

The $c\alpha$ Non Exclusion Principle and the vastly different internal electron and muon center of charge vacuum fluctuation geometry

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Abstract

The electronic and muonic hydrogen energy levels are **calculated** very accurately [1] in Quantum Electrodynamics (QED) by coupling the Dirac Equation four vector ($c\alpha, mc^2$) current covariantly with the external electromagnetic (EM) field four vector in QED's Interactive Representation (IR). The $c\alpha$ -**Non Exclusion Principle**($c\alpha$ -**NEP**) states that, if one accepts $c\alpha$ as the electron/muon velocity operator because of the very accurate hydrogen energy levels **calculated**, the one must also accept the resulting electron/muon internal spatial and time coordinate operators (ISaTCO) **derived directly** from $c\alpha$ without any assumptions. This paper does not change any of the accurate QED **calculations** of hydrogen's energy levels, given the simplistic model of the proton used in these calculations [1]. The Proton Radius Puzzle [2, 3] may indicate that new physics is necessary beyond the Standard Model (SM), and this paper describes the bizarre, and very different, situation when the electron and muon are located "inside" the spatially extended proton with their Centers of Charge (CoCs) orbiting the proton at the speed of light in S energy states.

The electron/muon center of charge (CoC) is a structureless point that vibrates rapidly in its inseparable, non-random vacuum whose geometry and time structure are defined by the electron/muon ISaTCO discrete geometry. The electron/muon self mass becomes finite in a natural way due to the ISaTCOs cutting off high virtual photon energies in the photon propagator. The Dirac-Maxwell-Wilson (DMW) Equations are derived from the ISaTCO for the EM fields of an electron/muon, and the electron/muon "look" like point particles in far field scattering experiments in the same way the electric field from a sphere with evenly distributed charge "e" "looks" like a point with the same charge in the far field (Gauss Law). The electron's/muon's three fluctuating CoC internal spatial coordinate operators have eight possible eigenvalues [4, 5, 6] that fall on a spherical shell centered on the electron's CoM with radius in the rest frame. The electron/muon internal time

operator is discrete, describes the rapid virtual electron/positron pair production and annihilation. The ISaTCO together create a current that produce spin and magnetic moment operators, and the electron and muon no longer have “intrinsic” properties since the ISaTCO kinematics define spin and magnetic moment properties.

1.0 Introduction

The Proton Radius Puzzle is qualitatively described well in Bernauer’s and Pohl’s Scientific American article [7] “You Would Be Forgiven for Assuming that We Understand the Proton”, which is very similar to the quantitative analyses given in this paper, especially the differences in the electron and muon internal spatial and time coordinate operators (ISaTCO) when they are located inside the spatially extended proton in hydro gen (see Figure 1). From this point on the word electron will mean any charged lepton, as the ISaTCO development is the same. However, the spatial extent and period of their CoC fluctuations is inversely proportional to their mass the electron, muon, and tau particle will have significantly different EM fields when they are inside the spatially extended proton.

The electron’s magnetic moment and spin are easily derived from their ISaTCOs’ “discrete internal current”, and the electron is not a “structureless” point particle with “intrinsic” properties, as it appears in the far field.. If one accepts $c\alpha$ as the velocity operator in the four current used in QED due to the accurate calculations that result, then one must accept he electron internal structure that is derived directly from the Dirac Equation’s velocity operator, $c\alpha$. This is called the $c\alpha$ -Non Exclusion Principle ($c\alpha$ -NEP), and violating this principle is totally illogical. The hydrogen atom is selected to illustrate the electron’s bizarre properties because it is

stable and the electron's internal structure is not destroyed. All the recent models of the extended electron with a point charge that follows a **continuous** path, such as a helical curve [see 8 for references], are incorrect, because the electron's internal space and time co-ordinates are discrete, as the ISaTCO are derived directly from the Dirac Equation. All the previous analyses of Zitterbewegung (ZBW) are not correct [see 9 for references], because one does not need to make additional, ad hoc assumptions about the electron's internal structure beyond how the Dirac Equation describes it with its velocity operator $c\boldsymbol{\alpha}$.

There is a misconception that the Dirac Equation is a single particle equation, and that ZBW discovered by Schrodinger [10] is a non-relativistic concept that is not incorporated in QED. In terms of Quantum Field theory (QFT), the electron is its CoC and well defined vacuum fluctuation that oscillates rapidly creating the electron EM field (see references in Schwartz [11]), and this paper will derive the ISaTCOs based on the $c\boldsymbol{\alpha}$ -NEP only without ad hoc assumptions. Accepting the physical description of the electron's internal structure resulting from its velocity operator, $c\boldsymbol{\alpha}$, cannot be refuted without refuting the very accurate predictions of QED. The QED solution of the electron internal "discrete position and time distribution" in hydrogen shows (see Figure 1) that the electron has non zero probability to be located inside the spatially extended proton. The energy levels in hydrogen are very accurately calculated by QED by importing the Dirac Equation four vector $(c\boldsymbol{\alpha}, mc^2)$ covariantly in QED's Interactive Representation (IR). Although the QED calculations are highly accurate, the physical description of the electron's ISaTCO described by the Dirac

Equation have been completely missed. In their section on the “Physical Origin of the Lamb Shift”, Eides et al. [1] state that “The **finite radius** of the electron generates a correction to the Coulomb potential...”, and then gives the equation for this correction. The QED “finite radius” described [1] is the ISaTCO radius. Eides et al. [1] go on to state: “the finite size of the electron (caused) by the QED radiative corrections leads to a shift of hydrogen energy levels”. It is $c\alpha$ that causes the “finite electron radius to Eides et al. [1] refer, not QED’s perturbation calculations.. This article should not be controversial because the $c\alpha$ -NEP cannot be refuted logically. QED’s covariant perturbation theory accurately calculates hydrogen energy levels, but misses the physical picture of the electron completely. QFT correctly addresses the vacuum fluctuations in general [11], but does not derive the electron’s ISaTCO with a point CoC located on a spherical shell in the rest frame and an oblate spheroidal shell in non rest frames. QED predicts the exact electron propagator, but QED fails to cut off the high energy virtual photons at the CoC shell defined by the ISaTCO, and the electron self energy becomes infinite in a natural way. This paper will derive the correct photon propagator defined by the electron’s ISaTCO, without ad hoc assumptions like Renormalization, Tree Theory, etc.. The electron described by the Dirac Equation and the ISaTCO is never one particle. It is a three, or five, or seven, or $2n-1$... particle model, with a point electron located at its CoC in the presence of a virtual electron/positron pair, whose positron annihilates the original electron leaving behind the electron from the virtual pair. There can be any number of virtual pairs present, but there is $1/137.04$ less likely to be two sets of virtual electron pairs than one, and $(1/137.04)^2$ less likely to have

three sets of electron/positron pairs than one, etc.. This virtual pair production and annihilation occurs as vacuum fluctuations on a very rapid time scale of $\sim 1.4 \cdot 10^{-21}$ sec. for the electron, and approximately 207 times more rapidly for the muon.

