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## A LOGICAL CONCEPTUALIZATION OF KNOWLEDGE ON THE NOTION OF LANGUAGE COMMUNICATION\*

**Abstract.** The main objective of the paper is to provide a conceptual apparatus of a general logical theory of language communication. The aim of the paper is to outline a formal-logical theory of language in which the concepts of the *phenomenon of language communication* and *language communication in general* are defined and some conditions for their adequacy are formulated. The theory explicates the key notions of contemporary syntax, semantics, and pragmatics. The theory is formalized on two levels: *token*-level and *type*-level. As such, it takes into account the dual – *token* and *type* – ontological character of linguistic entities. The basic notions of the theory: *language communication*, *meaning* and *interpretation* are introduced on the second, *type*-level of formalization, and their required prior formalization of some of the notions introduced on the first, *token*-level; among others, the notion of an *act of communication*. Owing to the theory, it is possible to address the problems of adequacy of both empirical acts of communication and of language communication in general. All the conditions of adequacy of communication discussed in the presented paper, are valid for one-way communication (sender-recipient); nevertheless, they can also apply to the reverse direction of language communication (recipient-sender). Therefore, they concern the problem of two-way understanding in language communication.

**Keywords:** act communication, language communication in general, token-type distinction, meaning, interpretation, problem of adequacy of communication, formal-logical theory of language communication.

### 1. Introduction

The key issue of modern pragmatics as a part of semiotics is communication, whose main task is the transmission, processing, and transformation of information. It does not mean, however, that we fully understand what communication is and what the conditions of its proper flow are. The problem of communication is as old as mankind and has been present in many different fields ever since, for example: in cultural systems, sign systems (in-

cluding language systems), but also in market systems, bank systems, and recently emerged computer networks.

The discovery and cognition of reality is best realized through the processes of cognition, whose result is knowledge of a conceptual space. It is expressed and represented in language and transferred to others in *acts of communication* by means of concrete, material language expressions – *token-expressions* (see Diagram 1).

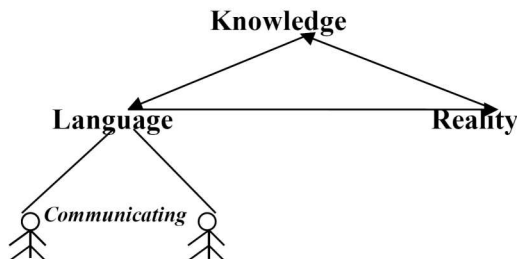


Diagram 1

Acts and processes of communication take place not only among people, but also among any communication channels, organization units, which are the subjects of this communication, for example: groups of people, firms, political parties, governments and so on. In communication acts, a very important role is played by the knowledge of objects represented by means of words and other signs. It can also be influenced by cultural, psychological, sociological, political, and technical factors. In this paper I concentrate on the representation of knowledge that takes place in language systems of communication.

The aim of the paper is to outline a logical theory of language in which the *phenomenon of language communication* and *language communication in general* are defined and some conditions for their adequacy are formulated.

Assimilation and transfer of knowledge about objects to other people is possible owing to the cognitive-communicative function of language. The transfer of verbal knowledge takes place in *acts of communication* by means of concrete, material language expressions (*token-expressions*). In formal considerations, first, we want to provide definitions of an *act of language communication* and the related notions such as: *using linguistic tokens* and *interpreting linguistic tokens* in order to formulate some general conditions for the correct course of the act of communication, i.e. to consider its *adequacy* and indicate some general causes of *verbal miscommunication*.

*A Logical Conceptualization of Knowledge on the Notion of Language...*

The notion of an *act of communication* has to be differentiated from the notion of *language communication* which is a basic concept of logical pragmatics and of the logic of language in general.

Answering the following questions:

*what is language communication as such?*

and

*what are the conditions for correct communication?*

i.e. considering the problem of its adequacy is a primary task for a general theory of language communication.

A logical conceptualization of the knowledge on the notion of *language communication* and such related notions as *meaning* and *interpretation* of language expressions involved in communication cannot be performed unless certain philosophical assumptions concerning the nature of these notions and of the expressions themselves are adopted, and unless some prior assumptions are made on the selection of primitive notions and the method of defining.

In the paper, an axiomatic theory of language communication *TLC*, as a semantic-pragmatic theory, independent of extra-logical factors, is outlined. First, in Section 2, some aspects that we take into account in formalization of the theory *TLC* will be discussed. The theory has to be based on a theory of syntax *T*. Some foundations of the syntax theory *T* will be presented in Section 3. According to the *token-type* distinction of language objects originated from Ch. S. Peirce (1931–1935) it is formalized on two levels: *token* and *type*. The proposed theory *TLC* will be developed in Section 4 as an expansion of the syntax theory *T* to the semantic-pragmatic theory in which – on the *token*-level – the concept of *act of language communication* by means of *token*-expressions (understood as physical, material, empirical, enduring through time-and-space objects) will be defined, and the problem of adequacy of communicating by means of such expressions will be considered, while – on the *type*-level – the notion of *language communication* by means of *type*-expressions (understood as abstract, ideal objects, classes of *token*-expressions) and such related notions as *meaning* and *interpretation* will be defined and some conditions of adequacy for such communication will be formulated. The paper ends with Section 5, in which we differentiate the earlier given conditions for adequacy and general, logical factors for verbal miscommunication and misunderstanding from the extra-logical (e.g., psychological, sociological, political) ones.

## 2. Three Aspects in Formalization of the *TLC* Theory

The presentations of an axiomatic formal-logical theory *TLC* as a semantic-pragmatic theory, independent of any extra-logical factors, psychological or sociological or communication channels, which – on the one hand – can enhance understanding, but – on the other – can interfere with it, will be however based on some assumptions.

Although *TLC* will concern communication by means of expressions of any language, it will take into consideration, to a certain degree, the following three aspects:

- (1) the cognitive-communicative function of natural language, according to its genesis,
- (2) the so-called functional approach to logical analysis of this language, and the one connected with it:
- (3) two understandings of a manner of use and a manner of interpreting language expressions in communication.

