The Typical Principle

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Abstract

If a proposition is typically true, given your evidence, then you should believe that proposition; or so I argue here. In particular, in this paper, I propose and defend a principle of rationality—call it the 'Typical Principle'—which links rational belief to facts about what is typical. As I show, this principle avoids several problems that other, seemingly similar principles face. And as I show, in many cases, this principle implies the verdicts of the Principal Principle: so ultimately, the Typical Principle may be the more fundamental of the two.

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Appendix

1 Introduction

Typicality facts—that is, facts about what is typical—are often good guides to rational belief. For instance, take the fact that typically, gases expand to evenly fill their containers.

Amber, a scientist, knows that this is true. So when shown a gas in a box, Amber should believe that this particular gas will indeed expand in that way. The fact that gases typically expand, in other words, is a good guide to rational beliefs about this particular gas's expansion.

In what follows, I propose a principle of rationality—the 'Typical Principle'—which captures this. To a first approximation, the Typical Principle says the following: if you know something is typical, given your evidence, then you should believe that something.[1] For example, if you know that gases typically expand—in the sense that gases expand in nearly all possibilities with which your evidence is compatible—then you should believe that this particular gas will expand. If you know that mixing baking soda and vinegar typically leads to an explosion, in all possibilities compatible with your evidence, then you should believe that this particular mixture of baking soda and vinegar will result in an explosion too.[2]

The Typical Principle bears several striking connections to a famous principle which also constrains the rationality of certain doxastic states. Very roughly put, that other principle—called the 'Principal Principle'—says the following: if you know the chance of something, then so long as you have no evidence to the contrary, your credence in that something should equal the value of that chance. As I discuss later, in a wide range of cases, the Typical Principle implies the Principal Principle: the former, that is, implies the verdicts of the latter. So the Typical Principle captures much of the content of the Principal Principle; and

[1]For implicit endorsements of something like the Typical Principle, see (Frigg [2011]; Goldstein [2012]; Lazarovici [forthcoming]; Lazarovici and Reichert [2015]; Shafer [2011]). For somewhat more explicit endorsements, see (Cournot [2019]; Maudlin [2020]; Wilhelm [2022]).

[2]The relationship between the Typical Principle, and the principle of induction, is complicated. In some ways, these principles are similar: for instance, inductive inferences are based on observations of many cases; and we often learn typicality facts—which then support inferences, through the Typical Principle—by observing many cases too. But there are also some important differences. Perhaps the main difference concerns the role that observations of cases can play in supporting the inference in question. In particular, the fact that proposition p is typical could make it rational for an agent to believe that p is true, even if that agent has never actually observed cases of p. For example, suppose that Amber has never actually seen any gases at all, and so has never actually seen any cases of expanding gases. Then Amber cannot use induction to rationally believe that this particular gas, the one in the box, will expand. But perhaps Amber knows—because a reliable source told her, or because she deduced it from some theory which she accepts, or because of something else—the following typicality fact: gas expansion is typical. Then Amber can use the Typical Principle to rationally believe that the gas in front of her will expand.

for that reason, the Typical Principle may be the more fundamental principle of rationality. Nevertheless, as I argue, these principles differ from one another in at least three ways: one is about belief while the other is about credence; one appeals to typicality while the other appeals to chance; and each has implications that the other does not. So ultimately, neither principle can be completely eliminated in favor of the other. They express connected, but distinct, constraints on rationality.

In Section 2, I explain what typicality is. In Section 3, I present a precise version of the Typical Principle. In Section 4, I discuss how the Typical Principle avoids some problems that another, somewhat similar principle faces. Finally, in Section 5, I discuss the relationship between the Typical Principle and the Principal Principle.

2 Overview of Typicality

Intuitively, something is typical just in case nearly all things, of a certain sort, are a certain way. Gases typically expand because nearly all gases expand. Mixing baking soda and vinegar typically leads to an explosion because, nearly every time that baking soda and vinegar mix, an explosion results. Typicality facts, in other words, are facts about how nearly all situations go. Typicality facts describe what is nearly always the case.

The literature features several analyses of typicality.[3] In this paper, I focus on the following 'propositional analysis'.[4] Let p be a proposition, let Γ be a set of possible worlds, and let Γ_p be the set of all worlds in Γ at which p holds. Then according to the propositional analysis, p is typically true in Γ if and only if Γ_p is nearly as big as Γ is. It is typical that p obtains in Γ, in other words, just in case the set of worlds in Γ at which p obtains is roughly as large as Γ itself.

The propositional analysis appeals to the sizes of sets: it says that p is typically true

[3]For other analyses, see (Frigg [2009]; Frigg and Werndl [2012]; Goldstein [2012]; Wilhelm [2022]).
[4]For a related analysis, see (Crane and Wilhelm [2020]).

in Γ just in case Γ and Γ_p have nearly the same size. In order to quantify those sets' sizes, mathematical measures can be used. Intuitively, a mathematical measure is just a function which assigns sizes to sets.[5] Think of mathematical measures as quantifying, or 'measuring', how big those sets are.

For an example application of the propositional analysis, recall the gas in a box. Let Γ be the set of all possible trajectories of the gas: since each element of Γ represents one possible way for the gas to evolve, each element in Γ may be interpreted—for present purposes—as a possible world. Let p be the proposition that the gas is in equilibrium after a short amount of time; so Γ_p is the set of possible trajectories for which after that short amount of time, the gas is in equilibrium. Then as can be shown, given a particularly natural measure of the sizes of subsets which Γ contains—a measure given, in part, by the physical laws[6]—Γ_p is nearly as big as Γ is. Therefore, by the propositional analysis, p is typically true in Γ: typically, in other words, the gas quickly evolves to equilibrium.