In this paper the Dirac-Maxwell-Wilson (DMW) Equations are derived for the electric and magnetic fields of an electron in the hydrogen atom, and the impact of the electron's CoC vacuum fluctuations is shown to be negligible a few Compton wavelengths from the electron's Center of Mass (CoM). The electron appears as a "structureless point particle" in the far field, as observed in the electron/proton scattering experiments used to probe the proton's internal structure (see [12] for further references). Trying to observe the electron's vacuum fluctuations structure externally with high energy photons would destroy the electron, creating other particles from the inelastic interactions.. It is fortunate that the electron is stable in hydrogen and that it is actually located "inside" the proton in the S-Level energy states some of the time, or the impact of electron's vacuum fluctuations would not be measured in hydrogen. If the electron were located only at its Bohr orbit, the impact of the electron's vacuum fluctuations on hydrogen energy levels would never be observed. In addressing the electron/muon "Proton Radius Puzzle" this paper will show how significantly different the electron and muon EM fields are when the electron and muon CoMs are located "inside" the proton, due to the highly different spatial and time extent of their vacuum fluctuation geometries. The spatially extended proton, with the electron and muon CoCs traveling at the speed of light around their CoMs "inside" the proton, is a bizarre picture, but is predicted

accurately by QED and QFT. The muon CoC is ~ 207 times closer to the proton's center when the electron and muon CoMs are located inside the proton. This bizarre situation is absolutely confirmed by QED's very accurate estimates of the hydrogen and muon energy levels [1], but it shows the need for a much more accurate model of the proton's internal structure. In [9] Pohl and Bernauer state: "If the muon is 200 times closer to the proton, it should also be spending considerably more time *inside* the proton (indeed, the probability is increased by a factor of 200^3 , or 8 million). Thus, in turn changes the Lamb shift of the atom by 2 percent – a relatively huge amount that should be easy to spot." If the proton radius is scaled up to 0.5 cm. (about half the size of a pea), the muon CoC would be approximately 6 m. from the proton's CoM at the origin, while the electron's CoC would be over 1200 m. from the proton's CoM at the origin. In both cases the extended proton is well "within" the electron or muon CoC shell, and the proton's quarks experience significantly different electromagnetic (EM) field fluctuations from induced the electron or muon. Even though this situation is chaotic, QED calculates the electron and muon interactions with their $c\alpha$ vacuum fluctuations very accurately in estimating the resulting hydrogen energy levels. QED does not predict the electron and muonic proton radius differences accurately. The latest measurements by Pohl et al. [2, 3] for the proton radius for muonic and electronic hydrogen are approximately 0.84 fm and 0.87 fm respectively. The proton is not a central force in the near field due to three quarks exchanging gluons inside a very complex proton spatial structure, and the $2S_{1/2} - 2P_{1/2}$ hydrogen energy levels would likely not be degenerate at the Dirac Equation level for a more accurate model of the proton.

There are no classical analogs to the electron position probability distributions in the hydrogen atom. The electron velocity operator $c\alpha$ defines its CoC vacuum fluctuations, and has eigenvalues $\pm c$. Negative energies and virtual electron/positron pair production occur within the electron internal structure defined by the ISaTCO in highly non classical terms. This paper will stress that you cannot separate the electron or muon CoC from their ISaTCO defined vacuum fluctuations.

In the next section the time dependence and geometrical distribution of electron discrete ISaTCO will be derived in the Heisenberg Representation following Sakurai [13] and the author's three previous papers [4, 5, 6]. The following two states cannot be differentiated:

1. The electron or muon are structureless point particles with "intrinsic" spin and magnetic moments, and infinite self-energy when viewed **from the far field**.
2. The specific electron and muon ISaTCO generate a current distributed on a spherical shell of a specific radius in the rest frame, and, by Gauss' Law, one cannot differentiate this case from a static point particle **in the far field**.
- 3.

Hu states [14]: "It is a general belief that the electron is a point-like particle with almost no measurable dimensions, and that the electron does not possess any sub-constituent. This belief is largely based on the results of Møller and Bhabha scattering, obtained from many experiments performed in the last twenty years at PETRA (Positron-Electron Tandem Ring Accelerator Facility at DESY laboratory in Hamburg), PEP(The Positron-Electron Project, a collaborative effort of SLAC and

Lawrence Berkeley Laboratory), TRISTAN (The e+e- Collider at National Laboratory for High Energy Physics, KEK in Japan), and LEP (The Large Electron Positron Collider at CERN). In the experiments conducted at PEP in the tests of leptonic substructure, Bender et al. stated, "experimentally there is no indication that the electron has structure [composite of more fundamental particles]. Lower limits for cutoff parameters are in the 100—200 GeV range, corresponding to an electron size of less than 10^{-16} cm. Therefore, if the electron is a composite particle its constituents are strongly bound, giving the electron the observed point-like quality at experimentally accessible energies (67). As a matter of fact the results obtained from the scattering experiments have been a "no-go" sign to any attempt of the so called large- electron theories, where the size of the electron was in an order of its Compton wavelength."

These far field electron scattering experiments in the far field are meaningless for the ISaTCO internal structure of the electron. In fact, stating that the electron is a structureless point particle with "intrinsic" properties violates the $c\alpha$ -NEP, and one cannot accept $c\alpha$ in the QED four current, and reject the direct derivation of the ISaTCO from $c\alpha$ as the Dirac Equation velocity operator. The analysis of the electron and muon physical pictures when inside the proton in hydrogen implies much work needs to be done in deriving a more accurate model of the proton structure to develop a QED solution with this improved proton model.

A recent paper [15] models the strong force distribution of the proton, and a similar analysis is needed for the EM distribution within the proton structure to include the

three charged quarks. The newly proposed Electron-Ion Collider (EIC) is being built to probe the proton internal structure with beams of electrons (see <https://doi.org/10.17226/25171> for a complete description of the EIC), and the June 2019 Scientific American article “The Deepest Recesses of the Atom” for the basic physics behind the need for the EIC. According to these references there are $\sim 10^{80}$ protons and neutrons, and an equal number of electrons, in the universe, but we don’t yet know where the proton gets its mass or spin. While the proton consists of three quarks and gluons and their virtual anti particles, the three quarks’ total mass is much lighter than the proton, accounting for \sim “2 percent” of the proton mass and less than “30% of the proton’s spin” according to these EIC references. One point of this article is that the electron and muon in hydrogen already probe the internal structure of the spatially extended proton’s complex internal structure for S energy levels, and electronic hydrogen is absolutely stable. If and when the EIC is finally built and producing data in 2030, perhaps more can be done to probe the proton internal structure. In the mean time, with the theory developed in this article and an improved proton model, QED can be used to address the Proton Radius Puzzle. The author feels strongly that the EIC will be valuable, but the electron model of a structureless point particle with intrinsic properties is certainly inaccurate for the electron and muon in hydrogen where their NR-VFGs are an inseparable part of the particle and are always present.

