Supervaluationism and logical revisionism

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Abstract

In the literature on supervaluationism, a central course of concern has been the acceptability, or otherwise, of its alleged logical revisionism. I attack the presupposition of this debate: arguing that when properly construed, there is no sense in which supervaluational consequence is revisionary. I provide new considerations supporting the claim that the supervaluational consequence should be characterized in a 'global' way. But *pace* Williamson[1] and Keefe[2], I argue that supervaluationism does not give rise to counterexamples to familiar inference-patterns such as *reductio* and *conditional proof*. My case rests on a disputable assumption. I strengthen the argument, therefore, by showing that even if this assumption is denied, the case for supervaluational consequence being *damagingly* revisionary is undermined.

* Thanks to the members of *Arché* centre in St Andrews, and the *CMM* in Leeds, for discussion. Thanks in particular to JC Beall, Ross Cameron, Andrew McGonigal, Joseph Melia, Greg Restall, Achille Varzi and Elia Zardini for discussion. Many thanks also to Jamie Stark for typesetting work.

1 Williamson, T., *Vagueness* (London: Routledge, 1994)

2 Keefe, R., *Theories of Vagueness* (Cambridge: Cambridge University Press, 2000)

In the literature on supervaluationism, a central source of concern has been the acceptability, or otherwise, of its alleged logical revisionism. Timothy Williamson claims that supervaluationism gives rise to:

> …breakdowns of the classical rules of contraposition, conditional proof, argument by cases and *reduction ad absurdum* in the supervaluationist logic of 'definitely'.[3]

Williamson is unhappy with such revisionism:

> Conditional proof, argument by cases and reduction ad absurdum play a vital role in systems of natural deduction, the formal systems closest to our informal deductions….Supervaluationists have naturally tried to use their semantic apparatus to explain other locutions. If their attempts succeed, our language will be riddled with counterexamples to the four rules.[4]

Others, accepting the case for logical revisionism, have argued that the upshot is unobjectionable. Thus Keefe:

> A number of commentators have emphasized how supervaluationist logic…fails to preserve certain rules of inference or classical principles about logical consequence…

[3] Williamson, *op cit.*, pp 151-152
[4] *Ibid.*

> How important is the failure…of certain classical principles governing logical consequence?...My reply is that the described features of supervaluationism are acceptable…[5]

The case for revisionism that these authors present depends on the presence, in the language at hand, of the supervaluationist notion 'Definitely' (the '*D*' operator)[6]. Roughly, 'Definitely *p*' says that, no matter how we sharpen the indeterminacy in our language, *p* always holds. In this, it is an object-language reflection of the supervaluationist's notion of truth – 'supertruth': the idea being that '*p*' is supertrue if it is true no matter how we sharpen our language. In a supervaluational language without the *D*-operator and its relatives, there is no special threat to the classical modes of inference. Once it is added, it is alleged that the following results hold, providing counterexamples to the respective classical mode of inference[7].

Contraposition

- $p \models_{sv} Dp$
- $\neg Dp \not\models_{sv} \neg p$

Conditional proof

- $p \models_{sv} Dp$
- $\not\models_{sv} p \supset Dp$

Argument by cases

- $p \models_{sv} Dp \vee D\neg p$
- $\neg p \models_{sv} Dp \vee D\neg p$
- $\not\models_{sv} Dp \vee D\neg p$

[5] Keefe, *op cit* pp 176-178
[6] Though as both authors emphasize, related notions would also lead to complaints
[7] The following are taken from Williamson, *op cit.*

Reductio

- $p \wedge \neg Dp \models_{sv} \bot$
- $\not\models \neg(p \wedge \neg Dp)$

Here, for example, is Keefe on contraposition:

> In any specification-space where *A* is super-true, *DA* is also super-true since *DA* is defined as true whenever *A* is true on all specifications. But it is not typically the case that $\neg DA \models_{sv} \neg A$…in a specification space where *A* is true on some specifications and false in others, $\neg DA$ is super-true, while $\neg A$ is not.[8]

Below, I argue that these results do not hold if the supervaluationist's framework is properly chosen. My positive case against revisionism will rest on a deniable premise concerning logicality. I strengthen the result, therefore, by showing that even if one denies my premise, and (further) can sustain the counterexamples *stricto sensu*, then the 'revisionism' that ensues in not damaging; indeed, whether the failure of these modes of inference really deserves the name 'revisionism' becomes a terminological issue.

1 Responses

I distinguish two ways for the supervaluationist to respond to the alleged counterexamples to classical modes of inference given above. The first is to claim *contra* Williamson and Keefe, that supervaluational consequence is thoroughgoingly classical, at least in the standard, single-conclusion setting. The second is to accept

[8] Keefe, *op cit.*, p 176.

that the examples arise and, with Keefe, to argue that the revisionism induced is not objectionable. It is the first line of response that concerns me at present.

I can think of three ways of making the case that supervaluationism requires no departure from classical logic.

1. In order to deny that the examples arise, one might give a non-standard treatment of the connectives involved, $\supset, \vee$ and so forth. So one would make a case that, properly understood, $\models_{sv} p \supset Dp$ holds, so that the validity of $p \models_{sv} Dp$ does not lead to a failure of conditional proof.
2. One might try to undermine the case by characterising $\models_{sv}$ in such a way that results such as $p \models_{sv} Dp$ does not hold, so that we can retain the standard treatment of connectives, and still not fall into revisionism.
3. One might make a case that the examples given above are not revisions of classical logic at all, because classical logic fails to sustain the relevant inferences. This would involve claiming that conditional proof, argument by cases and the rest are not universally valid, even for the thoroughgoing classicist.

I will focus here on (2). I argue that the natural generalization of the classical characterization of logical consequence will give a version of $\models_{sv}$ that does not lead to departures from classical logic: indeed, I will argue that none of the cases that Williamson and Keefe cite provide counterexamples to the rules they mention.

Furthermore, I do this while accepting much of Williamson's setting[9]: in particular, his rejection of 'local' characterizations of consequence in favour of 'global' characterizations.

2 The setting

Let me begin by outlining the treatment of consequence I favour. With WilliamsonI shall characterize consequence model-theoretically. The first challenge therefore is to say what a supervaluationist model looks like.

A supervaluationist model structure for a language *L* will consist, at minimum, of a domain of individuals *D*, a set of "delineations" Δ, and appropriate accessibility relations on the domain. In addition, there will be an interpretation function *f*, which will assign to expressions classical extensions. Formally, the model-structure so characterized is exactly analogous to that appropriate to a possible-worlds treatment of a modal language.