A brief notational aside: in what follows, let $Typ_X(p)$ be the fact that proposition p is typically true in set X. Throughout the examples to come, X will generally be a subset of the set of all physically possible worlds. The sizes of those subsets—including sets like X—are always quantified using specific mathematical measures: generally, measures which the physical laws invoke.[7]

In what follows, I focus on the propositional analysis of typicality because it is especially convenient. But nothing hangs on that choice. The same basic points could be made using another analysis instead: the various principles and conditions would just need to be reworded.

So to summarize: typicality is about how nearly all situations go. A proposition is

[5] Formally, mathematical measures are functions—satisfying various conditions—that map sets to numbers. For a detailed presentation, see (Tao [2011]).

[6] The measure is called the 'modified Lebesgue measure'. For discussion of it, and how it can be used to show that Γ and Γ_p are roughly the same size, see (Boltzmann [2015]; Goldstein [2001]; Lazarovici and Reichert [2015]).

[7] For more discussion of the mathematical measures which are used to express typicality facts in physics, see (Frigg [2009]; Frigg and Werndl [2012]).

typically true, in some set of possibilities, just in case that proposition obtains in nearly all of the possibilities in that set. Typicality facts, in other words, are facts about what is nearly always the case.

3 The Typical Principle

In this section, I formulate the Typical Principle. To start, I discuss a key notion which the Typical Principle invokes. Then I present the principle itself: it says that agents are rationally required to believe propositions which typically obtain; subject to certain important caveats, however.

The key notion, which figures in the Typical Principle, is the notion of total evidence. An agent's evidence is, basically, what that agent may use to form rational beliefs about the world. Your evidence, for instance, consists of those propositions which contribute to determining what you should believe.[8] An agent's total evidence is a proposition describing everything which the propositions in their evidence describe. Your total evidence, for instance, is the conjunction of all the propositions which are in your evidence.

Now for the Typical Principle. Basically, it says that typicality facts—defined in terms of total evidence—provide rationality constraints on beliefs.

Typical Principle

Let A be an agent, let e be A's total evidence, and let p be a proposition. Let Γ be the set of all physically possible worlds, and let Γ_e be the set of all worlds in Γ at which e holds. Suppose that $Typ_{\Gamma_e}(p)$ is both true and supported by e: so p is typically true in Γ_e, and this typicality fact is among A's total evidence. Then in order to be rational, A should believe that p is true.

[8]Evidence is often analyzed in terms of other doxastic notions. For instance, some analyze evidence in terms of knowledge: a proposition is in an agent's evidence if and only if that agent knows that proposition (Williamson [2000]).

In other words, suppose proposition p typically holds in the set of all worlds which are (i) physically possible, and (ii) compatible with agent A's evidence. And suppose A's evidence includes this typicality fact. Then rationality requires A to believe that p.

For example, once again, consider the gas in a box. Let e be Amber's total evidence, and let p be the proposition that in a relatively short amount of time, the gas will evolve to equilibrium. Suppose e includes the fact that initially, the gas is in thus-and-so macrostate; so Γ_e is the set of all physically possible worlds in which thus-and-so is the initial macrostate of the gas.[9] In addition, suppose that e supports the following typicality fact: typically, in the set of all possible trajectories compatible with the gas's initial macrostate being thus-and-so,[10] the gas quickly evolves to equilibrium. So $Typ_{\Gamma_e}(p)$ is both true and supported by e. Then by the Typical Principle, in order to be rational, Amber should believe p: in other words, Amber should believe that this particular gas will indeed evolve to equilibrium in a relatively short amount of time.

Here is a nice feature of the Typical Principle: it never implies that you should believe mutually exclusive propositions. To see why, let A be an agent with total evidence e, and let p be a proposition which is typically true in the set Γ_e of all physically possible worlds where e holds. Suppose that this typicality fact—namely, $Typ_{\Gamma_e}(p)$—is among A's evidence. Then according to the Typical Principle, A should believe that p is true. Now let r be any proposition which is mutually exclusive with p. Since p and r are mutually exclusive, it is not the case that r is typically true in Γ_e: for if p is true at nearly all of the worlds in Γ_e, and if p and r are mutually exclusive, then it is not the case that r is true at nearly all of the worlds in Γ_e. So the Typical Principle does not imply that A should believe that r. And that is, of course, a good thing: since A should believe that p, it is not the case that A should

[9]Strictly speaking, this only follows given a further simplifying assumption: e does not include any propositions apart from (i) the gas's initial macrostate being thus-and-so, and (ii) the typicality fact, mentioned below, which e supports.

[10]For the sake of the example, take the set of all those trajectories—which begin in that initial macrostate—to be the set of all physically possible worlds compatible with that macrostate.

also believe that r, as p and r exclude one another.

The Typical Principle, when combined with another principle of rationality, implies a version of the lottery paradox. To see why, suppose that Amber is playing a lottery consisting of 1000 tickets, exactly one of which is the winner. And consider the 'conjunction principle' of rationality: if agent A should believe proposition x and A should also believe proposition y, then A should believe the conjunction $x \wedge y$. For simplicity, suppose that Γ_e—the set of physically possible worlds compatible with Amber's evidence—consists of the world w_1 in which ticket 1 is the winner, the world w_2 in which ticket 2 is the winner, and ..., and the world w_{1000} in which ticket 1000 is the winner. And for each n from 1 to 1000, let p_n be the proposition that the nth ticket loses. Then each p_n is typically true in Γ_e: that is, for each such n, $Typ_{\Gamma_e}(p_n)$ holds. So by the Typical Principle, for each n from 1 to 1000, Amber should believe p_n: for each such n, in other words, Amber is rationally required to believe that ticket n loses. But then by the conjunction principle, Amber should believe the conjunction $p_1 \wedge p_2 \wedge \cdots \wedge p_{1000}$ as well: in other words, Amber should believe that every ticket loses the lottery. But that is implausible. For Amber knows the lottery set-up, so she knows that one of the tickets will win.