Let us expand on these aspects.

### 2.1. The cognitive-communicative function of natural language according to its genesis

Given the genesis of natural language, one can easily observe that it was formed in the process of cognition and communication between people who made use of material, concrete signs. Accordingly, we make the assumption that the primitive linguistic entities applied in communication acts between their senders and recipients are material creations, e.g. given sounds, written signs, physical objects somehow placed in time and space, concrete objects which have some referents attributed to them, and which are called *tokens*. According to the well-known *token-type* distinction made by CH.S. Peirce

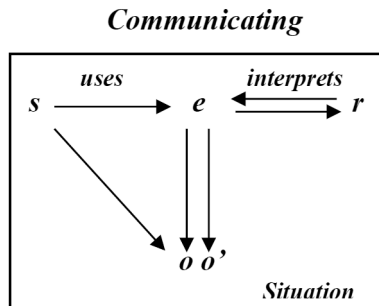


Diagram 2

(1931–1935), we differentiate *token*-signs (signs-examples) from *type*-signs, which are abstract, ideal linguistic objects, and whose physical representations are just *tokens*.

In acts of communication (see Diagram 2) the sender *s* calls, uses a token *e* of a sign with reference to a broadly conceived object *o*, while the recipient *r* interprets it in compliance or in discordance with the sender's intention, as object *o* or another object *o'*. Compliance produces understanding, while discordance produces misunderstanding.

## **2.2. The functional approach to natural language analysis**

As we have already seen, in order to explain the notion of communicating we had to introduce the terms *using* and *interpreting*, which entailed the use of concrete entities, i.e. *tokens*, and the inclusion of situational contexts accompanying them. This shows how *tokens* function in communication acts. Even though we are not going to refer to situational contexts in our theoretical considerations, the context is always present in such acts.

In the proposed theory *TLC*, the basic semantic-pragmatic notions, including the notion of *language communication* and the related concepts – *meaning* and *interpretation* – are defined by means of *expression-types*, and yet their definitions involve such primitive notions of the theory as *using* and *interpreting* expression-*tokens*.

The formal conception of language communication has some connections with the understanding of meaning as a manner of use of expressions and interpretation as a manner of interpreting expressions.

Speaking about the functional approach to natural language analysis, we have to take into consideration the *manner of use* and the *manner of interpreting* language expressions. The latter will be regarded as a special case of the former.

## **2.3. Two understandings of *manner of use* and *manner of interpreting* language expressions in communication**

The functional approach to natural language analysis involves speaking about two meanings of the terms: 'a *manner of use*' and 'a *manner of interpreting*'.

After the approach of J. Pelc (1971; 1979), we distinguish two understandings of these terms:

- in the first of them, the *manner of using* (*use*) and the *manner of interpreting* (*int*) occur only in given circumstances, in specific language-situational-contexts, and concern expression-*tokens* only,

- in the second – the *manner of Use (usage)* and the *manner of interpreting (Int)* characterize the meaning of the expression and the interpretation of the expression, respectively; these manners are somehow built into this meaning and this interpretation, respectively. In this case an expression can be treated as isolated, static, out of context, e.g. as an entry in a dictionary. It is then an expression-*type*, a class of its concrete occurrences, a distributive set of expression-*tokens* used either to represent a given object, or in concrete acts of communication in specific linguistic-situational contexts, with reference to only one broadly conceived object or to a set of objects of the same kind.

For example, two single *tokens* of the expression-*type* ‘scientist’, having an established meaning (the *manner of Use*) or a specific interpretation (the *manner of Int*) in English, can be used in a similar linguistic-situational context either with reference to a given scientist, e.g. the one which I am pointing to, or with reference to two different scientists, e.g. in a situation of teaching a student the meaning of the word ‘scientist’ through a definition and pointing to two different scientists.

The relation *use* and its sub-relation *int*, concerning all the relations of physical object-based reference of expression-*tokens* made by users of language, will be primitive notions of the theory *TLC* proposed here. The relation *Use* (resp. the relation *Int*) is, on the other hand, a relation defined by means of the relation of *use* (resp. the relation *int*) and applied by users of language for expression-*types*. The difference between these relations is explained by the fact that two persons can *Use* the same expression-*type* by means of its two different *tokens*.

The notion of an expression is a syntactic one and must be defined on the basis of a theory of syntax.

### 3. Language Syntax; Theory *T*

#### 3.1. Two levels of formalization of syntax of language

The theory of syntax *T* is formalized on two levels: *token*-level and *type*-level. According to the *token-type* distinction by Peirce (1931–1935), any language *L* is characterized as a construct of a double ontological nature: both as

- a language of expression-*tokens* (at the *token*-level)
- and as
- a language of expression-*types* (at the *type*-level).

The theory  $T$  is first formalized on the *token*-level as the theory of *token*-syntax describing  $L$  as a language of expression-*tokens*, and then, on the *type*-level, as the theory of *type*-syntax describing  $L$  as a language of expression-*types*. The theory of *type*-syntax is an extension of the theory of *token*-syntax.

*Tokens* are primitive objects of the theory  $T$ . They are intuitively understood as concrete, material, empirical objects, enduring through time and space and perceived by sight. They are usually inscriptions, but do not have to be inscriptions. They can be on paper, a notice board, a blackboard, a computer screen, a stone, etc.; they may be configurations of such things as jigsaw-puzzle pieces, leaves, stones, stars, or smoke signals, or illuminated advertisements, and so on.

*Types* are derived objects of the theory  $T$  defined by means of *tokens*. They are understood as sets (classes) of *tokens* bearing an *identifiability* relation to each other, i.e. *types* are ideal, abstract entities.

### 3.2. Identifiability of linguistic *tokens*

The *relation of identifiability*  $\sim$  of *tokens* (a primitive notion of the theory  $T$ ) is determined by pragmatic factors and not by physical similarity, and it is understood very broadly. For instance, inscriptions printed in different types but consisting successively of the same letters of the alphabet may be *identifiable*, e.g. the word-*tokens*:

DUBROVNIK

**Dubrovnik**

*Dubrovnik*

written in capital letters, in bold with bigger typeface or in italics, respectively, can be regarded as *identifiable* words.

