Tenable conditionals

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1 Ramsey tests

When should we believe a indicative conditional, and how much confidence in it should we have? Here's one proposal: one *supposes actual* the antecedent; and sees *under that supposition* what credence attaches to the consequent. Thus we suppose that *Oswald did not shot Kennedy*; and note that under this assumption, Kennedy was assassinated by someone other than Oswald. Thus we are highly confident in the indicative: if Oswald did not kill Kennedy, someone else did.

When should we believe a counterfactual conditional, and how much confidence in it should we have? Here's one proposal: one *counterfactually supposes* the antecedent; and sees *under that supposition* what credence attaches to the consequent. Thus we suppose that *Oswald had not shot Kennedy*; and note that, given this, Kennedy probably would not have been assassinated at all. Thus we are highly confident in the counterfactual: if Oswald had not killed Kennedy, no-one else would have.[1]

Let's use $P_A(B)$ as shorthand for 'the probability of B under the supposition-as-actual A'; and $P^A(B)$ for 'the probability of B' under the supposition-as-counterfactual of A. The Ramsey tests we have just appealed to can then be written as follows:

1. $P_A(B) = P(A \rightarrow B)$
2. $P^A(B) = P(A \mathbin{\Box\!\!\rightarrow} B)$

One possible defence of each equation is to take the probability of conditionals as given, and understand the suppositional probabilities in terms of it. The opposite thought is to take the suppositional probabilities as basic, and use them as a tool for understanding the logic and semantics of each kind of counterfactual.

If one is tempted by the latter project, then the natural thing to look for is some independent fix on the two kinds of supposition in question. Some may claim that this is supererogatory—after all, the supposition in question seems a perfectly familiar sort of mental process, and we surely can surely operate with it perfectly well with no more guidance than the little introduction I gave above.

This may well be so. But even if no 'conceptual analysis' of supposition is needed, it's worth thinking about what *suppositional probability should be*. If someone claims to assign *crazy* suppositional probabilities, we'd like to be able to explain where he was going wrong. So—if we can find it—we gain much by finding some independent traction on P_A and P^A.

In the case of supposition-as-actual (A-supposition) there is a well-trodden route to go down. A-suppositional probability is taken to be *conditional probability*, understood via the 'ratio formula':

[1] For discussion of counterfactuals and credences, see Moss (manuscript), who connects credences in counterfactuals to chance in ways that suggest the connection articulated below (though this is not explicitly endorsed by Moss).

3 $P_A(B) = P(B|A) = P(AB)/P(B)$

Such an equation is perhaps not exceptionless. In particular, when A is probability zero, the right hand side goes undefined. Many feel, however, that the left hand side can take definite values. But within its range of operation, this characterization is overwhelmingly endorsed.

What of supposition-as-counterfactual (C-supposition)? Here is there is far less discussion, but we can find a host of options by looking at the literature on the foundations of causal decision theory (Joyce, 1999). The most impressive, and simplest, it seems to me, is the identification of C-supposition probability with one's *expectation of conditional chance*—via a 'conditionalized version' of the principal principle:

4 $P^A(B) = \sum_x x \cdot P(Ch(B|A) = x)$

Again, this principle is not exceptionless. This time, the most important exceptions arise when P contains 'inadmissible information' (see Lewis (1980)). But again, in cases with 'no funny business', the constraint seems overwhelmingly plausible.

Putting these together, we get the following pair:

5. $P(B|A) = P(A \to B)$
6. $\sum_x x \cdot P(Ch(B|A) = x) = P(A \mathrel{\Box\!\!\to} B)$

Disaster strikes. (5), the 'conditional construel of conditional probability' (CCCP) is subject to an apparently devastating array of no go results and impossibility theorems.[2] (6) is subject to much the same problems.[3] So the the Ramsey test equations seem in trouble. Joyce (1999) calls the former 'the most comprehensibly refuted thesis in philosophy'.

One reaction at this point is to look again at the right-hand side of the equations (1) and (2) above. It *seems* to be relating suppositional probabilities to the unconditional probability of some proposition. But can't we see the right hand sides instead as a stylistic variant of the left hand side? Can't we, in other words, see *conditionals* as devices for expressing A- or C-suppositional probabilities, dressed up in the garbs of a declarative sentence? This is Adams' approach to indicative conditionals (Adams, 1975); and the extension to counterfactual conditionals (via C-supposition understood as corresponding to expectations of conditional chance) is advocated by Skyrms (1994).

The obvious problem with this approach is its radically revisionary nature. This approach makes the conditional an orthographic accident, and doesn't give conditional 'sentences' truth-conditions at all. To one wedded to a truth-conditional semantics for natural language, this is terrible.

[2]See Hájek & Hall (1994) for a review of the literature, which was kicked off by Lewis (1976).
[3]See (Williams, manuscripta), (Briggs, manuscript).

Another reaction is to ditch (3) and (4). Perhaps the direction of explanation flows instead from right to left in (1) and (2). It might be suggested that our best grip on *A*- and *C*-supposition comes from our grip on indicative and counterfactual conditionals. In the case of counterfactual conditionals, this is close to the view advocated by Joyce (1999).[4] Given a 'worlds-semantics' for each kind of conditional (Lewis, 1973; Stalnaker, 1968), it turns out that there are systematic strategies for finding readings of P_A and P^A ('imaging probabilities') that play the Ramsey-role for the respective conditionals.[5]

I think this approach is highly problematic. To be sure, there are *definitions* of P_A and P^A that do the job. But presumably if the Ramsey test itself does not illuminate the conditionals, what we should do instead is look to the sort of *independently motivated* accounts of indicative and counterfactual closeness that are given in the literature for illumination. And when we think through the implications of many such treatments (e.g. Lewis, 1979; Williams, 2008; Nolan, 2003) it becomes clear that whatever these quantities are, they're not good candidates for probabilities under supposition.[6]

The strategy I will pursue here is to take the triviality results head on. I will argue that we have the resources to resist, in a principled and non-ad hoc way, the various impossibility/triviality results in the literature. So the paper will amount to a defence of 'Ramsey conditionals'—conditionals that satisfy the Ramsey identities given above.

2 Preliminaries

Much of our task will be engaged in sorting through the various problems that have been raised for Ramsey conditionals. Each is independently resistable—and the resources I will appeal to are not novel. But the greatest danger to Ramsey conditionals, I think, is the sense that to defend them is to try to push forward a degenerating research project. Impossibility result has followed no go theorem has followed triviality argument—and the possible escape routes for friends of Ramsey conditionals has each time been narrowed. It is easy to picture a desperate defender of Ramsey conditionals, leaping from monster-barring hypothesis to monster-barring hypothesis, losing friends and influence along the way.

Let me set my stall out straight away, therefore, on a number of points.

First, the version of the Ramsey test(s) I am interested in defending is *restricted* to cases where the antecedent and consequent are not themselves conditional. This seems to me a principled restriction. The primary *data* that convinced people that something like the Ramsey test was compelling for indicatives (at least) involves 'simple conditionals' of exactly this form. We don't have to stray far from this (to right-embedded indicatives of the kind described by

[4]Joyce (1999) favours weakening the Ramsey test somewhat, to the following pair of 'Ramsey bounds':

1* $P_A(B) \geq P(A \to B)$

2* $P^A(B) \geq P(A \mathbin{\Box\!\!\rightarrow} B)$

Anyone denying 'Stalnaker's assumption' would be well advised to go for the above formulation.