The conjunction principle, rather than the Typical Principle, is ultimately responsible for this paradox. There are two reasons why. First, many versions of this paradox are formulated using the conjunction principle along with other principles of rationality, such as the Principal Principle discussed in Section 5. So in order to avoid all versions of the lottery paradox, rejecting the Typical Principle is insufficient. Second, the conjunction principle—just taken on its own—has extremely implausible implications: it generates the preface paradox, for instance (Makinson [1965]). This other paradox cannot be avoided by rejecting the Typical Principle, since the conjunction principle alone suffices to generate it.

So in response to the lottery paradox, we should reject the conjunction principle rather than the Typical Principle. It is true that the Typical Principle, in combination with the conjunction principle, has an unintuitive implication. But the source of that unintuitive implication is, ultimately, the conjunction principle.[11]

Note that in the Typical Principle, Γ is the set of all physically possible worlds. Γ is not allowed to be just any old set of possibilities; Γ includes all and only those possibilities which are compatible with the physical laws. So the Typical Principle only constrains rational beliefs on the basis of facts about what is typical among physically possible worlds.

This is a feature of the Typical Principle, not a bug. To see why, note that for many choices of a set Γ of metaphysically possible worlds, and for many choices of a mathematical measure which assigns sizes to subsets of Γ, it follows that for many propositions p and e, p is typically true in Γ_e. In other words, for many choices of Γ and many choices of a mathematical measure, $Typ_{\Gamma_e}(p)$ holds. Nevertheless, for many such choices, the resulting typicality fact $Typ_{\Gamma_e}(p)$ fails to be a good guide to rational belief. For many such choices of Γ, and many such choices of a mathematical measure, are gru-ish and gerrymandered. The corresponding typicality facts are artificial, and as a result, not good guides to rationality.

For example, let Γ be the set of all metaphysically possible worlds. Let w be a world in Γ at which electrons are positively charged. Consider the mathematical measure which assigns 1 to any subset of Γ containing w and assigns 0 to any subset of Γ lacking w. Let p be the—obviously false—proposition that electrons are positively charged. Let e be a tautology; so $\Gamma_e = \Gamma$. Then as a simple proof shows, $Typ_{\Gamma_e}(p)$ holds: typically, electrons are positively charged in Γ_e. Nevertheless, for any given agent A whose total evidence is e, rationality does not require A to believe p. A is not rationally required, in other words, to believe that electrons are positively charged. So for the above choice of Γ and the above choice of mathematical measure, facts like $Typ_{\Gamma_e}(p)$ are not good guides to rationality.

That is why in the Typical Principle, Γ is the set of physically possible worlds. And that is why the relevant mathematical measures, used to express typicality facts, should be those measures which the physical laws invoke. Other sets of possible worlds, and other mathematical measures, can be used to express typicality facts. But those typicality facts

[11] For more discussion of the lottery paradox and the preface paradox, and how they run afoul of the conjunction principle, see (Hawthorne [2009]).

are gru-ish, gerrymandered, and generally inapt guides to rational belief.

It is important that, in the Typical Principle, the total evidence e of the agent must support the typicality fact $Typ_{\Gamma_e}(p)$, in order for rationality to require that the agent believe p. The reason is straightforward: if an agent's total evidence did not support the fact that a given proposition p is typical—in other words, if that typicality fact were not in that agent's evidence—then that agent may not be rationally required to believe that p holds. For example, suppose that Amber has lived her whole life in a hot, dry climate. Then Amber travels to the Rocky Mountains, and for the first time, touches snow. The proposition that the snow will melt on her fingers is typically true, of course, in all physically possible worlds compatible with Amber's total evidence. But since Amber does not know this—as this typicality fact is not in Amber's evidence—it may be rationally permissible for Amber to believe that the snow will not melt after being touched. And the Typical Principle respects that.[12]

Overall, the Typical Principle does a great job of capturing the insight that rational beliefs are often constrained by typicality facts. Recall the intuitive observation with which this paper began: if you know that some proposition is typical, given your evidence, then you are rationally required to believe that proposition. The Typical Principle says exactly that. So clearly, the Typical Principle is a very attractive principle of rationality.

4 Problems Avoided

There is another significant reason to like the Typical Principle: it avoids some problems which a similar principle, closer to what is usually discussed in the literature, faces. In this section, I explain why. To start, I present a version of that other principle. Then I describe the problems. Finally, I explain why the problems do not arise for the Typical Principle.

In a few places, principles akin to the Typical Principle have been proposed. An early

[12]Thanks to an anonymous reviewer for this example.

example is due to Cournot ([2019], p. 43), who claims that events with zero probability—in other words, atypical events—are physically impossible, and so should not be expected. Similarly, Maudlin writes that agents ought to expect that which is typical ([2020], p. 246). And I have written that if something is typical, then agents are rationally required to expect it (Wilhelm [2022], p. 569).

There are two main differences between the Typical Principle and those other principles. First, the Typical Principle explicitly builds facts about agents' total evidence into its formulation. Those other principles, however, are not so explicit about the role that total evidence might play in constraining rational belief about facts which typically obtain. Second, to avoid the sorts of bizarre implications discussed at the end of Section 3, the set Γ in the Typical Principle is restricted to physically possible worlds. Some of those other principles, however, do not seem to adopt any analogous restrictions.