The crucial difference between my setting, and those of Williamson and Keefe, is that I assume that within the model structure of the intended model there will be delineations that are 'extreme' – relative to which a 6'8" man is short, for example. There are several reasons for wanting to have such delineations within our model structure. Consider the following supervaluationist treatment of comparatives[10].

[9] Williamson gives arguments for the revisionary consequences. I do not dispute that the results follow from the framework for supervaluationism that he sets up – I dispute elements of that framework on which the cogency of his arguments turn.

[10] Compare Lewis, D. K., 'General semantics'. *Synthese*, **22** (1970): 18-67. Reprinted with postscript in Lewis, *Philosophical Papers I* (Oxford: Oxford University Press, 1983) 189-229; Kamp, J. A. W., 'Two theories about adjectives'. In Keenan, E., (ed.) *Formal Semantics of Natural Language* (Cambridge: Cambridge University Press, 1975). Reprinted in Davis and Gillion (eds.). *Semantics: A reader* (Oxford: Oxford University Press, 2004) 541-562.

> ‘*A* is *F*-er than *B*’ is true at delineation d iff the set of delineations where ‘*A* is *F*’ is true is a proper superset of the set of delineations where ‘*B* is *F*’ is true.

This will be untenable unless we have available extreme delineations. For otherwise, the set of delineations making-true ‘*A* is tall’ (where *A* is 6’8”) will be the same as that making-true ‘*B* is tall’ (where *B* is 6’10”). On the treatment of comparatives given above, this will mean that ‘*B* is taller than *A*’ will be declared false, which is absurd[11].

Nevertheless, we want ‘a 6’8” man is tall’ to be true, and ‘a 6’8” man is short’ to be false, on the intended model. Due to the presence of extreme delineations, we cannot characterize supertruth in the simplest way, i.e. saying that *S* is supertrue iff on every delineation *d*, *S* is true relative to *d*. Some extra machinery is called for.

We first require models to pick out a subset of the delineations – a subset we will call the *sharpenings*. *S* will be supertrue (at a model) if it is true relative to each of the sharpenings of that model. A sentence will be supertrue *simpliciter* if it is supertrue at the *intended* model.

Second, we introduce an accessibility relation, ‘*S*-access’, on the space of delineations of the model structure, and let ‘*Dp*’ be true at a delineation *d* in a model if ‘*p*’ is true at all delineations *S*-accessed by *d*. On the intended model, the set of sharpenings will *S*-access each other[12]. Our models will therefore take the form:

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[11] Williamson and Keefe, op cit. cite such cases as objections to the treatment of comparatives, under their assumption that all delineations are non-extreme.
A second example: my desk is definitely flat. But in some extreme sense, it is not flat – it is less flat than an oil slick, for example. To give a treatment on which ‘this is definitely flat, but in some extreme sense, it is not flat’ will come out true, we need to appeal to extreme delineations “accessed” by ‘in some extreme sense’.
Yet another reason for wanting such delineations is the need to treat higher-order vagueness within the supervaluationist setting. See Williamson, *op cit.* sec. 5.7.

[12] Another constraint on *S* will be, presumable, that it is reflexive. More constraints will presumably flow from an adequate account of higher-order vagueness.

$$m = \langle D_m, \Delta_m, S_m, f_m, s_m \rangle$$

where D_m is the domain, Δ_m the set of all (extreme and non-extreme) delineations, S_m an accessibility relation, and s_m a subset of the delineations – the sharpenings of the model[13].

Once we have dropped the idea that delineations correspond to the intuitive notion of 'precisification of the language', there seems no reason to put *any* constraint what delineations there can be. We may as well let there by a delineation corresponding to *any* assignment of semantic values to non-logical terms. Only some among these arbitrary delineations will be sharpenings, and *S*-access each other on the intended interpretation[14]. I assume in what follows, therefore, that the delineations in the intended model of our language are arbitrary, though only some among them are sharpenings[15].

Given this setting, we can then distinguish two forms of consequence. The first is *local* consequence:

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[13] We can, of course, *introduce* an operator '*D**' such that *D*p* is true iff *p* is true on all delineations *whatsoever*. The interest of such a notion is questionable, particularly in a framework involving 'arbitrary delineations' rather than merely extreme ones. It also will not threaten any of the logical results later in the essay: the proof given in the appendix shows that the logic of this operator will be S5, just as is with the analogous introduction of modal operators. Thanks to an anonymous referee for raising this issue.

[14] It may be that there is semantic work for the notion of an 'extreme' but not arbitrary delineation. E.g., running the story about comparatives above with arbitrary delineations in place of extreme delineations would not be plausible: there are *arbitrary* delineations where 'tall' holds of all and only 6'10" men. Thus, the set of arbitrary delineations where a 6'8" man falls under 'tall' will not be a *proper* subset of the arbitrary delineations where a 6'10" man falls under 'tall'. However, for such semantic purposes, we need only introduce a new accessibility relation, *E*-access, whereby only the delineations I earlier called 'extreme' *E*-access each other. We would then reformulate the account of comparatives given earlier: '*x* is taller than *y*' is true at an (arbitrary) delineation d iff the set of delineations *E*-accessible from *d* where '*x* is tall' is true is a subset of those *E*-accessible from *d* where '*y* is tall' is true.

[15] Though this seems the natural setting to me, I do not believe the arguments below depend on accepting it as opposed to one where a delineation is at worst 'extreme'. All that needs to be maintained is that *truth* is to be analysed not as *truth on all delineations* but as *truth on all sharpenings*.

$\Gamma \models_{\text{local}} \Phi$ iff

On all models *m*, and all $d \in \Delta_m$, f_m makes Γ true relative to *d* only if f_m makes Φ true relative to *d*

Williamson rejects this characterization on behalf of supervaluationists. He takes it that consequence should be characterized in terms of *truth* preservation under arbitrary reinterpretations. For standard supervaluationists, then, it should be characterized in terms of supertruth preservation. But given this, local validity looks suspect:

> The problem for supervaluationists is that supertruth plays no role in the definition of local validity. Yet they identify truth with supertruth; since validity is necessary preservation of truth, they should identify it with necessary preservation of supertruth. That amounts to an alternative definition…[16]

This alternative is a *global* consequence[17]:

$\Gamma \models_{\text{global}} \Phi$ iff
On all models *m*, Γ is supertrue-at-*m* only if Φ is supertrue-at-*m*

The notion of supertruth – truth at all sharpenings – is the central notion used to define supertruth on a model. Hence global consequence meets Williamson's constraints.

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[16] Williamson, *op cit. p xxx*
[17] Williamson does not mention the relativization of truth to a model in the above. But I take it that he will wish to generalize over models in the final characterization, on pain of admitting water = H_2O as a logical validity.
If we added into our models a set of possible worlds, and a specification of one among these as 'actual', then generalizing over all models would amount to requiring *necessary* truth preservation, as Williamson requires.