[5]See Lewis (1976) and Joyce (1999) for more discussion.

[6]See (Williams, manuscriptb) for discussion.

McGee (1985)) to find cases which intuitively *do not* satisfy the Ramsey test.[7] So appeals to intuitions support the Ramsey test for 'simple' conditionals; its extension to all conditionals seems to me a rather unwarranted. (I'll say more about why this restriction seems interesting and principled below).

Second, I do not assume that any probability function whatsoever describes the credences of a rational agent. I'm happy to contemplate the possibility, for example, that rationally permissible credences should be *regular*—shouldn't assign extremal credences except to non-contingent propositions. And I'm happy to contemplate an ideal whereby our beliefs about our current credences need to mesh with those credences (a formal version of the ban on 'Moore paradoxical' beliefs—perhaps articulated via a limiting case of the 'reflection principles' of van Fraassen). Coherence requirements go beyond what is ruled out by the probability axioms. I don't see anything wrong in principle with such theses—logic is not the whole of rationality. This will be crucial; later I argue that certain probability functions are 'incoherent' simply because conditional and unconditional credences don't line up properly.

Third, conditionalization is not sacrosanct as a principle of belief updating. To be sure, there are arguments for it (e.g. diachronic dutch books). But it's a piece of theory as much as anything else. Most fundamentally, even if we accept that *all rational updating is conditionalization*, we needn't accept that *updating by any proposition A is rationally permissible*. Suppose an ideal agent receives what we would prima facie describe as the information that *A*—say that there is a ball in front of her. Simply updating on that information (or some phenomenalistic variant) will lead us into odd places—for example, it can lead us to violate 'Moorean' belief reflection requirements. The 'total information' by which the belief state is updated goes beyond *A*. Hence, *A* alone might simply be not available as a piece of 'total information' for *ideal* agents, simply because such agents must also ensure that they're meeting all their other obligations too.

Furthermore, it has been often suggested that conditionalization is too blunt a tool for the array of ways in which information can reach us. For example, we might gain information about what our *probability distribution* across some partition should be, rather than a single proposition holds. A generalization of standard conditionalization—Jeffrey conditionalization—is designed to accommodate this.

Combining these two points, I will later be using a generalization of conditionalization *updating by minimizing information-loss subject to a constraint*. The constraint might be that a certain proposition gets probability 1 (in which case this recipe coincides with conditionalization) or that a certain probability distribution is enforced (in which case it coincides with Jeffrey conditionalization). Or, to pick up on our earlier example, it might be that *A* is to get probability 1 *and that Moorean connections between beliefs, and beliefs about those beliefs* be sustained. Once we have more than just *propositional* constraints on 'rational credences', such a generalization seems *exactly* what we need in order to make rational updating formally tractable.

Fourth, we need to be very careful in thinking through the domain of 'worlds' over which we represent credences. Possible worlds are the philosophers' tool—they shouldn't rule the roost.

[7] McGee's cases are embedded conditionals such as 'If that's a fish, then if it has lungs, it's a lungfish'. Thinking through the details, one sees that the intuitively high probability of this conditional is in tension with the general Ramsey identity. See Williams (2009) for discussion.

Some would have us believe that in all metaphysically possible worlds, Hesperus is Phosphorus. If they are right, then the natural line to take in representing credences is to *not* work with metaphysically possible worlds, but with 'epistemically possible worlds' of some variety instead, of which some vindicate the identity and some do not. Without this, we couldn't assign a 50/50 credence to 'Hesperus is Phosphorus', which is very strange. (I know that some philosophers, following Stalnaker (1984), are convinced that discipline imposed by working exclusively with metaphysical possibilities is useful. That is a bold and provocative line to take; perhaps it is right, but it's a funny place to *start*). Likewise, if distinctions drawn by an agent's credences turn out to be intuitively 'too fine' to be captured by the space of possible worlds we start out with, we should have no compunction in expanding credal space to better represent their state of mind. If we need 'metaphysically impossible worlds' to do this, what's the problem? The constraint is that we know what we're working with, and that we make sure that important theoretical connections don't go astray. Beyond that, there's no sin in cutting one's worldly cloth to suit one's explanatory purposes.

All these four theses I am quite prepared to defend independently of any thought of conditionals. If I appeal to them at some point, I don't see anything unmotivated or ad hoc in doing so. On the contrary, I think the burden is on others to point to why the otherwise reasonable tactics are in this instance problematic.

3 Outline of what is to come

With all the cards on the table, I lay out the plan of campaign, and relate what I will say to the impossibility results that face Ramsey conditionals.

The first issue I discuss (**section 3**) is an additional motivation for the restricted form of Ramsey identity I endorse above. I relate this to the role that conditionals play in providing qualitative reports of the quantitative rationalizations of behaviour captured by decision theory.

Some of the 'no go' results for Ramsey conditionals are avoided simply by this restriction of the thesis in question. Stalnaker shows that (given a suitably strong logic) the Ramsey test can't be met in full generality. But such results do not speak to the tenability of Ramsey test for simple conditionals. I've argued that this restricted version of the thesis is not ad hoc—the evidence that supports the Ramsey test motivates no stronger thesis. I argue briefly below that the *function* for which Ramsey conditionals are needed are met by the restricted version of the thesis.

After this, I turn to defending Ramsey conditionals. The first piece of the jigsaw is the van Fraassen tenability results. I'll explain how these show that the Ramsey test for *simple* conditionals can be satisfied—first in the case of indicatives (**section 4**), and then in the case of counterfactuals (**section 5**).

The van Fraassen constructions essentially involve distributing credence over structures richer than 'metaphysically possible worlds'. As highlighted above, I don't think of this as any kind of problem—if the space of metaphysical possibilities isn't rich enough for our purposes, we simply introduce a more fine-grained backdrop. Indeed, one way of thinking of the van

Fraassen worlds is simply as maximally consistent sets of sentences, where those sentences include *conditionals*. It would be rather odd if we didn't allow ourselves such resources in representing an agent's beliefs about conditionals. Some of the most general 'impossibility results'—in particular, Hajek's 'wallflower argument' (Hájek, 1989), are simply inapplicable in this setting. The diagnosis of their failure is that they try to foist upon us what seems to me an unmotivated constraint to work only with a set of worlds, individuated in terms of what non-conditional truths hold at them.

The second compulsory question for a friend of Ramsey conditionals is how they behave under the impact of new information—how belief update works. In **section 6** I lay out a recipe for updating on arbitrary information, that ensures that updating never takes us from Ramsey-friendly credences to a credence set that violates the identities.

This story address dynamic triviality results—from the arguments of Lewis (1976), to the very general 'peturbation' results of Hájek (1994). The characteristic feature of these results is that they show that the Ramsey test cannot hold of a pair of probability functions, P and P', related in some specified way. My primary response to this is that Lewis and Hájek describe credences that don't correspond to the state of mind of any rational agent. This would cut little ice if I declared all *reasonable candidates* to be the resultant credences of updating on information that A to be irrational. But the story sketched in this section addresses this challenge, by showing that we say how update goes.