Those other principles differ from the Typical Principle—and from each other too—in a number of ways. But they all say something roughly along the following lines.

Simple Principle

Let A be an agent, let p be a proposition, let Γ be any given set of possible worlds, and let $Typ_{\Gamma}(p)$ be the fact that p is typically true in Γ. Suppose A knows that $Typ_{\Gamma}(p)$ holds. Then in order to be rational, A should believe that p is true.

In other words, the Simple Principle is just the Typical Principle without any qualifications concerning evidence or physically possible worlds. The Simple Principle basically just says that if something is typical in some given set of possibilities, and you know as much, then you should believe that something.

The Simple Principle faces the following problem: some propositions are known by agents to be typically true in a given set of worlds—the set of physically possible worlds, even—and yet those agents are not rationally required to believe those propositions. In fact, in some cases, agents are rationally required to believe that those propositions do not obtain. And in cases like these, the Simple Principle is false.

For example, recall Amber and the gas in a box.[13] Suppose that somehow or other, Amber learns that the gas's initial microstate is such that the gas will not evolve to equilibrium: perhaps one of Amber's colleagues meticulously prepared the gas to be in an anti-entropic macrostate like that, and then told Amber as much. Then obviously, it would not be rational for Amber to believe that this gas will evolve to equilibrium quickly. For as it happens, the gas will not evolve in that way; and Amber knows it.

The Simple Principle does not respect this fact about rationality. To see why, let Γ be the set of all possible trajectories of the gas. Let p be the proposition that the gas quickly evolves to equilibrium. Then p is typically true in Γ; in other words, $Typ_{\Gamma}(p)$ holds. In addition, Amber knows this typicality fact, since she is a competent scientist. So the Simple Principle says the following: in order to be rational, Amber should believe that p is true. In other words, according to the Simple Principle, Amber is rationally required to believe that the gas will quickly evolve to equilibrium. But that is obviously incorrect. For as mentioned above, Amber knows that—because of her colleague's intervention—the gas will not evolve to equilibrium quickly.

Therefore, in this case, the Simple Principle is false. For the Simple Principle fails to account for the specifics of Amber's total evidence. In particular, Amber's total evidence includes the fact that the gas will not evolve to equilibrium. So obviously, Amber is not rationally required to believe that the gas will evolve to equilibrium quickly. But because evolving in that way is typical, in the set of all possible trajectories of the gas, the Simple Principle says otherwise. Therefore, in this case, the Simple Principle is not a good guide to rational belief.

Note that the Typical Principle avoids this problem. To see why, let p be the proposition that the gas quickly evolves to equilibrium, and let e be Amber's total evidence. Then it is

[13] This example is based on a case discussed by Uffink ([2007], p. 980).

not the case that typically, p is true in Γ_e. In fact, at every possibility in Γ_e, p fails to obtain: for since e includes the proposition that the gas does not evolve to equilibrium, p is false at every world compatible with e. So $Typ_{\Gamma_e}(p)$ is false. And so the Typical Principle does not imply that Amber is rationally required to believe p: for p does not typically obtain in the set Γ_e of all physically possible worlds compatible with Amber's evidence.

There is another problem which the Simple Principle faces: a very general version of the problem discussed at the end of Section 3. To see the issue, let p be any proposition whatsoever which obtains in a large number of possible worlds. Let Γ be the set of all worlds w such that (i) p obtains in w, and (ii) w is not the actual world. Then $Typ_{\Gamma}(p)$ holds. So by the Simple Principle, any agent who knows $Typ_{\Gamma}(p)$ should believe that p is true. But that is clearly false: if an agent knows only that p is typically true in the set of all non-actual worlds where p is true, it certainly does not follow that this agent should believe p.[14]

The Simple Principle also faces a related, and even more unattractive, problem: as stated, the Simple Principle implies that agents should have contradictory beliefs. To see why, let p be any proposition such that (i) p obtains in a large number of possible worlds, and (ii) $\neg p$ obtains in a large number of possible worlds too. Let Γ_p be the set of all worlds in which p holds, and let $\Gamma_{\neg p}$ be the set of all worlds in which $\neg p$ holds. Then take any agent A who knows both that $Typ_{\Gamma_p}(p)$ and that $Typ_{\Gamma_{\neg p}}(\neg p)$. The Simple Principle implies that in order to be rational, A should believe that p is true, and A should also believe that $\neg p$ is true. In other words, according to the Simple Principle, A is rationally required to be inconsistent. And that is, of course, false.

All these are significant points in favor of the Typical Principle. Many propositions which typically obtain, relative to some set of possibilities or other, should not be believed. The Simple Principle does not accommodate this. The Typical Principle does. And so the Typical Principle avoids the problems mentioned above.

[14]To make this example especially compelling, let p be the proposition that the world is non-actual.

5 The Typical Principle and the Principal Principle

There is a subtle relationship between the Typical Principle and the Principal Principle. Basically, given certain plausible assumptions, the Typical Principle implies the Principal Principle in a wide range of cases. Nevertheless, as I argue, the principles differ in several important respects. So ultimately, it seems that neither principle can be completely dropped in favor of the other: they play related, but importantly distinct, roles in a comprehensive theory of rationality.

Before beginning, it is worth presenting a precise version of the Principal Principle. In what follows, I focus on the version below.[15]

Principal Principle

Let A be an agent, let p be a proposition, let t be a time, and let x be a number in the unit interval. Let Ch_t be a chance function defined, at time t, over an algebra of propositions which contains p. Let e be A's total evidence at t,[16] and suppose that e is admissible with respect to the proposition that $Ch_t(p) = x$.[17] Finally, let Cr_0 be A's initial credence function. Then in order for A to be rational, it should be the case that

$$Cr_0\big(p \mid e \,\&\, Ch_t(p) = x\big) = x$$

[15]This version is based on a principle due to Lewis ([1980], p. 270). One of the main differences, between this version and Lewis's version, concerns the proposition e upon which the initial credence function is conditionalized. In Lewis's version, e may be any admissible proposition whatsoever; in the version of the Principal Principle on which I focus here, e is the agent's total evidence.