If a tau particle version of hydrogen could be produced it could also exhibit unique TNR-VFG inside the proton when its CoC is 17.7 times closer to the proton’s center

than the muon's CoC. Perhaps, a third value of the proton radius could be measured from various S-P level transitions in the same way that the muonic and electronic hydrogen unique proton radii are measured.

2.0 Geometrical properties of vacuum fluctuations that the electron and muon exhibit within the hydrogen atom

Although "electron" is used in the rest of this paper, the equations apply equally to the muon by changing the mass. The Dirac Equation CoC velocity operator, $c\alpha$, must be accepted as is, even though far from the classical picture of continuous time and space CoM dynamics due to the $c\alpha$ - NEP discussed in the previous section. Even though $c\alpha$ is a 4×4 complex matrix representing the electron's velocity, and goes against one's classical intuition, one must accept the electron CoC with its very well defined vacuum fluctuations defined by the Dirac Equation. The electron position ICOs are derived directly from $c\alpha$, and are discrete and relativistic, while externally the electron's CoM is consistent with classical mechanics. The electron or muon CoC vibrating rapidly internal to the CoC shell have a very short range impact, and appears to be a point particle with intrinsic properties in the far field, as exists in the electron/proton scattering experiments. Within the hydrogen atom the electron's internal, discrete geometrical CoC structure is measured in a very stable fashion by the proton, and the electron's ICO operators create an unusual EM environment when the electron CoM is near or "inside" the proton, as allowed by the QED predicted position probabilities in hydrogen's S energy levels depicted in Figure 1.

A. The Electron's CoC position operator defining its deterministic vacuum fluctuation geometry

Let us start with the expression for the electron's CoC ICO from Sakurai [13 (p 116)].

Bold symbols will signify 3-D vectors in this paper, while scalars are given by non bold symbols. In the Heisenberg representation

$$x_k(t) = x_k(0) + c^2 p_k H_D^{-1} t + (i\hbar/2)(\alpha_k(0) - c p_k H_D^{-1}) H_D^{-1} e^{-2iH_D t/\hbar} \quad (1)$$

where

$$H_D = c \boldsymbol{\alpha} \cdot \mathbf{p} + \beta m c^2 \quad (2)$$

and

$$H_D^{-1} = H_D / E^2(p) \quad (3)$$

The center of mass coordinate \mathbf{x}^{CoM} is given by

$$x_k^{\text{CoM}}(t) = x_k(0) + c^2 p_k H_D^{-1} t \quad (4)$$

and the Heisenberg representation of the CoC coordinates, where operators are time independent, is given by:

$$x_k^{\text{CoC}}(t=0) = x_k^{\text{CoM}}(t=0) + (i\hbar/2)(\alpha_k - c p_k H_D^{-1}) H_D^{-1} \quad (5)$$

Now let ($\mathbf{p}^{\text{CoM}} = p_3 \hat{\mathbf{k}}$) for an electron's CoM moving in the z direction. A

straightforward calculation inserting Equation (3) into (5), and using standard anti commutation rules yields:

$$x_3^{\text{CoC}}(p_3) - x_3^{\text{CoM}} = (\hbar m c^3 / 2 E^2) \gamma_3 \quad (6)$$

where γ_k with $k = 0, 1, 2, 3$ are the normal DE 4x4 complex matrices, and should not be confused with the Special Relativity (SR) "gamma" factor γ . In the electron's rest frame ($\mathbf{p}^{\text{CoM}} = \mathbf{p} = \mathbf{0}$), and dropping the CoM momentum superscript for \mathbf{p} for simplicity for the remainder of the paper, we obtain

$$x_3^{\text{CoC}}(p=0) - x_3^{\text{CoM}}(p=0) = (\hbar / 2mc)\gamma_3 \quad (7)$$

$x_3^{\text{CoC}}(p_3)$ is the electron's intrinsic coordinate in the direction of the motion of its center of mass motion $\mathbf{p} = p_3 \hat{\mathbf{k}}$. The Dirac Equation defines a CoC momentum operator naturally as $mc\boldsymbol{\alpha}$, and the internal space/time structure of the electron is quantized or discrete [4, 5, 6].

The role of β multiplying the rest mass m in Equation (2) accounts for interaction with the negative electron energy states, and it will appear again in the definition of spin angular momentum.

The Dirac Equation's electron coordinate operator magnitude in the direction of motion is contracted by the square of the SR contraction factor γ since

$$\left| x_3^{\text{CoC}}(p_3) - x_3^{\text{CoM}}(p_3) \right| = \hbar mc^3 / 2E^2 = \hbar / 2mc\gamma^2 \quad (8)$$

where the SR contraction factor of a rigid body is given by $\gamma = 1 / (1 - (v_3/c)^2)^{1/2}$.

The symbol " β " = v^{CoM}/c is used in this paper for both the SR velocity factor and the fourth gamma matrix, γ_4 , but the equations make it clear where the scalar or the 4x4 complex matrix are used. The electron described by the Dirac Equation includes the ICO vacuum fluctuations, and is far from a structureless point particle, and contraction of the electron's CoC current shell for CoM momentum along the z-axis ($\mathbf{p} = p_3 \hat{\mathbf{k}}$) is non zero in directions perpendicular to the electron's CoM motion. A

straight forward calculation for $k=1$ from Equation (5) with $\mathbf{p} = p_3 \hat{\mathbf{k}}$ yields (the $k=2$ coordinate is similar)

$$x_1^{\text{CoC}}(p_3) - x_1^{\text{CoM}}(p_3) = ch/2E^2 \left[p_3^{\text{CoM}} c \Sigma_2 + \gamma_1 mc^2 \right] \quad (9)$$

Thus, in the perpendicular directions to the electron's CoM motion, the ICO not only contracts in magnitude, but also has an additional term $(ch/2)(p_3 c/E)\Sigma_2$ related to the electron spin $((h/2)\Sigma_2)$ in the perpendicular direction. A simple calculation yields

$$\left| x_1^{\text{CoC}}(p_3) - x_1^{\text{CoM}}(p_3) \right| = (hc/2)E(p) \quad (10a)$$

$$= \hbar/2mc \gamma \text{ for } \mathbf{p} = p_3 \mathbf{k} \quad (10b)$$

Therefore, the coordinates of its CoC current shell are contracted by the exact SR contraction of $1/\gamma$ in the direction **perpendicular** to the electron's CoM motion, and are caused by the nature of the non random vacuum fluctuations.

