once we've made the "realism" assumptions Joyce and Huber suggest---and which may seem particularly worrying if we want to give a Kolodny-style gloss on the enterprise. APO agents, we can agree, believe that C dominates B, where B is an improbabilistic belief state, and C is the probability that accuracy dominates according to the Brier score. It's true, that in virtue of their omniscience on matters a priori, that APO agents will believe:

C dominates B *given that accuracy is articulated via the Brier Score*.

But APO agents (or pure enquirers) need the unconditional belief that C accuracy-dominates B (i.e. that C is epistemically better than C) if our argument is to work as stated. Can we make the required move? Even assuming it's true that accuracy is given by the Brier score, why do we assume that APO agents are aware of this? True, it's not as if *empirical* information has any obvious role to play in finding out what the accuracy norm is. But that simply raises the question of whether there's *any* way to find out what the accuracy norm is. We may, in the spirit of realism, postulate that epistemic value has a determinate shape; but realism alone doesn't entitle us to the assumption that what that shape is, is epistemically accessible.

How should we respond to this? Well, we might argue directly that the One True accuracy measure is a priori accessible---perhaps they will be able to *know* that it is the Brier Score (for example). I don't think this is out of the question, but I prefer a different approach.

Consider an analogy to *moral* oughts. Inflicting suffering is bad. In virtue of this, it is plausible one morally ought not take action A if one believes that A will inflict suffering. This remains so even if in fact no suffering would result from A (more carefully, I think there's at least *one reading* on which one morally ought not to A, even if there's a second, non-information-dependent, reading on which it's morally ok). By contrast, consider a case where someone performs act A, which they *know* will inflict suffering, while *believing* that what they do is ok because they have whacky beliefs about right and wrong (that causing suffering is a good thing, for example). There's no reading on which it's *morally permissible* for them to perform A, I claim. You might well think that they're not *irrational* – there's no internal tension in their attitudes, since they're doing what's good-by-their-lights – but that's quite a different claim. Conclusion: deontic modals aren't relative to beliefs about *value,* even if they get relativized to what factual information one has available.

A similar distinction can be made in our case. It's one thing to accuse an APO agent of irrationality---of having a suboptimal-by-their-own-lights belief state. It's another to accuse them of having a belief state that (relative to their information state) they ought not to have. For the latter, where V are in fact the relevant facts about epistemic value, it suffices that the APO agent knows that *if V are the value facts,* then belief state x is epistemically better than their own belief state B. And this a modification of the core argument can deliver:

Core Argument (revised):

0. F is the accuracy measure. (Premise).
1. B is not a probability. (Premise)

2. B is F-accuracy dominated. (domination theorem, 1).
3. There is a specific probability (call it C), such that an APO agent will know of C that it is more F-accurate than B. (from 2).
4. If F is the accuracy measure, and one knows of C that it is more F-accurate than B, then one ought not to have belief state B. (Premise).
5. An APO agent ought not to have belief state B. (0,3,4)

One consequence of these considerations is that the accuracy-domination argument will *not* automatically reduce the "irrationality" of violating logical norms to a broader species of irrationality (of conflict between what one does, and what is best-by-one's-own lights)----that requires the *additional* premise about the a priori accessibility of the correct scoring rule. But even without that premise, our argument *does* make a strong case one *ought not* to have improbabilistic credences.

To bring this back to the original discussion of the grade and type of normativity of logic: recall the that the Williamson-eseque structure of "derivative norms" springing from "fundamental norms" only partially subjectivized the norm---whether the agent believed they knew the proposition mattered, but whether they thought of knowledge rather than coherence-with-others as the fundamental norm was not factored in. This fits nicely with what I've just been urging. But Kolodny's formulations were to look *either* to the non-subjectivized "oughts" themselves, or to what the agent thinks "ought" to be the case. So epistemic limitations over the accuracy-measure are more threatening for those who favour Kolodny's strategy for explaining away the appearance of normativity.

6. The original Bronfman objection.

I earlier set aside the original version of Bronfman's objection, directed against those who think there's no fact of the matter about which scoring rule describes accuracy (albeit that accuracy-domination is provable for each one). So our results above assume determinacy in the accuracy norm. Can we get anything similar if we loosen that assumption, and allow indeterminacy in which scoring rule gives the accuracy measure? The worry, recall, is that the precisifications of the score might give conflicting advice about which credence is the one that accuracy-dominates; each might condemn the others' recommendation.

Surprisingly (and some might think worryingly), if the argument above is valid, it looks like we can argue for probabilism even in this setting.[21] For simplicity, suppose that it is indeterminate whether F or G correctly describes the accuracy measure. For improbabilistic B, and accuracy measures F and G that allow an accuracy domination theorem, we can treat the earlier argument as a conditional proof of the (material) conditionals:

[21] Thanks here to Hannes Leitgeb, who urged me to think about this kind of extension of the earlier argument.