One might wonder about whether the intuitive connection between validity and necessary truth-preservation must be maintained at all costs. Would anything go wrong if the supervaluationist appealed to local consequence? Things would indeed go wrong: in the following section I point to considerations showing that $|=_{\text{global}}$ is indeed the proper explication of $|=_{sv}$.

3 Reasons to reject local consequence

There has been some debate in the literature about the relative merits of defining consequence in local and global ways. I side with those would take *global* characterization to be the proper one. The defenders of local consequence claim it avoids the revisionism that allegedly follows from the global characterization. But I will argue that the local consequence follows the classical paradigm *too closely*[18].

The case is easiest to make by considering one of the most persuasive ways of introducing a sorites paradox. Take a paradigmatically red colour-patch *A*, and a paradigmatically blue colour patch *Z*. Ask one's audience to endorse both the following:

1. *A* is red
2. *Z* is not red

[18] Delia Fara argues that supervaluationists have independent reason to give up rules such as contraposition etc (Fara. D., 'Gap principles, penumbral consequence, and infinitely higher-order vagueness'. Published under the name "Delia Graff" in Beall, J. C., (ed.) *Liars and Heaps: New essays on paradox* (Oxford: Oxford University Press, 2003)). I think her arguments are resistable, but I will not defend this here. The present section, though, makes an analogous move: the supervaluationist should not buy into a consequence relation that is extensionally equivalent to the full (multi-conclusion) classical relation.

Now present the audience with a series of conjunctive statements concerning a sorites series $A = A_1, A_2, ..., A_n = Z$, and ask them to *reject* each of them:

- A_1 is red $\wedge$ A_2 is not red
- A_2 is red $\wedge$ A_3 is not red
- ...
- A_{n-1} is red $\wedge$ Z is not red

Rejecting each of these premises is *prima facie* highly plausible. It is one of the advantages of supervaluationism that it can preserve this intuition: none of the conjunctive statements is supertrue (at best they are neither true nor false). And given that one should reject *p* if *p* fails to be true, each of the conjunctions is rejectable[19].

Often, the next move is to invite one's audience to move from the *rejection* of each of the conjunctions to that *acceptance* of their negations. Granted (1) and (2), and each of the negated conjunctions, standard (single-conclusion) classical logic allows one to derive a contradiction. The supervaluationist resists the sorites paradox by resisting the move from *rejecting p* to *accepting* $\neg p$: for in case *p* is neither true nor false, then we should reject both *p* and $\neg p$, and accept neither.

However, there is a standard way of extending logics so as to capture the logical relationship in this case more directly. One way of expressing the practical

[19] Some caveats. (1) This advantage does not extend to the *disjunction* of all of the individual conjunctions, which the supervaluationist takes to be supertrue, yet is intuitively repugnant. (2) Some resist the identification of truth with supertruth: see McGee, V., and McLaughlin, B., 'Distinctions without a difference'. *Southern Journal of Philosophy*, sup XXXII (1994), 203-251. If so, one would need to invoke the principle that one should reject *p* if and only if *p* fails to be *determinately* true to get this result. Notice, though, that if lack of determinate truth norms rejection in this way, the rest of my argument against local consequence goes through. Since these theorists tend to like local consequence, I suggest that they had better give up this account of rejection: if so, the supervaluationist who identifies truth with supertruth can plausibly claim an advantage, in that they give a theory which makes the intuitive reaction to the conjunctions above – viz. rejection – the right one.

significance of a logical consequence $\Gamma \models \Phi$ is as showing that the following combination of attitudes is (logically) incoherent: accepting each of the premises (the sentences in Γ) and rejecting the conclusion (Φ). In a situation where one rejects *multiple* sentences, then it is natural to look slightly more general consequence relation: $\Gamma \models \Delta$ is to show that one cannot coherently accept everything in Γ and reject everything in Δ[20]. The characterization of multi-conclusion consequence in the classical case is just a straightforward generalization of the usual definition: holds if and only if every model that makes everything in Γ true makes something in Δ true.

In this 'multiple conclusion' setting, one can enquire directly about the coherence of the pattern of acceptance and rejection that was motivated above (recall it is a *good-making* feature of supervaluationism that it is able to preserve this highly intuitive pattern). This is to ask whether the following holds:

A is red, Z is not red $\models_{sv}$.

(A is red $\wedge$ A_2 is not red), (A_2 is red $\wedge$ A_3 is not red),…, (A_{n-1} is red $\wedge$ Z is not red)

If $\models$ is fully classical then the answer is straightforward: the argument is just as valid as the traditional formulation of the sorites with multiple negated-conjunction

[20] For an articulation and defence of multiple conclusion systems, see Restall, G., 'Multiple conclusions'. In Hajek, P., Valdes-Villanueva, L., and Westerstahl, D., (eds) *Logic, Methodology and Philosophy of Science: Proceedings of the Twelfth International Congress* (London: King's College Publications, 2005) 189-205; Hyde, D., 'From heaps and gaps to heaps of gluts' *Mind* **106** (1997), 641-660; Weatherson, B., 'Many many problems' *Philosophical Quarterly* **53** (2003): 481-501, Varzi *op cit*. Restall argues *inter alia* for their utility in analyzing supervaluational 'denial'. Hyde discusses the departures from classical logic induced by global consequence in a multiple conclusion setting. Weatherson appeals to the fact that local consequence induces no revision in multiple conclusion settings as a good-making feature of that account. Varzi gives a detailed survey of logical options available to the supervaluationist, including their standing in the relevant multiple conclusion settings.

premises. That this is true follows from a result that is easy to verify: that in a classical setting, $\Gamma, \neg\Phi \models \Delta$ iff $\Gamma \models \Delta, \Phi$ [21].

But it would be a disaster if the *supervaluationist* agreed on this score. For that would be to grant that all the premises of a sorites argument are acceptable, all the conclusions rejectable, and the argument is valid! The vital connection between questions of validity and questions of the coherence of attitudes would be broken.