The Dirac Equation predicts that the magnitude of the electron's ICOs will contract to a point particle as $v_3 \rightarrow c$ with an ellipsoidal or oblate spheroid shape, where the ellipsoid's semi minor axis is in the direction of the electron's CoM motion. The Harmonic Oscillator nature of the CoC intrinsic coordinates is now clear:

$$\Delta X_k^{\text{CoC}}(p_k) = x_k^{\text{CoC}}(p_k) - x_k^{\text{CoM}}(p_k) = ch/2E^2 \left[(\Sigma x p c)_k + \gamma_k mc^2 \right] \quad (11a)$$

and their two time derivatives

$$\dot{\Delta X}_k^{\text{CoC}}(p_k) = c \alpha_k \text{ and } \ddot{\Delta X}_k^{\text{CoC}}(p_k) = -(\omega_o^2) [\Delta X_k^{\text{CoC}}(p_k)] = 0 \quad (11b)$$

Thus, the Dirac Equation predicts directly that the electron's CoC coordinate operators, $\Delta X_k^{CoC}(p_k)$, form three independent sets of one dimensional Harmonic Oscillators for any arbitrary CoM momentum \mathbf{p} , and this is caused by the nature of the ZBW vacuum fluctuations. These electron CoC not only oscillates at a very rapid rate of $\omega_o = 2E/h$, but there are only eight discrete ICO eigenvalues, $\pm h/2E(p)\gamma$, in the two directions perpendicular to the CoM motion and $\pm h/2E(p)\gamma^2$ in the direction of CoM motion. In the electron's rest frame, the CoC ICO eigenvalues are $\Delta X_k^{CoC}(p_k = 0) = \pm h/2mc$ for $k = 1, 2, 3$. The Dirac Equation does not reveal why these internal coordinates are discrete, or why the restoring Harmonic Oscillator potential exists

$$V_k(\Delta X_k^{CoC}(p_k)) = (1/2)K\Delta X_k^{CoC}(p_k)^2 = mc^2(1 - p_k^2c^2/E(p^2)) \quad (12)$$

where the restoring "spring constant" $K = 4mE^2(p)/h^2$. The electron vacuum fluctuations producing the ICOs defined above are the cause of this internal electron harmonic oscillator restoring force, and the Dirac Equation predicts the electron's CoC's chaotic rapidly oscillating motion that creates a current over time. The discrete vacuum fluctuations energy levels of the electron are deduced from the electron coordinate harmonic oscillator are given by

$$E_n = (n + 1/2) h \omega_o = (2n + 1) E(p) \quad (13)$$

where the ground state energy $E_0 = E(p)$, and the energy of the excited states are $E_1 = 3E(p)$, $E_2 = 5E(p)$, etc.. Thus, the Dirac Equation implies directly that the first

excited Harmonic Oscillator state of the free electron requires the absorption of a real photon with energy of at least $2E(p)$ to elevate the electron to an energy level of $3E(p)$. This is the nature of the electron's CoC 's vacuum fluctuations. In the electron rest frame ($\mathbf{p} = 0$) a photon of energy $2mc^2$ is necessary to elevate the electron to its first excited state, and, in QED or Dirac's Hole Theory description, this is exactly the amount of energy necessary to raise the energy level of a virtual negative energy electron across the "forbidden" region of $2mc^2$. In Dirac's Hole Theory, a positive energy electron and a positron (corresponding to the "Hole" in the completely filled sea of negative energy electron) are produced. An external EM field is required to supply the real photon of energy $2mc^2$, but the electron CoC vacuum fluctuations produce virtual electron/positron pairs, which are formed and annihilated quickly to establish the specific electron vacuum fluctuation geometry defined above for the electron in the hydrogen atom. The electron's instantaneous CoC position is a single discrete point that is in a rapid oscillation about its CoM creating a current over time with the Harmonic Oscillator frequency of $\omega_o = 2E/ \hbar$, and within the hydrogen atom, for averaging times large compared to its ZBW period of $\sim 1.4 \cdot 10^{-21}$ sec.. This is a very specific, non random property of the impact of the electron's CoC vacuum fluctuations. The electron appears over time as a uniform current, caused by the "spinning" charge e on the CoC shell. The electron internal time operator is also discrete [5,6], so the current set up by the electron's CoC quantized internal space and time position sequence only seems continuous when measured over times long compared to the electron's non random CoC vacuum fluctuation period. The above description of the electron in hydrogen is absolutely consistent with modern QED

calculations of hydrogen energy levels [1] and QFT [11], but the above geometrical distribution of the electron's CoC is missed completely in QED and QFT. No one can "picture the quantized nature of the electron internal position and time operators discussed in this section classically.

B. The CoC $c\alpha$ Induced Vacuum Fluctuation Contribution to the Lamb Shift

In a classical analysis of the Darwin term for the non relativistic solution of the hydrogen atom, Sakurai [9] derives the expression for the "ZBW" (Sakurai's term) correction to the potential from a point proton to an electron whose coordinates "fluctuate" over $|\Delta\mathbf{x}| \approx h/mc$, assuming a long time average compared to 10^{-21} sec

$$(V(\mathbf{x} + \Delta\mathbf{x}) - V(\mathbf{x})) \approx [e^2 (\Delta\mathbf{x})^2 / 6] \delta^3(\mathbf{x}) \quad (14)$$

Sakurai [9] states that Equation (14) above is the classical derivation of the ZBW term apart from the "incorrect" numerical term "1/6" instead of the correct "1/8" term. However, if one inserts the Dirac Equation's intrinsic coordinates, where $(\Delta\mathbf{R}^{CoC})^2 = 3h^2 / 4m^2c^2 = a^2$, the correct numerical factor of "1/8" is obtained for the Darwin term. The Dirac Equation predicts the electron CoC vacuum fluctuation induced "radius²" of $a^2 \equiv 3h^2 / 4m^2c^2$ in the rest frame, and, in QFT terminology, one can say the SG vacuum fluctuations near an electron's CoC yields a very specific radius of the fluctuations that agrees exactly with the Darwin Term. The Dirac Equation's ICOs established by the Dirac Equation are required to give the correct answer. Eides et al. [1] derive Equation (14) as the largest contribution to the Lamb Shift, but only note that it is due to the fluctuating "electron coordinate",

without mentioning ZBW or the Dirac Equation from which the fluctuations originate.

