Reply to Sullivan

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In ‘An A-theory without tense operators’, Meghan Sullivan vigorously contests the received view that an A-theory of time is adequately expressible only in a language with sentential tense operators. She develops and defends an interesting alternative A-theory in a language with tense predicate modifiers instead. Her argument intersects *Modal Logic as Metaphysics* at several points. The proper formulation of A-theoretic doctrines such as presentism is sensitive to the background quantified temporal logic, and in particular to the dispute between permanentism and temporaryism, the temporal analogue in the book of the modal dispute between necessitism and contingentism: the permanentist asserts, and the temporaryist denies, that always everything is always something. Moreover, I sketch a conception of radical change, analogous to radical contingency, adequately expressible only in A-theoretic language (Williamson 2013, pp. 403-22).

I agree with Sullivan that an A-theory is adequately expressible in a language with temporal predicate modifiers. However, I will question whether the move to such a language makes as much difference as Sullivan suggests, and whether it has the advantages she claims for it.

1. *Operators and modifiers*

The first point to notice is that sentence operators are in effect a special case of predicate modifiers. In section 4 of her paper, Sullivan symbolizes predicate modifiers 'with non-negative superscripts and subscripts'; M^n_m takes an *n*-place predicate as input and returns an *m*-place predicate as output. So, in particular, a predicate modifier M^0_0 takes a 0-place predicate as input and returns a 0-place predicate as output. But a 0-place predicate is tantamount to a sentence: for an *n*-place predicate yields a sentence when completed with *n* singular terms, so in particular a 0-place predicate yields a sentence when completed with 0 singular terms, a null operation. Thus in effect the modifier M^0_0 takes a sentence as input and returns a sentence as output. In other words, M^0_0 is a sentence operator. The upshot is that Sullivan's preferred form of language with temporal predicate modifiers already permits the category of temporal operators.

Of course, Sullivan could add the artificial stipulation that the predicate modifier M^n_m is permitted only if *m* and *n* are positive, not just non-negative. But how much difference would that make? Suppose for the time being that the language contains a λ-operator that enables us to form a one-place predicate λ*v*(*A*) given an individual variable *v* and a formula *A*. Sullivan omits such an operator from her language for simplicity, but seems to have no objection to it in principle: 'It is a straightforward matter to develop an explicit version of the system using type-theoretic semantics or lambda-abstraction'. Indeed, languages without something like a λ-operator are drastically restricted in the use they can make of

their predicate modifiers, since the latter cannot be applied to complex predicates. A λ-operator would solve problems of formalization Sullivan discusses in section 4.

Let us then temporarily admit an atomic sentence operator *O* into the language, in order to show that its effect can be simulated by a predicate modifier $O^1{}_1$. For we can define the 1-place predicate $O^1{}_1(P)$ as equivalent to $\lambda v(O(Pv))$ for all 1-place predicates. Here I depart from Sullivan's practice of writing the argument of a predicate modifier as if it were the formula consisting of the predicate and its arguments, which makes the predicate modifier look like a sentence operator, contrary to her intention; the present notation is more perspicuous. Let *A* be any formula, and *v* any individual variable. Thus the predicate $O^1{}_1(\lambda v(A))$ is equivalent to $\lambda v(O(\lambda v(A)v))$, which is in turn equivalent to $\lambda v(O(A))$, because $\lambda v(A)v$ is equivalent to *A* by the standard λ-conversion principle. Consequently, the formula $O^1{}_1(\lambda v(A))v$ is equivalent to the formula $\lambda v(O(A))v$, which is in turn equivalent to $O(A)$ by the λ-conversion principle. In other words, the effect on a formula of applying λv, followed by the predicate modifier $O^1{}_1$, followed by predication of *v*, is equivalent to that of the original sentence operator *O*. If we now expel *O* from the language, because it is a sentence operator (and non-truth-functional: Sullivan's language still has the usual truth-functional sentence operators), and even drop the λ-operator, we can still retain $O^1{}_1$ as an atomic predicate modifier with the same semantic effect as before. In brief, a sentence operator can always be dressed up as a predicate modifier.

The point can be illustrated with an example that Sullivan gives for another purpose, the sentence 'John is apparently drinking'. She treats 'apparently' as a predicate modifier, but we can think of it as derived from the sentence operator 'it appears that' (*O*). Then we

can understand her sentence as 'John is such that it appears that he is drinking', with the overall form $\lambda v(O(Dv))j$.

Of course, the application of λv to A is vacuous when v does not occur free in A, but such vacuous applications are harmless and normally permitted; any standard semantics for the λ-operator handles them smoothly. Even if they were banned, we could easily circumvent the difficulty by using a trivial equivalent of A in which v does occur free, such as $v=v \rightarrow A$.

A philosopher who took the sentence operator O to be *meaningless* would protest that the original definition of $O^1{}_1$ in terms of O conferred no meaning on $O^1{}_1$. But Sullivan nowhere suggests that temporal operators are meaningless, and that extreme view is hardly plausible.