[16]So e describes everything which A has learned from the initial time to time t.

[17]Intuitively, e is admissible with respect to $Ch_t(p) = x$ just in case an agent may know e and yet still use $Ch_t(p) = x$—in conjunction with the Principal Principle—to determine the rational credence in p. More precise characterizations of admissibility are, notoriously, difficult to formulate. But one plausible and precise characterization, due to Meacham, is as follows: basically, e is admissible with respect to $Ch_t(p) = x$ just in case the conjunction of e and $Ch_t(p) = x$ is logically equivalent to a disjunction of some law-history propositions, where a law-history proposition is itself a conjunction of (i) a proposition specifying a physical law, and (ii) a proposition which specifies everything that happens, up to some fixed time, in some possible world ([2010], p. 418). Put another way, e is admissible relative to $Ch_t(p) = x$ just in case their conjunction does not say any more than is said by descriptions of laws and possible histories of the world; in other words, admissibility is a matter of saying no more and no less than the laws, and the possible histories up to the time in question, say.

In other words, an agent's initial credence in a proposition—conditional on the chance of that proposition obtaining, along with everything else that the agent knows—should equal the chance of that proposition.

In a wide range of cases, the Typical Principle implies the Principal Principle. The precise statement of this result, and the details of the proof, are in the appendix. For now, however, the following rough description will suffice. Two popular principles imply a certain link between rational beliefs and rational conditional credences: one principle – called 'Chance Updating' in the appendix – describes how rational credences should be updated, while the other principle – called 'Doxastic Connection' in the appendix – connects rational beliefs about the chances of propositions to rational credences concerning those propositions. Plausibly, both principles hold; so suppose that they do, and therefore, suppose that the implied link holds as well. Then take any chance proposition; that is, any proposition ascribing a chance to another proposition at a time. In addition, take any agent whose total evidence is admissible with respect to the chance proposition in question. Finally, suppose that the chance proposition typically obtains in the set of all physically possible worlds compatible with the agent's total evidence. Then the verdicts of the Principal Principle, regarding this chance proposition, follow from the Typical Principle. In other words, to summarize: given certain plausible principles about how rational beliefs and rational credences relate, the Typical Principle implies the Principal Principle for chance propositions that typically obtain.

It is worth briefly explaining why this is true.[18] A chance proposition is, to be a bit more precise, a proposition of the form $Ch_t(p) = x$: this proposition says that at time t, x is the chance of proposition p. The following link, between rational belief and rational credence, is extremely plausible: if you have no inadmissible information, and if you are rationally required to believe the chance proposition that $Ch_t(p) = x$, then your initial credence in p—conditional on that chance proposition, and conditional on your evidence—should equal x as well. In other words, given that all your evidence is admissible, your beliefs about chances

[18] Again, for the formal details, see the appendix.

should guide your credences.[19] Now take any chance proposition $Ch_t(p) = x$ which typically obtains in the set of all physically possible worlds compatible with your evidence. Then the Typical Principle implies the following: you should believe that $Ch_t(p) = x$ is true. So by the above link between rational belief and rational credence, if you have no inadmissible information, then the following holds: your initial credence in p—conditional on the relevant chance proposition along with your evidence—should equal x. Therefore, your credences should conform to the relevant instance of the Principal Principle.

There is no analogously plausible derivation of the Typical Principle from the Principal Principle. The most natural candidate derivation is based on a link between rational belief and rational credence which differs from, though is analogous to, the link mentioned above. Roughly put, the relevant link is this: if you are rationally required to have credence x in proposition p at time t, then you should believe that the chance of p is x at t.[20] In other words, if a particular credence in a proposition is rational, then you should believe that the proposition has a corresponding chance of obtaining. The problem, of course, is that this is clearly false. For as many examples illustrate, agents are often rationally required to have credences in non-chancy propositions. For instance, perhaps you should have credence $\frac{1}{2}$ that Goldbach's conjecture—every even number greater than two equals the sum of two primes—is true. Nevertheless, you are not rationally required to believe that the chance of Goldbach's conjecture being true is $\frac{1}{2}$: plausibly, propositions about mathematics are not the sorts of propositions which have objective chances.

None of this implies, of course, that there is no derivation whatsoever of the Typical Principle's verdicts from the Principal Principle. My claim here is more modest: one particularly natural derivation is unsuccessful, because it assumes an implausible link between rational credence and rational belief. This provides some evidence for the view that no such derivation exists. But it is an open question whether the Principal Principle implies the

[19]This link is, in some respects, similar to the Principal Principle. But ultimately, the two are quite different; and it is a mistake, which is made periodically in the literature, to conflate them. For discussion, see the appendix.

[20]For a more precise characterization of this link, see the appendix.

Typical Principle given some other link between beliefs and credences.

Nevertheless, it is worth making the following observation. Suppose that this asymmetry, between the Typical Principle and the Principal Principle, is no illusion. In other words, suppose there are no plausible background assumptions which entail that, for some wide range of propositions, the Principal Principle implies the verdicts of the Typical Principle. Then there is clearly a sense in which the Typical Principle is the more fundamental of the two. For it covers much of the same ground that the Principal Principle covers, insofar as it implies many of the Principal Principle's verdicts. And yet the reverse is not true. The Principal Principle does not imply many of the Typical Principle's verdicts; the Principal Principle does not cover the ground covered by the latter. So the Principal Principle is less fundamental.