C. The electron's magnetic moment

The intrinsic magnetic moment of the electron in the hydrogen atom in the rest frame is generated by the Dirac Equation's electron's vacuum fluctuations as a localized current distribution, but with $c\alpha$ as the CoC velocity operator, and is defined as:

$$\mathbf{m}^{\text{CoC}} = 1/2 \left[\int d^3x (\mathbf{x} \otimes \mathbf{J}^{\text{CoC}}) \right] = eh/2m(\beta\Sigma) \int d^3x (\mathbf{x} \otimes \mathbf{J}^{\text{CoC}}) = eh/2m(\beta\Sigma) \quad (15)$$

\mathbf{m}^{CoC} is proportional to β times the normal expression for electron spin. If $\mathbf{S} = (eh/2m)\beta\Sigma$ is the spin operator, then $\mathbf{L} + \beta\mathbf{S}$ is the constant of the motion. In the Dirac Equation mass is given by βm to account for positive and negative energies, and likewise $\beta\mathbf{S}$ accounts for positive and negative energies for electron spin. The fact that the non-relativistic solution of the hydrogen atom using the Dirac Equation always uses $(eh/2m)\beta\Sigma$, and not $(eh/2m)\Sigma$, supports this concept. Equation 3.243 in Sakurai [13] shows that the electron helicity is constant only if there is no external electric field, but the electron can never escape its own self electric field. Thus, the Dirac Equation implies that the electron g-factor is not exactly 2, without referring to QED. QED is an excellent and efficient way to calculate the deviation of the g-factor from 2, and the author is not proposing a second method to calculate it. The physical cause of the electron anomalous magnetic moment is the interaction with its own electric field, proven by Sakurai's result [13] in his Equation 3.243, but he couldn't deduce the physical cause without the theory developed here.

3.0 The Dirac-Maxwell –Wilson Equations

If one accepts that $c\alpha_k$ is the velocity operator for the electron's CoC velocity, then the Dirac-Maxwell-Wilson DMW Equations follow directly. The electron's CoC motion is chaotic due to electron's non random vacuum fluctuations, and discrete in time due to the same vacuum fluctuations in the hydrogen atom [5,6].

The Dirac Equation EM field operators are developed for the electron's rest frame to describe the non-relativistic EM field with vacuum fluctuation, since the CoM for the electron in the hydrogen atom is a continuous average of the CoC motion. The DMW Equations can be extended to non rest frames using Equations (5) through (10a/10b), but this is unnecessary for the electron or muon in hydrogen.

For any observation point (x, y, z) the free electron's scalar potential will have, with equal probability, one of the eight possible values

$$\Phi_k(r, \theta, \phi, a, \theta_k, \phi_k) = \int_0^{2\pi} d\phi \int_0^\pi \sin \theta d\theta \int_0^\infty r^2 dr (e\delta^3(\mathbf{r}' - \Delta\mathbf{R}_k^{CoC})\mathbf{I}) / (\mathbf{r}^2 + a^2 - 2ar \cos(\Theta_k^r))^{0.5} \quad (16)$$

The “k” index always refers to one of eight electron/muon ICO eigenvalues, and the “prime index” is the integration variable over the Dirac Equation's CoC charge density. The unprimed indices are for the observation point. The 4x4 identity matrix \mathbf{I} will not be shown although it will understood to multiply all expressions for the electron's scalar potential electric field. It is easy to show that

$$\cos(\Theta_k^r) = [\sin \theta \cos \phi \sin \theta_k \cos \phi_k + \sin \theta \sin \phi \sin \theta_k \sin \phi_k + \cos \theta \cos \theta_k] \quad (17)$$

The integration over the quantized angular coordinates is not trivial like it was for the smooth ICOs in a previous article [4]. As stated in the previous section, the Dirac Equation's ICOs are analogous to the quantized spin angular momentum operators in Dirac's theory, where the quantization about the "z-axis" is valid no matter where one chooses this axis for the electron in the hydrogen atom. In fact, the geometry of the eight, equally probable Dirac Equation ICOs naturally defines two spin states depending on the sign of $\cos \theta_k$ in Equation (17). For example, if one arbitrarily assigns the ICO eigenvalue $(\hbar/2mc, \hbar/2mc, \hbar/2mc)$ as $k=1$, the ICO eigenvalue $(-\hbar/2mc, \hbar/2mc, \hbar/2mc)$ as $k=2$, the ICO eigenvalue $(\hbar/2mc, -\hbar/2mc, \hbar/2mc)$ as $k=3$, and the Dirac Equation ICO eigenvalue $(-\hbar/2mc, -\hbar/2mc, \hbar/2mc)$ as $k=4$, then the $k = 1,2,3,4$ ICO eigenvalues geometrically are arbitrarily called "spin up" in this paper. A little geometry shows that $\cos \theta_k = + (1/3)^{0.5}$ and that $\sin \theta_k = + (2/3)^{0.5}$ for $k=1,2,3,4$. Using the term "spin down" to describe the ICOs with negative "z coordinate", the $k=5,6,7,8$ terms still have $\sin \theta_k = + (2/3)^{0.5}$, but have $\cos \theta_k = - (1/3)^{0.5}$. Even though the CoC shell is spherical in the electron's rest frame, the quantization of the ICOs defined by the ZBW vacuum fluctuations resulting from the Dirac Equation make it easier to use Cartesian coordinates. The distance between any Dirac Equation ICO eigenvalue and the field observation point (x, y, z) given in Equation (1) are defined for "spin up" and "spin down" as Equation (18):

$$(\mathbf{r}^2 + a^2 - 2ar \cos(\Theta_k^r))^{0.5} = (\mathbf{r}^2 + a^2 - r(\hbar/mc)[\sin \theta \cos \varphi \sqrt{2} \cos \varphi_k + \sin \theta \sin \varphi \sqrt{2} \sin \varphi_k + / - \cos \theta])^{0.5} \quad (18)$$

where the “+/-” in front of the $\cos \theta$ term represents spin up (k=1,2,3,4) and spin down (k=5,6,7,8), respectively. Also, $\cos \varphi_k = +1/\sqrt{2}$ for k= 1, 4, 5, and 8 and $-1/\sqrt{2}$ for k= 2, 3, 6, 7, while $\sin \varphi_k = +1/\sqrt{2}$ for k= 1, 2, 5, 6 and $-1/\sqrt{2}$ for k= 3, 4, 7, 8. For example the k = 1 distance is:

$$(\mathbf{r}^2 + a^2 - 2ar \cos(\Theta_1^r))^{0.5} = (\mathbf{r}^2 + a^2 - (h/mc)[x + y + z])^{0.5} \quad (19)$$