We will assume that the relation of the *identifiability*  $\sim$  of *tokens* is an equivalence relation.

The *expressions* of language  $L$  are defined separately on the *token*-level and on the *type*-level. They are suitable *concatenations* of *tokens* or *types*. The *relation of concatenation of tokens* is another primitive notion of the theory  $T$ .

### 3.3. Concatenations

*Concatenations of tokens* are complex words of language  $L$  obtained from two words of the *vocabulary* of language  $L$  – the next primitive notion of theory  $T$ . Concatenations on the *token*-level may be, but do not have to be, sequences of two *tokens*. Intuitively, a concatenation of two written *tokens*  $a$

and  $b$ , for example in a European language, is a written *token*  $c$  that is made up by adding the written *token*  $b^*$ , identifiable with  $b$ , to the token  $a^*$ , identifiable with  $a$ , on the right.

For example, the concatenations of the following word-*tokens*:

$C$   
 $o$   
 $n$                     FORMAL METHODS  
 $f$   
 $e$   
 $r$   
 $e n c$   
 $e$

the second and the first, is the name-*token*:

*Formal Methods Conference*

and any name-*token* identifiable with it, in particular the token aligned vertically:

FORMAL METHODS  
CONFERENCE

or any token written on each poster on the conference.

So, the relation of *concatenation* defined by *tokens* is not a set-theoretical function and the relation of *identifiability* is not a relation of physical similarity. These two relations and the *vocabulary of tokens* are primitive notions of the theory of words which is included in the theory of syntax  $T$ . They are formalized on the *token*-level. All of them satisfy some specific axioms of the theory.

### 3.3. Well-formed expressions

The most important notion of the theory of syntax  $T$  is the notion of a *well-formed expression* of language  $L$  (for short: *wfe*). The theory  $T$  can be built as a theory of language syntax in which (see Wybraniec-Skardowska, 1991) all *wfes* are generated by a categorial grammar (see K. Ajdukiewicz, 1935; Y. Bar-Hillel, 1950, 1953, 1964; J. Lambek, 1958, 1961; R. Montague, 1970a, b, c, 1974; M. J. Cresswell, 1973, 1977; W. Marciszewski, 1988; W. Buszkowski, 1988, 1989 and others). On the basis of the theory  $T$  we can reconstruct such a grammar. The notion of a *wfe* is defined firstly on the *token*-level and then on the *type*-level. Then the set  $S$  of all *wfe-tokens* is formally defined as the smallest set including the vocabulary of tokens and closed with respect to syntactic connection rules.



The set  $S$  of all well-formed expression-*types* (for short: *wfe-types*) is defined as the quotient family of the set  $S$  of all *wfe-tokens* determined by the relation  $\sim$  of identifiability:

$$\underline{S} = S / \sim .$$

Hence, we get that:

$$\underline{p} \in \underline{S} \text{ iff } \exists p \in S (\underline{p} = [p]_{\sim} = \{q \in S : q \sim p\}).$$

So, any well-formed expression-type  $\underline{p}$  is an equivalence set of all *wfe-tokens* identifiable with a *wfe-token*  $p$ .

In the following sections, we will use *wfe-types* not only as elements of the set  $\underline{S}$  but also all non-empty subtypes of *wfe-types* of this set. By *wfe-types* of  $L$  we will mean all elements of the set  $S^*$ :

$$S^* = \{\underline{e} \subseteq \underline{p} : \underline{e} \neq \emptyset \wedge \underline{p} \in \underline{S}\},$$

i.e. all non-empty sets of identifiable *wfe-tokens*.

## 4. A Theory of Language Communication – Theory *TLC*

### 4.1. *Token-level*

Because the formal theory *TLC* should define the notion of *language communication*, its conceptual apparatus has to refer to the notions of *meaning* and *interpretation* of language expressions and to empirical *acts of communication* among people. So, on the *token-level* its conceptual apparatus has to include the notions of *using* and *interpreting token-expressions* by users of language  $L$ . Thus, we accept the postulate that in communication acts the sender, in order to send the message, applies the function *use* connected with the object reference of a *wfe-token*, whereas the recipient, in order to receive the message, applies another function – the function *int* of interpreting tokens.

#### 4.1.1. Primitive notions of *TLC*

Primitive notions of the theory *TLC* are:

- the set *User* of all users of a given language  $L$ ,
- the set *Ont* of all extra-linguistic objects described by  $L$ ,
- the two-place operation *use* of using the *wfe-tokens* of  $L$ .
- the two-place operation *int* of interpreting the *wfe-tokens* of  $L$ .

The first two primitive notions are understood very broadly. The set *User* of users of language  $L$  can be composed of current as well as former

or future users of this language. We do not make any assumptions, either, about the ontological nature of objects of the set *Ont*. They can be not only material objects, but also, for instance, fictional or abstract creations described by language *L*.

Of course, the sets *User* and *Ont* are non-empty sets:

AXIOM (*sets: User, Ont*):  $User \neq \emptyset$  and  $Ont \neq \emptyset$ .

We understand the operation (relation) *use* as an operation producing, calling, using, exposing or interpreting *wfe-tokens* in order to refer them to corresponding objects of the set *Ont*. We can also call this operation a *function of object reference of wfe-tokens* by users of language *L*.

The operation *int* occurs when we speak about communication by means of expression-tokens. This operation will be a restriction of the former one.

The operations *use* and *int* satisfy the following axioms:

AXIOMS (*using*): *use* is a partial function of

$$User \times S \rightarrow Ont,$$

$$Dom_1(use) = User \text{ and } Dom_2(use) \subset S.$$

AXIOM (*interpreting*): *int* is a partial function of the function *use*, i.e.

$$\emptyset \neq int \subseteq use \text{ and } Dom_2(int) \subseteq Dom_2(use) \subset S.$$

The expression:  $use(u, e) = o$ , where  $u \in User$ ,  $e \in S$ ,  $o \in Ont$  is read: *the user u uses (makes or exposes) the wfe-token e to refer to the object o*. This object *o* is called the *referent* of the *wfe-token e* assigned by its user *u*. Similarly, the expression  $int(u, e) = o$  is read: *the user u interprets (understands) the wfe-token e as a sign-token of the object o*. The object *o* is called the *interpretandum* of the *wfe-token e*.