This directly motivates accepting global rather than local consequence as the correct formulation of supervaluational consequence. For global consequence classes the argument invalid: on the intended model, the premises are all (super)true, and none of the conclusions are (super)true. So the intended model itself provides a countermodel to the putative consequence. However, local consequence bids us look at what happens *at each delineation*. And whenever the premises are all true at a delineation, at least one of the conclusions will be true. So logical consequence agrees with classical consequence that the multiconclusion formulation of the sorites argument is valid: an absurd result from the supervaluationist point of view[22]. There are two main ways the defender of local consequence might respond. The first is to reject the coherence of multi-conclusion logic[23]. The second is to question the *interpretation* of multi-conclusion logic presupposed above, whereby a valid argument shows us that one cannot simultaneously accept all its premises and reject

[21] See Restall, *op cit.*, p 14

[22] This point can be made without appeal to multiple conclusion logics for the *dual* of supervaluationism: subvaluationism (Hyde, *op cit.*). The subvaluationists definition of Φ being truth-on-a-model just requires that Φ be true on *some* sharpening. This means that all the premises of a standard 'long' sorites series will be subtrue (for example, the combination of (1) and (2) and all the negated conjunctions). The standard subvaluationist line is that the 'long' sorites fails to be *valid*. But this is only true if they work with a globally characterized consequence relation: for the argument is *locally valid*. This means the sorites will be a valid argument from true premises, with absurdity as its conclusion. This seems *clearly* unacceptable: the subvaluationist should stick with global validity.

[23] This seems mysterious to me: after all, I can *define* multi-conclusion logic and tell you its intended significance: what could be incoherent about it?

all its conclusions[24]. Such moves seem unnatural to me, especially when there's a straightforward alternative available that makes the problem go away: global consequence[25].

4 The *prima facie* case against revisionism

Global consequence diverges from classical logic when we consider arguments with multiple conclusions: and this is a *very good thing*. But does $\models_{\text{global}}$ induce logical revisionism even when only a single conclusion is involved? This would be the seemingly 'worrying' kind of revisionism that Keefe and Williamson focus upon, giving counterinstances to such entrenched inferential moves as *reductio* and *conditional proof*. In this section, I will show that *under a certain assumption* (single-consequence) supervaluational consequence is non-revisionary. In the following sections I will critically examine the assumption.

Consider the alleged result that $p \models_{sv} Dp$ .When $\models_{sv}$ is read as $\models_{\text{global}}$, we can find counterinstances. The very simplest case (there are many others) is the following: let the set of sharpenings contains a single delineation δ, relative to which *p* is true (thus *p* is supertrue at that model). Assume further that the accessibility relation *S* relates δ to a delineation δ′, and *p* is false relative to δ′. Then '*Dp*' is not true at δ, and hence not supertrue-at-*m*. Thus we have a counter-model to the claim that *Dp* is a consequence of *p*. The upshot is that this sequent cannot play a role in showing that contraposition, or conditional proof, fails.

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[24] Bu there *do* seem to be constraints of logical coherence on patterns of acceptance and rejection, and multi-conclusion logic seems the *obvious* way to capture them. At minimum, If one rejects this framework, one owes some substitute account.

[25] Achille Varzi, *op cit.* has suggested a fourth alternative: that the defender of local consequence might respond by giving a non-standard 'local' reading of what it is for an argument to be sound. But the simple fact that there are valid arguments from true premises to untrue conclusions seems bad enough to me.

Counter-models can be found to the other examples too. For example, consider the alleged result that $p \wedge \neg Dp \models_{sv} \bot$[26]. The model above is a counter-instance to this also; for relative to the model just described, the premise is supertrue, but the conclusion superfalse. Indeed, all the results cited fall to such considerations. In an appendix to this paper, I give a simple argument that shows that (for single-premise consequence) global supervaluational consequence coincides with classical consequence.

Game over? Not quite. An opponent need only concede that *some* relation has been constructed which (in single-conclusion cases) behaves exactly like classical consequence. It is still open to the sceptic to argue that there is something illegitimate about the construction: one which means that it doesn't deserve the name 'supervaluational consequence'. In particular, in setting out my proposal for supervaluational consequence, I let the meaning of '*D*' vary from model to model. In the intended interpretation, to be sure, it behaves in a familiar way. But in some unintended interpretations, the value assigned to '*D*' will not correspond to anything like the intuitive notion *definitely*[27].

It is familiar that when offering a semantic characterization of consequence, some interpretations of the language have to be declared 'inadmissible'. An interpretation of English on which 'or' is interpreted as meaning *and*, would be an interpretation on which 'A' might be true, while 'A or B' false. Nobody takes such interpretations to cast doubt on the validity of or-introduction. Familiarly, we disregard them by a pair of moves. (1) We restrict attention to admissible interpretations of the language: we

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[26] Williamson, *op cit.*, p 297, footnote 5.19, cites this as a robust result, one from which counterexamples to the above modes of inference all follow.

[27] That observation alone is not enough to cause concern. Few object, for example, when we show the invalidity of |= *Hesperus* = *Phosophorus* by appealing to a model where 'Hesperus' and 'Phosphorus' get assigned distinct referents, and *a fortiori*, one at least is assigned something other than its intended referent.

say that conclusion follows from premises only when all admissible interpretations of the language which make the premises true, make the conclusion true also. (2) We declare that interpretations that do not assign 'and' its intended meaning are inadmissible.

When characterizing the consequence relation $\models_{global}$, I presupposed that '*D*' is not in the same boat as 'or' in this regard. I took it that I did not need to pause to argue that the interpretations I appealed to did not vary the meaning of '*D*'. But one could take issue with this, and insist that interpretations that do not assign '*D*' its intended meaning are inadmissible. This may then change the extension of the consequence relation: let us call the new relation $\models^{+}_{global}$.

For all I have said so far, $\models^{+}_{global}$ may vindicate all the inferences familiarly imputed to global supervaluational consequence, including in particular the move from p to Dp and the move from $p \wedge \neg Dp$ to absurdity. If so, then by the familiar moves, $\models^{+}_{global}$ fails to sustain conditional proof, reductio and the rest. Supervaluational consequence, understood as $\models^{+}_{global}$, may yet be revisionary.

Two questions therefore arise: first, does $\models^{+}_{global}$ reinstitute the failures of classical inferential patterns that orthodoxy ascribes to supervaluational consequence? And second, supposing this is so, is $\models^{+}_{global}$ a better candidate than $\models_{global}$ for capturing supervaluational consequence?

5 Revisionism and $\models^{+}_{global}$: a vexed issue

We are concerned in this section with the formal behaviour of $\models^{+}_{global}$, and specifically, whether it leads to failures of conditional proof, reductio and the rest.

Here is an argument that it does. First, assume that the logic for "Definitely" is S5, as urged by Keefe[28]. Then, on the intended interpretation, every sharpening S-accesses all and only other sharpenings. Plausibly, the relationship just described between S-accessibility and sharpenings must hold on *every* model that does not shift the meaning of "Definitely", and only such models are taken into consideration in defining $|=^{+}_{global}$. Given this, it is easy to see that $p \; |=^{+}_{global} \; Dp$[29]. So, under our assumption, $|=^{+}_{global}$ will lead to failures of conditional proof, for we also have: $|\neq^{+}_{global} \; p \supset Dp$[30].