where $r^2 = x^2 + y^2 + z^2$, a^2 is a constant in the rest frame, and sign preceding x, y, and z in Equation (15) are given by the above definitions in deriving Equation (19) from (18). For k = 2 through 8 the signs of the x, y, and z terms in Equation (19) change as specified in Equation (18) with the specific values. Thus, the electron’s eight electric field operators for each of the equally probable Dirac Equation ICO eigenvalues are given by the operator $\mathbf{E}_k = -\nabla \Phi_k$, where the variable dependencies are now clear and will not be listed in every equation. For k=1 $\mathbf{E}_1 = -\nabla \Phi_1$ and

$$\mathbf{E}_1 = e[(x - h/2mc)\mathbf{i} + (y - h/2mc)\mathbf{j} + (z - h/2mc)\mathbf{k}] / (\mathbf{r}^2 + a^2 - (h/mc)[x + y + z])^{1.5} \quad (20)$$

where \mathbf{i} , \mathbf{j} , and \mathbf{k} are unit vectors in the x, y, and z directions. For k = 2 through 8 the signs of the x, y, and z terms in the denominator and the $h/2mc$ terms in the numerator of Equation (20) change as specified above. An instantaneous measurement of \mathbf{E}_k will yield one of eight possible values with equal probability with average value over time

$$\mathbf{E}_{ave} = 1/8 \sum_{k=1}^{k=8} \mathbf{E}_k \quad (21)$$

Unfortunately, Equations (20) and (21) do not simplify into a single simple expression, but the far field ($r \gg a$) expressions are the expected ones. In the far field, $\mathbf{E}_{\text{ave}} \approx e\mathbf{r}/r^3$ consistent with the static point model.

There are large fluctuations in the electron's electric field when the electron or muon CoM is located at or near $\mathbf{r} = \mathbf{0}$ "inside" the proton. One can easily see from Equation (20) that there would be very small fluctuations in the electron's electric field in the hydrogen atom, if the electron were always located near the Bohr orbit, which is in the far field of the proton as discussed above. The solution to the wave function for the hydrogen atom depicted in Figure 1 clearly show that the electron and muon can be located near and at the origin "inside" the proton for the S states. In this case the electron and muon non random vacuum fluctuations impact the proton in a fundamentally different way. They both have the same negative charge oscillating rapidly around the proton governed by non random vacuum fluctuation geometries, but the muon's CoC is ~ 207 times closer to the proton when both have their CoM positions at the origin inside the proton. These electric field oscillations back on the proton from the electron and muon when their CoM positions are near or inside the proton are violent, chaotic, and very complex. The proton is comprised of three quarks with fractional charges that interact via the EM interaction by the exchange of photons and by strong interaction by the exchange of gluons as described by Quantum Chromodynamics (QCD). It is unknown if the proton's three quarks exhibit non random vacuum fluctuations [15], but, considering the presence of the non random vacuum fluctuation geometries for the electron and muon in

hydrogen, it is possible that similar vacuum fluctuation geometries are present for the three charged quarks within the proton. The hydrogen energy levels calculated by QED that compare accurately with theory use an inaccurate proton model when the electron or muon CoM is “inside” or near the proton’s location. The author cannot yet prove that the very different vacuum fluctuation geometries produced by the electron and muon near the proton location would cause a difference in the measured energy levels or proton charge radius, but very different muon and electron electric fields are produced at the proton location in the hydrogen atom.

Although Equation (20) shows that $\mathbf{E}_{\text{ave}} = 0$ at $r=0$, the location of the CoM. The magnitude of \mathbf{E}_{ave} increases as $\sim 1/a^3$ with increasing r near the origin. The electron’s electric field magnitude never becomes infinite near the origin as it does in the single static point model, and, fortunately, the proton “measures” this impact accurately through the non random vacuum fluctuations contribution to hydrogen energy levels.

The Dirac Equation’s CoC shell defines the current operator as

$$\mathbf{J}_k^{\text{CoC}} = e\delta^3(\mathbf{r}' - \Delta\mathbf{R}_k^{\text{CoC}})\mathbf{c}\boldsymbol{\alpha} \quad (22)$$

where $\mathbf{c}\boldsymbol{\alpha}$ is the Dirac Equation velocity operator, which is unique because it is the same for all eight ICO positions. The Dirac Equation’s CoC and Equation (22) define the electron’s vector potential operator in the rest frame by

$$\mathbf{A}_k(r, \theta, \varphi, a, \theta_k, \varphi_k) = e\mathbf{a} / (\mathbf{r}^2 + a^2 - 2ar \cos(\Theta_k^r))^{0.5} \quad (23)$$

The CoC ICOs will not allow one to simplify the Dirac Equation velocity operator $c\boldsymbol{\alpha}$ into a diagonal form, even in the electron's rest frame. There are two current components suggested by the expectation value of $c\boldsymbol{\alpha}$ for non-relativistic physical environments. The average value of $c\boldsymbol{\alpha}$ is just a steady current moving at the speed of the electron's CoM [4,5, 6], while there is a rapidly rotating current producing the point dipole moment in the far field. Defining $\mathbf{B}_1 = \nabla \otimes \mathbf{A}_1$ we get

$$\mathbf{B}_{11} = e[(\alpha_3(y - h/mc) - \alpha_2(z - h/mc)] / (\mathbf{r}^2 + a^2 - (h/mc)[x + y + z])^{1.5} \mathbf{i} \quad (24a)$$

$$\mathbf{B}_{12} = e[(\alpha_1(z - h/mc) - \alpha_3(x - h/mc)] / (\mathbf{r}^2 + a^2 - (h/mc)[x + y + z])^{1.5} \mathbf{j} \quad (24b)$$

$$\mathbf{B}_{13} = e[(\alpha_2(x - h/mc) - \alpha_1(y - h/mc)] / (\mathbf{r}^2 + a^2 - (h/mc)[x + y + z])^{1.5} \mathbf{k} \quad (24c)$$

for the three components of the $k=1$ for the electron magnetic field operator \mathbf{B}_1 .

The $k=2$ through 8 expressions for \mathbf{B}_k are obtained in the same way as for the electric field by inserting the correct sign for x , y , and z in the denominator and the correct sign in front of $h/2mc$ in the numerator of Equations (24a), (24b), and

(24c). Taking the time average $\mathbf{B}_{ave} = 1/8 \sum_{k=1}^{k=8} \mathbf{B}_k$ does not result in a very simple

expression with clear physical interpretation, but the ICOs produce the most accurate estimate of the single electron's magnetic field, assuming only that the Dirac Equation's geometric estimate of its vacuum fluctuations is valid.