In the final section, (**section 7**), I turn to a long-delayed issue—what the actual truth values of Ramsey conditionals built by the van Fraassen construction are. I offer a smorgasboard of options. One—molinist—perspective takes the points of the van Fraassen construction to describe 'metaphysically possible worlds'—one of which is actual. All conditionals have definite truth-values—though their truth-values do not supervene on the non-conditional facts. I think molinism is a rather difficult account to understand—not only because of its apparent alethic extravagence, but also because it makes the rational force of the Ramsey test hard to understand.

A second perspective holds that the points of the van Fraassen construction cut finer than the metaphysically possible worlds. So, in the general case, we will have a *set* of van Fraassen worlds, each equally 'conditional enrichments' of the actual worlds. This situation poses a puzzle: should we say that a conditional is true, or false, when it is true at some enrichment of actuality, and false at another? I investigate what sort of view we need to take of this kind of 'indeterminacy', to make our views cohere.

4 Motivations for the restricted Ramsey test

I said above that I would defend the Ramsey tests *restricted* to the case of simple conditionals. It is important to me that this is a motivated restriction. One rationale for this is that our evidence motivates no stronger thesis. Still, one might think this is ad hoc (imagine someone defending the claim that emeralds are green at least until the year 2010, on the basis that our evidence does not support any stronger claim).

One reason I am profoundly comfortable with this thesis, is that it discharges what I see as the primary function of Ramsey conditionals. A primary role for conditionals—why they *matter* to us—is because of the role they play in theorizing about value and utility, as regimented via various *decision theories*.

Let us assume that decision theoretic *pure acts* and *states of affairs* can be characterized in purely factual (non-conditional) terms. Then the operative notions in the most elegant decision theories (I argue) are exactly the notions $P_A(B)$ (for evidential decision theory) and $P^A(B)$ (for causal decision theory)—where A expresses a possible act, and B a possible outcome of an action. The function I see for Ramsey conditionals is as allowing the communication of information about these key components in decision theory. If $P^A(B) = P(A \Box\!\!\rightarrow B)$, then by asserting a counterfactual conditional one conveys information critical to the evaluation of expected utility—the *efficacy* of a course of action. If $P_A(B) = P(A \rightarrow B)$, then by asserting an indicative conditional one conveys information critical to the evaluation of expected value—the *auspiciousness* of a course of action. I argue elsewhere that the availability of Ramsey conditionals is what we need, to connect qualitative practical reasoning (typically invoking conditionals) with the quantitative underpinnings decision theory describes.[8] But we only need this for conditionals connecting up acts (as antecedents) and outcomes (as consequents). If these are both factual, as I suggested, then so long as conditionals, of each kind, pass the Ramsey test for *factual* antecedents and consequents, they've fulfilled their function.

One might resist the above characterization, arguing that we have an interest in calculating the utility or value of more than just *acts*. What of the desirability of Party X winning the election? This is not a 'possible act', but decision theory, I believe, should aim to give a story about how its desirability relates to one's intrinsic values. Again, what of complex, conditional acts (something nearer to what we might call 'plans'—for example *making coffee if the coin lands heads; making tea if it lands tails*). Here we have a non-factual proposition—but it seems like the sort of thing we should assess for efficacy and desirability.

It may appear I am committed to the problematic thesis that the locus of well-defined value or utility be *factual* propositions. This, it seems, is what *no truth-conditions* theorists (in the tradition of Adams and Skyrms) will have to say. Presumably, they will have to provide some kind of ersatz for desires and acts naturally spelled out in terms of conditionals.

It is in advantage of the view I develop that I don't have to take this ultra-strong position (though of course, if the Adams/Skyrms tradition have a decent response to the above puzzle, I could always piggyback on it). Grant therefore, that conditionals express a proposition. Utilities and/or values are defined over all propositions, conditional and non-conditional, via the decision theoretic equations (phrased not in terms of conditionals in the first instance, but in terms of A-suppositional and C-suppositional probabilities). What the restriction of the Ramsey test to simple conditionals gives me is a constraint on *reporting* the factors that underpin such attributions of utility and value in conditional form.

So we go beyond the standard no truth value tradition. When the acts and outcomes in play are factual, we have something more: we can use (Ramsey) conditional locutions in *practical reasoning explanations*—to report the factors that determine the expected value or utility. If the Ramsey test holds only for simple conditionals, we cannot assume they play this role when

[8] See Elstein & Williams (manuscript)

the acts or outcomes are themselves conditional.

Now, I don't deny it would be *very nice* if we could give a unified account here, but I do not regard it as that much of a cost if, when we move beyond factual acts and outcomes, we have to find new, more perspicuous ways of reporting the decision-theoretic underpinnings. Furthermore, it doesn't *appear* that the conditional locutions we in fact have *do* communicate the relevant (suppositional) information—for with complex conditionals, unlike simple ones, the Ramsey test doesn't have much intuitive plausibility. So there doesn't seem to me any data to explain away here—while it might be nice to have a tool for every job in the vicinity, it seems we don't.[9]

5 The van Fraassen construction

van Fraassen indicatives

We start with an agent with opinions about matters of fact. Because they stand in various proportions to one another, this leaves him with various *conditional probabilities*. Our agent also has *dispositions to adopt certain credences upon A-supposing* which (in line with the majority) I'll take to be normed by her conditional probabilities.[10]

It's frustrating to have suppositional beliefs, playing an important role in one's mental economy, but not have any way to communicate about them. So the speaker starts using an indicative conditional $A \rightarrow B$, with the rider that its acceptability should go by the extent to which the B is likely, on the supposition-as-actual that A.

Many theorists can buy into this cartoon picture. But the question is: where to go next. In the 1970's, Stalnaker sketched a project which started from the sort of 'cognitive role' of conditionals just described, and invited us to use this a way of deciding what the proper logic and semantics of conditionals was—it was whatever it needed to be to vindicate the cognitive role. It is in this spirit that we turn to the van Fraassen construction. To begin with, over what domain are probabilities of conditionals defined? Maybe we can make do with the space of all 'factual possibilities' over which the agent's base probabilities are defined. But that's not the obvious starting point (nor is it necessarily a principled one—see my initial comments on methodology). The obvious route is to look at a space of logical possibilities from which we can read off the truth-values of simple conditionals. To do this, we work not with the factual possibilities alone, but with pairs of factual possibilities *paired with* total orderings of the space of possibilities—and use Stalnakerian truth-conditions for conditionals to use the total orderings to read off the truth-values of a simple conditional at an arbitrary pair.[11] More

[9]If I were convinced that there was no principled stopping point here, and the game was full Ramsey or nothing, then I would turn to the work of Stalnaker & Jeffrey (1994) rather than van Fraassen (1976) for resources to pursue the project.

[10]Note that these suppositional tendancies will be fact-to-fact—i.e. of the 'simple' kind focused on earlier.