It follows from the second axiom that every user of *L* uses at least one *wfe-token* of *L* to refer to an object. Not every *wfe-token* must have a referent. From the third axiom it follows that the operation *int* of *interpreting tokens* is narrower than the operation *use* of *using tokens*. This is because the pair  $\langle \text{a user, a token} \rangle$ , which has a referent, may have no corresponding interpretandum when, for instance, this *token* cannot be received or was used with the intention of being interpreted by a recipient, but he/she cannot interpret it. The fact is, however, that each pair that has an interpretandum also has the same referent.

The notion *int* of interpreting tokens emerges when we speak about communication by means of expression-tokens. From the axioms, we immediately get:

COROLLARY 1 *a.*  $\forall u \in Dom_1(int) \forall e \in Dom_2(int) (int(u, e) = use(u, e)),$   
*b.*  $\exists u \in User \exists e \in S \exists o \in Ont (use(u, e) = o = int(u, e)),$   
*c.*  $Dom_1(int) \subseteq Dom_1(use) \subseteq User.$

Thus (see part *c.*), *interpreting tokens* is a particular case of *using tokens*.

On the basis of part *a.* we can state that if we limited both domains of the operation *use* using *wfe-tokens* to the domain of operation *int* interpreting *wfe-tokens* of *L*, then these two operations would not be discernible; then every user using any expression-*token* to refer to an object is a person who also interprets this expression as this object. Such a situation is not specific of communicating by means of *tokens*, but it follows from part *b.* that there exists at least one user of *L* who uses and interprets a *token* in a given act of communication by means of this *token* as the same object.

#### 4.1.2. Act of Communication

The notion of *communication act* is new in *TLC*. An *act of communication* is defined as a triple satisfying of some conditions:

DEFINITION 1a) (*act of communication*):

$\langle s, e, r \rangle \in acom$  iff  
 $s, r \in User \wedge e \in S \wedge \exists o, o' \in Ont (use(s, e) = o \wedge int(r, e) = o').$

Its first element *s* (the sender) and the third of its elements *r* (the recipient) are users of language *L*, the second element *e* is a *wfe-token* of *L* and there exist objects  $o, o' \in Ont$  such that the sender *s* of the expression *e* uses the expression *e* to refer to the object *o* (the referent) and the recipient *r* of the expression *e* interprets this expression as a sign-token of the object *o'* (the interpretandum) (see Diagram 3a).

Communication acts can be carried out by means of two different expression-tokens of the same *wfe-type* (see Diagram 3b), if the sender *uses* a token and the recipient interprets another *token* the same expression-*type*; this is so in e-mail, microphone or telephone communication.

So, the more general definition of an *act of communication* is in accordance with:

DEFINITION 1b) (*act of communication*):

$\langle s, e, r \rangle \in ACom$  iff  $s, r \in User \wedge$   
 $\wedge \exists \underline{e} \in \underline{S}^* (e \in \underline{e} \wedge \exists e' \in \underline{e} \exists o, o' \in Ont (use(s, e) = o \wedge int(r, e') = o')).$

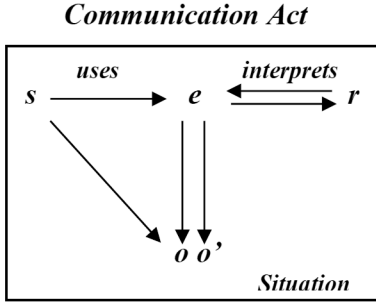


Diagram 3a

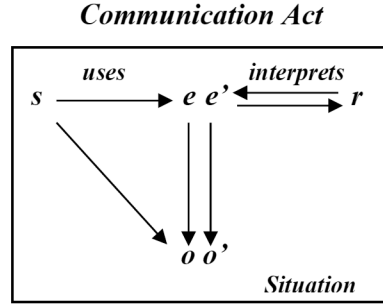


Diagram 3b

It is easy to see that any act of communication by means of one *token* is also an act of communication by means of two expression-*tokens*. So, we have

COROLLARY 2.  $\langle s, e, r \rangle \in \text{acom} \Rightarrow \langle s, e, r \rangle \in \text{ACom}$ .

Examples of communication acts include: making an announcement, this present paper, a specific question, e.g. in a discussion, etc.

#### 4.1.3. Adequacy of Communication Acts

The problem of adequacy of an act of communication by means of a *wfe-token* consists in its effectiveness. A communication act is effective if using the *token* by its sender and interpreting the *token* or a *token* identifiable with that *token* by its recipient are in agreement, i.e. the referent to which the sender uses the token and the interpretandum as an object of interpreting the *token* or a *token* identifiable with that *token* by its recipient, are the same. In other words, a communication act is effective when an *understanding* takes place between its sender and its recipient.

Two definitions of an act of communication by means of *wfe-tokens* will bring us to two definitions of the notion of *understanding* (see Diagrams 4a and 4b).

