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# Relevance without Minimality

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## I. Introduction

C5P1 A notion that comes up everywhere in philosophy is that of a circumstance “contributing” to a result or outcome—or being a “factor” in, or “helpful” or “relevant” to, the result or outcome.<sup>1</sup> One is looking in most cases for a *Q* that is *wholly* helpful: free of irrelevant accretions making no real difference.

C5P2 Causes should bear positively on their effects. Material to which an effect is not beholden should be kept as far as possible out of the cause. An argument’s premises, or the assumptions employed in a proof, should help to make the case for its conclusion. If a premise can be dropped without invalidating the argument, it probably shouldn’t have been there in the first place.

C5P3 Grounds should contribute to what they ground, both in toto and throughout. That it would redress an injustice is a reason for  $\varphi$ -ing only if its redressing the injustice counts in favor of  $\varphi$ -ing. Insofar as other properties of  $\varphi$ -ing (it is normally done at night) do not count in its favor, these other properties do not form part of the reason for  $\varphi$ -ing. An observation does not confirm a hypothesis if it is irrelevant to whether the hypothesis is true. One would not expect an irrelevant observation to figure in the evidence for that hypothesis.<sup>2</sup>

C5P4 This last example, of confirming *P* or figuring in the evidence for it, helps to clarify the kind of relevance at issue. Hempel distinguishes three notions of confirmation—absolute, comparative, and quantitative—to focus attention on the first (Hempel 1945). Quantitative confirmation theory tries to develop measures of the extent to which *Q* confirms *P*. Comparative confirmation theory tries to make sense of *Q* confirming *P* *more* than *Q'* confirms *P*. Absolute confirmation

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<sup>2</sup> I am hedging a bit because *Q* need not be “intrinsically” relevant to be relevant in the circumstances.

is a binary affair, both in involving two elements— $Q$  and  $P$ —and allowing only two verdicts:  $Q$  confirms  $P$ , or else it fails to confirm  $P$ . Hempel mentions comparative and quantitative confirmation only to set them aside for a later stage of the investigation.

C5P5 Relevance in the sense of this chapter is a binary affair too.  $Z$  contributes to  $Y$ , or it does not, period.<sup>3</sup> Not a lot will be said about comparative helpfulness, or degrees of helpfulness. Various other subtleties will be set aside as well. Our focus will be on actual, rather than generic, or potential helpfulness. This means, first, that  $Z$  is helpful to  $Y$  only if both obtain.<sup>4</sup> Also that a factor that normally works against  $Y$ — $Y$  holds, if it does, *despite* this factor—may yet be helpful to it on a particular occasion, and vice versa. Likewise a normally neutral  $Z$  may join forces with  $Y$ 's friends on some occasions, and its enemies on others.

C5P6 Plan of the chapter: Relevance is usually explained in terms of notions like *minimality*, *difference-making*, *essentiality*, and *non-redundancy*. This sort of explanation is reviewed in the next two sections, first from an analytic, then a quasi-historical perspective. We will see that it does not get to the heart of things.  $Z$  can still contribute to  $Y$  even if  $Z$  does not figure indispensably in the conditions for  $Y$ , and even where minimality considerations do not apply. The problem is seen to have hyperintensional aspects. A diagnosis is attempted and a solution sketched in terms of “focused” minimality, or minimality where a certain subject matter is concerned.

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## II. Dependence

C5P7 One imagines to begin with that  $Z$  contributes to  $Y$  just if  $Y$  counterfactually depends on  $Z$ , that is,  $Y$  would not have obtained if not for  $Z$ . Writing  $\gg$  for the counterfactual conditional and using an upper bar for negation:

C5P8 [C1]  $Z$  contributes to  $Y$  just if:  $\bar{Z} \gg \bar{Y}$ .

C5P9 This will not long satisfy us, for a couple of reasons. One is that  $Z$  and  $Y$  will in some applications (grounding, entailment,...) be necessary truths.  $\bar{Z} \gg \bar{Y}$  is in that case a counterpossible conditional. These are at least as theoretically elusive as positive relevance, and raise a lot of the same problems, for example to do with hyperintensionality.

C5P10 The second reason not to rest too much on counterfactuals comes from the theory of causation.  $Z$  can contribute causally to  $Y$  even if  $Y$  would still have

<sup>3</sup> Helpfulness may be contingent on other facts, but they are not among its relata.

<sup>4</sup> Just as both need to obtain for  $Y$  to hold *despite*  $Z$ . Helpfulness in our sense is something like the opposite of despitensess.

obtained (on some alternative basis) in  $Z$ 's absence. The mismatch is often explained as follows.<sup>5</sup>  $Y$  depends on  $Z$  if an  $X$  obtains with four properties:

- C5P11 (i)  $X$  contains  $Z$ .  
 C5P12 (ii)  $X$  suffices for  $Y$  ( $X \Rightarrow Y$ ).  
 C5P13 (iii)  $X \setminus Z$  does not suffice for  $Y$  ( $(X \setminus Z) \not\Rightarrow Y$ ).  
 C5P14 (iv)  $Y$  is not overdetermined—there is no backup condition  $B$  (actual or counterfactual) that would do the job if  $X$  didn't.

C5P15 But, granted that  $Y$  does not *depend* on  $Z$  if  $Y$  is overdetermined, why should this undermine  $Z$ 's claim to be making a contribution? Whether there are, or would be, other contributors about, even ones sufficient for  $Y$ , seems just irrelevant to the issue of whether  $Z$  itself contributes to  $Y$ . This suggests our focus should not have been on dependence, but a deeper fact (defined by (i)–(iii)) that sometimes *makes for* dependence:

C5P16 [C2]  $Z$  contributes to  $Y$  just if:  $Z$  is part of an actual  $X$  such that  $X \Rightarrow Y$ , but  $(X \setminus Z) \not\Rightarrow Y$ .

C5P17 Merely counterfactual backups drop out of the picture on this approach. Alternative *actual* backups are taken in stride and seen as posing no threat.  $X$  need not be in any sense unique, on [C2], for  $Z$  to qualify as helpful to  $Y$  by figuring essentially in  $X$ .  $Z$  achieves relevance by pulling an “almost” sufficient condition  $A$  (aka  $X \setminus Z$ ) over the finish line:  $A$  does not itself suffice for  $Y$ , but  $A+Z$  suffices.

C5P18 A problem remains.  $Z$  could pull  $A$  over the finish line even if it was partly irrelevant to  $Y$ , provided it was also partly relevant. Did Socrates die because he drank hemlock in a toga? Of course not. But drinking-hemlock-in-a-toga is by [C2]'s lights just as helpful to his death as drinking hemlock. If adding hemlock-drinking to the right sort of insufficient condition  $A$  yields a sufficient condition for death, then adding hemlock-drinking-in-a-toga does too.

C5P19 Here we can just double down on the idea behind [C2]. Rather than requiring only of  $Z$  that it be essential to  $X$  qua basis for  $Y$ , we should ask *everything* in  $X$  to be essential to it qua basis for  $Y$ .

C5P20 [C3]  $Z$  contributes to  $Y$  just if: an  $X$  obtains such that  $Z \leq X$ ,  $X \Rightarrow Y$ , and  $\forall U < X$  ( $U \not\Rightarrow Y$ ).

C5P21 The new requirement reaches down to  $Z$ 's parts, since these will also be part of  $X$  by transitivity of part/whole. Given that an  $X$  including the toga can be cut down to a no less sufficient  $U$  leaving the toga out, drinking hemlock in a toga

<sup>5</sup> Kment (2014), Strevens (2007).

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does not count by the new rule as a factor in Socrates' death. [C3] says in effect that  $Z$  contributes if it is contained in a *minimal* sufficient condition for  $Y$ , a sufficient condition  $X$  whose proper subconditions are always insufficient. This, the minimal sufficiency model of relevance, is what we are going to make trouble for in this chapter. It admits, like any philosophical model, of various refinements.<sup>6</sup> But we will not bother too much about these, since they do not affect the problem we're coming to. That problem runs deep and is not easily tweaked away.

C5P22 The problem formally speaking is that not everything *has* a minimal basis. We do not want to conclude from the fact that sufficient conditions for  $Y$  always contain smaller such conditions that none of these are wholly, pervasively helpful to  $Y$ . Especially if nothing counts as helpful at all except by participating in a sufficient  $X$  that is helpful through and through.

C5P23 The problem intuitively is that  $X$ , to be wholly helpful, need only be wholly *welcome* from  $Y$ 's perspective. So far is this from requiring  $X$ 's parts to be one and all essential to  $X$ , qua sufficient condition for  $Y$ , they can be one and all *inessential*.  $X$  can be composed of elements that would none of them be missed, though of course large enough combinations of them would be missed.