[11]We constrain this by the 'centering' assumption—the first element of the o-ordering at w is w itself—-'every world is closest to itself'. Note the ambiguity in what is ordered here—is it the *factual possibilities* or the *possibility-ordering pairs* themselves? In van Fraassen's construction, it is effectively the former; but we then have a canonical way for reading off an ordering of the latter—allowing conditionals of arbitrary complexity to

specifically, the conditional $A \to B$ is true at $\langle w, o \rangle$ if the 'closest world' to w at which A is true, is one in which B is true. Regarding the ordering as an ω-sequence of worlds, this is to say that $A \to B$ is true at $\langle w, o \rangle$ exactly when the first world in the ordering o at which A is true, B is true also. (Note the restriction to 'factual' antecedents and consequents—we'll address this in a moment).

So we have a space to work with—the ordered pairs $\langle w, o \rangle$. Two questions arise. First, what are the *actual truth values* of conditionals? Granted we have a recursive story about their truth values at $\langle w, o \rangle$ pairs—but how do we 'deparameterize' to say something about their truth or falsity simpliciter? I delay answering this question till later, since (important though it is) little in the van Fraassen construction turns on it. The second is: what *credences* should we have over this space of worlds? Once we have an answer to this question, we can read off credences for conditionals, since we can identify the probability of a conditional with the credence assigned to the set of $\langle w, o \rangle$ pairs at which it is true.

The heart of van Fraassen's constructions is a recipe for taking us from a probability function defined over the 'base space' of factual possibilities w, to an 'extended probability' function over $\langle w, o \rangle$. As noted previously, this gives us credences over simple conditionals—but we haven't yet said anything about credences in conditionals in general. But van Fraassen offers a way of reading off a given ordering of 'factual worlds' w an ordering $\bar{o}$ over $\langle w, o \rangle$ itself. Defining the truth-values of conditionals at $\langle w, o \rangle$ Stalnaker-style via the ordering $\bar{o}$, the recipe for distributing credence fix credences in all conditionals, simple or complex (and compounds thereof). Details are given in an appendix, but for present purposes we can treat them as a 'black box'. The important point is that we can pick out, uniquely, an extended probability that (i) disturbs not at all the probabilities assigned to factual propositions; (ii) satisfies the Ramsey identity for simple conditional; and (iii) gives a principled rule for assigning credences in any conditional or compound thereof.

van Fraassen counterfactuals

What of counterfactual conditionals? The desiderata here is that conditionals match up to expected chance.

We start with a space of 'factual worlds'. Again, the basic idea is to represent credences in counterfactuals over a space that fixes the truth-value of all counterfactuals, $\langle w, o \rangle$—with the orderings now interpreted as counterfactual rather than indicative closeness.[12]

Our method of construction differs. We assume that each factual world w comes with a chance function Ch_w, assigning weights to a field of subsets of the factual worlds. Just as we previously extend by the van Fraassen recipe a probability function P to an probability over the extended space, we expand a given Ch_w to a extended chance assignment Ch^*_w over the whole space of world-ordering pairs. We secure therefore the analogous result:

$$Ch^*_w(A \mathrel{\Box\!\!\to} B) = Ch_w(B|A)$$

receive truth-values at world-ordering pairs.

[12]Again, we can extract an ordering over this space, from an ordering o over factual worlds.

This shows us how the chance of a counterfactual could equal the conditional chance. What we want, of course, is the *credence* in a conditional equaling the *expectation* of conditional chance. So we need to start with our *credences* over the factual worlds, and extend this over the world-counterfactual-ordering pairs. I will assume that there is 'no inadmissible information' involved, and so the base credences line up with expected chance (by the principal principle).

To get the idea here, let's pretend we have a finite space of worlds to begin with. Then the idea will be to 'fracture' the credence assigned to w in the base case, and divide it amongst the various orderings $\langle w, o\rangle$. The obvious idea is to do this relative to the (extended) chances that obtain at that world, $P^*(\langle w, o\rangle) = P(w) \cdot Ch^*_w(\langle w, o\rangle)$. More generally, where ϕ is some proposition over the extended space, we set:

$$P^*(\phi \wedge w) = P(w) \cdot Ch^*_w(\phi)$$

Recall that $\sum_w P(w) f(w) = \sum_x x \cdot P(f = k)$, where $f = k$ is the proposition true at w iff $f(w) = k$. Letting $Ch^*(\phi) = k$ be the proposition true at w iff $Ch^*_w(\phi) = k$, we have:

$P^*(\phi)$	$=$	$\sum_w P^*(\phi \wedge w)$	since w form a partition
	$=$	$\sum_w P(w) \cdot Ch^*_w(\phi)$	construction of P^*
	$=$	$\sum_w x.P(Ch^*(\phi) = x)$	by the above.

So we have derived the principal principle for the extended space. For the special case where ϕ is a simple counterfactual, we have:

$$P^*(A \mathrel{\Box\!\!\rightarrow} B) = \sum_w x.P(Ch^*(A \mathrel{\Box\!\!\rightarrow} B) = x) = \sum_w x.P(Ch^*(B|A) = x)$$

(the final identity is secured by the earlier result relating chances of counterfactuals to conditional chances).

So as promised, the credence in a simple counterfactual is the expected conditional chance. It's interesting to note that this argument works *only* when the conditions for applying the principal principle (relative to factual propositions) are met. And this seems right. In the case where I have 'inadmissible information'—say, the oracle tells me that the chancy coin flip is going to land heads—then my probability for 'if it were flipped, it'd land heads' should be near 1, rather than fixed to the corresponding conditional chance.

6 Updating

The van Fraassen constructions can be seen as a black box—put in a credence assignment, or credence assignment and information about chances—-and one gets out an extended credence passing the Ramsey test (wherever that is applicable).

They address, therefore, the static impossibility results. Stalnaker's no go results don't apply, due to the restricted nature of the thesis defended. The arguments of (Hájek, 1989) don't get traction, since our credal space is richer than he allows. We haven't yet addressed the static worries of Williams (2009)—we'll deal with that when we come to consider what we should say about the truth values of conditionals in the setting currently under discussion.

But another famous class of impossibility results turned on updating beliefs. So I will now discuss how belief change should be handled.

Here's one take on the above. Agents, fundamentally, are interested in factual (non-conditional) issues. This fixes their conditional credences and expectations of conditional chance. And indicatives and counterfactuals are in the first instance simply devices for expressing such desires in sentential form. We go beyond the truth-value theorists in saying that there are propositions that the conditionals express—and this means we can allow free compounds of conditionals, and use standard machinery to give an account of the desirability of a given conditional truth, the utility of conditional act, and so forth. (In this, we go far beyond what no truth value theorists can give). But the overarching constraint—the explanatory basis—is the Ramsey test for simple conditionals.

Lewis's triviality proofs (and their successors) show us that if the Ramsey test for an indicative is met (even for a single, simple conditional $A \rightarrow B$) by a probability function P, then it must be violated in one of the two probability functions P_B and $P_{\neg B}$. But (thinks Lewis) the Ramsey test wasn't supposed to be a local matter—it was supposed to be *generally* true. The Ramsey constraint for "If I draw red, I'll draw diamonds" shouldn't be so fragile that it would be violated if we happened to receive the information that *I draw diamonds*. Hájek (1994) shows that a similar situation (again with a single, simple conditional) happens on *all sorts* of ways of updating information.