DEFINITION 2a) (*understanding*):

$$\text{und}_e(s, r) \text{ iff } s, r \in \text{User} \wedge e \in S \exists o \in \text{Ont} (\text{use}(s, e) = o = \text{int}(r, e)).$$

DEFINITION 2b) (*understanding*):

$$\text{Und}_e(s, r) \text{ iff } s, r \in \text{User} \wedge \exists \underline{e} \in \underline{S}^* (e \in \underline{e} \wedge \exists e' \in \underline{e} \exists o \in \text{Ont} (\text{use}(s, e) = o = \text{int}(r, e'))).$$

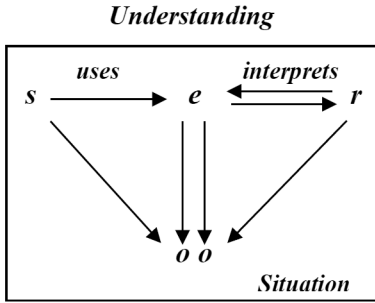


Diagram 4a

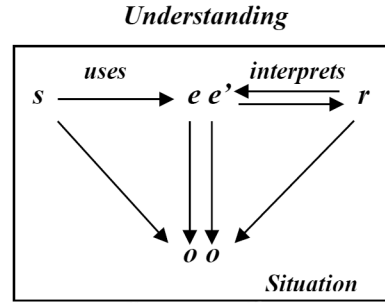


Diagram 4b

Abbreviations ‘ $und_e(s, r)$ ’ and ‘ $Und_e(s, r)$ ’ are used here for the expressions: ‘Between  $s$  and  $r$  in an act of communication by means of the *wfe-token*  $e$  or the *tokens*:  $e$  and some identifiable *token*  $e'$ , respectively, there exists understanding’. The object which is both the referent and the interpretandum in the act of communication determined by  $Und_e(s, r)$ , is called the *object of understanding*.

It is quite obvious that if there exists understanding in the first sense, then there exists understanding in the second sense, and the following conclusions are valid:

- COROLLARY 3
- a)  $und_e(s, r) \Rightarrow Und_e(s, r)$ ,
  - b)  $und_e(s, r) \Rightarrow \langle s, e, r \rangle \in acom$ ,
  - c)  $Und_e(s, r) \Rightarrow \langle s, e, r \rangle \in ACom$ ,
  - d)  $\exists u \in User \exists e \in S (\langle u, e, u \rangle \in acom \wedge und_e(u, u))$ ,
  - e)  $acom \neq \emptyset \wedge ACom \neq \emptyset$ .

Point d) of the above corollary states that there exists at least one user of the language  $L$  who takes part in an act of communication by means of an expression-token simultaneously as the sender and the recipient, and understanding takes place in the act. So, we have e): the sets of all communication acts, in both senses, are nonempty.

#### 4.1.4. Miscommunication: Misunderstanding

If, in an act of communication by means of a *wfe-token*, understanding does not take place between its sender and its recipient, then the act of communication is *not adequate* and we may speak about *miscommunication*. It occurs if *misunderstanding* takes place in this act or if an attempted act of communication fails because of *non-understanding* between the sender and the recipient.

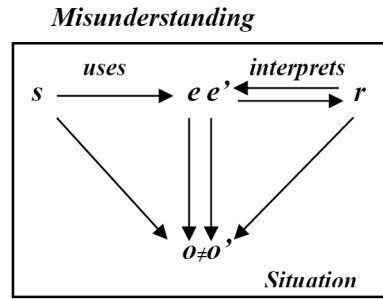
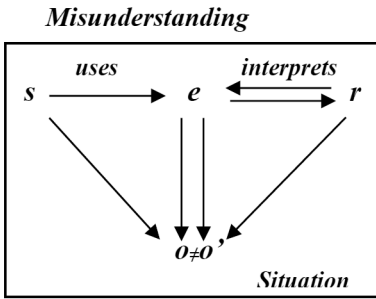
From two definitions of a communication act, we will obtain two definitions of *misunderstanding* and two definitions of *non-understanding*.

DEFINITION 3a) (*misunderstanding*):

$$misund_e(s, r) \text{ iff } s, r \in User \wedge e \in S \exists o, o' \in Ont (use(s, e) = o \neq o' = int(r, e)).$$

DEFINITION 3b) (*misunderstanding*):

$$Misund_e(s, r) \text{ iff } s, r \in User \wedge \wedge \exists \underline{e} \in \underline{S}^* (e \in \underline{e} \exists e' \in \underline{e} \exists o, o' \in Ont (use(s, e) = o \neq o' = int(r, e'))).$$



If the sender of the expression-token  $e$  uses this expression to refer to an object and the recipient interprets this or another expression-token  $e'$  of the same type as another object, then there exists a *misunderstanding* between the sender and the recipient in the act of communication by means of the expression  $e$  (see Diagrams 4a and 5a).

#### 4.1.5. Miscommunication: Non-understanding

If the sender of the expression  $e$  uses  $e$  to refer to a referent but the recipient is unable to interpret the expression  $e$  or an expression identifiable with that expression, then there follows a *non-understanding* (see Diagrams 6a and 6b). Thus, symbolically:

DEFINITION 4a). (*non-understanding*):

$$non-und_e(s, r) \text{ iff } s, r \in User \wedge e \in S \wedge \wedge \exists o \in Ont (use(s, e) = o \wedge \forall o' \in Ont (\neg int(r, e) = o')).$$

DEFINITION 4b). (*non-understanding*):

$Non-und_e(s, r)$  iff  $s, r \in \mathbf{User} \wedge \exists \underline{e} \in \underline{S} (e \in \underline{e} \wedge \wedge \exists e' \in \underline{e} \exists o \in \mathbf{Ont} (use(s, e) = o \wedge \forall o' \in \mathbf{Ont} (\neg int(r, e') = o')))$ .

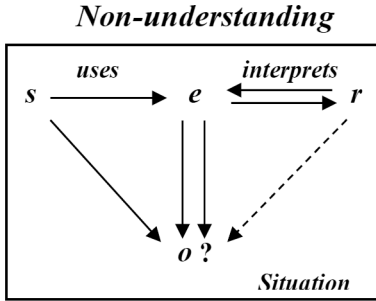


Diagram 6a

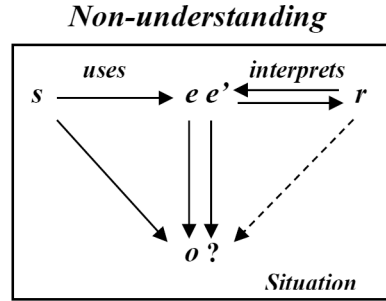


Diagram 6b

## 4.2. Type-level

### 4.2.1. Communication by means of expression-*types*

Empirical communication by means of expression-*tokens* has to be distinguished in a given community of *Users* from communication by means of *wfe-types*. On the *type-level* we expand the conceptual apparatus of the *TLC* with new notions. The most important one is the notion of *communication by means of types*. It is determined as a value of an *operation communication C* defined on expression-*types*.