Regardless of whether this is indeed the case, however, neither principle can be outright eliminated: neither can be dropped, that is, in favor of the other. For there are at least three significant differences between them. First, these principles are about different doxastic states. Second, these principles reference different sorts of objective, worldly facts. Third, each principle has implications that the other does not. Let us consider each of these in turn.

Regarding the first difference: the doxastic states, which the two principles constrain, are different. The Typical Principle constrains rational belief, while the Principal Principle constrains rational credence. And beliefs and credences are different from one another. Beliefs are all-or-nothing, for instance; credences come in degrees. So credences and beliefs are often taken to be distinct kinds of doxastic states. And even when one is analyzed in terms of the other, there are still differences between them: beliefs are taken to be high credences, for instance; or credences are taken to be beliefs about probabilities.[21] So regardless, the Typical Principle and the Principal Principle are about different 'regions' of doxastic space: the former is about the belief 'region', and the latter is about the credence 'region'.

Regarding the second difference: the two principles constrain rationality using differ-

[21] For discussion of these views, and some problems with them, see (Jackson [2020]).

ent sorts of worldly facts. Whereas the Typical Principle uses typicality facts as guides to rationality, the Principal Principle uses chance propositions as guides to rationality. As has been argued elsewhere, typicality and probability are distinct (Goldstein [2012]; Lazarovici and Reichert [2015]; Wilhelm [2022]). They obey different formal conditions: probability measures must be upward continuous, for instance, while measures of typicality need not be. What it is to be probable is distinct from what it is to be typical: only probability, for instance, is intimately related to randomness. And they support different sorts of explanations: some probabilistic explanations are not typicality explanations, and some typicality explanations are not probabilistic explanations. So these principles use different worldly facts, in the constraints that they impose on rationality.[22]

Regarding the third difference: each principle has implications which the other principle lacks. For example, here is an implication of the Typical Principle which the Principal Principle does not have. Let p be any proposition which typically obtains in the set of all physically possible worlds compatible with Amber's evidence. Suppose that Amber's evidence includes this typicality fact. And suppose that p is not a chance proposition. Then the Typical Principle implies that in order to be rational, Amber should believe that p is true. But the Principal Principle does not imply anything like this. In fact, since p is not a proposition about the chances at some time, the Principal Principle does not place any constraints on the rationality of Amber's doxastic states—concerning p—at all.

Now for an implication of the Principal Principle which the Typical Principle does not have. Let $Ch_t(p) = x$ be the proposition that at time t, x is the chance of proposition p. Suppose that Amber's total evidence e is admissible with respect to this chance proposition. And suppose that this chance proposition is not typically true at all physically possible worlds compatible with Amber's evidence: perhaps it only obtains at half of those worlds, of which the actual world is one. Then the Principal Principle implies that in order for Amber to be rational, it should be the case that $Cr_0\big(p \mid e \,\&\, Ch_t(p) = x\big) = x$. But the Typical Principle

[22] For more on the relationship between probability and typicality, see (Allori [2022]; Hubert [2021]).

does not imply anything like this. In fact, since $Ch_t(p) = x$ is not typically true in the set of all physically possible worlds compatible with Amber's evidence, the Typical Principle does not place any constraints on the rationality of Amber's doxastic states—concerning the proposition that $Ch_t(p) = x$—at all.

So the Typical Principle and the Principal Principle are importantly distinct. In a wide range of cases, the former implies the latter. That is, when supplemented with some plausible assumptions about how belief and credence relate, the Typical Principle implies the verdicts of the Principal Principle in cases involving chance propositions that typically obtain in the set of all physically possible worlds compatible with a given agent's total evidence. But the Typical Principle and the Principal Principle articulate constraints on different doxastic states: beliefs for the former; credences for the latter. Those principles articulate those constraints by invoking different portions of reality: typicality facts, for the Typical Principle; chance propositions, for the Principal Principle. And there are many cases in which one principle provides rationality constraints while the other does not: only the Typical Principle delivers verdicts for propositions which do not invoke chances; only the Principal Principle delivers verdicts for propositions which do not typically obtain. Altogether, then, these principles are quite different from one another.

Because of that, both principles should be endorsed. Neither should be eliminated in favor of the other. It is true that the Typical Principle, when supplemented with some plausible assumptions, implies many of the Principal Principle's verdicts. And I take this to be defeasible evidence that the Typical Principle is the more fundamental of the two. But these two principles still differ in significant ways: they concern different sorts of doxastic states; they appeal to different sorts of worldly facts; and each constrains rational agents in ways that the other does not. So these principles cover important, but different, ground in the overall theory of rationality.

6 Conclusion

The Typical Principle expresses a precise, intuitive, attractive rationality constraint. Roughly put, it says that if a proposition typically obtains, given your evidence, then you should believe that proposition. This principle avoids some problems that other, analogous principles face. In addition, the Typical Principle complements the Principal Principle quite nicely, and there is reason to think that it is the more fundamental principle of rationality. So it is worth endorsing the Typical Principle.

Appendix

In this appendix, I prove the result mentioned in Section 5: given certain plausible assumptions, the Typical Principle implies the verdicts of the Principal Principle for a wide range of chance propositions. Then I present the assumption which, for that same range of chance propositions, can be used to prove that the Principal Principle implies the verdicts of the Typical Principle. Finally, I explain why this latter assumption is implausible.

A brief aside: I focus on deriving the version of the Principal Principle from Section 5. Other versions of the Principal Principle could be derived instead.[23] I focus on the version from Section 5 simply because it is closest to Lewis's original formulation.