C5P24 Consider the (utterly banal) idea of "extra help." Extra help is a contradiction in terms on the minimal sufficiency model. For suppose  $Z$  was extra, or not strictly needed;  $X/Z$  would have been enough. Then the result  $X$  of adding  $Z$  to  $X \setminus Z$  was not minimally sufficient. So  $Z$  did not contribute? This runs completely counter to intuition. When the winning team in a tug of war is larger than necessary, we don't feel that some strange magic occurred, in which a team achieved victory with no help from its members. Yet we should feel this, if particular rope-tuggers, to be helpful, must pull an otherwise losing team over the line.

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**III. History**

C5P25 Let's re-approach the question "historically" (note the scare quotes). When did minimality pressures begin to make themselves felt? When do we first encounter the point just noted—that  $Z$ , to be welcome from  $Y$ 's perspective, need not be a *sine qua non* of  $Y$ , even in the relaxed sense of being a *that-without-which- $X$ -would-not-suffice*?

C5P26 Sufficiency had a long run in philosophy before anyone thought to worry about irrelevant add-ons. There was the Principle of Sufficient Reason. Causes were events given which the effect was sure to follow. Validity was a matter of premise-truth sufficing for the truth of the conclusion. Grounds for a higher-level fact were, and sometimes still are, items or conditions prior to that fact and sufficient for it.

<sup>6</sup> For instance one might want to add, for certain applications, a "contiguous chain" requirement along the lines of Kim (1973).

- C5P27 These proposals put a lower bound on, say, the cause, but not an upper bound, since  $X \Rightarrow Y$  is monotonic in  $X$  ( $X^+$  suffices if  $X$  does). The analysanda are more discerning. Socrates died not because he drank the hemlock in a toga, but because he drank the hemlock. The existence of even primes is grounded in 2 being an even prime, not that together with 9 being an odd non-prime.
- C5P28 If sufficiency allows causes to get too big, we might think of asking  $X$  also to be *necessary* for  $Y$ . Hume considers this in the *Treatise* but rejects it on the basis that effects need not have been caused at all, let alone by their actual causes:
- C5P29 If we define a ‘cause’ to be *An object precedent and contiguous to another, and where all the objects resembling the former are similarly precedent and contiguous to objects that resemble the latter*, we can easily grasp that there is no absolute or metaphysical necessity that every beginning of existence should be preceded by such an object. (Hume 1740/2003, Bk I, §14, “Of the Idea of Necessary Connexion”)
- C5P30 The issue for us is *natural* necessity, not metaphysical, and  $Y$ ’s specific cause rather than its being caused at all. But specific causes are not naturally necessary either, for Hume. From *Objects resembling  $X$  are always succeeded by objects resembling  $Y$* , it does not follow that *Objects resembling  $Y$  are always preceded by objects resembling  $X$* .<sup>7</sup>
- C5P31 Hume does appreciate, even in the *Treatise*, that causes as he officially defines them are liable to be overloaded with extraneous detail. We find him in the very next section (*I, 15*, “Rules by which to judge of causes and effects”) looking for ways to block this:
- C5P32 where several different objects produce the same effect, it must be by means of some quality, which we discover to be common amongst them...in order to arrive at the decisive point, we must carefully separate whatever is superfluous, and enquire by new experiments, if every particular circumstance of the first experiment was essential to it.
- C5P33 He suggests here a different way of keeping  $X$  within bounds. Rather than requiring causes to be necessary—so that  $Y$  no longer *holds* given just part of  $X$ —we ask them only to be non-redundant— $Y$  is not *ensured* by just part of  $X$ . This becomes in the *Enquiry* (Hume 1740/2006) a full-blown proportionality requirement:
- C5P34 we must proportion the [cause] to the [effect] and can never be allowed to ascribe to the cause any qualities, but what are exactly sufficient to produce the effect.<sup>8</sup>

<sup>7</sup> Similarly a truth does not have only one possible truthmaker, and there is more than one possible reason for doing a thing.

<sup>8</sup> “A body of ten ounces raised in any scale may serve as a proof, that the counterbalancing weight exceeds ten ounces; but can never afford a reason that it exceeds a hundred” (Hume 1740/2003).

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C5P35 A proportional cause is an  $X$  such that  $X$  suffices for  $Y$  and *nothing less suffices*.<sup>9</sup>

C5P36 [P]  $X$  is proportional to  $Y$  ( $X \propto Y$ ) iff:

C5P37 (i)  $X$  suffices for  $Y$  ( $X \Rightarrow Y$ ).

C5P38 (ii) For all  $X' \leq X$ , if  $X' \Rightarrow Y$ , then  $X' = X$ .

C5P39 Of course we are often interested in “contributory” causes that are not sufficient, and hence not proportional. But Hume has an easy way to bring these on board.  $Z$  contributes to  $Y$  if it is *contained* in a proportional cause  $X$  of  $Y$ :

C5P40 [H]  $Z$  contributes to  $Y$  ( $Z \rightarrow Y$ ) iff an  $X$  obtains such that  $Z \leq X$  and  $X \propto Y$ .

C5P41 By the *Humean Package* (HP), we’ll mean these two ideas together. The first idea:  $X$  is proportional to  $Y$  just if it is minimally sufficient for  $Y$ . The second:  $Z$  contributes to, or is relevant or helpful to,  $Y$  just if  $Z$  is contained in a proportional  $X$ . What the two together offer is an account of relevance in terms of the *prima facie* much clearer notions of sufficiency and minimality.

C5P42 The Humean Package has a lot going for it. It is powerful and illuminating and deals correctly with a great many cases. And it’s adaptable. [P] and [H], since they do not contain the word “cause,” offer a general template that is potentially of very wide application. Indeed it is hard to think of an area of philosophical inquiry that hasn’t employed the template. The Hypothetico-Deductive model of confirmation is Humean in spirit;  $E$  confirms  $H$  just if  $H$  figures essentially in some suitable  $E$ -entailing body of information. An action’s right-making features, on one account, are those included in a condition that is minimally sufficient for its rightness. Theories of presupposition have been trading in recent years on the “relevance,” explained in difference-maker terms, of an embedded sentence’s truth-value to the truth-value of the whole.<sup>10</sup> “ $Q$  is a difference-making ground for  $P$ ” is defined by Krämer and Roski like this:

C5P43 for some scenario  $S$  which contains a full ground of  $P$ ,  $S$  minus the fact that  $Q$  does not contain a full ground of  $P$ . (Krämer and Roski 2017, with inessential relettering)

C5P44 How can the fact  $Q$  that 5 is prime be relevant to the fact  $P$  that there are primes, when that fact cannot fail to obtain? A scenario  $S$  consisting precisely of 5 and its indivisibility by 2, 3 and 4 contains a full ground of  $P$ . No lesser scenario contains a full ground, and in particular  $S$  minus the fact  $Q$  of 5’s indivisibility by

<sup>9</sup> “Proportionality” in this chapter is only loosely analogous to the notion in Yablo (1992b) and (1992a).

<sup>10</sup> Schlenker (2008). A close cousin of the minimality problem is raised in Schlenker (2009, 52–3). (Thanks here to Danny Fox.)

2 does not fully ground *P*. 5's oddness contributes to the existence of primes because it makes the difference between a minimal ground for primes' existence and a near-ground.

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#### IV. Extra Help

C5P45 And yet a *non*-minimal condition *X*—one with elements that it doesn't need, to suffice for *Y*—can still be wholly, entirely helpful. Extra help is still help, and sometimes it is the only kind of help around. This goes back in a way to Zeno. A solid sphere takes up space. It has measure 1, say. The sphere's component points are helpful, surely? They are clearly helpful en masse—en masse they just *are* the sphere. And it is hard to see how they could be helpful together, if they were irrelevant individually. Still, since each point has measure 0, they would none of them be missed. None of the sphere's component points lies in a minimal subregion of measure 1, because there *are* no minimal subregions of measure 1.<sup>11</sup>

C5P46 Hume might have known, when he wrote the *Enquiry*, of Zeno's paradox of measure. He would not have known of the next example, as it grows out of events taking place that same year (1748). God is pleased, let's imagine, if he is praised infinitely many days. Being praised *every* day should be pleasing, surely.<sup>12</sup> But no, not if we go by the Humean Package. The reason was noted in effect by John Newton in *Amazing Grace*:<sup>13</sup>

C5P47 When we've been here ten thousand years

C5P48 Bright shining like the sun.

C5P49 We've no less days to sing His praise

C5P50 Than when we'd just begun.