The *operation communication C* is a function defined as follows:

DEFINITION 5 (*operation communication*):

$$C : \underline{S}^* \rightarrow 2^{\mathbf{User} \times S \times \mathbf{User}}$$

$$C(\underline{e}) = \{ \langle s, e, r \rangle : s, r \in \mathbf{User} \wedge e \in \underline{e} \wedge \langle s, e, r \rangle \in ACom \}$$

for every *wfe-type*  $\underline{e}$  of language *L*.

The value  $C(\underline{e})$  of the function *C* for the expression-*type*  $\underline{e}$  is called *communication by means of the expression-type*  $\underline{e}$ . *Communication C*( $\underline{e}$ ) *by means of the expression-type*  $\underline{e}$  is the relation  $\mathbf{User} \times S \times \mathbf{User}$  consisting of all ordered triples, such that the first element (the sender) uses a *wfe-token* of  $\underline{e}$  and the third component (the recipient) interprets a *token* of  $\underline{e}$  in an act of communication. So, communication  $C(\underline{e})$  by means of the expression-*type*  $\underline{e}$  is the set of all communication acts by means of expression-*tokens* of the type  $\underline{e}$ .

It includes the set of all communication acts by means of only one *token* of the *type*. Moreover, it follows from earlier corollaries that there exists a *wfe-type*  $\underline{e}$  such that communication  $C(\underline{e})$  by means of type  $\underline{e}$  is a nonempty set. Thus we arrive at:

COROLLARY 4 a)  $\{\langle s, e, r \rangle : s, r \in \mathit{User} \wedge e \in \underline{e} \wedge \langle s, e, r \rangle \in \mathit{acom}\} \subseteq C(\underline{e})$ ,  
 b)  $\exists \underline{e} \in \underline{\mathcal{S}}^* (C(\underline{e}) \neq \emptyset)$ .

#### 4.2.2. Using *types* and Interpreting *types*

Users that participate in acts of communication belonging to language communication by means of an expression-*type*  $\underline{e}$  are also *Using* the expression-*type*  $\underline{e}$ : senders *Use* this *type* while recipients *Interpret* it. The relation *Use* of *Using* expression-*types* and its sub-relation *Int* of *Interpreting* expression-*types* are new notions of *TLC*. They are binary relations satisfying some axioms and defined by means of relations *use* and *int* for tokens, respectively:

AXIOM (*Use*):  $\mathit{Use} \subseteq \mathit{User} \times \underline{\mathcal{S}}^*$ ,

AXIOM (*domain of Int*):  $\mathit{Dom}_1(\mathit{Int}) \subseteq \mathit{Dom}_1(\mathit{int}) \subseteq \mathit{User} = \mathit{Dom}_1(\mathit{use})$ .

The relation *Use* is defined as follows:

DEFINITION 6 (*Using types*):

$$u \mathit{Use} \underline{e} \text{ iff } \exists e \in \underline{e} \exists o \in \mathit{Ont} (\mathit{use}(u, e) = o).$$

According to this definition, the user  $u$  *Uses* the *wfe-type*  $\underline{e}$  iff the user  $u$  *uses* a *wfe-token* of the type  $\underline{e}$  to refer to some referent.

The definition of relation *Int* is dual to the definition of the relation *Use*,

DEFINITION 6<sup>i</sup> (*Interpreting types*):

$$u \mathit{Int} \underline{e} \text{ iff } \exists e \in \underline{e} \exists o \in \mathit{Ont} (\mathit{int}(u, e) = o).$$

and it says that the user  $u$  *Interprets* the *wfe-type*  $\underline{e}$  iff the user  $u$  *interprets* a *wfe-token* of the type  $\underline{e}$  as some interpretandum.

Because  $\mathit{int} \subseteq \mathit{use}$ , i.e. the relation *int* of interpreting tokens is included in the relation *use* of using tokens, the relation *Int* of interpreting *types* is included in the relation *Use* of using types (see Corollary 5a); however, from the Axiom given above for the relation *Int* for types, it follows that the user who *Uses* a *type* does not need to be the one who *Interprets* it.



Because communication  $C(\underline{e})$  by means of the type  $\underline{e}$  is a nonempty set, the above definitions lead to Corollary 5b) and the comment found at the top of this subsection is justified:

COROLLARY: 5 a)  $Int \subseteq Use$ .

b)  $int = use \Rightarrow Int = Use$ .

c)  $\langle s, e, r \rangle \in C(\underline{e}) \Rightarrow s Use \underline{e} \wedge r Int \underline{e}$ .

d)  $Use \neq \emptyset, Int \neq \emptyset$ ,

Point d) of the above corollary immediately follows from point c).

#### 4.2.3. Problem of Adequacy of Language Communication

*Adequate, effective, successful communication* in a community of *Users* by means of the expression-type  $\underline{e}$  is based on the agreed *meaning*  $\mu(\underline{e})$  of the expression-type  $\underline{e}$  used by users who are senders of *tokens* of  $\underline{e}$  in acts of communication, and based on the correlation  $\mu(\underline{e})$  with the *interpretation*  $\iota(\underline{e})$  of the expression-type  $\underline{e}$  interpreted by users who are recipients of these tokens in the acts (cf. Wybraniec-Skardowska 2015). Compatibility of the meaning and the interpretation of the expression-type  $\underline{e}$  leads to *understanding* between senders and recipients (see Diagram 7).