If F is the accuracy measure,
then an APO agent ought not to have belief state B.

If G is the accuracy measure,
then an APO agent ought not to have belief state B.

On a supervaluation-style treatment of indeterminacy we have:[22]

Either F is the accuracy measure or G is the accuracy measure.

But then disjunction elimination gives us the unconditional:

An APO agent ought not to have belief state B

So we have an argument for the epistemic badness of improbabilities, even when the relevant measures are indeterminate.

(I believe the arguments as presented are valid. Two concerns might be raised---one over the logic of the indicative conditionals especially with deontic modals in the consequent; the other over the logic of indeterminacy. In each case, the use of the metarules conditional proof and disjunction elimination have been questioned. However, I don't think we need worry about this. On the former front, the whole argument can be recast in terms of material conditionals rather than English indicatives, thus bypassing delicate issues about the interaction of deontic modality and natural language conditionals. On the latter, it's true that disjunction elimination and conditional proof are not *generally* valid in certain kinds of supervaluational logics. However, restricted versions of both *are* valid (essentially, those where no special "indeterminacy exploiting" move is made in the subproofs), and this is sufficient for our purposes.)

If one thinks that the conclusion of this argument is implausibly strong, one might wonder whether this undermines the plausibility of one of our premises. But I suspect that this is the wrong diagnosis, for it seems to me that what we have here is a *general* puzzle about indeterminate value. Suppose we have acts A, B, C, and moral-theory-1 says that A is optimal, B ok, and C evil; while moral-theory-2 says that C is optimal, B ok, and A evil. If it is indeterminate which of the moral theories is true, by the disjunction-elimination pattern we can argue that B ought not to be done; even though there's no action that *determinately* is better than it. This has, for me, exactly the same strangeness that we feel in the Bronfman case---and if indeterminacy in value is possible at all, it's worth thinking through how to react.

In other work, I've explored how indeterminacy interacts with assignments of value. There's certainly conceptions of indeterminacy available on which the argument by cases above is valid, and the proper conclusion is indeed that agents ought not to have ought not to have dominated belief states. I've explored one such framework elsewhere (and with quite independent motivations). The core idea is that when a subject is certain that p is indeterminate, then the subject is free to *groundlessly opt* to

[22] Of course, on a law-of-excluded-middle rejecting interpretation of indeterminacy (in a intuitionistic or Kleene logic, for example) the argument will be unsound at this point.

either judge that p, or judge that ~p.[23] In the case at hand, indeterminacy over which accuracy measure is the correct one, makes it *permissible* to groundless opt for one candidate or another to guide one's epistemic evaluations. Since improbabilistic beliefs are bad no matter which one chooses, the defence of probabilism stands despite the indeterminacy.

There's no consensus in the literature on indeterminacy and vagueness about how to think of indeterminate belief, desire and value, and so we can't appeal to an "off the shelf" model to resolve these sort of questions. I've pointed to one model where the Bronfman objection *wouldn't* be an obstacle to the accuracy-domination argument for probabilism. It would be interesting to see presented a model of indeterminate value on which it *is* an obstacle. At that point, we could evaluate the success of the argument (on the assumption that accuracy measures are indeterminate) by tackling the broader question of which conception of indeterminate value is right.

Conclusion

Our starting point was the question: why be logical? The Joycean gradational results (based on a Brier-score articulation of accuracy) allow a substantive answer to this question. Agents like us should be probabilists because any failure to do so means we fail to match our beliefs to the evidence; but further, we should be *disposed* to meet logical constraints on partial beliefs since only belief states meeting this condition have the virtue of reflective stability. Reflective stability, I've argued, is a commitment of the pure enquirer; and departures from the rules binding on the pure enquirer count as epistemic vices.

The Joyce argument is at its strongest when we assume that there is some determinate scoring rule that describes accuracy, which leads to an accuracy-domination theorem. Perhaps a Joyce-style project of providing independent constraints on legitimate accuracy measures can help with this, but in principle our justification for the assumption need not be *independent* of a commitment to probabilism (indeed, one of the most persuasive arguments for me that there is some such accuracy measure is by inference to the best explanation from probabilism itself). I've argued that we do *not* need to assume that it's even knowable what accuracy measure is the right one, in order for this argument to go through. The assumption of determinacy may be unnecessary – I've pointed to one conception of indeterminacy where this is so.

[23] See in particular Williams "Decision making under indeterminacy" *Philosophers' Imprint,* forthcoming. The discussion there is framed in terms of desirability rather than objective moral or epistemic value, but the structure should be quite analogous. I think this model is the proper conception of indeterminacy for anybody who favours a non-epistemic but classical logic and semantics of the relevant indeterminacy. The treatment of vague desirability in Williams "Non-classical probability and logic" (op cit) provides further models for thinking about vague desirability.