<sup>11</sup> Skyrms (1983) is an interesting discussion. "Zeno's paradox of measure rests on the following premises:

- (I) Partition: [the sphere] can be partitioned into an infinite number of parts such that
- (II) Measurability: the concept of magnitude applies to the parts.
- (III) Invariance: the parts all have equal positive magnitude, or zero magnitude.
- (IV) Archimedean Axiom: there are no infinitesimal magnitudes.
- (V) Ultra-Additivity: the magnitude of the whole is the sum of the magnitudes of the parts.

Ancient responses focused largely on (I) and (II). Doctrines of finite indivisible magnitudes (certainly Epicurus and probably Democritus and Leucippus) rejected (I). Aristotle rejected (I) and (II). It is possible that a doctrine of infinitesimal indivisible magnitudes was also current (possibly held by Xenocrates, possibly by Democritus) which rejected (IV). (III) could have also been challenged by a holder of a doctrine of infinitesimal magnitudes. (V), Ultra-Additivity, appears to have been accepted without question by every party to the dispute. It is ironic that it is just here that the standard modern theory of measure finds the fallacy" (235).

<sup>12</sup> Assume the future is infinite.

<sup>13</sup> Newton's religious phase began in 1748, when his ship nearly went down in a storm off the coast of Ireland. He was returning from Africa, where he had been first a slave trader, then himself enslaved to a colleague's African wife. The ship miraculously righted itself and Newton promised to change his ways. He was ordained as an Anglican priest in 1764. *Amazing Grace* was written a few years later.

- C5P51 Singing *every* day is out of proportion with the effect, on Hume’s definition, since God would still be pleased if we waited 10,000 years before beginning. And of course the same is true for any other set of days one might choose. There *is* no least infinite set of days. Every praise-day is helpful to the cause, but not because it figures in a minimal sufficer.
- C5P52 Minimality had better not be required for relevance, because you can’t always get it. It is not always required, in fact, even when you *can* get it. The pope’s crown was once supposedly made of three smaller crowns. Suleiman the Magnificent, not to be outdone, had *four* crowns in his crown. Suleiman’s crown was wholly relevant to *There are crowns*. But you could lop the upper sub-crowns off and still have a sufficient condition for the sentence’s truth. Here we *can* point to a minimal sufficient basis for *There are crowns*. But there is no reason to do so. Suleiman’s total crown is no less helpful for being four times larger than necessary.
- C5P53 The US Senate cannot conduct certain kinds of business unless fifty-one members are present (a quorum); it is not “in order” without a quorum. Suppose that fifty-two senators are present on a given occasion. They all arrived at the same time and the situation is in other ways symmetrical. The presence of these senators—the Gang of 52, call them—seems wholly helpful to order obtaining. True, there is a Gang of 51 present as well which also suffices, in fact there are fifty-two such gangs. Somehow though this does not detract from our initial judgment. The Gang of 52 is wholly relevant despite the fact that not all its members had to be there.

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## V. Previous Proposals

- C5P54 That *X* can still be wholly relevant to *Y*, even if not all of it is needed, has not gone unnoticed. Fine makes the point in connection with truthmaking (proportional truthmakers in his system are *exact* rather than *inexact*) (Fine 2017c).<sup>14</sup> Humeans in effect take the exact verifiers to be the minimal inexact verifiers, those that inexactly verify without properly containing an inexact verifier.
- C5P55 But whilepiling on random extras ruins an exact verifier,<sup>15</sup> piling on additional exact verifiers does not:
- C5P56 Given the facts *f*, *g*, *h*,..., we take there to be a *composite* fact or *fusion* *f•g•h*...that is the ‘factual conjunction’ of the component facts *f*, *g*, *h*,...,

<sup>14</sup> “With inexact verification, the state should be at least partially relevant to the statement; and with exact verification, it should be wholly relevant. Thus the presence of rain will be an exact verifier for the statement ‘it is rainy’; the presence of wind and rain will be an inexact verifier for the statement ‘it is rainy’, though not an exact verifier” (Fine 2017c).

<sup>15</sup> “On our understanding of verification as relevant verification, it should *not* be supposed that if *f* verifies a truth *A* then *any* ‘larger’ fact *f•g* must also verify *A*” (Fine 2012b, 7, emphasis added; see also Fine 2012a).



obtaining just in case all of the component facts obtain; and we shall suppose that whenever the facts  $f, g, h, \dots$  verify the truth  $A$ , their fusion  $f \cdot g \cdot h \cdot \dots$  also verifies  $A \dots$  (Fine 2012b, 7)

C5P57 The presence  $r \cdot w$  of rain and wind exactly verifies *It is rainy or windy* without minimally verifying it. Rain and wind are more than  $R \vee W$  needs, but there is nothing in  $r \cdot w$  that is irrelevant to the statement's truth.

C5P58 Kratzer's theory of exemplification strikes a similar note. The fact of two teapots exemplifies *There are teapots*, she says, despite its non-minimality. The fact of a teapot and a dog does not. Why is the extra dog more of a problem than the extra teapot? The parts of a  $P$ -exemplifying situation  $s$  must "earn their keep" by figuring crucially, not perhaps in  $s$  itself, but in a minimal  $P$ -verifying part of  $s$ .

C5P59  $s$  exemplifies  $P$  iff for all  $s'$  such that  $s' \leq s$  and  $P$  is not true in  $s'$ , there is an  $s''$  such that  $s' \leq s'' \leq s$ , and  $s''$  is a minimal situation in which  $P$  is true. (A minimal situation in which  $P$  is true is a situation that has no proper parts in which  $P$  is true.) (Kratzer 2002: 660)

C5P60 The fact of two teapots exemplifies *There are teapots* ( $P$ ) despite its non-minimality because everything in it is part of some minimal  $P$ -verifier or other. Here  $s$  and  $P$  are like our  $X$  and  $Y$  and exemplification is like being-sufficient-for-and-wholly-helpful-to.

C5P61 Kratzer's theory does loosen the bonds between relevance and minimality. But minimality is still playing its same old role one level down; a non-minimal verifier needs to contain minimal verifiers. It is a problem, then, if "a statement may have inexact verifiers without having any minimal verifiers" (Fine 2017c).<sup>16</sup>

C5P62 Similarly a cause might still be wholly helpful to an effect, even if all its sufficient parts contain smaller such parts all the way down. Imagine a detector that buzzes

<sup>16</sup> Kratzer is aware of this. Her example is *There are infinitely many stars* ((7) in her paper):

If the proposition expressed by (7) is the proposition  $P$  that is true in any possible situation in which there are infinitely many stars, we are in trouble. [The] definition would predict that there couldn't be a fact that makes  $P$  true .... Situations with five or six stars, for example, ... are not part of any minimal situation in which  $P$  is true (Kratzer 2002, 662)

She notes that that (7) has a reading "that the German sentence (8) brings out more clearly":

(8) Sterne gibt es unendlich viele.  
Stars are there infinitely many.  
As for stars, there are infinitely many of them.

In (8), the common noun "Stern" has been topicalized. The proposition expressed by (8) might now be taken to be the proposition  $Q$  that is true in a situation  $s$  iff (i)  $s$  contains all the stars in the world of  $s$ , and (ii) there are infinitely many stars in  $s$ . Consequently, if  $Q$  is true in a world at all, there is always a minimal situation in which it is true, hence there is always a fact that exemplifies it (Kratzer 2002, 662).

But, although (7) can be understood so that it comes out with minimal verifiers, this is not the only way of understanding it. And there might be other examples where topicalization is not an option.



when presented with infinitely many point particles, or with a single particle of any positive mass.<sup>17</sup>

C5P63 Infinitary relevance can *sometimes* be dealt with as follows.<sup>18</sup> Take again Zeno's Paradox of Measure. How do the individual points in a sphere contribute to its volume, when each point is of measure zero? Well, the points are *collectively* relevant, and none is more relevant than any other. Perhaps  $Y$  is wholly helpful to  $X$  if:

- C5P64 (1)  $Y$  subdivides into the  $Y_i$ 's.
- C5P65 (2)  $X$  fails if *all* the  $Y_i$ 's fail.
- C5P66 (3) one  $Y_i$  is as relevant to  $X$  as another.

C5P67 Or, looking back at Amazing Grace, we might reason like this. The *number* of praise-days does not shrink if we add one more day, but the *set* does shrink. And cardinality considered as a measure on sets is a coarsening of membership; size in the how-many sense is monotonically grounded in size in the membership sense. Perhaps  $Y$  is wholly helpful to  $X$  if:

- C5P68 (1)  $X$  is to the effect that  $Y$  is at least so big by a certain measure.
- C5P69 (2) That measure is monotonically grounded in another, finer measure.
- C5P70 (3) Each  $Y_i$  bears on  $Y$ 's size by this finer measure.