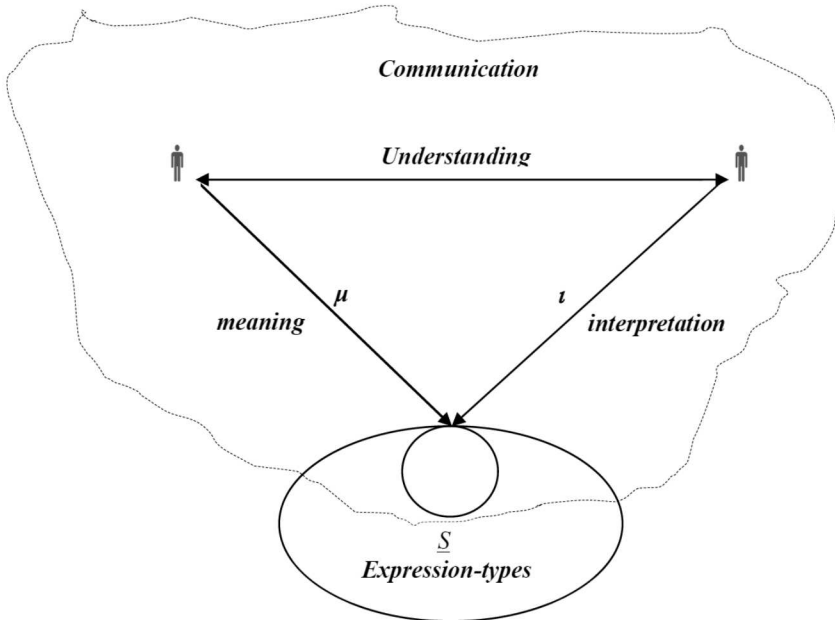


Diagram 7

A disagreement between the meaning and the interpretation of the expression-*type* leads to *misunderstanding*, while ignorance of the interpretation of the expression-*type* leads to *non-understanding*.

#### 4.2.4. Notions Relating to Language Communication

It is obvious that the conceptual apparatus of the theory *TLC* has to be enriched by notions concerning *meaning* and *interpretation* of language expression-*types*.

As we said before, these notions will be characterized in relation to the understanding of *meaning* as a *manner of Using* (*usage*) expression-*types* and *interpretation* as a *manner of Interpreting* (*Int*) these expressions; these manners are in a way built into this *meaning* and this *interpretation*, respectively.

*Interpretation* indicates the *meaning* or *meanings* of a given expression-*type* and cannot be identified with its *meaning*. Let us also note that the notion of *interpretation* does not need to be connected with sign-based systems of communication only; in semantics, it plays a special, central role.

The notion of *meaning* is defined by means of the *relation*  $\cong$  of *having the same manner of Using* *wfe-types* and the notion of *interpretation* – by means of the *relation*  $\cong_i$ . of *having the same manner of Interpreting* (*understanding*) *wfe-types* (see Wybraniec-Skardowska, 2007a, b; 2015, 2016). The definitions of these relations are as follows:

DEFINITION 7 (*having the same manner of Using types*):

$$\underline{e} \cong \underline{e}' \text{ iff } \forall u \in \text{User} [(u \text{ Use } \underline{e} \Leftrightarrow u \text{ Use } \underline{e}') \wedge \\ \wedge \forall o \in \text{Ont} (\exists e \in \underline{e} (\text{use}(u, e) = o) \Leftrightarrow \exists e' \in \underline{e}' (\text{use}(u, e') = o))].$$

DEFINITION 7<sup>i</sup> (*having the same manner of Interpreting types*):

$$\underline{e} \cong_i \underline{e}' \text{ iff } \forall u \in \text{User} [u \text{ Int } \underline{e} \Leftrightarrow u \text{ Int } \underline{e}' \wedge \\ \wedge \forall o \in \text{Ont} (\exists e \in \underline{e} (\text{int}(u, e) = o) \Leftrightarrow \exists e' \in \underline{e}' (\text{int}(u, e') = o))].$$

Two *wfe-types*  $\underline{e}$  and  $\underline{e}'$  have the same manner of *Using* (resp. of *Interpreting*) *wfe-types* if and only if every user of language *L* **uses** (resp. **interprets**) the other one every time he/she **uses** (resp. **interprets**) either of them, and every object is a referent (resp. an interpretant) of some *token* of the *type*  $\underline{e}$  (*used/interpreted* by the user) iff it is a referent (resp. an interpretant) of some *token* of the other *type*  $\underline{e}'$  (*used/interpreted* by the user).

The *relation*  $\cong_i$  *having the same manner of Interpreting types* is given if its arguments belong to  $\text{Dom}_2(\text{Int})$ . So, we adopt the following axiom:

AXIOM (domain of  $\cong_i$ ):  $\cong_i \subseteq (Dom_2(Int) \times Dom_2(Int)) \cap \cong$ .

And, the relation  $\cong_i$  is a sub-relation of the relation  $\cong$ , and it can easily be proved that it is a nonempty relation (from Corollary 5c:  $Int \neq \emptyset$ , and because it is a reflexive relation).

THEOREM 1: The relations  $\cong$  and  $\cong_i$  are equivalence relations in the set  $\underline{S}^*$ .

Definitions of meaning and interpretation of the wfe-type  $\underline{e}$  are the following:

DEFINITIONS 8 (*meaning and interpretation*):

$$\text{a) } \mu(\underline{e}) = [\underline{e}]_{\cong} \quad \text{and} \quad \text{b) } \iota(\underline{e}) = [\underline{e}]_{\cong_i}.$$

The definition of interpretation  $\iota(\underline{e})$  of the *wfe-type*  $\underline{e}$  is dual to the definition of meaning  $\mu(\underline{e})$  of the expression. According to these definitions: Meaning  $\mu(\underline{e})$  and interpretation  $\iota(\underline{e})$  of the *wfe-type*  $\underline{e}$  is the equivalence class of all expressions possessing the same manner of *Using* or, respectively, *Interpreting (understanding)*, as the expression  $\underline{e}$ , and can be intuitively understood as a common property of all *wfe-types* having the same manner of *Using* or, respectively, *Interpreting* as the expression-type  $\underline{e}$ . The property can be called the *manner of using* or, respectively, the *manner of interpreting* of the expression-type  $\underline{e}$ . In this way, we are referring here to ideas originating from Ludwig Wittgenstein (1953) and Kazimierz Ajdukiewicz (1931, 1934), that is to understanding of the meaning as a manner of its *Use/Interpreting*.