A more subtle objection is that λ-conversion may fail in a contingentist or temporaryist setting when a modal or temporal operator intervenes between an occurrence of the λ-operator and an occurrence of a variable that it binds (Stalnaker 2003; intentional operators such as 'John believes that' may also make difficulties for λ-conversion, but are not the main concern of Sullivan's paper). For instance, where P is a past tense operator, it may be held that $\lambda v(P(Dv))j$ ('John is such that he was drinking') entails $\exists v\, j=v$ ('John is something'), because predication entails being, but that $P(Dj)$ ('In the past, John was drinking') has no such entailment (see Williamson 2013, pp. 148-58 and 172-88 for discussion). Fortunately, we can finesse that issue by assigning a necessary and permanent being such as the null set as the value of the variable v, in which case $O^1{}_1(\lambda v(A))v$ is anyway equivalent to $O(A)$.

Thus predicate modifiers are hardly an *alternative* to sentence operators, for they subsume the latter. It is therefore no surprise that predicate modifiers give A-theorists the expressive power they need. Conversely, predicate modifiers by themselves give no more expressive power than variable-binding temporal operators do. For consider the predicate modifier M^1_1. Let Ov be a sentence operator binding the variable v such that $Ov(A)$ is equivalent to $M^1_1(\lambda v(A))v$. Then, for any 1-place predicate F, since $\lambda v(Fv)$ is equivalent to F by a trivial instance of λ-conversion, $M^1_1(F)v$ is equivalent to $M^1_1(\lambda v(Fv))v$, which is in turn equivalent to $Ov(Fv)$. The argument can easily be generalized to predicate modifiers M^n_m for any natural numbers m and n.

The foregoing arguments do not involve assimilating sentence operators to predicate modifiers applied to a truth predicate for propositions, a device Sullivan contemplates on behalf of A-theorists who wish to characterize changes in ontology without treating them as changes in a given thing. No truth predicate was used in the constructions above; nor was any quantifier over propositions. If one wants to generalize about change, one can go second-order, and quantify into predicate or sentence position without assuming that there are such *things* as properties or propositions (Williamson 2013, pp. 257-61).

The foregoing arguments also do not involve treating existence as a monadic property of objects. Thus they do not violate the view that 'Existence is not a monadic property of objects'. That is the second in a package of three views Sullivan describes as 'neo-Quinean', but since Quine rejected the postulation of properties, he would not have regarded the formulation as apt: given that there are no properties, it is trivial that existence is not a property. However, with the λ-operator, we can of course formulate the monadic predicate of objects $\lambda v(\exists x\, v{=}x)$, which deserves to be called an 'existence

predicate' given the third view in the 'neo-Quinean' package, that 'Existence is properly expressed by the existential quantifier ($\exists x$) and the identity relation (=)'.

So far, nothing much seems to be at issue in the A-theorist's choice between sentence operators and predicate modifiers. We must now examine Sullivan's reasons for thinking otherwise.

2. *Objections to operators*

Sullivan gives three reasons why A-theorists might be dissatisfied with using temporal sentence operators to articulate their account of time and change. I will consider each reason in turn.

Sullivan's first reason is that A-theorists may hold that 'their logical primitives ought to reflect their views about the structure of reality', but it is not clear what it could be 'for an intensional operator [such as a temporal operator] to reflect some aspect of reality'. Thus, by using an ideology of temporal operators, A-theorists risk losing out to B-theorists, whose ideology 'merely requires the more familiar object/property distinction'.

If there is a problem here, the shift from sentence operators to predicate modifiers does not solve it. For it is no clearer how an intensional predicate modifier can reflect some aspect of reality than it is how an intensional sentence operator can do so. Of course, if predicates stand for properties or relations, then since the result of applying a predicate modifier to a predicate is itself a complex predicate, it stands for a property or relation. But

that does not meet the original challenge, which concerned 'logical primitives', not complex expressions. The logical primitive here is the predicate modifier itself, not the result of applying it to a predicate. The predicate modifier does not stand for a first-order property or relation. Perhaps it stands for a higher-order property or relation. But any such account that works for predicate modifiers M^n_m will generalize to sentence operators. For it will explain how the relevant aspect of reality relates the input *n*-place property or relation uniquely to the output *m*-place property or relation, for various natural numbers *m* and *n*. A corresponding account will then explain in particular how the aspect of reality relevant to a sentence operator relates an input 0-place property or relation to an output 0-place property or relation. What is a 0-place property or relation? It is a *state of affairs*, in a sense to be refined by analogy with whatever theory of properties and relations is in play. If aspects of reality can map properties or relations to properties or relations, then an aspect of reality can map states of affairs to states of affairs. This is just the metaphysical analogue of points made about semantics in the previous section.