This discussion proceeded on the assumption that the logic of *D* was S5. The more popular view, at least in the context of supervaluational treatments of vagueness, is to take it that "Definitely" obeys some weaker logic: that it can be indefinite whether or not *Dp* holds, for example[31]. I contend, however, that once we have only these weaker constraints on the accessibility relation, the usual arguments for revisionism lapse, even if we are holding the meaning of "Definitely" invariant.

To see this, consider the putative validity that Williamson takes to provide the strongest case for revisionism: the entailment of absurdity by $'p \wedge \neg Dp'$. I will construct a toy model for a vague language which provides a countermodel to the claim that $p \wedge \neg Dp \; |=^{+}_{global}$. I hope my reader will agree that nothing in the model is unfaithful to the intended sense of "Definitely". If so, then this countermodel cannot

[28] Keefe, *op cit.*, ch7.
[29] *Proof.* Take any model that makes-true *p*. Then *p* holds on every sharpening. But that means that *p* holds on all delineations which any sharpening can *S*-access: and that is just to day that *Dp* holds at every sharpening and so is true on the model.
[30] *Proof.* Consider any model where *p* holds at some sharpenings but not at others. Then $\neg Dp$ holds at all sharpenings. So $p \supset \neg Dp$ fails at those sharpenings where *p* holds. So the conditional is not (super)true on the model.
[31] The coherence of such a statement requires a logic for "Definitely" weaker than S4. See Williamson, *op cit.*, ch5, *passim*, for the claim that the logic for definitely should be not sustain the characteristic S4 or S5 principles.

be regarded as inadmissible. The construction turns crucially on an aspect of the supervaluational framework advocated in this paper not shared by Williamson's model. I take it that the difference reflects favourably on the present approach.

Suppose that *A, B, C, D* and *E* are colour patches, with each successively redder than the preceding patches. Suppose that δ_{AB}, δ_{BC}, δ_{CD} and δ_{DE} are four delineations of a language containing the predicate 'red', corresponding to four placements of the red/non-red cut off in this series (thus δ_{AB} places the cut off between *A* and *B*, and so on). Let us further suppose that the meaning-fixing facts are such that δ_{AB} and δ_{DE} are unintended. δ_{BC} and δ_{CD} are the two sharpenings of this interpreted language (see figure 1). That means that "*A* is red" and "*B* is red" will each be supertrue on this model, since both sharpenings place the cut-off for red no earlier than *B*.

Let us further suppose that each cut-off thinks that itself and its neighbours are sharpenings of the language. Thus δ_{BC} *S*-accesses itself, δ_{AB} and δ_{CD} (but not δ_{DE}). δ_{CD} *S*-accesses itself, δ_{BC} and δ_{DE} (but not δ_{AB}) (see figure 2).

On such a model, *A* and *B* will be red, while *D* and *E* will be not-red. On the model, it will be indefinite whether *C* is red[32]. But in addition, we have 'higher order vagueness'. On the model, *A* is not only red, it is definitely red, and *E* is not only fails to be red, it is definitely not red. However, while *B* is red, it is not the case that the model counts it as definitely red: the sharpening δ_{BC} accesses a delineation δ_{AB} at which it doesn't count as red[33].

Consider, then the sentence "*B* is red and *D* is not red". This is false at δ_{AB} and δ_{DE}, and is true at δ_{BC} and δ_{CD}. Further, "It is determinate that *B* is red and *D* is not red" is false not only at δ_{AB} and δ_{DE}, but also at δ_{BC} and δ_{CD}, for each of them sees a

[32] For this to be the case, it has to be that 'it is indefinite whether *C* is red' is true at each sharpenings, i.e. δ_{BC} and δ_{CD} This requires that each see delineations where *C* is red, and delineations where *C* is not red. Since each sees itself and the other, and since they classify *C* opposingly, this condition is met.

[33] Care is needed! It is not the case that the model counts it as definitely red, but neither does it count it as an indefinite case, for *one* of the sharpenings (δ_{CD}) counts it as definitely red.

non-sharpening precisification (a different one in each case) that makes the sentence false. With 'p' = '*B* is red and *D* is not red', it follows that $p \wedge \neg Dp$ is true at this model.

I need not argue that this is the *intended* model for some language containing "Definitely". All that is needed here is the concession that there is no *a priori* reason for ruling out this kind of situation, given the meaning of "Definitely". For once we admit that $p \wedge \neg Dp$ can be true in some model that preserves the meaning of "Definitely", then the claim that sentences of that form entail absurdity must be given up; and with it, the argument that $p \wedge \neg Dp \models^{+}{}_{\text{global}}$ leads to failures of conditional proof, reductio and the rest[34].

I contend, therefore, that Williamson's route from supervaluationism to logical revisionism is blocked, *even if* we treat 'Definitely' as a logical constant[35].

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[34] Further, once one appreciates the aspect of the model that lets $p \wedge \neg Dp$ be true on this model – the fact that each sharpenings accesses at least one non-sharpening delineation, one can start to construct models that are less unrealistic. For example, one unrealistic aspect of the above model is that it contains only four delineations, and two sharpenings. But we can make the same case with a model containing infinitely many of both. For each real number *r*, let δ_r be a delineation that places the cut-offs of 'red' at a wavelength of *r*nm (of light with wavelength less than *r*nm will not count as red at δ_r). There will be a set of admissible cut-offs, corresponding to a range of sharpenings: suppose the sharpenings are $\{\delta_r : r > r > r\}$. Let $|r - r| = d$. Let us define the accessibility relation *S*-access as follows: δ_x accesses δ_y iff $|x - y| < d$. It is easy to check that this is a reflexive, symmetric but intransitive accessibility relation, and that every sharpening will *S*-access a non-sharpening. This is just a generalization of the simpler model given in the text. For example, light of wavelength r_1 will be red, since it counts as red on each sharpening. Light of wavelength r_2 will count as non-red, since it is non-red on each sharpening. 'Light of wavelength r_1 is red and light of wavelength r_2 is non-red' will be true. But since every sharpening sees some (non-sharpening) precisification where this is false, 'Determinately, light of wavelength r_1 is red and light of wavelength r_2 is non-red' will be false at every sharpening, and so the negation is supertrue. So again, we have a model making true an instance of $p \wedge \neg Dp$.