I want to insist that the result of conditioning P with the information *I draw diamonds* doesn't describe the credences of a rational agent. On the present theory, one is rationally required to match one's credence in a simple conditional $A \rightarrow B$ to the corresponding conditional probability. Probability functions that violate this are as 'incoherent' as those that violate the probabilistic constraints linking the probability of $A \wedge B$ to the probabilities of A and the probability of B (of course we sometimes violate such constraints—-but it's a pro tanto flaw in us when we do). This I think of as analogous to the earlier cases or violations of Moorean harmony between beliefs and beliefs about our beliefs—if we treat the *prima facie* information A that we received as *total* information, then constraints on ideal rationality will be violated. In the earlier case, it was Moorean harmony. In our current case, it is the constraints connecting beliefs in conditionals and conditional beliefs.

So *simply* noting that the Ramsey test is violated in some probability assignment isn't terribly worrying. But Lewis and Hájek have a stronger case than this. Perhaps some probability functions are incoherent in this way—but we don't want to lose any grip on how rational agents update by saying so! Lewis can challenge the friend of the Ramsey test: what do *you* think should happen, when we apparently receive information like *the draw will be diamonds*? Hájek's result effectively shows us that a whole range of answers will get us into trouble.

Here is one, limited, kind of dynamics we could give that would guarantee that we never violate the Ramsey test.[13] Start with any credences over factual propositions—-which via van Fraassen extends to credences over conditionals. Say we receive the information that *A*—where *A* is factual. How to react? Here's a recipe: look at the original base credence over factual propositions, conditionalize on *A*, and rebuild via van Fraassen to discover the new conditionals. But such a recipe is restricted—it will work only where we the proposition we start with isn't a conditional. But how shall we react when we learn conditional information (from testimony, say)?

A more general perspective is available. Here's the picturesque version.

Think of the space of all probability functions spread out in front of you. Most are incoherent—even if they meet the standard probability axioms, they violate the Ramsey test. But there is a dusting of 'coherent spots'—probability functions for which the Ramsey test is met.[14] Some of these are described by the van Fraassen constructions we just went through—they are those where, when you restrict attention to factual propositions, and expand via the van Fraassen construction, you get back to where you started. What Lewis and Hajek show is that, typically, updating (in all sorts of ways) on factual information *A* will lead you from a coherent spot to an incoherent spot—one where the Ramsey test is violated. What should we say instead?

Since we're dealing with updating on propositions subject to *coherence constraints* that go beyond those captured in the standard probabilistic axioms, it is natural to look for a general strategy to describe how updating (with prima facie input *A*) should work when these extra constraints are in place. Here is where we turn to the generalization of conditionalization mentioned earlier.

Information-loss updating proceeds as follows. Take belief space *B*—the set of all *probabilistically* coherent credences. Now represent total information received as a constraint on belief space—a subset *C* of *B*. Intuitively, *C* includes all those probability functions that incorporate the total information received in some manner or other. But which of the ways of incorporating the information shall we choose? The information loss method says that (i) there is a privileged way to *measure* the distance between two probability functions—a measure that captures the *information-lost* in going from the first to the second.[15] (ii) rational updating, subject to the constraint *C*, should move to a probability function that meets *C* which is *minimally different from the starting point*—that loses the least information. Putting these together, the idea is that we move from prior credence *b* to posterior credence b' which is the *nearest* credence to *b* within *C*.

Where the constraint is that $P(A) = 1$, for a proposition *A*, it turns out that b' is the result of conditioning *b* with *A*. Hence the recipe is a generalization of conditionalization. It stands in a similar relationship to Jeffrey conditionalization. But it is far more flexible. Suppose one wished to represent updating for Moore-ideal subjects. Then we will first look at the set *M* of credences that are Moore-ideal, as well as meeting the probability axioms. Rather than

[13] Drawn from Leuenberger (manuscript).

[14] Distinguish *Ramsey coherence*—that the Ramsey test is met (for simple conditionals) from *van Fraassen coherence*—that credences over conditionals and their compounds are related to credences over factual propositions via a van Fraassen construction. I will assume here that van Fraassen coherence is the constraint in play—pending better understanding of what the credences in arbitrary compounds should be.

[15] The Kullback-Leiber information gauge is the following: $\sigma(P, Q) = \Sigma_{w\in\Omega}(Q(W)\log(Q(W)/P(W)))$

updating on the constraint that $P(A) = 1$, we update on the constraint that $M \wedge P(A) = 1$.[16] If we move to a nearest credence meeting this constraint, we have guaranteed that the credences will be Moore ideal as well as incorporating the information that A (hence, our 'total information' gained in the update will include that one believes that A).[17]

The very same recipe works in the current case. Rather than updating on factual information A alone (i.e. updating by minimal information loss subject to the constraint that $P(A) = 1$) we add in the constraint of rationality that the probability of conditionals are the conditional probabilities—in whatever form we think that constraint is plausible as a rational requirement. Notice that *any* content—conditional or non-conditional—can be updated on in this way. We therefore have a universal, independently motivated recipe for updating, generalizing conditionalization, that will ensure that the Ramsey identities are never violated. Of course, the recipe would be pretty awful if there weren't enough belief states around that satisfied the Ramsey test—but we are assured by the van Fraassen tenability results that there is at least one for each distribution of credence across 'factual' propositions. So I don't take this to be a great worry.

Notice that nothing in this response to the dynamic triviality proofs so far requires that conditionals be 'context sensitive' in any way. The truth values at each $\langle w, o \rangle$ pair are invariant—it's the *probability* that is shifting around under updating. But as we'll see, when we come to address the question of what the *actual* truth values of conditionals are, we'll need to revisit this question.

7 Truth values of conditionals

The above construction is all very well, but it's noticeable that we've said nothing about the truth-status of indicative or counterfactual conditionals *given how the world actually is*. Sure, we know what their truth value is at a pair of the form $\langle w, o \rangle$, where w is a 'factual' world and o is an ordering of all worlds. But what about actuality itself?

A natural thought goes as follows. Reality picks out some *factual* world—@, let us say—as 'actual'. In typical circumstances, we'd have a characterization of truth-values relative to this or that world. And the truth-status of p would then simply be its truth-status relative to @. However, in the current setting, there's an extra parameter—the choice of an *ordering*. And about this, we've said nothing as yet. And without a fix on what ordering to concentrate on, we haven't fixed the actual truth-values.

[16]That is, the relevant constraint is $C := \{b \in B : b(A) = 1 \wedge b \in M\}$.

[17]It is not *immediate* that the recipe picks out a *unique* credence state—though this will follow whenever the constraints in play are convex (closed under linear mixes). But this doesn't strike me as too worrying. I don't see an a priori reason to think that rationality has to narrow down our options uniquely. (See the current literature on mushy credences for discussion of related issues).

Sophisticated no truth value theorists

I think van Fraassen's own position is to try to finesse this question. He emphasizes the role of *logic* and *credences* in conditionals—and this his construction delivers. But why go beyond this and ask for a characterization of truth? Why not say that their degree of 'acceptibility' in a given context (given by rational credences) is sufficient.