The derivation relies on two principles about beliefs and credences at various times. The first principle, versions of which are endorsed throughout the literature,[24] connects rational credences at the initial time to rational credences at later times.

<u>Chance Updating</u>

Let A, p, t, x, Ch_t, Cr_0, and e be as defined in the Principal Principle. In addition, let

[23] For instance, a version due to Hall ([1994]), and a version due to Ismael ([2008]), could also be derived.
[24] For endorsements of updating principles like this one, see (Meacham [2016]; Pettigrew [2020]).

Cr_t be the credence function which agent A has at time t. Then in order for A to be rational, it should be the case that

$$Cr_0\big(p \mid e \;\&\; Ch_t(p) = x\big) = Cr_t(p)$$

In other words, an agent's initial credence in a proposition—conditional on the chance of that proposition obtaining at a later time, along with the agent's total evidence—should equal the agent's credence in that proposition at the later time.

The second principle provides a natural connection between rational belief and rational credence.[25]

<u>Doxastic Connection</u>

Let A, p, t, x, Ch_t, and Cr_t be as defined above. Suppose that A's total evidence is admissible with respect to the proposition that $Ch_t(p) = x$. Then the following holds: if A should believe that $Ch_t(p) = x$, then it should be the case that $Cr_t(p) = x$.

In other words, if an agent should believe that a particular proposition has a particular chance of obtaining (at a time), then that agent's credence in that proposition should equal that chance (at that time).

One might object that any derivation of the Principal Principle, which assumes Doxastic Connection, is circular. For Doxastic Connection, one might claim, is basically just another version of the Principal Principle. The two are pretty similar, after all, since both say that a certain sort of credence should match a certain sort of chance. So there is something problematically circular, one might claim, about deriving the Principal Principle from a set of assumptions which includes Doxastic Connection.

[25]This principle is often implicitly endorsed, or even taken to follow from the Principal Principle itself: see (Ismael [2008], p. 293; Lewis [1980], pp. 270-271). In addition, this principle follows from many other standard principles in the literature: for instance, it follows from the combination of (i) Lockean views about belief, and (ii) standard updating principles (Clarke [2013]; Foley [1992]).

That is not correct, however. For there are significant differences between Doxastic Connection and the Principal Principle. Perhaps the most important one is this: while Doxastic Connection links rational credence and rational belief, the Principal Principle links rational credence and chance. So whereas Doxastic Connection links two different kinds of mental states, the Principal Principle links one kind of mental state—namely, credence—to facts about the non-mental world. To put it another way: the Principal Principle is an externalist constraint on rationality, since it says how mind-external facts (about chance) constrain rational credence; Doxastic Connection is an internalist constraint on rationality, since it says how mind-internal facts (about rational beliefs concerning chance) constrain rational credence. So a derivation of the Principal Principle from a set of assumptions, one of which includes Doxastic Connection, is not problematically circular. For that derivation shows how a standard externalist constraint on rationality – namely, the Principal Principle – follows from (i) some internalist constraints on rationality, and (ii) another externalist constraint on rationality – namely, the Typical Principle.

Now for the derivation itself. Let A, p, t, x, Ch_t, Cr_0, and Cr_t be as defined above. Suppose that Chance Updating and Doxastic Connection both hold. Then the theorem below holds as well.

Theorem 1. *Let e be A's total evidence. Let the proposition $Ch_t(p) = x$ be typically true in the set Γ_e of all physically possible worlds at which e holds. In addition, suppose that (i) this typicality fact is included in e, and (ii) e is admissible with respect to the proposition that $Ch_t(p) = x$. Then the Typical Principle implies that in order for A to be rational, it should be the case that $Cr_0\big(p \mid e \;\&\; Ch_t(p) = x\big) = x$.*

Proof. To start, note that Chance Updating and Doxastic Connection jointly imply the following conditional.

(COND) If A should believe that $Ch_t(p) = x$, then in order for A to be rational, it should be the case that $Cr_0\big(p \mid e \;\&\; Ch_t(p) = x\big) = x$.

Now to prove the theorem. As mentioned above, e includes the fact that $Ch_t(p) = x$ is typically true in Γ_e. So by the Typical Principle, in order to be rational, A should believe that $Ch_t(p) = x$. And so by (COND), in order for A to be rational, it should be the case that $Cr_0\big(p \mid e \,\&\, Ch_t(p) = x\big) = x$. □

In other words, suppose the chance proposition $Ch_t(p) = x$ is typically true in the set of all physically possible worlds compatible with an agent's total evidence. And suppose that the agent's total evidence is admissible with respect to that chance proposition. Then given some plausible principles linking beliefs and credences, the Typical Principle implies the relevant verdicts of the Principal Principle. In slogan form: in a wide range of cases, the Principal Principle follows from the Typical Principle.

Chance Updating and Doxastic Connection cannot be used to show that in the same range of cases, the Typical Principle follows from the Principal Principle. A modified version of Doxastic Connection, however, can be used to show as much. The modified version is below.

> Doxastic Connection 2
>
> Let A, p, t, x, Ch_t, and Cr_t be as defined above. Suppose that A's total evidence is admissible with respect to the proposition that $Ch_t(p) = x$. Then the following holds: if it should be the case that $Cr_t(p) = x$, then A should believe that $Ch_t(p) = x$.

Along with Chance Updating, Doxastic Connection 2 implies the following conditional.

> (COND 2) If it should be the case that $Cr_0\big(p \mid e \,\&\, Ch_t(p) = x\big) = x$, then A should believe that $Ch_t(p) = x$ holds.

As a simple proof demonstrates, (COND 2) can be used to show that in a wide range of cases, the Principal Principle implies the verdicts of the Typical Principle.