[35] The critical difference between the characterization of $\models^{+}{}_{\text{global}}$ given here and the kind of characterization of $\models_{sv}$ that Williamson offers, is the role given to the *sharpenings*. In Williamson's setup, supervaluational models do not contain a set of sharpenings: rather, they contain a designated 'base point'. The sharpenings are then *defined* as the set of delineations accessible from the base point. That way of formulating matters therefore *guarantees by definition* that there is at least one sharpening which sees all and only other sharpenings (though in general, sharpenings other than the base point may see non-sharpenings). By contrast, if one looks back to the models sketched above, we see that no choice of 'base point' will deliver δ_{BC} and δ_{CD} or $\{ \delta_r : r_1 \geq r \geq r_2\}$, as the sharpenings.

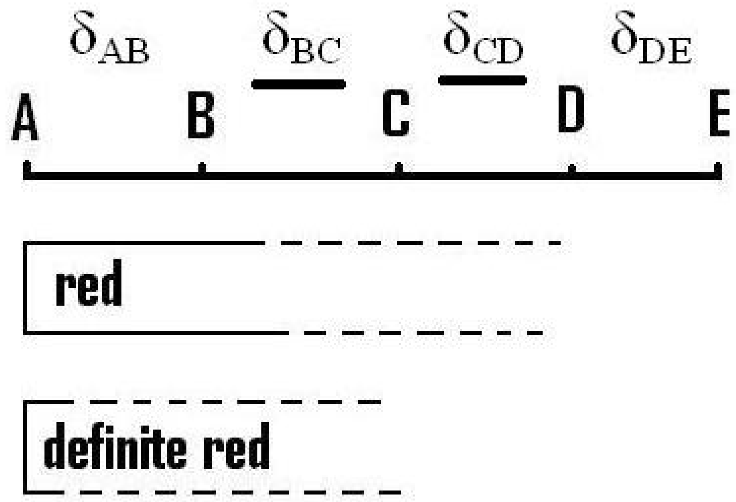

Figure 1: Sorites series and intuitive verdicts of redness/definite redness of patches. Delineations for red placed; sharpenings for ‘red’ underlined.

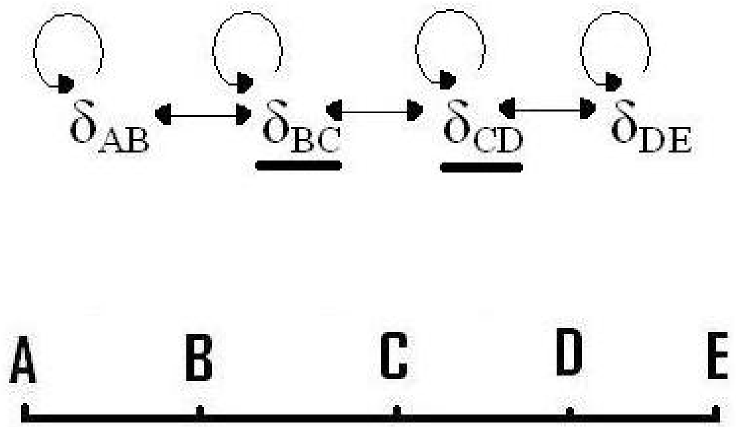

Figure 2: Model structure, including accessibility relations and sharpenings (underlined).

Why not then simply argue that $|=^{+}_{global}$, like $|=_{global}$ is non-revisionary? I have three reasons for continuing my examination:

1. As mentioned above, there is still a case for revisionism if the logic of "Definitely" is S5. And that position has some advocates in the literature: most notably, Keefe.
2. The case for the non-revisionism of $|=^{+}_{global}$ is perhaps not conclusive. Perhaps some inventive defender of the orthodox position could make a case that the toy model constructed above, and all others like it, are unfaithful to the intended sense of "Definitely" (I have no idea what such a case would look like, but cannot rule it out).
3. All that has been shown here is that one argument (albeit the most famous and influential one) for revisionism fails. Unlike the case of $|=_{global}$, I have offered no *positive* argument that $|=^{+}_{global}$ is non-revisionary.

For there reasons, further discussion is warranted, and *for the sake of argument* we can assume that $|=^{+}_{global}$ is revisionary.

6 Definitely and Logicality

Terms which are required to keep their intended meanings across reinterpretations in characterizing logical consequence, are known as *logical constants*. $|=^{+}_{global}$ is defined by insisting that admissible models always feature "Definitely" with its intended

The 'base point' formulation Williamson gives, then, is no innocent part of his setup. Furthermore, it is an entirely artefactual one: the *only* use for a base point is in defining the set of sharpenings; it is the latter notion that plays a crucial role in characterizing truth, consequence and the like. The setup given here, with sharpenings taken as basic rather than artificially defined, is the more natural.

meaning; for $|=_{\text{global}}$, this constraint is lifted. We can thus summarize the issue as follows: in characterizing supervaluational consequence as $|=_{\text{global}}$, I supposed that *D* was not a logical constant. One who characterizes supervaluational consequence as $|=^{+}_{\text{global}}$, supposes that *D* is a logical constant.

The case for supervaluational consequence sustaining counterexamples to conditional proof, reductio, and the rest, thus at best turns on a highly controversial issue in the philosophy of logic: the nature of the logical constraints.

Some terms are *paradigmatically* logical: 'and', 'or', 'all' are traditional examples. It is plausible that it is a constraint on a successful theory of logicality that it categorize these as logical constraints. But, moving beyond this traditional set, it is not clear that any informed consensus emerges. One looks in vain in the literature on the logic of vagueness for explicit discussion of the putative statue of 'Definitely' as a logical constraint. And as with modal and tense operators, I do not see a pre-theoretic case for treating 'Definitely' as a *logical* constant.

One might, indeed, turn the considerations given above around, to construct a *prima facie* case against the logicality of "Definitely". The logical constants, after all, are those terms whose meanings we keep constant in order to generate the logical consequence relation. And while we might have little pretheoretic grip on which terms should count as logical constants when we move beyond propositional and predicate logic, the entrenchment of this or that pattern in our inferential practice surely gives us traction on which inferences we are pretheoretically treating as valid. It is a presupposition of the debate thus far that conditional proof, reductio and the rest are entrenched in inferential practice. Suppose now it turns out that treating some expression *C* as a logical constant would mean admitting counterexamples to those rules. Absent further considerations, the appropriate response is to take it that we are (implicitly) treating *C* as non-logical.

At this point in the dialectic, this is exactly the situation that faces us with "Definitely"[36]. Our pretheoretic grip on the goodness of inferential patterns such as *reductio* creates the *prima facie* case that "Definitely" is not a logical constant. This situation should not be overstated: if we had strong theoretical reason to treat "Definitely" as a logical constant, the above considerations will not put up a great deal of resistance. But I contend it does shift the burden of proof on the defender of $\models^{+}_{global}$ to justify their commitments. The defender of $\models_{global}$ wins by default.