In Equations (20) and (24 a, b, c) $\mathbf{B}_k = (\mathbf{V}_{op} / c) \otimes \mathbf{E}_k$, where $\mathbf{V}_{op} = c\boldsymbol{\alpha}$ in analogy with [1], with the Dirac Equation CoC velocity operator replacing the vector CoM velocity used in Classical Electrodynamics. Equations (16) through (24) and all the associated information in this section define the DMW Equations for the electron or

muon within hydrogen, which obey Maxwell's Equations only in the far field. Currently, there is no way to determine if the DMW Equations apply to a free electron or muon due to the very short range non random vacuum fluctuation impact. All experiments done to date, such as the electron/proton scattering experiments [12, 14] are done in the electron's far field where the electron's NR-VFG has little impact. In the far field the electron or muon appears to be a point particle with "intrinsic" properties, but this paper shows that this point particle assumption is not true in the hydrogen atom. To penetrate the electron's or muon's CoC shell to measure the impact of specific vacuum fluctuations for a free particle, the high energy required will the electron or muon decaying into other particles. The DMW Equations are the most accurate description of the electron or muon in the hydrogen atom.

The electron's Lorentz Force operator is $\mathbf{F}_k = e(\mathbf{E}_k + \boldsymbol{\alpha} \otimes \mathbf{B}_k)$, and from Equations (20) and (24)

$$\mathbf{F}_k = (e^2 / R_k^3)(-\Delta \mathbf{R}_k + (i(\boldsymbol{\Sigma} \otimes \Delta \mathbf{R}_k))) \quad (25)$$

where $\Delta \mathbf{R}_k = (x - \varepsilon_k^x \hbar / mc)\mathbf{i} + (y - \varepsilon_k^y \hbar / mc)\mathbf{j} + (z - \varepsilon_k^z \hbar / mc)\mathbf{k}$ and

$$R_k^3 = (\mathbf{r}^2 + a^2 - 2ar \cos(\Theta_k^r))^{3/2} = (\mathbf{r}^2 + a^2 - (h / mc)[\varepsilon_k^x x + \varepsilon_k^y y + \varepsilon_k^z z])^{3/2}$$

The Lorentz Force operator is a direct consequence of the Dirac Equation's SG vacuum fluctuations, but the Dirac Equation does not specify the physical basis for the CoC dynamics based on continual virtual electron/positron pair interactions in the vacuum within the CoC shell. In deriving Equation (25) the electric field part of

the Lorentz Force is repulsive, but is overcome by the magnetic field ICs to be a net restoring force directed toward the CoM. The second term in Equation (25) is a “tangential” force perpendicular to the electron’s IC and spin, and the total Lorentz Force operator ensures the electron’s CoC remains on the CoC shell and is not driven inward toward the CoM. Perhaps Equation (25) can be understood in QFT as sort of a “Casmir effect” where the totality of wavelengths outside the CoC shell produce a net inward force against the wavelengths that are “cutoff” at higher wavelengths inside the electron’s CoC shell.

4.0 Finite Electron Self Mass in Hydrogen

Since the development of the electron ICs is absolutely consistent with modern QED calculations on hydrogen energy levels [1], the CoC’s $c\alpha$ derived ICs establish a natural cut off energy [4, 5, 6] that is consistent with the geometrical CoC shell description of the electron given in the sections above. The fully covariant QFT estimate of electron self mass in the rest frame was derived by Weinberg [16], and can be calculated using the radius a of the CoC shell, and is:

$$\Delta m = (3\alpha / 2\pi) \ln(h / m_0 c a) = (.068679\alpha) m_0 \quad (26)$$

In QED self-mass is “logarithmically divergent,” and much less divergent than the “linear divergence” of Classical Electrodynamics or the QM estimate. In QED the photon propagator is incorrect, because it is not cut off by the physical geometrical description of the electron SG vacuum fluctuation shell. It is not correct to say that the electron has a finite radius, because the CoC derived from the Dirac Equation is a point. It is the CoC embedded in the vacuum fluctuations in QFT terms that produce

a specific geometrical extended shell on which there is a current. Nothing in this article implies that the electron has a spatially distributed charge distribution at a single time. The electron CoC vacuum fluctuations create a vibrating charge speed that creates a current over time. The very small QED derived self mass shown in Equation (26) is finite, and occurs in a natural fashion once the physical geometrical description of the electron as given in this paper is known. The Dirac Equation, and, therefore QFT and QED in the IR (which incorporates $c\alpha$ in the four vector current), predicts all interactions, including self interactions, occur on the CoC shell. Feynman self mass diagrams should have the virtual photon emission and absorption distance no closer than the distance between adjacent points in the electron's eight quantized ICOs. Renormalization and tree diagrams to overcome QED's self mass ultra violet infinities are unnecessary and ad hoc. It's unfortunate that one can't turn off the electron charge to experimentally test the result in Equation (26).

5.0 Conclusions and proposed model complexity to address the electrodynamic interaction between the proton quarks/gluons and the electron/muon CoC in hydrogen.

The known parameters predicted by QED for the hydrogen atom are the energy levels and a probability distribution of the electron and muon positions in these energy levels, as illustrated in Figure 1. The electron's $c\alpha$ induced fluctuations are fully incorporated in QED, and accurate energy levels are predicted. The previous sections show that both the electron and muon ICOs define these vacuum

fluctuations with a specific geometry within the hydrogen atom. These non random electron and muon vacuum fluctuations are caused by virtual pair production and subsequent pair annihilation at time scales of a period $\sim 1.4 \cdot 10^{-21}$ sec. (and ~ 207 times more rapid for the muon), that is unimaginably short in duration. The electron and muon position probability shown in Figure 1 increases as the electron or muon position moves further from the proton center, until it peaks at the Bohr orbit where the electron and muon CoC fluctuations are negligible in the proton far field, as shown by the DMW Equations (see Equations 16, 20a, 20b, and 20c). The Proton Radius Puzzle probably indicates that a more precise model of the proton is required for the case when the electron or muon CoM is very near or inside the proton.

There is no deterministic theory, and perhaps there never can be, due to the probabilistic nature of Quantum Mechanics, for the forces that cause the electron and muon to move dynamically to satisfy the QED probability for positions in various energy states. QED does not address the complex model of the proton with three fractionally charged quarks held in place by the strong force caused by the exchange of gluons against both attractive and repulsive EM forces between quarks inside the proton. The position of the quarks within the proton may be perturbed by the relatively weaker EM interaction with the electron and muon when the proton is at times “inside” the CoC fluctuations of a negative charge moving at the speed of light.