This is a strategy familiar with the tradition of Adams (1975) (very familiar for indicatives, less prominent for counterfactuals—see *inter alia* Edgington (1986, 1995, 1997a), Skyrms (1994)). The crucial distinction is one of ambition. Adams-style theorists typically rule compounds of conditionals out of court, at best paraphrasing them in terms of simple conditionals. Lacking conditional propositions, have trouble with embeddings in propositional attitude contexts. The 'Frege-Geach' problems of embeddings are a headache for the traditional version of this approach, but the van Fraassen-based theory has a better time of it. After all, it *does* assign acceptibilities to arbitrary compounds of conditionals. And, since conditionals are associated with propositions, other embedding problems may become easier to handle. The project thus is close to the one explicitly advocated by McGee (1989). But are we forced to this reading?

Conditional and non-conditional facts

The opposite thought is that our worries about whether there's a *problem* of fixing truth values is ill-founded. Perhaps the 'facts' simpliciter do serve to determine an ordering of worlds. So for each w, we can 'read off' an ordering $o = f(w)$, and the actual truth of $A \rightarrow B$ then turns on whether it is true at $\langle @, f(@) \rangle$. (And we say this both for the case of indicatives, and the case of counterfactuals).

What should we say about the points $\langle w, o \rangle$ that are not of the form $\langle w, f(w) \rangle$? If are convinced that conditional facts supervene on non-conditional facts, we might say that these describe metaphysically impossible situations. The constructions above then require one to devote credence to epistemic possibilities that are not metaphysically possible.

Another option is to deny the supervenience of the conditional on the non-conditional (that is, conditionals can, in Dummett's terminology, be 'barely true'). There are indeed conditional facts, but it's genuinely possible for the physical facts to be held fixed, and the conditional facts to vary independently. Reality on this view has conditional as well as non-conditional aspects—and reality picks out an actual ordering just as directly as it picks out the world-element @.

In either case, consistent to say that there's some (relatively simple and illuminating) recipe in terms of purely non-conditional information, that tells us which o is appropriate at each world. Such an 'illuminating explanation' might be in the tradition of Lewis (1979).[18] Equally, we might take the facts about what the conditional facts are at actuality as *brute*—not admitting of any illuminating explanation (compare Hawthorne, 2005).

[18]If we endorse supervenience, this make take the form of a *reduction* of ordering facts to non-conditional facts. Without supervenience, we might regard such statements as 'laws of conditionals' (Chalmers (cf. 1996) on consciousness).

All this is *consistent* with the probability assignments given above. We might think there are facts about which conditionals are true, but be quite unopinionated about what these facts are (or, rather, opinionated in exactly the way described by the van Fraassen construction). But it's not wholly unproblematic. To begin with, if we think some illuminating explanation can be given about how the conditionals relate to the non-conditional facts, then presumably we have some idea how it should go. This amounts to taking a particular stance about how one should divide credence over the orderings consistent with a world *w*. But of course, once we place constraints like this, it becomes an open question whether van Fraassen's construction meets them. So one cost of thinking that there's some illuminating account to be given, is that one opens again the question of why the van Fraassen description of credences over conditionals should be taken to describe our credences. The idea of some illuminating relationship (even reduction) of the conditional to the non-conditional, fits badly with the sort of rhetoric I used above to motivate the construction—and the question is whether some replacement can be found.

Whatever the philosophical costs of treating conditional facts as 'barely true'—there being brute facts about which ordering is actually correct—at least it doesn't generate this sort of tension with the van Fraassen construction. But it does lead to other perplexing questions. What is it about these barely true facts, that means that we are under an obligation to meet the Ramsey test with respect to them? Why should we align credences in indicatives with conditional credences, for example? Above, I made an analogy with the case of conjunction—-violations of the Ramsey test should be as 'incoherent' as assigning higher credence to a conjunction than to either conjunct. But once one gives an account of what truth-function conjunction is, and assumes that credences aim at truth, then it becomes understandable *why* this constraint is in place.[19] But if all we say about conditional facts is that they brutely supervene on the non-conditional, it's obscure why this should be a requirement of rationality. (The dialectic here is rather similar to the accusations thrown against 'primitivists' about chance—theorists like Lewis ask why chance should have the distinctive cognitive role it is, if it is just some primitive operator attaching in various quantities to various contingent prospects).

Matters are slightly better for the case of counterfactuals. Here, the fundamental hypothesis connects up chances of counterfactuals with conditional chances. And even if counterfactual facts are brute, one might take this to describe a law-like connection between two kinds of fact. With that in place, we reduce the problem of why credences in counterfactuals should match expected chance, to the general issue of why chances norm credences. Everyone owes an answer to that—and one would hope that whatever answer one gives would generalize to this case.[20]

[19]The most attractive presentation of this I know is in Joyce (1998). The idea there is that violations of standard probability axioms (such as induced by the situation just described) makes one's credences *needlessly* less accurate than they need to be.

[20]Compare Moss (manuscript), whose work in defending a chance-credence-counterfactual link was one of the reasons I became interested in these questions.

One worry here is whether our stories about 'chance' would generalize—after all, we defined the chance of counterfactuals earlier via the van Fraassen construction—but who's to say they deserve that name, once it's made clear that it has to play a normative role vis a vis credence?

Primitive indeterminacy

Maybe the right thing to say about the orderings is that there *is* one that correctly describes the actual world, but there's *no fact of the matter* which one it is (McGee & McLaughlin, 1994; Barnes & Williams, 2010). We can define truth, as above, simply as truth at the actual world. Since there's no fact of the matter which world is actual, there will be no fact of the matter whether ordinary conditionals are true.

Perhaps we can thereby cut down on the 'metaphysical extravagance' of saying that the actual world fixes all conditional facts—for our idea now is exactly it does not so fix it. (Of course, some might worry that the notion of 'no fact of the matter' being appealed to is *itself* metaphysically extravagant). But, more importantly I think, the explanatory task identified above is not allayed. Why should our credence in conditionals with status be so precisely constrained?

Supervaluational truth value gaps

A quite different sort of approach is to reject the assumption that the actual world fixes a particular ordering o. One sort of precedent for dealing with cases where we have 'dangling parameters' is to go supervaluationist. Rather than aiming to pick out one special glowing ordering, we give characterizations of truth and falsehood, ranging over all of them. Thus we might say:

- $A \rightarrow B$ is true (in the present context) if it is true relative to $\langle @, o \rangle$, for each o.
- $A \rightarrow B$ is true (in the present context) if it is true relative to $\langle @, o \rangle$, for each o.

When $A \rightarrow B$ is true at some ordering (paired with the actual world) and false at others, then it will be neither true nor false. A similar definition could be given for $A \mathrel{\Box\!\!\rightarrow} B$.