Whether or not the above is accepted, it is appropriate to consider whether there *is* any theoretical reason to treat "Definitely" as a logical constant. Unfortunately, surveying the literature on logical constants will not help us much, for every consideration we might appeal to is hotly contested.

Among those who are happy with talk of a unique set of logical constants, it is controversial what sort of criteria mark off logical from non-logical constants[37]. There are those who seek to defend and extend Tarski's permutation criterion for logical constanthood. There are those who favour instead inferential criteria: logical constants being terms who meaning is exhausted by a specification of introduction and

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[36] I am, of course, supposing something that was left open in the previous section: that treating *D* as a logical constant does lead to counterexamples to conditional proof, and the rest. But of course, if the reasoning above is accepted, then we can argue disjunctively: either treating *D* as a logical constant leads to no revision, or if it does, we have a *pirma facie* case against treating it as a logical constant. Either way, absent further argument, supervaluational consequence will be non-revisionary. I'm also supposing something that will be later called into question: that *reductio* and the like are indeed entrenched in inferential practice. The point made here is essentially dialectical: if it is the case that whether or not supervaluational consequence is damagingly revisionary turns on whether or not we treat 'Definitely' as a logical constant, then that itself creates a *prima facie* case for the conclusion that it should not be so treated. Thanks to Ross Cameron for discussion of these issues.

[37] I am indebted to John MacFarlane's work on this topic, in particular MacFarlane, J. G., *What does it mean to say that logic is formal?* University of Pittsburgh, Pittsburgh. PhD Dissertation, 2000 and MacFarlane, J. G., "Logical Constants", in Zalta, E. N., (ed.) *The Stanford Encyclopedia of Philosophy*. http://plato.stanford.edu/archives/win2005/entries/logical-constants/. See particularly the latter for an overview of the debate and the labels used below.

elimination rules. There are those who see the demarcation as *pragmatically* drawn: as the *minimal* set of constants that are adequate for certain theoretical purposes: for the deductive systematization of science, for example. And the status of "Definitely" may well be an issue on which different demarcation criteria return different verdicts[38].

Further, it is controversial, even amongst those who buy into one this or that criterion, how to apply that criteria to 'hard' cases (really, anything beyond the 'traditional' logical constants of predicate logic). Within the permutation criterion tradition, for example, MacFarlane formulates an account of logicality that is applicable to rich languages: such as ones featuring modal and tense operators or the supervaluational setting we are presently working with. But the question of whether operators such as "Always", "Necessarily", "Actually" and "Definitely" count as logical constants turns out to be underdetermined by the permutation criterion alone. This illustrates just how far one can push an account of logicality, without have a definitive answer to the 'hard cases'[39].

As well as disagreements about the demarcation principle for logical constants, and disagreements about the applications of such a principle, there are, in addition, there those who would reject the very starting point: the idea that there is a privileged set of terms – *the logical constants* – to be theorized about. Consider, for example, a view considered explicitly by Tarski:

> Perhaps it will be possible to find important objective arguments which will enable us to justify the traditional boundary between logical and extra-logical

[38] We can hardly expect, for example, the pragmatic criterion to classify "Definitely" as logical (in that literature, even the logicality of the first-order existential quantifier is thrown into question).

[39] For what it is worth, I believe that MacFarlane's full story will probably classify "Definitely" and "Actually" as logical constants. But, as he himself emphasizes, there are several places for one sympathetic to his approach to the permutation criterion to resist this contention (see in particular, his discussion of the question of what to count as 'intrinsic structure').

> expressions. But I also consider it to be quite possible that investigations will bring no positive results in this direction, so that we shall be compelled to regard such concepts as logical consequence, analytical statement, and tautology as relative concepts which must, on each occasion, be related to a definite, although in greater or less degree arbitrary, division of terms into logical and extra-logical.[40]

In the present case, this position would admit *both* $\models^{+}_{global}$ and $\models_{global}$ as legitimate consequence relations, but insist that question of which is "really" logical consequence is misguided[41].

There is little hope, therefore, of extracting an answer to the question of whether "Definitely" is logical, without engaging four-square with this literature. I have no space to do this here. In light of the *prima facie* case given earlier, I regard $\models_{global}$ as the pretheoretic frontrunner, and if this is sustained, then supervaluational consequence is straightforwardly non-revisionary. However, we cannot ignore the other possibilities canvassed, though no argued for, above: that best theory will end up classifying 'Definitely' as a logical constant, or that the very question of whether or not 'Definitely' is a logical constant is misconceived. I therefore strengthen my case by arguing in the following section that *even if* 'Definitely' is a logical constant, and even supposing that $\models^{+}_{global}$ does provide counterexamples to conditional proof and the life, then using the resources constructed in this paper we can show that supervaluational consequence is not *damagingly* revisional in the way often suggested[42].

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[40] Tarski, A. "What are logical notions?" *History and Philosophy of Logic* (1986) 143-154. Transcript of a 1966 talk, ed. Corcoran, J. (p. 420)

[41] I mention this position as illustrative of a range of non-demarcation positions: it is the one I find most attractive. Another option is provided by 'debunkers' such as Etchemendy (Etchemendy, J., *The Concept of Logical Consequence* (Cambridge, MA: Harvard University Press, 1990))

[42] I will assume that showing that the relevant candidates for being 'logical consequence' are either non-revisionary, or not damagingly revisionary, will quiet the worries of those who favour non-demarcation views on logicality such as Tarski-inspired relativism.

I concentrate in what follows in the (alleged) revisionary implications for *inferential practice* of failures of the kinds of reasoning mentioned above. A quite different sort of concern is that

Non-damaging revisionism

I begin by pointing to an underlying assumption of the debate thus far. In stating, for example that '$|=^{+}_{global}$ fails to sustain conditional proof', we are assuming that a consequence relation *x* sustains conditional proof (for example) if and only if the following holds:

If $A, B \models_x C$, then $A \models_x B \supset C$

Let us call this mode of inference CP. CP is, I take it, the orthodox notion of 'conditional proof' within metalogic.[43]

However, we should note that the following pattern (which I will call CP*) is sustained:

If $A, B \models_{global} C$, then $A \models^{+}_{global} B \supset C$

That is, any pattern of reasoning which is valid by the lights of $\models_{global}$, will make the corresponding conditional a $\models^{+}_{global}$-validity. It would be pointless to start a dispute about whether this pattern should be called 'conditional proof'. But the fact CP* holds – whatever we call it – is highly relevant to the overall dialectic. To being with, it is relevant to the question of how extensive a revision of classical logic $\models^{+}_{global}$ induces, assuming turns out to produces counterinstances to CP in the first place. In the light of

probabilities (and by extension, degrees of belief) will have to be treated non-classically if global consequence makes p&~Dp a contradiction. I will not deal with this interesting issue here.

