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ON THE THEORY OF LABELS-TOKENS

This note is based on a lecture delivered at the Conference on the Scientific Research of the Mathematical Center of Opole, Turawa, May 10-11th, 1980. A somewhat extended version will be published in the Proceedings of the Conference. At the same time it is an abstract of a part of a planned larger paper, which will involve the theory of labels-tokens.

The problem of constructing an axiomatic theory of labels-tokens was formulated by Professor Jerzy Słupecki. A certain solution to this problem is given in [1], another version of it is presented in this paper.

1. The theory of labels-tokens, to be denoted here by the symbol "$TLTk$", is a description of concrete, visually perceptible objects, i.e. of material objects. Clearly, to explain the properties such objects have is more complicated than just to settle the properties of abstract objects. Some difficulties are faced already when we try to explain the intuitional sense of the primitive notions of the theory $TLTk$.

The theory $TLTk$ is based on four primitive concepts: the set of all labels-tokens, the set of all words-tokens, the equiformity relation and the concatenation relation. These concepts will be denoted, by the symbols: "Lb", "W", "$\approx$" and "c", respectively.

Labels-tokens are for instance definite expressions of a given language written with a pen, ball point pen or pencil at a definite place in a notebook or printed at a definite place in a given copy of a book. Each part of label-token (the word "part" has here the mereological meaning). Concrete labels need not be products of man, they may be products of Nature, e.g. twines of branches or stems, or configurations of stones.

The *set W of all concrete words-tokens* is the subset (proper or not) of the set Lb. Not all properties of words-tokens need to be properties of labels-tokens. Such a property is e.g. belonging to a definite syntactic category. Also, some axioms of the theory $TLTk$ are true for all words-tokens, but are not true for labels-tokens. Such axioms will be given further on (p. 2) (see A10-A12).

The *equiformity relation* is a binary relation on Lb. The expression

$$p \approx q$$

is to be read: the labels-tokens p and q are equiform. Examples of equiform labels-tokens are: the label being the antecedent of implication A10 given below and the label being the consequent of implication A7. Equiform labels may appear at different places. Therefore equiformity of labels is not equality of labels.

Finally, *concatenation relation* is a ternary relation on Lb. The expression

$$c(p, q, r)$$

is to be read: the label-token r is a concatenation of the labels-tokens p and q.

In the European ethnic graphic languages the concatenation of the labels p and q is each label obtained from the label equiform to the label p by adding at the right side, in the immediate neighborhood and at the same level of it, the label equiform to the label q.

In particular, the concrete label of the English language

coffee

(and each equiform to it) obtained from the labels-tokens

fee
cof

is their concatenation.

On the other hand, for instance in the Hebrew language, the concatenation of two labels is obtained differently. Quite a lot of expressions-tokens of the language of the theory $TLTk$ I use also do not possess the property.

2. The theory $TLTk$ is based on the classical first-order functional calculus with identity and on the set theory.

In the axioms of the theory $TLTk$, the variables denoted by the letters $p, q, r, s, t, u, v \ldots$ range over the elements of Lb and the variable X over the family 2^{Lb}. I shall also use the defined symbol "V" to denote a definite subset of the set W of all words-tokens called the *vocabulary*:

$$v \in V = v \in W \wedge \sim \bigwedge_{p,q \in W} c(p,q,v).$$

According to the definition, the vocabulary V is the set of those words-tokens which are not concatenations of any pairs of words-tokens.

The following expressions are the *specific axioms* of the theory $TLTk$:

A1a. $p \approx q$,
  b. $p \approx q \Rightarrow q \approx p$,
  c. $p \approx q \wedge q \approx r \Rightarrow p \approx r$,
A2. $\bigvee_{r} c(p,q,r)$,
A3. $c(p,q,r) \Rightarrow \sim (r \approx p) \wedge \sim (r \approx q)$,
A4. $p \approx r \wedge q \approx s \wedge c(p,q,t) \wedge c(r,s,u) \Rightarrow t \approx u$,
A5. $c(p,q,s) \wedge c(s,r,t) \wedge c(q,r,v) \wedge c(p,v,u) \Rightarrow t \approx u$,
A6. $c(p,q,t) \wedge c(r,s,t) \Rightarrow (p \approx r \Leftrightarrow q \approx s)$,
A7. $c(p,q,s) \wedge s \approx r \Rightarrow c(p,q,r)$,
A8. $c(p,q,t) \wedge c(r,s,u) \wedge t \approx u \Rightarrow p \approx r \vee \bigvee_{v}[c(r,v,p) \vee c(p,v,r)]$,
A9a. $V \neq \emptyset$,
  b. $W \subseteq Lb$,
A10. $c(p,q,r) \Rightarrow (r \in W \Leftrightarrow p,q \in W)$,
A11. $p \in W \wedge p \approx q \Rightarrow q \in W$,
A12. $X \subseteq Lb \wedge V \subseteq X \wedge \bigwedge_{p,q \in X} [c(p,q,r) \Rightarrow r \in X] \Rightarrow W \subseteq X$.

(Axioms A1a-c state that the equiformity is an equivalence relation on Lb. Axioms A2-A8 settle the properties of the concatenation relation, axioms A8-A12 characterize the sets V and W).

3. Like it was shown in [1], basing on $TLTk$ it is possible to construct a theory of labels-types (abstract labels). Abstract labels are equivalence classes of the equiformity, determined by labels-tokens. The theory of labels-types is a theory generated from $TLTk$ by adding to it some definitions. Moreover, it does not require neither new primitive terms or axioms.

All axioms of metascience – the theory constructed by A. Tarski [2] – are theorems of it.

References

[1] G. Bryll and S. Miklos, *Teoria konkretnych i abstrakcyjnych słów (The theory of concrete and abstract words)*, **Zeszyty Naukowe Wyższej Szkoły Pedagogicznej w Opolu, Matematyk** XX (1977), pp. 63–76.

[2] A. Tarski, **Pojęcie prawdy w językach nauk dedukcyjnych**, Warszawa 1933.

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