C5P71 Both ideas are worth pursuing. I don't want to pursue them here, for a couple of reasons. The first is that they seem insufficiently general. (Why should  $X$  be to the effect that  $Y$  is "at least so big by a certain measure"?) The second is that relevance presents, as we are about to see, logical challenges that they're ill equipped to meet.

C5S6

## VI. Hyperintensionality

C5P72 One problem for the Humean package ( $[H]$  and  $[P]$ ) is that minimality is not always available. Another is that minimality is not always even desirable. Now we turn to a third, importantly different, problem.

C5P73 Humean proportionality is *prima facie* intensional: if  $X$  and  $Y$  are necessarily equivalent to  $X^*$  and  $Y^*$ , then  $Y$  is proportional to  $X$  only if  $Y^*$  is proportional to  $X^*$ .

<sup>17</sup> Yablo (2017a).

<sup>18</sup> These ideas were prompted by an observation of Williamson's about content-parts in propositional logic.  $B$  is analytically contained in  $A$ , I had proposed, if  $\forall\alpha \exists\beta \beta \subseteq \alpha$  and  $\forall\beta \exists\alpha \beta \subseteq \alpha$ ; the Greek letters range over minimal models of  $A$  and  $B$ . Williamson pointed out (p.c., 2006) that containment continues to make sense in infinitary settings where  $A$ 's models are all non-minimal. *Infinitely many atomic truths exist* appears, for instance, to contain *Atomic truths exist*. (For analytic containment see Angell 1989, Correia 2004, Yablo 2014, 59 and Fine 2015).

For proportionality is defined by  $[P]$  in terms of sufficiency and minimality, which are themselves intensional.<sup>19</sup>

C5P74 Is the relation of “being entirely relevant to” intensional? It is not. An example on the  $X$  side: *His praise is sung infinitely many days* ( $X$ ) is true in the same worlds as *His praise is sung infinitely many days after 12019* ( $X^*$ ). Singing every day starting now ( $Y$ ) is wholly helpful to  $X$ , but overkill when it comes to  $X^*$ . Singing today is absolutely beside the point when it comes to singing infinitely often in the distant future.

C5P75 An example on the  $Y$  side:<sup>20</sup> In Alternative Eden, there are infinitely many apples on the Tree of Life, but only one, BadApple, on the Tree of Knowledge of Good and Evil. Eve can't recall God's precise instructions, and decides to check it out with the serpent:

C5P76 EVE. What did God allow me to do again?

C5P77 SERPENT. Hmmmm, I'm not sure, but I remember it was equivalent to this:

C5P78 *You eat infinitely many apples.*

C5P79 [Eve eats all the apples and is expelled from Eden.]

C5P80 EVE, FURIOUS. Why did you say God had allowed me to eat infinitely many apples!?!?

C5P81 SERPENT. Wait, I said it was *equivalent* to that. And it was. *You eat infinitely many apples  $\vee$  such that  $\vee \neq$  BadApple* ( $Y$ ) holds in the same worlds as *You eat infinitely many apples, period* ( $Y^*$ ). One apple cannot make the difference between an infinite set and a finite one.

C5P82 Let  $X$  be *Eve did as she was told*.  $Y$ 's truth is wholly helpful to  $X$ , given that God had allowed Eve to eat infinitely many apples other than BadApple. There is no disobedient way of doing that (see below for “ways”). Whereas  $Y^*$ 's truth is not wholly helpful to  $X$ , since there are ways for  $Y^*$  to hold that have Eve disobeying God.

C5S7

## VII. Mereology

C5P83 What is it for  $X'$  to be  $\leq X$  in  $[P]$ ? You might think that  $X' \leq X$  iff  $X$  implies, or necessitates,  $X'$ . But although this is how content-parts have sometimes been understood, the view quickly runs into trouble.<sup>21</sup> For one thing it allows  $X$  to be knocked out of proportion with  $Y$  by  $X \vee S$ , provided that  $S$  too is sufficient for  $Y$ . Which is the wrong result.

<sup>19</sup> “Prima facie” because the part-relation  $<$  plays a role as well. It could turn out that necessary equivalents are not freely substitutable on the right-hand side of  $U < Y$ . Best to ignore this as it's not the issue here. (See Section VII.)

<sup>20</sup> Fine (2017). <sup>21</sup> Gemes (1994, 1997), Fine (2013), Yablo (2014), Fine (2015).

C5P84 Socrates' drinking the hemlock on an empty stomach ( $X$ ) suffices, let's assume, for his death ( $Y$ ).  $X$  is proportional to the death only if nothing less suffices. Yet something less is bound to suffice, if  $\leq$  is just the converse of implication. For let  $S$  be any other sufficient basis for death, say, falling off a high cliff. Then  $X \vee S$  is a weaker sufficient condition for  $Y$  than  $X$  is.  $X \vee S$  knocks  $X$  out of proportion with  $Y$ , if  $\leq$  means is-implied-by. So it had better mean more than that. The answer we'd like to give is that  $X \vee S$ , although *weaker* than  $X$ , is not *contained* in  $X$ . To be proportional to  $Y$ ,  $X$  should have no proper *parts* sufficient for  $Y$ .

C5P85 This notion of content-part is not available to the Humean, as it is hyperintentional, e.g.  $P \vee Q$  is part of  $P \vee PQ$  but not of  $P$ . Hyperintentionality, looking ahead a bit, is going to be explained with *ways*. The difference between  $X$  and  $X^*$ —*His praise is sung infinitely many days starting now* and *His praise is sung infinitely many days after 12019*—is that while they hold in the same worlds, they don't hold in the same ways in those worlds. Not a single way of singing infinitely many days after 12019 involves singing tomorrow. Singing tomorrow may well have a role to play, however, in how God's praise is sung infinitely many days starting now. Ways are the key as well to content-parts:

C5P86  $X' \leq X$  iff:

C5P87 (i) Every way for  $X$  to hold implies a way for  $X'$  to hold.

C5P88 (ii) Every way for  $X'$  to hold is implied by a way for  $X$  to hold.<sup>22</sup>

C5P89 They bear too, finally, on the problem we are mainly concerned with in this chapter, the problem of minimality. Details will have to wait; suffice it for now to say that although the Humean Package faces a number of challenges, they all push in the same theoretical direction.

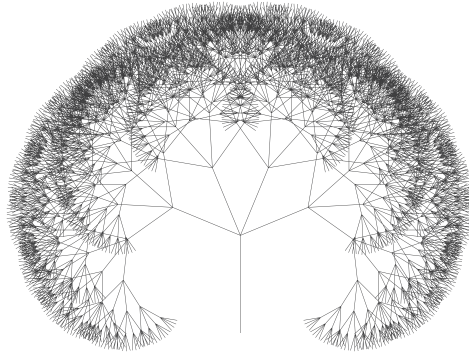
C5S8

## VIII. Bottomless Kinds

C5P90 A fractal is a geometrical figure containing isomorphic copies of itself; these will then contain isomorphic copies of *themselves*, and so on all the way down. An example is tree  $t$  below.

C5P91 Fractals are counterexamples par excellence to the minimality requirement. The fact that  $t$  exists ( $[t \text{ exists}]$ , for short) is as helpful as it could be to *There are fractals*. You are not going to find a better candidate for a proportional, discerning, basis for the truth of *There are fractals* than the existence of  $t$ .

<sup>22</sup> Gemes (1994, 1997), Yablo (2014), Fine (2015), Yablo (2016), Fine (2017). (Where I have "implies," others say "contains." This is not the place.)