One trouble with this is *just how many* truth-value gaps there will be. The orderings featuring in the van Fraassen construction are quite arbitrary. No matter how crazy it might seem to call w the 'closest A-world' to @, there'll be some ordering on which it is. For contingently false antecedents, where the antecedent does not necessitate the consequent, there'll always be a 'falsifying' ordering around, and always a 'truth-making' ordering to be had. So non-necessitating indicatives and counterfactuals with contingently false antecedents with be universally gappy, on this proposal. When the antecedent necessitates the conclusion, the indicative and counterfactual will be true. When the antecedent is impossible, there'll be vacuously true. And where the antecedent is contingently true, the truth value is fixed by the truth value of the consequent. Notice that this distribution of supervaluational truth-statuses is the same for both counterfactuals and indicatives.[21]

[21] This is the case at least if we start off with the same modal space for each. If we do not, the relevant sense of 'necessitates' may be different—e.g. if indicatives are built on a space of possibilities compatible with what some agent knows; in which case, when a material conditional is known to be true, the corresponding indicative is also. For something like this epistemic treatment, see Weatherson (2001). For the van Fraassen construction to

There are reasons for worrying whether this sort of treatment fits rather badly with the Ramsey test. For if we assign little credence to A (which is contingent), then our credence that $A \to B$ is true (where A doesn't necessitate B) will be very low)—for the only circumstances where it'll be true are where both A and B hold together. However, $P(B|A)$ might be very high, and hence (by the Ramsey test) $P(A \to B)$ will be very high. So one will be highly confident both in the proposition p, and in the claim that this proposition is not true. This is one form of the 'static' challenge to the equation, based on indeterminacy, given in Williams (2009). The tension here springs from the thought that truth norms belief—so it'd be bad to invest credence in something one is well aware is untrue.[22]

Supervaluational degrees of truth

Briggs (manuscript) develops an alternative proposal, in a setting similar to this one, for which I have considerable sympathy (I'll construct my own version here). Rather than talk in supervaluationist fashion of truth and falsity simpliciter, look at the *amount* of precisifications that make true a given conditional. Call this its 'degree of truth'. We then get a sort of 'degree supervaluation' treatment of indeterminacy favoured by (Edgington, 1997b; Kamp, 1975; Lewis, 1970)).[23]

One question to ask anybody adapting supervaluationism is: what do you mean by 'amount of precisifications'? What do we do, count them? How do we do that if there are infinitely many? What is needed is a *measure* over the space of precisifications, to make such notions well-defined. Luckily, in the case at hand, we have measures ready-to-go. At the actual world, we have a particular chance-ordering. This induces by the van Fraassen method a measure over the orderings (one we've been calling the chance-measure). So simply let *this* measure the precisifications. As chances evolve over time, so will the degree of truth of counterfactuals, but that seems perfectly acceptable. And (at least when we have no inadmissible information) we get the pleasant property that credences of counterfactuals aim to match the degree of truth of that counterfactual.[24]

Yet there are some obvious issues with this. Given @, we might have A true and B false. Yet, compatibly with these being the way things turn out, the current conditional chance of B on A

work, it better be that the probability assigned to the whole 'space' is 1. If we want to identify it with epistemic space, in Weatherson's way, we would do better to work with evidential probabilities (Williamson, 2000) rather than doxastic ones.

[22] An interesting recent suggestion (developed independently by Cantwell and Rothschild) is that our 'effective' credence in Q should match the conditional probability $P(Q \text{ is true}|Q \text{ is true or false})$—that is, we simply disregard truth value gaps when calculating credence. This dovetails exactly with the Ramsey test—for on the above suggestion (except in extreme cases, which work anyway) the probability of $A \to B$ being true, conditionally on its having a truth-value, is exactly $P(A \to B|A)$. And on any 'centered' conditional, this will equal $P(B|A)$. Cantwell and Rothschild use a three-valued logic to frame their proposal, which gives a similar result for the probability of a simple conditional. But the proposals will come apart in a couple of ways. Firstly, the logic of conditionals is a classically-based Stalnaker logic. In their setting, we'd have some non-classical many valued logic operative. Secondly, the above proposal has a non-truth-functional treatment of truth-values, so that e.g. $A \to A$ will be true even if A is false, whereas in their setting it will be truth-valueless. Thirdly, there's no guarantee that probabilities of compounds of conditionals coincide on the two proposals.

[23] In a language without determinacy-operators, degree supervaluational *logic* is classical. But in richer settings, it gets very strange. See Williams (manuscriptc) for some further details about the (peculiar) logic that arises.

[24] See **?** for more on the idea of 'aim of credence' in these non-classical settings.

might be arbitrarily high. So there'd be little chance of modus ponens preserving 'degree of truth' (unless, of course, we fix the degree-of-truth of factual propositions be their chances—but that strains credulity rather). A second problem arises when we consider inadmissible information. If my guru tells me that $A \wedge B$, even though the conditional chance of B on A is low, then I should give the counterfactual $A \,\square\!\!\rightarrow B$ high credence—a knowing violation of the principle that I should match my credence to its degree of truth.

In order to finesse the second issue, we could introduce a large degree of context sensitivity into the formulations. Relative to a given stock of 'evidence' E, we might use the measure over orderings given by the van Fraassen chances conditional on E, rather than the van Fraassen chance alone. Degrees of truth so defined will norm belief—but they'll be very context sensitive. And the problems with modus ponens remain.

For indicatives, it seems like our only way of replicating these results will be to define degrees of truth directly in terms of the van Fraassen credences over conditionals. And so our 'degrees of truth' will be even more context sensitive. One wonders, indeed, what is gained by offering such definitions, rather than simply sticking with the original construction and adopting the rhetoric of the no truth valuers.

8 Normative silence

Let me put one last possibility on the table—the one I (tentatively) like best. It is easiest to fit with the supervaluationist truth-value gap proposal described earlier.

Recall one worry (from Williams (2009)) was that most conditionals will be (knowably) untrue—and hence apparently should not be believed. This idea makes the critical assumption that untruth is normatively on a par with falsity, so far as one's credence in *p* goes. **?** (in a quite different setting) argues that truth value gaps need not be construed in this fashion. One should not believe the false; one should not disbelieve the true—but truth-value gaps are also *alethic norm gaps*—there's nothing *general* to be said about what to believe or disbelieve, if the question at hand is gappy.

Nothing *general* to be said—but Maudlin suggests that we develop (or discover in existing practice a commitment to) various 'local rules' to cover the cases. Such a local rule might reinstitute something like the 'rejectionist' picture of gaps given before (i.e. it says to treat untruth on a par with falsity). But in general, the rules are only constrained by very general desiderata—of simplicity, completeness, consistency with the truth and falsity norms, familiarity (interacting with logical connectives in a way that mimics the way truth does).

Adapted to our setting, the idea would be this: the truth and falsity of conditionals sets the boundaries within which we operate. But then the *rules that van Fraassen gives* exactly describe local norms—tell us how to fix credences in conditionals when they are knowably gappy. The rules are *well motivated*, after all—for they mean that we can use conditionals to express conditional credences or expectations about chance. Why wouldn't we use them?

The attraction of this view is that it gives an *general rationale* for the way that the semantic status plays into the familiar facts about how credences should be shaped. It does this subtly—by providing a picture on which conditionals in the crucial cases have a status whereby it is a purely conventional matter what credences to assign to them. The story has elements of several of the preceding pictures—it's not too far away from the sophisticated no truth value picture, except that it gives a non-ad-hoc way of explaining how some conditionals (the boundary cases) get genuine truth values. It also has elements of Briggs' suggested degree theoretic approach—if one wished to articulate the local rules in terms of induced 'truth values', one couldn't do better than her definitions—but there are firm, contextually stable definitions of the underlying semantic statuses of truth, falsity, and gappiness.

I suggest, therefore, that the probabilistic constraints on indicative and counterfactual conditionals can be reconciled with an internally consistent, non ad hoc, and precise story about their logic, their truth-status, and a general story about updating. These are *useful*, well-behaved devices. It is another question whether they are the best candidates for interpreting the conditional connectives of English or other natural languages—that I haven't broached here. But if the probabilistic constraints are to be taken as our guide, we have no better starting point.