The degeneracy of the hydrogen atom energy levels in the Dirac Equation solution of the hydrogen atom for the $2S_{1/2} - 2P_{1/2}$ and similar states with the same energy level and total spin, but different orbital energy levels, is caused by the assumption of a central force for the proton. The author is developing a model of the proton with the three fractionally charged quarks held in place by the strong force based on Shanahan et al [15] and its references, but with the electron and muon CoC oscillating around the proton center at the speed of light. For the more probable positions of the electron and muon CoM further away from the proton towards their Bohr orbit, the impact of vacuum fluctuations is minimal and the electron and muon seem like point, structureless particles (or CoC shells) from the far field location of the proton. Additional factors complicating this model of the proton EM interaction with the electron and muon are:

1. It is unknown if the quarks inside the proton exhibit their own specific vacuum fluctuations (see [15] for an analysis of the Foldy-Darwin Term)
2. When the entire, spatially extended proton is “inside” the electron and muon CoC oscillations caused by vacuum fluctuations, the EM interaction is the sum of three separate interactions with each of the three proton quarks, or a complex four body problem.
3. The electrodynamic forces of the very different muon and electron CoC distances from the origin need to be estimated when the spatially extended proton is inside the muon and electron CoC shells. The average muon attraction to the proton when its CoM is at the origin is $\sim (207)^2 \sim 4 \times 10^4$ stronger than that of the electron when its CoM is at the origin. It is not

known what impact this will have on the average proton electromagnetic charge distribution, or “proton radius”.

The fact that the proton “measures” the vacuum fluctuation impact of the electron and muon within the hydrogen atom is a most unique situation. If one attempted to penetrate the electron or muon CoC shells at close range with a highly energetic photons or charged particles, the inelastic scattering debris of the collision would not reveal the valuable information that the hydrogen atom provides. It is not known yet if impact of vacuum fluctuations within the hydrogen atom changes the proton charge radius significantly for muonic and electronic hydrogen, but calculating the QED estimate of the electron and muon ICOs with a more accurate proton model must be completed to determine the impact on the Proton Radius Puzzle. The electron and muon are defined by the Dirac Equation to have discrete position and time ICOs, and their true digital nature is beyond our classical comprehension. The DMW Equations provide the most accurate estimate of the electron’s or muon’s EM fields in the hydrogen atom.

Appendix A. Test proposed for a relativistic entangled electron/positron pair exchanging information slower than instantaneously through their correlated non random vacuum phase fluctuations?

Previous sections show that the electron or muon eight ICO eigenvalues are equally likely, and the electron occupies both spin states equally until an external EM is applied, resulting in the electron being spin up or down with equal probability. An

electron or muon CoC can be in any of the 8 possible ICO eigenvalues with equal probability, and does not exist in one spin state or the other, but in both at once.

Quantum Entanglement: Since $c\alpha$ induced fluctuations exist for an electron or muon in hydrogen, one can make a testable speculation [5, 6] about the cause of Quantum Entanglement for the special case of the electron/positron (or muon/anti muon) entangled pairs. If the positron is a negative electron traveling backward in time, its phase becomes “phase locked” 180° out of phase with the electron as they travel away from each other in a vacuum that contains their phase fluctuations. Changes in the electron or positron phase travel at a speed of c^2/v^{CoM} , which is nearly instantaneous for $v^{CoM} \ll c$. Changing the electron or positron spin reverses their phase, which is almost instantaneously sensed by the other particle as a phase reversal through the fluctuating vacuum. Although this explanation for the quantum entanglement for an entangled electron/positron pair is a speculation, it can be tested. If the electron/positron pair traveled apart at a speed of $.5c$, the phase speed carrying the information between particles would not be instantaneous, but would be $2c$. Only, if the entangled electron/positron pair was nearly at rest would the information transfer be instantaneous. The results in this paper show the electron CoC and its vacuum fluctuation is a single entity and inseparable from one another unless the electron is destroyed by an external EM field. The speculation above asserts that an electron positron entangled pair may also possess a very specific non random phase for the entangled pair. The test proposed above is conceptually straight forward, but the relatively high speed of separation to measure non instantaneous information exchange may be difficult to measure experimentally

due to difficulty keeping the electron/positron pair entangled over a large enough separation to measure the difference in information velocity from instantaneous.

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Figure 1 Caption: Electron Position Probability Distribution in the Hydrogen Atom

The Hydrogen Energy Levels are estimated very accurately, but the positions of the electron and proton are not known deterministically

If the Hydrogen atom had classical Bohr orbits, there would be no Darwin Term, and ZBW would be hidden

Quantum numbers



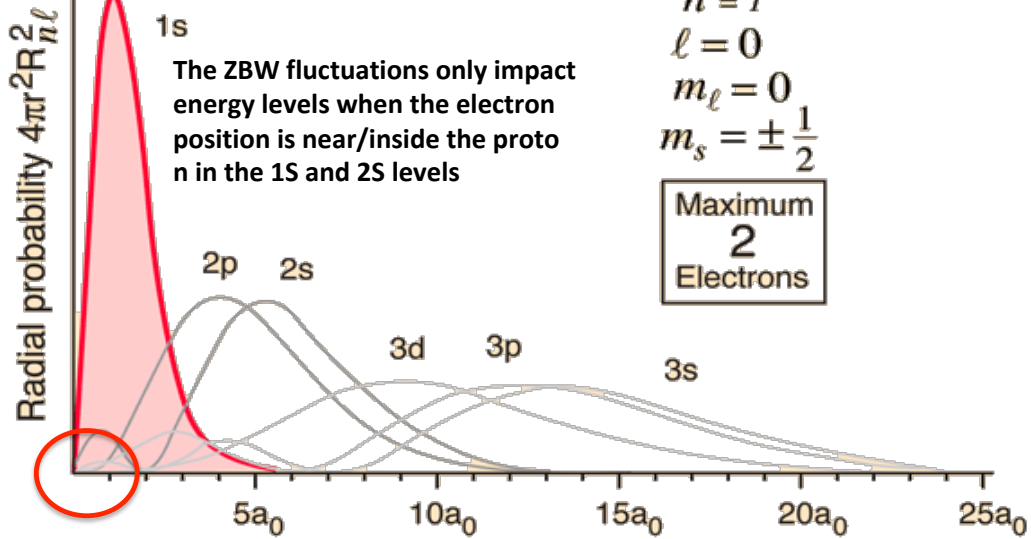
$$n = 1$$

$$\ell = 0$$

$$m_\ell = 0$$

$$m_s = \pm \frac{1}{2}$$

Maximum
2
Electrons



The ZBW fluctuations only impact energy levels when the electron position is near/inside the proton in the 1S and 2S levels

Bohr Radius 5.25×10^{-11} m