- C5P92 A fact that is clearly out of proportion with *There are fractals* is [*t exists and Sparky is a dog*]. What is the difference exactly? You can throw the Sparky conjunct out, of course, and still be left with a fact sufficient for the existence of fractals. But one can also throw out part of the fact that *t* exists. For the immediate right subtree *u* of *t* is also a fractal, and *t*'s existence consists in the joint existence of *u* and *v* (*v* is the rest of *t*). It is not clear as yet why [*u exists and v exists*] would be more proportional to *There are fractals* than [*t exists and Sparky is a dog*], or for that matter [*u exists and Sparky is a dog*], given that the second conjunct is in each case dispensable.
- C5P93 Call a kind *K* bottomless if to be a *K* is to contain smaller *K*s. If *K* is bottomless, then clearly, a minimal *K* is not to be expected. Are there other bottomless kinds, besides *fractal*?
- C5P94 A set is infinite iff all of its members can be paired off 1–1 with its members other than *x*, for some *x* in the set. Suppose that *S* is equipotent in that sense with  $S_1 = S \setminus \{x\}$ , and let *y* be a member of  $S \setminus \{x\}$ . Then if  $y \in S_1$ , it follows on standard assumptions that  $S_1$  is equipotent with  $S_2 = S_1 \setminus \{y\}$ , and so on without limit. *Infinite set* is thus a bottomless kind.
- C5P95 A property is *dissective* if a thing cannot instantiate it unless all its parts do.<sup>23</sup> This does not ensure bottomlessness all by itself, but it does if we add that proper parts always exist. Sellars uses the notion to illustrate his distinction between the “scientific” and “manifest” images of reality:
- C5P96 Color expanses in the manifest world consist of regions which are themselves color expanses.<sup>24</sup>
- C5P97 The manifestly colored expanses form a bottomless kind for Sellars. (Of an especially pure sort. Fractals can contain non-fractals, but the parts of a blue expanse are all blue.) Aristotelian water is supposed to be dissective, and a stretch

<sup>23</sup> Goodman (1951).<sup>24</sup> Sellars (1963), 35.

of continuous motion always subdivides into smaller stretches of continuous motion. A minimal verifier of *The particle was moving continuously at noon* is not to be hoped for.

C5P98 Why do people think that  $X$  cannot be wholly relevant to  $Y$ , if less than  $X$  suffices, when these problems are so obvious? A condition may retain its hold on us, it is true, even after we see that it cannot always be met. A set of everything is impossible, too, but that doesn't make it any less "what we wanted." Logicians *regret* the unavailability of a universal set; they look for ways of approximating or simulating it.<sup>25</sup> There is nothing to regret in the fact that we can't lay our hands on a minimal fractal.

C5S9

### IX. Schematization

C5P99 The Humean Package is more of a schema than a claim.  $Z$  is *causally* relevant to  $Y$  iff it's part an  $X$  that *causally* suffices for  $Y$ , where nothing less causally suffices.  $Z$  is *ground*-relevant to  $Y$  (it is a difference-making ground; see Section III) iff it is part of a full ground  $X$  of  $Y$  such that nothing less than  $X$  fully grounds  $Y$ .  $Z$  helps to justify  $Y$  iff it is part of an  $X$  that fully justifies  $Y$ , but ceases to do so when anything is deleted. The "generic" notions of sufficiency ( $\Rightarrow$ ), proportionality ( $\propto$ ) and helpfulness ( $\rightsquigarrow$ ) in  $[H]$  and  $[P]$  should be read as shorthands for particular flavors  $\Rightarrow^k$ ,  $\propto^k$  and  $\rightsquigarrow^k$  of these notions:

C5P100  $[H] Z \rightsquigarrow^k Y$  iff  $Z \leq X$  for some (actual)  $X$  such that  $X \propto^k Y$ .

C5P101  $[P] X \propto^k Y$  iff (i)  $X \Rightarrow^k Y$ , (ii) for all  $X' \leq X$ , if  $X' \Rightarrow^k Y$ , then  $X' = X$ .

C5P102 The superscripted "arrows"  $\Rightarrow^k$ ,  $\propto^k$  and  $\rightsquigarrow^k$  stand ambiguously for the various sorts of sufficiency and relevance with which philosophers have concerned themselves: causal, logical, modal, nomological, explanatory, evidential and so on.

C5P103 Dividing things up in this way doesn't *help* with minimality, but it does bring out the problem's breadth, and the kind(s) of trouble we're in if we can't figure it out. *He is praised infinitely many days* has an unending chain of progressively weaker sufficers: he is praised every day from today on ( $X_0$ ), every day from tomorrow on ( $X_1$ ), ..., every day from day  $n$  on ( $X_n$ ), and so on. The sufficiency in this case is ground-flavored. Writing  $\Rightarrow^g$  for "is sufficient in the manner characteristic of a full ground for," we have:

C5P104  $X_0 \Rightarrow^g Y$   
 $X_1 \Rightarrow^g Y$   
 $X_2 \Rightarrow^g Y$

<sup>25</sup> This is part of the attraction of plural quantification.

$X_3 \Rightarrow^g Y$

....

C5P105  $X_n \Rightarrow^g Y$

....

C5P106 But it holds of no  $X_i$  on the list that  $X_i \propto^g Y$ , for each  $X_i$  has a proper part  $X_{i+1}$  such that  $X_{i+1} \Rightarrow^g Y$ . If indeed proportional grounds in the sense of [P] don't exist, then (to go by [H]) it is never true that  $Z \sim^g Y$ . (Difference-making grounds have to be parts of proportional grounds.) It somehow contributes nothing to *He is praised on infinitely many days* that he is praised today, or throughout 12019.

C5P107 That was the ground-theoretic variation on our descending chain theme. For the causal variation, let  $Y$  be *God is pleased*. It holds of no  $X_i$  on the list that  $X_i \Rightarrow^c Y$ , since each  $X_i$  has a proper part  $X_{i+1}$  such that  $X_{i+1} \Rightarrow^c Y$ . And so it is never true, by the same reasoning as before, that  $Z \sim^c Y$ . God does to be sure wind up pleased if praised every day. It's just that this is not due even in part to the praise received on any specific day(s).<sup>26</sup> Examples of the same sort can be given for any variety of relevance: moral, evidential, nomological, etc.

C5P108 The Humean Package—a theory H of relevance built on the back of a theory P of proportionality—evidently needs work.<sup>27</sup> That work begins, in Section XI, with a particular *kind* of non-Humean proportionality, the kind  $\propto^t$  appropriate to ways of being true. The hope is that other sorts of proportionality can be recovered from it, and other sorts of relevance ( $\sim^c$ , etc.) from them.

C5S10

## X. Ways and Worlds

C5P109 Parthood and proportionality are hyperintensional notions; so is indifference which we'll be getting to soon. To do them justice, we will have to expand our toolkit ("The possible worlds apparatus can only draw intensional, not hyperintensional, distinctions."<sup>28</sup> Lucky for us, the role traditionally played by worlds is better played in any case by *ways*.<sup>29</sup> And ways are hyperintensional right out of the box.  $P$  is true in the same worlds as  $(P \equiv Q) \vee (P \equiv \neg Q)$ , but the latter has different ways of being true.<sup>30</sup> Events that necessarily co-occur may have different ways of occurring, e.g., this ball bouncing off that one, and that one bouncing off this. A solid figure occupies almost all of the open sphere  $\{ \langle x, y, z \rangle \mid x^2 + y^2 + z^2 < 1 \}$  in

<sup>26</sup> A case with fewer distractions: A buzzer sounds at the weigh station when a truck enters weighing over 70,000 pounds. The buzzer goes off "for no particular reason," judging by [P] and [H]. Reasons have to be drawn from conditions minimally causally sufficient for the effect. And numbers don't exist that are minimally larger than 70,000.

<sup>27</sup> Our focus has been, and will continue to be, on P. But H has issues of its own; they will come up briefly at the end.

<sup>28</sup> Berto (2017).

<sup>29</sup> Yablo (2014, 2017b).

<sup>30</sup> The second can be true by way of  $P$ 's truth and  $Q$ 's falsity.

a world  $w$  just if it occupies almost all of the closed sphere  $\{ \langle x, y, z \rangle \mid x^2 + y^2 + z^2 \leq 1 \}$ . But these outcomes obtain in different ways. Occupying all of the closed sphere is a way of occupying almost all of it, but not of occupying almost all of the open sphere.<sup>31</sup>

C5P110 What is meant by a “way  $\omega$  for  $P$  to hold”? I don’t know how to define the notion, and won’t even try. There is nothing scandalous about this. Does Lewis put possible worlds semantics on hold until he can define “ $P$ -world  $w$ ”? Not at all. Two issues have to be distinguished in his view:

C5P111 (1) What is a “world in which proposition  $P$  is true”?

C5P112 (2) What is the “proposition  $P$  expressed by  $P$ ”?

C5P113 (1) is trivial, if propositions are sets of worlds.  $P$  is true in  $w$  iff  $w$  is a member of  $P$ . (2), the problem of associating propositions with sentences, is nothing special to do with worlds, nor does the worldly view of propositions make it more difficult. One approach lets the intensions of atomic expressions be given outright; semantics comes in to derive the intensions of complex expressions. Or, atomic intensions could be assigned on some kind of covariational basis. Lewis himself works backwards from sentence intensions, using reference magnetism as a tie-breaker.<sup>32</sup>

C5P114 None of this need change if propositions are made up of non-worldly circumstances  $c$ .  $S$  is true in  $c$  just if  $c$  belongs to  $S =$  the proposition that  $S$ . There will be time enough later to explain how a sentence comes to express this proposition rather than that. This is par for the course in semantics. Propositions for Kratzer are sets of situations. Propositions for Humberstone are sets of possibilities. Propositions for new-style expressivists like Yalcin and Moss are sets of probability-measures.<sup>33</sup> Propositions in truthmaker semantics are sets of ways.