The contrast with the B-theoretic ideology is in any case overblown. For the B-theorist's 'logical primitives' include not only singular terms and predicates, standing for objects and properties or relations (or indeed states of affairs, in the case of primitive sentence constants), but also primitive logical devices such as conjunction, negation, and the universal quantifier. It is not immediately obvious which aspects of reality those logical devices reflect. But suppose that, from a sufficiently abstract perspective, we can understand extensional sentence operators like conjunction and negation as reflecting aspects of reality. Then, from an equally abstract perspective, why can't we understand intensional sentence operators such as the temporal ones as also reflecting aspects of reality?

Thus Sullivan's first reason for the A-theorist to switch from sentence operators to predicate modifiers is not compelling.

Sullivan's second reason for the switch is that 'Priorian tense logic has difficulty accounting for true claims involving cross-time relations and inferences that involve temporal anaphora'. There are indeed such difficulties, but how does the shift to predicate modifiers help resolve them? As she explains in second 4 of her paper, Sullivan's preferred method is to add quantification over times to the language. I agree with her that, on an appropriate understanding of what times are, such an addition is consistent with the spirit of an A-theory. But then what is to stop A-theorists who stick with temporal sentence operators from adding quantification over times to their language too?

Of course, adding quantification over times will be of little help if the time variables do not engage with the atomic sentences in any non-trivial way. But one can solve that problem by also adding to the language temporal sentence operators of the form 'at *t*', where '*t*' is a time variable open to quantification, as in hybrid logic (Williamson 2013, pp. 417-22). Such an A-theorist understands 'John is sitting at *t*' as 'at *t*: John is sitting'. The point of the operator 'at *t*' is *not* to supply a value for an implicit 'when?' argument place in 'John is sitting', just as the point of the operator 'according to Mary' in 'According to Mary, John is sitting' is not to supply a value for an implicit 'according to whom?' argument place in 'John is sitting'. For an A-theorist such as Prior, the sentence 'John is sitting' expresses the *same* complete proposition when uttered at different times (holding other things fixed, such as the reference of 'John'). The hybrid sentence operator 'at *t*' maps one complete proposition to another, just as the sentence operator 'according to Mary' does. By contrast, a B-theorist may postulate an implicit time variable in 'John is sitting', whose value 'at *t*' can

fix. Once one has such hybrid temporal operators, their temporal variable can be bound and quantified on, to solve problems of cross-time relations and temporal anaphora. What matters is the availability of quantification over times and devices with an effect like that of 'at *t*', not the shift from sentence operators to predicate modifiers. Thus Sullivan's second reason for the A-theorist to make the shift is also not compelling.

Sullivan's third reason for the switch, and the problem which she regards as 'the most severe' of the three, is the derivability of Barcan formulas and their converses in quantified modal and temporal logics, which makes such logics unsuitable for contingentists. Here she rather overstates her case. Some axiomatized quantified modal logics lack the Barcan and converse Barcan theorems and are sound and complete with respect to a possible worlds model theory with variable domains (see Hughes and Cresswell 1996, pp. 289-311 for an introduction). For the temporal case, Sullivan adverts to derivations of the Barcan and converse Barcan formulas in a natural axiomatic system of quantified temporal logic by John Burgess, which make such logics unsuitable for temporaryists (2009, pp. 33-4). But of course Burgess does not claim, absurdly, that those formulas are derivable in *all* axiomatic systems of quantified temporal logic. He sketches a Kripke-style model theory for quantified temporal logic with variable domains and notes that 'a perspicuous axiomatic proof procedure delivering as theorems just the closed formulas valid for this model theory is lacking' (2009, p. 37). The overall situation in quantified modal and temporal is that excluding the Barcan and converse Barcan formulas tends to make for more messy complications than including them, at the level of proof theory, though not at the level of model theory. I argue that contingentists are forced to take a more instrumentalist line towards the Kripke-style model theory than are necessitists (Williamson 2013, pp. 134-9), but that is not the issue Sullivan is raising.

Since the problem Sullivan is raising for quantified temporal logic with sentence operators concerns proof systems, one might expect her to present a proof system for quantified temporal logic with predicate modifiers, to show how the switch enables one to avoid the Barcan and converse Barcan theorems perspicuously, without messy complications. However, her paper does not attempt to sketch or cite any proof system. It is thus unclear what bearing the switch from sentence operators to predicate modifiers is supposed to have on the proof-theoretic problem.

After various reflections, Sullivan concludes that 'the operator-free A-theory fits best with a permanentist ontology' (as she puts it in her section 1), and hints at an analogous operator-free form of necessitism. But since the proof-theoretic problem only arose in the first place for those unwilling to accept necessitism and permanentism, even under pressure, this concession seems to undermine her third reason for switching from sentence operators to predicate modifiers. For once one has accepted those metaphysical views, the proof-theoretic problem should not worry one, so one can comfortably return to the operator formulations after all.

In consequence, Sullivan's three reasons for A-theorists to switch from sentence operators to predicate modifiers are not compelling, individually or together. That supports the conclusion of section 1 above, that the switch does not make very much difference. We can stick with the operator formulations.

References

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