[43] Compare McGee and McLaughlin 'Logical commitment and semantic indeterminacy: reply to Williamson'. Linguistics and Philosophy 27(1) 2004 pp.123-136, esp section 3. Though formulated in different terms, the point I make in this section is closely related to their claim that there is an *inferential* version of conditional proof which is available to the supervaluationist even if the full *metatheoretic* one is unavailable.

it sustaining CP*, I suggest it would not induce a very extensive revision at all. Perhaps 'conditional proof' strictly so-called, fails; but every time we move from a $\models_{global}$-valid argument to the validity of a conditional, we are following a good inference pattern. Even if $\models^{+}_{global}$ turns out to be the one true consequence relation, the only instances of conditional proof that we have to 'give up' are those that go from $\models^{+}_{global}$ valid arguments that are not $\models_{global}$-valid, to the corresponding conditional.

So we have a systematic derived inference pattern – CP* - that just like conditional proof, allows us to draw 'categorical conclusions…on the basis of hypothetical reasoning'[44]. Analogous results for all the other disputed inference patterns[45].

The sense in which the failures of CP or 'conditional proof proper' *proper* should count as a 'revision' must now be reconsidered. In a highly theoretical sense, it may be that $\models^{+}_{global}$ is revisionary, where this is just to express the claim that CP is sustained by the classical logical consequence relation, but not (we are assuming) by $\models^{+}_{global}$. But I do not think that it was revisionism in this theoretical sense that was supposed to be the *damaging* indictment of supervaluationism. Williamson, in particular, talks of the revisionary implications of supervaluational consequence for *inferential practice* ('…supervaluations invalidate our natural mode of deductive thinking')[46]. But for this charge to be sustained, a case would have to be made that

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[44] Williamson, *op cit.*, p 152

[45] *Proof.* Whenever $\models_{global} \Phi$, we have $\models^{+}_{global}$ Φ. Suppose that the $\models_{global}$ –validity of arguments $A_1,\ldots A_n$ entails the $\models_{global}$-validity of argument C. By the above observation, C will be $\models^{+}_{global}$ –valid also. So the rule that takes us from that $\models_{global}$ –validity arguments $A_1,\ldots A_n$ to the $\models^{+}_{global}$ –validity of C will hold. Reductio, argument by cases and the rest, as well as conditional proof, fit this pattern.

[46] Williamson, *op cit.*, p 152

conditional proof proper, and not just CP*, underlies practice. That is, we would have to show that inferential practice mandates moving from $\models^{+}_{global}$-valid but $\models_{global}$-invalid arguments, to the corresponding conditional. No such case has been made, and I contend that none is available. If so, then supervaluational consequence is *revisionary of classical logical theory* but not *revisionary of inferential practice*. In the light of its conservatism with respect to inferential practice, I contend that whatever revisionary consequences it has will be undamaging.

Even if $\models^{+}_{global}$ rather than $\models_{global}$ is that correct supervaluational consequence relation, the characterization of $\models_{global}$ has instrumental value: it allows one to formulate rules such as CP* which undermine the Williamson case that supervaluational consequence is revisionary in a damaging sense.

7 Conclusion

One headache for supervaluationists in recent times has been the alleged logical revisionism induced by their semantic framework. I say that there is a natural framework for supervaluationists that undermines the arguments for 'damaging' revisionism offered in the literature. Moreover, the setup – involving 'extreme' delineations – is one that supervaluationists have independent reasons to accept.

The arguments are undermined in one of two ways. The most straightforward case arises if one accepts my assumption that 'Definitely' is no logical constant. If so, supervaluational consequence demonstrably matches classical single-conclusion consequence (and in particular, it sustains conditional proof, reductio et al are retained).

For a setting wherein ‘Definitely’ is treated as a logical constant, the issue is more vexed. No clean arguments either for or against the logical revisionism *stricto sensu* are forthcoming.

Questions of logicality are notoriously hard to arbitrate. It is important, therefore, that we see that even if ‘Definitely’ does turn out to be a logical constant and that counterexamples to classical modes of inference *stricto sensu* arise, that this in no ways threatens the cogency of inferential practice.

Supervaluational consequence is not revisionary. And even if it is, it is not damagingly so.

A Appendix

One can think of global supervaluational logic, as defined in this paper, as a modal logic where on each model, sentences are true if they are true at *a set of actually worlds A* rather than at a single actual world *a*.

Call modal models of the second kind *classical models*, and modal models of the former kind *extended models*. $\Gamma \models_c \Phi$ holds when every classical model *M* such that every formula in Γ is true-on-*M*, is such that Φ is true-on-*M*. $\Gamma \models_{sv} \Phi$ holds when every extended model M' such that every formula in Γ is true-on-M', is such that Φ is true-on-M'.

The operator 'definitely' is to be thought of as a necessity operator whose accessibility relation (in the simplest cases) is an equivalence relation which on the *intended model* relates distinct worlds iff they are both in *A*. But the accessibility relation governing 'definitely' may by very different in *unintended interpretations*.

If we can show that this alteration to modal framework does not change the extension of the consequence relation, then we shall have sustained the central claim of this paper: that global supervaluational consequence is non-revisionary. For any inferential rules that fails in the extended setting would have to also fail in the setting of a classical modal logic. I now argue that the consequence relation defined via extended models does indeed coincide with the consequence relation defined via classical models.

First, notice that for every classical model, there is an extended model that makes true exactly the same formulas: If *a* is the actual world of a model *M*, then let $A := \{a\}$ be the set of actual worlds of an extended model M'. Clearly *M* and M' make true all the same formulae. This immediately gives us the following:

$$\Gamma \models_{sv} \Phi \Rightarrow \Gamma \models_{c} \Phi$$

So we will have shown the two notions equivalent if we establish the following:

$$\Gamma \not\models_{sv} \Phi \Rightarrow \Gamma \not\models_{c} \Phi$$

Suppose that $\Gamma \not\models_{sv} \Phi$ holds. Then there must be some extended model *M** such that every formulae in Γ was true-on-*M** but Φ was not. For each $\Psi \in \Gamma$ to be true-on-*M**, Ψ has to be true at each world in A_{M*}. For Φ to fail to be true-on-*M**, it has to be false at some world in A_{M*} : call this world *a**. Now consider the classical model *M*** which differs from the extended model *M** just by replacing the set of actual worlds A_{M*} by the single actual world *A**. By construction, every $\Psi \in \Gamma$ is true at *a** and Φ is false at *a**. Consequently, every $\Psi \in \Gamma$ is true-on-*M*** and Φ is false-on-*M***. Thus, *M*** provides a countermodel to $\Gamma \not\models_{c} \Phi$. QED