C5S11

## XI. The Only In Part By Test

C5P115 Suppose I am right that way-for-it-to-be-that- $P$  is on a par methodologically with world-in-which- $P$ . Neither requires for semantic purposes an analysis. There is still the question of *which* unanalyzed notion is intended. Here my job is in one respect easier than Lewis’s: “way for  $P$  to be the case” is a familiar, commonsensical notion. But it’s in another respect harder. Ways are a miscellaneous lot, and

<sup>31</sup> Continuous motion occurs in the same worlds as continuous motions. But for this particle to move continuously from noon to one, and then that one from three to four, is a way for continuous motions to occur. Distinct particles moving at different times is overkill when it comes to continuous motion. (Kit Fine’s example.)

<sup>32</sup> Lewis (1974, 1983). <sup>33</sup> Yalcin (2012), Moss (2018).



I need to direct your attention to an elusive sub-genre. Our target in the end is ways for it to be that  $P$ , but it helps to look more generally at ways for a thing  $x$  to  $\varphi$ . (Ways for it to be that  $P$  fall out as the case where  $x$  is a world and to  $\varphi$  is to be the kind of world that verifies  $P$ .)

C5P116 So, let's try it. Here to get us going are some paradigms of ways for  $x$  to do a certain thing, or instantiate a certain property:

C5P117 Disjuncts: For  $x$  to sing is a way for  $x$  to sing or dance.

C5P118 Instances: For  $x$  to sing is a way for something to sing.

C5P119 Determinates: For  $x$  to yodel is a way for  $x$  to sing.

C5P120 And here are some foils, that is, paradigms of failure to be a way:

C5P121 Conjunctions: To sing and dance is NOT a way of singing.

C5P122 Generalizations: For everyone to sing is NOT a way for  $x$  to sing.

C5P123 Manners: To yodel badly is NOT a way of singing.

C5P124 Prequels: Practicing is NOT a way of getting to Carnegie Hall.<sup>34</sup>

C5P125 Prerequisites: To dance is NOT a way of persisting over time.<sup>35</sup>

C5P126 Some principles will help us to sort these cases out. There will be no epiphanies. But we should get some amount of insight into why the line is drawn where it is.

C5P127 A first condition on  $\psi$ , if it is to count as a way of  $\varphi$ -ing, involves the notion of *only*  $\varphi$ -ing. Why is singing and dancing not a way of singing? A way of singing is not a *further* thing one does, in addition to singing. Singing of its nature has to be done in some way or other (by yodeling, say), just as eating involves there being something or other that one eats. This is why to eat carrots, or yodel, is not to do a further thing besides eating, or singing. To sing and dance *is* in part to do a further thing, namely dance.<sup>36</sup> That one also dances means that one isn't only singing. Our first test, then, is:

<sup>34</sup> Dancing is not in our sense a way of failing the course, even if the instructor disapproves of dancing, or dancing cuts into your study time.

<sup>35</sup> An instantaneous entity cannot dance.

<sup>36</sup> The idea is from Kratzer (1989): "One evening in 1905, Paula painted a still life with apples and bananas. She spent most of the evening painting and left the easel only to make herself a cup of tea, eat a piece of bread, discard a banana or look for an apple displaying a particular shade of red. Against the background of this situation, consider the following two dialogues that might have taken place the following day."

*Dialogue with a Pedant*

PEDANT: What did you do yesterday evening?

PAULA: The only thing I did yesterday evening was paint this still life over there.

PEDANT: This cannot be true. You must have done something else like eat, drink, look out of the window.

PAULA: Yes, strictly speaking, I did other things besides paint this still life. I made myself a cup of tea, etc.

- C5P128 Only-Way: To  $\psi$  is a way of  $\phi$ -ing only if:  $x \psi$ -d is compatible with  $x$  only  $\phi$ -d.
- C5P129 *Al yodeled* is compatible with *Al only sang*. But *Al sang* is likewise compatible with *Al only yodeled*, and singing is not a way of yodeling. One thing we could say here is that  $\psi$ -ing should necessitate  $\phi$ -ing (as singing does not necessitate yodeling). But singing-while-persisting- over-time, which does necessitate singing, also passes the ONLY test. And we don't want to say that singing-while-persisting is a way of singing.
- C5P130 The difference is that one doesn't persist *by* singing, or for that matter sing by singing and dancing. Whereas one does sing by yodeling, and sing or dance by singing. And one arranges for someone to sing, by arranging for Bert to do so. This gives us a second condition:
- C5P131 By-Way: To  $\psi$  is a way of  $\phi$ -ing only if:  $x \phi$ -s *by*  $\psi$ -ing.
- C5P132 This still lets too much in. One can fail the course by dancing—when one ought to be studying—and to dance is not in the relevant sense a way of failing the course. If the answer to “How do I get to Carnegie Hall?” is “Practice!,” still practicing is not a way of getting to Carnegie Hall. The dancing is more like a cause, or facilitator, of—let us say, *prequel* to—failing the course, and the practicing is a prequel to Carnegie Hall. Here is a third principle aimed at prequels:
- C5P133 Way-In: To  $\psi$  is a way of  $\phi$ -ing only if:  $x \phi$ -s *in*  $\psi$ -ing.
- C5P134 One may fail a course by dancing, but not (certain courses aside) in the act of dancing. No one gets in the act of practicing to Carnegie Hall. Singing and yodeling are different in this respect. Hank Williams sang not only by yodeling, but in the act of yodeling.
- C5P135 Only one of the foils remains to be dealt with. Yodeling badly is not supposed to be a way of singing. But can't a Hank Williams impersonator sing both by, and in, yodeling badly? Intuitions may differ on this, but let's allow it is possible. A different explanation will then be needed of why yodeling badly does not count as a way of singing. I know what I *want* to say: as long as Bert is yodeling, how well or badly he does it is irrelevant to whether he sings. This does not get us very far, however. For have no account as yet of (ir)relevance; the whole point of this

*Dialogue with a Lunatic*

LUNATIC: What did you do yesterday evening?

PAULA: The only thing I did yesterday evening was paint this still life over there.

LUNATIC: This is not true. You also painted these apples and you also painted these bananas. Hence painting this still life was not the only thing you did yesterday evening. (608)

The pedant is technically correct, if a bore. But Kratzer rightly objects to the lunatic that Paula “didn't paint apples and bananas *apart* from painting a still life. Painting apples and painting bananas was part of her painting a still life” (608).

chapter is that (ir)relevance begins to elude our grasp once we see that it cannot be captured Hume-style in terms of minimality.

C5P136 Ah, but what if the particular *type* of irrelevance now at issue was independently identifiable, without getting into grander issues about relevance as such? Ways in the relevant sense are intimately related to *parts*; and relevance has a counterpart virtue on the side of parts that is easier to get a grip on. (1)–(6) show that to be *G* is part of being *F*, in many cases, just if to be  $\bar{G}$  is a way of being  $\bar{F}$ :<sup>37</sup>

C5P137 (1) To be  $\left(\frac{R}{\bar{R}}\right)$  is  $\left(\frac{\text{part}}{\text{a way}}\right)$  of being  $\left(\frac{R \wedge S}{R \vee S}\right)$ .

C5P138 (2) To be  $\left(\frac{R \vee S}{\bar{R} \wedge \bar{S}}\right)$  is NOT  $\left(\frac{\text{part}}{\text{a way}}\right)$  of being  $\left(\frac{S}{\bar{S}}\right)$ .

C5P139 (3) To be ....  $\left(\frac{R \vee S}{\bar{R} \wedge \bar{S}}\right)$  is  $\left(\frac{\text{part}}{\text{a way}}\right)$  of being  $\left(\frac{R \vee S}{R \equiv S}\right)$ .

C5P140 (4) To be  $\left(\frac{R \supset S}{R \wedge \bar{S}}\right)$  is NOT  $\left(\frac{\text{part}}{\text{a way}}\right)$  of being  $\left(\frac{R \wedge S}{R \vee \bar{S}}\right)$ .

C5P141 (5) To be  $\left(\frac{R \supset S}{R \wedge \bar{S}}\right)$  is  $\left(\frac{\text{part}}{\text{a way}}\right)$  of being  $\left(\frac{R \equiv S}{R \neq S}\right)$ .