9 Conclusion

The Ramsey test for indicative conditionals has a bad reputation; and similar troubles afflict the counterfactual version. But there are reasons for wanting 'Ramsey conditionals'—in a version stronger than standard 'no truth value' approaches delivers.

van Fraassen (1976) provides a key to constructing tenable Ramsey conditionals. He showed that from any 'base probability', a conditional meeting the Ramsey test can be constructed—and as we've seen something similar goes for the counterfactual version. I have argued there's nothing ad hoc about the assumptions we must make (e.g. about the space of possible worlds over which credences in conditionals are defined). And I've elaborated the account in two ways: saying something about how updating works when Ramsey conditionals in play; and giving an interpretation of the truth-values of conditionals (in terms of supervaluational gaps) that differs markedly from van Fraassen's preferred view. But such indeterminacy must be treated with care—I've briefly outlined one view of indeterminacy which fits with the package being developed here.

A The van Fraassen construction

Let W be a set of worlds, and Ω a field of subsets of W, over which a probability P is defined.

Extending the space

Consider the space W^{ω}—the set of ω-sequences of worlds drawn from W. If $\pi \in W^{\omega}$, we write $\pi(m)$ for the mth member of the sequence. We can think of W^{ω} equivalently as consisting of the ordered 'world, world-sequence' pairs $\langle w, \pi \rangle$, where w is simply $\pi(0)$. This is technically trivial, but helps indicate the interpretation—the points fix both a 'factual world' (to fix non-conditional truths) and an 'ordering of factual worlds' (to fix conditional truths).

Take a propositional language (already interpreted over W) supplemented by a conditional $\rightarrow$. For atomic sentences A, we say A is true at $\langle w, \pi \rangle$ iff it is true at w. We give the standard recursive clauses for conjunction, disjunction, negation etc—hence a non-conditional or 'factual' sentence will be true at $\langle w, \pi \rangle$ iff it is true at w.

For conditionals we use a Stalnaker characterization. The sequences π aren't Stalnaker orderings, since they may contain repetitions (though they clearly determine an ordering—just order each worlds by the order of their first appearance in the sequence (and any worlds that do not show up will be 'inaccessible'. We might then like to offer the following kind of characterization of conditional truth conditions:

> $A \rightarrow B$ is true at $\langle w, \pi \rangle$ is true iff B is true at the first A-world in the π-ordering.

But the ordering is defined over W, rather than W^*, so the RHS is making use of a notion of 'truth at w' rather than 'truth at $\langle w, \pi \rangle$'. This wouldn't matter if we were only concerned about conditionals with factual antecedent and consequent, since those are true at $\langle w, \pi \rangle$, for any π, iff they are true at w in the underlying space. But if we to handle conditional antecedents and consequents we need more.

Van Fraassen offers the following extension. Let pi_m be the truncation of pi, obtained by removing the first m terms. Thus if $\pi = \langle \pi(0), \pi(1), \ldots, \pi(m-1), \pi(m), \pi(m+1) \ldots \rangle$ then $\pi_m = \langle \pi(m), \pi(m+1) \ldots \rangle$. Call $\bar{\pi} = \{\pi_1, \pi_2, \ldots\}$ the ordering of W^{ω} *induced by* π.

With this in hand, we give the following definition:

> $A \rightarrow B$ is true at $\langle w, \pi \rangle$ is true iff B is true at the first $\langle w', \pi' \rangle$ at which A is true in the $\bar{\pi}$-ordering. (in the case where there is no point in the ordering at which A is true, let the conditional be vacuously true).

Extending the space

We now have a space—-W^ω—at which all conditionals get definite truth values. The second step is to extend an arbitrary probability measure P defined over W (or strictly, over the field of subsets Ω of W) to an algebra over W^ω.

Our field of subsets of W^ω will be the following (which we write Ω_ω. If A_i is a element of Ω for each $i \leq n$, then let $\langle A_1, \ldots, A_n \rangle$ be the set of $\pi \in W^\omega$ the ith member of which is drawn from A_i. Ω^ω is generated by such sets.

We define P^ω over Ω^ω as the product measure. For sets of the kind just defined, we have $P^\omega\langle A_1, \ldots, A_n\rangle = P(A_1) \cdot P(A_2) \cdot \ldots \cdot P(A_n)$. The measure is extended to other elements of Ω^ω by additivity. It's easy to see that this probability extends P, relative to the injection of A to $\langle A \rangle$.

We can then give an illuminating characterization of the subset of Ω^ω at which a simple conditional is true. Recall that this will be true at a point iff the first A-world in the relevant ordering at that point. Let A^i be $\langle \neg A, \neg A, \ldots, \neg A, A \wedge B \rangle$, a sequence of length $i+1$. Notice that, by construction,

$$P^\omega(A^i) = (P(\neg A))^i \cdot P(A \wedge B)$$

Notice that any $\pi \in A^i$ will be a sequence the first i of whose members make A false, and whose $i+1$th member makes AB true. This means that it contains all those points that 'make true' the conditional $A \to B$ at the ith position. Every point at which the conditional is true is either (i) vacuous; or (ii) one of the members of A^i, for some i. Thus, $A \to B$ is the infinite union of the A^i—plus the vacuous cases.

The vacuous cases are those sequences which never make A true at any stage. They are thus the intersection I of the following sets: $\bar{A}_i := \langle \neg A, \neg A, \ldots, \neg A \rangle$ (where the sequence has i members). Clearly $P(I)$ is bounded above by $P(\bar{A}_i)$ for each i. Now, $P^\omega(\bar{A}_i) = P(\neg A_i) \cdot \ldots P(\neg A_i)$. And hence for every case where $P(\neg A) = k \neq 1$,[25] we have $P^\omega(\bar{A}_i) = k^n$. Hence $P^\omega(I) \leq k^n$ for all n, where $k < 1$—from which it follows that

$$P(I) = 0.$$

$$\begin{array}{rcll} P^\omega(A \to B) & = & P^\omega(I \vee \bigvee_i A^i) & \text{definition of } \to \\ & = & P^\omega(I) + \sum_i P^\omega(A^i) & \text{definition of } P^\omega \\ & = & 0 + \sum_i (P(\neg A))^i P(A \wedge B) & \text{above results} \\ & = & P(A \wedge B) \sum_i (P(\neg A))^i & \text{rewriting} \\ & = & P(A \wedge B)\frac{1}{1-P(\neg A)} & \text{summing series} \\ & = & P(A \wedge B)\frac{1}{P(A)} & \text{basic probability} \\ & = & P(B|A) & \text{ratio formula} \end{array}$$

This delivers our result—P^ω is the extension of the probability measure P over W to W^ω—and for factual conditionals the probability of the conditional is the conditional probability (in all cases where the latter is defined).[26]

[25] Of course, if $P(\neg A) = 1$, then $P(I) = 1$, and hence $P(A \to B) = 1$. But this is exactly the case which the standard Ramsey test doesn't cover—conditionals with probability zero antecedents.

[26] van Fraassen (1976) shows in addition that the identity holds for once-embedded conditionals.

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