The problem with this derivation, of course, is that Doxastic Connection 2 is completely implausible. For Doxastic Connection 2 implies that if you have any rational credence in any proposition at any time, then you are rationally required to believe that the proposition has a particular chance of obtaining—equal to that credence—at that time. And that is false. As mentioned in Section 5, you may currently have a particular rational credence in Goldbach's conjecture. Nevertheless, it does not follow that the current chance of Goldbach's conjecture being true equals your credence.

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References

Allori, V. [2022]: 'The paradox of deterministic probabilities', *Inquiry*. doi:10.1080/0020174X.2022.2065530

Boltzmann, L. [2015]: 'On the Relationship between the Second Fundamental Theorem of the Mechanical Theory of Heat and Probability Calculations Regarding the Conditions for Thermal Equilibrium' (K. Sharp and F. Matschinsky, *trans*), *Entropy*, **17**, 1971–2009.

Clarke, R. [2013]: 'Belief Is Credence One (In Context)', *Philosophers' Imprint*, **13**, pp. 1–18.

Cournot, A. A. [2019]: *Exposition of the Theory of Chances and Probabilities* (O. Sheynin, *trans*), New Castle: Scholars' Press.

Crane, H., and Wilhelm, I. [2020]: 'The Logic of Typicality', in V. Allori (*eds*), *Statistical Mechanics and Scientific Explanation*, Singapore: World Scientific, pp. 173–229.

Foley, R. [1992]: 'The Epistemology of Belief and the Epistemology of Degrees of Belief', *American Philosophical Quarterly*, **29**, pp. 111–24.

Frigg, R. [2009]: 'Typicality and the Approach to Equilibrium in Boltzmannian Statistical Mechanics', *Philosophy of Science*, **76**, pp. 997–1008.

Frigg, R. [2011]: 'Why Typicality Does Not Explain the Approach to Equilibrium', in M. Suárez (*ed*), *Probabilities, Causes and Propensities in Physics*, New York, NY: Springer, pp. 77–93.

Frigg, R., and Werndl, C. [2012]: 'Demystifying Typicality', *Philosophy of Science*, **79**, pp. 917–29.

Goldstein, S. [2001]: 'Boltzmann's Approach to Statistical Mechanics', in J. Bricmont, D. Dürr, M. C. Galavotti, G. Ghirardi, F. Petruccione, and N. Zanghì (*eds*), *Chance in Physics*, Heidelberg: Springer, pp. 39–54.

Goldstein, S. [2012]: 'Typicality and Notions of Probability in Physics', in Y. Ben-Menahem and M. Hemmo (*eds*), *Probability in Physics*, New York, NY: Springer-Verlag, pp. 59–71.

Hall, E. [1994]: 'Correcting the Guide to Objective Chance', *Mind*, **103**, pp. 505–17.

Hawthorne, J. [2009]: ‘The Lockean Thesis and the Logic of Belief’, in F. Huber and C. Schmidt-Petri (*eds*), *Degrees of Belief*, New York, NY: Springer, pp. 49–74.

Hubert, M. [2021]: ‘Reviving Frequentism’, *Synthese*, **199**, pp. 5255–84.

Ismael, J. [2008]: ‘Raid! Dissolving the Big, Bad Bug’, *Noûs*, **42**, pp. 292–307.

Jackson, E. G. [2020]: ‘The relationship between belief and credence’, *Philosophy Compass*, **15**, e12668.

Lazarovici, D., and Reichert, P. [2015]: ‘Typicality, Irreversibility and the Status of Macroscopic Laws’, *Erkenntnis*, **80**, pp. 689–716.

Lazarovici, D. [forthcoming]: ‘Typicality versus Humean probabilities as the Foundation of Statistical Mechanics’, in B. Loewer, B. Weslake, and E. Winsberg (*eds*), *The Probability Map of the Universe*. Boston, MA: Harvard University Press.

Lewis, D. [1980]: ‘A Subjectivist’s Guide to Objective Chance’, in W. L. Harper, R. Stalnaker, and G. Pearce (*eds*), *Ifs*, Boston, MA: D. Reidel, pp. 267–97.

Makinson, D. C. [1965]: ‘The Paradox of the Preface’, *Analysis*, **25**, pp. 205-07.

Maudlin, T. [2020]: ‘The Grammar of Typicality’, in V. Allori (*ed*), *Statistical Mechanics and Scientific Explanation*, Singapore: World Scientific, pp. 231–51.

Meacham, C. J. G. [2010]: ‘Two Mistakes Regarding the Principal Principle’, *The British Journal for the Philosophy of Science*, **61**, pp. 407–31.

Meacham, C. J. G. [2016]: ‘Understanding Conditionalization’, *Canadian Journal of Philosophy*, **45**, pp. 767–97.

Pettigrew, R. [2020]: ‘What is conditionalization, and why should we do it?’, *Philosophical Studies*, **177**, pp. 3427–463.

Shafer, G. [2011]: ‘A betting interpretation for probabilities and Dempster-Shafer degrees of belief’, *International Journal of Approximative Reasoning*, **52**, pp. 127–36.

Tao, T. [2011]: *An Introduction to Measure Theory*, Providence, RI: American Mathematical Society.

Uffink, J. [2007]: ‘Compendium of the Foundations of Classical Statistical Physics’, in J.

Butterfield and J. Earman (*eds*), *Philosophy of Physics*, Amsterdam: Elsevier, pp. 923–1074.

Wilhelm, I. [2022]: ‘Typical’, *The British Journal for the Philosophy of Science*, **73**, pp. 561–81.

Williamson, T. [2000]: *Knowledge and its Limits*, New York, NY: Oxford University Press.