C5P142 (6) To be  $\left(\frac{R \equiv S}{R \neq S}\right)$  is NOT  $\left(\frac{\text{part}}{\text{a way}}\right)$  of being  $\left(\frac{R \wedge S}{\bar{R} \vee \bar{S}}\right)$ .

C5P143 So, (1) to be red is part of being red and square, and its negation non-red is a way of being the negation of red and square, viz. non-red or non-square. (2) To be red or square is no part of being square, and to be neither red nor square is not a way of being non-square. (3) To be red inclusive-or square is part of being red exclusive-or square, and to be neither red nor square is a way of being red iff square. (4) To be non-red or square is not part of being red and square, while to be red and non-square is not a way of being non-red or non-square. (5) To be square if red is part of being square iff red, and to be red and non-square is a way of failing to be square iff red. (6) To be square iff red is NOT part of being square and red, and failing to be square iff red is not a way of being non-red or non-square.

C5P144 Parts and ways are shaping up to be *duals* (like  $\forall/\exists$ , or  $\square/\diamond$ ). That  $\psi$  is necessary for  $\varphi$  iff  $\varphi$  suffices for  $\psi$  has as its hyperintensional counterpart that  $\psi$ -ing is part of  $\varphi$ -ing just if  $\bar{\psi}$ -ing is a way of  $\bar{\varphi}$ -ing. Rearranging and swapping positives for negatives, we get the following as our final necessary condition on ways:

C5P145 Part-Way: To  $\psi$  is a way of  $\varphi$ -ing only if: to  $\bar{\psi}$  is part of what is involved in  $\bar{\varphi}$ -ing.

<sup>37</sup>  $\underline{\vee}$  is exclusive disjunction.

- C5P146 This catches irrelevant or non-contributing manners that the other conditions are apt to miss. Bad manners give rise under negation to sore-thumb disjuncts that prevent  $\bar{\psi}$  from being part of  $\bar{\phi}$ . Take yodeling badly. It is a way of singing only if part of what is involved in *not* singing is *not* yodeling badly: either not yodeling at all, or else yodeling well. Not yodeling may indeed be part of what it takes not to sing. But to yodel well, one must sing! This makes it hard to see yodeling-well as caught up (even disjunctively) in not-singing. The latter to contain not-yodeling-or-yodeling-well should intuitively contain yodeling-well. But it's *inconsistent* with yodeling-well.
- C5P147 Yodeling badly is by our fourth condition not a way of singing. This is good since yodeling-badly was a foil, not a paradigm. Part-Way looks favorably on our paradigms. To sing remains a way of singing or dancing, since to do neither is in part not to sing. Yodeling remains a way of singing, since not to sing is in part not to yodel. For Bert to sing remains a way for someone to sing, since part of what is involved in no one's singing is for Bert in particular not to sing.
- C5P148 Four necessary conditions have been laid down: Only-Way, By-Way, Way-In and Part-Way. They constitute together the *Only-In-Part-By* test for way-hood. I say "test" because the conditions do not pretend to get at what ways really are. They aim only to sort the cases out properly, and they seem so far to succeed at this. Questions can be raised about all of them, and there are other conditions that might be considered as well. But not here. Our topic is relevance and it is time to get back to it.

C5S12

## XII. Indifference

- C5P149 *God is pleased* has an unending chain of progressively weaker sufficers: He is praised every day from today on, every day from tomorrow on, . . . , every day after 12019, and so on. The weaker ones are no better, if all God wants is to be praised infinitely many days. The chain's endlessness would be unfortunate, if knocking off initial segments brought a feeling of progress, of getting closer to God's real reason for being pleased. But we never do get closer and progress is never made.
- C5P150 Cantor was disappointed in what we now call the infinite numbers. No  $\aleph_\alpha$  could satisfy him, because there was always a bigger one down the road; and bigger, in the infinity department, is better. A *truly* infinite number would be as large as possible. This is why he called the  $\aleph_\alpha$ s "transfinite," reserving "infinite" for a (putative) number too big for his system.<sup>38</sup> Cantor preferred larger numbers because they had more of what he wanted: size. If we perceive no advantage in  $X'$  (singing every day after 12019) as the cause of God's pleasure, over  $X$  (singing

<sup>38</sup> Hallett (1986). To continue with the coincidences, Cantor was hospitalized for depression in 1899. *That Obscure Object of Desire*, or the novel it is based on, appeared the year before.

every day henceforth), it stands to reason that  $X'$  does not have more of what we wanted. It does no better proportionality-wise than  $X$  did, which means that  $X$  did no worse.

C5P151 The idea that smaller is not necessarily better—that  $X$  and  $X'$  are “the same to  $Y$ ,” in symbols,  $X' \equiv_Y X$ —seems worth looking into.<sup>39</sup> Trouble is, we don’t know what “same to  $Y$ ” means as yet. If we did, we would take our existing account of proportionality:

C5P152 [P1]  $X \propto^k Y$  iff (i)  $X \Rightarrow^k Y$ , (ii) for all  $X' \leq X$ , if  $X' \Rightarrow^k Y$ , then  $X' = X$ .

C5P153 and replace the final identity with  $X' \equiv_Y^k X$  to obtain: |

C5P154 [P2]  $X \propto^k Y$  iff (i)  $X \Rightarrow^k Y$ , (ii) for all  $X' \leq X$ , if  $X' \Rightarrow^k Y$ , then  $X' \equiv_Y^k X$ . )

C5P155 For a sense of how this might work, let  $Y$  be *There are infinitely many whatnots*.  $Y$  is about size in the *how-many* sense, not the *inclusion* sense. That is why there is nothing to be gained proportionality-wise by deleting one of the whatnots.  $Y$  doesn’t *care* about, it is not *concerned* with, the kind of size where subsets are smaller. When we fix our attention on the *how-many* notion of size, we find that  $X'$  offers no advantages over  $X$ .

C5S13

### XIII. Aboutness

C5P156 Time to take stock.  $Z$  contributes <sup>$k$</sup>  to  $Y$  just if it is part of an  $X$  that is proportional <sup>$k$</sup>  to  $Y$ .  $X$  is proportional <sup>$k$</sup>  to  $Y$  if  $Y$  does not care about any differences that might obtain between  $X$  and those of its proper parts  $X'$  that also suffice <sup>$k$</sup>  for  $Y$ . (Not caring is expressed by  $X' \equiv_Y^k X$ .)  $X$  does not have to be minimal in all respects to be proportional to  $Y$ , the thought is, just the respects that matter, the ones  $Y$  is concerned about.

C5P157 The “concern” is metaphorical, you’ll be glad to hear. But the “about” and the “mattering” are not; they will be cashed out in terms of ways of being true. How is it that  $P \vee \neg P$  is about a different matter than  $Q \vee \neg Q$ , when they are true in the same worlds? Well, they are true in different ways in those worlds. Why does the subject matter of  $P \& Q$  include the subject matter of  $P$ , but not that of  $(P \& Q) \vee R$ , when  $|P \& Q|$  (writing  $|S|$  for the set of  $S$ -worlds) is a subset both of  $|P|$  and  $|(P \& Q) \vee R|$ ? Well,  $(P \& Q) \vee R$  holds, sometimes, in ways not implied by any way for  $P \& Q$  to hold; the same cannot be said of  $P$  in relation to  $P \& Q$ .

<sup>39</sup> A better notation, since they could conceivably be the same to  $Y$  as *causes*, but different as *grounds*, or *reasons*, would be  $X' \equiv_Y^k X$ .

C5P158 This section attempts to make the notion of subject matter a bit precise—as precise as it needs to be for the proposed application to “minimality in the respects that matter.” We start by asking, what are subject matters considered as entities in their own right? This sets up a second question: what is *the* subject matter of a particular sentence *S*?

C5P159 A subject matter **m**—the **number of stars**, is Lewis’s example—is given by specifying all the ways things can be where **m** is concerned. The ways they can be number-of-stars-wise are for there to be no stars, or one star, or two stars, or etc. Formally we can think of **m** as a collection of set-of-worlds propositions **P**. **m** = {**A**, **B**, **C**, ...} just if **A**, **B**, **C**, ... constitute between them all the ways that things can play out **m**-wise.

C5P160 Subject matters can be more or less fine-grained. **The number of stars** is coarser-grained than **which stars exist** (henceforth **the stars**). It is finer-grained, though, than **whether the number of stars is prime**. **m** is as fine-grained as **n** when each **n**-cell subdivides into **m**-cells, and finer-grained when this holds in one direction only. The reader can check that this definition “works” if:

delivers the right results

C5P161 **The stars** =

C5P162  $\{|Nothing\ is\ a\ star|, |The\ only\ star\ is\ Sol|, \dots, |The\ stars\ are\ Sol, Polaris, Vega, \dots|, \dots\}$ .