It is easy to see that we have:

THEOREM 2 a)  $\iota(\underline{e}) \subseteq \mu(\underline{e})$ .

$$\text{b) } int = use \Rightarrow \iota(\underline{e}) = \mu(\underline{e}).$$

So, the notion of meaning is stronger than the notion of interpretation.

#### 4.2.5. Dual Conceptual Counterparts

It should be observed that the notions of the system:

$$(*) \quad use, Use, \cong, \mu,$$

have, within *TLC*, dual counterparts in the system:

$$(**) \quad int, Int, \cong_i, \iota.$$

All the notions of the system (marked with two asterisks) have dual definitions towards the corresponding definitions of the theory *TLC* concerning the notions of the first system (\*).

So, all theorems of the theory *TLC* formulated for the notions of (\*) remain valid if we replace the notions of this system (\*) with their dual counterparts of (\*\*). The close relationships between the semantic-pragmatic notions of the systems (\*) and (\*\*) cause these notions to be often regarded as identical. However, each relation or function of the system (\*\*) is only a sub-relation of its counterpart in the system (\*) and not of all theorems of *TLC* concerning the notions of this system **which** have their dual counterparts.

The meaning  $\mu(\underline{e})$  of a wfe-type  $\underline{e}$  and the interpretation  $\iota(\underline{e})$  of the type  $\underline{e}$  may differ. If that is the case, the communication  $C(\underline{e})$  by means of the wfe-type  $\underline{e}$  does not have to be *adequate*.

Using the notions of *meaning* and *interpretation* we can define the notion of *adequacy of language communication*.

#### 4.2.6. Adequacy of Language Communication

As it has already been mentioned, in language communication, interpretation indicates the meaning or meanings of the expression-*type* which intermediates in this communication. An expression-*type* may have more than one meaning. If it has more meanings, they are determined by subtypes of the expression, as for example, for the terms: ‘key’ or ‘bank’.

We will adopt the following definition of *adequacy of communication*:

DEFINITION 9 (*adequacy of language communication*):

If  $\underline{e}$  has  $n$  ( $n \geq 1$ ) meanings determined by its subtypes  $\underline{e}_1, \underline{e}_2, \dots, \underline{e}_n$  then  $C(\underline{e})$  is an *adequate communication* iff

$$\forall k = 1, \dots, n \ (\underline{e}_k \text{ has determined interpretation and } \iota(\underline{e}_k) = \mu(\underline{e}_k)).$$

From the definition of adequacy of communication by means of wfe-*type* we obtain some conditions of adequacy of language communication:

COROLLARY 6:

- a) If  $\underline{e}$  has  $n$  ( $n \geq 1$ ) meanings determined by its subtypes  $\underline{e}_1, \underline{e}_2, \dots, \underline{e}_n$ , then  $C(\underline{e})$  is not an *adequate language communication* iff  $\exists k = 1, \dots, n$  ( $\underline{e}_k$  does not have a determined interpretation or  $\iota(\underline{e}_k) \neq \mu(\underline{e}_k)$ ).
- b) If  $\underline{e}$  has an established meaning and  $\underline{e}$  has a determined interpretation then  $C(\underline{e})$  is an *adequate language communication* iff  $\iota(\underline{e}) = \mu(\underline{e})$ .

- c) If  $\underline{e}$  has an established meaning and  $\underline{e}$  does not have a determined interpretation, then  $C(\underline{e})$  is not an adequate language communication.
- d) If  $\underline{e}$  has an established meaning,  $\underline{e}$  has a determined interpretation and  $\iota(\underline{e}) \neq \mu(\underline{e})$ , then  $C(\underline{e})$  is not an adequate language communication.

We see that the accord of meaning and interpretation is a necessary condition of adequate language communication by means of expression-*type* of  $L$ .

The two next theorems provide us with some sufficient conditions for adequacy of communication by means of types.

**THEOREM 3:** If  $int = use$  and  $\underline{e}$  has an established meaning and a determined interpretation then  $C(\underline{e})$  is an adequate language communication.

The above theorem follows from Theorem 2b and Corollary 6b.

## 5. Summary

The main objective of the work presented was to provide a conceptual apparatus of a general logical theory of language communication. The outlined axiomatic theory explicates the key notions of contemporary syntax, semantics and pragmatics.

The theory is formalized on two levels: *token*-level and *type*-level. As such, it takes into account the dual – *token* and *type* – ontological character of linguistic entities.

The basic notions of the theory: language communication, meaning and interpretation are introduced on the second, *type*-level of formalization, and they require prior formalization of some of the notions introduced on the first, *token*-level; among others, the notion of an act of communication.

Owing to the theory, it is possible to address the problems of adequacy of both empirical acts of communication and of language communication in general.

However, so far it has not been possible to theoretically capture the intuitive relationships between the adequacy of language communication and the correctness of its communication acts.

The paper is only an attempt at providing a conceptual apparatus for the theory. One cannot expect it to offer strong theorems as yet, although it seems that the theorems concerning the relationships between adequacy of language communication and adequacy of its communication acts should function well enough.

All the general conditions of adequacy of language communication discussed in the presented paper were shown as if they were valid for one-way communication (sender–recipient); nevertheless, they can also apply to the reverse direction of language communication (recipient–sender). Therefore, they concern the problem of two-way understanding in language communication.

Finally, it can be noted that the conceptual apparatus of the theory can be enriched through the introduction of notions concerning some specific forms of communication, such as discourse and dialog.

#### N O T E

\* The sketch of the paper was presented by me and Jacek Waldmajer at IASS-AIS 9<sup>th</sup> World Congress of Semiotics, *Communication : Understanding, Misunderstanding*, 11–17 June 2007, Helsinki–Imatra, Finland. Some assumptions of this paper was also presented by the author and Jacek Waldmajer at the 37<sup>th</sup> Poznań Linguistic Meeting, 20–23 April 2006; see also my and Jacek Waldmajer paper (2008). The present version of the paper was delivered by me at *Formal Methods and Science in Philosophy Conference*, 4–6 May 2017, Dubrovnik, Croatia.

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