C5P163 **The number of stars** =

C5P164  $\{|\exists_0 x\ star(x)|, |\exists_1 x\ star(x)|, \dots, |\exists_k x\ star(x)|, \dots\}$ .

C5P165 **Whether the number of stars is prime** =

C5P166  $\{\cup_{prime(k)} |\exists_k x\ star(x)|, \cup_{-prime(k)} |\exists_k x\ star(x)|\}$ .

C5P167 Next the subject matter of particular sentences. *S*’s subject matter **s** is made up of *S*’s various ways of being true; it is the set of all set-of-worlds propositions **P** such that *S* is true in way **P** in some world *w*. If *Stars exist* has a way of being true for each possible non-empty roster of stars, its subject matter will be what above we called **the stars**, except that the first, star-less, cell must be dropped since *Stars exist* is false in that cell.

C5S14

#### XIV. Every Bit as Sufficient

C5P168 The idea behind P2: *Z* is relevant<sup>*k*</sup> to *Y* just if *Z* is part of an *X* that is proportional<sup>*k*</sup> to *Y*—an *X* no proper part of which “undercuts” *X* by sufficing<sup>*k*</sup> for *Y* on a more economical basis. But, when *does* a still-sufficient proper part of *X* undercut *X* in this way? How indeed can *X* not be undercut by *X*’, if *X*’ is every bit as sufficient for *Y*? Ways were supposed to shed light on this.

C5P169 For *X*’ to be “every bit as sufficient” as *X* for *Y* seems at first to mean that *Y* holds in as high a proportion of *X*’-worlds as *X*-worlds, viz. all of them.

But another, more discerning reading is possible when statements holding in the same worlds can hold in a greater or lesser variety of ways.

C5P170 Alice has three children and Bert has two. Is Alice any more of a parent than Bert? Certainly it is no more *true* of Alice that she is a parent. But she has some sort of advantage parental-status-wise; for the truth of *Alice is a parent* is more thoroughly witnessed than that of *Bert is a parent*. The advantage is clearest if the witnesses to the one truth form a proper subset of the witnesses to the other. Alice has two children with Bert, let's say, and one not with Bert. Now she becomes, in addition to being more often a parent, more *richly* or *comprehensively* a parent than Bert is. Her status as parent is witnessed by all the children witnessing Bert's parental status, and other children as well.

C5P171 This is how *X* can avoid being undercut by *X'*, though *Y* is no less definitely true in *X'*-worlds than *X*-worlds: *Y* is not as richly provided for in *X'*-worlds as in *X*-worlds. *God is praised every day from now on* is better proportioned to *God is pleased* than *God is praised and dogs bark every day from now on*, since the effect is just as richly guaranteed whether dogs bark or not. Can we maintain on a similar basis that *God is praised every day from tomorrow on* is better proportioned to *God is pleased* than *God is praised every day from now on*? We cannot, for the effect is *more* richly guaranteed in worlds where the praise starts today. Where it's truth that is more richly guaranteed, we'll speak of relative truthiness.

C5P172 Suppose *P* and *Q* are both true in *w*. *P* is as truthy there as *Q* if every way *Q* holds in *w* is also a way that *P* holds in *w* ( $\| Q \| ^w \subseteq \| P \| ^w$ ); it is truthier if *Q* holds in additional ways besides. So, for instance, *A* is as truthy as *AVB* in worlds where *B* is false. But the disjunction is truthier in worlds where *A* and *B* are both true; *A* is true in *w* in a proper subset of the ways in which *AVB* is true. Let's now add a transworld version. Writing  $\| P \| ^u$  for *P*'s ways of being true in *u*, *P* is as truthy in *u* as *Q* is in *v* just if  $\| Q \| ^v$  is a subset of  $\| P \| ^u$ . And *P* is truthier in *u* if  $\| Q \| ^v$  is a proper subset of  $\| P \| ^u$ . Imagine that goats eat cans in both worlds but abstain from bottles in *v*. Then *Goats eat cans* is exactly as truthy in *u* as *Goats eat cans or bottles* is in *v*. If on the other hand goats eat bottles in *v* as well as cans, the disjunction is truthier in *v* than *Goats eat cans* is in *u*.

C5P173 Now we are ready to say why a still-sufficient part *X'* of *X* sometimes, but not always, knocks *X* out of proportion with *Y*. *X'* undercuts *X* if it guarantees *Y*, not only as *surely* as *X* does, but also as *fully* as *X* does. *X'* does not undercut *X* if *X* compensates for its extra strength by guaranteeing *Y* more fully. Guaranteeing more fully is a matter of  $\exists W (W \Rightarrow^k Y)$  being truthier on the whole in *X*-worlds than *X'*-worlds; there is for each *X*-world an *X'*-world where it's less truthy, but not vice versa. The official definition is as follows where  $\uparrow^k Y$  is short for  $\exists W (W \Rightarrow^k Y)$ , that is, *Y* is guaranteed<sup>k</sup>:

C5P174 [S]  $X$  guarantees<sup>k</sup>  $Y$  more fully than  $X'$  does iff:

C5P175 (i)  $X \Rightarrow^k Y$  and  $X' \Rightarrow^k Y$

C5P176 (ii)  $(\forall X$ -worlds  $u)(\exists X'$ -world  $v) [\uparrow^k Y$  is truthier in  $u$  than in  $v]$ , but

C5P177 (iii)  $\neg(\forall X'$ -worlds  $v)(\exists X$ -world  $u) [\uparrow^k Y$  is truthier in  $v$  than in  $u]$ .

C5P178 Return with [S] in mind to *God is praised every day from today on* ( $X$ ) and *God is praised every day from tomorrow on* ( $X'$ ). They both causally guarantee *God is pleased* ( $Y$ ). But  $X$  provides a fuller guarantee, since *God's pleasure is guaranteed* is truthier in  $X$ -worlds where God's praises are sung every day than  $X'$ -worlds where the praise begins tomorrow.<sup>40</sup>  $X$  guaranteeing  $Y$  more fully,  $X'$  is in no position to knock it out of proportion with  $Y$ .

C5S15

### XV. Super-Humeanism

C5P179 Hume's idea was to explain of proportionality as minimal sufficiency. Perhaps there is something right about this after all, *if* we are careful about what is being minimized, subject to which constraints. A proportional  $X$  is one that is minimal among conditions guaranteeing  $Y$  as fully as  $X$  does:

C5P180 [P3]  $X$  is proportional<sup>k</sup> to  $Y$  ( $X \propto^k Y$ ) iff:

C5P181 1.  $X$  guarantees<sup>k</sup>  $Y$  ( $X \Rightarrow^k Y$ ).

C5P182 2.  $(\forall X' \leq X)$  [if  $X'$  guarantees<sup>k</sup>  $Y$  as fully as  $X$ , then  $X' = X$ ].<sup>41</sup>

C5P183 The super-Humean package is [P3] plus:

C5P184 [H]  $Z$  is helpful<sup>k</sup> to  $Y$  ( $Z \rightarrow^k Y$ ) iff an  $X$  obtains such that  $Z \leq X$  and  $X \propto^k Y$ .

C5P185 Helpfulness is a simple, deep, and elusive idea. Hume thought he had explained it with minimality. But the explanation didn't work, because some  $X$ s are helpful all the way down. A variant using *focused* minimality—minimality where a certain subject matter is concerned—seems to do better.

<sup>40</sup> Objection: Let  $X''$  be *God is cursed today and praised from tomorrow on*.  $X''$  suffices<sup>c</sup> for *His pleasure is guaranteed*. If  $[X'' \Rightarrow^c Y]$  and the like are allowed as truthmakers for  $\uparrow^k Y$ , then  $\uparrow^k Y$  isn't truthier in  $u$  than  $v$  after all; it is made true in  $v$ , but not  $u$ , by  $[X'' \Rightarrow^c Y]$ . Reply: Why think  $[X'' \Rightarrow^c Y]$  is a way for  $\uparrow^k Y$  to be true? The *Only- In- Part- By* test suggests otherwise. *I saw to it that God was cursed and then praised* undercuts *I ONLY saw to it that God's pleasure was guaranteed*. And I don't arrange for that guarantee *BY* arranging that God be cursed today and praised thereafter; God was pleased *despite* the cursing.

<sup>41</sup> This is a version of [P2] if " $X' \equiv^k_Y X$ " is suitably unpacked.  $X'$  and  $X$  are the same to  $Y$  if  $X'$ , although weaker than  $X$ , also provides less of a guarantee.



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